Homework 1: Linear programs

Due date: 11:00pm on Monday February 6, 2017
See the course website for instructions and submission details.

1. **[5 pts] Warm-up.** Model the following problem in JuMP.

$$\begin{aligned} \underset{x_1,x_2,x_3}{\text{maximize}} \quad & 5x_1 - x_2 + 11x_3 \\ \text{subject to:} \quad & 2x_1 \geq x_2 + x_3 \\ & 0 \leq x_j \leq 3, \ j \in \{1,2,3\} \end{aligned}$$

Solve this problem using `Clp`, `ECOS`, and `SCS` solvers. Compare the answers found by each solver: which solver is more accurate? which is fastest (use the `@time` macro)? can you speculate as to why?

2. **[10 pts] Standard form with equality constraints.** Rather than using the standard LP form we saw in class, some prefer using a form where all variables are nonnegative, all constraints are equality constraints, and the cost function is a minimization. So a general LP would look like:

$$\begin{aligned} \text{minimize} \quad & c^\mathsf{T} x \\ \text{subject to:} \quad & Ax = b \\ & x \geq 0 \end{aligned} \tag{1}$$

Consider the following LP:

$$\begin{aligned} \underset{z_1,z_2,z_3,z_4}{\text{maximize}} \quad & 3z_1 \ - \ z_2 \\ \text{subject to:} \quad & -z_1 \ + \ 6z_2 \ - \ z_3 \ + \ z_4 \ \geq \ -3 \\ & 7z_2 \qquad\qquad + \ z_4 \ = \ 5 \\ & \qquad\qquad\quad z_3 \ + \ z_4 \ \leq \ 2 \\ & -1 \leq z_2 \leq 5, \quad -1 \leq z_3 \leq 5, \quad -2 \leq z_4 \leq 2 \end{aligned}$$

a) Transform the above LP into the equality-constrained standard form of (1). What are A, b, c, and x? Be sure to explain how the decision variables of your transformed LP relate to those of the original LP.

b) Solve both versions of the LP using JuMP and show that you can recover the optimal z and objective value by solving your transformed version of the LP.

3. [10 pts] Crop planning. Farmer Jane owns 45 acres of land. She is going to plant each with wheat or corn. Each acre planted with wheat yields $200 profit; each with corn yields $300 profit. The labor and fertilizer used for each acre are given in the table below. One hundred workers and 120 tons of fertilizer are available.

	Wheat	Corn
Labor	3 workers	2 workers
Fertilizer	2 tons	4 tons

a) How should Jane plant her crops to maximize profit? Model and solve this problem using JuMP.

b) Solve the problem graphically and confirm that you obtain the same solution.

4. [10 pts] Alloy blending. The company Steelco has received an order for 500 tons of steel to be used in shipbuilding. The steel must have the following characteristics:

Chemical Element	Minimum Grade (%)	Maximum Grade (%)
Carbon (C)	2	3
Copper (Cu)	0.4	0.6
Manganese (Mn)	1.2	1.65

The company has seven different raw materials in stock that may be used for the production of this steel. The following table lists the grades, available amounts and prices for all materials:

Raw Material	C%	Cu%	Mn%	Availability in tons	Cost in $/ton
Iron alloy 1	2.5		1.3	400	200
Iron alloy 2	3		0.8	300	250
Iron alloy 3		0.3		600	150
Copper 1		90		500	220
Copper 2		96	4	200	240
Aluminum 1		0.4	1.2	300	200
Aluminum 2		0.6		250	165

Determine the composition of the steel that minimizes the production cost.