DISCRETE STRUCTURES

Lecture no. 1

21/10/22

Pg. 1

Mathematics is the queen of all sciences

Discrete → countable.

data can be stored in sets, matrix, lists.

1. SETS

24/10/22.

sets are unordered collection of distinct objects.

A = { set of words }.

*interest numerical sets

Numerical Sets

$A = \{ w_1, w_2, w_3, \dots \}$.

w_i is a set of member / element of set A.

$\boxed{w_1 \in A}$ reads as 'belongs to'

natural language

formal language $\in$ = epsilon.

$w_5 \notin A$ does not belongs to.

1. set of natural

Numerical Sets

- Discrete (countable)
- Continuous (uncountable / measureable).

Discrete Sets.

① set of Natural Numbers

$N = \{1, 2, 3, \dots\}$

② set of whole Numbers

$W = \{0, 1, 2, 3, \dots\}$

set builder notation : to define sets in formal lang.

③ set of integers

$Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

| → such that

④ set of odd numbers.

$O = \{ x \mid x \% 2 == 1 \}$

⑤ set of even numbers

$E = \{ x \mid x \% 2 == 0 \}$

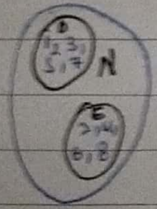
⑥ set of prime numbers

$P = \{ x \mid x \% y \,!= 0, \; y < x, \; y \in Z - \{0, \pm 1\} \}$

VENN DIAGRAMS: visual representation of sets

Subsets

disjoint → no common



odd or even are disjoint

→ $N = E \cup O$ $\cup$ → union

→ $O \subset N$ $\subset$ → subset

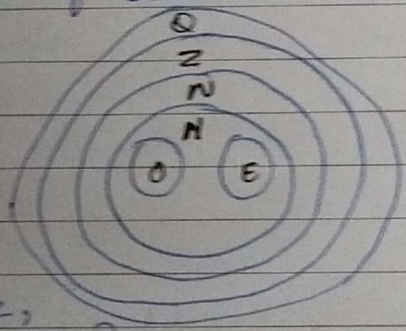
odd numbers are subset of natural numbers / Every odd number is a natural number.

Natural number contains odd numbers

→ $N \supset O$

natural number is a superset of odd number.

$N \subset W \subset Z$



2. CONTINOOUS

① Rational Numbers (Q)

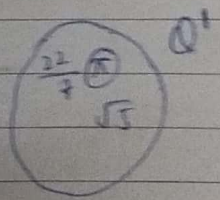
$Q = \{x \mid x = p/q,\ p \in Z,\ q \in Z - \{0\}\}$

if quotient is continuous or repeated

② Irrational Numbers (Q')

$Q \cap Q' = \phi$

Disjoint sets



REAL NUMBERS.

$R = Q \cup Q'$ set of real numbers.

Operations On Sets:

27/10/22

①

M = {Students who studies mathematics}

$n(M) = 15$ → no. of elements in M is 15

$\boxed{total = 100}$

. no. of elements in a set is called as cardinality of a set.

P = {Students who studies physics}

universal set = total = U .

① ① Complement ' $M' = U - M$

② Intersection $\cap$

③ Union $\cup$

④ Difference $-$

M' = {Those who do not study mathematics}

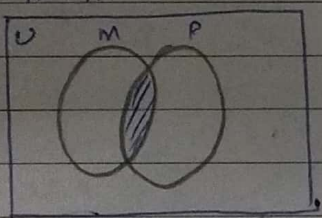
$M \cap P$ = {Those who study both maths & physics}

$P \cap M'$ = ~~$M \cap P$~~ {Those who only study physics but not maths}

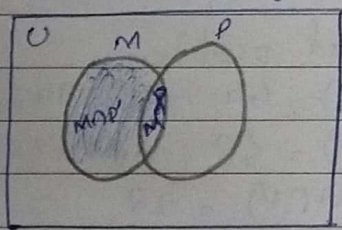
$M \cup P$ = { atleast maths / physics }

Venn diagram.

