ECE 532 - HW5

FALL 2017

1 (b) $\hat{w}_\lambda = \underset{w}{\operatorname{argmin}} \; \| y - X w \|_2^2 + \lambda \|w\|_2^2$

$$\Rightarrow \quad \hat{w}_\lambda = (X^T X + \lambda I)^{-1} \cdot X^T y$$

writing $X = U \Sigma V^T$, consider $X^T X$

$$X^T X = (U \Sigma V^T)^T (U \Sigma V^T)$$

$$= V \Sigma \underbrace{U^T U}_{= I} \Sigma V^T$$

$$= V \Sigma^2 V^T$$

$$\therefore \quad \hat{w}_\lambda = (V \Sigma^2 V^T + \lambda I)^{-1} \cdot (U \Sigma V^T)^T \cdot y$$

$$= (V \Sigma^2 V^T + \lambda V \cdot V^T)^{-1} \cdot \underbrace{(U \Sigma V^T)^T}_{= V \Sigma U^T} \cdot y$$

$$= V (\Sigma^2 + \lambda I)^{-1} \cdot \underbrace{V^T (V}_{= I} \Sigma U^T) \cdot y$$

$$= V (\Sigma^2 + \lambda I)^{-1} \cdot \Sigma U^T \cdot y$$