

1.

$$A = \begin{bmatrix} \underline{a}_1 & \underline{a}_2 \\ 3 & 1 \\ 0 & 3 \\ 0 & 4 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

(a)

$$\text{Let } \underline{u}_1 = \frac{\underline{a}_1}{\|\underline{a}_1\|_2} = \frac{1}{\sqrt{3^2}} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{residual } \underline{a}_2' = \underline{a}_2 - \underline{u}_1 (\underline{u}_1^T \underline{a}_2)$$

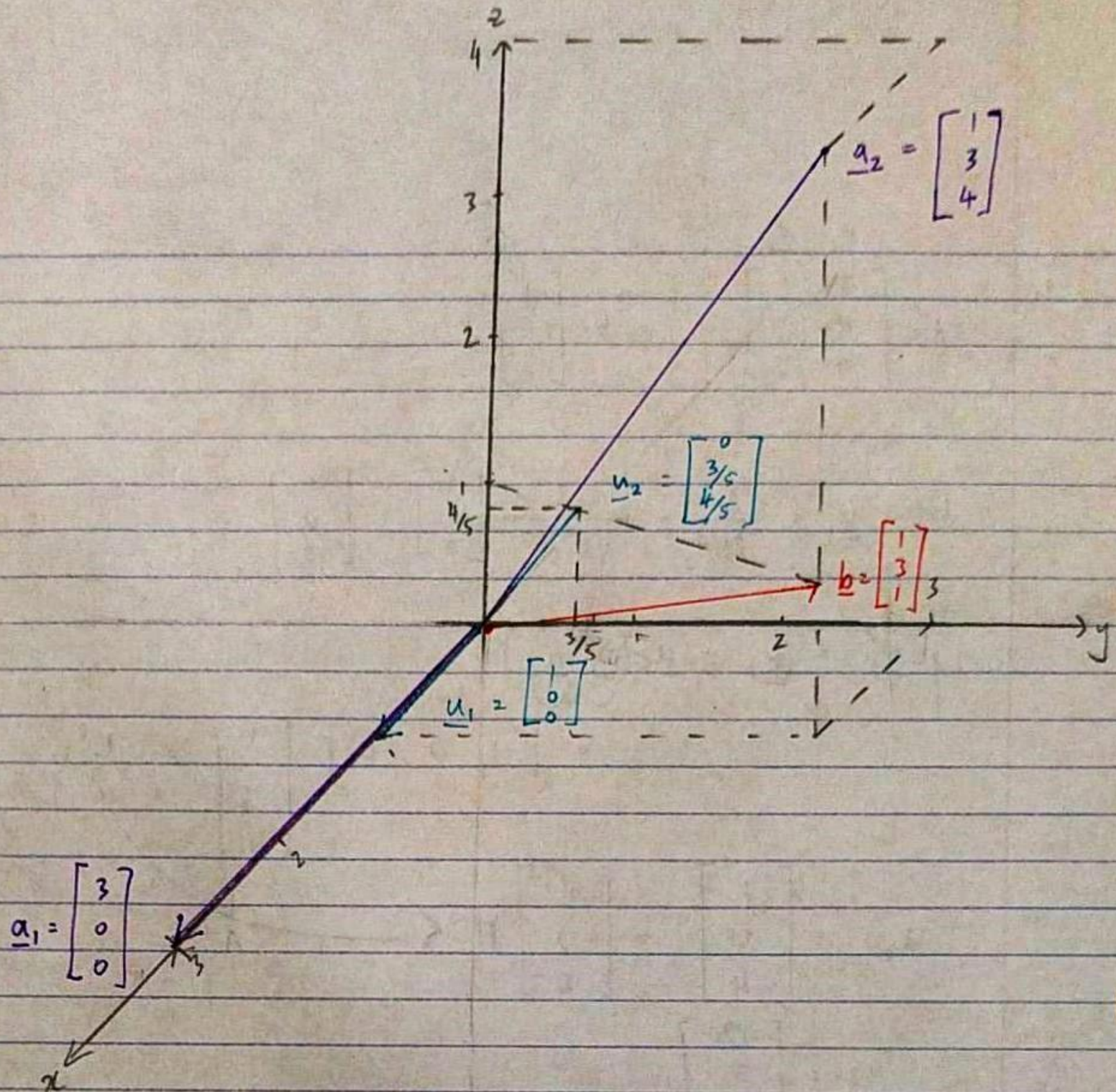
$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = 1$$

