KMeans_PCA.ipynb
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You may use the MNIST dataset or any dataset for Face Images or Flower Images for this assignment.

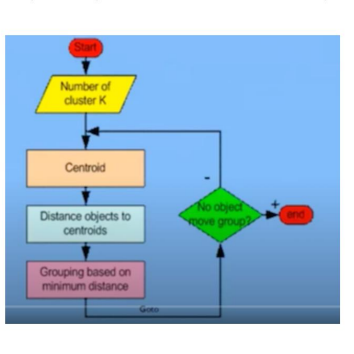
TASK 1) Implement k-means clustering. Analyse the clusters formed for various values of k. Display the centroids of the clusters. DO NOT USE IN_BUILT ROUTINE for k-means clustering.

2) Implement Dimensionality reduction using PCA. Analyse the reconstruction error for various values of k. Display the Eigen Vectors. DO NOT USE IN_BUILT ROUTINE for implementing PCA. However you can use in-built routines for computing Eigen vectors and Eigen values.

Note that when you apply PCA on images, you are dealing with data with a very high dimensionality, i.e. d>m, where m is the number of images in the training dataset. Therefore you need to apply the technique given in Section 23.1.1. of the Textbook.

Further, for images, you have the red, green and blue components for the color at every pixel location. You can simply consider the average of the three color components. This average is the intensity value at the pixel. The feature vector is constructed using the intensity values of the all the pixels in the image.

Prepare a report with all the results and steps of implementation.



Task 1 :

1. Kmeans clustering
2. Clusters formed for diff K
3. centroids of the clusters

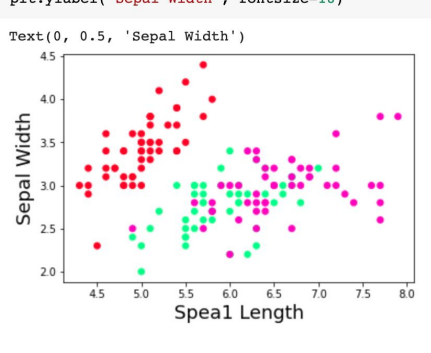
Dissimilar examples should be part of different clusters, Similar examples should be part of same cluster. K means tries to put the data into the number of clusters we tell it to. For a given K, randomly chose k data points to be the initial clyster centers, Assign each data point to each clyster center, Re Compute the cluster centers with previous clustering. If converging criteria does not met repeat again.

K means works through the following iterative process: Pick a value for k (the number of clusters to create) Initialize k 'centroids' (starting points) in your data Create your clusters. Assign each point to the nearest centroid. Make your clusters better. Move each centroid to the center of its cluster. Repeat steps 3–4 until your centroids converge.

Using Inbuilt Functions

```
[44] from sklearn import datasets
     import matplotlib.pyplot as plt
     import pandas as pd
     from sklearn.cluster import KMeans
     iris = datasets.load_iris()
     X = iris.data[:, :2]
     y = iris.target
     plt.scatter(X[:,0], X[:,1], c=y, cmap='gist_rainbow')
     plt.xlabel('Speal Length', fontsize=18)
     plt.ylabel('Sepal Width', fontsize=18)
```

```
Text(0, 0.5, 'Sepal Width')
```




```
[45] km = KMeans(n_clusters = 3, n_jobs = 4, random_state=21)
     km.fit(X)
```