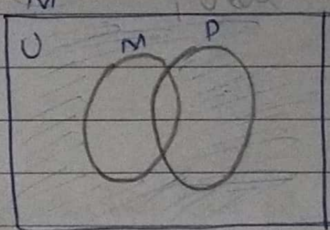
$M \cap P$

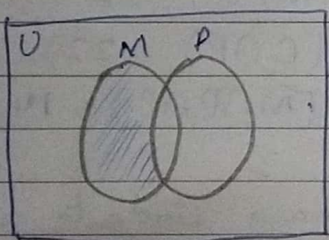


$M \cap P'$ → only maths = $M - M \cap P$



M'





U - M

$M \cup P = M + P - (M \cap P)$

M = {1, 3, 5, 7, 9, 10} P {2, 4, 6, 8, 9, 10}

$M \cup P$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$M \cap P$ = {9, 10}

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10} - {9, 10}

$U = \{1,2,3,4,5,6\}$

$M = \{1,2,3,4\}$

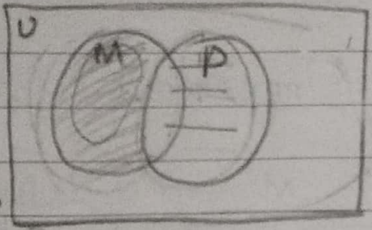
$P = \{3,4,5,6\}$

$P' = U-P = \{1,2,3,4,5,6\} - \{3,4,5,6\}$

$P' \{1,2\}$

$M \cap P' = \{1,2,3,4\} \cap \{1,2\}$

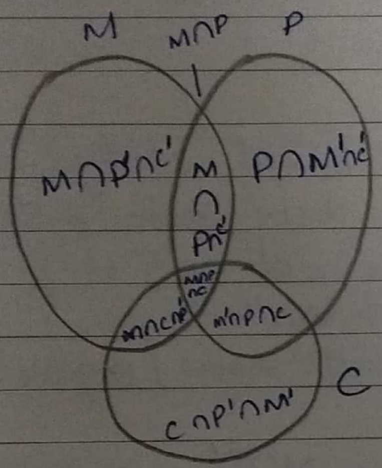
$M \cap P' = \{1,2\}$

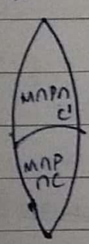


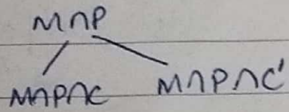
$M \cap (U-P)$

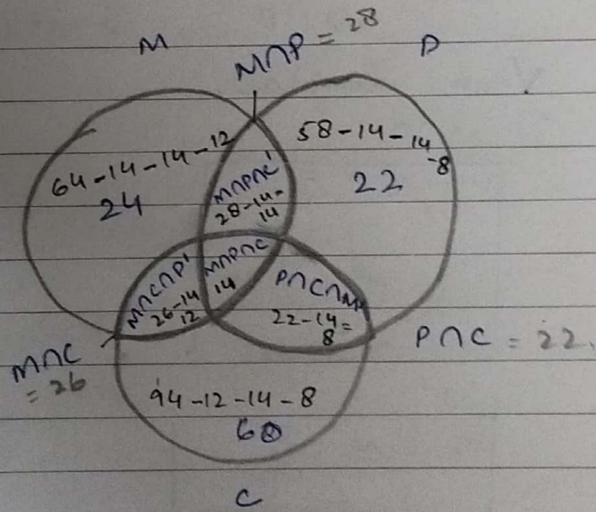
31/10/22.

Three Sets Venn Diagram.









online word problem:

$n(M) = 64 \quad n(C) = 94$

$n(P) = 58$

$n(M \cap P) = 28$

$n(M \cap C) = 26$

$n(C \cap P) = 22.$

$n(M \cap P \cap C) = 14$

Q: How many students are there who had taken only one course

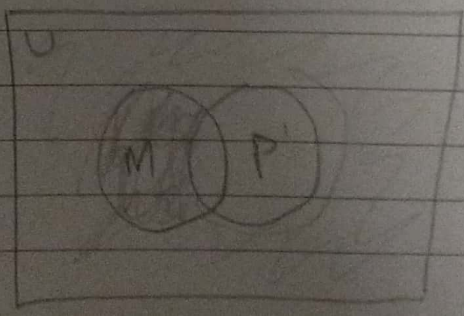
$\text{Total} = M \cap P' \cap C' + M' \cap P \cap C' + M' \cap P' \cap C.$

