

6/DEC/2022

STACK

LIFO

Top (open end).
Bottom (closed).

Insertion: Push (element) → on Top.

Deletion: Pop () → from Top.

PUSH..

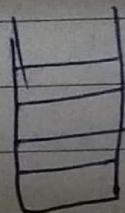
$$\text{Top} = -1$$

Push (A)

if ($\text{Top} \neq \text{size} - 1$)

$$\text{Stack}[0] = A$$

Top++



Top = -1

Top = size - 1

overflow

Top = size

Top = 0

overflow

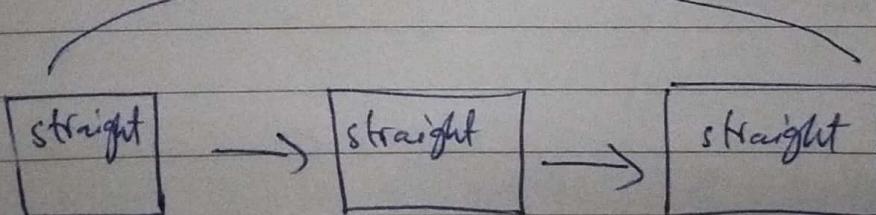
else

stack is full / stack overflow.

PEEK → it points the top element in stack..

Eg: Backtracking ⇒ puzzles, games, undo

home



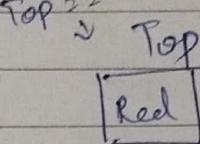
Linked List / Stack:

→ $\text{Top} == \text{null}$ {empty stack}

 └ Insertion (this condition makes normal insertion)
 └ Deletion (stack underflow).

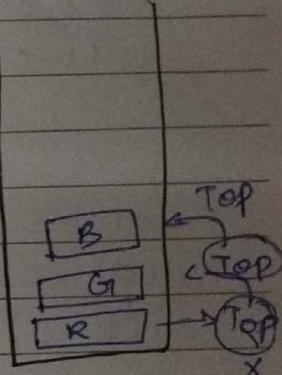
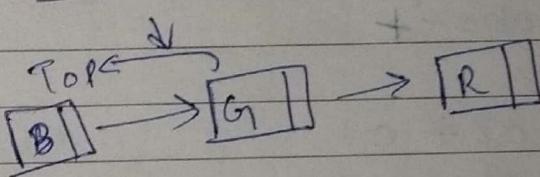
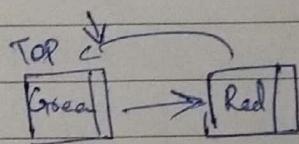
→ $\text{Top}^X \rightarrow \text{count} == n$
 └ Insertion → (Stack overflow).

$\text{Top}^{22} \text{ null}$



Insertion at start

$\text{Push}()$

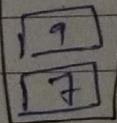


$$x = 7$$

$$y = 9$$

$\text{push}(x)$

$\text{push}(y)$



$$x = \text{pop}() \Rightarrow x = 9$$

$$y = \text{pop}() \Rightarrow y = 7$$

$\text{print}(x, y)$

Finals

$$g = 64$$

$$e = 32$$

$$x = 16$$

$$y = 8$$

$$n = 8$$

al ah av ee*

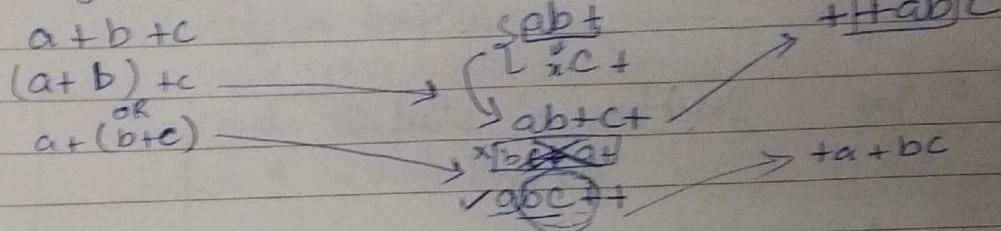
ah bh ya*

bl bu ya

bh gu

✓

Infix (Human)	Post fix (computer)	Prefix
In-between operands	after operands	before operands
operand op operand	operand operand op.	op operand operand
a op. b	a b op	op a b
requires precedence	doesn't	doesn't
a + b	a b +	+ ab.
(high level)		(low level)



$$\begin{array}{ll} a+b-c & \\ (a+b)-c & \rightarrow ab+c- \\ a+(b-c) & \rightarrow abc-+ \end{array}$$

$$\begin{array}{ll} a * b + c & \rightarrow ab * c + \\ a * (b+c) & \rightarrow abc + * \end{array}$$

Precedence a. Association
 1. \wedge L \rightarrow R.
 3. $*$ /
 4. + -

$$a^\wedge b^* c \quad (R \Rightarrow L)$$

$$a^\wedge (b^* c)$$

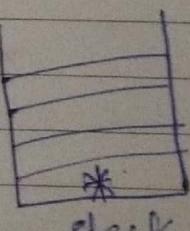
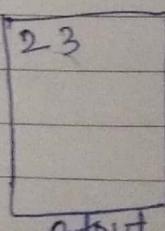
$$\begin{array}{l} ① 2 * 3 + 4 / 5 \\ \quad 23 * 45 / + \\ \quad \rightarrow 2345 / * + \end{array}$$

$$\begin{array}{r} 23 * \\ . 6 4 5 / \\ \hline 60.1 + \end{array}$$

23^v

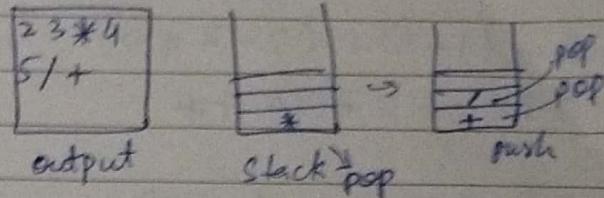
$\swarrow 23 * 45 / +$ (Post fix)