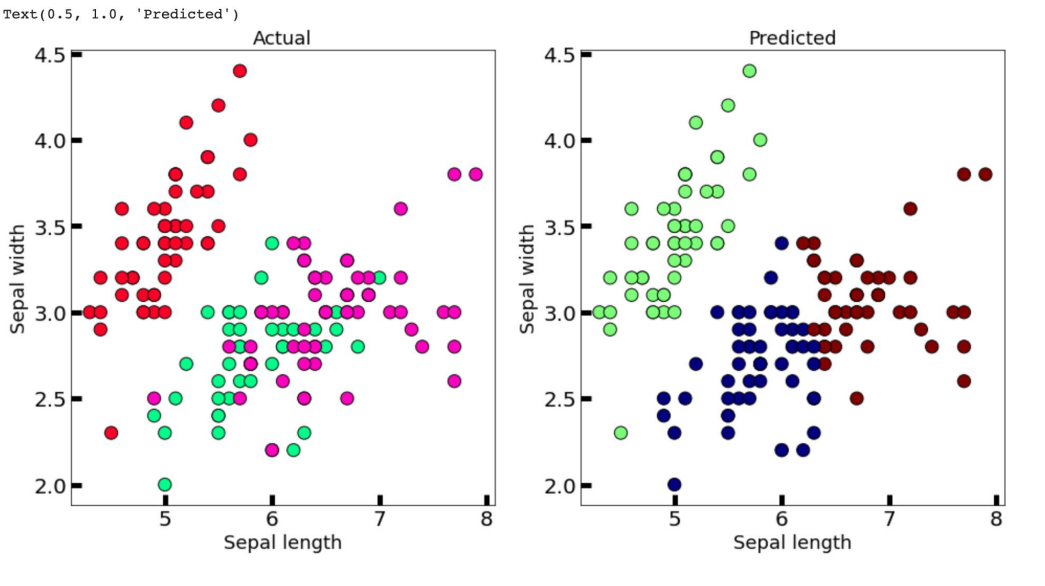
```
KMeans(algorithm='auto', copy_x=True, init='k-means++', max_iter=300,
       n_clusters=3, n_init=10, n_jobs=4, precompute_distances='auto',
       random_state=21, tol=0.0001, verbose=0)
```

```
[47] centers = km.cluster_centers_
     print(centers)
```

```
[[5.77358491 2.69245283]
 [5.006      3.428     ]
 [6.81276596 3.07446809]]
```

```
[48] new_labels = km.labels_
     # Plot the identified clusters and compare with the answers
     fig, axes = plt.subplots(1, 2, figsize=(16,8))
     axes[0].scatter(X[:, 0], X[:, 1], c=y, cmap='gist_rainbow',edgecolor='k', s=150)
     axes[1].scatter(X[:, 0], X[:, 1], c=new_labels, cmap='jet',edgecolor='k', s=150)
     axes[0].set_xlabel('Sepal length', fontsize=18)
     axes[0].set_ylabel('Sepal width', fontsize=18)
     axes[1].set_xlabel('Sepal length', fontsize=18)
     axes[1].set_ylabel('Sepal width', fontsize=18)
     axes[0].tick_params(direction='in', length=10, width=5, colors='k', labelsize=20)
     axes[1].tick_params(direction='in', length=10, width=5, colors='k', labelsize=20)
     axes[0].set_title('Actual', fontsize=18)
     axes[1].set_title('Predicted', fontsize=18)
```

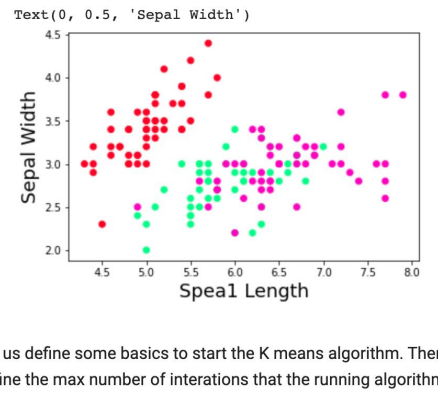
```
Text(0.5, 1.0, 'Predicted')
```



Not using Inbuilt Functions

```
[118] from sklearn import datasets
      import matplotlib.pyplot as plt
      import pandas as pd
      from sklearn.cluster import KMeans
      iris = datasets.load_iris()
      X = iris.data[:, :2]
      y = iris.target
      plt.scatter(X[:,0], X[:,1], c=y, cmap='gist_rainbow')
      plt.xlabel('Speal Length', fontsize=18)
      plt.ylabel('Sepal Width', fontsize=18)
```

```
Text(0, 0.5, 'Sepal Width')
```

