Date 20

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REG. NO: CS211246

CLASS: BSCS-4C

COURSE: DESIGN & ANAYSIS OF ALGORITHMS

COURSE INSTRUCTOR:

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ASSIGNMENT # 02

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Question No. 1

Solve the following recurrence relations

1. $T(n) = T(n-1) + n$.

Recursive algorithm that loops through the input to eliminate one item.

$\Rightarrow T(n) = T(n-1) + n$.

$\Rightarrow T(n) = T(n-2) + (n-1) + n$

$\Rightarrow T(n) = T(n-3) + (n-2) + (n-1) + n$

if it runs k times.

$\Rightarrow T(n) = T(n-k) + (n-2) + (n-1) + n$

$\Rightarrow T(n) = (n-k) + (n-(k-1)) + (n-(k-2)) + \cdots + (n-1) + n$.

$\Rightarrow T(n-1) = T(n-1-1) + (n-1)$

$T(n-1) = T(n-2) + (n-1)$

$\Rightarrow T(n-2) = T(n-2-1) + (n-2)$

$T(n-2) = T(n-3) + (n-2)$

Since $T(0) = 1$.

So, $n - k = 0$

or $n = k$.

$\Rightarrow T(n) = T(n-n) + (n-n+1) + (n-n+2) + \cdots + (n-1) + n$.

$\Rightarrow T(n) = T(0) + 1 + 2 + \cdots + (n-1) + n$.

$\Rightarrow T(n) = 1 + 1 + 2 + \cdots + (n-1) + n$.

$\because 1 + 2 + \cdots + (n-1) + n = \frac{n(n-1)}{2}$

i.e. Sum of n natural numbers

$\Rightarrow T(n) = 1 + \frac{n(n+1)}{2}$.

$T(n) = 1 + \frac{n^2 + n}{2}$

$\Rightarrow \underline{O(n^2)}$

3. $T(n) = T(n/2) + n$.

Recursive algorithm that halves the input but must examine ~~the~~ every item in the input

$\Rightarrow T(n) = T(n/2) + n$ $\Rightarrow T(n/2) = T(n/2/2) + (n/2)$

$\Rightarrow T(n) = T(n/4) + n/2 + n$ $T(n/2) = T(n/4) + n/2$

$\Rightarrow T(n) = T(n/8) + n/4 + n/2 + n$ $\Rightarrow T(n/4) = T(n/4/2) + (n/4)$

and it runs k times $T(n/4) = T(n/8) + n/4$

$\because 2^3 = 8 \Rightarrow 2^k$

$\Rightarrow T(n) = T(n/2^k) + n/2^{k-1} + n/2^{k-2} + \cdots + n/2 + n$

Since $T(1) = 1$

$n/2^k = 1$

or $n = 2^k$.

$\Rightarrow T(n) = T(2^k/2^k) + n/2^{k-1} + n/2^{k-2} + \cdots + n/2 + n$

$\Rightarrow T(n) = T(1) + n/2^{k-1} + n/2^{k-2} + \cdots + n/2 + n$.

$\Rightarrow T(n) = 1 + n\left(1/2^{k-1} + 1/2^{k-2} + \cdots + 1/2 + 1\right)$

Here $\Rightarrow 1/2^{k-1} + \cdots + 1/2 \approx 1$

$\Rightarrow T(n) = 1 + n(1+1)$.

$\Rightarrow T(n) = 1 + 2n$.

$O(n)$

4. $T(n) = 2T(n/2) + 1$

Recursive algorithm that splits the input into 2 halves and does a constant amount of other work.

$\Rightarrow T(n) = 2T(n/2) + 1$

$\Rightarrow T(n) = 2[2T(n/4) + 1] + 1$

$\Rightarrow T(n) = 4T(n/4) + 2 + 1$

$\Rightarrow T(n) = 4[2T(n/8) + 1] + 2 + 1$

$\Rightarrow T(n) = 8T(n/8) + 4 + 2 + 1$

and it runs k times.

$= 2^3 = 8 \Rightarrow 2^k$

$\Rightarrow T(n) = 2^k T(n/2^k) + 7$

here, if $n = 8$ then $n - 1 = 7$.

$\Rightarrow T(n) = 2^k T(n/2^k) + n - 1$

$\because T(1) = 1$

so $n/2^k = 1$

or $n = 2^k$.

$\Rightarrow T(n) = n \cdot T(2^k/2^k) + n - 1$

$\Rightarrow T(n) = n \cdot 1 + n - 1$

$\Rightarrow T(n) = n + n - 1$

$\Rightarrow T(n) = 2n - 1$.

