

Appendix A. Results of Monte Carlo experiment

Table A1. RMSE of direct impacts without omv. $\beta = (0.2, 0.5)^T$, $\gamma = (0, 0)^T$.

ρ	δ	λ	θ	OLS		SLX		SAR		SEM		SAC		SDM		SDEM	
				x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
0.0	0, 0	0.0	0, 0	0.0321	0.0350*	0.0322	0.0350	0.0321	0.0351	0.0321	0.0350	0.0323	0.0353	0.0321*	0.0351	0.0322	0.0350
0.5	0, 0	0.0	0, 0	0.0348	0.0376	0.0346	0.0373	0.0337*	0.0365*	0.0361	0.0514	0.0339	0.0375	0.0346	0.0374	0.0341	0.0369
0.0	0.4, 0.7	0.0	0, 0	0.0308*	0.0281*	0.0325	0.0363	0.0308	0.0304	0.0308	0.0282	0.0310	0.0354	0.0326	0.0363	0.0325	0.0363
0.5	0.4, 0.7	0.0	0, 0	0.0433	0.1459	0.0325	0.0353	0.0322*	0.0313*	0.0335	0.0471	0.0323	0.0338	0.0326	0.0347	0.0325	0.0348
0.0	0, 0	0.5	0, 0	0.0345	0.0372	0.0346	0.0373	0.0346	0.0427	0.0317*	0.0346*	0.0325	0.0373	0.0346	0.0372	0.0341	0.0370
0.5	0, 0	0.5	0, 0	0.0432	0.0456	0.0433	0.0455	0.0421	0.0663	0.0377	0.0623	0.0348*	0.0436	0.0433	0.0470	0.0380	0.0416*
0.0	0.4, 0.7	0.5	0, 0	0.0368	0.0379	0.0324	0.0352	0.0336	0.0655	0.0323*	0.0334*	0.0325	0.0344	0.0326	0.0348	0.0325	0.0347
0.5	0.4, 0.7	0.5	0, 0	0.0570	0.1547	0.0360	0.0363	0.0367	0.0513	0.0360	0.0392	0.0341*	0.0351*	0.0366	0.0352	0.0350	0.0353
0.0	0, 0	0.0	0.1, 0.8	0.0334	0.0372	0.0322	0.0350*	0.0333	0.0411	0.0336	0.0463	0.0317*	0.0504	0.0323	0.0350	0.0322	0.0350
0.5	0, 0	0.0	0.1, 0.8	0.0388	0.0441	0.0348	0.0377	0.0363	0.0875	0.0517	0.2033	0.0361	0.0422	0.0347	0.0373*	0.0344*	0.0402
0.0	0.4, 0.7	0.0	0.1, 0.8	0.0375	0.2843	0.0325	0.0363	0.0313	0.1256	0.0347	0.1820	0.0303*	0.1016	0.0325	0.0363*	0.0325	0.0363
0.5	0.4, 0.7	0.0	0.1, 0.8	0.0684	0.5806	0.0329	0.0397	0.0341	0.0861	0.0507	0.1096	0.0320*	0.1068	0.0326	0.0347*	0.0328	0.0360
0.0	0, 0	0.5	0.1, 0.8	0.0357	0.0393	0.0346	0.0373	0.0345	0.0478	0.0351	0.1047	0.0352	0.0390	0.0349	0.0379	0.0341*	0.0370*
0.5	0, 0	0.5	0.1, 0.8	0.0466	0.0510	0.0434	0.0458*	0.0396	0.0749	0.0555	0.2369	0.0390	0.0853	0.0440	0.0537	0.0388*	0.0583
0.0	0.4, 0.7	0.5	0.1, 0.8	0.0428	0.2866	0.0324*	0.0352	0.0325	0.0644	0.0347	0.0482	0.0327	0.0568	0.0327	0.0349	0.0325	0.0347*
0.5	0.4, 0.7	0.5	0.1, 0.8	0.0781	0.5844	0.0364	0.0404	0.0375	0.0533	0.0512	0.1381	0.0375	0.0532	0.0372	0.0356*	0.0355*	0.0370
Average				0.0434	0.1518	0.0348	0.0379	0.0347	0.0587	0.0386	0.0875	0.0336*	0.0517	0.0350	0.0377*	0.0340	0.0379
				0.0976		0.0363		0.0467		0.0631		0.0427		0.0363		0.0359*	

* Lowest RMSE for x_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($x_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Table A2. RMSE of indirect impacts without omv. $\beta = (0.2, 0.5)^T$, $\gamma = (0, 0)^T$.

ρ	δ	λ	θ	SLX		SAR		SAC		SDM		SDM	
				x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
0.0	0, 0	0.0	0, 0	0.0903	0.0929	0.0121*	0.0302*	0.0366	0.0914	0.0909	0.0927	0.0907	0.0928
0.5	0, 0	0.0	0, 0	0.1371	0.1807	0.0449*	0.0843*	0.0876	0.2049	0.1766	0.1796	0.1302	0.2053
0.0	0.4, 0.7	0.0	0, 0	0.0745	0.0610	0.0113*	0.0280*	0.0240	0.0587	0.0748	0.0610	0.0748	0.0609
0.5	0.4, 0.7	0.0	0, 0	0.1143	0.1090	0.0406*	0.0644*	0.0572	0.1091	0.1368	0.0988	0.1098	0.1315
0.0	0, 0	0.5	0, 0	0.1292	0.1307	0.1728	0.4297	0.0570*	0.1415	0.1754	0.1789	0.1136	0.1165*
0.5	0, 0	0.5	0, 0	0.2150	0.2402*	0.5339	1.3185	0.1666	0.4097	0.4925	0.6443	0.1535*	0.2453
0.0	0.4, 0.7	0.5	0, 0	0.1082	0.0838	0.1377	0.3076	0.0317*	0.0760*	0.1363	0.0980	0.0968	0.0773
0.5	0.4, 0.7	0.5	0, 0	0.1837	0.1526*	0.3835	0.8000	0.0882*	0.1904	0.3674	0.3380	0.1374	0.1802
0.0	0, 0	0.0	0.1, 0.8	0.0903	0.0929	0.0467*	0.6617	0.1640	0.1566	0.0909	0.0953	0.0907	0.0928*
0.5	0, 0	0.0	0.1, 0.8	0.1648	0.5762	0.0820*	1.1564	0.3560	0.3315	0.1781	0.1999*	0.1771	0.7493
0.0	0.4, 0.7	0.0	0.1, 0.8	0.0745	0.0610	0.0341*	0.4238	0.1511	0.0726	0.0749	0.0618	0.0748	0.0609*
0.5	0.4, 0.7	0.0	0.1, 0.8	0.1373*	0.3220	0.1817	0.3776	0.3614	0.3485	0.1381	0.1104*	0.1500	0.4798
0.0	0, 0	0.5	0.1, 0.8	0.1292	0.1307	0.1388	0.2855	0.2987	0.2354	0.1831	0.4253	0.1136*	0.1165*
0.5	0, 0	0.5	0.1, 0.8	0.2328	0.5913	0.5372	0.3217*	0.4937	0.4227	0.5462	1.6884	0.2103*	0.8949
0.0	0.4, 0.7	0.5	0.1, 0.8	0.1082	0.0838	0.1610	0.1220	0.2438	0.1807	0.1404	0.2072	0.0968*	0.0773*
0.5	0.4, 0.7	0.5	0.1, 0.8	0.1983	0.3353*	0.5526	0.6058	0.5483	0.6037	0.3837	0.8393	0.1870*	0.6627
Average				0.1367	0.2028*	0.1919	0.4386	0.1979	0.2271	0.2116	0.3324	0.1254*	0.2652
				0.1697*		0.3153		0.2125		0.2720		0.1953	

