LINEAR ALGEBRA (L.A)

07-03-2023

Book: Elementary Linear Algebra, 9th Ed. by Howard Anton

MATRIX ALGEBRA

MATLAB

Wolfram Alpha

- order: rows x column.
- Addition / Subtraction -> order of 1st & 2nd matrix should be same
  order ① == order ②.
- Matrix identity = I.
  $A_{2\times2} \cdot \boxed{I_{2\times2}} = A_{2\times2}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

additive identity
$2 + ⓪ = 2$
Multiplicative identity
$2 \times ① = 2$

- Additive Identity = O = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
  $A_{2\times2} + O_{2\times2} = A_{2\times2}$.
- Multiplicative Identity = I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
- $\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}$ row vector $\downarrow$ (1×3) $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ column vector, (3×1) => multiplication permissible.
  => $(2\times1)+(3\times3)+(1\times4) = \boxed{15}$
- Scalar Multiplication
  $\boxed{6} \begin{bmatrix} 2 & 3 \\ 0 & 8 \end{bmatrix} \Rightarrow 6 \times \begin{bmatrix} 2 & 3 \\ 0 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 6\times2 & 6\times3 \\ 6\times0 & 6\times8 \end{bmatrix} \Rightarrow \begin{bmatrix} 12 & 18 \\ 0 & 48 \end{bmatrix}$
- Transpose (A^T) => Rows becomes columns & columns becomes rows.
  Ex: $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $A = A^T$
  Symmetric Matrix => if ~~Exam~~ $A = A^T$, the A is symmetric

13-03-23

Linear Equation => $2x - y = 0$.

Non-linear Equation => $\sqrt{x} - xy = 18$ or $x^2 y = 19$.

$x^{1/2}$

$x + y = 5$
$x - y = 1$
} Gauss Elimination Method to solve } Gauss made it.

$2x + 3y - 3z = 18$ —— ①

$-x + 18y + 12z = 30$. —— ②

$9x + 0 \cdot y + 18z = 13$. —— ③

Coefficient Matrix

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 2 & 3 & -3 \\ -1 & 18 & 12 \\ 9 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 30 \\ 13 \end{bmatrix}$$

Column vector.

$[1 \quad 3 \quad 4]$ → row vector.
order = 1×3.

ROW Operation

rows can change position.

a row can be multiplied by any scalar e.g. $2 * R_2$

Augmented Matrix:

$$\begin{bmatrix} 2 & 3 & -3 & 18 \\ -1 & 18 & 12 & 30 \\ 9 & 0 & 18 & 13 \end{bmatrix} \begin{array}{l} \to R_1 \\ \to R_2 \\ \to R_3 \end{array}$$

Row operations are applied on augmented matrices.

Element/Entry.

Position = a_{31}

$2 \times (1/2) = 1$

scalar → Inverse; multiplicative identity.

solution to any equation is finding unknown.

$$[A]^{-1} [A] = I$$

$$\boxed{[A]^{-1} [A]} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [A]^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$I \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [A]^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

GUASSIAN ELIMINATION

$$\left[\begin{array}{cccc} \boxed{1x & 3y & 4z & 6} \\ 0x & \boxed{1y & 9z & 8} \\ 0x & 0y & \boxed{1z & -1} \end{array}\right]$$

Back Substitution

$\boxed{z = -1}$

$\boxed{y = 17}$

$\boxed{x = \quad}$

Echelon Form

$$\begin{bmatrix} 1 & 4 & 5 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- diagonals have 1
- last row have max zeros.
- next coming row will have zeros > prev row.

Reduced Echelon form

$$\begin{bmatrix} 1x & 0 & 0 & = 8 \\ 0 & 1y & 0 & = 3 \\ 0 & 0 & 1z & = -1 \end{bmatrix}$$

$x = 8$

$y = 3$

$z = -1$

Guass-Jordan.

when we convert augmented matrix to row-echelon form then it is Guassian elimination.

when we convert into reduced row ~~then it is Guass~~ Jordan elimination

$$x_1 + 2x_2 + 3x_3 + 2x_4 = -1$$
$$-x_1 - 2x_2 - 2x_3 + x_4 = 2.$$
$$2x_1 + 4x_2 + 8x_3 + 12x_4 = 4.$$

$$\begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array} \begin{array}{c} \begin{matrix} -2 & -4 & -6 & -4 & 2 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 3 & 2 & -1 \\ -1 & -2 & -2 & 1 & 2 \\ 2 & 4 & 8 & 12 & 4 \end{bmatrix} \end{array} \approx \begin{array}{l} R_2 + R_1 \\ R_3 + (-2)R_1. \end{array}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 & -1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 2 & 8 & 6 \end{bmatrix} \approx R_3 + (-2) R_2.$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 & -1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix} \approx \tfrac{1}{2} R_3 \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 & -1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1. & 2 \end{bmatrix}$$

Ex: $x_1 + x_2 + x_3 = 2$
$x_2 - x_3 = -1$
$-2x_2 - 3x_3 = -8$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & -3 & -8 \end{bmatrix}.$$

$R_1 + (-1)R_2$
$R_3 + (2)R_2$

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & & & \\ & & & \end{bmatrix}$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & -3 & -8 \end{bmatrix}$$

- $-1(R_2) + R_1 \rightarrow R_1$
- $2(R_2) + R_3 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -5 & -10 \end{bmatrix}$$

- $R_3/-5$

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

- $-2(R_3) + R_1 \rightarrow R_1$.

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

- $R_3 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 = -1$
$x_2 = 1$
$x_3 = 2$ ✓

Solve the system of linear equations using Guassian elimination

$$4x + 8y - 4z = 4$$
$$3x + 8y + 5z = -11$$
$$-2x + y + 12z = -17$$

$$\begin{bmatrix} 4/4 & 8/4 & -4/4 & 4/4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

- $1/4 \times R_1 \rightarrow R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

- $(-3)R_1 + R_2 \rightarrow R_2$.
- $(2)R_1 + R_3 \rightarrow R_3$.

$$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

- $(1/2) R_2$.

$$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

- $(-2)R_2 + R_1 \rightarrow R_1$
- $(-5) R_2 + R_3 \rightarrow R_3$.

$$\Rightarrow \begin{bmatrix} 1 & 0 & -9 & +15 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & -10 & 20 \end{bmatrix}$$

- $(-1/10) R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -9 & +15 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$9(R_3) + R_1 \rightarrow R_1$
$-4(R_3) + R_2 \rightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1_x & 0 & 0 & -3 \\ 0 & 1_y & 0 & 1 \\ 0 & 0 & 1_z & -2 \end{bmatrix}$$

$x = \cancel{4}\ -3$.
$y = 1$
$z = -2$.

Q

$x + y + z = 6 \Rightarrow 1 + 2 + 3 = 6$ ✓

$2x - y + 2z = 6 \Rightarrow 2(1) - (2) + 2(3) = 6$ ✓

$-3x + y + 3z = 8 \Rightarrow -3(1) + 2 + 3(3) = 8$ ✓

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 2 & 6 \\ -3 & 1 & 3 & 8 \end{array}\right].$$

- $-2(R_1) + R_2 \rightarrow R_2$
- $3(R_1) + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 0 & -6 \\ 0 & 4 & 6 & 26 \end{array}\right]$$

- $(1/-3) R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 4 & 6 & 26 \end{array}\right]$$

- $(-1)R_2 + R_1 \rightarrow R_1$
- $(-4)R_2 + R_3 \rightarrow R_3$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 6 & 18 \end{array}\right]$$

- $1/6(R_3)$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right].$$

- $(-1)R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x = 1, \; y = 2, \; z = 3$

$$\begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

in left most column, top first entry is not non-zero so to make it a non-zero digit, replace R_1 by R_2.

$$\begin{bmatrix} 2 & 4 & -10 & 6 & 12 & 28 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

- $(1/2)R_1$

$$\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

- $-2(R_1) + R_3 \rightarrow R_3$.

$$\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{bmatrix}$$

cover the top row & continue with sub-matrix.

RS_1, RS_2

- $RS_1(1/-2)$

$$\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{bmatrix}$$

cover submatrix's first row & continue.

$-5(RS_1) + RS_2 \rightarrow RS_2$

$$\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1/2 & 1 \end{bmatrix} RSS_1$$

cover submatrix's first row & continue

$RSS_1 \times 2$

- $RSS_1 \times 2$

$$\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

row echelon form!

- $(7/2)R_3 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

- $(-6)R_3 + R_1 \rightarrow R_1$

$$\begin{bmatrix} 1 & 2 & -5 & 3 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

- $5(R_2) + R_1 \rightarrow R_1$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 & 7 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

reduced row echelon form.

Ex: 5

$$\begin{aligned} x_1 + 3x_2 - 2x_3 \quad + 2x_5 &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\ 5x_3 + 10x_4 \quad + 15x_6 &= 5 \\ 2x_1 + 6x_2 \quad + 8x_4 + 4x_5 + 18x_6 &= 6 \end{aligned}$$

Augmented matrix.

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix}$$

- $-2(R_1) + R_2 \to R_2$, $-2(R_1) + R_4 \to R_4$.

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{bmatrix}$$

$(-1) R_2 \to R_2$,

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{bmatrix}$$

$(-5)R_2 + R_3 \to R_3$, $(-4)R_2 + R_4 \to R_4$

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \end{bmatrix}$$

interchange R_3 to R_4 in vice versa

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$1/6 (R_3)$.

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$-3(R_3) + R_2 \to R_2$

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

now, we have.

$$\begin{aligned} x_1 + 3x_2 + 4x_4 + 2x_5 &= 0 \\ x_3 + 2x_4 &= 0 \\ x_6 &= 1/3. \end{aligned}$$

$\to x_1 = -3x_2 - 4x_4 - 2x_5$

$\to x_3 = -2x_4$

$\to x_6 = 1/3.$

we can solve above eq's by parametizing variables on R.H.S

$\boxed{x_1 = -3r - 4s - 2t}$

$\boxed{x_3 = -2s}$ $\boxed{x_2 = r}$

$\boxed{x_6 = 1/3.}$ $\boxed{x_4 = s}$ $\boxed{x_5 = t}$

$2(R_2) + R_1 \to R_1$

$$\begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

CamScanner

VECTORS, MATRIX, System of equations,
20-03-2023
Determinant, INVERSE of a Matrix, Rank of a Matrix.

↳ $\boxed{\text{Cramer's Rule}}$

$$a x_1 + b x_2 = c_1 \qquad 2\times 2.$$
$$c x_1 + d x_2 = c_2.$$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ unknown, const $= \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.

Determinant of A

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \boxed{ad - bc}$ $\qquad x_1 = \dfrac{\begin{vmatrix} c_1 & b \\ c_2 & d \end{vmatrix}}{|A|} = \dfrac{\#}{\#} = \#$

if $|A| = 0$, then there is no solution for the given equation

$A^{-1} = \dfrac{1}{|A|} \text{Adj}(A).$ $\qquad x_2 = \dfrac{\begin{vmatrix} a & c_1 \\ c & c_2 \end{vmatrix}}{|A|}$

$(a)(b) = 0$ $\qquad \boxed{a, b \in \mathbb{R}}$

(i) $a = 0$
(ii) $b = 0$.
(iii) $a = b = 0$.

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Null matrix.

"CONCEIVE, BELIEVE, ACHIEVE"

$Ax + By = c_1$ — ①
$Cx + Dy = c_2$ — ②
} General form of linear equation

$y = 3x + 4$

$y = \underbrace{\boxed{m}}_{\text{slope}}x + c$

① point of intersection gives you a solution.

② parallel lines → no solution

e.g. $y = \boxed{3}x - 8$

$y = \boxed{3}x + 18$ slope same = parallel

$m_1 = m_2$ ⇒ parallel, no solution

③. $x + y = 3$ — ①

$2x + 2y = 6$ — ⑪.

$2(x + y) = 6$

$x + y = 6/2 \Rightarrow \boxed{x + y = 3}$ two lines overlap, so it has infinitely many solutions or many points of intersection.

Inverse of a matrix.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

or

$$\underbrace{[A^{-1}][A]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A^{-1} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix},$$

$\therefore A^{-1} = \frac{1}{|A|} Adj(A)$

$$I \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

$0 = -2$. false statement

So, the system has no solution (inconsistent).

QUIZ NEXT WEEK / ASSIGNMENT.

$$A^{-1} = \frac{1}{|A|} \text{adj}(A).$$

$A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}$. adj of $A = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$.

