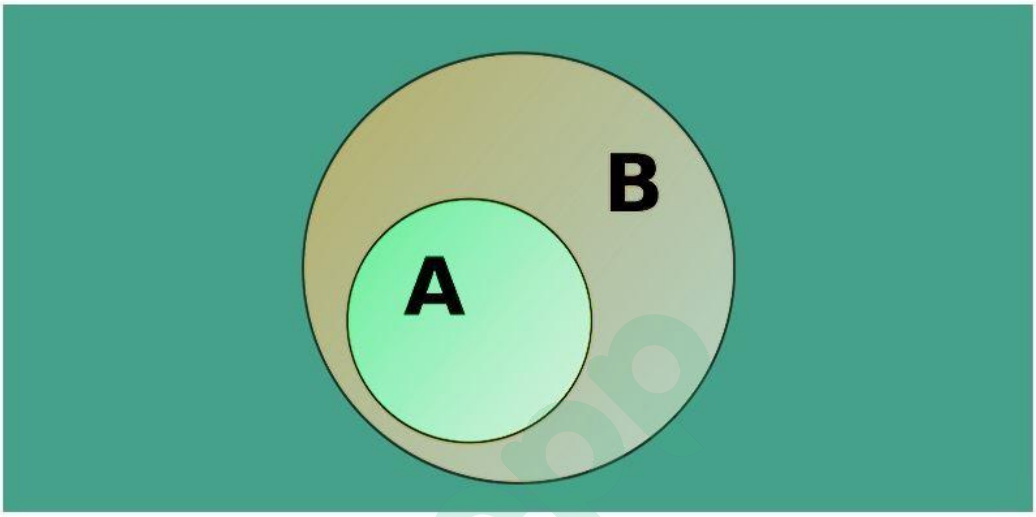
EVENTS IN PROBABILITY

What are Events in Probability?

A probability event can be defined as a set of outcomes of an experiment. In other words, an event in probability is the subset of the respective sample space.

The entire possible set of outcomes of a random experiment is the sample space or the individual space of that experiment. The likelihood of occurrence of an event is known as probability. The probability of occurrence of any event lies between 0 and 1.



The sample space for the tossing of three coins simultaneously is given by:

S = {(T , T , T) , (T , T , H) , (T , H , T) , (T , H , H) , (H , T , T) , (H , T , H) , (H , H, T) ,(H , H , H)}

Suppose, if we want to find only the outcomes which have at least two heads; then the set of all such possibilities can be given as:

E = { (H , T , H) , (H , H ,T) , (H , H ,H) , (T , H , H)}

Thus, **an event is a subset of the sample space, i.e., E is a subset of S**.

There could be a lot of events associated with a given sample space. For any event to occur, the outcome of the experiment must be an element of the set of event E.

What is the Probability of Occurrence of an Event?

The number of favourable outcomes to the total number of outcomes is defined as the probability of occurrence of any event. So, the probability that an event will occur is given as:

P(E) = Number of Favourable Outcomes/ Total Number of Outcomes

Types of Events in Probability

Impossible and Sure Events

If the probability of occurrence of an event is 0, such an event is called an **impossible event** and if the probability of occurrence of an event is 1, it is called a **sure event**. In other words, the empty set ϕ is an impossible event and the sample space S is a sure event.

Simple Events

Any event consisting of a single point of the sample space is known as a **simple event** in probability. For example, if S = {56 , 78 , 96 , 54 , 89} and E = {78} then E is a simple event.

Compound Events

Contrary to the simple event, if any event consists of more than one single point of the sample space then such an event is called a **compound event**. Considering the same example again, if S = {56 ,78 ,96 ,54 ,89}, E_1 = {56 ,54 }, E_2 = {78 ,56 ,89 } then, E_1 and E_2 represent two compound events.

Independent Events and Dependent Events

If the occurrence of any event is completely unaffected by the occurrence of any other event, such events are known as an **independent event** in probability and the events which are affected by other events are known as **dependent events**.

If the probability of occurrence of an event A is not affected by the occurrence of another event B, then A and B are said to be independent events.

Consider an example of rolling a die. If A is the event 'the number appearing is odd' and B be the event 'the number appearing is a multiple of 3', then

$P(A) = 3/6 = 1/2$ and $P(B) = 2/6 = 1/3$

Also A and B is the event 'the number appearing is odd and a multiple of 3' so that

$P(A \cap B) = 1/6$

$P(A | B) = P(A \cap B) / P(B)$

$= \frac{\frac{1}{6}}{\frac{1}{3}} = 12$

$P(A) = P(A | B) = 1/2$, which implies that the occurrence of event B has not affected the probability of occurrence of the event A .