* Lowest RMSE for x_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable (Wu); δ = autocorrelation in the covariates ($x_k = f(Wx_k)$); λ = autocorrelation in the disturbances (Wu); θ = spatial spillover effects of covariates (WX); γ = strength of omv.

Table A3. RMSE of direct impacts with omv. $\beta = (0.2, 0.5)^T$, $\gamma = (0.3, 0)^T$.

OLS			SLX		SAR		SEM		SAC		SDM		SDEM	
ρ	δ	λ	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
0.0	0, 0	0.0	0.3011	0.0350*	0.3010*	0.0350	0.3014	0.0350	0.3021	0.0354	0.3010	0.0352	0.3010	0.0350
0.5	0, 0	0.0	0.3148	0.0379	0.3152	0.0373	0.3152	0.0366*	0.3162	0.0374	0.3154	0.0374	0.3112	0.0369
0.0	0.4, 0.7	0.0	0.3014	0.0281*	0.3007	0.0363	0.3015	0.0301	0.3014	0.0330	0.3007*	0.0364	0.3007	0.0363
0.5	0.4, 0.7	0.0	0.3668	0.1462	0.3137	0.0353	0.3153	0.0311*	0.3156	0.0326	0.3147	0.0348	0.3137	0.0348
0.0	0, 0	0.5	0.3151	0.0373	0.3153	0.0373	0.3213	0.0424	0.3056	0.0391	0.3154	0.0374	0.3128	0.0370
0.5	0, 0	0.5	0.3499	0.0466	0.3504	0.0456	0.3655	0.0655	0.3301	0.0474	0.3583	0.0469	0.3226	0.0416*
0.0	0.4, 0.7	0.5	0.3464	0.0384	0.3141	0.0352	0.3121	0.0651	0.3130	0.0355	0.3147	0.0347	0.3141	0.0347
0.5	0.4, 0.7	0.5	0.4507	0.1559	0.3466	0.0363	0.3527	0.0511	0.3345	0.0352*	0.3531	0.0353	0.3318	0.0353
0.0	0, 0	0.0	0.3009	0.0372	0.3010	0.0350*	0.3018	0.0408	0.2884*	0.0505	0.3009	0.0351	0.3010	0.0350
0.5	0, 0	0.0	0.3149	0.0445	0.3152	0.0377	0.3184	0.0876	0.3297	0.0449	0.3155	0.0373*	0.3094	0.0403
0.0	0.4, 0.7	0.0	0.3159	0.2843	0.3007	0.0363	0.2929	0.1314	0.3051	0.1820	0.3007	0.0363*	0.3007	0.0363
0.5	0.4, 0.7	0.0	0.3896	0.5807	0.3135	0.0397	0.3195	0.0902	0.2530*	0.1114	0.3147	0.0347*	0.3131	0.0360
0.0	0, 0	0.5	0.3149	0.0395	0.3153	0.0373	0.3210	0.0478	0.2762*	0.1057	0.3158	0.0380	0.3128	0.0370*
0.5	0, 0	0.5	0.3499	0.0522	0.3505	0.0459*	0.3641	0.0754	0.2205*	0.2382	0.3593	0.0534	0.3181	0.0584
0.0	0.4, 0.7	0.5	0.3609	0.2867	0.3141	0.0352	0.3128	0.0669	0.2900*	0.0463	0.3147	0.0349	0.3141	0.0347*
0.5	0.4, 0.7	0.5	0.4732	0.5848	0.3464	0.0404	0.3603	0.0545	0.2388*	0.1409	0.3523	0.0356*	0.3283	0.0372
Average			0.3479	0.1522	0.3196	0.0379	0.3235	0.0595	0.3148	0.0544	0.3217	0.0377*	0.3128	0.0379
			0.2501			0.1787		0.1915			0.1846	0.1797		0.1754*

* Lowest RMSE for \mathbf{x}_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Table A4. RMSE of indirect impacts with omv. $\beta = (0.2, 0.5)^T$, $\gamma = (0.3, 0)^T$.