$|A| = 16 - 12 = \boxed{4}$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} 8/4 & -3/4 \\ -4/4 & 2/4 \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} 2 & -3/4 \\ -1 & 1/2 \end{bmatrix}.$$

To verify, multiply A with A^{-1}, if result matrix is I, then it is right: $A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

Doing row operations

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 4 & 8 & 0 & 1 \end{array}\right]$$ we've to shift unit to left side.

$\frac{1}{2}(R_1)$

$$= \left[\begin{array}{cc|cc} 1 & 3/2 & 1/2 & 0 \\ 4 & 8 & 0 & 1 \end{array}\right]$$

$-4(R_1) + R_2 \rightarrow R_2$

$$= \left[\begin{array}{cc|cc} 1 & 3/2 & 1/2 & 0 \\ 0 & 2 & -2 & 1 \end{array}\right]$$

$\frac{1}{2}(R_2)$.

$$= \left[\begin{array}{cc|cc} 1 & 3/2 & 1/2 & 0 \\ 0 & 1 & -1 & 1/2 \end{array}\right]$$

$-3/2(R_2) + R_1 \rightarrow R_1$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 2 & -3/4 \\ 0 & 1 & -1 & 1/2 \end{array}\right]$$

$\uparrow A^{-1}$

Cofactors.

$$A = \begin{array}{c|ccc} & C_{11} & C_{12} & C_{13} \\ & & & \\ C_{21} & & & \\ C_{31} & & & \end{array}\begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & -2 \\ 1 & -3 & 5 \end{bmatrix}\begin{array}{c} \\ C_{23} \\ C_{33} \end{array}$$

C_{22}, C_{32}

$(-1)^{i+j}\ C_{ij}$
$(-1)^{1+1}\ C_{11}$
$(-1)^{2}\ C_{11}$
$+C_{11}$

$(-1)^{i+j}\ C_{ij}$
$(-1)^{1+2}\ C_{12}$.
$(-1)^{3}\ C_{12}$
$-C_{12}$

matrix of cofactors $\rightarrow$ adj of A.

· if determinant of a matrix is 0, then it has no solution.

Verify that $|A| = |A^T|$

$$A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix} \qquad A^T = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}.$$

$|A| = -8 - 3 = -11$ $\qquad A^T = -8 - 3 = -11.$

2.2 By inspection, solve the equation.

$$\begin{vmatrix} x & 5 & 7 \\ 0 & x+1 & 6 \\ 0 & 0 & 2x-1 \end{vmatrix} = 0.$$

$$x \begin{vmatrix} x+1 & 6 \\ 0 & 2x-1 \end{vmatrix} = 0.$$

$$x\left[(x+1)(2x-1) - 6\times 0\right] = 0$$

$$x(x+1)(2x-1) = 0.$$

$x=0$, $x=-1$, $x=1/2$. ✓

$|A+B| \neq |A| + |B|$.

$|AB| = |A|\,|B|$.

Rank of a matrix.

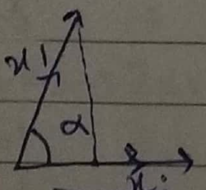
$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

no. of non-zero rows is Rank of a matrix.

Rank (A) = 3.

VECTORS

Scalar Product of Vectors.



$\boxed{x_2 \cos\alpha}$ projection of x_1 onto x_2.

$$\vec{x_1} \cdot \vec{x_2} = \vec{x_1} \cdot \vec{x_2} \cos\alpha.$$

$$\| \vec{x_1} \| \, \| \vec{x_2} \| \cos\alpha.$$

length, in algebra it is norm "$\| \;\|$"

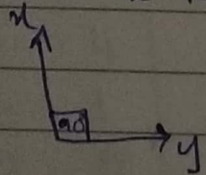
norm of $\vec{x_1} = \| \vec{x_1} \|$

let $x_1 = 2i + 3j$

Norm $= \| x_1 \| = \sqrt{2^2 + 3^2}$

- if $\vec{x} \cdot \vec{y} = 0$ the angle b/w vectors is $90°$ or orthogonal

$\cos 90° = 0$.



Inner product = scalar

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\underset{1\times3}{x^T y = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}} \Rightarrow \underset{1\times1}{\left[x_1 y_1 + x_2 y_2 + x_3 y_3 \right]}.$$

Outer Product

$$\underset{(3\times1)}{x}\ \underset{(1\times3)}{y^T} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 y_1 & x_1 y_1 & x_1 y_3 \\ x_2 y_1 & x_2 y_2 & x_2 y_3 \\ x_3 y_1 & x_3 y_3 & x_3 y_3 \end{bmatrix}$$

PAPERWORK

$2x_1 + 3x_2 + 6x_3 = 6$

$9x_1 \quad 10x_3 = 10$

$10x_1 + 3x_2 + 100x_3 = 90.$

write Augmented Matrix form

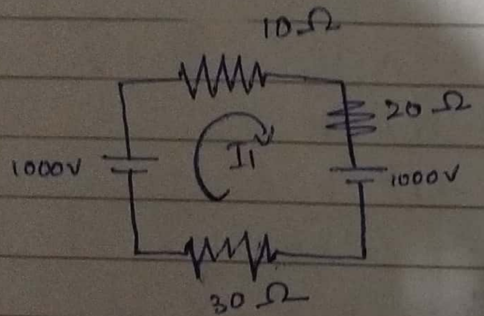
$$\left[\begin{array}{ccc|c} 2 & 3 & 6 & 6 \\ 9 & 0 & 10 & 10 \\ 10 & 3 & 100 & 90 \end{array}\right]$$

Krichoff's current law : $I = I_1 + I_2 + I_3.$

" Voltage Law $= V = IR$

Ω ohm unit

algebric sum +ve 4 -ve both are considered



$10i_1 + 20(i_1 + i_2) + 30i_1 = 1000 - 1000.$

$15i_2 + 20(i_2 + i_1) + 40i_2 + 5(i_2 + i_3) = 2000 - 1000.$

$25i_3 + 35i_3 + 5(i_3 + i_2) = 2000 - 2000.$

$60i_1 + 20i_2 = 0$

$20i_1 + 80i_2 + 5i_3 = 1000.$

$5i_2 \quad 65i_3 = 0$

$$\left[\begin{array}{ccc|c} 60 & 20 & 0 & 0 \\ 20 & 80 & 5 & 1000 \\ 0 & 5 & 65 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 60 & 20 & 0 & 0 \\ 20 & 80 & 5 & 1000 \\ 0 & 5 & 65 & 0 \end{array}\right]$$

$\frac{20}{3} + 80^{\times 3}$ $\quad \frac{20+240}{3} \rightarrow \frac{220}{3}$

$(1/60) R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 1/3 & 0 & 0 \\ 20 & 80 & 5 & 1000 \\ 0 & 5 & 65 & 0 \end{array}\right]$$

$\frac{-5}{11} + 65$

$-20(R_1) + R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1/3 & 0 & 0 \\ 0 & 220/3 & 5 & 1000 \\ 0 & 5 & 65 & 0 \end{array}\right]$$

$3/220 (R_2) \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1/3 & 0 & 0 \\ 0 & 1 & 3/44 & 150/11 \\ 0 & 5 & 65 & 0 \end{array}\right]$$

$-1/3(R_2) + R_1 \rightarrow R_1, \; -5(R_2) + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/44 & -50/11 \\ 0 & 1 & 3/44 & 150/11 \\ 0 & 0 & 2845/44 & -750/11 \end{array}\right]$$

$44/2845 (R_3) \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/44 & -50/11 \\ 0 & 1 & 3/44 & 150/11 \\ 0 & 0 & 1 & -600/569 \end{array}\right]$$

$-3/44 (R_3) + R_2 \rightarrow R_2, \; \frac{1}{44}(R_3) + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2600/569 \\ 0 & 1 & 0 & 7800/569 \\ 0 & 0 & 1 & -600/569 \end{array}\right]$$

$i_1 = -2600/569 = -4.56$

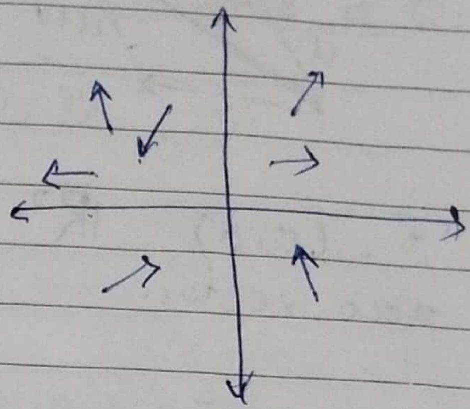
$i_2 = 7800/569 = 13.70$

$i_3 = -600/569 = -1.05$

3-4-23

VECTOR SPACE

2D 3D



2D $\mathbb{R}^2$ (x, y) . 3D $\mathbb{R}^3$ (x, y, z)

$\mathbb{R}^n$

length of a vector = norm of vector.

$\text{norm}(\vec{v}) = \|\vec{v}\|$

$\vec{i} = i \cdot i$

$= \|i\| \, \|\vec{i}\| \cos 0$

$= (1)(1)(1)$

$= 1$

$\vec{V} \cdot \vec{V} = V\hat{n} \cdot V\hat{n}$ let $\vec{v} = |\vec{v}|\hat{n}$

$= V^2(\hat{n} \cdot \hat{n})$. $= v\hat{n}$

$= V^2(1)$

$\sqrt{\vec{V} \cdot \vec{V}} = \sqrt{V^2} \rightarrow$ norm.

$\sqrt{\vec{V} \cdot \vec{V}} = V.$

$V(v_1, v_2) \in \mathbb{R}^2$

$V(v_1, v_2, v_3) \in \mathbb{R}^3$.

$V(v_1, v_2, \dots, v_n) \in \mathbb{R}^n$.

$|\vec{V}| = \sqrt{V_1^2 + V_2^2}$ => length or norm of $\vec{V}$.

components of a vector.

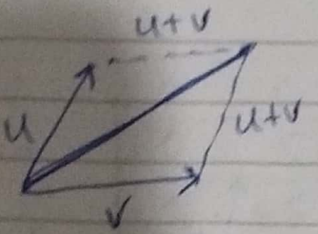
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 6 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$ column vector.

diff b/w matrix in vector.

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$u + v = (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n)$ => Resultant vector

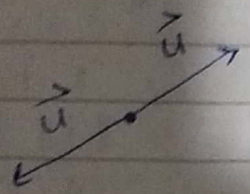
$cu = cu_1, cu_2, \dots cu_n$.



Lower dimension wale
higher dimension wale
ko nhi dekh skty.

$\vec{0} = (0,0)$ $\mathbb{R}^2$
zero vector.

origin is reference



$\vec{u} + (-\vec{v}) = \vec{0}.(0, 0, 0, \ldots, 0_n) \in \mathbb{R}^n$
To converge to origin, add vector to its additive inverse.

THEOREM 4.1

a) $u + v = v + u$ commutative
b) $u + (v + w) = (u + v) + w$ associative
c) $u + 0 = 0 + u$ additive identity.
d) $u + (-u) = 0.$ additive inverse
e) $c(u + v) = cu + cv$
f) $(c+d)u = cu + du.$
g) $c(du) = (cd)u.$
h) $1u = u$ identity vector $\Rightarrow 1.$
unit vector $\Rightarrow$

$u = (2, 5, -3)$, $v = (-4, 1, 9)$, $w = (4, 0, 2)$.
$2u = (4, 10, -6)$
$3v = (-12, 3, 27)$
$2u - 3v + w \Rightarrow (4, 10, -6) - (-12, 3, 27) + (4, 0, 2)$
$\Rightarrow (16, 7, -33) + (4, 0, 2)$
$\Rightarrow (20, 7, -31)$

Dot product will give scalar (because it $\theta = 0$).

$u = (1, -2, 4)$ and $v = (3, 0, 2)$

$u \cdot v = 3 - \cancel{0} + 8$

$u \cdot v = \cancel{9}\ 11$.

•

① $u \cdot v = v \cdot u$

② $(u+v) \cdot w = \cancel{u \cdot (v+w)} /\ u \cdot w + v \cdot w$.

③ $cu \cdot v = c(u \cdot v) = u \cdot cv$.

