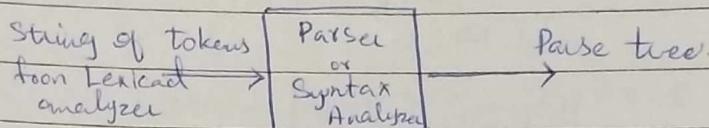


# COMPILER CONSTRUCTION PRACTICE

Date 7-2-2024  
M T W T F S

## Ch: 3 :: SYNTAX ANALYSIS

- Second phase of compiler.



### Types

- ① Universal.
- ② Top-Down.

Builds parse tree from top (root) to the bottom (leaves).

- ③ Bottomup.

Builds parse tree from the leaves and works their way up to the root.

In any case, input to the parser is scanned from left-to-right, one symbol at a time.

### Grammars:

- A powerful tool for describing & analyzing the correct syntax of a language.
- A set of rules by which valid sentences in a language are constructed.
- Non-terminal: a grammar symbol that can be replaced/replaced/expanded to a sequence of symbols. They are syntactic variables, intermediate.
- Terminal: an actual word of a language, symbols that cannot be replaced. They are tokens, leaves in a parse tree.
- Production: A grammar rule that describes how to replace/expand a symbol/non-terminal
- Start Symbol: A designation of one of the non-terminal

Example: All non-empty strings that start and end with the same symbol.

$$S \rightarrow aXa \mid bXb \mid a \mid b \\ X \rightarrow ax \mid bx \mid \epsilon$$

Example: All strings with more a's than b's.

$$S \rightarrow Aa \mid MS \mid SMA \\ A \rightarrow Aa \mid \epsilon \\ M \rightarrow MM \mid bMa \mid aMb \mid \epsilon$$

Example: Abbaababab      bbaabab

$$S \rightarrow MS \\ \rightarrow aMbs \\ \rightarrow abMabs \\ \rightarrow abbMaabs \\ \rightarrow abbaabS \\ \rightarrow abbaabMS \\ \rightarrow abbaabbambs \\ \rightarrow abbaababS \\ \rightarrow abbaababAa \\ \rightarrow abbaababab$$

$$S \rightarrow MS \\ \rightarrow bMas \\ \rightarrow bbMaas \\ \rightarrow bbaas \\ \rightarrow bbaams \\ \rightarrow bbbaabMas \\ \rightarrow bbcaabas$$

can't be passed

Example: All palindromes (a palindrome is a string that reads the same forwards & backwards).

$$S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon$$

① Define a grammar for the language of strings with one or more a's followed by zero or more b's.

$$S \rightarrow aA \\ A \rightarrow aA \mid bA \mid \epsilon$$

② Define a grammar for the even-length palindromes.

$$S \rightarrow aSa \mid bSb \mid \epsilon$$

③ Define a grammar for strings where the number of a's is equal to number of b's.

$$S \rightarrow aB \mid bA \quad S \rightarrow SaSBs \mid SbSAs \mid \epsilon \quad S \rightarrow Sa\bar{S}bs \\ B \rightarrow bS \mid b \bar{a}b \\ A \rightarrow aS \mid a \bar{b}AA$$

ababa

$$S \rightarrow ab \quad \checkmark$$

④ Define a grammar for strings where the number of a's is not equal to the number of b's. (Hint: think about it as two separate cases).

$$S \rightarrow SaSBs \mid SbSAs \mid a \mid b \times$$

$$S \rightarrow S_1 \mid S_2$$

$$\begin{cases} S_1 \rightarrow aS_1 \mid S_1a \mid a. & \text{? a's are} \\ S_1 \rightarrow bS_1S_1 \mid S_1bS_1 \mid S_1S_1b. \end{cases} \quad \text{greater}$$

$$S_2 \rightarrow bS_2 \mid S_2b \mid b. \quad \text{? b's are}$$

$$S_2 \rightarrow aS_2S_2 \mid S_2bS_2 \mid S_2S_2b. \quad \text{greater}$$

The grammar defines simple arithmetic expressions.

$$\text{expression} \rightarrow \text{expression} + \text{term}$$

$$\text{expression} \rightarrow \text{expression} - \text{term}$$

$$\text{expression} \rightarrow \text{term}$$

$$\text{term} \rightarrow \text{term} * \text{factor}$$

$$\text{term} \rightarrow \text{term} / \text{factor}$$

$$\text{term} \rightarrow \text{factor}$$

$$\text{factor} \rightarrow (\text{expression})$$

$$\text{factor} \rightarrow \text{id}$$

OR

$$E \rightarrow E + T \mid E - T \mid T$$

$$T \rightarrow T * F \mid T / F \mid F$$

$$F \rightarrow (E) \mid \text{id}$$

A grammar for balanced parenthesis

$$S \rightarrow (S)S \mid \epsilon$$

### Ambiguity:

A grammar which generates more than one parse tree for a given string is said to be ambiguous.