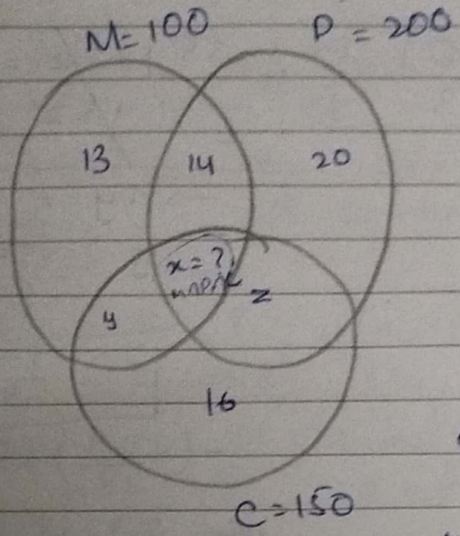
$T = 24 + 22 + 60$

$\boxed{T = 106}$

Book: Discret Maths. by Rozen

Book of Proof





$n(M \cup P \cup C) = 450$

$n(M \cap P \cap C) =$ ___

$M \to 100 = 13 + 14 + x + y.$

$100 = 27 + x + y$ — (i)

$P \to 200 = 14 + 20 + x + z$

$200 = 34 + x + z$ — (ii)

$C \to 250 = 16 + x + y + z$ — (iii)

$450 = 13 + 14 + x + y + 20 + x + z + \ldots$

$450 = x + y + z + 63$ — (iv)

~~Simpli~~

Solving eq's simultaneously

eq (i) → $\boxed{x + y = 73}$ — (i)   eq (2) = $x + z = 166$ eq (3) $x + y + z = 234$

$x + z = 166$ — (ii)

$x + y + z = 234$ — (iii)

$73 + z = 234$

$z = 234 - 73$

$\boxed{z = 161}$ put z in eq (ii)

$x + 161 = 166$

$x = 166 - 161$

$\boxed{x = 5}$ put in eq (i)

$5 + y = 73$

$y = 73 - 5$

$\boxed{y = 68}$

$x = 5,\ y = 68,\ z = 161$

$(x, y, z) = (5, 68, 161)$ Ans

Flow to Solve Three Linear Equations with three unknown using Elimination Method.

$2x - 5y + 3z = 10$ — (i)

$3x - 2y + 4z = 5$ — (ii)

$5x - 3y + 5z = 4.$ — (iii)

1. take any two pair of equations and eliminate any one variable from them.

taking eq. (i) & (ii)

$3(2x - 5y + 3z = 10) \Rightarrow \cancel{6x} - 15y + 9z = 30$

$2(3x - 2y + 4z = 5) \Rightarrow \cancel{6x} - 4y + 8z = 10$

$(-)\quad (+)\quad (-)\quad (-)$

$\boxed{-11y + z = 20}$ (A)

similarly take eq (2) & (iii)

$5(3x - 2y + 4z = 5) \Rightarrow \cancel{15x} - 10y + 20z = 25$

$3(5x - 3y + 5z = 4) \Rightarrow \cancel{15x} - 9y + 15z = 12$

$-\quad +\quad -\quad -$

$\boxed{-y + 5z = 13}$ (B)