$\Rightarrow O(n)$

$\Rightarrow T(n/2) + 2T(n/2) + 1$

$T(n/2) = 2T(n/4) + 1$

$\Rightarrow T(n/4) = 2(T(n/4)/2) + 1$

$T(n/4) = 2T(n/8) + 1$

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QUESTION NO. 2

Given the merge sort Algorithm, dry-run. the complete algorithm and show each step for the given arrays.

1.

a) 5, 2, 4, 7, 1, 3, 2, 6

A =

1	2	3	4	5	6	7	8
5	2	4	7	1	3	2	6

MERGE-SORT(A, 1, 8)

1	2	3	4	5	6	7	8
5	2	4	7	1	3	2	6

MERGE-SORT(A, 1, 4) | MERGE-SORT(A, 5, 8).

1	2	3	4
5	2	4	7

5	6	7	8
1	3	2	6

MERGE-SORT(A,1,2) | MERGE-SORT(A,3,4) | MERGE-SORT(A,5,6) | MERGE-SORT(A,7,8)

1	2
5	2

3	4
4	7

5	6
1	3

7	8
2	6

MERGE-SORT(A,1,1) | MERGE SORT(A,2,2) | MERGE SORT(A,3,3) | MERGE SORT(A,4,4) | MERGE SORT(A,5,5) | MERGE SORT(A,6,6) | MERGE SORT(A,7,7) | MERGE SORT (A,8,8)

5 | 2 | 4 | 7 | 1 | 3 | 2 | 6

1	2
5	2

3	4
4	7

5	6
1	3

7	8
2	6

MERGE(A,1,1,2) | MERGE(A,3,3,4) | MERGE(A,5,5,6) | MERGE(A,7,7,8)

5 | 2 | 2 — i=1, j=1, k=1

5 | 2 | 2 5 — i=1, k=2

1	2
2	5

4 | 7 | 4 — i=1, j=1, k=3

4 | 7 | 4 7 — i=1, j=1, k=4

3	4
4	7

1 | 3 | 1 — i=1, j=1, k=5

1 | 3 | 1 3 — j=1, k=6

5	6
1	3

2 | 6 — i=1, j=1

2 — k=7

2 | 6 — j=1, k=8

7	8
2	6

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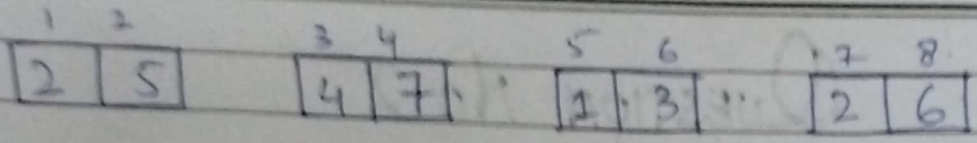

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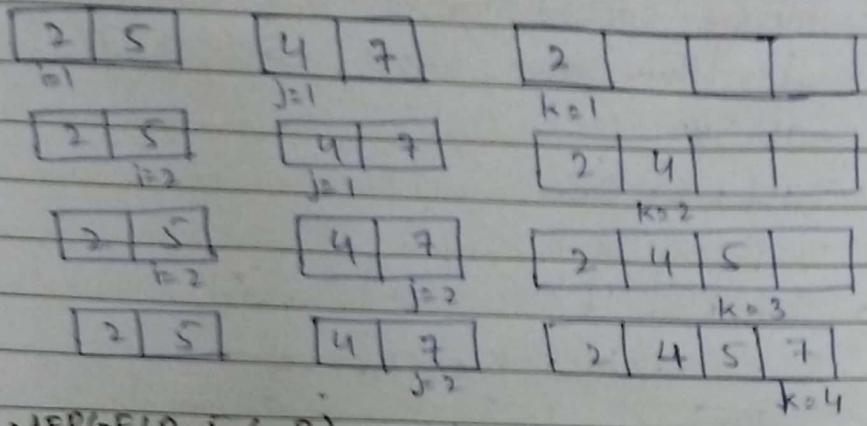