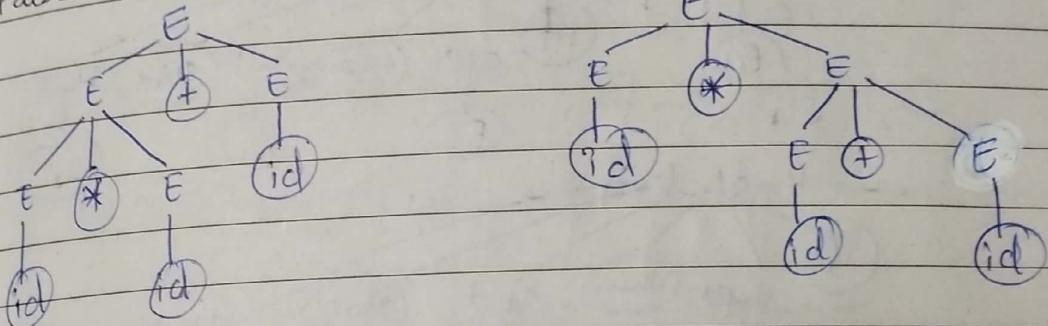
Example.

Grammatical rules:

$$E \rightarrow E + E \mid E * E \mid - E \mid ( E ) \mid id$$

String  $id * id + id$ .

Parse Tree



Hence, the grammar is ambiguous.

Solution for ambiguity:

Associativity of operators:

Mostly operators: +, -, \*, / are left-associative.

Left-associative means, if an operand has same operators on its left & right  
 e.g.  $9 + 5 + 2$ , 5 has + operator on left & right, 5 first belongs to its left side operator.

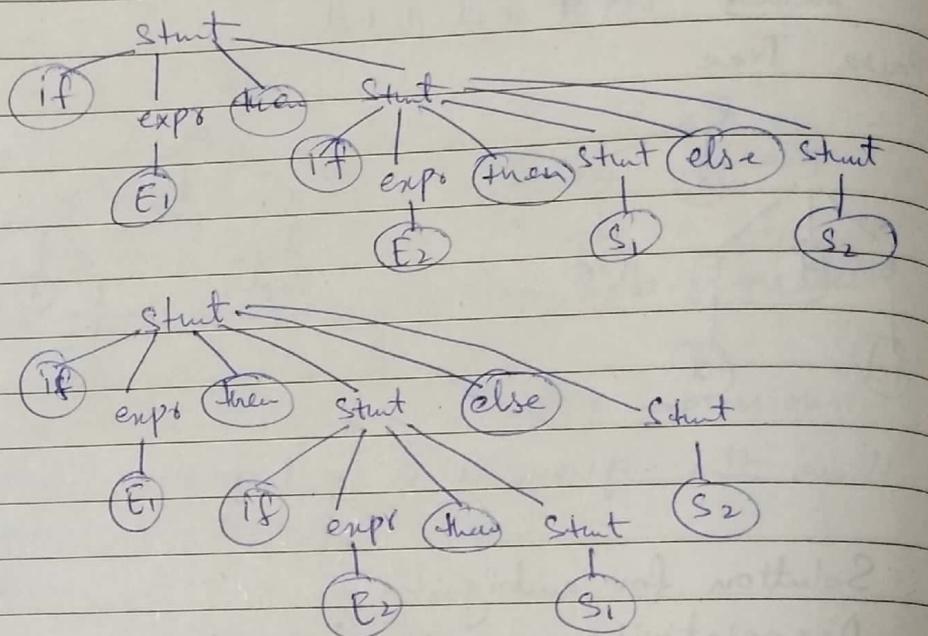
Precedence of Operators:

\* & / operators have more precedence than + & -  
 Operators on the same line have same associativity by precedence.