PAPERWORK

$5(-11y + z = 20) \Rightarrow -55y + \cancel{5z} = 100$

$-y + 5z = 13 \Rightarrow -y + \cancel{5z} = 13$

$+ \quad - \quad -$

$-54y = 87$

$-y = \frac{\cancel{87}^{29}}{\cancel{54}_{18}} = \boxed{\frac{-29}{18}}$

$\boxed{y = -\frac{29}{18}}$

$\cancel{-\frac{29}{18} + 5z = 13}$

$\cancel{5z = 13 + \frac{29}{18}}$

$\cancel{5z = \frac{263}{18}}$

$\cancel{z = \frac{263}{18 \times 5}}$

$\cancel{\boxed{z = \frac{263}{90}}}$

$-\left(-\frac{29}{18}\right) + 5z = 13$

$\cancel{13} \; \frac{29}{18} + 5z = 13$

$5z = 13 - \frac{29}{18}$

$z = \frac{205}{18 \times 5}$

$\boxed{z = \frac{41}{18}}$

put y & z in eq ①

$2x - 5y + 3z = 10$

$2x - 5\left(-\frac{29}{18}\right) + 3\left(\frac{41}{18}\right) = 10$

$2x + \frac{145}{18} + \frac{123}{18} = 10$

$2x = 10 - \frac{145}{18} - \frac{123}{18}$

$2x = -\frac{44}{9}$

$\boxed{x = -\frac{22}{9}}$

$\boxed{(x, y, z) = \left(-\frac{22}{9}, -\frac{29}{18}, \frac{41}{18}\right)}$

②

3/11/22

Cartesian Product of two or more sets:

CS144

$N = \{n_1, n_2, n_3, \dots\} \rightarrow$

$S = \{s_1, s_2, s_3, \dots\}$

The cartesian product of two sets A and B is denoted by

$A \times B = \{(n_1, s_1), (n_1, s_2), (n_1, s_3), \dots (n_1, s_n)$
(A cross B) $(n_2, s_1), (n_2, s_n), \dots \}$.

cardinality $\rightarrow$ no. of elements in a set.

$n(N) = \underline{p}$ — p and q are no. of elements.

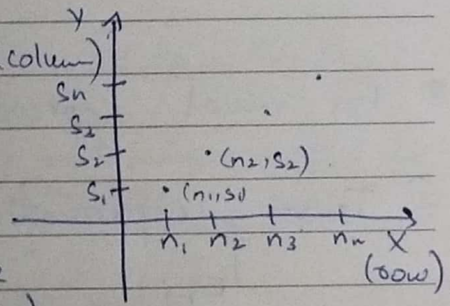
$n(S) = \underline{q}$

$p * q \rightarrow$

cartesian product is list of list

$A \times B \neq B \neq A$.

$(x, y) \neq (y, x)$
axis replaced



$(x, y) \rightarrow$ ordered pair

$(c_1, p_1, r_1, s_1) \rightarrow$ tuple (more than 2 in a pair).

Cartesian Product of two or more sets.

7/11/22.

$A_1, A_2, A_3, \dots, A_k$

$A_1 = \{a_{11}, a_{12}, a_{13}, \dots, a_{1m}\}$.

(in a_{11}: set number, element number)

$n(A_1) = m$.

$A_2 = \{a_{21}, a_{22}, a_{23}, \dots, a_{2p}$

$n(A_2) = p$. / $A_3 = \{a_{31}, a_{32}, a_{33}, \dots, a_{3q}\}$

$n(A_3) = q$.

$A_1 = \{a_{11}, a_{12}, a_{13}\}$

$A_2 = \{a_{21}, a_{22}, a_{23}\}$

$A_3 = \{a_{31}, a_{32}, a_{33}\}$.

$A_1 \times A_2 \times A_3 = \{(a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), \dots, (a_{11}, a_{21}, a_{3q}),$
$(a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), \dots, (a_{11}, a_{22}, a_{3q}),$
$(a_{11}, a_{23}, a_{31}), (a_{11}, a_{23}, a_{32}), \dots, (a_{11}, a_{23}, a_{3q})$

PAPERWORK

7/11/22

Example

$A_1 = \{a_{11}, a_{12}\} \Rightarrow n(A_1) = 2$

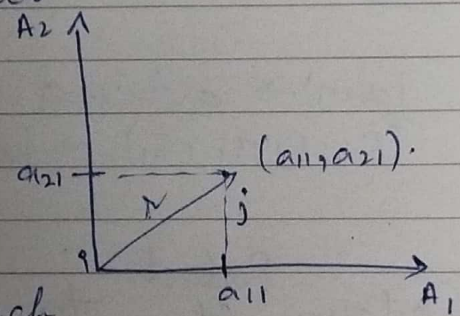
$A_2 = \{a_{21}, a_{22}, a_{23}\} \Rightarrow n(A_2) = 3 \rightarrow p$