If A and B are independent events, then $P(A | B) = P(A)$

Using Multiplication rule of probability, $P(A \cap B) = P(B) .P(A | B)$

$P(A \cap B) = P(B) .P(A)$

Note: A and B are two events associated with the same random experiment, then A and B are known as independent events if $P(A \cap B) = P(B) .P(A)$

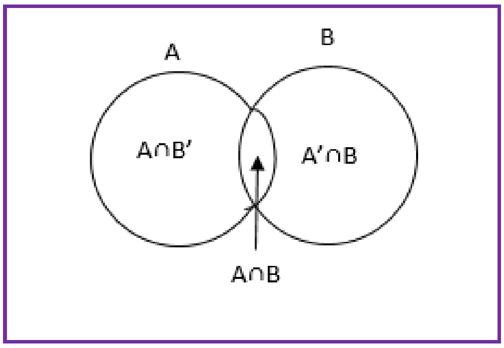
Independent Events Venn Diagram

Let us proof the condition of independent events using a Venn diagram.

Theorem: If X and Y are independent events, then the events X and Y’ are also independent.

Proof: The events A and B are independent, so, $P(X \cap Y) = P(X)\,P(Y)$.

Let us draw a Venn diagram for this condition:



From the Venn diagram, we see that the events $X \cap Y$ and $X \cap Y'$ are mutually exclusive and together they form the event X.

$X = (X \cap Y) \cup (X \cap Y')$

Also, $P(X) = P[(X \cap Y) \cup (X \cap Y')]$ or, $P(X) = P(X \cap Y) + P(X \cap Y')$

or, $P(X) = P(X)\,P(Y) + P(X \cap Y')$

or, P(X inter Y’) $= P(X) - P(X)\,P(Y) = P(X)\,(1 - P(Y)) = P(X)\,P(Y')$

Example: Let X and Y are two independent events such that P(X) = 0.3 and P(Y) = 0.7. Find P(X and Y), P(X or Y), P(Y not X), and P(neither X nor Y).

Solution: Given $P(X) = 0.3$ and $P(Y) = 0.7$ and events X and Y are independent of each other.

P(X and Y) $= P(X \cap Y) = P(X)\,P(Y) = 0.3 \times 0.7 = 0.21$

P(X or Y) $= P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = 0.3 + 0.7 - 0.21 = 0.79$

P(Y not X) $= P(Y \cap X') = P(Y) - P(X \cap Y) = 0.7 - 0.21 = 0.49$

And P(neither X nor Y) $= P(X' \cap Y') = 1 - P(X \cup Y) = 1 - 0.79 = 0.21$

Mutually Exclusive Events

If the occurrence of one event excludes the occurrence of another event, such events are mutually **exclusive events** i.e. two events don’t have any common point. For example, if S = {1 , 2 , 3 , 4 , 5 , 6} and E_1, E_2 are two events such that E_1 consists of numbers less than 3 and E_2 consists of numbers greater than 4.

So, E1 = {1,2} and E2 = {5,6}

Then, E1 and E2 are mutually exclusive.

In probability, the specific addition rule is valid when two events are mutually exclusive. It states that the probability of either event occurring is the sum of probabilities of each event occurring. If A and B are said to be mutually exclusive events then the probability of an event A occurring or the probability of event B occurring is given as P(A) + P(B), i.e.,

P (A or B) = P(A) + P(B)

Some of the examples of the mutually exclusive events are:

- When tossing a coin, the event of getting head and tail are mutually exclusive. Because the probability of getting head and tail simultaneously is 0.
- In a six-sided die, the events “2” and “5” are mutually exclusive. We cannot get both the events 2 and 5 at the same time when we threw one die.
- In a deck of 52 cards, drawing a red card and drawing a club are mutually exclusive events because all the clubs are black.

If the events A and B are not mutually exclusive, the probability of getting A or B is given as:

P (A or B) = P(A) + P(B) – P (A and B)

Mutually Exclusive Events Probability Rules

In probability theory, two events are mutually exclusive or disjoint if they do not occur at the same time. A clear case is the set of results of a single coin toss, which can end in either heads or tails, but not for both. While tossing the coin, both outcomes are collectively exhaustive, which suggests that at least one of the consequences must happen, so these two possibilities collectively exhaust all the possibilities. Though, not all mutually exclusive events are commonly exhaustive. For example, the outcomes 1 and 4 of a six-sided die, when we throw it, are mutually exclusive (both 1 and 4 cannot come as result at the same time) but not collectively exhaustive (it can result to distinct outcomes such as 2,3,5,6).

From the definition of mutually exclusive events, certain rules for the probability are concluded.