## Ambiguous Grammatical.

stmt → if expr then stmt ①  
   | if expr then stmt else stmt ②  
   | other ③

String: if E<sub>1</sub> then if E<sub>2</sub> then S<sub>1</sub> else S<sub>2</sub>.



Two parse trees, grammar is ambiguous.

- The idea is that a stmt appearing between a then and an else ② must be 'matched', i.e. the interior stmt ① must not end with an unmatched then
- A matched stmt is either an if - then - else statement containing no open statements or it is any other kind of unconditional statement.

## Unambiguous Grammatical.

stmt → matched-stmt | open-stmt

matched-stmt → if expr then matched-stmt else matched-stmt | other

open-stmt → if expr then stmt |

if expr then matched-stmt else open-stmt

## Ambiguity Practice

①  $bExp \rightarrow bExp \text{ AND } bExp$   
 $bExp \text{ OR } bExp$   
 $bExp \text{ NOT } bExp$  | True | False.

After removing ambiguity:

$bExp \rightarrow bExp_1 \text{ OR } bExp_1$  |  $bExp_1$ .  
 $bExp_1 \rightarrow bExp_1 \text{ AND } bExp_2$  |  $bExp_2$ .  
 $bExp_2 \rightarrow \text{NOT } bExp_2$  | True | False.

②  $R \rightarrow R + R$  |  $R \cdot R$  |  $R^*$  | a | b | c

After removing ambiguity:

|   |   |                                     |
|---|---|-------------------------------------|
| $R \rightarrow R + R_1$   $R_1$         | { | $R \rightarrow RR'$   a   b   c     |
| $R_1 \rightarrow R_1 \cdot R_2$   $R_2$ |   | $R' \rightarrow +R$   $\cdot R$   * |
| $R_2 \rightarrow R_2^*$   a   b   c     |   |                                     |

&  $A \rightarrow AA$  |  $(A)$  |  $\epsilon$

$rep \rightarrow rep^* | rep | rep rep$   
 $| rep^* | (rep) | letter$

$rep \rightarrow rep^* | rep^2.$  |  $rep^2.$

$rep^2 \rightarrow rep^2 rep^3$  |  $rep^3.$

$rep^3 \rightarrow rep^3 rep^4$  |  $rep^4.$

$rep^4 \rightarrow ("rep^4")$  | letter.

$(ab|b)^*$

## Elimination of Left Recursion.

A grammar is left-recursive if it has a non-terminal A such that  $A \rightarrow A\alpha$ .

Top-down parsing cannot handle left-recursion.

General form of left-recursive Grammar.

$$A \rightarrow A\alpha | B$$

Here  $\alpha$  and  $B$  can be a terminal or non-terminals.

This form of production could be replaced by the non-left-recursive productions.

$$A \rightarrow A\alpha | B \rightarrow A \rightarrow BA^1 \\ A^1 \rightarrow \alpha A^1 | \epsilon$$

This rule by itself suffices for many grammars.

Example: Arithmetic Expressions Grammar.

$$E \rightarrow E + T \quad | \quad E - T \quad | \quad T \rightarrow \text{left recursive}$$

$$T \rightarrow T * F \quad | \quad T / F \quad | \quad F \rightarrow \text{left recursive.}$$

$$F \rightarrow (E) \quad | \quad \text{id}$$

$$E \rightarrow E + T \quad | \quad E - T \quad | \quad T \quad \therefore A \rightarrow A\alpha | B$$

$$E \rightarrow T E' \quad A = E$$

$$E' \rightarrow +TE' \quad | \quad -TE' \quad | \quad \epsilon \quad \alpha = +T, -T$$

$$B = T$$

$$T \rightarrow T * F \quad | \quad T / F \quad | \quad F$$

$$T \rightarrow FT' \quad A = T$$

$$T' \rightarrow *F \quad | \quad /F \quad | \quad \epsilon \quad \alpha = *F, /F$$

$$B = F$$

Now, the non-left-recursive Grammar is.