SLX			SAR		SAC		SDM		SDEM	
ρ	δ	λ	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
0.0	0, 0	0.0	0.0903	0.0929	0.0288*	0.0290*	0.0690	0.0693	0.0907	0.0927
0.5	0, 0	0.0	0.2049	0.1807	0.3058	0.0818*	0.3601	0.1662	0.3477	0.1797
0.0	0.4, 0.7	0.0	0.0745	0.0610	0.0265*	0.0266*	0.0486	0.0482	0.0747	0.0611
0.5	0.4, 0.7	0.0	0.2153	0.1091	0.2993	0.0615*	0.3197	0.0945	0.3266	0.0988
0.0	0, 0	0.5	0.2492	0.1307	0.4181	0.4194	0.1948*	0.1961	0.3467	0.1791
0.5	0, 0	0.5	0.5124	0.2405*	1.5642	1.2857	0.7623	0.5244	1.5075	0.6378
0.0	0.4, 0.7	0.5	0.2529	0.0839	0.3516	0.3032	0.1300*	0.1194	0.3261	0.0983
0.5	0.4, 0.7	0.5	0.5703	0.1533*	1.2656	0.7828	0.5845	0.2854	1.3652	0.3336
0.0	0, 0	0.0	0.0903	0.0929	0.0562*	0.0666	0.4080	0.2711	0.0910	0.0954
0.5	0, 0	0.0	0.1, 0.8	0.1685	0.5761	0.1710	1.1624	0.4710	0.3491	0.1994*
0.0	0.4, 0.7	0.0	0.1, 0.8	0.0745*	0.0610	0.4442	0.4065	0.1267	0.0750	0.0618
0.5	0.4, 0.7	0.0	0.1, 0.8	0.1833	0.3220	0.8884	1.2033	0.2167	0.3272	0.1098*
0.0	0, 0	0.5	0.2492	0.1307	0.4806	0.2946	0.7523	0.1748	0.3824	0.4252
0.5	0, 0	0.5	0.1, 0.8	0.2492	0.1307	0.4806	0.7523	0.1748	0.3824	0.4252
0.0	0.4, 0.7	0.0	0.1, 0.8	0.4647	0.5912	1.8671	1.7147	0.4123	1.5691	1.6597
0.5	0, 0	0.5	0.1, 0.8	0.2529	0.0839	0.5336	0.1372	0.6841	0.3490	0.2067
0.0	0.4, 0.7	0.5	0.1, 0.8	0.2529	0.0839	0.5336	0.1372	0.6841	0.3490	0.2067
0.5	0.4, 0.7	0.5	0.1, 0.8	0.5318	0.3355*	1.8952	0.5415	1.8538	1.3240	0.8206
Average			0.2616	0.2028*	0.6669	0.4365	0.6659	0.2376	0.5533	0.3287
			0.2322		0.5517		0.4518		0.4410	
										0.2253*

* Lowest RMSE for \mathbf{x}_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Table A5. Bias of direct impacts without omv. $\beta = (0.2, 0.5)^\top$, $\gamma = (0, 0)^\top$.

			OLS		SLX		SAR		SEM		SAC		SDM		SDEM		
ρ	δ	λ	θ	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2		
0.0	0, 0	0.0	0, 0	-0.0006	0.0027	-0.0007	0.0027	-0.0004	0.0030	-0.0007	0.0028	-0.0000*	0.0045	-0.0007	0.0026*	0.0027	
0.5	0, 0	0.0	0, 0	-0.0003	0.0029	-0.0002	0.0031	-0.0005	0.0031	-0.0172	-0.0378	-0.0001*	0.0045	-0.0001	0.0033	-0.0009*	
0.0	0.4, 0.7	0.0	0, 0	-0.0002	0.0029	-0.0011	0.0021	-0.0001*	0.0031	-0.0002	0.0029	-0.0002	0.0028	-0.0011	0.0021*	0.0022	
0.5	0.4, 0.7	0.0	0, 0	0.0209	0.1401	-0.0010	-0.0021*	0.0000*	0.0035	-0.0080	0.0301	0.0001	0.0042	-0.0006	0.0023	0.0035	
0.0	0, 0	0.5	0, 0	0.0001*	0.0034	-0.0001	0.0034	0.0073	0.0223	-0.0011	0.0023*	0.0009	0.0075	-0.0002	0.0033	0.0031	
0.5	0, 0	0.5	0, 0	0.0007	0.0041*	0.0007*	0.0044	0.0192	0.0524	-0.0232	-0.0530	0.0024	0.0112	0.0033	0.0106	-0.0072	
0.0	0.4, 0.7	0.5	0, 0	0.0006	0.0040	-0.0006	0.0023*	-0.0060	-0.0570	-0.0008	0.0029	-0.0002*	0.0026	-0.0006	0.0025	0.0025	
0.5	0.4, 0.7	0.5	0, 0	0.0222	0.1422	-0.0003*	-0.0018	0.0035	-0.0376	-0.0176	-0.0168	0.0003	0.0046	0.0011	0.0037	0.0004*	
0.0	0, 0	0.0	0.1, 0.8	-0.0010	0.0020*	-0.0007	0.0027	-0.0025	-0.0182	-0.0044	-0.0229	-0.0032	0.0363	-0.0006*	0.0026	0.0027	
0.5	0, 0	0.0	0.1, 0.8	-0.0007	0.0015*	-0.0002	0.0024	-0.0068	-0.0781	-0.0400	-0.1998	0.0039	-0.0071	-0.0001*	0.0034	-0.0144	
0.0	0.4, 0.7	0.0	0.1, 0.8	0.0140	0.2816	-0.0011	0.0021	-0.0003*	0.1214	0.0032	0.1737	-0.0137	0.0976	-0.0011	0.0019*	0.0022	
0.5	0.4, 0.7	0.0	0.1, 0.8	0.0420	0.5754	-0.0012	-0.0164	0.0004*	0.0794	-0.0366	-0.1011	-0.0005	0.1023	-0.0005	0.0023*	0.0067	
0.0	0, 0	0.5	0.1, 0.8	-0.0003	0.0026*	-0.0001*	0.0034	0.0033	-0.0298	-0.0138	-0.0981	0.0058	-0.0032	0.0003	0.0070	0.0031	
0.5	0, 0	0.5	0.1, 0.8	0.0003*	0.0027*	0.0007	0.0036	0.0092	-0.0617	-0.0464	-0.2344	0.0075	-0.0718	0.0049	0.0280	-0.0410	
0.0	0.4, 0.7	0.5	0.1, 0.8	0.0148	0.2828	-0.0006	0.0023*	-0.0016	0.0562	-0.0087	0.0237	-0.0061	0.0478	-0.0006*	0.0041	-0.0025	
0.5	0.4, 0.7	0.5	0.1, 0.8	0.0432	0.5776	-0.0006*	-0.0160	0.0061	0.0405	-0.0394	-0.1326	0.0065	0.0402	0.0014	0.0047*	-0.0085	
Average (absolute bias)				0.0101	0.1268	0.0006*	0.0044*	0.0042	0.0417	0.0163	0.0709	0.0032	0.0280	0.0011	0.0053	0.0022	0.0065
				0.0684		0.0025*			0.0230		0.0436		0.0156		0.0032		0.0043

* Lowest bias for \mathbf{x}_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Table A6. Bias of indirect impacts without omv. $\beta = (0.2, 0.5)^\top$, $\gamma = (0, 0)^\top$.