- ① variable
 ② operator \leftarrow stack

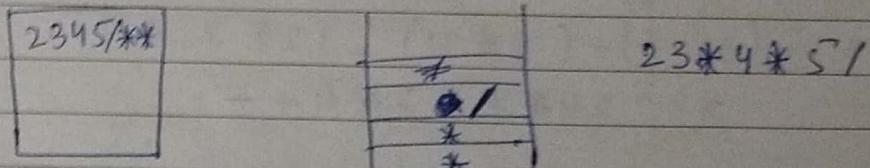


$$① 2 * 3 + 4 / 5 \Rightarrow 23 * 45 / + \checkmark$$

- ① variable
② operator



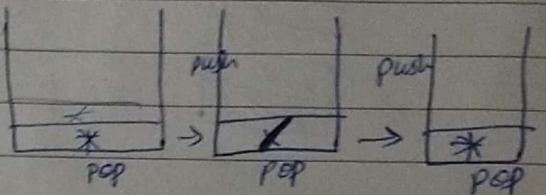
$$② 2 * 3 * 4 / 5 \Rightarrow 2345 / * * \times$$



$$2 * 3 / 4 * 5$$

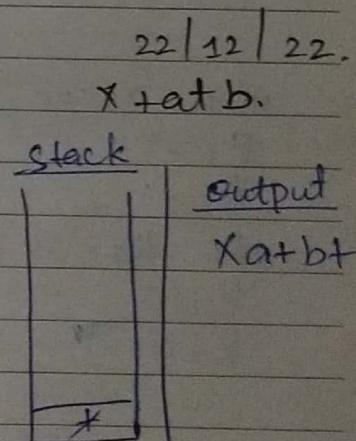
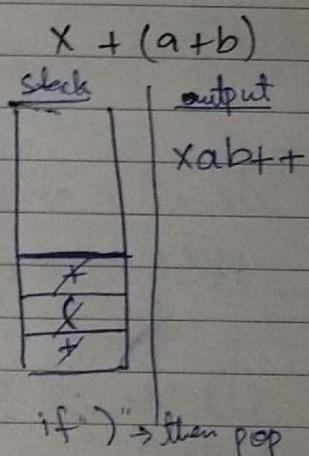
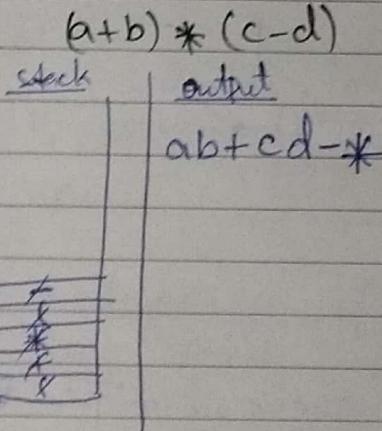
$$23 * 4 / 5 *$$

$$\begin{array}{|c|} \hline 23 * 4 \\ \hline 15 * \\ \hline \end{array}$$



$$3) 2 + 3 * 4 / 5$$

$$234 * 5 / +$$



if ") " then pop

$> =$ pop
push()

$$① (a+b) + (c+d)$$

$$ab+cd++$$

$$② ((a+b) * c) + d$$

$$ab+c * d + *$$

$$③ \{a * (b+c) / d * (4-z)\} < push()$$

$$abc + * d / 4z - *$$

Expression Evaluation:

$$1. \quad 5 + (1 - 3) + (2 + 7 * 9) . \quad = 68$$

$\overset{3}{\cancel{5}} - \overset{3}{\cancel{2}} + 68$

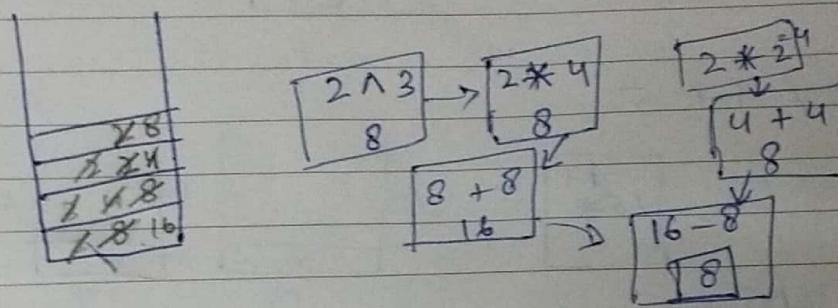
$5 1 3 - + 2 7 9 * + +$

$$2. \quad (5 * 3) * 2 + (4 - 3) . \quad = 31$$

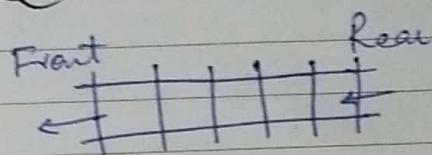
$\overset{30}{\cancel{5}} * \overset{1}{\cancel{3}} * 2 + 4 - 3$

$$3. \quad (2^3) + (2 * 4) - (2 * 2 + 4) . \quad = 8$$

$2^3 + 2 * 4 + 2 * 2 + 4 -$



QUEUE: FIFO



Delete \rightarrow Front / Head
 Insert \rightarrow Dequeue()
 Insert \rightarrow Queue(u).
 \rightarrow Rear / Tail.
 Peek(.) .

when queue is empty.