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \quad | \quad -TE' \quad | \quad \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' \quad | \quad /FT' \quad | \quad \epsilon$$

$$F \rightarrow (E) \quad | \quad \text{id}$$



## Eliminating left recursion.

$$E \rightarrow \text{int} \mid \text{int} * E \mid \text{int} / E \mid E - (E)$$

- ①  $\text{expr} \rightarrow \text{expr} + \text{term} \mid \text{term}$   
 $\text{term} \rightarrow \text{term} * \text{factor} \mid \text{factor}$   
 $\text{factor} \rightarrow \text{factor} * \mid \text{primary}$   
 $\text{primary} \rightarrow a \mid b$

## Eliminate left recursion.

$$\text{② } \text{expr} \rightarrow \underbrace{\text{expr}}_A + \underbrace{\text{term}}_\alpha \mid \underbrace{\text{term}}_{B'}$$

$$\therefore A \rightarrow A\alpha \mid B'$$

$$\text{expr} \rightarrow \text{term} \text{ expr}'$$

$$\text{expr}' \rightarrow + \text{term} \mid \in$$

$$\begin{array}{l} A \rightarrow B A' \\ A' \rightarrow \alpha A' \mid \in \end{array}$$

$$\text{③ } \text{term} \rightarrow \underbrace{\text{term}}_A \alpha \text{ factor} \mid \underbrace{\text{factor}}_{B'} *$$

$$\text{term} \rightarrow \text{factor term'}$$

$$\text{term'} \rightarrow \text{factor} \mid \in$$

$$\text{④ } \text{factor} \rightarrow \underbrace{\text{factor}}_A \alpha \mid \underbrace{\text{primary}}_{B'}$$

$$\text{factor} \rightarrow \text{primary} \text{ factor'}$$

$$\text{factor'} \rightarrow * \text{ factor' } \mid \in$$

Grammar:

$$S \rightarrow Aa \mid b$$

$$A \rightarrow Ac \mid Sd \mid \in$$

→ contains left-recursion.

S contains recursion as well.

$$A \rightarrow Ac \mid Aa \mid bd \mid \in$$

→ first substitute.

now remove L.R.

S in A's production

$$A \rightarrow bd A'$$

$$A' \rightarrow cA' \mid adA' \mid \in$$

$$S \rightarrow Aa \mid b$$

$$A \rightarrow bd A'$$

$$A' \rightarrow cA' \mid adA' \mid \in$$

## Left-Factoring:

- Useful for producing a grammar suitable for predictive, or top-down parsing.
- When the choice b/w two alternative A-productions isn't clear, we can rework the productions to defer the decision until enough of the input has been seen that we can make right choice for..

$$\text{Ex: } A \rightarrow aAb \mid aAc.$$

which production to choose isn't clear.

on reading a, we can't be assured that we're choosing the right production.

General form for which may require left-factoring

$$A \rightarrow \alpha \beta_1 \mid \alpha \beta_2 \quad \alpha, \beta_1 \text{ and } \beta_2$$

↓

$$A \rightarrow \alpha A'$$

can be any terminals

$$A' \rightarrow \beta_1 \mid \beta_2$$

or non-terminals

Example:

$$\text{Grammar: } S \rightarrow i E t S \mid i E t S e S \mid a.$$

$$E \rightarrow b.$$

$$S \rightarrow i E t S S' \mid a. \quad \alpha \Rightarrow i E t.$$

$$S' \rightarrow e S \mid E$$

$$\beta_1 \Rightarrow S$$

$$E \rightarrow b.$$

$$\beta_2 \Rightarrow S e S$$

Example: Left factor the following grammar.

$$E \rightarrow \text{int} \mid \text{int} + E \mid \text{int} - E \mid E - (E).$$

$$E \rightarrow \text{int } E' \mid E - (E)$$

$$\alpha \Rightarrow \text{int}$$

$$E' \rightarrow E \mid + E \mid - E$$

$$\beta_1 = E$$

$$\beta_2 = + E$$

$$\beta_3 = - E$$

Q: left factor the given grammar

$$r\text{exp}^* \rightarrow r\text{exp}^* + r\text{term} \mid r\text{term}$$

$$r\text{term} \rightarrow r\text{term} \ r\text{factor} \mid r\text{factor}$$

$$r\text{factor} \rightarrow r\text{factor} * \mid r\text{primary}$$

$$r\text{primary} \rightarrow a \mid b$$

No left factoring needed.