$A_3 = \{a_{31}, a_{32}, a_{33}, a_{34}\} \quad n(A_3) = 4 \rightarrow p$

$A_1 \times A_2 \times A_3 = \{(a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{21}, a_{33}),$
$(a_{11}, a_{21}, a_{34}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}),$
$(a_{11}, a_{22}, a_{33}), (a_{11}, a_{22}, a_{34}), (a_{11}, a_{23}, a_{31}),$
$(a_{11}, a_{23}, a_{32}), (a_{11}, a_{23}, a_{33}), (a_{11}, a_{23}, a_{34}),$
$(a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{21}, a_{33}),$
$(a_{12}, a_{21}, a_{31}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}),$
$(a_{12}, a_{22}, a_{34}), (a_{12}, a_{22}, a_{34}), (a_{12}, a_{23}, a_{31}),$
$(a_{12}, a_{23}, a_{33}), (a_{12}, a_{23}, a_{33}), (a_{12}, a_{23}, a_{34})\}$

Total tuple in a cartesian product $= m * p * q$

① For visual representation → vector



② Tensor → representation of tuple.

T_{kij}

k ⇒ no. of sets.

i and j ⇒ no. of element in each set respectively.

∴ Finite and Infinite Sets.

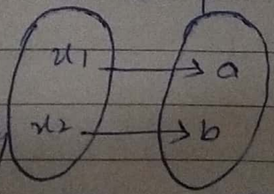
- countably infinite or enumerable sets
- uncountably (measureable). infinite or denumerable sets

function

• every input has an output, one input has only one output

one to one.

distinct inputs has distinct outputs respectively



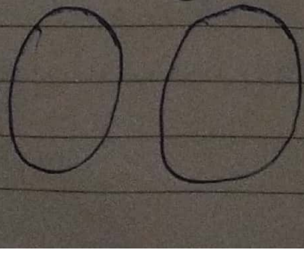
$\forall x_1, x_2 \in D_f$

$x_1 \neq x_2$

$f(x_1) \neq f(x_2)$.

onto

every output has corresponding at least one input



Bijective (one to one and onto)

$2x = 3y - 5$

Turing Machine

10/11/2022

FUNCTIONS:

- a function is a relation b/w two variables independent & dependent variable.

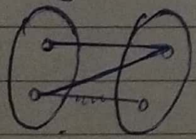
f : Doman $\rightarrow$ ~~co~~ domain / range

$\ni$ (such that)

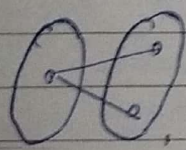
$\frac{f(\underset{\text{input}}{x})}{\text{output}}$ $\rightarrow$ it read as "f" of x.

Conditions

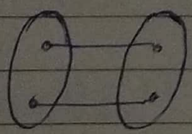
① every input must come up with an unique output.



unique
not distinct



not unique output for an input
it is just a relation, not a function
But outputs are distinct



unique
distinct

$\exists \rightarrow$ there exist
$\forall \rightarrow$ for all / every

1) Every input must has an output

Domain of f = D_f

* can't check if it is a function

(i) $\forall x \in D_f \; \exists \; f(x)$

(ii) $x_1, x_2 \in D_f \quad f(x_1) \quad f(x_2)$.

$x_1 = x_2 \rightarrow f(x_1) \neq f(x_2)$.

- injective (one to one)
- surjective (onto).

$\forall \; x_1, x_2 \in D_f$

$x_1 \neq x_2 \qquad f(x_1) = f(x_2)$.

PAPERWORK

14/11/22

Q Let $f: N \to R$ $f(x) = x^2 - 1$. $\Rightarrow y = f(x)$.
find whether it is onto or not?

Solve

$y = x^2 - 1$

Transform the equation in the form of $x = g(y)$.

$y = x^2 - 1$

$x^2 = y + 1$

$x = \pm\sqrt{y+1}$

let $y = 2 \in R \Rightarrow x = \pm\sqrt{3} \notin N \Rightarrow$

$\exists\ y \in R \ni x = g(y) \in N$.

(There exist) (belongs to) (~~such that~~)

Hence, f is not onto function.

! under root of real number is also a real number and every ~~natural~~ real number is not a natural number, hence, it is not an onto function.

-: Counting Techniques :-

① 1 pencil → 2Rs.
5 pencils → 10 Rs.