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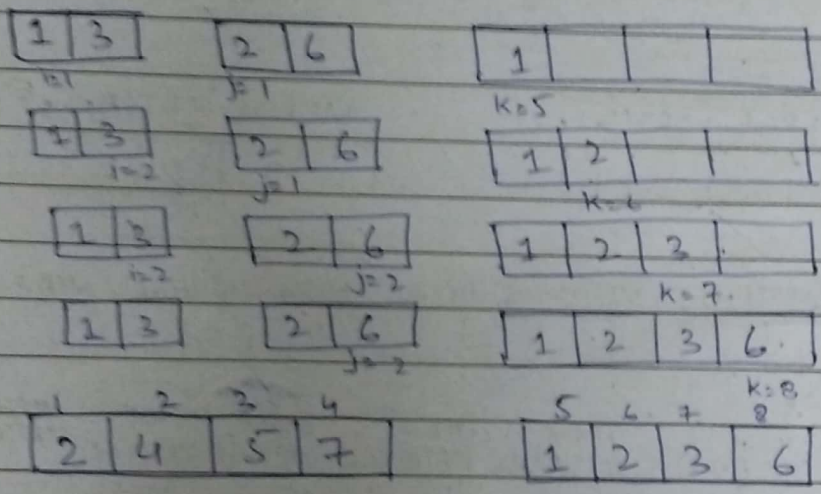

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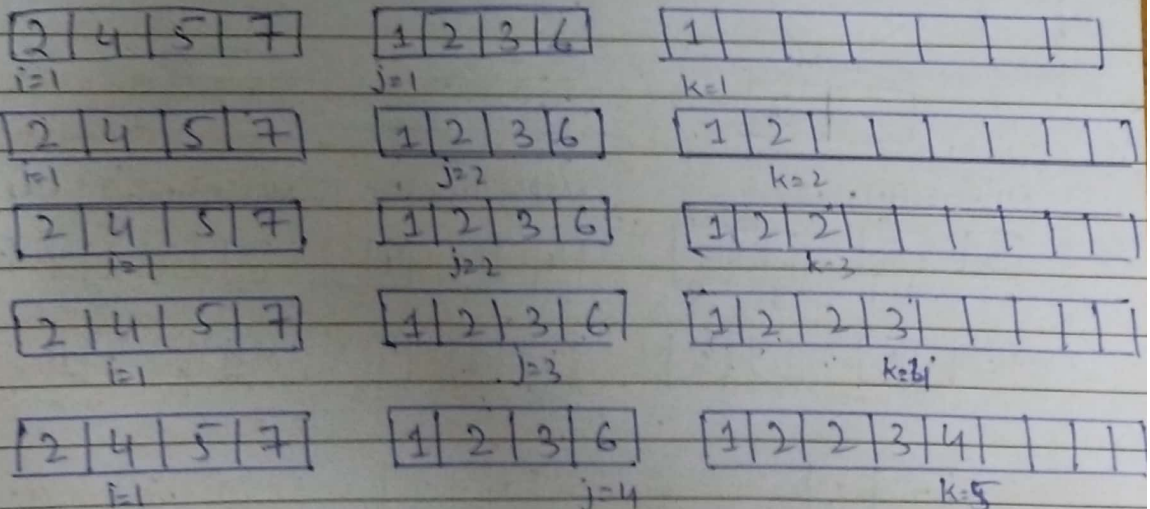

