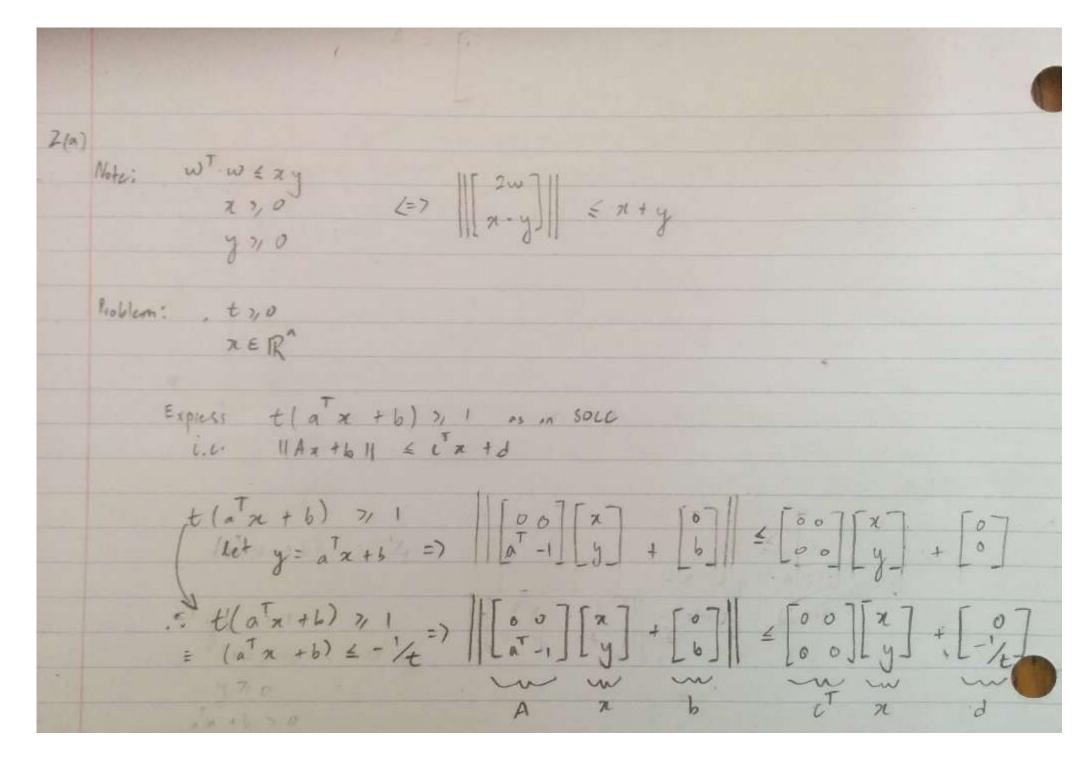
2. Hyperbolic program [10 pts]

In this problem, we start with a problem that doesn't appear to be convex and show that it is in fact convex by converting it into an SOCP.

a) Recall from class that for any $w \in \mathbb{R}^n$ and $x, y \in \mathbb{R}$, the following constraints are equivalent:

$$w^{\mathsf{T}}w \le xy, \quad x \ge 0, \quad y \ge 0 \qquad \Longleftrightarrow \qquad \left\| \begin{bmatrix} 2w \\ x-y \end{bmatrix} \right\| \le x+y$$

Suppose we have an optimization problem with variables $t \ge 0$ and $x \in \mathbb{R}^n$. Express the constraint: $t(a^Tx + b) \ge 1$ as a second-order cone constraint. Specifically, write the constraint in the form $||Ax + b|| \le c^Tx + d$. What are A, b, c, d, x?



b) Consider the following hyperbolic optimization problem (note the nonlinear objective):

minimize
$$\sum_{i=1}^{p} 1/(a_i^\mathsf{T} x + b_i)$$
 subject to
$$a_i^\mathsf{T} x + b_i > 0, \qquad i = 1, \dots, p$$

$$c_j^\mathsf{T} x + d_j \ge 0, \qquad j = 1, \dots, q$$

Write this optimization problem as an SOCP. Hint: the first part of this problem is very relevant!

(6)	$ \begin{array}{cccc} ynin & \sum_{i=1}^{p} (a_i^T x + b_i) \\ x & i=1 \end{array} $
	s.t. aix + 6; >0, fo, i=1,,p
SOLUTION:	\mathbf{x}_{i} \mathbf{z}_{i} (ω_{i})
	022 - 1 = wi for i=1, p => 1 = wi(ai + bi) for i=1,,
	$ai^{\dagger}n + bi$ > 0, for $i = 1,, p$ of the same form as part (a) $cj^{\dagger}n + dj$ > 0, for $j = 1,, q$
	1,0