			SLX		SAR		SAC		SDM		SDEM				
ρ	δ	λ	θ	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2		
0.0	0, 0	0.0	0, 0	0.0064	0.0060	0.0013*	0.0030*	0.0105	0.0264	0.0067	0.0058	0.0065	0.0058		
0.5	0, 0	0.0	0, 0	-0.0457	-0.1252	0.0030*	0.0110*	0.0175	0.0474	0.0120	0.0138	-0.0637	-0.1689		
0.0	0.4, 0.7	0.0	0, 0	0.0055	0.0022	0.0006*	0.0008*	0.0043	0.0078	0.0057	0.0023	0.0055	0.0021		
0.5	0.4, 0.7	0.0	0, 0	-0.0358	-0.0696	0.0016*	0.0051*	0.0073	0.0159	0.0098	0.0059	-0.0514	-0.1060		
0.0	0, 0	0.5	0, 0	0.0086	0.0108	0.1679	0.4225	0.0243	0.0613	0.0127	0.0130	0.0068*	0.0076*		
0.5	0, 0	0.5	0, 0	-0.0421*	-0.1169*	0.5107	1.2887	0.0857	0.2210	0.1971	0.4513	-0.0786	-0.2038		
0.0	0.4, 0.7	0.5	0, 0	0.0070	0.0053	0.1335	0.3030	0.0104	0.0233	0.0102	0.0059	0.0067*	0.0044*		
0.5	0.4, 0.7	0.5	0, 0	-0.0331	-0.0641	0.3629	0.7805	0.0256*	0.0600*	0.1546	0.2556	-0.0672	-0.1484		
0.0	0, 0	0.0	0.1, 0.8	0.0064*	0.0060	-0.0427	-0.6605	0.1543	-0.1062	0.0065	0.0068	0.0065	0.0058*		
0.5	0, 0	0.0	0.1, 0.8	-0.0095	-0.5606	-0.0322	-1.1496	0.3229	-0.1349	0.0119*	0.0165*	-0.1349	-0.7393		
0.0	0.4, 0.7	0.0	0.1, 0.8	0.0055*	0.0022	0.0223	-0.4209	0.1448	-0.0178	0.0056	0.0025	0.0055	0.0021*		
0.5	0.4, 0.7	0.0	0.1, 0.8	-0.0777	-0.3093	0.1544	-0.3544	0.3395	0.2994	0.0096*	0.0057*	-0.1110	-0.4720		
0.0	0, 0	0.5	0.1, 0.8	0.0086	0.0108	0.1286	-0.2720	0.2828	0.1264	0.0570	0.3836	0.0068*	0.0076*		
0.5	0, 0	0.5	0.1, 0.8	-0.0958*	-0.5523	0.5008	0.0649*	0.4480	-0.0739	0.3238	1.6066	-0.1637	-0.8841		
0.0	0.4, 0.7	0.5	0.1, 0.8	0.0070	0.0053	0.1526	-0.0948	0.2333	0.1365	0.0413	0.1804	0.0067*	0.0044*		
0.5	0.4, 0.7	0.5	0.1, 0.8	-0.0750*	-0.3038*	0.5222	0.5486	0.5172	0.5282	0.2208	0.7967	-0.1425	-0.6537		
Average (absolute bias)				0.0350*	0.1344	0.1711	0.3988	0.1643	0.1179*	0.0678	0.2345	0.0540	0.2135		0.1338
				0.0847*		0.2849		0.1411		0.1512					

* Lowest bias for \mathbf{x}_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Table A7. Bias of direct impacts with omv. $\beta = (0.2, 0.5)^\top$, $\gamma = (0.3, 0)^\top$.

ρ	δ	λ	θ	OLS		SLX		SAR		SEM		SAC		SDM		SDEM	
				x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
0.0	0, 0	0.0	0, 0	0.2994	0.0027	0.2993*	0.0027	0.2997	0.0030	0.2993	0.0028	0.3004	0.0040	0.2993	0.0027*	0.2993	0.0027
0.5	0, 0	0.0	0, 0	0.3128	0.0029	0.3133	0.0032	0.3133	0.0032	0.2718*	-0.0384	0.3143	0.0043	0.3135	0.0034	0.3093	-0.0009*
0.0	0.4, 0.7	0.0	0, 0	0.2998	0.0029	0.2989	0.0021*	0.2999	0.0030	0.2998	0.0029	0.2998	0.0026	0.2989*	0.0022	0.2989	0.0022
0.5	0.4, 0.7	0.0	0, 0	0.3647	0.1401	0.3120	-0.0021*	0.3137	0.0034	0.2933*	0.0267	0.3139	0.0036	0.3130	0.0024	0.3120	0.0035
0.0	0, 0	0.5	0, 0	0.3132	0.0033	0.3134	0.0034	0.3195	0.0217	0.2889*	0.0022*	0.3036	0.0145	0.3135	0.0033	0.3109	0.0031
0.5	0, 0	0.5	0, 0	0.3469	0.0041*	0.3477	0.0045	0.3635	0.0514	0.2435*	-0.0534	0.3277	0.0255	0.3557	0.0105	0.3204	-0.0072
0.0	0.4, 0.7	0.5	0, 0	0.3443	0.0041	0.3124	0.0023*	0.3104	-0.0566	0.3050*	0.0028	0.3112	-0.0064	0.3130	0.0025	0.3124	0.0025
0.5	0.4, 0.7	0.5	0, 0	0.4470	0.1423	0.3447	-0.0017	0.3508	-0.0373	0.2661*	-0.0194	0.3325	0.0012	0.3512	0.0036	0.3299	0.0004*
0.0	0, 0	0.0	0.1, 0.8	0.2990	0.0020*	0.2993	0.0027	0.2999	-0.0177	0.2956	-0.0229	0.2865*	0.0354	0.2992	0.0027	0.2993	0.0027
0.5	0, 0	0.0	0.1, 0.8	0.0015*	0.0015*	0.3133	0.0024	0.3164	-0.0782	0.2442*	-0.2015	0.3277	-0.0150	0.3135	0.0034	0.3075	-0.0145
0.0	0.4, 0.7	0.0	0.1, 0.8	0.3140	0.2816	0.2989	0.0021	0.2912	0.1274	0.3032	0.1737	0.2454*	0.1169	0.2990	0.0019*	0.2989	0.0022
0.5	0.4, 0.7	0.0	0.1, 0.8	0.3857	0.5754	0.3117	-0.0163	0.3177	0.0838	0.2505*	-0.1030	0.3000	0.1109	0.3131	0.0022*	0.3114	0.0067
0.0	0, 0	0.5	0.1, 0.8	0.3128	0.0026*	0.3134	0.0034	0.3191	-0.0299	0.2743*	-0.0991	0.3244	-0.0094	0.3139	0.0070	0.3109	0.0031
0.5	0, 0	0.5	0.1, 0.8	0.3465	0.0027*	0.3477	0.0037	0.3620	-0.0624	0.2184*	-0.2358	0.3566	-0.0739	0.3566	0.0274	0.3158	-0.0412
0.0	0.4, 0.7	0.5	0.1, 0.8	0.3585	0.2828	0.3124	0.0023*	0.3112	0.0591	0.2881*	0.0200	0.3038	0.0554	0.3130	0.0040	0.3124	0.0025
0.5	0.4, 0.7	0.5	0.1, 0.8	0.4680	0.5776	0.3445	-0.0159	0.3584	0.0421	0.2365*	-0.1355	0.3584	0.0404	0.3504	0.0047*	0.3264	-0.0089
Average (absolute bias)				0.3453	0.1268	0.3177	0.0044*	0.3217	0.0425	0.2736*	0.0713	0.3129	0.0325	0.3198	0.0052	0.3110	0.0065
				0.2361	0.1611	0.1821				0.1724		0.1727		0.1625		0.1588*	