④ $u \cdot u \geq 0$, and $u \cdot u = 0$ iff $u = 0$.

it's always +ve.

let $\vec{u} = u_1 \vec{i} + u_2 \vec{j} + u_3 \vec{k}$

$\vec{u} \cdot \vec{u} = (u_1)(u_1) + (u_2)(u_2) + (u_3)(u_3)$

$\vec{u} \cdot \vec{u} = u_1^2 + u_2^2 + u_3^2 = \sqrt{\vec{u} \cdot \vec{u}}$. (norm in terms of dot product

$\sqrt{\|u\|^2} = \sqrt{u_1^2 + u_2^2 + u_3^2}$

$u = (1, 3, 5)$ and $v = (3, 0, 1, 4)$

$\|u\| = \sqrt{1^2 + 3^2 + 5^2}$

$= \sqrt{1 + 9 + 25}$

$= \sqrt{35}$

$\|v\| = \sqrt{3^2 + 0^2 + 1^2 + 4^2}$

$= \sqrt{9 + 1 + 16}$

$= \sqrt{26}$

$\neq \cancel{4}$

- A unit vector is a vector whose norm is 1.
- If v is a non zero vector, then the vector is a unit vector in the direction of v.

$\hat{u} = \dfrac{\vec{u}}{\|u\|} = \dfrac{20\hat{n}}{20} = \hat{n}$.

$\boxed{\hat{u} = \dfrac{\vec{u}}{\|u\|}}$

u-cap

Ex: 03.

a) Show that the vector (1,0) is a unit vector.

$\vec{v} = (1,0)$.

$$\hat{v} = \frac{1i + 0j}{\sqrt{1^2+0^2}} = 1$$

b) Find the norm of the vector (2,-1,3). Normalize this vector.

$\vec{v} = (2,-1,3)$

$\|v\| = \sqrt{2^2+(-1)^2+3^2}$

$\|v\| = \sqrt{4+1+9}$

$\|v\| = \sqrt{14}$.

$$\hat{V} = \frac{2i - j + 3k}{\sqrt{14}} = \frac{2}{\sqrt{14}}i - \frac{1}{\sqrt{14}}j + \frac{3}{\sqrt{14}}k.$$

$$\sqrt{\left(\frac{2}{\sqrt{14}}\right)^2 + \left(\frac{-1}{\sqrt{14}}\right)^2 + \left(\frac{3}{\sqrt{14}}\right)^2}.$$

$$\sqrt{\frac{4}{14} + \frac{1}{14} + \frac{9}{14}}$$

$$\sqrt{\cancel{14}/\cancel{14}}$$

$\boxed{1}$ Hence, Proved, it is a normalized vector.

Angle Between Vectors (in $\mathbb{R}^2$)

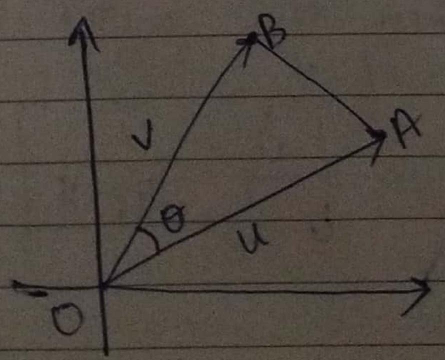
$\vec{U} \cdot \vec{V} = \|U\|\|V\| \cos\theta$.

$$\frac{\vec{U} \cdot \vec{V}}{\|u\| \|v\|} = \cos\theta.$$

$$\theta = \cos^{-1}\left(\frac{\vec{U} \cdot \vec{V}}{\|u\| \|v\|}\right)$$

$0 \leq \theta \leq \pi$

→ angle b/w two vectors



θ will be in radians.

⇒ π radians = 180°

Example. 4.

Determine the angle b/w the vectors $u = (1,0,0)$ & $v = (1, 0, 1)$ in $\mathbb{R}^3$.

$$\|u\| = \sqrt{1^2+0^2+0^2} = 1.$$

$$\|v\| = \sqrt{1^2+0^2+1^2} = \sqrt{2}.$$

$$u \cdot v = 1+0+0$$

$$u \cdot v = 1;$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right).$$

$$\theta = 45$$

$$\theta = \frac{\pi}{4} \text{ radians.}$$

ORTHOGONAL VECTOR.

θ b/w two vectors is $90°$ $(\pi/2)$.
if their dot product is 0, two vectors are orthogonal, since $\cos 90 = 0$.

$u \cdot v = 0$ orthogonal.

vector should be non-zero.

Ex. 5

Show that the following pairs of vectors are orthogonal.

(a) $(1,0)$ & $(0,1)$.

$$u \cdot v = (1\times 0) + (0\times 1) = 0. \rightarrow \text{orthogonal}$$

b) $(2,-3,1)$ & $(1,2,4)$

$$= (2\times 1) + (-3\times 2) + (1\times 4)$$

$$= 2 - 6 + 4 = 0$$

PAPERWORK

CamScanner

Ex. 6

Determine a vector in $\mathbb{R}^2$ that is orthogonal to $(3,-1)$. Show that there are many vectors so that they all lie on a line.

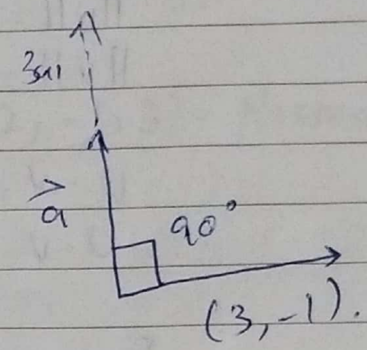
Suppose $\vec{a} = (a_1, a_2) \cdot (3,-1)$

$\vec{0} = (a_1, a_2) \cdot (3,-1)$.

$(0,0) = 3a_1 - a_2$.

$a_2 = 3a_1$

$(a_1, 3a_1)$.



Distance b/w Points.

$$d(x,y) = \sqrt{(x_1 - y_1)^2 \cdots (x_n - y_n)^2}$$

Ex: 7 :

Determine the distance b/w the points.

$x = (1, -2, 3, 0)$ and $y = (4, 0, -3, 5)$ in $\mathbb{R}^4$

$$d(x,y) = \sqrt{(1-4)^2 + (-2+0)^2 + (3-3)^2 + (0+5)^2}$$

$$= \sqrt{9 + 4 + 36 + 25}$$

$$= \sqrt{74}$$

LINEAR SPACE, SUBSPACE

10-03-2023

Machine Learning   Data Structures.

$\downarrow$

Pattern, Structures

$\rightarrow$ 2dim-in, out
$\downarrow$
matrix multiplic-

$\rightarrow$ Rotation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Subspaces

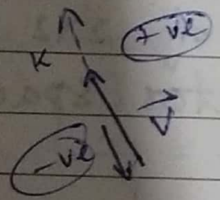
(A) $\boxed{\text{closed}}$ under $\oplus$

(B) $\boxed{\text{closed}}$ under scalar multiplication $k\vec{V}$

$\hookrightarrow k \in \mathbb{R}$

if subspace satisfies these two properties, then the subspace is also a linear space. (vector space)

- Let $\vec{U}, \vec{V} \in V \Rightarrow$ vector space
  and if $\vec{U} + \vec{V} \in V$, then vectors are closed under $\oplus$.
- $k \cdot \vec{V}$, $k \in \mathbb{R}$.



Example 1.

Let U be the subset of $\mathbb{R}^3$ consisting of all vectors of the form $(a, 0, 0)$ (with zeros as 2nd in 3rd components and $a \in \mathbb{R}$), i.e., $U = \{(a, 0, 0) \in \mathbb{R}^3\}$.

Show that U is a ~~sphere~~ subspace of $\mathbb{R}^3$.

Solution

$\begin{matrix} x & y & z \\ (a, & 0, & 0) \end{matrix}$

let $\vec{p}\,(a, 0, 0)$

$\vec{q}\,(b, 0, 0)$

$\because a, b \in \mathbb{R}$

① $(a, 0, 0) + (b, 0, 0) = (a+b, 0, 0)$. $(a+b) \in \mathbb{R}$.

closed under $\oplus$

② $k(a, 0, 0) = (ka, 0, 0)$ $a \in \mathbb{R} \Rightarrow ka \in \mathbb{R}$.

$k \in \mathbb{R}$

Ch: 3

Quiz 2: → $\boxed{\text{Ch: 4}}$ Solved problems.

Example 2.

Let V be a set of vectors of $\mathbb{R}^3$ of the form (a, a^2, b), namely $V = \{(a, a^2, b) \in \mathbb{R}^3\}$.

Solution

Let $\vec{p}$ (a, a^2, b)

$\vec{q}$ (c, c^2, f).

① $(a, a^2, b) + (c, c^2, f) = (\underbrace{a+c}, a^2+c^2, \underline{b+f})$.

$a+c \in \mathbb{R}$ also $b+f \in \mathbb{R}$ but $a^2+c^2 \neq (a+c)^2$

$a, c \in \mathbb{R}$ $\quad$ $b, f \in \mathbb{R}$

not a subspace.

Example 3.

Prove that the set W of 2x2 diagonal matrices is a subspace of the vector space M_{22} of 2x2 matrices

Solution

Let $u = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ & $v \begin{bmatrix} p & 0 \\ 0 & q \end{bmatrix} \in W$.

$\Rightarrow u + v = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} p & 0 \\ 0 & q \end{bmatrix} = \begin{bmatrix} a+p & 0 \\ 0 & b+q \end{bmatrix}$.

$\Rightarrow \underbrace{u+v}_{2\times2} \in W$.

Let $c \in \mathbb{R}$, $cu = c \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$\Rightarrow \begin{bmatrix} ca & 0 \\ 0 & cb \end{bmatrix}$.

$\Rightarrow \underbrace{cu}_{2\times2} \in W$

W is a subspace of M_{22}.

LINEAR COMBINATION OF VECTORS.

If w is a vector in vector space V, then w is said to be a linear combination of the vectors $v_1, v_2, \ldots v_r$ in V if w can be expressed as.

$\vec{U}, \vec{V}$

Linear combination of $\vec{u}$, $\vec{v}$

$\downarrow$

$c_1\vec{U} + c_2\vec{V}$

$w = k_1v_1 + k_2v_2 + \ldots k_r v_r$.

k = scalars → coefficients of LC.

a for n vectors.

$c_1U_1 + c_2U_2 + c_3U_3 + \cdots + c_nU_n$.

Vector spaces are represented by capital letters. V, U.

L.C of vectors in any set (suppose S), then S is subspace of V.

$W = \{(a, a, b) \mid a, b \in \mathbb{R}^2\} \subseteq \mathbb{R}^3$.

$(a, a, b) = a(1,1,0) + b(0,0,1)$.

$\therefore$ W is generated by $(1,1,0)$ a $(0,0,1)$.

e.g. $(2,2,3) = 2(1,1,0) + 3(0,0,1)$.

$(-1,-1,7) = -1(1,1,0) + 7(0,0,1)$.

Example 1

The vector $(5,4,2)$ is a linear combination of the vectors $(1,2,0), (3,1,4)$ a $(1,0,3)$, since it can be written $(5,4,2) = (1,2,0) + 2(3,1,4) - 2(1,0,3)$.

$$(5,4,2) = (1,2,0) + 2(3,1,4) - 2(1,0,3)$$

$$'' = (1 + (2\times3) - (2\times1), 2 + (2\times1) - (2\times0), 0 + (2\times4) - (2\times3))$$

$$'' = (1 + 6 - 2, 2 + 2 - 0, 0 + 8 - 6)$$

$$(5,4,2) = (5, 4, 2)$$

Example 2

Determine whether or not Show that the vectors $(3,-4,-6)$ cannot be expressed as a linear combination of the vectors $(1,2,3)$, $(-1,-1,-2)$ & $(1,4,5)$

Solution

Suppose, $c_1(1,2,3) + c_2(-1,-1,-2) + c_3(1,4,5) = (3,-4,-6)$

$$c_1 - c_2 + c_3 = 3$$
$$2c_1 - c_2 + 4c_3 = -4$$
$$3c_1 - 2c_2 + 5c_3 = -6$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 2 & -1 & 4 & -4 \\ 3 & -2 & 5 & -6 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \\ \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 1 & 2 & -10 \\ 0 & 1 & 2 & -15 \end{array}\right] \begin{array}{l} R_2 + R_1 \to R_2 \\ -R_2 + R_3 \to R_3 \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 3 & 7 \\ 0 & 1 & 2 & -10 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

$$c_1 + 3c_3 = 7$$
$$c_2 + 2c_3 = -10.$$
$$0 = -5 \; \times \text{ false}$$

The system has no solution.
Thus $(3,-4,6)$ is not a linear combination of the vectors $(1,2,3)$, $(-1,-1,-2)$ & $(1,4,5)$

Determine whether the matrix $\begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ is L.C of the matrices $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$, & $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ in the vector space of 2x2 M_{22}.