Q: Given Eliminate left recursion and left factor the grammar (if needed).

$$\textcircled{1} \quad S \rightarrow SS + \mid SS * \mid a$$

Removing left recursion.

$$S \rightarrow SS' \mid a$$

$$S' \rightarrow + \mid *$$

Left factoring

$$S \rightarrow a S''$$

$$S'' \rightarrow a S' S'' \mid \epsilon$$

$$S' \rightarrow + \mid *$$

$$S \rightarrow SSS' \mid a$$

$$S' \rightarrow + \mid *$$

$$S \rightarrow a S''$$

$$S'' \rightarrow (\textcircled{3}) S' S'' \mid \epsilon$$

$$\textcircled{2} \quad S \rightarrow OS1 \mid O1$$

Left factoring

$$S \rightarrow O S'$$

$$S' \rightarrow S1 \mid 1$$

$$\textcircled{3} \quad S \rightarrow O/S1 \mid O1$$

Not left recursive as no left factor is needed

$$\textcircled{3} \quad S \rightarrow + SS \mid * SS \mid a$$

Not left recursive as no left factoring needed

$$\textcircled{4} \quad S \rightarrow S(S)S \mid \epsilon$$

Removing left recursion.

$$S \rightarrow S'$$

$$S' \rightarrow (S) S S' \mid \epsilon$$

No left factoring needed.

$$Q: S \rightarrow aSSbS \mid aSaSb \mid abb \mid b$$

$$S \rightarrow aS' \mid b$$

$$S \rightarrow aS' \mid b$$

$$S' \rightarrow SSbS \mid SaSb \mid bb \rightarrow S' \rightarrow S'' \mid bb$$

$$S'' \rightarrow SbS \mid aSb$$



$$⑤ \quad S \rightarrow S + S \mid S \ S \mid (S) \mid \underline{S} \ * \mid a$$

## Left factoring

$$S \rightarrow S S_1 | (g) | a.$$

$S_1 \rightarrow +S \mid S \mid *$

Eliminating left recursion.

$$S \rightarrow (S) S' | a S'$$

$S' \rightarrow +SS' \mid SS' \mid *S' \mid \epsilon$

## TOP-DOWN PARSING

- Constructing a parse tree for the input string, starting from root by creating the nodes of the parse tree in preorder
  - Left-most derivation of an input string.
  - Recursive descent parsing, a general form of top-down parsing, which may require backtracking to find the correct production
  - Predictive parsing, is a special case of recursive descent parsing, where no backtracking is required.

It chooses correct production by looking ahead at the input (one at a time)

Ex: Grammae

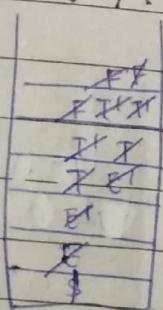
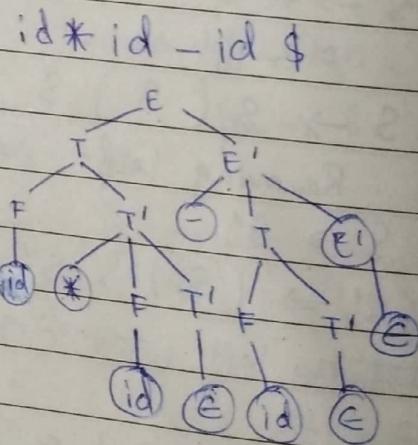
$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid e \mid -TE'$$

$$T \rightarrow FT^1$$

$$T' \rightarrow *RT' | \in | /RT'$$

$$F \rightarrow (E) | id$$



## LL PARSER.

Date \_\_\_\_\_ 20 \_\_\_\_\_

Predictive parsers (recursive descent parsers with no backtracking) can be constructed for a class of LL grammar.

LL Parser accepts LL grammar, a subset of context-free grammar with some restrictions  
LL parser is denoted by LL(k)

passing input from  
left to right  $\xrightarrow{\quad} \text{LL}(k)$

left most derivation

number of  
lookahead  
symbol.

The construction of this parser is aided by two functions FIRST() and FOLLOW() associated with grammar.

During top-down parsing, these functions allow us to choose which production to apply, based on the next input symbol

### FIRST()

First(A), A is any symbol in the grammar,  
terminal or non-terminal

$$\text{FIRST}(E) = \{ \in \}$$

$$\text{FIRST}(\text{terminal}) = \{ \text{terminal} \}$$

$$\text{Ex: } A \rightarrow C B$$

$$\text{FIRST}(A) = \{ C \}$$

### FOLLOW(r):

To compute Follow(A) for all non-terminals A.

Place \$ in the start non-terminal's FOLLOW(S).

