280823 - Mooring Systems
Lesson 1 - Line Modelling - Catenary

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19th September 2019

Outline

Catenary

Figure 1: Different catenaries and archs.

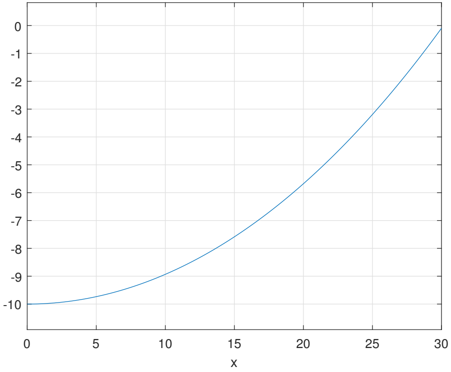
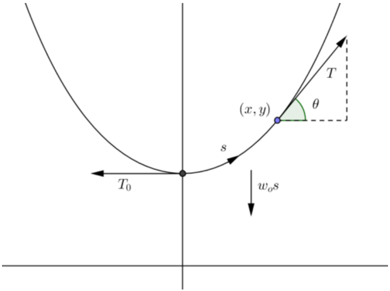


Figure 2: Catenary curve.

Equilibrium:

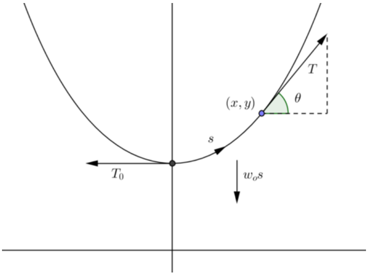
$$\sum F = 0 \rightarrow \begin{cases} T_H = T\cos\theta \\ T_V = T\sin\theta = \lambda g s = ws \end{cases} \tag{1}$$



Line Tension:

$$
T = \sqrt{T_H^2 + T_V^2}
$$

$$
\theta = \frac{dy}{dx} \tag{2}
$$

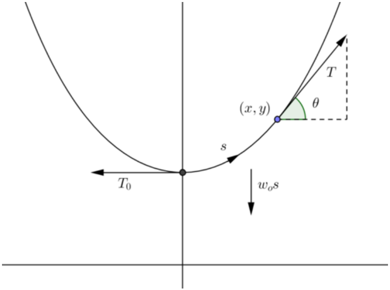


Slope:

$$\tan\theta = \frac{dy}{dx} = \frac{\lambda g s}{T_H} = \frac{s}{a}, \qquad a = \frac{T_H}{\lambda g} \tag{3}$$

Length Differential:

$$ds = \sqrt{dx^2 + dy^2} \tag{4}$$



Length Differential:

$$\begin{cases} \dfrac{ds}{dx} = \dfrac{\sqrt{a^2+s^2}}{a} \rightarrow x = a \operatorname{arcsinh}\left(\dfrac{s}{a}\right) + C_1 \\ \dfrac{ds}{dy} = \dfrac{\sqrt{a^2+s^2}}{s} \rightarrow y = \sqrt{a^2+s^2} + C_2 \end{cases} \tag{5}$$

References Axis:

$$\begin{cases} x = 0 & , \quad s = 0 \quad \rightarrow \quad C_1 = 0 \\ y = -H & , \quad s = 0 \quad \rightarrow \quad C_2 = -(H + a) \end{cases} \tag{6}$$

Catenary Curve Equation:

$$\begin{cases} s = a\sinh\dfrac{x}{a} \\ y = \sqrt{a^2 + (a\sinh\dfrac{x}{a})^2} - (H + a) \\ \\ y = a\left(\cosh\dfrac{x}{a} - 1\right) - H \end{cases} \tag{7}$$

Catenary Curve Equation:

$$\begin{cases} s = a \sinh \dfrac{x}{a} \\ y = \sqrt{a^2 + (a \sinh \dfrac{x}{a})^2} - (H + a) \\ \\ y = a\left(\cosh \dfrac{x}{a} - 1 \right) - H \end{cases} \tag{8}$$

Polynomial Approximation

Taylor Series (x=0) :

$$cosh(x) = \frac{e^{x} + e^{-x}}{2} = \cos(ix)$$

$$e^{x} = \sum_{k=0}^{\infty} \frac{x^{k}}{k!} = 1 + x + \frac{x^{2}}{2} + \frac{x^{3}}{6} + \frac{x^{4}}{24} + \dots \tag{9}$$

Types

Non-uniform:

$$
\begin{aligned}
&W_{QP} := \int_P^Q w \,\mathrm{d}s \\
&\sum F = 0 \rightarrow \begin{cases} T\cos\theta = T_H \\ T\sin\theta = \int_P^Q w\,\mathrm{d}s \end{cases} \\
&\frac{\mathrm{d}y}{\mathrm{d}x} = \tanh\theta = \frac{1}{T_H}\int_P^Q w\,\mathrm{d}s \\
&w = T_H \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{T_H}{\kappa\cos^2\theta}
\end{aligned}
\tag{10}
$$

Types

Suspended weight and negligible self-weight:

$$W_{QP} := f(x)$$

$$\sum F = 0 \rightarrow \begin{cases} T\cos\theta = T_H \\ T\sin\theta = f(x) \end{cases} \text{, e.g. } f(x) := wx \tag{11}$$

$$y(x) = \frac{w}{2T_H}(x^2 - x_1^2) + y_1$$

Problem 1:

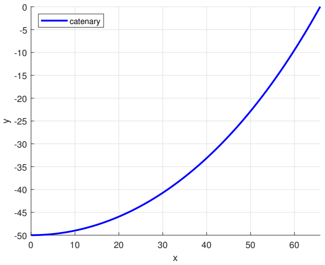


Figure 3: Problem 1's schematic

$T_H = 1000$ kN

$L = 65.8479$m

$g = -9.81 \frac{\text{m}}{\text{s}^2}$

$H = 50$ m

circular section

Bouyancy taken into account

r, s, θ, T?

Types

Problem 2:

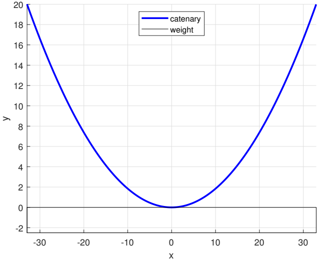


Figure 4: Problem 2's schematic

$$q = 0.1019 \frac{\text{kg}}{\text{m}}$$

$$L = 120 \text{ m}$$

$$H = 20 \text{ m}$$

$$T_y = qx \text{ N}$$

$$T_H, T_{max}, a?$$

THE END

References