Solution

Let, $c_1\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + c_2\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix} + c_3\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$.

By ~~adding~~ simplifying.

$$\begin{bmatrix} c_1 & 0 \\ 2c_1 & c_1 \end{bmatrix} + \begin{bmatrix} 2c_2 & -3c_2 \\ 0 & 2c_2 \end{bmatrix} + \begin{bmatrix} 0 & c_3 \\ 2c_3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} c_1+2c_2+0 & 0-3c_2+c_3 \\ 2c_1+0+2c_3 & c_1+2c_2+0 \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$$

$$\begin{bmatrix} c_1+2c_2 & -3c_2+c_3 \\ 2c_1+2c_3 & c_1+2c_2 \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$$

equality.

$$c_1 + 2c_2 = -1$$
$$-3c_2 + c_3 = 7$$
$$2c_1 + 2c_3 = 8$$
$$c_1 + 2c_2 = -1$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & -3 & 1 & 7 \\ 2 & 0 & 2 & 8 \\ 1 & 2 & 0 & -1 \end{array}\right]$$

By solving to reduced row echelon form

$c_1 = +3$, $c_2 = -2$ & $c_3 = 1$.

Therefore, $\begin{bmatrix} -1 & 7 \\ 8 & 1 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

SPANNING SETS

The vectors $v_1, v_2, \dots v_m$ are said to be span a vector space, if every vector in the space can be expressed as a linear combination of those vectors.

Ex: 7.

Show that the vectors $(1,2,0), (0,1,-1)$ & $(1,1,2)$ span $\mathbb{R}^3$.

<u>Solution</u> * is type k ① question mid or final mein bhi ayega.

Let x, y, z be an arbitiary element of $\mathbb{R}^3$.

Suppose $(x,y,z) = c_1(1,2,0) + c_2(0,1,-1) + c_3(1,1,2)$.

$\Rightarrow (x,y,z) = (c_1 + c_3,\ 2c_1 + c_2 + c_3,\ -c_2 + 2c_3)$

$$\Rightarrow \begin{cases} c_1 + c_3 = x \\ 2c_1 + c_2 + c_3 = y \\ -c_2 + 2c_3 = z \end{cases} \Rightarrow \begin{cases} c_1 = 3x - y - z \\ c_2 = -4x + 2y + z \\ c_3 = -2x + y + z \end{cases}$$

$\Rightarrow (x,y,z) = (3x-y-z)(1,2,0) + (-4x+2y+z)(0,1,-1) + (-2x+y+z)(1,1,2)$.

$\Rightarrow$ The vectors $(1,2,0), (0,1,-1)$ & $(1,1,2)$ span $\mathbb{R}^3$.

Ex : 8

Show that the following matrices span the vector space M_{22} of 2×2 matrices.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

<u>Solution</u>

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22}$ (an arbitiary element).

we can express this matrix as follow.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Assignment 2.

- Exercise set 4.1 pages 190:
  Qs. 1, 3, 5, 7.

- Quiz 2 ON 17-04-2023.

17-03-23

LINEAR DEPENDENCE & INDEPENDENCE

The set of vectors $\{v_1, v_2, \dots, v_m\}$ in a vector space V is said to be linearly independent if there exist scalars $c_1, c_2, \dots, c_m$ not all zero, such that $c_1 v_1, c_2 v_2, \dots, c_m v_m = 0$.

All scalars $c_1, c_2, \dots c_n$ or $k_1, k_2, \dots k_n \in \mathbb{R}$.

$$\underbrace{0.1}_{c_1} v_1 + \underbrace{0}_{c_2} v_2 + \underbrace{3}_{c_3} v_3 = 0$$

$$v_3 = \frac{-0.1 v_1}{3}.$$

v_1 is dependent on v_3

Example 1.

Show that the set $\{(1,2,3), (-2,1,1), (8,6,10)\}$ is linearly dependent in $\mathbb{R}^3$.

Solution

Suppose, $c_1(1,2,3) + c_2(-2,1,1) + c_3(8,6,10) = 0.$

$(c_1, 2c_1, 3c_1) + (-2c_2 + c_2 + c_2) + (8c_3 + 6c_3 + 10c_3) = 0$

$$c_1 - 2c_2 + 8c_3 = 0$$
$$2c_1 + c_2 + 6c_3 = 0$$
$$3c_1 + c_2 + 10c_3 = 0.$$

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$$\left[\begin{array}{ccc|c} 1 & -2 & 8 & 0 \\ 2 & 1 & 6 & 0 \\ 3 & 1 & 10 & 0 \end{array}\right] \quad \begin{array}{l} -2R_1+R_2 \to R_2 \\ -3R_1+R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 8 & 0 \\ 0 & 5 & -10 & 0 \\ 0 & 7 & -14 & 0 \end{array}\right] \quad (1/5)R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 8 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 7 & -14 & 0 \end{array}\right] \quad \begin{array}{l} 2R_2+R_1 \to R_1 \\ -7R_2+R_3 \to R_3. \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$c_1 + 4c_3 = 0 \Rightarrow c_1 = -4c_3$

$c_2 - 2c_3 = 0 \Rightarrow c_2 = 2c_3.$

Example 3

Consider the functions $f(x) = x^2+1$, $g(x) = 3x-1$, $h(x) = -4x+1$ of the vector space P_2 of polynomials of degree ≤ 2.

Show that the set of functions $\{f, g, h\}$ is linearly independent.

Suppose

$$c_1 f + c_2 g + c_3 h = 0.$$

Since for any real number x.

$$c_1(x^2+1) + c_2(3x-1) + c_3(-4x+1) = 0.$$

Consider three convenient values of x, we get.

$x=0$ $\quad c_1 - c_2 + c_3 = 0$

$x=1$ $\quad 2c_1 + 2c_2 - 3c_3 = 0$

$x=-1$ $\quad 2c_1 - 4c_2 = 0.$

Thus, the system has a unique solution.

$$c_1 = c_2 = c_3 = 0.$$

Thus $c_1 f + c_2 g + c_3 h = 0$ implies that the set $\{f, g, h\}$ is linearly independent.

THEOREM 4.7

A set consisting of two or more vectors in a vector space is linearly dependent if and only if it is possible to express one of the vectors as a linear combination of the other vectors.

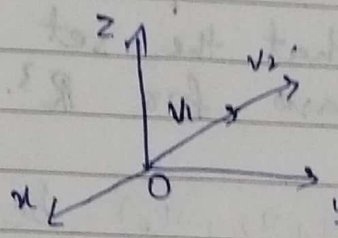
Ex: 4:

The set of vectors $\{v_1 = (1,2,1), v_2 = (-1,-1,0), v_3 = (0,1,1)\}$ is linearly dependent, since

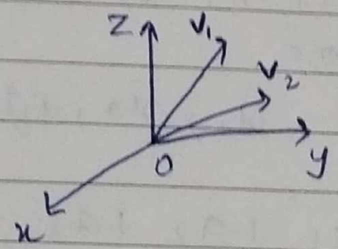
$$v_3 = v_1 + v_2 .$$

Thus v_3 is a linear combination of v_1 and v_2.

- Linearly dependent vectors lie on a line

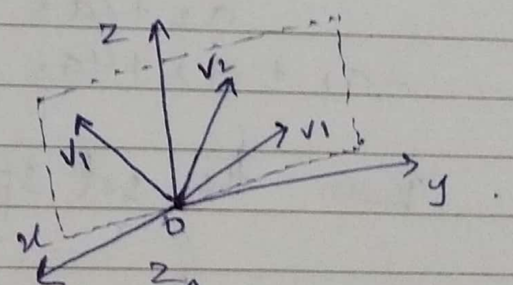


- Linearly independent vectors do not lie on a line

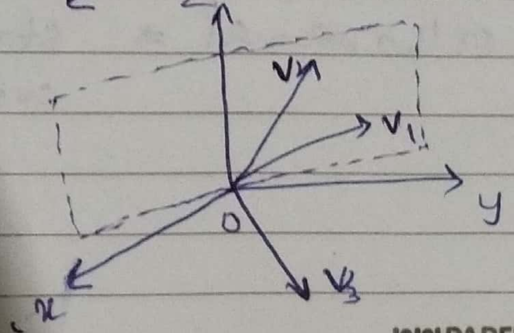


- Et

- Linearly dependent vectors lie in a plane

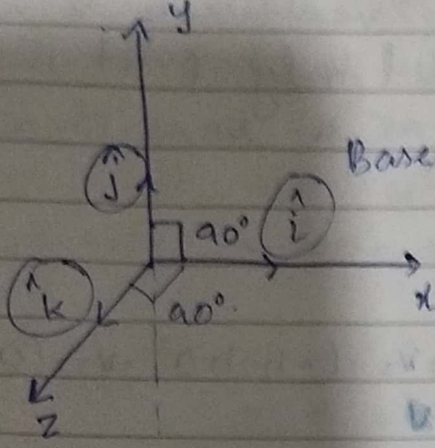


- Linearly independent vectors do not lie in a plane



aaptae 3 laws

Bases in Dimensions



unit vector $(\hat{i}, \hat{j}, \hat{k}) \rightarrow$ bases.

Bases:

Set of n vectors.
$\{(1,0,\ldots 0),(0,1,0\ldots 0)\ldots(0,\ldots,1)\}$ is a basis for $\mathbb{R}^n$. This basis is called the standard basis for $\mathbb{R}^n$
↓
har ek ki length 1 hoga

Example 1.

Show that the set $\{(1,0,-1),(1,1,1),(1,2,4)\}$ is a basis for $\mathbb{R}^3$.

Let (x_1, x_2, x_3) be an arbitrary element of $\mathbb{R}^3$.
Suppose.

$$(x_1, x_2, x_3) = a_1(1,0,-1) + a_2(1,1,1) + a_3(1,2,4).$$

$$\begin{aligned} a_1 + a_2 + a_3 &= x_1 \\ a_2 + 2a_3 &= x_2 \\ -a_1 + a_2 + 4a_3 &= x_3 \end{aligned} \Rightarrow \begin{aligned} a_1 &= 2x_1 - 3x_2 + x_3 \\ a_2 &= -2x_1 + 5x_2 - 2x_3 \\ a_3 &= x_1 - 2x_2 + x_3. \end{aligned}$$

Thus the set spans the space.

orthonormal $\Rightarrow$ standard basis (length 1)
every pair will be $90°$.

Graham - Shmidt Process

$v_1, v_2, \ldots, v_n$ Independent vectors

$\downarrow$ convert

$\frac{e_1}{|e_1|}, \frac{e_2}{|e_2|}, \ldots, \frac{e_n}{|e_n|}$. orthogonal basis

finding unit vector of orthogonal basis will give orthonormal.

Orthonormal Basis.

An orthogonal set in which each vector has norm 1.

QR - Decomposition.

$$A = Q R.$$

FINAL TERM

08-05-23.

Ex: 1.1 , Q: 27 in book. Mathematical Modeling

Suppose you are asked to find three real numbers such that the sum of the numbers is 12, the sum of two times the first plus the second plus two times the third is 5, and the third number is one more that the firs. Find (but do not solve) a linear equation system whose equation describe the three conditions.

$x_1 = ?$, $x_2 = ?$, $x_3 = ?$

$x_1 + x_2 + x_3 = 12.$

$2x_1 + x_2 + 2x_3 = 5.$

$x_3 = x_1 + 1 \Rightarrow -x_1 + 0 + x_3 = 1.$

Q: 25.

Suppose that a certain diet calls for 7 units of fat, 9 units of protein and 16 units of carbohydrates for the main meal, & suppose that an individual has three possible foods to choose from to meet these requirements.

Food 1: Each course contains 2 units of fat, 2 units of protein, and 4 units of carbohydrates.

$2x \rightarrow$ fat, $2x \rightarrow$ protein, $4x \rightarrow$ carbohydrates.

Food 2:

Each ounce contains 3 units of fat, 1 unit of protein, and 2 units of carbohydrates

$3y \rightarrow$ fat, $y \rightarrow$ protein, $2y \rightarrow$ carbohydrates

Food 3: Each ounce contains 1 unit of fat, 3 units of protein, and 5 units of carbohydrates.