$$\underline{a}_2' = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

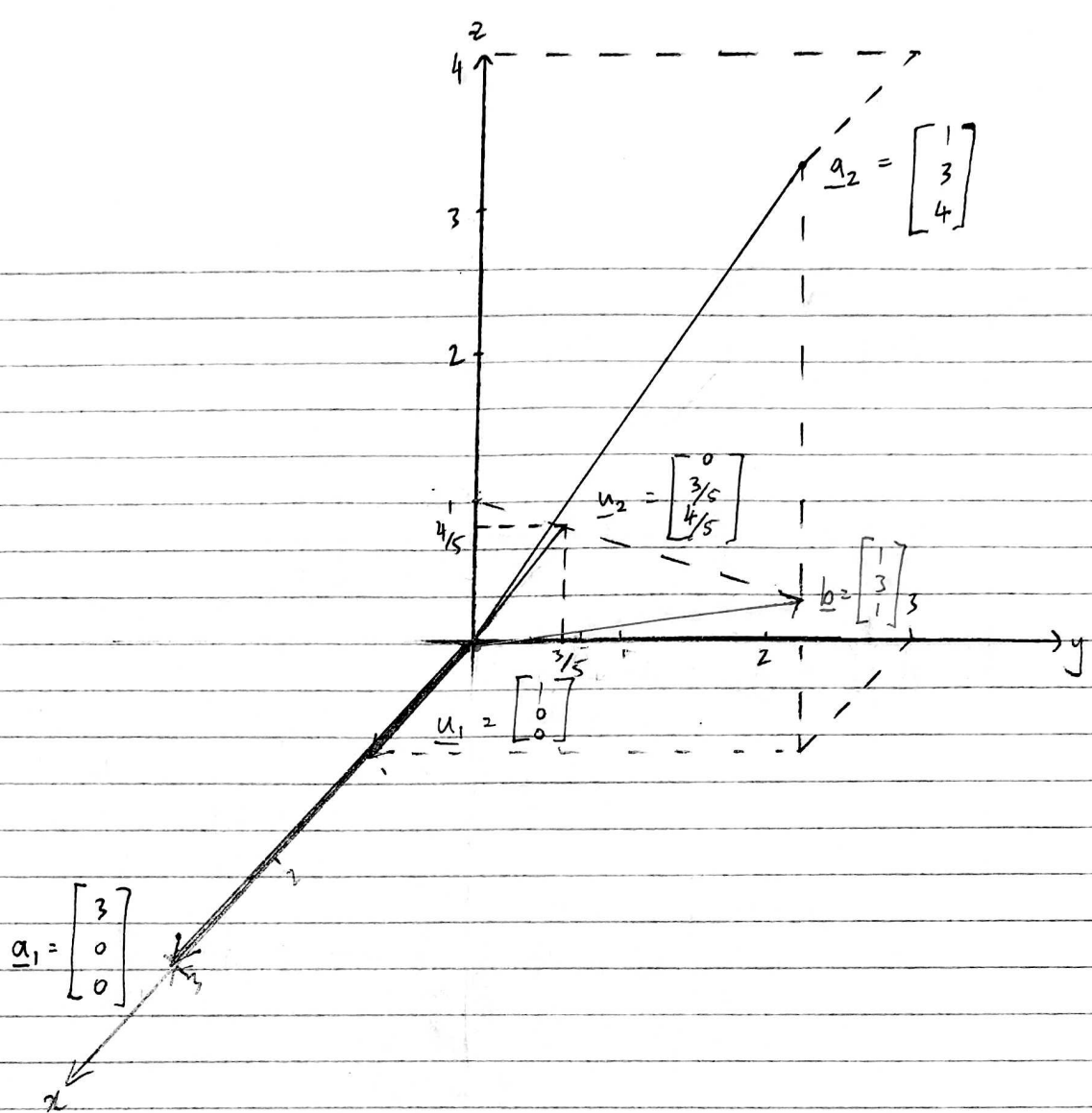
$$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = 1 \cdot \underline{u}_1$$

$$\underline{u}_2 = \frac{\underline{a}_2'}{\|\underline{a}_2'\|_2} = \frac{1}{\sqrt{(0)^2 + 3^2 + 4^2}} \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3/5 \\ 4/5 \end{bmatrix}$$

$$\therefore U = [\underline{u}_1 \quad \underline{u}_2] = \begin{bmatrix} 1 & 0 \\ 0 & 3/5 \\ 0 & 4/5 \end{bmatrix}$$



(b)



(c)

$$\hat{\underline{b}} = A(A^T A)^{-1} A^T \underline{b}$$

$$= \underline{U} \underbrace{(\underline{U}^T \underline{U})^{-1}}_{=I} \underline{U}^T \underline{b}$$

$$= \underline{U} \underline{U}^T \underline{b}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 3/5 \\ 0 & 4/5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3/5 & 4/5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$3 \times 3$                        $2 \times 3$                        $3 \times 1$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 3/5 \\ 0 & 4/5 \end{bmatrix} \begin{bmatrix} 1 \\ 13/5 \end{bmatrix}$$

$3 \times 2$                        $2 \times 1$

$$= \begin{bmatrix} 1 \\ 39/25 \\ 52/25 \end{bmatrix}$$