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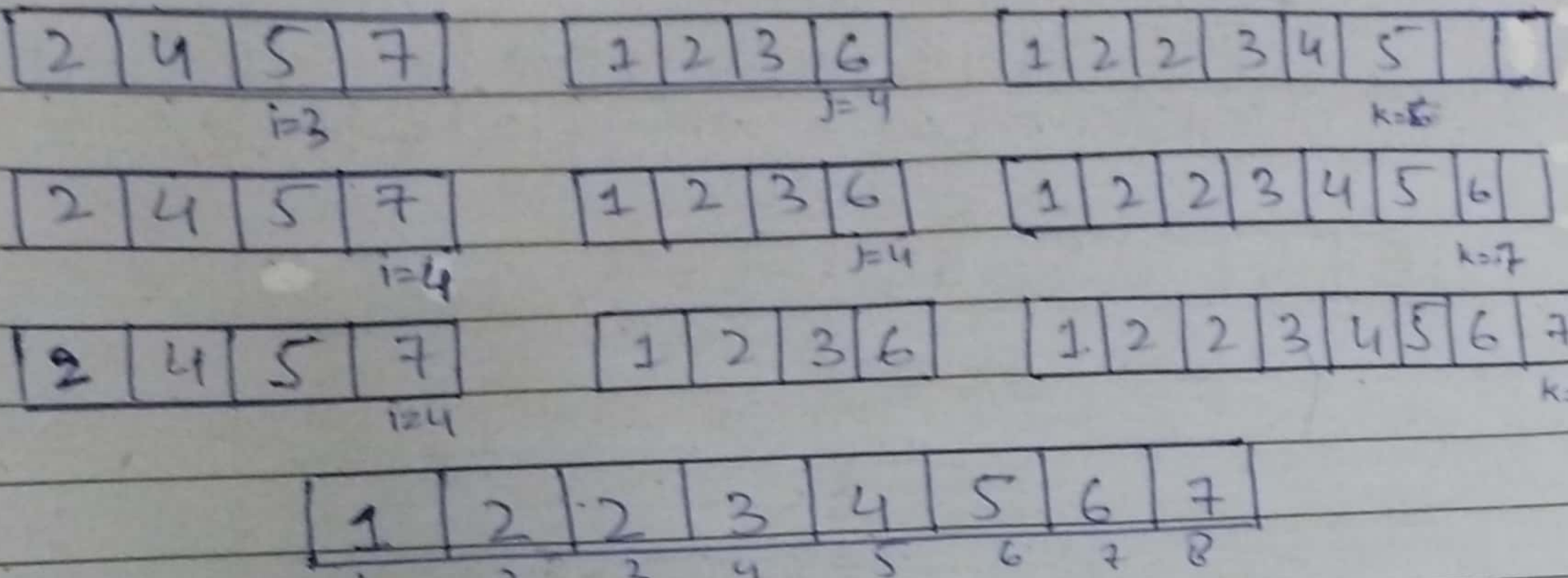
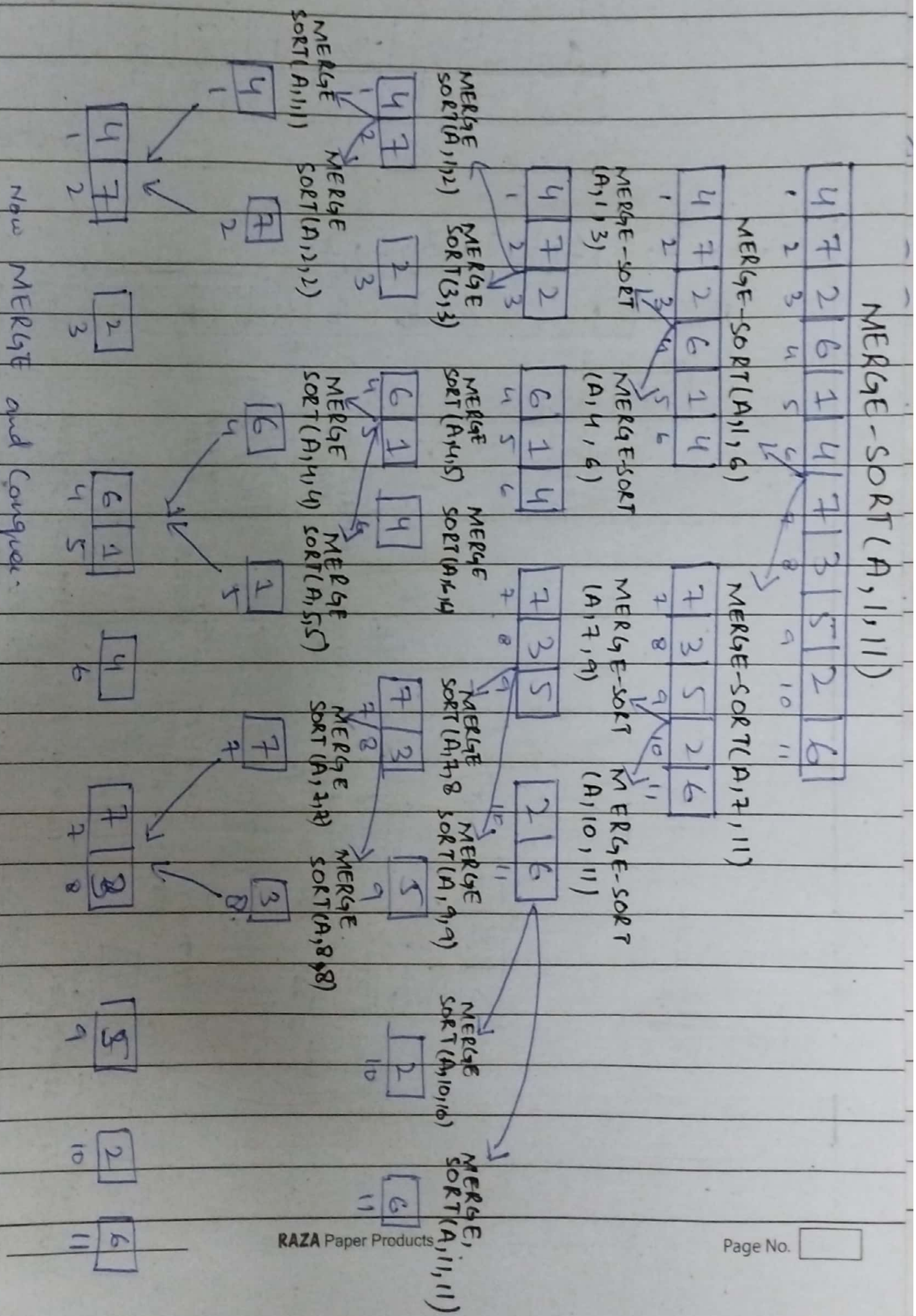


