CS/ECE/ME 532
Homework 0: Warm-up (not graded)

1. **Matrix-vector multiplication**. For each case below, calculate the products by hand and then verify your answer using code (Matlab, Python, or Julia).

   a) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

   b) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}^{\mathsf{T}}$

   c) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 3 \end{bmatrix}$

   d) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

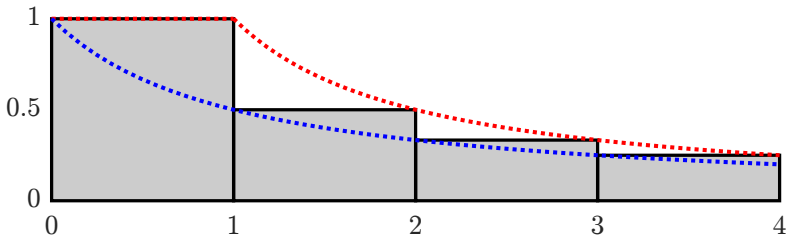
2. **Linear equations**. Consider the following equations:

$$\begin{aligned} x + 2y &= 4 \\ 2x + 3y &= 7 \end{aligned}$$

   a) What is the solution of this system of equations?

   b) What would happen if the 3 were changed to a 4?

3. **Harmonic series bounds**. Consider the diagram below.



   The k^{th} gray rectangle has an area of $\frac{1}{k}$ (width of 1 and height of $\frac{1}{k}$). Therefore:

$$\text{(area beneath blue curve)} \;\leq\; \underbrace{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}_{\text{area of gray rectangles}} \;\leq\; \text{(area beneath red curve)}$$

   a) Using the observation above as a hint, prove that the following is true:

$$\log(m+1) \;\leq\; 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{m} \;\leq\; 1 + \log(m) \qquad \text{for } m = 1, 2, \ldots$$

   b) Numerically verify that the inequality above is true when $m = 10^6$.

   c) Make a figure using Matlab, Python, or Julia that looks similar to the one above.