! multiplication is the fast process of addition

a, b, c → make a passwords with these characters.
no character can be repeated

_ _ _

Q. How many passwords of three different characters can be made using (a, b, c)? Repetition isn't allowed.

1. a b c
2. a c b
3. b a c
4. b c a
5. c a b
6. c b a

Ans) Total passwords $\boxed{6} \to 3!$

$\underset{\text{no. of characters}}{3} * \underset{\text{no. of ways}}{2} = 6$

—

Technique no. 1 (a, b, c, d) → 4!

① n objects → n!

"n" different objects can be arranged themselves in n! ways

$2-2=0$

$\cancel{2}! - 3! = 0$

permutation → change in order → increase in count

combination → arrangement doesn't matter in counting.

a, b, c, d

__ __ __

17/NOV/2022.

Permutation

Technique no. 01.

'n' different objects can be arrange themselves in n! ways.

Q. 1. In how many ways, three different people

(i) can arrange themselves in a row?

P_1, P_2, P_3

P_1	P_1	P_2	P_2	P_3	P_3	→ 3 choices (P_1, P_2, P_3)
P_2	P_3	P_1	P_3	P_1	P_2	→ 2 choices
P_3	P_2	P_3	P_1	P_2	P_1	→ 1 choice

$3 \times 2 \times 1$

$3! = 6$ different rows

(ii) Count only those lines in which two specified persons / friends are always next to each other.

count the given two persons as one object.

P_1, P_2, P_3 $\quad 3-2+1 = 2$

$P_1 P_2 \; P_3$

$P_2 P_1 \; P_3$

$P_3 \; P_1 P_2$

$P_3 \; P_2 P_1$

2 choices for arrangement

2! ways. ⇒ $2! = 2$

$2! * 2!$

$2! \to 2$

$+$

$2! \to 2$

$= 4$

Q.3 4 different persons. count lives in which three friends are always next to each other?

P_1, P_2, P_3, P_4

$\boxed{P_1\ P_2\ P_3}\ P_4$, $P_4\ \boxed{P_1\ P_2\ P_3}$

$\boxed{P_1\ P_3\ P_2}\ P_4$, $P_4\ \boxed{P_1\ P_3\ P_2}$

Two different objects can be arranged in 2! ways.
and three persons in the box can be arranged in 3! ways.

$\boxed{P_2\ P_1\ P_3}\ P_4$, $P_4\ \boxed{P_2\ P_1\ P_3}$

$\boxed{P_2\ P_3\ P_1}\ P_4$, $P_4\ \boxed{P_2\ P_3\ P_1}$

$\boxed{P_3\ P_1\ P_2}\ P_4$, $P_4\ \boxed{P_3\ P_1\ P_2}$

$\boxed{P_3\ P_2\ P_1}\ P_4$, $P_4\ \boxed{P_3\ P_2\ P_1}$

$\therefore$ Total Ways = 2! * 3!
2 × 6
$\boxed{\text{12 ways}}$

2! ↓ from one shape of box

3! arranging shape of box.

$\boxed{P_1 , P_2}$, $\boxed{P_3 , P_4}$

2! * 2! * 2!

① $(P_1, P_2) \rightarrow (P_3, P_4)$, (P_4, P_3)

② $(P_2, P_1) \rightarrow (P_3, P_4)$, (P_4, P_3)

2! * 2! * 2!
2 * 2 * 2
$\boxed{\text{8 ways}}$

Shaklein ++;

Permutation 21/NOV/2022

Q. How many passwords of three different letters (a, b, c). the password always start from 'a'?

a fix — 1 choice, 2 choice, 1 choice.

1 × 2 × 1 = 2 passwords can be made.

Q. How many three different digits code/natural
(i) numbers can be made using (0, 1, 2, 3, 4, 5)?

H	T	U
5 choices	5 choices	4 choices

5 × 5 × 4 = 100 different natural number can be formed

(ii) $203 < x < 432$.

H T U

= Natural number starts with 2 + Natural numbers start with 3 + Natural number start with 4

(a) 2 fix — 1 choice, 0 fix — 1 choice, 2 choices ⇒ 1 × 1 × 2 = 2

(b) 2 fix — 1 choice, 4 choices (1, 3, 4, 5), 4 choices (0, + 3 choices). 1 × 4 × 4 = 16.