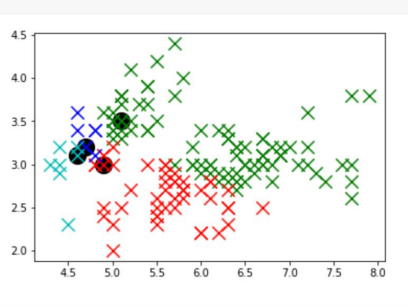


Let us define some basics to start the K means algorithm. There is a Threshold (theta) for movement of centroid in K means, total iterations define the max number of interations that the running algorithm takes place. The train/fit and predict functions will give the functionality of the K Means. The underlying difference between K Means and General Classification algorithms is that, in K Means we just focus whether we achieved the clusters which we targeted however in other clsutering algorithms we focus on more of finding how accurate the algorithm worked. Since it is unsupervised we have to use only training.

```
[119] k=4
      theta = 0.01
      tot_iter = 80
      colors = 10*["g", "r", "b", "c", "k"]

      def fit(data):
        centroids = {}
        for i in range(k):
          centroids[i]= data[i]
          # That is first k=4 centroids will be the starting k=4 of the dataset
        for i in range(tot_iter):
          labels = {}
          for i in range(k):
            labels[i] = []
          # feature set in data
          for fset in X:
            distances = [np.linalg.norm(fset-centroids[centroid]) for centroid in centroids]
            # Creating a list of distances where 0th element of the list will be
            # the distance to the 0th centroid with data elements.
            label = distances.index(min(distances)) # Classification
            labels[label].append(fset) # Says that feature set belongs to that centroid
          prev_centroids = dict(centroids)
          for label in labels:
            pass
            # centroids[label] = np.average(labels[label], axis=0)
            # Taking avergae of all the labels we have and assigning the centroid with that label
            # Finds the centroid for previous centroids labels
            # Finds the mean of all the features for any given class and redefines the centroid
          converged = True
          for clabel in centroids:
            original_centroid = prev_centroids[clabel]
            current_centroid = centroids[clabel]
            # Checking for the threshold theta by which the centroid should take movements
            if np.sum((current_centroid - original_centroid)/original_centroid * 100) > theta:
              converged = False
          if converged:
            break
        for centroid in centroids:
          plt.scatter(centroids[centroid][0], centroids[centroid][1], marker="o", color="k", s=150, linewidths=5)

        for label in labels:
          color = colors[label]
          for fset in labels[label]:
            plt.scatter(fset[0], fset[1], marker="x", color=color, s=150, linewidths=5)
      def predict(data):
        distances = [np.linalg.norm(data-centroids[centroid]) for centroid in centroids]
        label = distances.index(min(distance))
        return label
```

OLD CENTROIDS

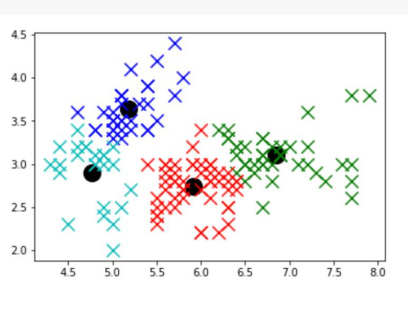
```
[94] fit(X)
```



```
[120] k=4
      theta = 0.01
      tot_iter = 80
      colors = 10*["g", "r", "b", "c", "k"]

      def fit(data):
        centroids = {}
        for i in range(k):
          centroids[i]= data[i]
          # That is first k=4 centroids will be the starting k=4 of the dataset
        for i in range(tot_iter):
          labels = {}
          for i in range(k):
            labels[i] = []
          # feature set in data
          for fset in X:
            distances = [np.linalg.norm(fset-centroids[centroid]) for centroid in centroids]
            # Creating a list of distances where 0th element of the list will be
            # the distance to the 0th centroid with data elements.
            label = distances.index(min(distances)) # Classification
            labels[label].append(fset) # Says that feature set belongs to that centroid
          prev_centroids = dict(centroids)
          for label in labels:
            # pass
            centroids[label] = np.average(labels[label], axis=0)
            # Taking avergae of all the labels we have and assigning the centroid with that label
            # Finds the centroid for previous centroids labels
            # Finds the mean of all the features for any given class and redefines the centroid
          converged = True
          for clabel in centroids:
            original_centroid = prev_centroids[clabel]
            current_centroid = centroids[clabel]
            # Checking for the threshold theta by which the centroid should take movements
            if np.sum((current_centroid - original_centroid)/original_centroid * 100) > theta:
              converged = False
          if converged:
            break
        for centroid in centroids:
          plt.scatter(centroids[centroid][0], centroids[centroid][1], marker="o", color="k", s=150, linewidths=5)

        for label in labels:
          color = colors[label]
          for fset in labels[label]:
            plt.scatter(fset[0], fset[1], marker="x", color=color, s=150, linewidths=5)
```

