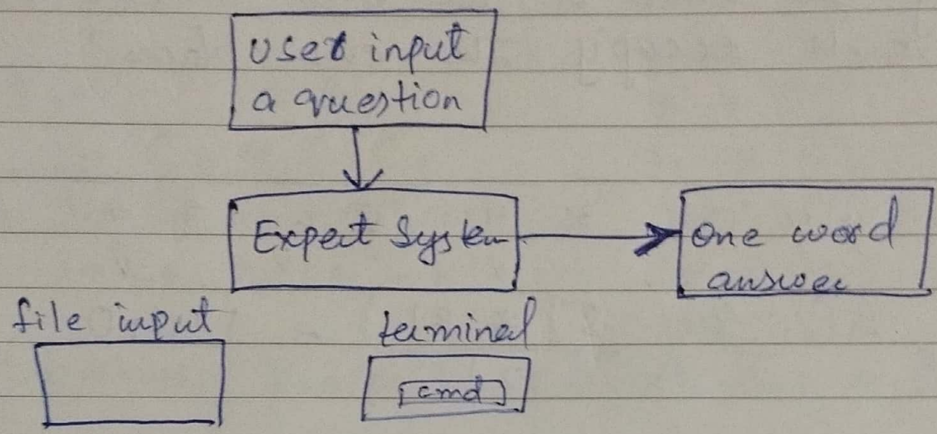
Propositional & Predicate Calculus.

Chapter #1 of Book by Rozen.

Goal: Student must be able to define the knowledge base of a domain specific Expert system in prolog language that can give one word answer of domain specific questions?



⇒

Domain Engineer → has domain specific knowledge.

↓

Knowledge Engineer → convert the knowledge acquired in the form of knowledge base. for computer

Data-driven approach

Hybrid approach.

Rule-based approach.

- unstructured text.

5/12/2022.

⇒ Install SWI Prolog

⇒ Domain specific Expert System: who can answer one word.

⇒ How to represent Declarative sentences?

<u>Propositions</u>:

Declarative sentences whose truth value is known i.e true or false.

<u>Ex</u>:

$1+2=3$ → truth value = true

$1+2=4$ → truth value = false

$x+5=3$ → this is not a proposition.

p : corona is a disease.

- Logical Connectives ⇒ operators to combine two or more propositions.

⇒ $\sim$ negation (NOT)
$\wedge$ conjunction (AND)
$\vee$ disjunction (OR)
$\rightarrow$ Implication
$\leftrightarrow$ Biconditional.

$\sim p$: corona is not a disease.

p	$\sim p$
T	F
F	T

Book Ex: 1
Q10 page 13.
p : The election is decided.
q : The votes have been counted.
Express the given compounds propositions in English.

(a) $\sim p$
The election is not decided.

(b) $p \vee q$
Either the election is decided OR the votes have been counted OR both.

(c) $\sim p \vee q$
Either the election is not decided OR the votes have been counted OR both.

(d) $\sim p \vee \sim q$
Neither the election is not decided NOR the votes have been counted OR both

5 12 22

Implication: → Conditional.

if p then q. ⇒ $p \to q$.

reads as p employs to q

Example:

if it is below freezing ~~and~~ then it is not snowing.

p: it is below freezing

~q: it is not snowing.

$p \to \sim q$.

if $\underbrace{\overset{\text{premise}}{\text{it is below freezing}}}_{p}$ then $\underbrace{\overset{\text{conclusion}}{\text{it is not snowing}}}_{\sim q}$

$p \to \sim q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Identification for Conditional Statements / Implication:

- if p then q ——— $p \to q$
- if p, q ——— $p \to q$
- "p" is sufficient for "q" — $p \to q$
- q if p. ——— $p \to q$
- "q" when p. ——— $p \to q$
- a necessary condition for p is q. — $p \to q$
- q unless ~p ——— $p \to q$
- p implies ~q. ——— $p \to q$
- p only if q. ——— $p \to q$
- a sufficient condition for q is p — $p \to q$
- "q" whenever p ——— $p \to q$.
- q is necessary for p — $p \to q$.
- q follows from p — $p \to q$

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Exercise

p: you have the flu.
q: you miss the final exam.
Translate into logical connectives.

you miss the final exam whenever you have the flu.

↳ $p \rightarrow q$

You have the flu. whenever you miss the final exam.

$q \rightarrow p$

$p \rightarrow q \not\equiv q \rightarrow p$ ∵ $\equiv$ ⇒ logically equivalent

$\sim p \vee \sim q \not\equiv \sim(p \wedge q)$?? "$\not\equiv$" ⇒ not logically equivalent.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \wedge q$	$\sim(p \wedge q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	F	T	T	T	F	T

Hence, $\sim p \vee \sim q \equiv \sim(p \wedge q)$

Q: 24 → page no 15:
Express the given statements into symbolic form.
a) I will remember to send you the address only if you sent me an email message.

Let r: I will remember to send you the address.
and s: you send me an email message.

$r \rightarrow s$

Paisa laya ?? Red Pen Rishta? ok Bero

PAPERWORK

(iii) To be a citizen of this ~~city~~ country, it is sufficient that you were born in the US.

Let t: To be a citizen of this country.
and m: you were born in the US.

$$t \rightarrow m.$$

Bicanditional $(p \leftrightarrow q)$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p).$$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

XNOR

Precedence of Logical Connectives.

$\sim$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Identification for Biconditional statements

- p is necessary and sufficient for q $p \leftrightarrow q$
- if p then q and conversely $p \leftrightarrow q$.
- p iff q $p \leftrightarrow q$

if and only if

5/12/22

Conditional statements.

Converse — $q \rightarrow p$

Inverse — $\sim q \rightarrow \sim p$

Contrapositive. — $\sim p \rightarrow \sim q$

Ex $\underbrace{\text{The home team wins}}_{p}$, whenever $\underbrace{\text{it is raining}}_{q}$?

$$q \rightarrow p$$

Converse:

$p \rightarrow q$: It is raining whenever the home team wins. OR if the home team wins then it is not raining.