$F = -1$

$R = -1$

(normal, circular, priority) queues.

when there is one element in queue.

$F = 0$

if ($Q == \text{empty}$)

$R = 0$

$F++$;

$R++$;

when we delete last element from queue, front and last element of will reset

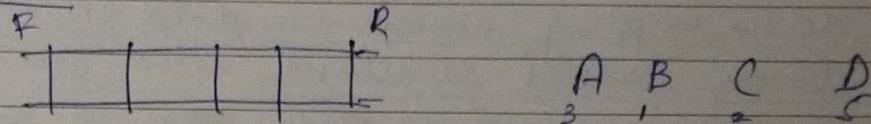
$F = -1$

$R = -1$

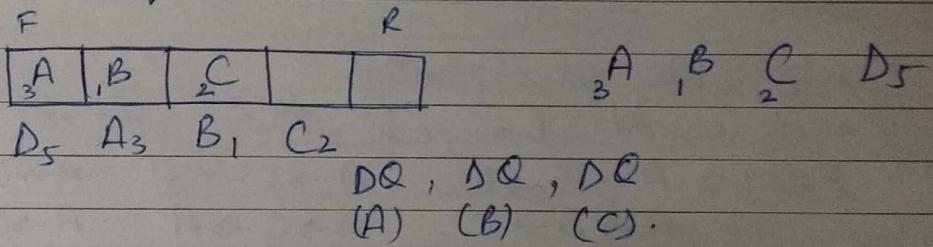
Types of Queue.

- ① Normal
- ② Circular
- ③ Priority.

Priority Queue:



- if element on front has lower priority than the element to be inserted then first dequeue all elements in queue then enqueue the high priority element and enqueue all elements in the same sequence they dequeued - If element on front has higher priority then just enqueue new element from rear.



$DQ \Rightarrow$ delete from front. the ENQ.

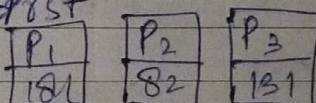
$ENQ(\text{Pointer}, \text{Data}) \Rightarrow$ for normal queue.

$ENQ(\text{Pointer}, \text{Data}, \text{Priority}) \Rightarrow$ for priority queue.
↓ sorted insertion.

$$P_1 = 181 \text{ KB}, P_2 = 182 \text{ KB}, P_3 = 131 \text{ KB}.$$

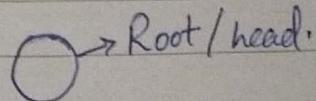
normal $[P_1 | P_2 | P_3]$

shortest job ~~first~~
Data priority

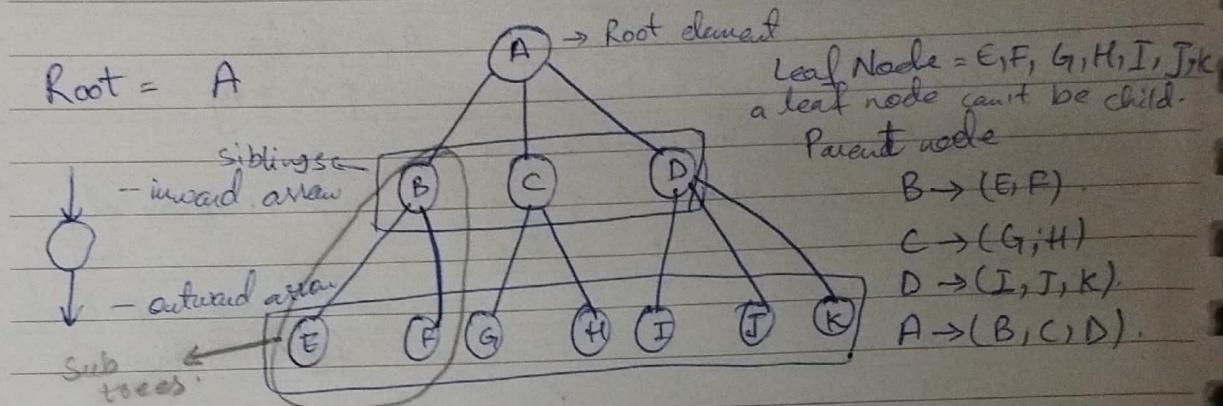


NON-LINEAR DATA STRUCTURES

1) TREE :

