ECE 532

HW4

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1(a)
$$\|y - Xw\|_2^2 + \lambda\|w\|_2^2$$
$$= \left\| \begin{bmatrix} y & - & Xw \\ \sqrt{\lambda} & & x \end{bmatrix} \right\|^2$$
$$= \left\| \begin{bmatrix} y \\ 0 \end{bmatrix} - \begin{bmatrix} X \\ \sqrt{\lambda} I \end{bmatrix} \right\|^2$$

$$\Rightarrow \begin{bmatrix} X \\ \sqrt{\lambda} I \end{bmatrix} \hat{w} = \begin{bmatrix} y \\ 0 \end{bmatrix} \Rightarrow B\hat{w} = z$$

$$\begin{bmatrix} X \\ \sqrt{\lambda} I \end{bmatrix}^T \begin{bmatrix} X \\ \sqrt{\lambda} I \end{bmatrix} \hat{w} = \begin{bmatrix} X \\ \sqrt{\lambda} I \end{bmatrix}^T \begin{bmatrix} y \\ 0 \end{bmatrix}.$$

$$(X^T X + \lambda I)\hat{w} = X^T y$$

(b) n $\boxed{X \in \mathbb{R}^{n \times p}}$ p $\quad n < p \Rightarrow$ more unknowns than equations "underdetermined"

There is always going to be a unique solution to this problem.

$\Rightarrow$ $A^T A \succeq 0$ (PSD)

and $\lambda I \succ 0$ (PD)

so $A^T A + \lambda I \succ 0$, hence it will always be invertible.