NEW CENTROIDS

```
[121] fit(X)
```



Note that some points which are classified green previouslt, are now classified according to the nearest centroid

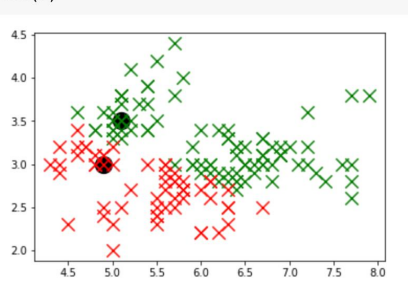
Lets try with smaller and larger cluster for values k=2 and k=6

K=2

[ ] ↳ 1 cell hidden

K=2 plot

```
[99] fit(X)
```

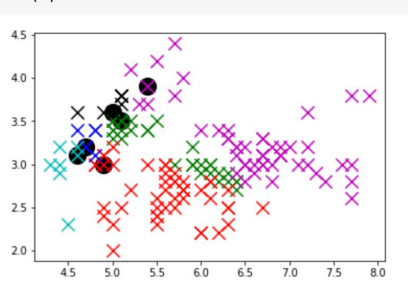


K=6

[ ] ↳ 1 cell hidden

K=6 Plot

```
[105] fit(X)
```



Note that for K=6 it is wrose than k=4

K-Means : A Try with MNIST dataset

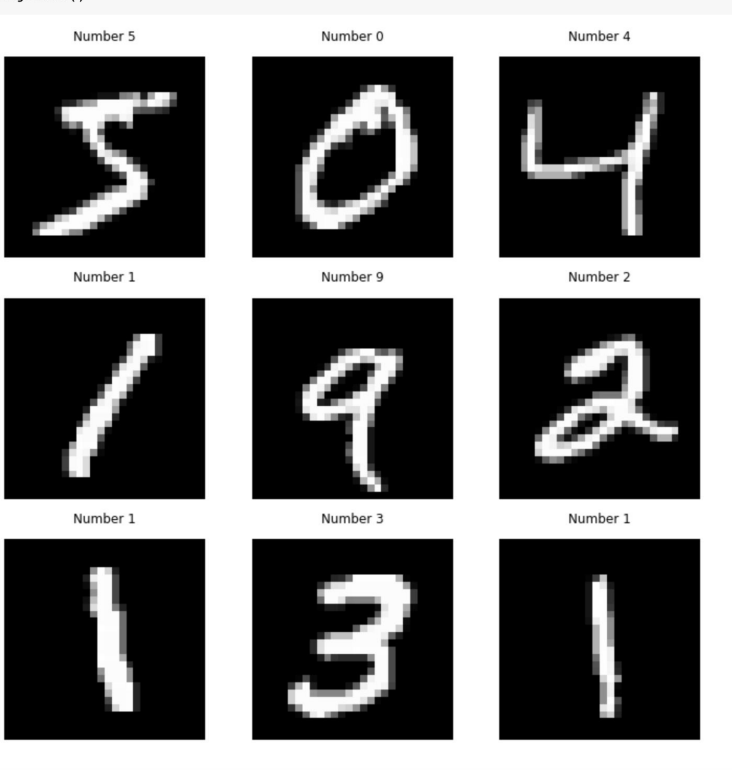
```
[223] # Import the MNIST dataset
      from keras.datasets import mnist

      (x_train, y_train), (x_test, y_test) = mnist.load_data()

      print('Training Data: {}'.format(x_train.shape))
      print('Training Labels: {}'.format(y_train.shape))
```

```
Training Data: (60000, 28, 28)
Training Labels: (60000,)
```

```
[224] print('Testing Data: {}'.format(x_test.shape))
      print('Testing Labels: {}'.format(y_test.shape))
```

```
Testing Data: (10000, 28, 28)
Testing Labels: (10000,)
```

```
[225] fig, axs = plt.subplots(3, 3, figsize = (12, 12))
      plt.gray()

      for i, ax in enumerate(axs.flat):
        ax.matshow(x_train[i])
        ax.axis('off')
        ax.set_title('Number {}'.format(y_train[i]))

      fig.show()
```




```
[226] # convert each image to 1 dimensional array
```

```
[227] X = x_train.reshape(len(x_train),-1)
      Y = y_train

      # normalize the data to 0 - 1 One hot

      X = X.astype(float) / 255.

      print(X.shape)
      print(X[0].shape)
```

```
(60000, 784)
(784,)
```