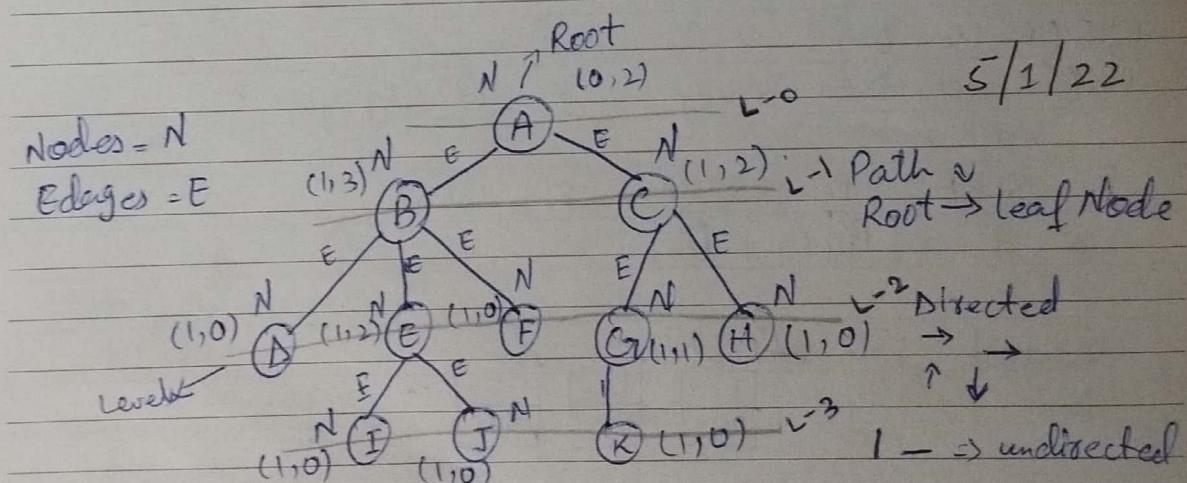


- We are mapping non-linear data structure onto linear data structures either array or linked list because we have no other way to save them.



- siblings have same priority.
 - parent or child know each others address.

Paths \Rightarrow A \rightarrow B \rightarrow E A \rightarrow C \rightarrow G A \rightarrow D \rightarrow I
 A \rightarrow B \rightarrow F A \rightarrow C \rightarrow H A \rightarrow D \rightarrow J
 A \rightarrow D \rightarrow K.



Patterson

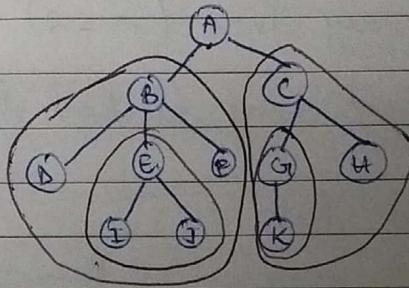
1. A - B - D
 2. A - B - E - I
 3. A - B - E - J
 4. A - B - F
 5. A - C - G - K
 6. A - C - H

- total paths = total dead nodes.

1 only root node can have
incoming 0 or else
there is issue in
tree

incoming arrows
outgoing arrows
 (I, O)

- Root node will have incoming = 0, otherwise not a tree or tree is not implemented correctly.
- The nodes that have outgoing = 0 are leaf nodes.
- Internal nodes are non-leaf nodes which have both incoming and outgoing - one or more child nodes.
- All nodes in same level are peers
- Levels are counted through edges.
- maximum no. of level is max. no. of edges.
- Siblings have same parents, peers have same levels.
→ Subtrees.



Ek node sy jitni edges nikalne woh ek subtree ban jati hai

- Depth Degree of tree i.e. max. no. of nodes

$\text{Degree}(A) = 2$. max. outgoing edges is
 $\text{Degree}(B) = 3$
 $\text{Degree}(C) = 2$
 $\text{Degree}(D) = 0$
 $\text{Degree}(E) = 2$
 " "
 " "

- Depth = Level hence can be counted through edges.

Depth of tree Root \rightarrow Leaf (max.) \therefore max. level?

Depth of a particular node is its level.

$\text{Depth}(C) = 1$

$\text{Depth}(F) = 2$.

$\text{Depth}(I) = 3$

- One node can't have two incoming arrows

- Height of Tree

\rightarrow from leaf to Root (out from every leaf to root, the maximum count will be its height).

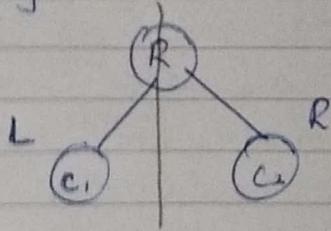
\rightarrow from leaf to node $\Rightarrow H(E) = 1$

- $n \leq 2$ (Binary)

Q Q

$n \leq 2$ (Binary)
Binary Trees.

: R = Root
C = Child.



10/01/2023

General Tree - n-ary tree ($n \Rightarrow$ no of children).
↓
Binary Tree ($n=2$)

it can be implemented using linked list or arrays.

→ 1D array

→ Doubly linked list

node {

data.

next / Right child

prev / Left child.

}

Binary \rightarrow sorted

Operations

1. → Construction of Tree.

2. → Insert Node (value).

3. → Delete Node (value)

4. → Search value.

 ↳ min value,

$LC < R < RC$

 ↳ max value

Construction of Binary Tree

→ node by node.

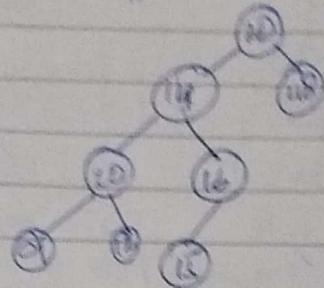
→ if root is greater than element ($R > E$),

 add element on left.

→ if root is lesser than element ($R < E$),

 add element on right.

20, 14, 48, 10, 15, 9



BST

Binary Search Tree.