$z \rightarrow$ fat, $3z \rightarrow$ protein,

$5z \rightarrow$ carbohydrates.

$$\boxed{\begin{aligned} 2x + 3y + z &= 7 \\ 2x + y + 3z &= 9. \\ 4x + 2y + 5z &= 16 \end{aligned}}$$

.64-20-80

Lower - Upper Decompisition.

Method: [A] decomposes to [L] an [U].

$$[A] = [L][U] = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$A = LU \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} d & e & f \\ ad & ae+g & af+h \\ bd & be+cg & b+ch+i \end{bmatrix}$$

computing both side

$\boxed{d=1}$, $\boxed{e=1}$, $\boxed{f=1}$.

$ad = 1$ — (1) $\quad ae+g = 2$ — (2) $\quad af + h = 2$ — (3).

$bd = 1$ — (4) $\quad be+cg = 2$ — (5) $\quad b+cj+i = 3$ — (6).

(1). $ad = 1$.
$a(1) = 1$
$\boxed{a=1}$

(4) $bd = 1$
$b(1) = 1$
$\boxed{b=1}$.

(2) $ae+g = 2$
$(1)(1) + g = 2$
$1 + g = 2$
$\boxed{g=1}$

(5) $be + cg = 2$.
$(1)(1) + c(1) = 2$
$1 + c = 2$
$\boxed{c=1}$

(3) $af + h = 2$.
$(1)(1) + h = 2$
$1 + h = 2$
$\boxed{h=1}$

(6) $b + cj + i = 3$
$(1) + (1)(1) + i = 3$.
$1 + 1 + i = 3$.
$2 + i = 3$
$\boxed{i=1}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Find an LU-Decomposition.

$$A = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} d & e & f \\ ad & ae+g & af+h \\ bd & be+cg & bf+ch+i \end{bmatrix}$$

Comparing:

$\boxed{d = 2}$ $\boxed{e = 6}$ $\boxed{f = 2}$.

$ad = -3$ — ① $\boxed{ae + g = -8}$ — ② $af + h = 0$ — ③

$bd = 4$ — ④ $be + cg = 9$ — ⑤ $bf + ch + i = 2$ — ⑥

① $ad = -3$
$a(2) = -3$
$\boxed{a = -3/2}$

④ $bd = 4$
$b(2) = 4$
$\boxed{b = 2}$

② $ae + g = -8$
$-\frac{3}{\cancel{2}}(\cancel{6}^{3}) + g = -8$
$-9 + g = -8$
$\boxed{g = 1}$

③ $af + h = 0$
$\left(\frac{-3}{\cancel{2}}\right)(\cancel{2}) + h = 0$
$\boxed{h = 3}$

⑤ $be + cg = 9$
$(2)(6) + c(1) = 9$
$12 + c = 9$
$c = 9 - 12$
$\boxed{c = -3}$

⑥ $bf + ch + i = 2$
$(2)(2) + (-3)(3) + i = 2$
$4 - 9 + i = 2$
$-5 + i = 2$
$\boxed{i = 7}$

$$\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix}$$

6. A = L · U

$$\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 6 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 2+0+0 & (1)(6)+(0)(1)+(0)(0) & (1)(2)+(0)(3)+(0)(7) \\ (-\tfrac{3}{2})(2)+(1)(0)+(0)(0) & (-\tfrac{3}{2})(6)+(1)(1) & (-\tfrac{3}{2})(2)+(1)(3)+(0)(7) \\ (2)(2) & (2)(6)+(-3)(1) & (2)(2)+(-3)(3)+(1)(7) \end{bmatrix}$$

$4-9+7$

$$\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$$

Hence, Proved!

ORTHONORMAL BASIS

set of vectors is orthogonal, if there dot products is zero (between each pair)

Theorem. if we normalize orthogonal vectors than they'll be orthonormal vectors.

Graham-Schmidt Process.

$$v_i = u_i - \frac{\langle u_i, v_1\rangle}{\|v_1\|^2} v_1 - \frac{\langle u_i, v_2\rangle}{\|v_2\|^2} v_2 - \dots - \frac{\langle u_i - v_{i-1}\rangle}{\|v_{i-1}\|^2} \times v_{i-1}$$

$$v_1 = u_1$$

$$v_2 = u_2 - \frac{\langle u_2, v_1\rangle}{\|v_1\|^2} v_1$$

$$v_3 = u_3 - \frac{\langle u_3, v_1\rangle}{\|v_1\|^2} v_1 - \frac{\langle u_3, v_2\rangle}{\|v_2\|^2} v_2$$

$u_1, u_2, u_3, \dots$ are linearly independent vectors.

$v_1, v_2, v_3, \dots$ are orthogonal basis.

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Question 2.

$v_1 = (2, -1, 0)$, $v_2 = (1, 1, 1)$.

$V_1' = V_1 \Rightarrow \boxed{V_1' = (2, -1, 0).}$

$$V_2' = V_2 - \frac{\langle V_2, V_1' \rangle}{\|V_1'\|^2} V_1'$$

$$V_2' = (1, +1, 1) - \frac{(2-1+0)}{(\sqrt{5})^2} (2, -1, 0)$$

$$V_2' = (1, 1, 1) - \left(\frac{2}{5}, \frac{-1}{5}, 0\right)$$

$1 - \frac{2}{5} = \frac{3}{5}$
$1 + \frac{1}{5}$

$$\boxed{V_2' = \left(\frac{3}{5}, \frac{6}{5}, 1\right)}$$

15-05-23.

Eigen Values, Rank of a Matrix:

Graham-Schmidt Process.

$$V_i = U_i - \frac{\langle U_i, V_1 \rangle}{\|V_1\|^2} V_1 - \frac{\langle U_1, V_2 \rangle}{\|V_2\|^2} V_2 - \dots$$

$\downarrow$

$A = QR$

and $R = Q^T \cdot A$

$Q = [q_1 \ q_2 \ q_3]$

$q_1 = v_1$, $q_2 = v_2$, $q_3 = v_3$

$\downarrow$ orthogonal

$q_1 = \frac{v_1}{\|v_1\|}$

$\downarrow$

orthonormal.

15/05/23

EIGEN VALUES, RANK OF A MATRIX

Rank of a Matrix

Assignment 3.
Quiz 3 next week

The non-zero rows of a matrix A that is in reduced row echelon form.

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

reduced echelon form.
Rank (A) = 3.

Example. Find the basis for the row space of the following matrix A, and determine its rank.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 1 & 1 & 5 \end{bmatrix}.$$

$-2R_1 + R_2$.
$-1R_1 + R_3$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

$-2R_2 + R_1$
$R_2 + R_3$

$$= \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 7$
$x_2 = -2$

Rank (A) = 2.

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"if you don't sacrifice for what you want, what you want will be the sacrifice."

Example

$$1x_1 + 2x_2 + 3x_3 + 5x_4 = b_1$$
$$2x_1 + 4x_2 + 8x_3 + 12x_4 = b_2 \qquad Ax = b$$
$$3x_1 + 6x_2 + 7x_3 + 13x_4 = b_3$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 5 & b_1 \\ 2 & 4 & 8 & 12 & b_2 \\ 3 & 6 & 7 & 13 & b_3 \end{array}\right] \quad \begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 5 & b_1 \\ 0 & 0 & 2 & 2 & -2b_1 + b_2 \\ 0 & 0 & -2 & -2 & -3b_1 + b_3 \end{array}\right] \quad (1/2)R_2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 5 & b_1 \\ 0 & 0 & 1 & 1 & -b_1 + \frac{b_2}{2} \\ 0 & 0 & -2 & -2 & -3b_1 + b_3 \end{array}\right] \quad \begin{array}{l} -3R_2 + R_1 \\ 2R_2 + R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & -2b_1 - \frac{3b_2}{2} \\ 0 & 0 & 1 & 1 & -b_1 + \frac{b_2}{2} \\ 0 & 0 & 0 & 0 & -5b_1 + b_2 + b_3 \end{array}\right]$$

Simplest Orthogonal Vectors

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{orthonormal basis unit vectors.}$$

Eigenvalues and Eigenvectors.

$$\begin{array}{r}150\\80\\50\\160\\\hline 340\end{array}$$

$[A]$, $x \in \mathbb{R}^2$:- $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. , $\lambda \in \mathbb{R}$

$\lambda = \lambda_1, \lambda_2$

$$[A]\, x = \lambda x \quad \text{—— (1)}$$
$$A x = \lambda I x \quad \text{—— (2)}$$
$$(A - \lambda I)\, x = 0 \quad \text{—— (3)}$$
$$B x = 0 \quad \text{—— (4)}$$

Yes, x lies in the nullspace B!

1st assumption.

$$[A]\, x = \lambda x$$
$$A x - \lambda x = 0$$
$$(A - \lambda)\, x = 0 \rightarrow$$ (Subtraction of a matrix with a scalar is not permissible).

$\downarrow$

$$(A - \lambda I)\, x = 0$$ $\rightarrow$ Subtraction of two matrices will again be another matrix

$\downarrow$

$$B x = 0$$

$\therefore B = A - \lambda I$

$$B = \begin{bmatrix} e & f \\ g & h \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$|B| = 0 \leftarrow B = \begin{bmatrix} e-\lambda & f \\ g & h-\lambda \end{bmatrix}$.

characteristic equation

Example $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$.

① $|B| = 0$

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 0.8 - \lambda & 0.3 \\ 0.2 & 0.7 - \lambda \end{vmatrix} = 0$$

$$(0.8 - \lambda)(0.7 - \lambda) - (0.3)(0.2) = 0.$$

$$0.56 - 0.8\lambda - 0.7\lambda + \lambda^2 - 0.06 = 0$$

$$0.5 - 1.5\lambda + \lambda^2 = 0.$$

$$\cancel{0.8 = 0.1\lambda}$$

$$\cancel{\lambda = \frac{0.5}{0.1}}$$

$$\lambda^2 - 1.5\lambda + 0.5 = 0.$$

$$\cancel{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \Rightarrow \cancel{\frac{-(-1.5) \pm \sqrt{(-1.5)^2 - 4(1)(0.5)}}{2(1)}}$$

$$\cancel{\frac{1.5 \pm \sqrt{2.25 - 2}}{2}}$$

$$\cancel{\frac{1.5 \pm 0.25}{2}}$$

$$\lambda^2 - 1.5\lambda + 0.5 = 0.$$

$$\lambda^2 - 1\lambda - 0.5\lambda + 0.5 = 0.$$

$$\lambda(\lambda - 1) - 0.5(\lambda - 1) = 0.$$

$$(\lambda - 1)(\lambda - 0.5) = 0.$$

$$\boxed{\lambda_1 = 1} \quad \boxed{\lambda_2 = 0.5}$$

Eigen values $\lambda_1 = 1$

$\lambda_2 = 0.5$

Step ② Eigen Vectors ?

$\because \quad Ax = \lambda x.$

$Ax = 1x \quad$ or $\quad Ax = 0.5x.$

$\therefore \quad (A - \lambda I)x = 0.$

$\lambda = 1 \quad (A - I)x = 0.$

$$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 0.8-1 & 0.3 \\ 0.2 & 0.7-1 \end{bmatrix}.$$

$$\begin{bmatrix} -0.2 & 0.3 \\ 0.2 & -0.3 \end{bmatrix}$$

$-0.2x_1 + 0.3x_2 = 0$

$0.2x_1 - 0.3x_2 = 0$

$\lambda = 0.5$

$$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} - 0.5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

$$\begin{bmatrix} 0.8-0.5 & 0.3 \\ 0.2 & 0.7-0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.3 & 0.3 \\ 0.2 & 0.2 \end{bmatrix}$$

$0.3x_1 + 0.3x_2 = 0$

$0.2x_1 + 0.2x_2 = 0.$

$\lambda = 1$

① $- \quad -0.2x_1 + 0.3x_2 = 0$

② $- \quad 0.2x_1 - 0.3x_2 = 0$

let ① $\quad 0.2x_1 = 0.3x_2$

if $x_1 = 0.3$ & $x_2 = 0.2$, they'll be identical.