Inverse:

$$\sim q \rightarrow \sim p.$$

Contrapositive:

$$\sim p \rightarrow \sim q.$$

★ $p \rightarrow q \equiv \sim q \rightarrow \sim p.$

any conditional statement is logically equivalent to its contrapositive.

8/Dec/2022

Logical Equivalence:

Two compound propositions are said to be logically equivalent ($\equiv$)

(i) if there truth tables are same.

(ii) By applying laws of equivalence.

$p \vee \sim p \equiv T$ — negation

Book pg. 27 Table: 6.

$p \wedge T \equiv p$ — identity — $p \vee F \equiv p$

$p \wedge F \equiv F$, $p \vee T \equiv T$

$p \vee q = p$ or q.

$\sim(\sim p) = p$.

8/12/22

Demorgan's law:

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$
$$\sim(p \wedge q) \equiv \sim p \vee \sim q.$$

Distributive law:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r).$$

Associative law:

$$p \vee (q \vee r) \equiv (p \vee q) \vee r \qquad \text{or } q \vee (p \vee r)$$
$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r \qquad \text{or } q \wedge (p \wedge r)$$

Implication Law:

$$p \to q \equiv \sim p \vee q.$$

Biconditional:-

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

⇒ Apply the Demorgan's law on the following statement.

(i) $$\sim[\underbrace{(p \to q)}_{A} \wedge \underbrace{(q \to r)}_{B}].$$

let A ∧ B

$$\sim[A \wedge B] \equiv \sim A \vee \sim B.$$

Hence

$$\sim[(p \to q) \wedge (q \to r)] \equiv \sim(p \to q) \vee \sim(q \to r)$$

(ii) Simplify:

$$\sim[(p \to q) \wedge (q \to r)].$$

$$\Rightarrow \sim[(\sim p \vee q) \wedge (\sim q \vee r)].$$

$$\Rightarrow \cancel{\sim[\{\ \wedge\ \} \vee \{\ \wedge\ \}]}$$

$$\Rightarrow \sim[\{(\sim p \vee q) \wedge \sim q\} \vee \{(\sim p \vee q) \wedge r\}].$$

$$\Rightarrow \sim[\{(\sim p \wedge \sim q) \vee \underbrace{(q \wedge \sim q)}_{F}\} \vee \{(\sim p \vee q) \wedge r\}].$$

$$\Rightarrow \sim[(\sim p \wedge \sim q) \vee \{(\sim p \vee q) \wedge r\}]$$

$\Rightarrow \sim[\sim(p \vee q) \vee \{(\sim p \vee q) \wedge r\}]$.

$\Rightarrow (p \vee q) \wedge \sim\{(\sim p \vee q) \wedge r\}$.

another approach:

$\Rightarrow \sim[(p \to q) \wedge (q \to r)]$.

$\Rightarrow \sim(p \to q) \wedge \sim(q \to r)$

$\Rightarrow \sim(\sim p \vee q) \vee \sim(q \vee r)$.

$\Rightarrow (p \wedge \sim q) \vee (q \wedge \sim r)$.

Book pg no. 35

Q.no. 14 — 33 (for practice purpose),

Tautology $\to$ always true. ex. $\to P \vee T = T$

Contradiction $\to$ always false. ex. $\to P \wedge F = F$

Contingency $\to$ True / false.

Q. 14 Determine whether it is tautology or not.

$[\sim p \wedge (p \to q)] \to \sim q$.

$\Rightarrow [\sim p \wedge (\sim p \vee q)] \to \sim q$

$\Rightarrow [(\sim p \wedge \sim p) \vee (\sim p \wedge q)] \to \sim q$.

$\bar{A}(\bar{A}+B)$

$\bar{A}\bar{A} + \bar{A}B$

$\bar{A} + \bar{A}B$

$\bar{A}(1+B)$

$\bar{A}$

$[\sim p] \to \sim q$

$\sim[\sim p] \vee \sim q$,

$p \vee \sim q \not\equiv T$

Pg. 35 Solve all

19/12/22

Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.

Solution

$p \leftrightarrow q$

$(p \rightarrow q) \wedge (q \rightarrow p)$

$(\sim p \vee q) \wedge (\sim q \vee p)$.

$A \wedge (B \vee C)$

$(A \wedge B) \vee (A \wedge C)$

prod. + prod.

Sum of product

$[(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$.

$[(\sim q \wedge \sim p) \vee \underbrace{(\sim q \wedge q)}_{\text{False}}] \vee [(p \wedge \sim p) \vee (p \wedge q)]$

$[(\sim q \wedge \sim p) \vee \text{False}] \vee [\text{False} \vee (p \wedge q)]$.

$(\sim q \wedge \sim p) \vee (p \wedge q)$

$(p \wedge q) \vee (\sim p \wedge \sim q)$.

- first order propositional logic

Non-Declarative Sentences.

is $x > 3$?

Predicate logic.

let $Q(x)$ represents $x > 3$

$Q(3)$ = false $(3 \not> 3)$

$Q(4)$ = true $(4 > 3)$.

p = person x like apple

q = person x like mango.

Like (x, y) represents person x like fruit y.

like (Tehuiat, apple) ⇒ Tehuial likes apple.

→ one argument

1st Argument P.L

→ multi argument

2nd Argument P.L.

Boot Que

Let $P(x)$ denotes $x > 3$.

what is the truth value of $P(5)$? True.

19/12/22

"Coding is to Programming what typing is to Writing"

Q. Let $Q(x,y)$ represents $x+y>3$.
what is the value of
$Q(1,2)$ = false
$Q(1,3)$ = true

Pg-53 first 4 questions

unstructured → structured

Que. Express the following statements into symbolic form using predicates. and logical connectives.