Sorted!

b) 4, 7, 2, 6, 1, 4, 7, 3, 5, 2, 6

	1	2	3	4	5	6	7	8	9	10	11
A =	4	7	2	6	1	4	7	3	5	2	6



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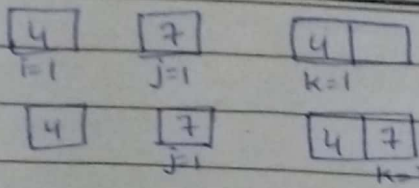
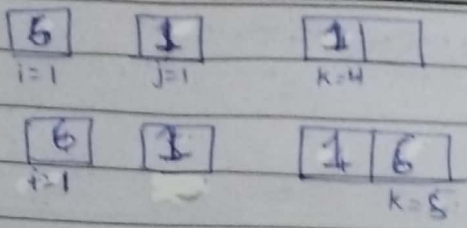
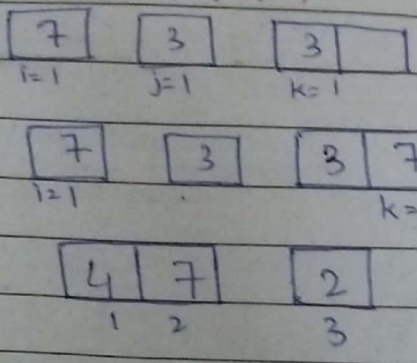
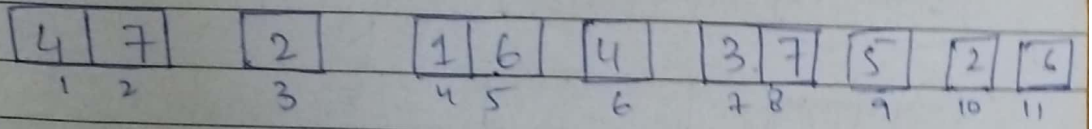


MERGE (A,4,4,5) Date 20



MERGE(A,7,7,8)





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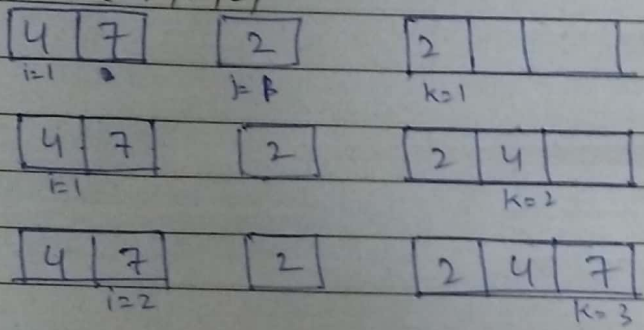

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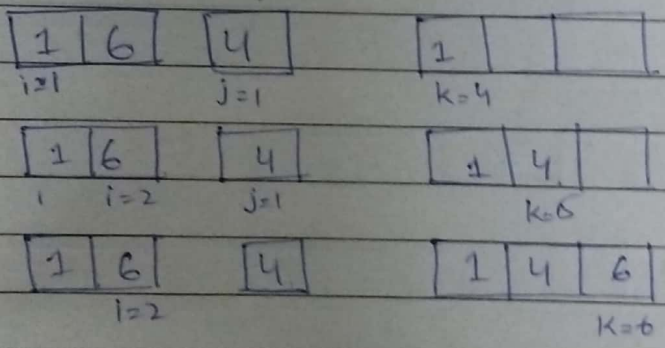
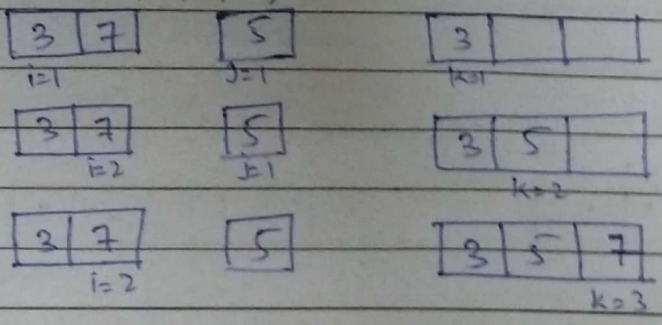


