CS/ECE/ME 532
Homework 6: more SVD

due: Friday October 28, 2016

1. **Normal equations.** Using the SVD, show that for any $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$, the normal equations $A^\mathsf{T} Ax = A^\mathsf{T} b$ always have at least one solution. What is this solution?

2. **Right inverses.** A *right inverse* of $A \in \mathbb{R}^{m\times n}$ is a matrix $B \in \mathbb{R}^{n\times m}$ such that $AB = I$.

   a) Prove that if the rows of A are linearly independent, A has a right inverse.

   b) Prove that if A has a right inverse, the rows of A are linearly independent.

   c) Suppose A has linearly independent rows. Find a parameterization of all right inverses of A.

3. **Recovering a blurred signal.** Many sensing and imaging systems produce signals that may be slightly distorted or blurred (e.g., an out-of-focus camera). In such situations, algorithms are needed to deblur the data to obtain a more accurate estimate of the true signal. Suppose our true signal is a vector $x \in \mathbb{R}^n$, and the blurring produces a new signal $b \in \mathbb{R}^n$ according to the model:

$$b_i = \frac{1}{k}\big(x_i + x_{i-1} + \cdots + x_{i-k+1}\big) + w_i \qquad \text{for } i = 1, \dots, n$$

   In other words, each b_i is the average of the past k values of x_i, plus some extra noise w_i. Note: in the above formula, treat x_j as zero when $j < 1$. The goal is to estimate x using b and A.

   a) We can write the above equations in the more compact form: $b = Ax + w$. Write code that generates the $A \in \mathbb{R}^{n\times n}$ matrix as a function of n and k.

   b) Suppose the true x is given in the file `xsignal.csv`. Generate b by using $k = 30$. To generate w, make each w_i normally distributed with standard deviation σ. For example, you can do this in matlab via: `w=sigma*randn(n,1)`. Plot x and b using $\sigma = 0.01$ and $\sigma = 0.1$.

   c) Reconstruct x in the three following ways, and for each one plot the true x and its reconstruction.

   (i) Ordinary least squares

   (ii) Truncated SVD; only keep the largest m singular values of A and try different values of m.

   (iii) Regularized (Tikhonov) least squares; try different values of the parameter λ.

   d) Experiment with different averaging functions (i.e., different values of k in the code) and with different noise levels (σ in the code). How do the blurring and noise level affect the value of the regularization parameters that produce the best estimates?