~~a) A student in your class has a dog, cat or a monkey.~~

(i) Let the domain consist of all students of your class.
(a) A student in your class has a dog, cat or a monkey.
Let $D(x)$ represents the student x has a dog,
$C(x)$ represents the student x has a cat or
$M(x)$ represents the student x has a monkey.

$C(x) \vee D(x) \vee M(x)$.

(ii) Let the domain consist of all persons.
(a) A student in your class has a dog, cat or a monkey.
Let $S(x)$ represent x is a student of this class.
✓ (i) $S(x) \wedge [C(x) \vee D(x) \vee M(x)]$ correct
(ii) $S(x) \rightarrow [C(x) \vee D(x) \vee M(x)]$.

$S(x)$	$C(x)$	$S(x) \rightarrow C(x)$	$S(x) \wedge C(x)$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

Quantification:- (i) universal quantification $\forall$

$P(x)$

All lions are fierce.

$\forall x\, P(x)$ = $P(x)$ is true when all the x follows the property | $p(x)$ is false, if anyone x donot satisfies the prop.

Let $P(x)$ represents all lions are fierce.

$\forall x\, P(x)$.

(ii) Existential quantifies $\exists$ (read as there exists/some/any).

$\exists x\, P(x)$ is true if any one x satisfy the property
is false when all x do not satisf the given prop.

(A) Q: Translate into symbolic form: if $x \in$ set of all lions

(i) all lions are fierce.

Let $P(x)$ represent lions are fierce.

$\forall x\, P(x)$.

(ii) Some lions are fierce.

$\exists x\, P(x)$.

(B).

Let $x \in$ set of all animals

Q: Translat into symbolic form:

(a) all lions are fierce.

Let $L(x)$ represents "x is a lion"
and $f(x)$ represents "x is fierce"

$$\forall x\, [L(x) \rightarrow F(x)]$$

(b) Some lions are fierce

$$\exists x\, [L(x) \wedge F(x)]$$

$L(x)$	$F(x)$	$\forall x$ $[L(x) \rightarrow F(x)]$	$\exists x$ $[L(x) \wedge F(x)]$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

22/12/22

Negation of Quantifiers.

$$\sim \forall x\, P(x) \equiv \exists x \sim P(x).$$

$$\sim \exists x\, P(x) \equiv \forall x \sim P(x).$$

Everybody send emails messages to person y.

Let send(x, y) represents person x send email messages to person y.

$$\forall x \; send(x, y).$$

	$\forall x\, send(x,y)$	$\sim \forall x\, send(x,y)$
when all of the persons send email to y	T	F
someone didnot send email	F	T

	$\forall x\, send(x,y)$	$\sim \forall x\, send(x,y)$
when all send $\forall x\, send(x,y)$.	T	F
some do not send $\exists x \sim send(x,y)$.	F	T
when not all send email to y. $\sim \forall\; send(x,y)$.	F	T

$\sim \forall x\, send(x,y)$	$\exists x \sim send(x,y)$
F	F
T	T
T	T

$\equiv$

Translate into symbolic form:

1. Everybody send email messages to y.
   - Translate: $\forall x \; send(x, y)$.
   - Negation: $\sim \forall x \; send(x, y) \equiv \exists x \sim send(x, y)$.
   - Translate: Somebody do not send email messages to y. ✓
     Nobody send email messages to y. ✗

2. Somebody send email messages to y.
   - Translate: $\exists x \; send(x, y)$
   - Negation: $\sim \exists x \; send(x, y) \equiv \forall x \sim send(x, y)$.
   - Translate: Everybody do not send email messages to y.

26.12.22

Nested Quantifiers:

→ Everybody send email messages to everyone.

- Let send (x,y) represents x send email messages to y.

$$\forall x \forall y \text{ send}(x,y)$$

→ $\forall x \forall y \text{ send}(x,y) \equiv \forall y \forall x \text{ send}(x,y)$

→ $\forall x \exists y \text{ send}(x,y) \not\equiv \exists y \forall x \text{ send}(x,y)$.

- nested loops

- Everybody send email messages to some one.

$$\forall x \exists y \text{ send}(x,y).$$

- Someone recieved email messages from everyone

$$\exists y \forall x. \text{ send}(x,y)$$

	T	F	
$\forall x \exists y \text{ send}(x,y)$	T	F	$\exists x \exists y \sim \text{send}(x,y)$
$\forall x \exists y \text{ send}(x,y)$	T	F	$\exists x \forall y \sim \text{send}(x,y)$.

T	F
$\exists y \forall x \text{ send}(x,y)$	$\exists y \exists x \sim \text{send}(x,y)$.
$\forall y \cancel{\forall x} \text{send}(x,y)$	$\exists y \forall x \sim \text{send}(x,y)$

$$\exists y \forall x \, p(x,y) \not\equiv \forall x \exists y \, p(x,y)$$

$$\forall x \exists y \, p(x,y) \not\equiv \exists y \forall x \, p(x,y.$$

Negation of Nested Quantifiers:

$$\sim [\forall x \forall y \, p(x,y)] \equiv \exists x \sim [\forall y \, p(x,y)].$$
$$\equiv \exists x \exists y \sim p(x,y).$$

$$\sim [\forall x \forall y \, p(x,y)] \equiv \exists x \exists y \sim p(x,y).$$

$\sim [\exists x \exists y\ p(x,y)] \equiv \forall x \sim [\exists y\ p(x,y)] \equiv \forall x \forall y \sim p(x,y)$

Make negation of the following statements:

→1. Everybody send email message to everyone.
Translate: $\forall x \forall y\ send(x,y)$
Negation: $\sim [\forall x \forall y\ send(x,y)]$.
$\equiv \exists x \exists y \sim send(x,y)$.
Translate:
Negation of 1: Someone don't send email messages to anyone.

Verb (subject, object) → to make predicate.

X was ill. He needs to go to the hospital.

D.B
Relational D.B
ELT
ETL.
Datawarehouses
Amazon S3 Bucket.
Data Pipelining.
Data replication.
Integration
destination.
ERP
Tableu.

