Homework 10: Nonlinear programs

Due date: 11:00pm on Tuesday May 1, 2018
See the course website for instructions and submission details.

Note: for this Homework set, I suggest using the `Ipopt` solver in JuMP. It has apparently been fixed recently and I verified that it works on my system (after running `Pkg.update()` and restarting Julia). Be sure to make use of the macros `@NLconstraint` and `@NLobjective` when specifying nonlinear constraints or objectives respectively.

1. **Hexagon construction.** The goal is to figure out the optimal geometry of a hexagon such that (1) the hexagon's area is maximized and (2) the hexagon's diameter does not exceed 1. The *diameter* of the hexagon is the largest distance between any pair of points belonging to the hexagon. There are many different ways to solve this problem. A few suggestions/hints:

   - Place one of the vertices at the origin so that you don't need as many decision variables.
   - A simple way of computing the area of a hexagon is to split it into four triangles that share a common vertex. The area of the hexagon is then the sum of the areas of the four triangles.
   - The optimal solution will look like a legitimate hexagon. It will not be degenerate (no coincident vertices), nor will it be perfectly regular (all sidelengths equal).

   Plot a picture of the optimal hexagon.

2. **Fertilizer influence model.** A series of experiments is conducted to determine the effect of a particular fertilizer on wheat crop yield. Here are the results: Such situations typically involve some sort

fertilizer rate (x)	crop yield (y)
-5	127
-3	151
-1	379
1	421
3	460
5	426

   of "diminishing returns". So there is a limit to how much the fertilizer can boost the yield. A popular model for this relationship is the following:

$$y = k_1 + k_2 \exp(k_3 x)$$

   Use nonlinear least squares to determine the values of (k_1, k_2, k_3) that provide the best fit to the data. We are expecting coefficients in the neighborhood of $(500, -200, -1)$.

   To display your result, produce a plot of the original data points and the best-fit curve.