$x_1 = 0.3 \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}$

$x_2 = 0.2$

$\lambda = 0.5$

$0.3x_1 + 0.3x_2 = 0 \Rightarrow 0.3(x_1 + x_2) = 0$

$0.2x_1 + 0.2x_2 = 0 \Rightarrow 0.2(x_1 + x_2) = 0$

$x_1 + x_2 = 0$

$x_1 = -x_2$

or $x_2 = -x_1$

infinitely many solution (parametric solutions)

if $x_1 = 1$, $x_2 = -1$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ -t \end{bmatrix}$

Original matrix $\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$

Eigen vectors $\begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Eigen values $(1, 0.5)$

Inverse of a 3X3 Matrix.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right]$$

Solve it by row operations, make left side reduced row echelon form and lastly the right side will give inverse of that matrix.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -2R_1+R_2 \\ 3R_1+R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 0 & 1 \end{array}\right] \quad 2R_2+R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 0 & 1/3 \end{array}\right] \quad \left(\frac{1}{3}\right) R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1/3 \end{array}\right] \quad -2R_3+R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -4 & 1 & -2/3 \\ 0 & 0 & 1 & 1 & 0 & 1/3 \end{array}\right]$$

To verify multiply matrix with ist inverse, if it results in identity matrix.

$$\begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix} \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -2/3 \\ 1 & 0 & 1/3 \end{bmatrix}.$$

$$\begin{bmatrix} -3+8-4 & 2-2-0 & 0+\frac{4}{3}-\frac{4}{3} \\ -6+12-6 & 4-3-0 & 0+2-2 \\ 9-24+15 & -6+6+0 & 0-4+5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Hence, Proved!

$$A \cdot A^{-1} = I$$

Q1: Find eigen-values and vectors using following matrices

$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

① $|B| = 0 \Rightarrow \begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \{(3-\lambda)(-1-\lambda)\} - (0)(8) = 0.$$

$$-3-3\lambda+\lambda+\lambda^2-0=0.$$

$$\lambda^2-2\lambda-3=0.$$

$$\lambda^2-3\lambda+\lambda-3=0$$

$$\lambda(\lambda-3)+(\lambda-3)=0.$$

$$(\lambda+1)(\lambda-3)=0$$

$$\lambda_1=-1, \lambda_2=3$$

Eigen values
$\lambda_1 = -1$
$\lambda_2 = 3$

PAPERWORK

CamScanner

Step ② Eigen vectors.

$$Ax = \lambda x.$$

$\lambda = -1 \Rightarrow Ax = (-1)x$ $\qquad\qquad$ $\lambda = 3 \Rightarrow Ax = 3x.$

$$\therefore (A - \lambda I)x = 0.$$

for $\lambda = -1$.

$$(A - (-1)I)x = 0.$$

$$(A + I)x = 0.$$

$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} \cancel{8}1 & 0 \\ 8 & 0 \end{bmatrix} \Rightarrow \begin{matrix} \text{if } x_1 = 0 \\ 8x_1 = 0. \end{matrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$12x_1 = 0$

$\boxed{x_1 = 0}$

origin

for $\lambda = 3$.

$$(A - 3I) = 0.$$

$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 8 & -4 \end{bmatrix} = 0 \Rightarrow \begin{matrix} 0 + 0 = 0. \\ 8x_1 - 4x_2 = 0. \end{matrix}$$

$\Downarrow$

$8x_1 = 4x_2.$

if $x_1 = 4$ $\quad$ $x_2 = 8.$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}.$$

Original Matrix	Eigen Vectors	Eigen Values
$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$	$(-1, 3)$.

CamScanner

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

① $|B| = 0$.

$$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = 0.$$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = 0.$$

$$\{(1-\lambda)(2-\lambda)\} - (3)(2) = 0.$$

$$2 - \lambda - 2\lambda + \lambda^2 - 6 = 0.$$

$$\lambda^2 - 3\lambda - 4 = 0.$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0.$$

$$\lambda(\lambda-4) + (\lambda-4) = 0.$$

$$(\lambda+1)(\lambda-4) = 0.$$

$$\lambda_1 = -1, \quad \lambda = 4.$$

Eigen values $\lambda_1 = -1$

$\lambda_2 = 4$.

for $\lambda = -1$.

$$(A - (-1)I)x = 0.$$

$$(A + I)x = 0$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0.$$

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} = 0.$$

$2x_1 + 2x_2 = 0 \Rightarrow 2(x_1+x_2) = 0$ $\qquad x_1 = -x_2$

$3x_1 + 3x_2 = 0 \Rightarrow 3(x_1+x_2) = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

for $\lambda = 4$.

$(A - 4I)x = 0$.

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 0.$$

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} = 0.$$

$-3x_1 + 2x_2 = 0$. $\quad 3x_1 = 2x_2$.

$3x_1 - 2x_2 = 0$. $\quad x_1 = 2 \quad x_3 = 3$.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}. \quad \cancel{A}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{matrix} \lambda \\ -\lambda \end{matrix}$$

$1 - 2 = \lambda$

$3 - 2 = -\lambda$

$-1 = \lambda$

$1 = -\lambda$

Original Matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

Eigen Vectors $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Eigen Values. $(-1, 4)$

$$\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}.$$

$Ax = \lambda x.$

$$\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$5 + 1$

$1 + 3$

$$\begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix}$$

① $|B| = 0$

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0.$$

$$\begin{vmatrix} 5-\lambda & -1 \\ 1 & 3-\lambda \end{vmatrix} = 0.$$

$\{(5-\lambda)(3-\lambda)\} - (-1)(1) = 0.$

$15 - 5\lambda - 3\lambda + \lambda^2 + 1 = 0.$

$\lambda^2 - 8\lambda + 16 = 0.$

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0.$

$(\lambda - 4) = 0 \quad (\lambda - 4) = 0.$

$\lambda = 4.$

Original form

$$\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}.$$

$(A - \lambda I)x = 0.$

$(A - 4I)x = 0.$

Eigen Vectors

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

Eigen Values.

(4)

$$\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = 0.$$

$x_1 - x_2 = 0$

$x_1 - x_2 = 0$

$x_1 = x_2$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

22-05-23

LINEAR TRANSFORMATIONS

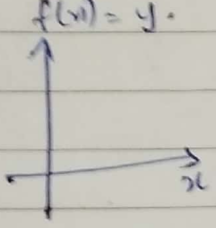
$\int \Delta [x] \Rightarrow \int 1 \cdot dx \Rightarrow x$

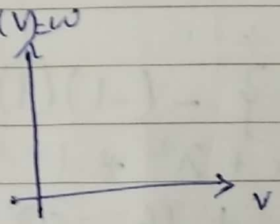
that means.

$\int, \Delta$ are inverses of each other.

$\cancel{f^{-1}}, \cancel{f}$

$f(x) = y.$





T is transformation

- for T, it is not obvious that it will always be linear

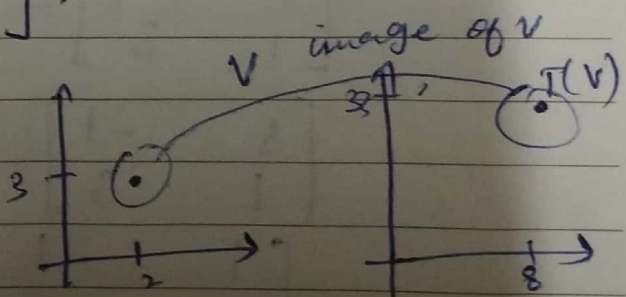
Suppose, $V = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $T = \begin{bmatrix} 1 & 2 \\ 3 & 9 \end{bmatrix}$

an you apply transformation

$T(V) = \begin{bmatrix} 1 & 2 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$

$T(V) = \begin{bmatrix} 2+6 \\ 6+27 \end{bmatrix}$

$T(V) = \begin{bmatrix} 8 \\ 33 \end{bmatrix}$



$$T^{-1} = \frac{1}{|T|}\begin{bmatrix} & \\ & \end{bmatrix} \qquad |T| = \begin{vmatrix} 1 & 2 \\ 3 & 9 \end{vmatrix} \Rightarrow 9-6=3.$$

$$T^{-1} = \cancel{\frac{1}{3}}\begin{bmatrix} 9/3 & -2/3 \\ -3/3 & 1/3 \end{bmatrix}$$

$$T^{-1} \; T(V)$$

$$\begin{bmatrix} 9/3 & -2/3 \\ -3/3 & 1/3 \end{bmatrix}\begin{bmatrix} 8 \\ 33 \end{bmatrix}.$$

$$\begin{bmatrix} \left(\frac{9}{3}\times 8\right) - \frac{2}{3}\times 33 \\ \left(-\frac{3}{3}\times 8\right) + \left(\frac{1}{3}\times 33\right) \end{bmatrix} \Rightarrow \begin{bmatrix} 24-22 \\ -8+11 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Conclusion:: $T(V) = V\,T.$ or $W = VT$

and

$V = T^{-1}\, T(V)$. or $V = T^{-1}W$.

Ex:1 (A function from $\mathbb{R}^2$ into $\mathbb{R}^2$).

$T: R^2 \to R^2 \qquad v = (v_1, v_2) \in \mathbb{R}^2.$

$T(v_1, v_2) = (v_1 - v_2,\ v_1 + 2v_1).$

(a) Find the image of $v = (-1, 2)$.

$V = (-1, 2).$

$T(-1, 2) = (-1-2,\ -1 + 2(-1)).$

$T(-1, 2) = (-3, -3) = w$

(b) Find the preimage of $w = (-1, 11)$.

$w = (-1, 11), \qquad V = T^{-1}\, w.$

(b) $T(v) = w = (-1, 11)$.

$T(v_1, v_2) = (v_1 - v_2,\ v_1 + 2v_2) = (-1, 11)$

$v_1 - v_2 = -1 \quad \times 2$

$v_1 + 2v_2 = 11$

$3v_2 = 12$

$\boxed{v_2 = 4}$.

$2v_1 - 2v_2 = -1$

$v_1 + 2v_2 = 11$

$3v_1 = 10$

$v_1 =$

$v_1 - 4 = -1$

$v_1 = -1 + 4$

$\boxed{v_1 = 3}$

$v_1 = 3,\quad v_2 = 4$.

$v = (3, 4)$

Thus, $(3, 4)$ is the preimage of $w = (-1, 11)$.

Linear Transformation (L.T).

V, W; vector space.

$T: V \to W$: V to W linear transformation.

① $T(u+v) = T(u) + T(v)$, $\forall u, v \in V$

② $T(cu) = c\,T(u)$, $\forall c \in \mathbb{R}$.

if these conditions holds, then it is L.T.

$T(u+v) = T(u) + T(w)$ — Addition in V; Addition in W

$T(cu) = c\,T(u)$. — Scalar Mult. in V; Scalar Mult. in W.

(2) linear operator: $T V : V$

↳ identity.

Ex:2: (Verifying a linear transformation T from R^2 to R^2
$T(v_1, v_2) = (v_1 - v_2, v_1 + 2v_2)$.

Proof:

Suppose $u = (u_1, u_2)$ & $v = (v_1, v_2)$: vector in R^2, $c \in R$

① Vector addition:

$u + v = (u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$.

apply T now.

$T(u+v) = T(u_1 + v_1, u_2 + v_2)$.

$= T((u_1+v_1) - (u_2+v_2), (u_1+v_1) + 2(u_2+v_2))$

$= T((u_1 - u_2) + (v_1 - v_2), (u_1 + v_1) + (2u_2 + 2v_2))$

$= T((u_1 - u_2) + (v_1 - v_2), (u_1 + 2u_2) + (v_1 + 2v_2))$

$= (u_1 - u_2, u_1 + 2u_2) + (v_1 - v_2, v_1 + 2v_2)$

$= T(u) + T(v)$

Hence, 1st hold.

②. Scalar multiplication.

$cu = c(u_1, u_2) = (cu_1, cu_2)$

apply $T \Rightarrow T(cu) = T(cu_1, cu_2)$

Ex:3

(a) $f(x) = \sin x$.

$\sin(x_1 + x_2) \neq \sin(x_1) + \sin(x_2)$

let $x_1 = \pi/2$, $x_2 = \pi/3$.

$\sin(\pi/2 + \pi/3) \neq \sin(\pi/2) + \sin(\pi/3)$

$\Rightarrow \sin x$ is not a L.T.

(b) $f(x) = x^2$.

$(x_1 + x_2)^2 \neq x_1^2 + x^2$.

$(1+2)^2 \neq 1^2 + 2^2$

$9 \neq 5$ $\quad$ x^2 is not L.T.

(c) $f(x) = x+1$

$(x_1+1 + x_2+1) \cancel{f} = (x_1+1) \cancel{f} + (x_2+1) \cancel{x}$

$x_1 = 1$, $x_2 = 2$.