Getting Failed when trying to run method fit(X) which is defined in earlier sections

The Error is expected to come since it is very large computation

```
[137] k=10
      theta = 0.01
      tot_iter = 80
      colors = 10*["g", "r", "b", "c", "k", "m", "y", "w"]

      def fit(data):
        centroids = {}
        for i in range(k):
          centroids[i]= data[i]
          # That is first k=4 centroids will be the starting k=4 of the dataset
        for i in range(tot_iter):
          labels = {}
          for i in range(k):
            labels[i] = []
          # feature set in data
          for fset in X:
            distances = [np.linalg.norm(fset-centroids[centroid]) for centroid in centroids]
            # Creating a list of distances where 0th element of the list will be
            # the distance to the 0th centroid with data elements.
            label = distances.index(min(distances)) # Classification
            labels[label].append(fset) # Says that feature set belongs to that centroid
          prev_centroids = dict(centroids)
          for label in labels:
            # pass
            centroids[label] = np.average(labels[label], axis=0)
            # Taking avergae of all the labels we have and assigning the centroid with that label
            # Finds the centroid for previous centroids labels
            # Finds the mean of all the features for any given class and redefines the centroid
          converged = True
          for clabel in centroids:
            original_centroid = prev_centroids[clabel]
            current_centroid = centroids[clabel]
            # Checking for the threshold theta by which the centroid should take movements
            if np.sum((current_centroid - original_centroid)/original_centroid * 100) > theta:
              converged = False
          if converged:
            break
        for centroid in centroids:
          plt.scatter(centroids[centroid][0], centroids[centroid][1], marker="o", color="k", s=150, linewidths=5)

        for label in labels:
          color = colors[label]
          for fset in labels[label]:
            plt.scatter(fset[0], fset[1], marker="x", color=color, s=150, linewidths=5)
```

```
[140] fit(X)
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:34: RuntimeWarning: divide by zero encountered in true_divide
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:34: RuntimeWarning: invalid value encountered in true_divide
---------------------------------------------------------------------------
KeyboardInterrupt                         Traceback (most recent call last)
/usr/local/lib/python3.6/dist-packages/zmq/backend/cython/checkrc.pxd in zmq.backend.cython.checkrc._check_rc()

KeyboardInterrupt:
```

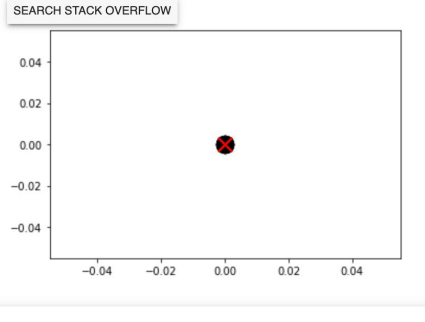
SEARCH STACK OVERFLOW

```
Exception ignored in: 'zmq.backend.cython.message.Frame.__dealloc__'
Traceback (most recent call last):
  File "zmq/backend/cython/checkrc.pxd", line 13, in zmq.backend.cython.checkrc._check_rc
KeyboardInterrupt
---------------------------------------------------------------------------
KeyboardInterrupt                         Traceback (most recent call last)
<ipython-input-140-76392c5d9b19> in <module>()
----> 1 fit(X)
```

9 frames

```
/usr/local/lib/python3.6/dist-packages/matplotlib/artist.py in sticky_edges(self)
   1050         self.stale = True
   1051
-> 1052     @property
   1053     def sticky_edges(self):
   1054         """

KeyboardInterrupt:
```