29/12/2022

Rules of Inference:

① P if T
$P \rightarrow q$ if T — modus ponen or Law of Detachment
∴ q will be true.

$[p \wedge (p \rightarrow q)] \rightarrow q$, this is a tautology.

$\Rightarrow \sim [p \wedge p \rightarrow q] \vee q$
$\Rightarrow [\sim p \vee \sim (p \rightarrow q)] \vee q$
$\Rightarrow [\sim p \vee \sim (\sim p \vee q)] \vee q$
$\Rightarrow [\sim p \vee (p \wedge \sim q)] \vee q$
$\Rightarrow [(\sim p \vee p) \wedge (\sim p \vee \sim q)] \vee q$.
$\Rightarrow [T \wedge (\sim p \vee \sim q)] \vee q$
$\Rightarrow (\sim p \vee \sim q) \vee q$.
$\Rightarrow (\sim p) \vee (\sim q \vee q)$
$\Rightarrow \sim p \vee T$
$\Rightarrow T$

Hence, Proved!

$(p \wedge q) \rightarrow p$
$\sim (p \wedge q) \vee p$
$\sim p \vee \sim q \vee p$
$\sim p \vee p \vee \sim q$
$T \vee \sim q$
$\boxed{T}$

PAPERWORK

fruit (apple)
fruit (mango).

29/12/22

like (usman, apple)
like (saad, mango).
like (usman, X) :- fruit (X, yummy).

? fruit (apple).
true
? like (daniyal, x)
x = apple.

Taste (x, yummy) :- like (daniyal, X).

like (daniyal, apple) = true
? taste (apple, yummy)
= true

2-Jan-2023

modus tollen

$\sim q$ if T
$p \to q$ if T
$\therefore \sim p$ T

because $[\sim q \wedge (p \to q)] \to \sim p \equiv T$

Hypothetical Syllogism

$p \to q$ T
$q \to r$ T
$\therefore p \to r$ T

Ex # 1 Pg # 60 (Book).
If it's snows today, then we will go skiing
its snowing today are true.

P => fact
$P \to q$ => rule
$\therefore q$.

P: it's snowing today
q: we will go skiing

$\therefore$ Conclusion: we will go sking.

2/6/22

Ex# 2.

It is snowing is false.

If we will not go skiing ~~is also false~~ then it does not snow today is false.

p false $\therefore \sim p$ true

$\sim q \rightarrow \sim p$ false $\therefore \sim(\sim q \rightarrow \sim p)$ true

$\sim(q \vee \sim p)$ "

$\sim q \wedge p$ true

false

$\sim p$ true

$\sim q \wedge p$ true

Conclusion:

Since p is false, $\sim q \wedge p$ can never be true hence, it is an invalid argument.

$\sim p$ true

$\sim q \wedge p$ true X

$\sim q \wedge p$ $\therefore \sim p$ false

$\therefore p$ true

Book Ex #3.

It is below freezing now. $= f$

$\therefore$ It is either below freezing or raining now. (r)

which rule is this applies?

$$\frac{f}{\therefore f \vee r}$$ Addition.

Ex# 4. $f \wedge r$

It is below freezing and raining now.

$\therefore$ It is below freezing now.

$$\frac{f \wedge r}{\therefore f}$$ Simplification

pg# 71 Q.9 ~~a1~~.

For each of these sets of premises (facts), what relevant conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from premises)

a) 1. "If I take the day off, it either rains or snows".
2. "I took Tuesday off or I took Thursday off".
3. "It was sunny on Tuesday"
4. "It did not snow on Thursday."

Let p: I take the day off.
q_1: rains. q_2: snows.
t_1 = It is tuesday
t_2 = It is thursday
s = It was sunny.

1. $p \to (q_1 \vee q_2)$
2. $p \wedge (t_1 \vee t_2)$
3. $s \wedge t_1$
4. $\sim q_2 \wedge t_2$.

1. $p \to (q_1 \vee q_2)$
4. $\sim q_2 \wedge t_2$

$\sim q_2$ = true.
$\therefore q_2$ false.

q_2 false
$p \to (q_1 \vee q_2)$ true

$p \to q_1$ true.

2. $p \wedge (t_1 \vee t_2)$ true
3. $s \wedge t_1$ true

$\therefore t_1$ true.
$\therefore p \wedge t_1$ true.

2. $p \wedge (t_1 \vee t_2)$ true
4. $\sim q_2 \wedge t_2$ true

$\therefore t_2$ true
$\therefore p \wedge t_2$ true.

5/01/2

METHODS OF PROOFS:

$(p \to q)$.

Direct proof

Given "p" is true then prove that "q" will also be t

· if $\underbrace{n \text{ is even}}_{p}$, prove that $\underbrace{2n+6 \text{ is also even}}_{q}$.

Solve

n is even $\Rightarrow \forall k \in Z \ni$

$n = 2k$.

k	2k
1	2
2	4
3	6
4	8.

$n = 2k \Rightarrow 2n + 6$.

$\Rightarrow 2(2k)+6 \Rightarrow 4k+6 \Rightarrow 2(2k+3)$.

$\because k \in Z \Rightarrow 2k \in Z,\ 3 \in Z \Rightarrow 2k+3 \in Z$

$\Rightarrow 2\,[m]$ where $m = 2k+3 \in Z$

which is even, hence $2n+6$ is even

9/01/22

Methods of Proof:

Direct proof —— $p \to q$.

Indirect proof:

1. proof by contradiction.

p : true

by q is not true.

$\sim q$: false.

Prove that: by using proof by contradiction

if $\underbrace{n \text{ is even}}_{p}$ then $\underbrace{2n+8 \text{ is even}}_{q}$.

Solve

Assume that n is even but $2n+8$ is not even.

Because n is even $\Rightarrow \forall k \in Z \ni n = 2k$.