For any non-terminal  $A$ , for which you are finding its FOLLOW(A), check that on the expanded side of product what comes after that non-terminal 'A'.

If it is terminal, then add it in the set



- if it is a non-terminal, then everything in the  $\text{FIRST}(B)$ , will be in  $\text{FOLLOW}(A)$ .
- if there is nothing after the non-terminal A, then or if the non-terminal B after A has  $\epsilon$  in it  $\text{FIRST}(B)$ , then check the  $\text{FOLLOW}$  of that rule's non-terminal, it'll become  $\text{FOLLOW}(A)$ .

$$\text{Ex: } C \rightarrow aAB \quad C \rightarrow aA$$

in this production A has

$$B \text{ in } \text{FIRST}(B) = \{\epsilon\}.$$

then  $\text{FOLLOW}(A) = \text{FOLLOW}(C)$

in this production there

is nothing after A, the

$\text{FOLLOW}(A) = \text{FOLLOW}(C)$

- $\text{FOLLOW}(A)$  will not contain  $\epsilon$ .

Example: Grammar.

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' | \epsilon | -TE'$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' | /FT' | \epsilon$$

$$F \rightarrow (E) | \text{id.}$$

Find FIRST

$$\text{FIRST}(E) = \text{FIRST}(T) = \{(, \text{id}\}\}$$

$$\text{FIRST}(E') = \{+, -, \epsilon\}$$

$$\text{FIRST}(T) = \text{FIRST}(F) = \{(, \text{id}\}\}$$

$$\text{FIRST}(T') = \{*, /, \epsilon\}$$

$$\text{FIRST}(F) = \{(, \text{id}\}\}$$

Find FOLLOW

$$\text{FOLLOW}(E) = \{ \$, ) \}$$

$$\text{FOLLOW}(E') = \text{FOLLOW}(E) = \{ \$, ) \}$$

$$\text{FOLLOW}(T) = (\text{FIRST}(E') - \epsilon) \cup \text{FOLLOW}(E) \cup$$

$$\text{FOLLOW}(E') = \{ +, -, \$, ) \}$$

$$\text{FOLLOW}(T') = \text{FOLLOW}(T) = \{ +, -, \$, ) \}$$

$$\text{FOLLOW}(F) = \text{FIRST}(T') - \epsilon \cup \text{FOLLOW}(T) \cup$$

$$\text{FOLLOW}(F) =$$

How to check If a grammar is LL(1) or not.

Let  $G \Rightarrow A \rightarrow \alpha \mid \beta$ .

- First( $\alpha$ ) and FIRST( $\beta$ ) are disjoint sets, means their intersection is  $\emptyset$  or  $\text{FIRST}(\alpha) \cap \text{FIRST}(\beta) = \emptyset$
- If  $\epsilon$  is in  $\text{First}(\beta) \Rightarrow \text{FIRST}(\alpha) \cap \text{Follow}(A) = \emptyset$   
by same logic for  $\epsilon$  in  $\text{FIRST}(\alpha)$ .

### LL(1) Parsing Table.

- Make a table consisting of total non-terminals (rows) and total terminals/input symbols (columns), e.g. S
- Now check for the non-terminal (in a row) and terminal (in column), if that terminal is in the set of FIRST(S) then, write the rule which accepts that terminal in the cell.
- If  $\epsilon$  is in  $\text{FIRST}(S)$ , then check FOLLOW(S) to fill the cells with that rule (epsilon rule).

### LL(1) Parsing Table for Arithmetic Expression Grammar

| Non-Terminals |                     | Terminals / Input Symbols. |                       |                       |                       |   |                     |  |
|---------------|---------------------|----------------------------|-----------------------|-----------------------|-----------------------|---|---------------------|--|
|               |                     | id                         | .                     | +                     | -                     | * | /                   | ( ) \$                                       |
| E             | $E \rightarrow TE'$ |                            |                       |                       |                       |   |                     | $E \rightarrow TE'$                          |
| $E'$          |                     |                            | $E' \rightarrow +TE'$ | $E' \rightarrow -TE'$ |                       |   |                     | $E' \rightarrow E$ $E' \rightarrow \epsilon$ |
| T             | $T \rightarrow FT'$ |                            |                       |                       |                       |   | $T \rightarrow FT'$ |  |
| $T'$          | -                   | $T' \rightarrow E$         | $T' \rightarrow E$    | $T' \rightarrow *FT'$ | $T' \rightarrow /FT'$ |   | $T' \rightarrow E$  | $T' \rightarrow E$                           |
| F             | $F \rightarrow id$  |                            |                       |                       |                       |   | $F \rightarrow (E)$ |  |