Search() {

```
    while (current != null) {  
        if (current.data == key) {  
            return child  
        } else if (key < current.data) {  
            current = current.left  
        } else {  
            current = current.right  
        }  
    }
```

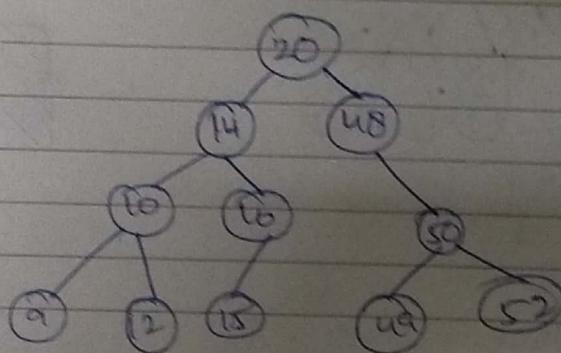
① Leaf node delete → Simple deletion

① Delete 12 ⇒ 10. Right child = null

Leaf (current.RC == null && current.LC == null).

② Node with one child (connect child to parent's parent).

③ Node with two children.



deleting any node.

max value from left sub tree.

or

min value from right sub tree

Tree min values are on left most side.

Tree max values are on right most side.

Tree Traversal Techniques

① Infix

② Postfix

③ Prefix

Infix

$a+b$

L R t R

Intra versal

Postfix

$ab+$

$L.R.Rt.$

Post traversal

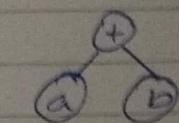
Prefix

$+ab$

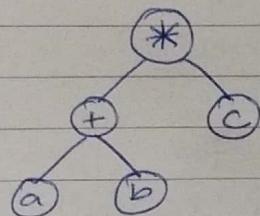
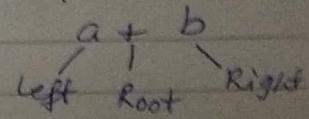
$Rt.LR$

Pre traversal

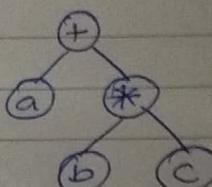
Prefix



expression tree

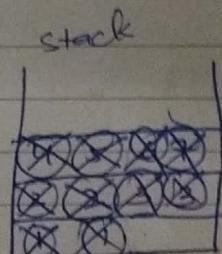
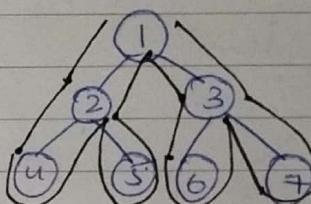


$(a+b)*c$



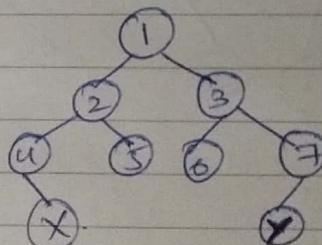
$a+(b*c)$

In expression trees
leaf are operands



Postfix = 4526731

Prefix = 1245367



Post fix = X4526Y731

Prefix = 124X536Y

Array Representation of Binary Tree..

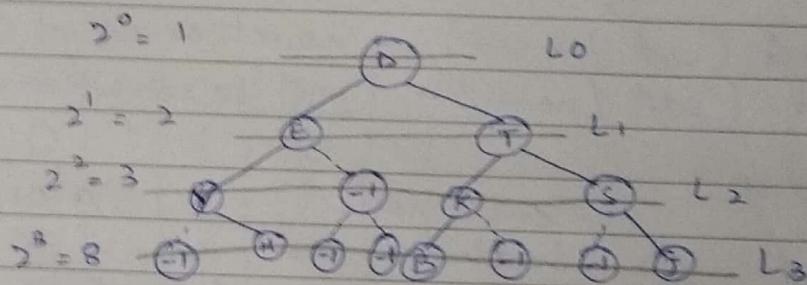
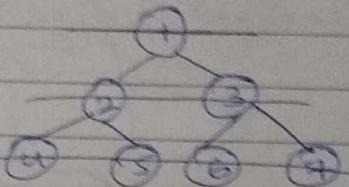
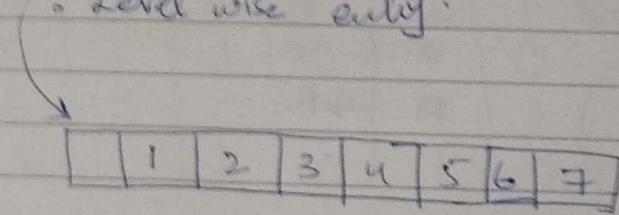
Left Child (k) = $k * 2$.

Right Child (k) = $k * 2 + 1$.

Parent (k) = $k / 2$

Total:

- $a[0]$ # of elements in tree
- Level wise entry.



$2^n = m \rightarrow n = \text{level}, m = \text{no. of nodes that should be present on that level.}$

D	E	T	Y	-1	K	S	-1	H	-1	-1	B	-1	-1	J

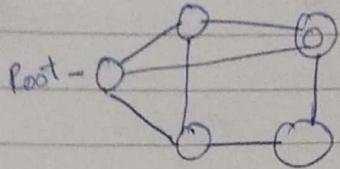
skew tree (jhuki hui tree)

Handled by tree balancing

- Tree is a special form of a graph in which there is only one inward arrow.
- ① Incoming \rightarrow 1 arrow
- ② No cycle formation (loop).