$2n + 8$ is not even

$\Rightarrow \exists k \in z \ni 2n+8 = 2k+1$.

$2n = 2k - 7$

$n = \dfrac{2k-7}{2}$

$n = \dfrac{2k-6-1}{2}$

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$n \Rightarrow \frac{2k-6}{2} - \frac{1}{2}$

$n \Rightarrow \frac{2(k-3)}{2} - \frac{1}{2}$ $\rightarrow m$

$n = 2m - \frac{1}{2} \in R . \notin Z.$

$\therefore n \neq 2k$ which is a contradiction.
that p is true.

$\Rightarrow \sim p$ = true.

$\therefore \sim p$ is true $\therefore \sim q$ true

if n is real number then $2n+8$ can be odd

★ 1 argument valid kab hogi, jab woh tautology hogi.

Proof by Mathematical Induction.

$T_1 \quad T_2 \quad T_3 \qquad T_n$ Sum of n natural numbers.

$\underbrace{1 + 2 + 3 + \dots + n}_{L.H.S} \Rightarrow \underbrace{\frac{n(n+1)}{2}}_{R.H.S.}$

1st Step

Let $P(n) = \frac{n(n+1)}{2}.$

Show that for $n=1$, $P(n)$ is true.

LHS	$\equiv$	$P(1) = \frac{1(1+1)}{2}$
1		$P(1) = \frac{1(\cancel{2})}{\cancel{2}}$

$\boxed{a+(n-1)d}$

$P(1) = 1$

$1 \equiv 1$ ✓

2nd Step

Assume that $P(n)$ is true for $n=k$. $K = W$

$K+1 = N$.

$1+2+3+\dots+k = \frac{k(k+1)}{2}.$

3rd Step

Prove that the given formula is true $\forall n$, i.e. $n = k+1$

$\left\{ \underbrace{1+2+3+\dots+k+k+1}_{LHS} = \underbrace{\frac{(k+1)(k+1+1)}{2}}_{RHS} \right.$

$\underbrace{1+2+3+\dots+k} + k+1$

$\frac{k(k+1)}{2} + k+1.$

k^2+3k+2

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= \frac{(k+1)(k+1+1)}{2}$$

19/01/2023.

The square of every integer is positive.

$\forall x \in Z \quad x^2 > 0.$

$\exists x \sim (x^2 > 0).$

$\exists x \, (x^2 < 0).$

. Some integer has -ve square.

PROOF BY CONTRAPOSITION.

1) $p \to q \equiv \sim q \to \sim p$

Prove that if $\underbrace{3n+2 \text{ is odd}}_{p}$, then $\underbrace{n \text{ is odd}}_{q}$;

From direct proof.

Given $3n+2$ is odd $\Rightarrow \exists k \in Z \ni$

$3n+2 = 2k+1$

$3n = 2k-1$

$n = \frac{2k-1}{3}.$

Direct proof.

$\sim q \to \sim p$

Assume that $\sim q$ is true $\Rightarrow n$ is even $\forall k \in Z$

$n = 2k$

put in $3n+2$.

$3(2k)+2$

$= 6k+2.$

$= 2(3k+1)$

$= 2$ (some integer).

$= 2m \in Z.$

$\Rightarrow$ It has been proved that if n is even then $3n+2$ is even. By contraposition if $3n+2$ is odd then n is odd.

PAPERWORK

Prove that   Vacuous Proof

if $\underbrace{n > 1}_{p}$ then $n^2 > n$ is True

Assume p is false $\Rightarrow$ $n > 1$ is false $\Rightarrow n \leq 1$

$n \leq 1$

$n(n \leq 1)$. $n \in Q$ or $n \in O$ $p \wedge v$.

$n^2 \leq n \quad \because n \leq 1$

$\Rightarrow$ if $n \leq 1$ then $n^2 \leq n$.

$\Rightarrow$ if $n > 1$ then $n^2 > n$.

Proof By Cases

We want to show that

$\Rightarrow [p_1 \vee p_2 \ldots p_n] \rightarrow q$

$\Rightarrow \sim [p_1 \vee p_2 \ldots p_n] \vee q$

$\Rightarrow [\sim p \wedge \sim p_2 \ldots \sim p_n] \vee q.$

$\Rightarrow (\sim p_1 \vee q) \wedge (\sim p_2 \vee q) \ldots (\sim p_n \vee q).$

$\Rightarrow (p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \ldots (p_n \rightarrow q).$

Show that.

$|x|\,|y| \leq |xy|$ $|(-x)(-y)|$

$x < 0 \quad y < 0$ $|+xy|$

$|x < 0| \quad |y < 0|$ xy.

$-(-x) \quad -(-y).$

$+x \quad + y.$

16/01/2023

Prolog

$\Rightarrow$ swipl

$\Rightarrow$ consult (file path) .

$\Rightarrow$ halt. # to terminate prolog.

:- , rule

, and opr

; or opr.

19/01/2023

Sir Usman Khalil.

Cartesian Product.

$A = \{a, b\}$ and $B = \{1, 2, 3\}$

$A \times B = \{a, b\} \times \{1, 2, 3\}$.

$A \times B = \{(a,1), (a,2), (a,3), (b,4), (b,5), (b,6)\}$.

cardinality of set A x Cardinality of set B

2 x 3

6 ordered pairs

$B \times A = \{1, 2, 3\} \times \{a, b\}$

$B \times A = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$A \times B \neq B \times A$.

Cartesian product also represents 2D array.

$A = \{0, 1\} = 2$, $B = \{1, 2\} = 2$, $C = \{0, 1, 2\} = 3$

$A \times B \times C = \{0, 1\} \times \{1, 2\} \times \{0, 1, 2\}$.