Addition Rule: $P(A + B) = 1$

Subtraction Rule: $P(A \cup B)' = 0$

Multiplication Rule: $P(A \cap B) = 0$

There are different varieties of events also. For instance, think a coin that has a Head on both the sides of the coin or a Tail on both sides. It doesn’t matter how many times you flip it, it will always occur Head (for the first coin) and Tail (for the second coin). If we check the sample space of such experiment, it will be either { H } for the first coin and { T } for the second one. Such events have single point in the sample space and are called **“Simple Events”**. Such kind of two sample events is always mutually exclusive.

Conditional Probability for Mutually Exclusive Events

Conditional probability is stated as the probability of an event A, given that another event B has occurred. Conditional Probability for two independent events B has given A is denoted by the expression P(B|A) and it is defined using the equation

$P(B|A) = P(A \cap B)/P(A)$

Redefine the above equation using multiplication rule: $P(A \cap B) = 0$

$P(B|A) = 0/P(A)$

So the conditional probability formula for mutually exclusive events is:

$P(B \mid A) = 0$

Example: What is the probability of a die showing a number 3 or number 5?

Solution: Let,

P(3) is the probability of getting a number 3

P(5) is the probability of getting a number 5

$P(3) = 1/6$ and $P(5) = 1/6$

So,

$P(3 \text{ or } 5) = P(3) + P(5)$

$P(3 \text{ or } 5) = (1/6) + (1/6) = 2/6$

$P(3 \text{ or } 5) = 1/3$

Therefore, the probability of a die showing 3 or 5 is 1/3.

Example: Three coins are tossed at the same time. We say A as the event of receiving at least 2 heads. Likewise, B denotes the event of getting no heads and C is the event of getting heads on the second coin. Which of these is mutually exclusive?

Solution: Firstly, let us create a sample space for each event. For the event 'A' we have to get at least two head. Therefore, we have to include all the events that have two or more heads.

Or we can write:

A = {HHT, HTH, THH, HHH}.

This set A has 4 elements or events in it i.e. n(A) = 4

In the same way, for event B, we can write the sample as:

B = {TTT} and n(B) = 1

Again using the same logic, we can write;

C = {THT, HHH, HHT, THH} and n(C) = 4

So B & C and A & B are mutually exclusive since they have nothing in their intersection.

Exhaustive Events

A set of events is called **exhaustive** if all the events together consume the entire sample space.

Complementary Events

For any event E_1 there exists another event E_1' which represents the remaining elements of the sample space S.

$E_1 = S - E_1'$

If a dice is rolled then the sample space S is given as S = {1 , 2 , 3 , 4 , 5 , 6 }. If event E_1 represents all the outcomes which is greater than 4, then E_1 = {5, 6} and E_1' = {1, 2, 3, 4}.

Thus E_1' is the complement of the event E_1.

Similarly, the complement of E_1, E_2, E_3………E_n will be represented as E_1', E_2', E_3'………E_n'

Events Associated with "OR"

If two events E_1 and E_2 are associated with **OR** then it means that either E_1 or E_2 or both. The union symbol **(U)** is used to represent OR in probability.

Thus, the event $E_1 \cup E_2$ denotes E_1 OR E_2.

If we have mutually exhaustive events E_1, E_2, E_3………E_n associated with sample space S then,

$E_1 \cup E_2 \cup E_3 \cup \ldots\ldots\ldots E_n = S$

Events Associated with "AND"

If two events E_1 and E_2 are associated with **AND** then it means the intersection of elements which is common to both the events. The intersection symbol **(∩)** is used to represent AND in probability.

Thus, the event $E_1 \cap E_2$ denotes E_1 and E_2.

Event E_1 but not E_2

It represents the difference between both the events. Event E_1 but not E_2 represents all the outcomes which are present in E_1 but not in E_2. Thus, the event E_1 but not E_2 is represented as

$E_1, E_2 = E_1 - E_2$

Solved Example

Q1. In the game of snakes and ladders, a fair die is thrown. If event E_1 represents all the events of getting a natural number less than 4, event E_2 consists of all the events of getting an even number and E_3 denotes all the events of getting an odd number. List the sets representing the following:

i) E_1 or E_2 or E_3

ii) E_1 and E_2 and E_3

iii) E_1 but not E_3

Solution:

The sample space is given as S = {1 , 2 , 3 , 4 , 5 , 6}

E_1 = {1,2,3}

E_2 = {2,4,6}

E_3 = {1,3,5}

i) E_1 or E_2 or E_3= E_1 E_2 E_3= {1, 2, 3, 4, 5, 6}

ii) E_1 and E_2 and E_3 = E_1 E_2 E_3 = ∅

iii) E_1 but not E_3 = {2}