GRAPH.

2D array



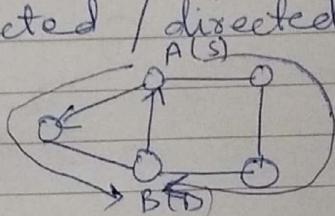
- multiple incoming & outgoing
- no head
- Source, Destination.
- Direction matters

- connections are edges.

- shortest distance / path

→ cost involved.

- undirected / directed.



src dest

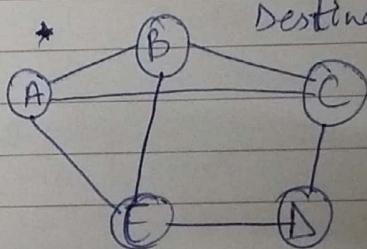
A to B

A → B

- undirected (cost / no cost) lowest or equal cost
- directed (cost / no cost). will be considered as 1

① Starting point

= Root



Source node

Destination node

19/01/2023

"A = src C = Des"

{ A - B - C

A - C

A - B - E - D - C

A - E - B - C

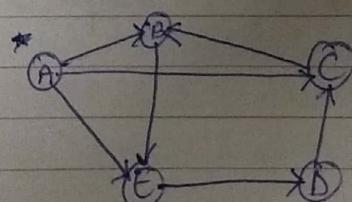
A - E - D - C

Edge is a connection b/w two nodes/vertex.

Graph = { V, E }

↓ vertex → Edges.

Directed
Undirected (Bidirectional).



A = src C = Des

A → C

A → B → E → D → C

A → E → D → C

Mixed Graph ⇒ both directed & undirected.

optimal ⇒ min, max.

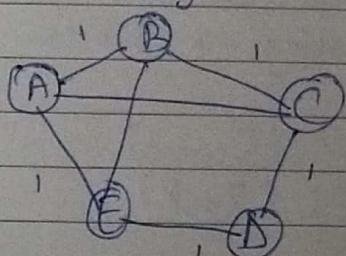
weighted (edges)
unweighted

shortest path algorithms
kruskal, prims, djextra.

① Adjacency Matrix ② Indices Matrix

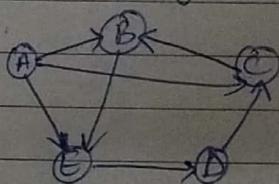
	A	B	C	D	E
A	0	1	1	0	1
B	1	0	1	0	1
C	1	1	0	1	0
D	0	0	1	0	1
E	1	1	0	1	0

bidirectional and unweighted



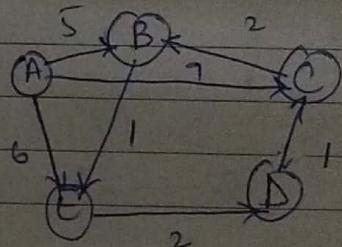
	A	B	C	D	E
A	0	1	1	0	1
B	0	0	0	0	1
C	0	1	0	0	0
D	0	0	1	0	0
E	0	0	0	1	0

disected and unweigted



disected by weighted

*	A	B	C	D	E
A	0	5	9	0	6
B	0	0	0	0	1
C	2	0	0	1	0
D	0	0	1	0	0
E	0	0	0	2	0



$$q = A \xrightarrow{9} C$$

$$q = A \xrightarrow{5} B \xrightarrow{1} E \xrightarrow{2} D \xrightarrow{1} C$$

$$XA \xrightarrow{6} E \xrightarrow{2} B \xrightarrow{1} C$$

$$q = A \xrightarrow{6} E \xrightarrow{2} D \xrightarrow{1} C$$

Graph Traversal

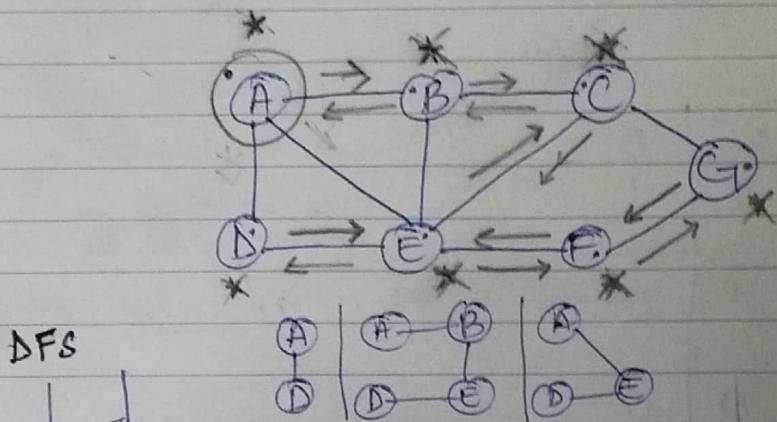
- ↳ Breadth First (BFS) Queue or stack.
- ↳ Depth First (DFS)

- ① Stack → Array, LL, Expression
- ② Queue → Array, LL, Implementation
- ③ Tree → LL, Array, BST, Insert, Search, Delete, Infix, Postfix, Prefix
- ④ Graph → LL, Array (2D) BFS, DFS, Dijkstra.
- ⑤ Hashing → Linear, Quad, Double Hash.