$= \{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$

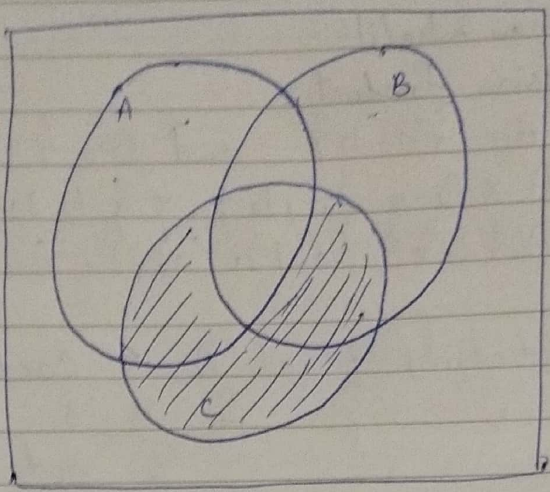
$A = \{1, 2\} = 2$, $B = \{3, 4, 5\} = 3$, $C = \{6, 7, 8\} = 3$.

$A \times B \times C = \{1, 2\} \times \{3, 4, 5\} \times \{6, 7, 8\}$

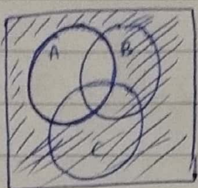
$= \{(1,3,6), (1,3,7), (1,3,8), (1,4,6), (1,4,7), (1,4,8), (1,5,6), (1,5,7), (1,5,8), (2,3,6), (2,3,7), (2,3,8), (2,4,6), (2,4,7), (2,4,8), (2,5,6), (2,5,7), (2,5,8)\}$.

PAPERWORK

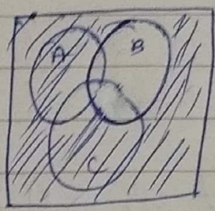
$(\overline{A} \cup \overline{B}) \cap C$



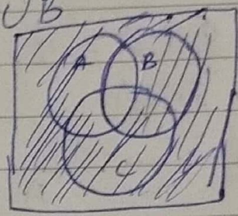
$\overline{A}$



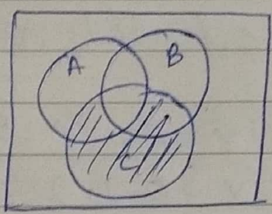
$\overline{B}$



$\overline{A} \cup \overline{B}$



$(\overline{A} \cup \overline{B}) \cap C$



$(\overline{\overline{A} \cup \overline{B}}) \cap C$

✓

23-01-2023.

1. Sequences and series
2. Recurrance and Relation } Algorithms complexity
3. Binomial theorem
4. Number theory
5. Graphs and Relations.

SEQUENCE AND SERIES:-

Sequence is a function in which a term is dependent on previous term

E.g. $2, 4, 6, 8, \ldots, T_n$ $2, 4, 8, 16, \ldots, T_n$

$T_1 \quad T_2 \quad T_3 \quad T_4$.

- n^{th} term / General term.

1. Arithmetic Sequence (Ap).
2. Geometric Sequence (Gp).
3. Harmonic Sequence (Hp).

Arithmetic Progression:

$\frac{1}{2}+\sqrt{3} \cdot 0+0$

$T_1 = a$ $\quad$ $T_1 = a$

$T_2 = a+d$ $\quad$ $T_2 = a+d$

$T_3 = a+d+d$ $\quad$ $T_3 = a+2d$

$T_4 = a+d+d+d$ $\quad$ $T_4 = a+3d$

$\vdots$

$T_n = a+(n-1)d.$

$1, 3, 5, 7, \ldots$

$T_{50} = ?$

First we will check that this sequence is A.P. If the difference b/w given sequence is same.

$3-1=2$

$5-3=2$

$7-5=2$ ✓ It's an A.P.

$T_n = a+(n-1)d$ $\quad \because a=1,\ d=2,\ n=50.$

$T_{50} = 1+(50-1)2.$

$T_{50} = 1+49\times 2$

$T_5 = 99.$

Series $\Rightarrow$ Sum of all terms in a sequence.

$1+3+5+7+\cdots+T_n.$

$S_n = ?$

$\frac{n(n+1)}{2}$

$1, 2, 3, , \ldots, n-2, n-1, n.$

$\underset{T_1}{1} + \underset{T_n}{n}$, $\quad \underset{T_2}{2} + \underset{T_{n-1}}{(n-1)} \Rightarrow n+1$, $\quad 3+n-2$

$n+1.$

$\frac{n(n+1)}{2}.$

$S_n = \frac{n}{2}[2a+(n-1)d].$

$S_{10} = \frac{10}{2}[2(1)+(10-1)2].$

$S_{10} = 100$

$S = \sum_{j=1}^{n}(a+jd) = na+d.$

$\sum_{j=1}^{n} j = na+d\left(\frac{n(n+1)}{2}\right)$

$a + a+d + a+2d + \cdots + a+(n-1)d.$

$\underbrace{a+a+a+\cdots+a}_{} + \underbrace{d+2d+\cdots+(n-1)d}_{Nd.}$ $N = n-1$

$\Rightarrow n \times a + \frac{N(N+1)d}{2}$

$\Rightarrow \quad '' \quad + \frac{(n-1)(n-1+1)d}{2}$

$\Rightarrow na + \frac{n(n-1)d}{2}.$

$\Rightarrow \frac{2na + nd(n-1)}{2}.$

$\Rightarrow \frac{n[2a + (n-1)d]}{2}.$

Geometric Sequence:-

$a, ar, ar^2, \cdots ar^{n-1}$

$T_n = ar^{n-1}$ $\quad n \geq 1$

$1, 2, 4, 8, \cdots$

This is G.P, if

$T_{50} = (1)(2)^{50-1}$

$T_{5} = 2^{49}.$

Sum of n-terms in G.P.