SEARCH STACK OVERFLOW



Use MiniBatchKmeans

Due to the size of the MNIST dataset, we will use the mini-batch implementation of k-means clustering provided by scikit-learn. This will dramatically reduce the amount of time it takes to fit the algorithm to the data. Ref: https://medium.com/datadriveninvestor/k-means-clustering-for-imagery-analysis-56c9976f16b6

```
[146] from sklearn.cluster import MiniBatchKMeans

      n_digits = len(np.unique(y_test))
      print(n_digits)

      # Initialize KMeans model

      kmeans = MiniBatchKMeans(n_clusters = n_digits)

      # Fit the model to the training data

      kmeans.fit(X)

      kmeans.labels_
```

```
10
array([6, 7, 3, ..., 6, 5, 9], dtype=int32)
```

```
[152] def infer_cluster_labels(kmeans, actual_labels):
        inferred_labels = {}

        for i in range(kmeans.n_clusters):

            # find index of points in cluster
            labels = []
            index = np.where(kmeans.labels_ == i)

            # append actual labels for each point in cluster
            labels.append(actual_labels[index])

            # determine most common label
            if len(labels[0]) == 1:
                counts = np.bincount(labels[0])
            else:
                counts = np.bincount(np.squeeze(labels))

            # assign the cluster to a value in the inferred_labels dictionary
            if np.argmax(counts) in inferred_labels:
                # append the new number to the existing array at this slot
                inferred_labels[np.argmax(counts)].append(i)
            else:
                # create a new array in this slot
                inferred_labels[np.argmax(counts)] = [i]

            #print(labels)
            #print('Cluster: {}, label: {}'.format(i, np.argmax(counts)))

        return inferred_labels
```

```
[153]
      def infer_data_labels(X_labels, cluster_labels):
          # empty array of len(X)
        predicted_labels = np.zeros(len(X_labels)).astype(np.uint8)

        for i, cluster in enumerate(X_labels):
            for key, value in cluster_labels.items():
                if cluster in value:
                    predicted_labels[i] = key

        return predicted_labels
```

```
[155]
      cluster_labels = infer_cluster_labels(kmeans, Y)
      X_clusters = kmeans.predict(X)
      predicted_labels = infer_data_labels(X_clusters, cluster_labels)
      print (predicted_labels[:20])
      print (Y[:20])
```

```
[7 0 4 1 7 2 1 2 1 6 3 1 3 6 1 7 2 1 6 7]
[5 0 4 1 9 2 1 3 1 4 3 5 3 6 1 7 2 8 6 9]
```

```
[156] kmeans = MiniBatchKMeans(n_clusters = 36)
      kmeans.fit(X)

      # record centroid values
      centroids = kmeans.cluster_centers_

      # reshape centroids into images
      images = centroids.reshape(36, 28, 28)
      images *= 255
      images = images.astype(np.uint8)

      # determine cluster labels
      cluster_labels = infer_cluster_labels(kmeans, Y)

      # create figure with subplots using matplotlib.pyplot
      fig, axs = plt.subplots(6, 6, figsize = (20, 20))
      plt.gray()

      # loop through subplots and add centroid images
      for i, ax in enumerate(axs.flat):

          # determine inferred label using cluster_labels dictionary
          for key, value in cluster_labels.items():
              if i in value:
                  ax.set_title('Inferred Label: {}'.format(key))

          # add image to subplot
          ax.matshow(images[i])
          ax.axis('off')

      # display the figure
      fig.show()
```

[ ]

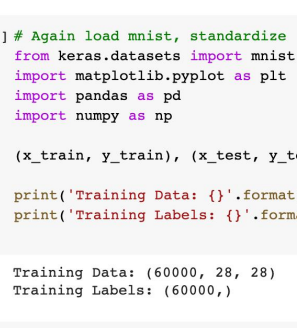
PCA

```
[200] # Again load mnist, standardize
      from keras.datasets import mnist
      import matplotlib.pyplot as plt
      import pandas as pd
      import numpy as np

      (x_train, y_train), (x_test, y_test) = mnist.load_data()

      print('Training Data: {}'.format(x_train.shape))
      print('Training Labels: {}'.format(y_train.shape))
```

```
Training Data: (60000, 28, 28)
Training Labels: (60000,)
```

```
[201] plt.imshow(x_train[2])
      print(y_train[2])
```

```
4
```



```
[202] X = x_train.reshape(len(x_train),-1)
      Y = y_train

      # normalize the data to 0 - 1 One hot

      X = X.astype(float) / 255.

      print(X.shape)
      print(X[0].shape)
      print(Y.shape)
```

```
(60000, 784)
(784,)
(60000,)
```

```
[203] new_labels = Y[0:15000]
      new_data = X[0:15000]
      print('the shape of sample data: '+ str(new_data.shape))
```

```
      the shape of sample data: (15000, 784)
```

```
[204] covar_matrix = np.matmul(new_data.T, new_data)
      print(covar_matrix.shape)
```

```
      (784, 784)
```

```
[205] # From scipy.linearalgebra we can have pre built libs of eigen value and eigen vectors
      from scipy.linalg import eigh
      # Higest eigen values and vectors
      # eigh returns values in ascending order
      values, vectors = eigh(covar_matrix, eigvals=(782,783))
      vectors = vectors.T
      print(vectors.shape)
```

```
      (2, 784)
```

```
[206] final_coordinates = np.matmul(vectors,new_data.T)
```

```
[207] print(final_coordinates.shape)
      # (2X784) X (784X15000)
```

```
      (2, 15000)
```

```
[208] new_labels.shape
```