Balancing of Tree

BFS (Queue)

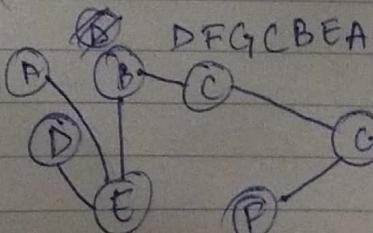
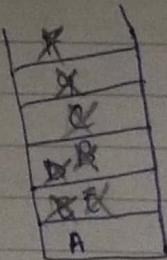
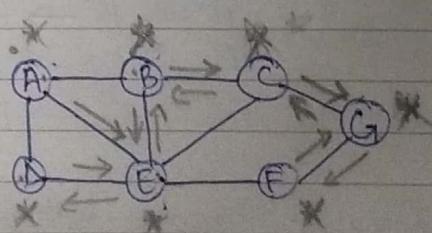
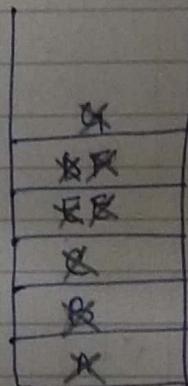
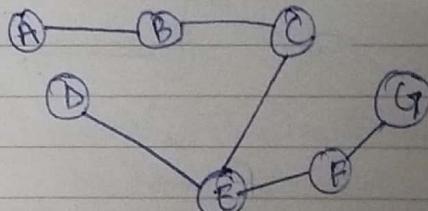
DFS (Stack).



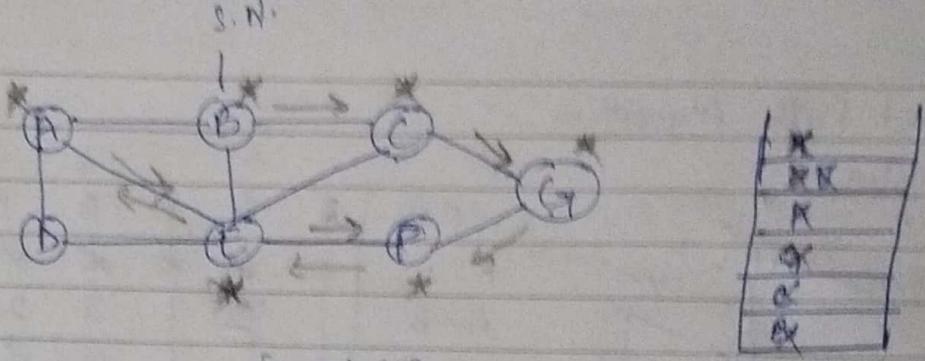
DFS



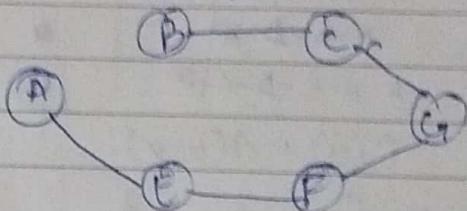
DGFEBCA



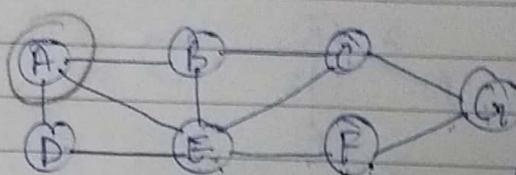
DFGCBEA



AEPGCB

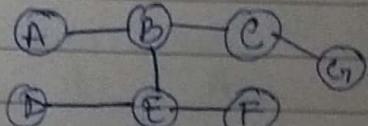
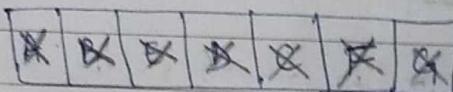


BFS

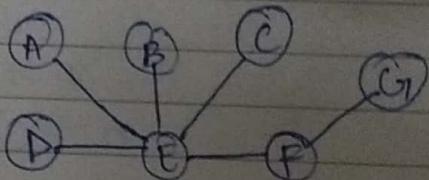
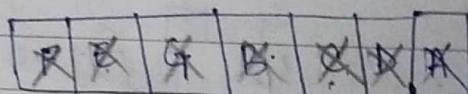