$S_n = \frac{a(1-r^n)}{1-r}$; $r < 1$

$S_n = \frac{a(r^n-1)}{r-1}$; $r > 1$

$S_{50} = \frac{1(2^{50}-1)}{2-1}$

Harmonic Sequence:-

$H.P = \frac{1}{A.P}$ $\quad \frac{1}{d} \ln\left(\frac{2a+(2n-1)d}{2a-d}\right)$

$\frac{1}{a}, \frac{1}{a+d}, \cdots, \frac{1}{a+(n-1)d}.$

$T_n = \frac{1}{T_n \text{ of } A.P}$ | Sum of Hp $= \frac{1}{S_n \text{ of } A.P}$

$2^n = 2, 4, 8$

$2^{n+1} =$

$4, 10, 28, \ldots$

$3^1 + 1 = 4, \quad 3^2 + 1 = 10, \quad 3^3 + 1 = 28.$

$T_n = 3^n + 1.$

$1, -2, 3, -4, 5, \ldots$

1

$1, -1, 1, -1, \ldots$

$T_n = -1(-1)^n$

$n = 1 \Rightarrow 1$

$n = 2 = -1$

Summation Notation:-

$1 + 2 + 3 + \ldots + n.$

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n.$$

$2 \times 1 + 2 \times 2 + 2 \times 3$

$2 \times (1 + 2 + 3)$

$$\sum_{i=1}^{n} ci = c \sum_{i=1}^{n} i$$

L.H.S $c + 2c + 3c + \ldots nc.$

$c\,[1 + 2 + 3 + \ldots n].$

$c \sum_{i=1}^{n} i$ Proved!

$$\left[\frac{n(n+1)}{2}\right]^2$$

$$\frac{n^2(n+1)^2}{4}$$

$$\frac{n^2(n^2+2n+1)}{4}.$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = (1)^2 + (2)^2 + (3)^2 + \ldots n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$ (Kitab mein sy confirm kr lo).

Calculate $\displaystyle\sum_{i=3}^{7} \sum_{j=3}^{5} (ij - 3i)$

$$\Rightarrow \sum_{i=3}^{7} \left[\sum_{j=3}^{5} ij - \sum_{j=3}^{5} 3i \right].$$

$$\Rightarrow \sum_{i=3}^{7} \left[i \sum_{j=3}^{5} j - 3i \sum_{j=3}^{5} (1) \right]$$

$$\Rightarrow \sum_{i=3}^{7} \left[i \left\{ \frac{5 \times \cancel{6}^{3}}{\cancel{2}_1} - \frac{\cancel{2} \times 3}{\cancel{2}} \right\} - 3i \left\{ \sum_{i=1}^{5} (1) - \sum_{j=1}^{2} (1) \right\} \right]$$

$$= \sum_{i=3}^{7} [i(12) - 3i \times 3]$$

$$= \sum_{i=3}^{7} (3i) \Rightarrow 3 \sum_{i=3}^{7} i \Rightarrow$$

$\because \sum_{i=1}^{n} j = \frac{n(n+1)}{2}.$

$\sum_{i=1}^{5} j = \sum_{i=1}^{5} j - \sum_{i=1}^{2} j$

$\sum_{i=1}^{5} j = \frac{5(5+1)}{2} - \frac{2(2+1)}{2}$

$3\left[\sum_{i=1}^{7} i - \sum_{1}^{2} i\right]$

$28 - 3$

3×25

$\boxed{75}$

PAPERWORK

BINOMIAL THEOREM:-

$$(a+b)^n = a^n + n(n-1)$$

$$(a+b)^n = nC_0\, a^n b^0 + nC_1\, a^{n-1} b^1 + nC_2\, a^{n-2} b^2 + \dots + nC_n\, a^0 b^n$$

where $nC_r = \dfrac{n!}{(n-r)! * r!} = \binom{n}{r}$

$$6C_2 = \frac{6!}{(6-4)! \times 2!} = \frac{6!}{4! \times 2!}$$

$$= \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!} \times 2!} = \frac{30}{2} = 15$$

A.

Evaluate $(1+99)^{53}$ ✓

26/01/23

① Propositional calculus.
- → how to remove implication in simplification
- → translation and negation of nested quantified statements (in reverse process).

- → Venn diagram
- → Functions (is or not, is onto or one-to-one
- → Sequences and series
- → Recurrence relation
- → Summation notation.
- → Binomial
- → Graph Theory.
- → Graph Relation
- → Number Theory (Encryption in Decryption).

CamScanner

26-01-23.

NUMBER THEORY:

- Encryption
- Decryption

PLOT

A	B	C	D	......	Z	(Enumeration).
1	2	3	4		26.	

For Example

$\boxed{16 \quad 12 \quad 15 \quad 20} \rightarrow M$

$C = M^e \bmod (n)$, where n is any 2 prime numbers.

code

$n = p * q$ such that (e, n) are relatively prime i.e $\gcd(e, n) = 1$

Modular exponentiation Algorithm

$3^{2000} \bmod 5 = ? \Rightarrow b^a \bmod m$

Power ko binary main convert krein.

$\rightarrow a = (a_0, a_1, a_2, \cdots a_k)_2$ Binary

let power = b mod m

let x = (output) = 1

$m = 5$

$b = 3$

$a = 2000$

	x	power
a_0	if(a_i==0) then no update x.	power = power * mod m
a_1	x=(x * power) mod m	
a_2		
⋮		
a_k		

Modular Arithmetic

$3 \bmod 5 = 5\overline{)3}$

$= 3.$

a mod n a % n = Rem

Q: Calculate $3^2 \bmod 5$

$9 \bmod 5 =$

$5\overline{)9} = 4.$

5 (4) R

Modular Exponetiation Algo.

$3^{2000} \bmod 5 = ?$

$32 \bmod 5 = 2$

$2^5 = 32$

~~$2 \bmod 1$~~

$2 \bmod 5 = 2$

$\overset{4}{1}\overset{2}{0}\overset{1}{1} = 5$

$2^5 \bmod 5 = ?$

1) power = b mod m = 2 mod 5 = 2

(a[0] == 0) ✓ = x = (x * power) mod m = 2

2) power = 2 * (2 mod 5) = 4

(a[1] == 0) ✗

3) power = 4 x (4 mod 5) = 16 (a[2] == 0) ✓ x = (32) mod 5 = 2

p x q is key.