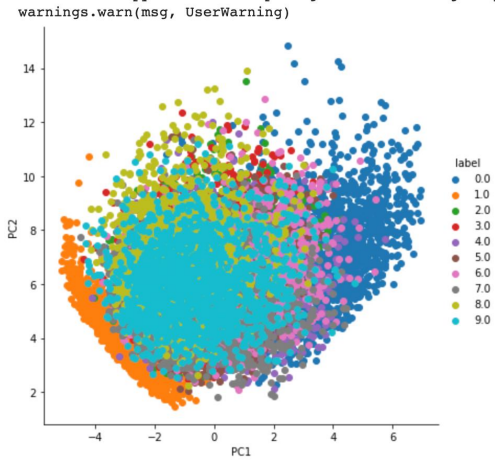
```
      (15000,)
```

```
[209] # Adding labels
      final_coordinates = np.vstack((final_coordinates, new_labels)).T
      df = pd.DataFrame(data=final_coordinates, columns=("PC1","PC2", "label"))
      print(df.head())
```

```
              PC1       PC2  label
      0 -0.272273  6.483883    5.0
      1  3.545998  6.941575    0.0
      2  1.478622  3.166248    4.0
      3 -3.305616  4.144610    1.0
      4 -1.533636  5.880873    9.0
```

```
[212] import seaborn as sb
      sb.FacetGrid(df, hue="label", size=6).map(plt.scatter, 'PC1', 'PC2').add_legend()
      plt.show()
```

```
      /usr/local/lib/python3.6/dist-packages/seaborn/axisgrid.py:316: UserWarning: The `size` parameter has been renamed to `height`; please update your code.
        warnings.warn(msg, UserWarning)
```



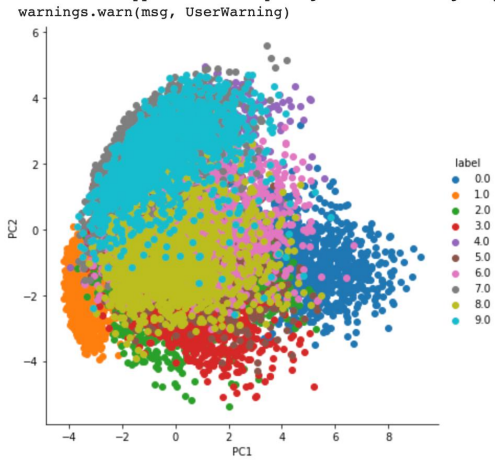
Note that labels 0 (dark blue) and 1 (orange) are completely separated. Moreover, we have just transformed our 784D dataset in 2D plane.
Great!

```
[219] # Now using PCA / Prebuilt function
      from sklearn import decomposition
      pca = decomposition.PCA()
      pca.n_components = 2
      pca_data = pca.fit_transform(new_data)
      print(pca_data.shape)
```

```
      (15000, 2)
```

```
[222] # Putting to vertical stack
      pca_data = np.vstack((pca_data.T, new_labels)).T
      pca_dataframe = pd.DataFrame(data=pca_data, columns=("PC1","PC2", "label"))
      sb.FacetGrid(pca_dataframe, hue="label", size=6).map(plt.scatter, 'PC1', 'PC2').add_legend()
      plt.show()
```

```
      /usr/local/lib/python3.6/dist-packages/seaborn/axisgrid.py:316: UserWarning: The `size` parameter has been renamed to `height`; please update your code.
        warnings.warn(msg, UserWarning)
```




```
[ ]
```