* Lowest bias for \mathbf{x}_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Table A8. Bias of indirect impacts with omv. $\beta = (0.2, 0.5)^\top$, $\gamma = (0.3, 0)^\top$.

ρ	δ	λ	θ	SLX		SAR		SAC		SDM		SDEM	
				x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
0.0	0, 0	0.0	0, 0	0.0064	0.0060	0.0032*	0.0031*	0.0176	0.0177	0.0067	0.0059	0.0065	0.0058
0.5	0, 0	0.0	0, 0	0.1588	-0.1251	0.2947	0.0113*	0.3220	0.0388	0.2989	0.0143	0.1165*	-0.1689
0.0	0.4, 0.7	0.0	0, 0	0.0055	0.0022	0.0018*	0.0011*	0.0083	0.0064	0.0057	0.0021	0.0055	0.0021
0.5	0.4, 0.7	0.0	0, 0	0.1858	-0.0696	0.2910	0.0058	0.3019	0.0143	0.2960	0.0058*	0.1471*	-0.1060
0.0	0, 0	0.5	0, 0	0.2131	0.0109	0.4112	0.4127	0.1437*	0.1455	0.2985	0.0137	0.1871	0.0077*
0.5	0, 0	0.5	0, 0	0.4664	-0.1164*	1.5406	1.2576	0.6997	0.4293	1.4320	0.4455	0.2638*	-0.2040
0.0	0.4, 0.7	0.5	0, 0	0.2286	0.0053	0.3459	0.2990	0.1003*	0.0930	0.2961	0.0061	0.2054	0.0045*
0.5	0.4, 0.7	0.5	0, 0	0.5404	-0.0639*	1.2476	0.7650	0.5493	0.2298	1.3194	0.2504	0.3272*	-0.1488
0.0	0, 0	0.0	0.1, 0.8	0.0064*	0.0060	0.0396	-0.6655	0.3985	-0.2498	0.0066	0.0068	0.0065	0.0058*
0.5	0, 0	0.0	0.1, 0.8	0.1050	-0.5605	0.4865	-1.1647	1.1389	-0.3839	0.2992	0.0162*	0.0445*	-0.7396
0.0	0.4, 0.7	0.0	0.1, 0.8	0.0055*	0.0022	0.1817	-0.4417	0.4020	-0.1071	0.0056	0.0024	0.0055	0.0021*
0.5	0.4, 0.7	0.0	0.1, 0.8	0.1439	-0.3093	0.8781	-0.4118	1.1915	0.1341	0.2960	0.0057*	0.0855*	-0.4725
0.0	0, 0	0.5	0.1, 0.8	0.2131	0.0109	0.4718	-0.2825	0.7329	-0.0184	0.3400	0.3834	0.1871*	0.0077*
0.5	0, 0	0.5	0.1, 0.8	0.4127	-0.5518	1.8377	0.0338*	1.6697	-0.1388	1.4996	1.5784	0.1775*	-0.8851
0.0	0.4, 0.7	0.5	0.1, 0.8	0.2286	0.0053	0.5274	-0.1162	0.6751	0.0508	0.3217	0.1799	0.2054*	0.0045*
0.5	0.4, 0.7	0.5	0.1, 0.8	0.4985	-0.3036*	1.8747	0.4823	1.8309	0.4265	1.2818	0.7785	0.2474*	-0.6565
Average (absolute bias)				0.2137	0.1343*	0.6521	0.3971	0.6364	0.1553	0.5002	0.2309	0.1387*	0.2138
				0.1740*		0.5246		0.3958		0.3656		0.1763	

* Lowest bias for \mathbf{x}_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Appendix B. Different combinations of autocorrelation

Table B1. Bias of direct impacts for different strengths of autocorrelation. $\beta = (0.2, 0.5)^\top$, $\gamma = (0, 0)^\top$, $\delta = (0, 0)^\top$, $\theta = (0.1, 0.8)^\top$.