ABEDCFG

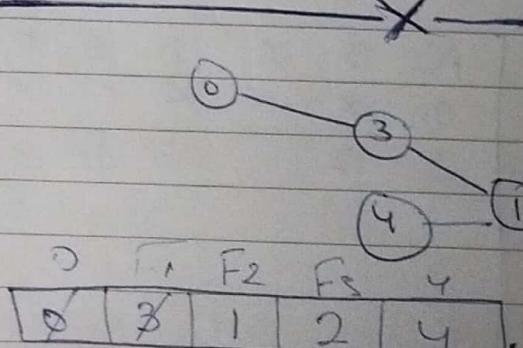
AECGF
AEBDCEFCA
AEBDCFG



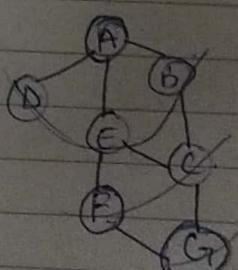
ABEDCFG



FEGBCDA

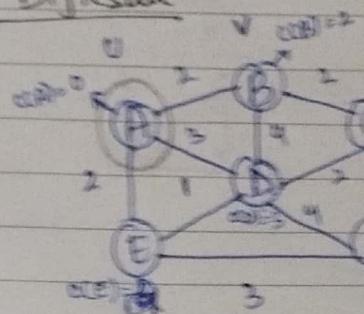


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Shortest Path Algorithm

Dijkstra

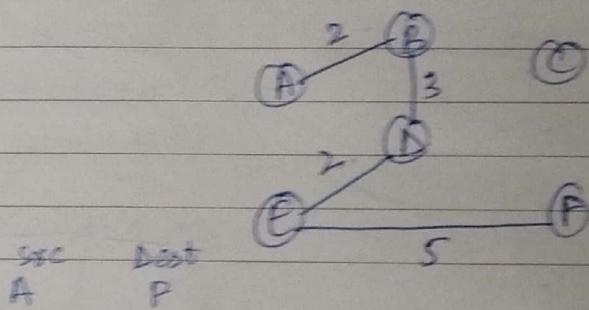


Path	A	B	C	D	E	F
A	0	∞	∞	∞	∞	∞
A-B		2	∞	∞	∞	∞
A-B-D			4	2	∞	∞
A-B-B-E				2	2	∞
A-B-E-D-C+F					2	5
					4	

$$C(v) = C(u) + d(u,v),$$

$$= 0 + 2 = 2$$

A - B - D - E - C - F



Do it again
Same

Path	A	B	C	D	E	F
A	0	∞	∞	∞	∞	∞
A-B		2	∞	∞	∞	∞
A-B-D			4	3	2	∞
A-B-D-E				4	2	∞
A-B-D-E-F					2	5
					3	

Path	A	B	C	D	E	F
A	0	∞	∞	∞	∞	∞
A-E		2	∞	∞	3	2
					3	5

HASH:

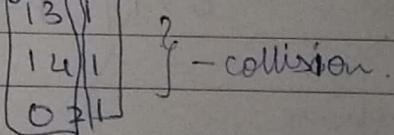
$R \leftarrow L_1 \leftarrow L_2 \leftarrow RAM \leftarrow HDD$			Division Method
Hash Table is 1D			↓ location/block
0	$(0 - m-1)$	$K \mod m = \text{block \#}$	$K \mod m$
any key	11	$h(K) = K \mod m$	11 $11 \mod 10 = 1$
2	32	\downarrow hash function	99 $99 \mod 10 = 9$
3			54 $54 \mod 10 = 4$
4	54		32 $32 \mod 10 = 2$
5			
6			
7			
8			
9	99		

Multiplication method or Folding technique or Mid square
(Co.)

Collision.

Tags. Index

method.



To resolve collision, there are two methods.

→ Open hashing → no boundary to get solution.

→ close hashing → bounded to get solution

K	h(K)	Prob	List of List	
			Index	Index
211065	5	0		21100
211100	0	0		211131 211041
211131	1	0	1	211133
211133	3	0	3	211034 211041
211034	4	0	4	211065
211017	7	0	7	
211041	1	1	1	
211184	4			
211197	7			

Open hashing / chaining

~~Close Hashing~~

- ① Linear Probing
- ② Quadratic Probing

Probing = no. of collisions.

Linear Probing :

$$h'(k) = (h(k) + \text{Prob}) \% m$$

0	100
1	131
2	041
3	133
4	021
5	065
6	
7	
8	
9	

k	$h(k)$	Prob
065	5	0
100	0	0
131	1	0
133	3	0
034	4	0
017	7	0
041	1	01
184		
147		
021	1	XZ3, 4

$$\begin{aligned} h'(k) &= (h(k) + P) \% m \\ &= (1+1) \% 10 \\ 2 \% 10 &= 2 \\ (1+3) \% 10 &= 4 \\ 4 \% 10 &= 4 \end{aligned}$$

Quadratic Probing

$$h'(k) = (h(k) + \text{Prob}^2) \% 10$$

Mid square method.

(even) $2(10)65$ (odd) $3(11)5$

$$10^2 \Rightarrow 100$$

$$[100 \% 10m]$$

$$4^2 \Rightarrow 16$$

$$[16 \% 10m]$$

Folding technique.

$$m = 10$$

$$[2][10]65$$

$$[3]45$$

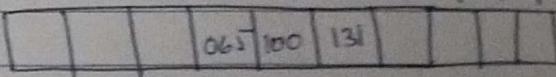
$$[96 \% 10m]$$

$$\begin{array}{r} 21 \\ 10 \\ +65 \\ \hline 96 \end{array}$$

$$\begin{array}{r} 03 \\ +45 \\ \hline 48 \end{array} \% 10m$$

$j \rightarrow \text{index at}$

k	hash 1	hash 2	$h(k) = h_1(k) + j h_2(k)$
065	$(2 * 65 + 3) \% 10 = 3$	$(3 * 65 + 1) \% 10 = 6$	$h_1(k) = (2k + 3) \% 10$
100	$(2 * 100 + 3) \% 10 = 3$	$(3 * 100 + 1) \% 10 = 1$	$h_2(k) = (3k + 1) \% 10$
131	$(2 * 131 + 3) \% 10 = 5$	$(3 * 131 + 1) \% 10 = 2$	0 1 2 3 4 5 6 7 8 9



$$h(k) = h_1(k) + j h_2(k)$$

$$h(k) = 3 + \Theta^*(6)$$

$$\boxed{h(k) = 3}$$

$$h(k) = 3 + 0 \times (1) = 3 \checkmark$$

$$h(k) = 3 + 1 \times (1) = 4 \checkmark$$

$$h(k) = 5 + 0 \times (2) = 5 \checkmark$$