MERGE(A,7,8,9)



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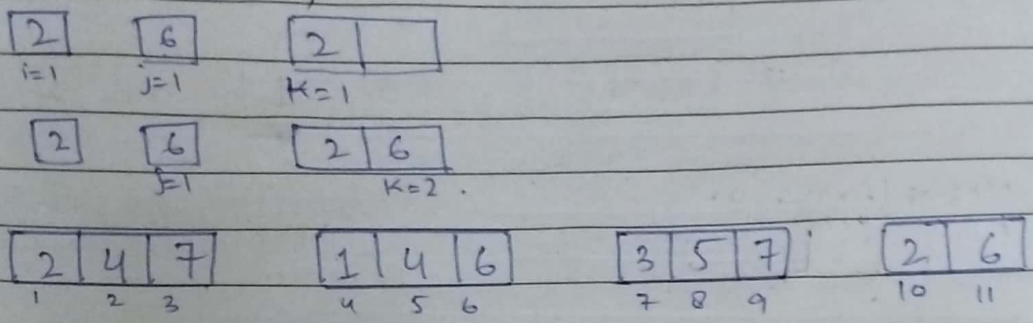
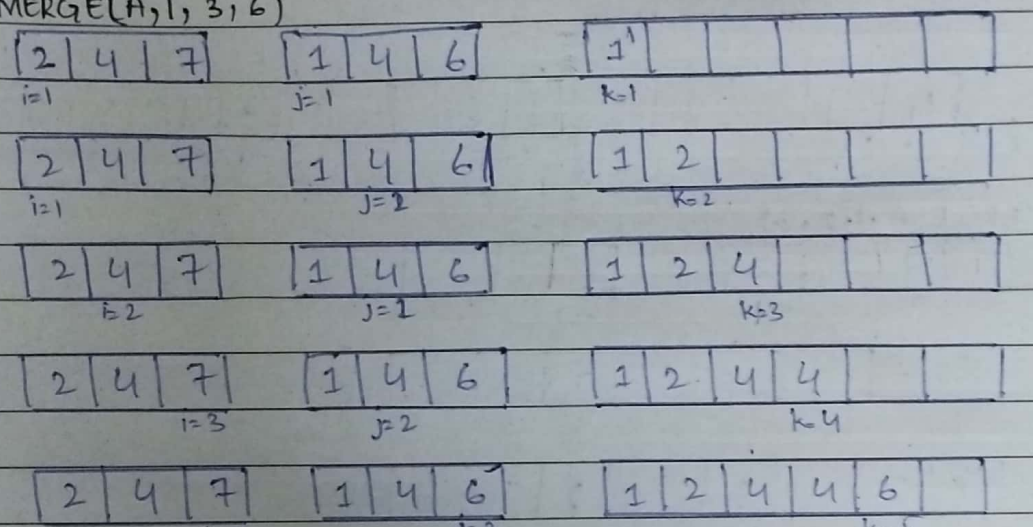
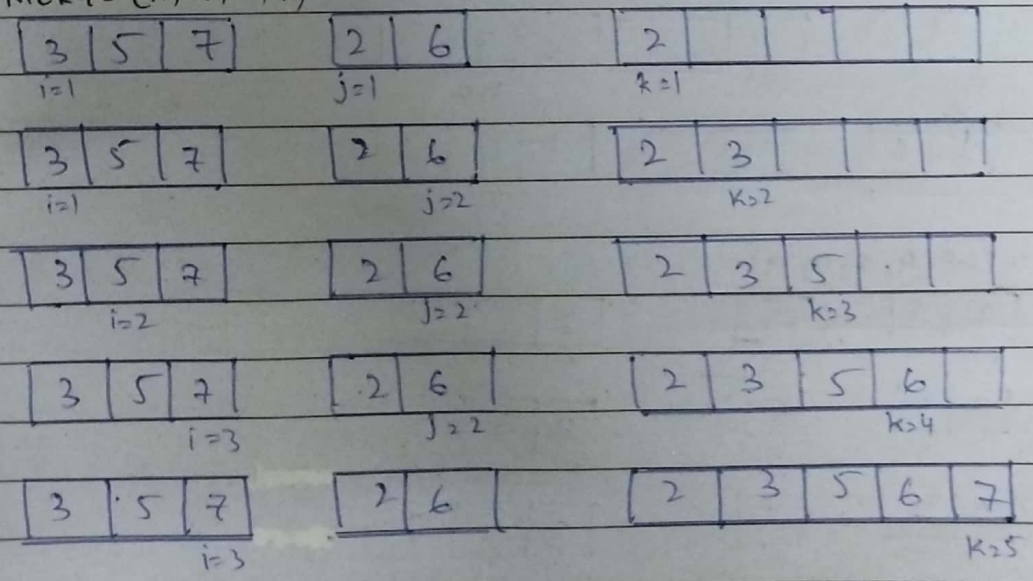
MERGE(A, 10, 10, 11)