ρ	λ	OLS		SLX		SAR		SEM		SAC		SDM		SDEM	
		x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2				
0.3	0.3	-0.0004	0.0022*	-0.0001*	0.0031	-0.0004	-0.0518	-0.0271	-0.1503	0.0050	-0.0142	0.0004	0.0068	-0.0017	-0.0055
0.5	0.3	-0.0002	0.0021*	0.0003	0.0030	-0.0000*	-0.0741	-0.0441	-0.2219	0.0055	-0.0464	0.0017	0.0119	-0.0065	-0.0300
0.7	0.3	-0.0000*	0.0014*	0.0007	0.0024	0.0009	-0.1104	-0.0711	-0.3317	0.0012	-0.1109	0.0048	0.0245	-0.0217	-0.0996
0.9	0.3	-0.0006*	-0.0037	0.0009	-0.0027*	-0.0080	-0.2325	-0.1360	-0.5993	-0.0270	-0.3120	0.0116	0.0494	-0.0761	-0.3347
0.3	0.5	0.0000*	0.0027*	0.0003	0.0036	0.0061	-0.0465	-0.0301	-0.1686	0.0088	-0.0334	0.0019	0.0152	-0.0026	-0.0111
0.5	0.5	0.0003*	0.0027*	0.0007	0.0036	0.0092	-0.0617	-0.0464	-0.2344	0.0075	-0.0718	0.0049	0.0280	-0.0086	-0.0410
0.7	0.5	0.0007	0.0024*	0.0013	0.0034	0.0142	-0.0871	-0.0725	-0.3389	-0.0005*	-0.1470	0.0119	0.0578	-0.0245	-0.1127
0.9	0.5	0.0005*	-0.0016	0.0018	-0.0006*	0.0039	-0.2090	-0.1364	-0.6015	-0.0356	-0.3655	0.0235	0.0998	-0.0773	-0.3407
0.3	0.7	0.0007*	0.0037*	0.0009	0.0045	0.0203	-0.0256	-0.0327	-0.1844	0.0166	-0.0452	0.0061	0.0381	-0.0037	-0.0174
0.5	0.7	0.0011*	0.0040*	0.0015	0.0049	0.0279	-0.0290	-0.0484	-0.2453	0.0129	-0.0899	0.0130	0.0680	-0.0106	-0.0510
0.7	0.7	0.0018	0.0043*	0.0023	0.0054	0.0363	-0.0438	-0.0737	-0.3452	0.0005*	-0.1747	0.0263	0.1230	-0.0265	-0.1226
0.9	0.7	0.0022*	0.0027*	0.0033	0.0037	0.0157	-0.1861	-0.1367	-0.6032	-0.0422	-0.4083	0.0360	0.1528	-0.0778	-0.3445
0.3	0.9	0.0021*	0.0069*	0.0021	0.0077	0.0599	0.0492	-0.0348	-0.1975	0.0430	-0.0172	0.0201	0.1160	-0.0047	-0.0232
0.5	0.9	0.0029*	0.0083*	0.0030	0.0092	0.0681	0.0507	-0.0499	-0.2540	0.0346	-0.0723	0.0328	0.1660	-0.0119	-0.0583
0.7	0.9	0.0042*	0.0110*	0.0044	0.0120	0.0654	0.0152	-0.0744	-0.3495	0.0148	-0.1717	0.0465	0.2142	-0.0274	-0.1284
0.9	0.9	0.0059*	0.0183*	0.0065	0.0191	0.0224	-0.1736	-0.1366	-0.6039	-0.0409	-0.4287	0.0449	0.1814	-0.0775	-0.3454
Average (absolute bias)		0.0015*	0.0049*	0.0019	0.0056	0.0224	0.0904	0.0719	0.3393	0.0185	0.1568	0.0179	0.0845	0.0287	0.1291
		0.0032*		0.0037			0.0564		0.2056		0.0877		0.0512		0.0789

* Lowest bias for x_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($x_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Table B2. Bias of indirect impacts for different strengths of autocorrelation. $\beta = (0.2, 0.5)^T$, $\gamma = (0, 0)^T$, $\delta = (0, 0)^T$, $\theta = (0.1, 0.8)^T$.

ρ	λ	SLX		SAR		SAC		SDM		SDEM	
		x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
0.3	0.3	-0.0260*	-0.1971	0.0890	-0.5762	0.2900	-0.0351*	0.0686	0.3528	-0.0420	-0.2831
0.5	0.3	-0.0978*	-0.5567	0.1863	-0.6482	0.3776	-0.1366*	0.1374	0.6562	-0.1525	-0.8284
0.7	0.3	-0.3245*	-1.6044	0.4664	-0.6929	0.4893	-0.6352*	0.3418	1.5363	-0.4851	-2.3461
0.9	0.3	-1.8563	-8.3520	0.7387	-3.6504*	-0.1147*	-6.0352	0.9216	3.9014	-2.3812	-10.6470
0.3	0.5	-0.0245*	-0.1936	0.2670	-0.1768	0.3690	0.0839*	0.1520	0.8308	-0.0477	-0.3147
0.5	0.5	-0.0958*	-0.5523	0.5008	0.0649*	0.4480	-0.0739	0.3238	1.6066	-0.1637	-0.8841
0.7	0.5	-0.3219*	-1.5977	1.0849	0.7313*	0.4992	-0.7792	0.8223	3.7944	-0.4983	-2.4074
0.9	0.5	-1.8524	-8.3378	1.4847	-1.9049*	-0.3411*	-6.8599	1.9225	8.3204	-2.3870	-10.6740
0.3	0.7	-0.0219*	-0.1870*	0.7086	0.8009	0.5559	0.4223	0.3612	2.0194	-0.0536	-0.3465
0.5	0.7	-0.0925*	-0.5435	1.2625	1.7761	0.6306	0.2161*	0.7878	3.9738	-0.1731	-0.9316
0.7	0.7	-0.3174*	-1.5845	2.2593	3.4158	0.6228	-0.6721*	1.7894	8.3603	-0.5078	-2.4529
0.9	0.7	-1.8458	-8.3083	2.2187	-0.2088*	-0.4630*	-7.3746	2.9508	12.8685	-2.3900	-10.6909
0.3	0.9	-0.0166*	-0.1651*	2.4503	4.6096	1.4205	2.1489	1.1525	6.6166	-0.0586	-0.3737
0.5	0.9	-0.0860*	-0.5139*	3.3502	6.4141	1.4331	1.7591	2.0378	10.4425	-0.1795	-0.9651
0.7	0.9	-0.3087*	-1.5387	3.9543	7.2462	1.2674	0.4890*	3.1944	15.0044	-0.5123	-2.4786
0.9	0.9	-1.8342	-8.2010	2.6037	0.6582*	-0.2434*	-7.1352	3.5207	15.1102	-2.3891	-10.6950
Average		0.5701*	2.6521	1.4766	2.0984*	0.5979	2.1785	1.2803	5.9622	0.7763	3.5824
(absolute bias)		1.6111		1.7875		1.3882*		3.6212		2.1794	

* Lowest bias for x_k within the parameter combination. Number of observations=900, repetitions=1000. ρ = autocorrelation in the dependent variable ($\mathbf{W}y$); δ = autocorrelation in the covariates ($x_k = f(\mathbf{W}x_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}u$); θ = spatial spillover effects of covariates ($\mathbf{W}X$); γ = strength of omv.

Appendix C. Lagrange multiplier tests

Table C1. Rejection rates of H_0 (Lagrange multiplier test). $\beta = (0.2, 0.5)^\top$, $\gamma = (0, 0)^\top$.