$(1+1 + 2+1) \cancel{f} = (1+1) + (2+1)$.

$f(x_1+x_2) = x_1+x_2+1$

$f(x_1) + f(x_2) = (x_1+1) + (x_2+1) = x_1+x_2+1$.

Since $f(x_1+x_2) \neq f(x_1) + f(x_2)$

$f(x) = x+1$ is not a L.T.

- Zero Transformation:.
  $T: V \to W$ $\quad$ $T(v) = 0$, $\forall v \in V$. (Transformation se origin pe move ki jye ga).
- Identity Transformation.
  $T: V \to V$ $\quad$ $T(v) = v$, $\forall v \in V$.
  e.g $f(x) = x$.
- Properties of L.T.
  $T: V \to W$, $u, v \in V$.
  1. $T(0) = 0$.
  2. $T(-v) = -T(v)$.
  3. $T(u-v) = T(u) - T(v)$.
  4. If $v = c_1v_1 + c_2v_2 + \dots + c_nv_n$ (linear combination).
     Then $T(v) = T(c_1v_1 + c_2v_2 + \dots + c_nv_2)$.
     $= c_1T(v_1) + c_2T(v_2) + \dots + c_nT(v_n)$.

Ex:4: Linear Transformation on Bases.
Let $T : R^3 \to R^3$ be a L.T such that.
$T(1,0,0) = (2,-1,4)$
$T(0,1,0) = (1,5,-2)$
$T(0,0,1) = (0,3,1)$.
Find $T(2,3,-2)$.

Solution

$(2,3,-2) = 2(1,0,0) + 3(0,1,0) - 2(0,0,1)$.
$(2,3,-2) = (2,0,0) + (0,3,0) + (0,0,-2)$.
$(2,3,-2) = (2,3,-2)$ ✓

$T(2,3,-2) = 2T(1,0,0) + 3T(0,1,0) - 2T(0,0,1)$.
" $= 2(2,-1,4) + 3(1,5,-2) - 2(0,3,1)$
" $= (4,-2,8) + (3,15,-6) + (0,-6,-2)$.
$T(2,3,-2) = (7,7,0) \in \mathbb{R}^3$.

solve book examples

Ex:5: A Linear Transformation defined by a matrix.
Then function $T : R^2 \to \mathbb{R}^2$ is defined as

$$T(v) = Av = \begin{bmatrix} 3 & 0 \\ 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

(a) Find $T(v)$, where $v = (2,-1)$.

$$T(v) = Av$$

$$T(v) = \begin{bmatrix} 3 & 0 \\ 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$T(v) = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}$$

Symbolic Proof is important

(b) Show that T is a linear Transformation from $\mathbb{R}^2$ into $\mathbb{R}^3$.

① $T(U+V) = A(U+V) = A(U) + A(V) = T(U) + T(V)$

Ex: A projection in $\mathbb{R}^3$ $\quad R^3 \to R^2$.

29-05-23

APPLICATIONS OF LINEAR ALGERA

P.529 (Ch:10)

→ Equation of LINE
CIRCLE
PLANE
SPHERE

→ QR-DECOMPOSITION.

① $Q = [q_1 \quad q_2 \quad q_3]$

② Gram-Schmidt Process → $\left[\frac{v_1}{\|v_1\|} \quad \frac{v_2}{\|v_2\|} \quad \frac{v_3}{\|v_3\|} \right]$

③ $R = Q^T A$.

in $\mathbb{R}^3$

→ u_1, u_2, u_3 ⇒ given vectors.

$u_1 = (1, 1, 1)$
$u_2 = (0, 1, 1)$
$u_3 = (0, 0, 1)$.

→ Finding Q.

→ Gram Schmidt

$\langle u_2, v_1 \rangle \Rightarrow u_2 \cdot v_1$

$\boxed{v_1 = u_1}$, $\boxed{v_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{(\|v_1\|)^2} v_1}$.

$\boxed{v_3 = u_3 - \frac{\langle u_3, v_1 \rangle}{(\|v_1\|)^2} v_1 - \frac{\langle u_3, v_2 \rangle}{(\|v_2\|)^2} v_2}$

→ $V = [v_1, v_2, v_3]$.

$$V = \begin{bmatrix} 1 & -2/3 & 0 \\ 1 & 1/3 & -1/2 \\ 1 & 1/3 & 1/2 \end{bmatrix}$$

$$\Rightarrow Q = \begin{bmatrix} \frac{v_1}{\|v_1\|} & \frac{v_2}{\|v_2\|} & \frac{v_3}{\|v_3\|} \end{bmatrix}.$$

$\|v_1\| = \sqrt{3}$
$\|v_2\| = \sqrt{6}/3$
$\|v_3\|$

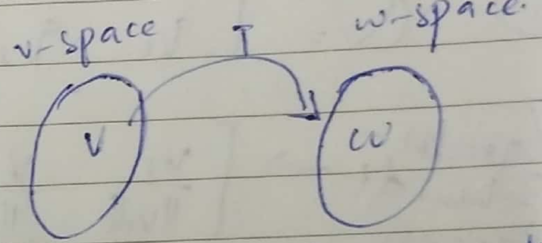
$$Q = \begin{bmatrix} 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \end{bmatrix}$$

Now find R.
then verify that $Q.R = A$.

Basic Transformations.

ROTATION.

$T(v) \Rightarrow$ Linear Transformation

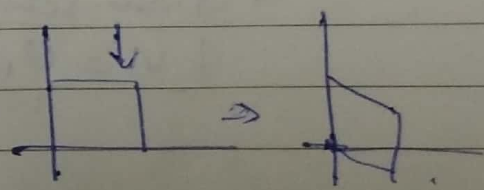


Matrix Multiplication
↳ Zoom in/out
↳ Rotation

Scaling.
Increase size in x or y direction

pg: 283 ⇒ book.
Geometry of Matrix operations on $\mathbb{R}^2$
✓ solve exercises as well.

Shearing.

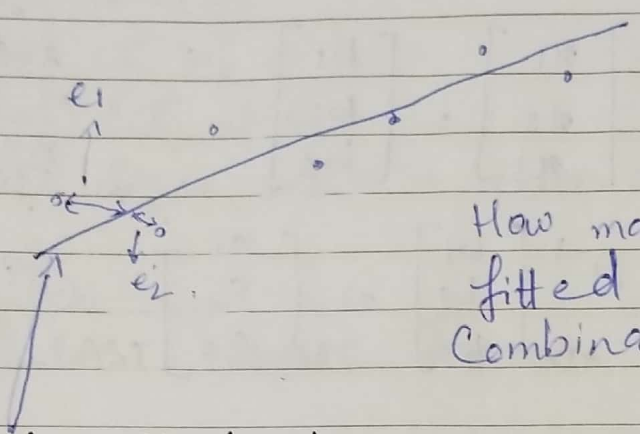


⇒ Skewing
Shearing forces
if we apply shear forces
its shape will be changed

Data Modeling

Linear Least-Squares Fitted Models

→ $y = ax+b$.



7 points.

How many straight lines can be fitted.

Combinations $\binom{7}{2} \Rightarrow {}^{7}C_2$.

Best fitted line ⇒ sum of squared error is minimum.

$$e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2 + e_6^2 + e_7^2$$

$e_5^2 = 0$ → because this point lies on the line.

$$\text{Min} = \sum_{i=1}^{7} e_i^2 \Rightarrow E$$

$$\frac{\partial E}{\partial a} = 0, \quad \frac{\partial E}{\partial b} = 0.$$

Non-Linear

How to Solve Least-Square. Go to book.

Linear Least Squares.

Ch: 9, 10, 11 Book Problems.

Find x that comes "closest" to satisfying $Ax \neq b$.

$Ax \neq b.$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$\underset{3\times3}{A} = \underset{3\times3}{Q}\underset{3\times3}{R}.$

$\underset{3\times3}{A}\underset{3\times1}{x} = \underset{3\times1}{b}$

$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$

$R \begin{bmatrix} 4+9 & 2+3 \\ 2+3 & 1+1 \end{bmatrix}$

$\begin{bmatrix} 13 & 5 \\ 5 & 1 \end{bmatrix}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}.$$

$3\times3 \quad 3\times1$

$x_1 \simeq c_1$

$x_2 \simeq c_2$

$x_3 \simeq c_3.$

$A = QR$

$R = A^{-T}Q$

$R = Q^TA$

$A = QR$

$A = QR$

$R = Q^TA \qquad R = \frac{A}{Q} = AQ^T$

$A^{-1}A = 2 \qquad 2 = -3$

$A = 2. \qquad 1 = A^{-1}2$

Composite Transformation.

Using matrix A. $\begin{bmatrix} 2 & 3 & 6 \\ 9 & 6 & 6 \\ 1 & 3 & 0 \end{bmatrix}$.

find (i) A^{-1}

(ii) Rank(A)

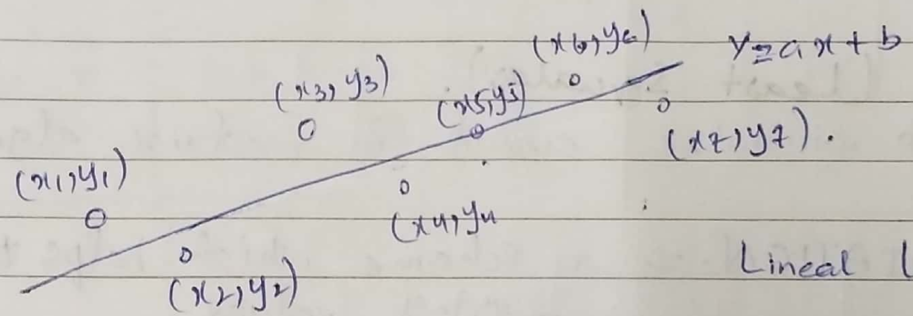
(iii) LU of A

(iv) QR of A

(v) Eigen values & vectors of A

5 - June - 23

LEAST SQUARE FITTING



Lineal least square

$^7C_2 = 21$.

Best fitted line.

$$\underbrace{\sum_{i=1}^{7} e_i^2}_{\text{Least}} = e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2 + e_6^2 + e_7^2.$$

$$\hat{y}_i = ax_i + b.$$

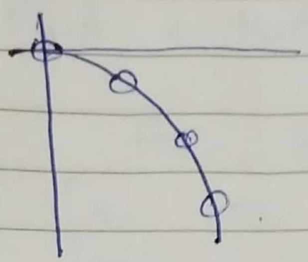
$$\Sigma x_i, \ \Sigma y_i, \ \Sigma x_i y_i.$$

Partialy differential

- (x_1, y_1)
- (x_2, y_2).

for $e_3 = y_3 - \hat{y}_3$

· Non-linear Least square Fitting.



in real life, non-linear is applied mostly. (more applications)

$p = -\frac{1}{2} g t^2$.
estimate g from fit.

↓
almost 0 error.

Pg: 381 (Least Squares).
How to minimize errors in matrix algebra.

IMPUTATION. ⇒ a scheme which helps to find/estimate missing values.

Example:

$$x_1 - x_2 = 4$$
$$3x_1 + 2x_2 = 1$$
$$-2x_1 + 4x_2 = 3.$$

$Ax = b$.

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$$

redundant system
⇒ 2 unknowns
⇒ 3 equations

convert the system.
find $A^T . A$

1 Question in final!

$$A^T \cdot A = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 1+9+4 & -1+6-8 \\ -1+6-8 & 1+4+16 \end{bmatrix}$$

$$\boxed{A^T \cdot A = \begin{bmatrix} 14 & -3 \\ -3 & 21 \end{bmatrix}} \quad 2\times 2. \quad \left(\begin{matrix} 2 \text{ unknown} \\ 2 \text{ equations} \end{matrix}\right)$$

$$A^T \cdot b = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$$

$$A^T \cdot b = \begin{bmatrix} 4+3-6 \\ -4+2+12 \end{bmatrix}$$

$$\boxed{A^T \cdot b = \begin{bmatrix} 1 \\ 10 \end{bmatrix}} \quad 2\times 2.$$

$$A^T \cdot Ax = A^T \cdot b.$$

$$\begin{bmatrix} 14 & -3 \\ -3 & 21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 14 & -3 & 1 \\ -3 & 21 & 10 \end{array}\right] \quad \begin{matrix} 1/14 \, R_1 \\ 3R_1 + R_2 \end{matrix}$$

$$\left[\begin{array}{cc|c} 1 & -3/14 & 1/14 \\ 0 & 285/14 & 143/14 \end{array}\right] \quad 14/285 \, R_2 \, , \, 3/14 \, R_2 + R.$$

$$\left[\begin{array}{cc|c} 1 & 0 & 17/95 \\ 0 & 1 & 143/285 \end{array}\right] \Rightarrow \boxed{\begin{matrix} x_1 = 17/95 \\ x_2 = 143/285 \end{matrix}}$$

$Ax = b$.

errors $b - Ax = 0$.