MERGE(A, 1, 3, 6)



MERGE (A, 7, 9, 11)



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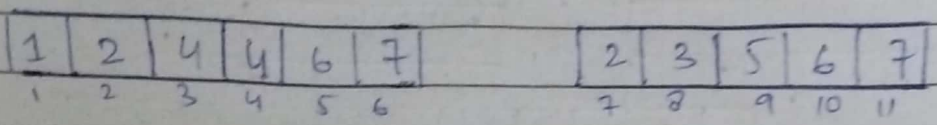

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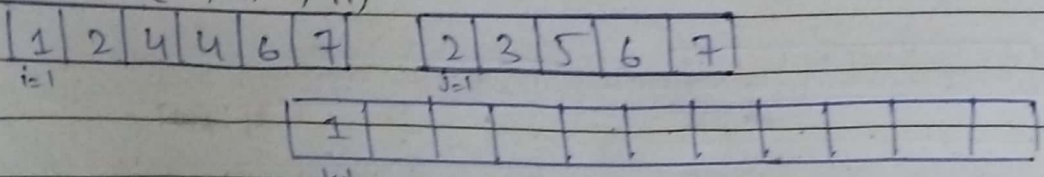

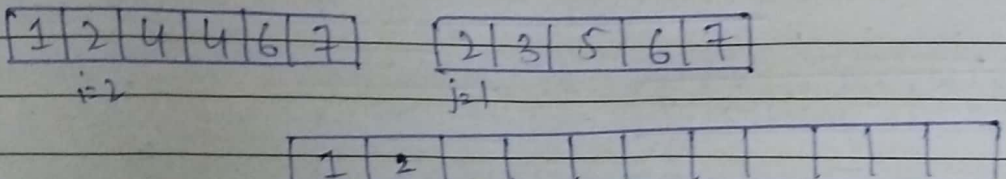

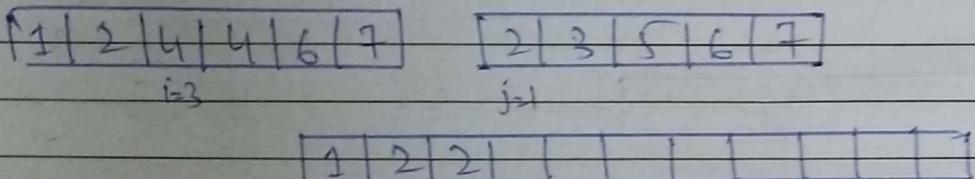

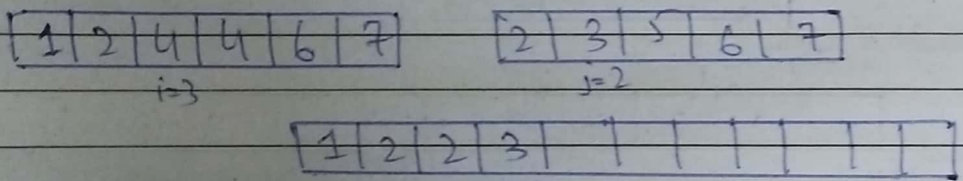

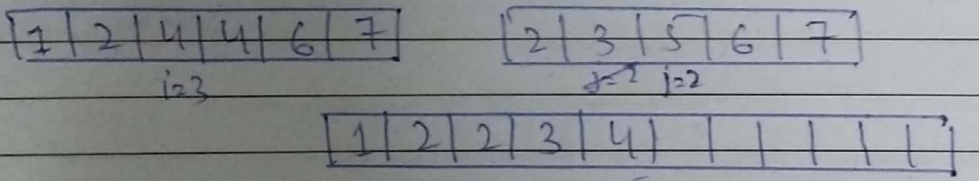

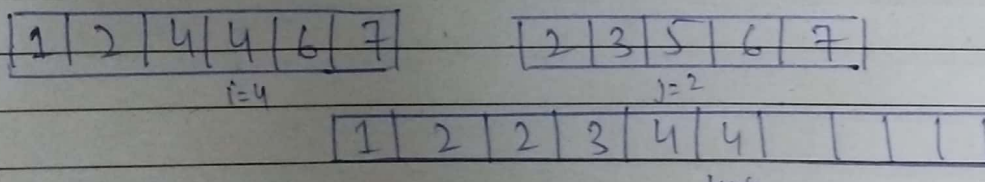

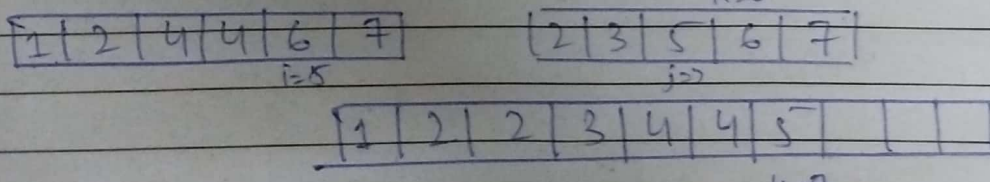

A,7,8)

MERGE SORT (A,8)

6

2

6

(A,7,7,8)

6

j=1

2

2 6

6