Ex: The following grammar, written abstractly for if-else statement:

$$\begin{aligned} S &\rightarrow ; E t S \quad S' \mid a \\ S' &\rightarrow e \quad S \mid \epsilon \\ E &\rightarrow b. \end{aligned}$$

Find FOLLOW FIRST

$$\text{FIRST}(S) = \{i, a\}.$$

$$\text{FIRST}(S') = \{e, \epsilon\}$$

$$\text{FIRST}(E) = \{b\}$$

Find FOLLOW

$$\begin{aligned} \text{FOLLOW}(S) &= (\text{FIRST}(S') - \epsilon) \cup \text{FOLLOW}(S) \cup \text{FOLLOW}(S) \\ &= \{e, \$\}. \end{aligned}$$

$$\text{FOLLOW}(S') = \text{FOLLOW}(S) = \{e, \$\}.$$

$$\text{FOLLOW}(E) = \{t\}.$$

Parsing Table.

| Non-terminals | Terminals.  |                   |                   |  |   |                           |
|---------------|---|-------------------|-------------------|--|---|---------------------------|
|               | i   | t                 | a                 | e  | b | \$                        |
| S             | $\begin{matrix} S \rightarrow \\ iEtSS' \end{matrix}$ |                   | $S \rightarrow a$ |  |   |                           |
| S'            |   |                   |                   | $\begin{matrix} S' \rightarrow eS \\ S' \rightarrow \epsilon \end{matrix}$ |   | $S' \rightarrow \epsilon$ |
| E             |   | $E \rightarrow b$ |                   |  |   |                           |

The grammar is not LL(1) parseable, since.

the rule  $S' \rightarrow eS \mid \epsilon$

if  $\epsilon \Rightarrow \text{FIRST}(B)$ , according to second point.

$$\{\epsilon\} \cap \{\{e, \$\}\} = \{\epsilon\}.$$

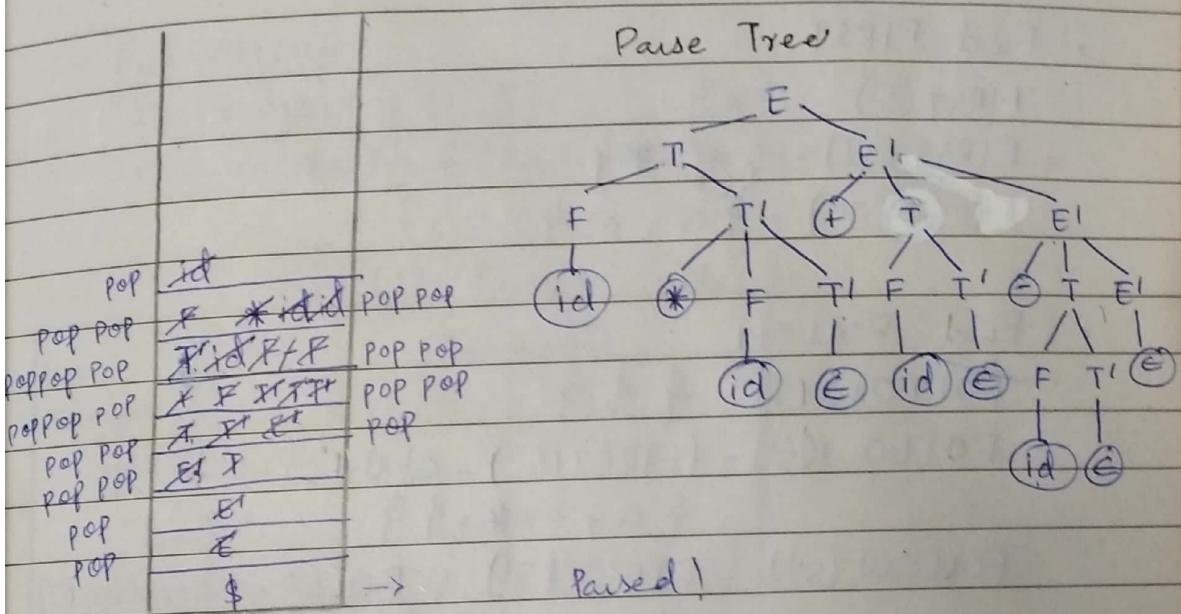
$$\text{and } \{\epsilon\} \neq \emptyset.$$



- How to parse an input string using LL(1) Parse Table.
  - An input string, ending with \$
  - A stack ; which has \$ initially at its top .
  - A Parsing Table .