(c) 3 fix — 1 choice, 5 choices, 4 choices. 1 × 5 × 4 = 20

(d) 4 fix — 1 choice, 3 choice (0, 1, 2) (3), 4 choices (0, 1). 1 × 3 × 2 = 12

4 fix — 1 choice, 1 choice, 2 choices. 1 × 1 × 2 = 2.

2 + 16 + 20 + 12 + 2 = 52 choices

PAPERWORK

(iii) even natural numbers. → from (0, 1, 2, 3, 4, 5).

___ ___ ___ = 75

5 choices 5 choices 3

0 fix

___ ___ ___

5 choice 4 choice 1 choice ⇒ $5 \times 4 \times 1 = 20$

2 fix (2, 4)

___ ___ ___

4 choices 4 choice 2 choice ⇒ $4 \times 4 \times 2 = 32$

$\boxed{52}$

(iv) count odd numbers.

16

1 fix

___ ___ ___

4 choices 1 choice

___ ___ ___

5 choice 5 choices 4 choices $5 \times 5 \times 4 = 100$

Total ⇒ 100 − even

100 − 52 = $\boxed{48}$

GROUP PERMUTATION.

1.	A	C	T
2.	A	T	C
3.	C	T	A
4.	C	A	T
5.	T	A	C
6	T	C	A

(A, A, T) ⇒ A A T

A T A

A T A

A A T $1+1+1=3$

T A A

T A A 3!

$n1$

$\frac{n!}{n1!}$

↓

(!)

• Same objects can be arrange themselves in $n1!\, n2! \ldots nk!$

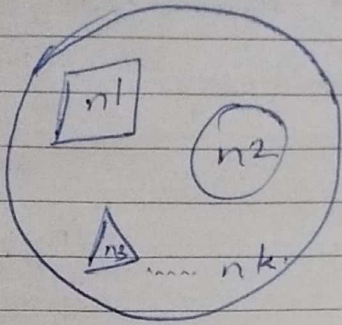
A A T

A T A

T A A 3!

→ (!) $\frac{n!}{n1! * n2! * , , * nk!}$

Group Permutation:
if there are n objects.



where $n = n_1 + n_2 + \cdots + n_k$.

$$\frac{n!}{n1! * n2! * \cdots nk!} = \binom{n}{n_1 * n_2 * \cdots n_k}$$

A A B C C C D D D D.

$$\frac{10!}{2! * 3! * 4!} \Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{2! * 3! * \cancel{4!}} \Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{2 \times 1 \times 3 \times 2 \times 1}$$

$$\Rightarrow \frac{10 \times 9 \times \cancel{8}^{4} \times 7 \times \cancel{6} \times 5}{\cancel{2} \times \cancel{6}} = 10 \times 9 \times 4 \times 7 \times 5 \quad \cancel{1720}$$

$$\boxed{12600}$$

24/11/22

GROUP PERMUTATION.

"n" = n_1 type + n_2 type + ... + n_k type

$$\binom{n}{n_1, n_2, \ldots n_k} = \frac{n!}{n_1! * n_2! * \cdots * n_k!}$$

Q.1 LETTER.

(i) How many strings of length "6" can be made using the alphabets in the word "LETTER".

Total characters = 6

no. of T = n_1 = 2.

no. of E = n_2 = 2.

$$\binom{6}{2, 2, 1, 1} = \frac{6!}{2! * 2!}$$

(ii) How many strings of length 6 that'll start with T.

(ii) How many strings of length 6 that'll start with T.

no. of characters $= n = 6$.

T is fixed, so $n = 5$.

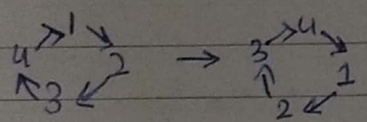
no. of E $= n_1 = 2$.

$$\frac{T_{fix}}{1\text{ choice}} \quad \frac{5!}{2!}$$

(iii) strings that'll start with L.

$$\frac{L_{fix}}{1\text{ choice}} \quad \frac{5!}{2! * 2!}$$

Circular Permutation:-



"n" different objects are to be arrange in a circle.
Number of ways of their arrangements $= (n-1)!$

Q. C_1 C_2 C_3 C_4.
In how many ways the computers can be arrange themselves in a ring network?

$\therefore (n-1)!$

$(4-1)!$

$3! = \underline{6 \text{ ways}}$.