ρ	δ	λ	θ	LM_λ	LM_ρ	LM_λ^*	LM_ρ^*	$LM_{\lambda\rho}$
0.0	0, 0	0.0	0, 0	0.0410	0.0440	0.0520	0.0500	0.0580
0.4	0, 0	0.0	0, 0	1.0000	1.0000	0.0920	0.7750	1.0000
0.8	0, 0	0.0	0, 0	1.0000	1.0000	0.3880	0.9990	1.0000
0.0	0.4, 0.7	0.0	0, 0	0.0440	0.0520	0.0550	0.0590	0.0550
0.4	0.4, 0.7	0.0	0, 0	1.0000	1.0000	0.1340	0.9900	1.0000
0.8	0.4, 0.7	0.0	0, 0	1.0000	1.0000	0.9950	1.0000	1.0000
0.0	0, 0	0.4	0, 0	1.0000	1.0000	0.7190	0.1140	1.0000
0.4	0, 0	0.4	0, 0	1.0000	1.0000	0.8560	0.6830	1.0000
0.8	0, 0	0.4	0, 0	1.0000	1.0000	0.8380	0.9780	1.0000
0.0	0.4, 0.7	0.4	0, 0	1.0000	1.0000	0.9430	0.1100	1.0000
0.4	0.4, 0.7	0.4	0, 0	1.0000	1.0000	0.9970	0.9560	1.0000
0.8	0.4, 0.7	0.4	0, 0	1.0000	1.0000	1.0000	1.0000	1.0000
0.0	0, 0	0.8	0, 0	1.0000	1.0000	0.9990	0.2860	1.0000
0.4	0, 0	0.8	0, 0	1.0000	1.0000	0.9960	0.5190	1.0000
0.8	0, 0	0.8	0, 0	1.0000	1.0000	0.7450	0.7410	1.0000
0.0	0.4, 0.7	0.8	0, 0	1.0000	1.0000	1.0000	0.3000	1.0000
0.4	0.4, 0.7	0.8	0, 0	1.0000	1.0000	1.0000	0.7280	1.0000
0.8	0.4, 0.7	0.8	0, 0	1.0000	1.0000	1.0000	0.9410	1.0000
0.0	0, 0	0.0	0.1, 0.8	0.4240	0.9910	1.0000	1.0000	1.0000
0.4	0, 0	0.0	0.1, 0.8	1.0000	1.0000	1.0000	1.0000	1.0000
0.8	0, 0	0.0	0.1, 0.8	1.0000	1.0000	1.0000	1.0000	1.0000
0.0	0.4, 0.7	0.0	0.1, 0.8	0.8320	1.0000	1.0000	1.0000	1.0000
0.4	0.4, 0.7	0.0	0.1, 0.8	1.0000	1.0000	0.9960	1.0000	1.0000
0.8	0.4, 0.7	0.0	0.1, 0.8	1.0000	1.0000	0.4970	1.0000	1.0000
0.0	0, 0	0.4	0.1, 0.8	1.0000	1.0000	0.9900	1.0000	1.0000
0.4	0, 0	0.4	0.1, 0.8	1.0000	1.0000	0.9530	1.0000	1.0000
0.8	0, 0	0.4	0.1, 0.8	1.0000	1.0000	0.8520	1.0000	1.0000
0.0	0.4, 0.7	0.4	0.1, 0.8	1.0000	1.0000	0.9560	1.0000	1.0000
0.4	0.4, 0.7	0.4	0.1, 0.8	1.0000	1.0000	0.0930	1.0000	1.0000
0.8	0.4, 0.7	0.4	0.1, 0.8	1.0000	1.0000	0.9600	1.0000	1.0000
0.0	0, 0	0.8	0.1, 0.8	1.0000	1.0000	0.2590	0.9930	1.0000
0.4	0, 0	0.8	0.1, 0.8	1.0000	1.0000	0.2370	0.9940	1.0000
0.8	0, 0	0.8	0.1, 0.8	1.0000	1.0000	0.1530	0.9940	1.0000
0.0	0.4, 0.7	0.8	0.1, 0.8	1.0000	1.0000	0.9880	1.0000	1.0000
0.4	0.4, 0.7	0.8	0.1, 0.8	1.0000	1.0000	1.0000	1.0000	1.0000
0.8	0.4, 0.7	0.8	0.1, 0.8	1.0000	1.0000	1.0000	1.0000	1.0000

Number of observations=900, repetitions=1000. LM = Lagrange multiplier test, LM^* = Robust Lagrange multiplier test, each for $H_0: \lambda = 0$, $H_0: \rho = 0$, $H_0: \lambda, \rho = 0$. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