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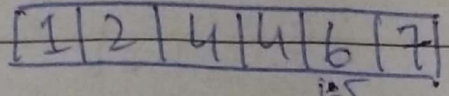

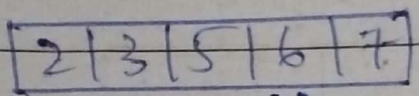

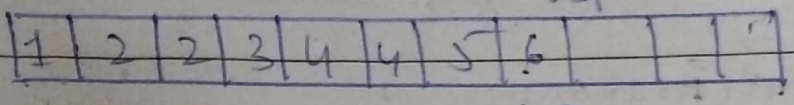

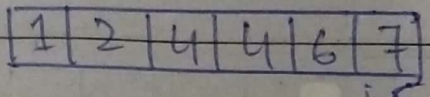

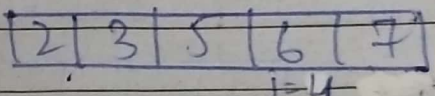

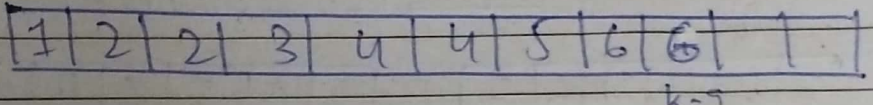

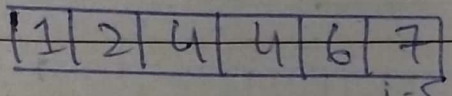

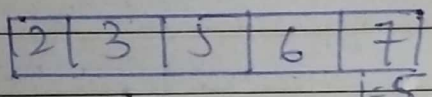

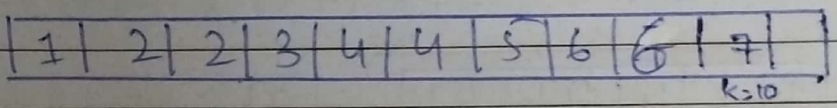

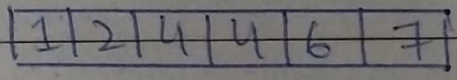

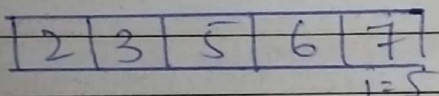

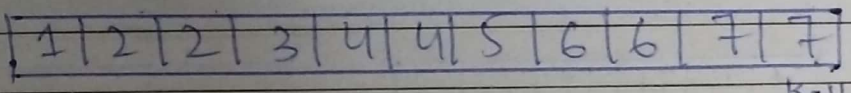

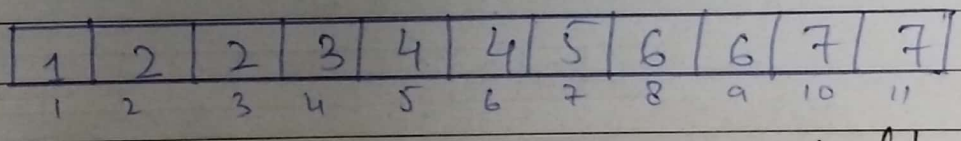

Sorted!