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 17/95 \\ 143/285 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} \frac{17}{95} - \frac{143}{285} \\ \left(3 \times \frac{17}{95}\right) + \left(\frac{2 \times 143}{285}\right) \\ \left(-2 \times \frac{17}{95}\right) + \left(\frac{4 \times 143}{285}\right) \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} -\frac{92}{285} \\ \frac{439}{285} \\ \frac{95}{57} \end{bmatrix} \rightarrow \frac{94}{57}.$$

$$-\frac{34}{95} + \frac{572}{285}$$

$$\text{Error} = \begin{bmatrix} 1232/285 \\ -154/285 \\ 4/3 \end{bmatrix} \text{ ??}$$

$$\|b - Ax\| = \sqrt{e_1^2 + e_2^2 + e_3^2}.$$

$$\text{''} = \sqrt{\left(\frac{1232}{285}\right)^2 + \left(-\frac{154}{285}\right)^2 + \left(\frac{4}{3}\right)^2}$$

$$\|b - Ax\| \cong 4.556$$

Pg: 529.

Equation of Line, Plane, Circle, Sphere.

Eq. of straight line

$\Rightarrow y = ax + b$ (x_1, y_1)

or $y = mx + c.$ (x_2, y_2)

$y_2 - y_1 = m(x - x_1).$

in Matrix algebra.

corresponding matrix for straight line.

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

for circle $x^2 + y^2 = r^2.$

$$\begin{vmatrix} x^2 & y^2 & 1 \\ '' & '' & 1 \\ '' & '' & 1 \end{vmatrix} = 0.$$

for plane $c_1 x + c_2 y + c_3 z = 3.$

for sphere $(x-a)^2 + (y-b)^2 + (z-c) = r^3$

Linear Interpolation.

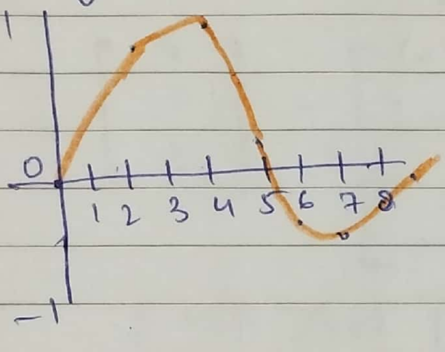
Quadratic

Cubic Spline Interpolation book pg. no. 540.

12-06-23

Cubic Splines, MARKOV Chains, Graph Theory

Scatterplot, Scattergram.



Scatterplot: Gives an idea about the behaviour of the system.

more than 100 ways to to plot graph.

function is determined using interpolation.

Scatter plot fitted by linear interpolation.
polynomial interpolation

$y = ax^1 + b \Rightarrow$ two points.
$y = ax^2 + bx + c \Rightarrow$ three points.
$y = ax^3 + bx^2 + cx + d \Rightarrow$ four points.

$(-3, 2), (0, 5), (3, 0), (-5, 0).$

find cubic spline.

$\therefore \quad y = ax^3 + bx^2 + cx + d.$

$(-3, 2) \Rightarrow 2 = a(-3)^3 + b(-3)^2 + c(-3) + d. \quad — ①$

$(0, 5) \Rightarrow 5 = a.(0)^3 + b.(0)^2 + c(0) + d. \quad — ②$

$(3, 0) \Rightarrow 0 = a(3)^3 + b.(3)^2 + c(3) + d. \quad — ③$

$(-5, 0) \Rightarrow 0 = a(-5)^3 + b(-5)^2 + c(-5) + d \quad — ④$

$Ax = b.$

$$\begin{bmatrix} -27 & 9 & -3 & 1 \\ 0 & 0 & 0 & 1 \\ 27 & 9 & 3 & 1 \\ -125 & +25 & -5 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

$$A^{-1} A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = A^{-1} \begin{bmatrix} 2 \\ 5 \\ 0 \\ 0 \end{bmatrix}.$$

find $A^{-1} \Rightarrow$

$$\left[\begin{array}{cccc|cccc} -27 & 9 & -3 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 27 & 9 & 3 & 1 & 0 & 0 & 1 & 0 \\ -125 & +25 & -5 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$\left(-\frac{1}{27}\right) R_1 \Rightarrow R_1$

$(-27) R_1 + R_3 \Rightarrow R_3$

$(125) R_1 + R_4 \Rightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & -1/3 & 1/9 & -1/27 & -1/27 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 18 & 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & -50/3 & 80/9 & -98/27 & -125/27 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & -1/3 & 1/9 & -1/27 & -1/27 & 0 & 0 & 0 \\ 0 & 18 & 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & -50/3 & 80/9 & -98/27 & -125/27 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] \left(\tfrac{1}{18}\right) R_2$$

$$\left[\begin{array}{cccc|cccc} 1 & -1/3 & 1/9 & -1/27 & -1/27 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/9 & 1/18 & 0 & 1/18 & 0 \\ 0 & -50/3 & 80/9 & -98/27 & -125/27 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} (1/3)R_2+R_1 \\ \left(\tfrac{50}{3}\right)R_2+R_3 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1/9 & 0 & -1/54 & 0 & 1/54 & 0 \\ 0 & 1 & 0 & 1/9 & 1/18 & 0 & 1/18 & 0 \\ 0 & 0 & 80/9 & -16/9 & -100/27 & 0 & 25/27 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] \left(\tfrac{9}{80}\right)R_3$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1/9 & 0 & -1/54 & 0 & 1/54 & 0 \\ 0 & 1 & 0 & 1/9 & 1/18 & 0 & 1/18 & 0 \\ 0 & 0 & 1 & -1/5 & -5/12 & 0 & 5/48 & 9/80 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right]$$

$(-1/9)R_3+R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1/45 & +1/36 & 0 & 1/144 & -1/9 \\ 0 & 1 & 0 & 1/9 & 1/18 & 0 & 1/18 & 0 \\ 0 & 0 & 1 & -1/5 & -5/12 & 0 & 5/48 & 9/80 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right]$$

$(1/5)R_4+R_3,\ (-1/9)R_4+R_2,\ (-1/45)R_4+R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1/36 & -1/45 & 1/144 & -1/9 \\ 0 & 1 & 0 & 0 & 1/18 & -1/9 & 1/18 & 0 \\ 0 & 0 & 1 & 0 & -5/12 & 1/5 & 5/48 & 9/80 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} +1/6 & -1/45 & 1/144 & -1/9 \\ +1/18 & -1/9 & 1/18 & 0 \\ -5/12 & 1/5 & 5/48 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{"} = \begin{bmatrix} +1/18 - 1/9 + 0 - 0 \\ +1/9 \quad -5/9 + 0 \quad +0 \\ -5/6 \quad +1 \quad +0 + 0 \\ 0 \quad +5 \quad +0 + 0 \end{bmatrix}$$

$$\text{"} = \begin{bmatrix} -1/18 \\ -4/9 \\ -1/6 \\ 5 \end{bmatrix}$$

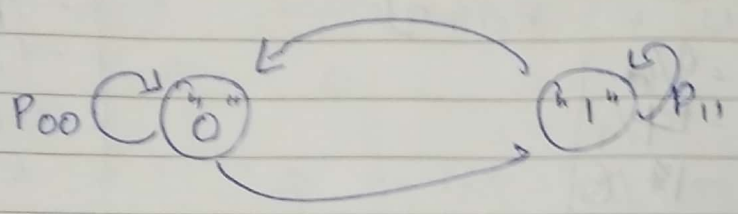
$$\begin{bmatrix} -1/18 \\ -4/9 \\ 1/6 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -1/18 \\ -4/9 \\ -1/6 \\ 5 \end{bmatrix} \quad \text{Ans}$$

A MARKOV CHAIN

Transition Probabilities Matrix.

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$



A 3-State Markov model of weather.

- Assume, weather can be rain or snow (state 1), cloudy (state 2), or sunny (state 3).
- Assume, weather of any day "t" is characterized by following three states.

Transition probabilities.

$$P = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$$

if $\boxed{P^3 = P^4}$ => saturation state, then it is not gonna change. but it is for that particular phase.

- Random process => The outcome is not sure.
- Bernoulli Trial => The outcomes are ~~two~~ two (determined)

a repitition of bernoulli trial is.

dt $\quad P_{(H)} = \{H\}. = 1/2$

$n \to \infty$ $\quad \downarrow \{H, T\}.$

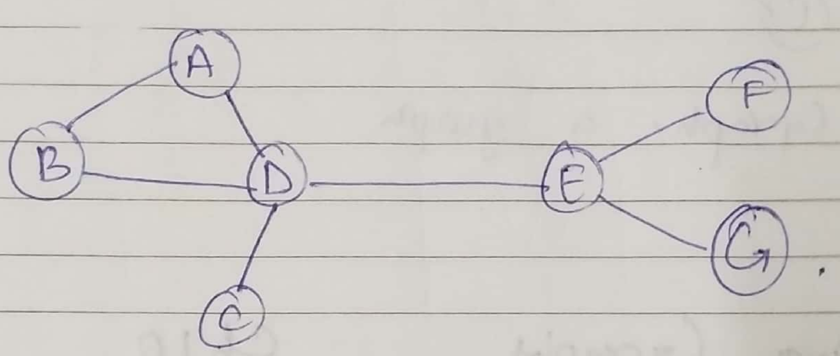
GRAPHS.

$G = (V, E)$.

Different types

1. Undirected – edges do not have direction (bidirectional)
2. Directed – edges do have direction (uni-directional)
3. Weighted – cost.
4. Unweighted

Path: A path is a list of vertices $p_1, p_2, \ldots p_k$ where there exists an edge $(p_i, p_{i+1}) \in E$



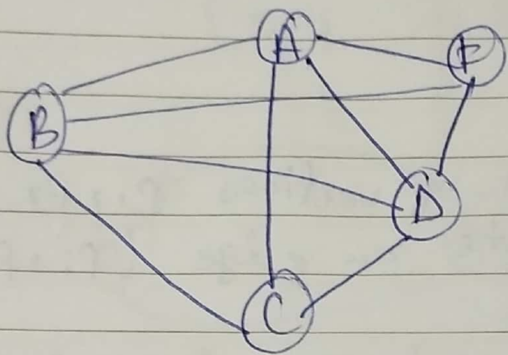
Cycle: A subset of edges that form a path & such that first & last nodes are same.

Connected: Every pair of vertices are connected by path (undirected)

Strongly connected: every two vertices are reachable by a path.

Tree is a type of graph if it does not contain cycle.

Complete graph: an edge exists b/w every node.

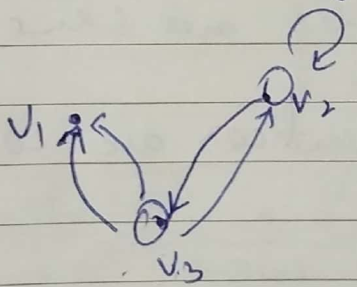


?? Bipartite Graph: a graph

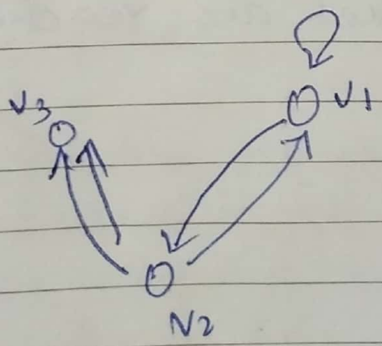
Representing Graphs

<u>Ch 10</u>

Adjacency List, Matrix.



$$\begin{array}{c|ccc} & V_1 & V_2 & V_3 \\ \hline V_1 & 0 & 0 & 0 \\ V_2 & 0 & 1 & 1 \\ V_3 & 2 & 1 & 0 \end{array}$$



$$\begin{array}{c|ccc} & V_1 & V_2 & V_3 \\ \hline V_1 & 1 & 1 & 0 \\ V_2 & 1 & 0 & 2 \\ V_3 & 0 & 0 & 0 \end{array}$$