PLOT

Encrypt P L.

16 12.

$M = 1612$, $n = 59$.

$e = 13$.

$C = M^e \bmod n$.

$C = (1612)^{13} \bmod (59)$.

Convert 13 into binary.

$13 = 1101$

$x = 1$

power = b mod m

power = 1612 mod 59 =

Powe = 19.

a_0	1	$x = x * \text{power} \bmod m$ $x = 1 * 19 \bmod 59$.	power = (power . power) mod 59. Power $= 19^2 \bmod 59 = 7$
a_1	0	$x = 19$	$P = (P \cdot P) \bmod 59$ $P = 7^2 \bmod 59 = 49$
a_2	1	$x = x * P \bmod m$ $x = 19 * 49 \bmod 59$ $x = 46$	$P = P^2 \bmod 59$. $P = (49)^2 \bmod 59$ $P = 41$
a_3	1	$x = x * P \bmod m$ $x = 46 * 41 \bmod 59$ $x = 57$	$P = P^2 \bmod 59$ $P = (57)^2 \bmod 59$. $P = 4$

$$S = \sum_{j=1}^{n} (a + jd) = \cancel{\sum} na + d \sum_{j=1}^{n} j = na + d \ \frac{n(n+1)}{2}$$

$$S = \sum_{j=1}^{n} (a + jd) = \sum_{j=1}^{n} a + \sum_{j=1}^{n} jd = na + d \sum_{j=1}^{n} j$$

$$\sum_{j=1}^{n} = 1 + 2 + 3 + 4 + \cdots + (n-2) + (n-1) + n.$$

$n+1$ $2+n-1$ $3+n-2$

$n+1$ $n+1$

$n(n+1)$

$\frac{n(n+1)}{2}$ → why divided by 2

$$S = \sum_{j=1}^{5} (2 + 3j) =$$

$$\Rightarrow \sum_{j=1}^{5} 2 + \sum_{j=1}^{5} 3j$$

$$\Rightarrow 2 \sum_{j=1}^{5} (1) + 3 \sum_{j=1}^{5} j \qquad \text{since } \sum_{j=1}^{n} = \frac{n(n+1)}{2}$$

$$\Rightarrow 2 \times 5 + 3 \left\{ \frac{5(5+1)}{2} \right\}.$$

$$\Rightarrow 10 + 3 \times 15$$

$$\Rightarrow 10 + 45$$

$$\Rightarrow 55$$

$$S = \sum_{j=3}^{5} (2 + 3j).$$

$$S = \left\{ \sum_{j=1}^{5} (2 + 3j) \right\} - \left\{ \sum_{j=1}^{2} (2 + 3j) \right\}. \qquad \frac{2(2+1)}{2} = 3$$

$$S = \left\{ 2 \sum_{j=1}^{5} 1 + 3 \sum_{j=1}^{5} j \right\} - \left\{ 2 \sum_{j=1}^{2} 1 + 3 \sum_{j=1}^{2} j \right\}.$$

$$S = \{2 \times 5 + 3 \times 15\} - \{2 \times 2 + 3 \times 3\}$$

$$S = (10 + 45) - (4 + 9)$$

$$S = 55 - 13$$

$$S = 42$$

$$S = \sum_{i=1}^{4} \sum_{j=1}^{2} (2i - j).$$

$$S = \sum_{i=1}^{4} \left[\sum_{j=1}^{2} 2i - \sum_{j=1}^{2} j \right].$$

$$S = \sum_{i=1}^{4} \left[2i \sum_{j=1}^{2} (1) - \sum_{j=1}^{2} j \right] \cancel{2(2+1}$$

$$S = \sum_{i=1}^{4} \left[2i(2) - \frac{\cancel{2}(2+1)}{\cancel{2}} \right].$$

$$S = \sum_{i=1}^{4} [4i - 3].$$

$$S = 4 \sum_{i=1}^{4} i - 3 \sum_{i=1}^{4} (1)$$

$$S = 4 \left(\frac{\cancel{4}^{2}(4+1)^{5}}{\cancel{2}} \right) - 3(4))$$

$$S = 4(10) - 12)$$

$$S = 40 - 12)$$

$$\boxed{S = 28}$$

$$S = \sum_{i=3}^{7} \sum_{j=3}^{5} (ij - 3i)$$

$$S = \sum_{i=3}^{7} \left[\sum_{j=3}^{5} (ij - 3i). \right].$$

$$S = \sum_{i=3}^{7} \left[i \sum_{j=3}^{5} j - 3i \sum_{j=3}^{5} (1) \right].$$

$$S = \sum_{i=3}^{7} \left[i \left\{ \sum_{j=1}^{5} j - \sum_{j=1}^{2} j \right\} - 3i \left\{ \sum_{j=1}^{5} (1) - \sum_{i=1}^{2} (1) \right\} \right]$$

$$S = \sum_{i=3}^{7} \left[i \left\{ \frac{5(5+1)}{\cancel{2}} - \frac{\cancel{2}(2+1)}{\cancel{2}} \right\} - 3i \{ 5 - 2 \} \right].$$

$$S = \sum_{i=3}^{7} [i(\underbrace{15 - 3}_{12}) - 3i(3)].$$

$$S = \sum_{i=3}^{7} 12i - 9i$$

$$S = \sum_{i=3}^{7} 3i \Rightarrow 3 \left[\sum_{i=1}^{7} i - \sum_{i=1}^{2} i \right]$$

$$= 3 \left[\frac{7(7+1)}{2} - \frac{\cancel{2}(2+1)}{\cancel{2}} \right]$$

$$= 3(28 - 3)$$

$$= 3(25)$$

$$= 75.$$

1, 2, 4 2^n ($n = 0, 1, 2, 3 \ldots$

1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0.