Appendix D. Different specifications of W

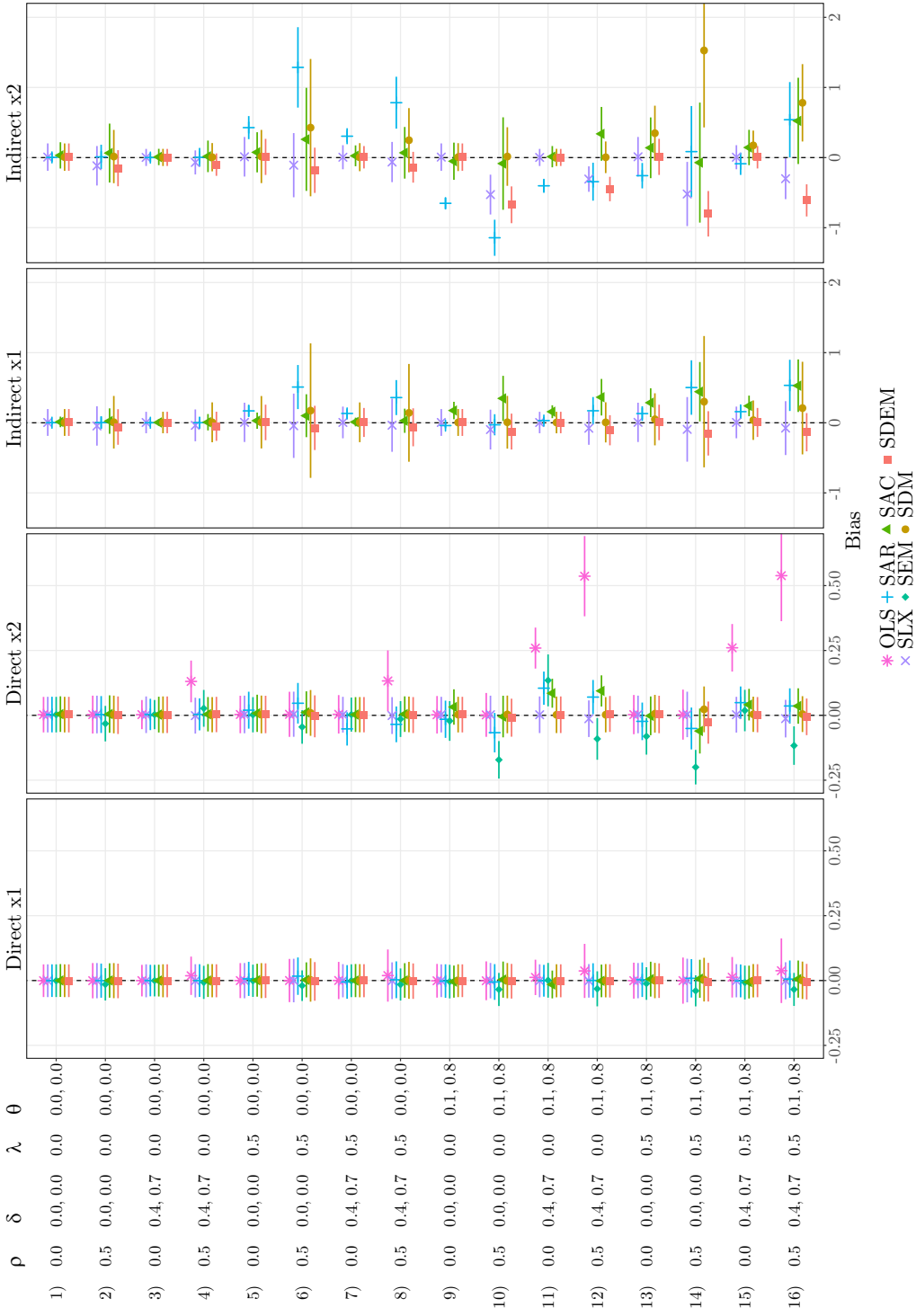


Figure D1. \mathbf{W} = 10-nearest neighbours, inverse distance weighted, row-normalized. Bias of impacts and 95% confidence interval of empirical standard deviation without omv: $\beta = (0.2, 0.5)^\top$, $\gamma = (0, 0)^\top$. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.

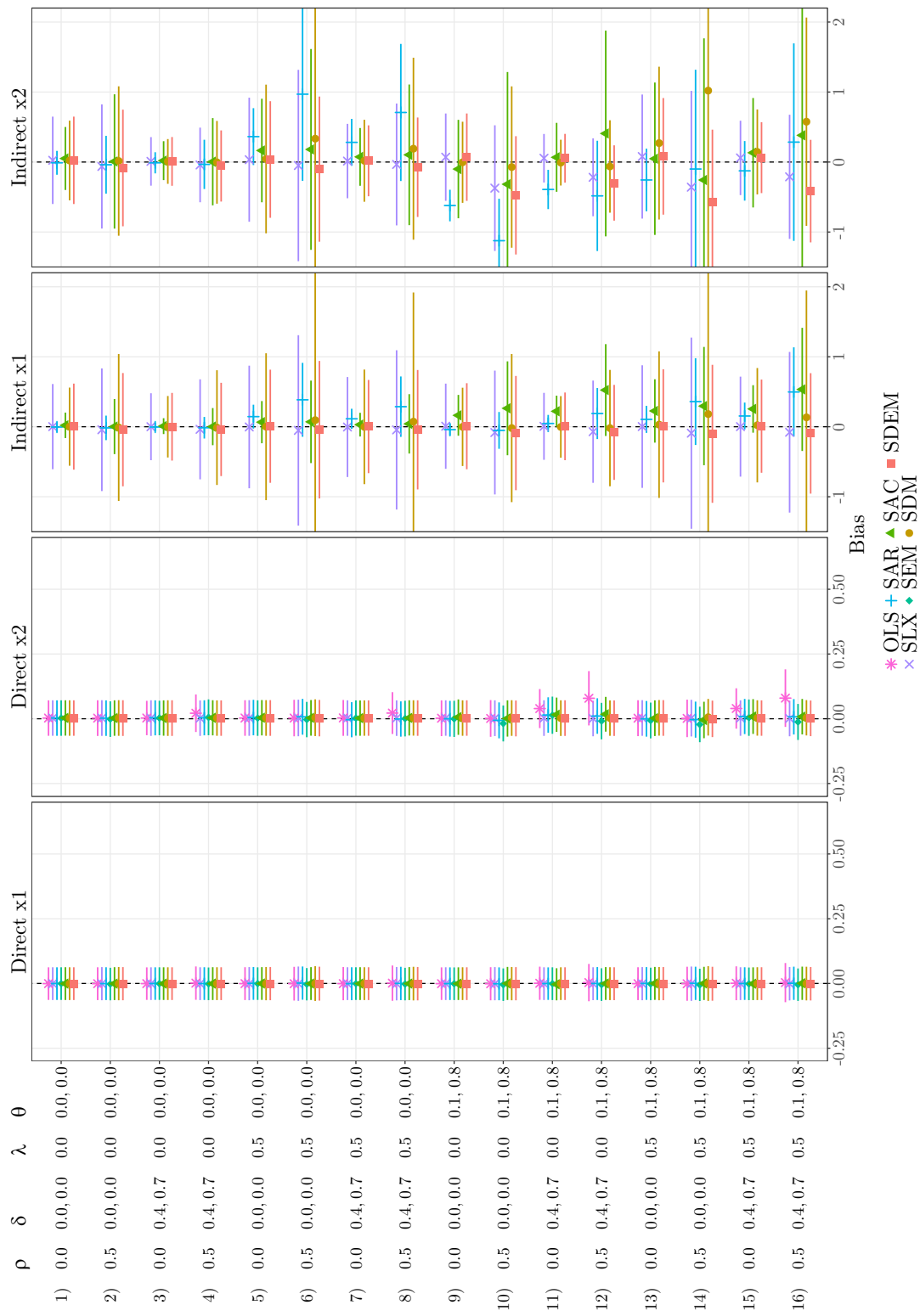


Figure D2. \mathbf{W} = inverse distance weighted neighbours (cut-off: 100 neighbours), maximum eigenvalue-normalized. Bias of impacts and 95% confidence interval of empirical standard deviation without omv: $\beta = (0.2, 0.5)^\top$, $\gamma = (0, 0)^\top$. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); γ = strength of omv.