Q from internet: WALLFLOWER.

(ii)

Bad main

it belongs to combinational logic

- In circular permutation, if the objects are distinct, so they canbe arrange in $(n-1)!$ ways.
- In circular permutation, if the objects are identical, so they can be arrange in $\frac{1}{2}[(n-1)!]$ ways

Q P1 P2 P3 P4 P5

In how many ways 5 different persons can be arrange in a ~~city~~ circle if two specified persons always come together.

P_1 $\boxed{P_2 \quad P_3}$ P_4 P_5

4 obj.

$3! * 2!$

28/NOV/22.

COMBINATION:

- counting problems in which arrangement does not matter.
- out of "n" different objects, "r" are to be selected

Ex:
In a class of 3 students, ~~three~~ two boys, we have to make CR's

(b_1, b_2, b_3).

$b_1 \ b_2 = b_2 b$ (count 1).

$b_1 \ b_3$

$b_2 \ b_3$

Out of "n" different object "r" are to be selected. The number of ways of their selection is denoted by ${}^nC_r$ or $\binom{n}{r}$ and are calculated as.

$${}^nC_r = \frac{n!}{(n-r)! * r!}$$

e.g. $${}^3C_2 = \frac{3!}{(3-2)! * 2!} =$$

$${}^3C_2 = \frac{3!}{1! * 2!} = \frac{3 \times \cancel{2!}}{\cancel{2!}} = 3$$

$${}^3C_2 = 3$$

PAPERWORK

CamScanner

Q. In how many ways, two CRs can be choose
(a) out of three girls and two boys?

3 Girls (g_1, g_2, g_3) — 2 Boys (b_1, b_2)

$g_1 g_2$ — $b_1 b_2$

$g_1 g_3$

$g_2 g_3$

$g_1 \to b_1$
$g_1 \to b_2$
$g_2 \to b_1$
$g_1 \to b_2$
$g_3 \to b_1$
$g_3 \to b_2$

3 G — 2 B

$2 \to {}^3C_2 * 0 \to {}^2C_0$

OR + $0 \to {}^3C_0 * 2 \to {}^2C_2$

OR + $1 \to {}^3C_1 * 1 \to {}^2C_1$

Total Ways $= 3C_2 * {}^2C_0 + 3C_0 * 2C_2 + 3C_1 * 2C_1$

OR

$= 5C_2 = \frac{5!}{(5-2)! * 2!} = \frac{5!}{3! * 2!} = 10$ ways

(b) if one girl and one boy is to be choosen?

3 Girls — 2 Boys

$3C_1$ — $2C_1$

$2C_1 + 2C_1 + 2C_1$

g_1 g_2 g_3 $\Rightarrow {}^3C_1 * 2C_1$

$\Rightarrow 3 * 2$

$= 6$ ways

Ans

(ii) One girl must be a CR, now how many way, another CR can be choosen.

$1C_1 * 4C_1 = 4$ ways

(iii) One girl must not be choosen as CR, so how many ways

$4C_2 = 6$ ways

(iv) if one girl must be encluded & there must be one girl & one boy CR has to be choosen?

$2C_1 * 2C_1 = 2 * 2 = 4$ ways

DISCRETE

Vowels = 3   Consonants = 5.

E = 2

I = 1   Total chars = 8.

Q. How many strings of length 8 can be made using the letters in the word DISCRETE, if vowels occupy the even places?

5 3 4 2 3 1 2 1

1C 2V 3C 4V 5C 6V 7C 8C

V V V V

$$^4C_3 * \left(\frac{5! * 3!}{2!} \right) = 1440$$

V V V

5 3 4 2 3 1 2 1

V V V

5 4 3 3 2 2 1 1

V V V

5 3 4 3 2 2 1 1

V V V

5 3 4 2 3 2 1 1

Mid Exam Topics

1. Venn diagrams
2. Permutation
3. Combination