Let's pause  $\rightarrow$  id \* id + id - id', using arithmetic grammar.

Input: id \* id + id - id \$



Stak.

- ① Put the start non-terminal at the top and check for its possible rules according to the lookahead "L" pointing on the input. Make nodes for the symbols recognized.
  - ② The rule, which is to be applied, place its symbol in the stack from right to left to make their nodes.
  - ③ after each placement in stack, check if a terminal is at top, if yes, pop it to move lookahead forward.  
POP the non-terminal, if it is done.

Grammatical Postfix expression with operand a

$$S \rightarrow SS+ | SS* | a.$$

- After Left-factoring:

$$S \rightarrow SSS' | a.$$

$$S' \rightarrow + | *$$

- After eliminating left-recursion:

$$S \rightarrow a S''$$

$$S \rightarrow a S''$$

$$S'' \rightarrow SS'S'' | \epsilon \Rightarrow S'' \rightarrow a S'' S'' | \epsilon$$

$$S' \rightarrow + | *$$

$$S' \rightarrow + | *$$

- Find FIRST

$$\text{FIRST}(S) = \{a\}$$

$$\text{FIRST}(S') = \{+, *\}$$

$$\text{FIRST}(S'') = \{a, \epsilon\}$$

- Find FOLLOW

$$\text{FOLLOW}(S) = \{\$\}$$

$$\text{FOLLOW}(S') = (\text{FIRST}(S'') - \epsilon) \cup \text{FOLLOW}(S'') \\ \{a, +, *, \$\}$$

$$\text{FOLLOW}(S'') = \text{FIRST}(S') \cup \text{FOLLOW}(S) \cup \text{FOLLOW}(S') \\ = \{+, *, \$\}$$

### LL(1) Parsing Table.

Non-Terminals

Terminals.

|     | a  | +                          | *                          | \$                         |
|-----|--|----------------------------|----------------------------|----------------------------|
| S   | $S \rightarrow a S''$                                    |                            |                            |                            |
| S'  |  | $S' \rightarrow +$         | $S' \rightarrow *$         |                            |
| S'' | $S'' \rightarrow a S'' S'$<br>$S'' \rightarrow \epsilon$ | $S'' \rightarrow \epsilon$ | $S'' \rightarrow \epsilon$ | $S'' \rightarrow \epsilon$ |

## Grammar

$S \rightarrow 0 S_1 | 0.$   
 after left-factoring  
 $S \rightarrow 0 S'$   
 $S' \rightarrow S_1 | 1.$

Find Follow FIRST.

$$\text{FIRST}(S) = \{0\}.$$

$$\text{FIRST}(S') = \text{FIRST}(S) \cup \{1\} = \{0, 1\}.$$

Find FOLLOW.

$$\text{FOLLOW}(S) = \{1, \$\}.$$

$$\text{FOLLOW}(S') = \text{FOLLOW}(S) = \{1, \$\}.$$

## LL(1) Parsing Table

|               |                      | Terminals.          |
|---------------|----------------------|---------------------|
| Non-Terminals | 0                    | 1.                  |
| S             | $S \rightarrow 0 S'$ |                     |
| S'            | $S' \rightarrow S_1$ | $S' \rightarrow 1.$ |

Example: Prefix expression with operand a.

$$S \rightarrow R + S S | * S S | a.$$

Find FIRST

$$\text{FIRST}(S) = \{+, *\}, a\}.$$

Find FOLLOW

$$\text{FOLLOW}(S) = \text{FIRST}(S) \cup \text{FOLLOW}(S) = \{+, *, a\}, \$\}.$$

## LL(1) Parsing Table

|              |                       | Terminal.             |                    |    |
|--------------|-----------------------|-----------------------|--------------------|----|
| Non-terminal | +                     | *                     | a                  | \$ |
| S            | $S \rightarrow + S S$ | $S \rightarrow * S S$ | $S \rightarrow a.$ |    |
|              |                       |                       |                    |    |

