Multivariate Calculus - Practice 16/11/22.

Activity # 1 (Functions Of Several Variables).

Exercise set 13.1

1-8 Functions of two variables.

1. Let $f(x,y) = x^2y+1$

Find (a) $f(2,1)$

$f(2,1) = (2)^2(1)+1$

$f(2,1) = 4+1$

$f(2,1) = 5$

(b) $f(1,2)$

$f(1,2) = (1)^2(2)+1$

$f(1,2) = 3$

(c) $f(0,0)$

$f(0,0) = (0)^2(0)+1$

$f(0,0) = 1$.

(d) $f(1,-3)$

$f(1,-3) = (1)^2(-3)+1$

$f(1,-3) = -3+1$

$f(1,-3) = -2$.

(e) $f(3a,a)$

$f(3a,a) = (3a)^2(a)+1$

$f(3a,a) = (9a^2)(a)+1$

$f(3a,a) = 9a^3+1$

(f) $f(ab, a-b)$

$f(ab, a-b) = (ab)^2(a-b)+1$

$f(ab, a-b) = (a^2b^2)(a-b)+1$

$f(ab, a-b) = a^3b^2 - a^2b^3 + 1$

2. Let $f(x,y) = x + \sqrt[3]{xy}$

Find (a) $f(t,t^2)$

$f(t,t^2) = t + \sqrt[3]{(t)(t^2)}$

$f(t,t^2) = t + \sqrt[3]{t^3}$

$f(t,t^2) = t + t$

$f(t,t^2) = 2t$

(b) $f(x,x^2)$

$f(x,x^2) = x + \sqrt[3]{(x)(x^2)}$

$f(x,x^2) = x + \sqrt[3]{x^3}$

$f(x,x^2) = x + x$

$f(x,x^2) = 2x$.

(c) $f(2y^2, 4y)$

$f(2y^2, 4y) = 2y^2 + \sqrt[3]{(2y^2)(4y)}$

$f(2y^2, 4y) = 2y^2 + \sqrt[3]{8y^3}$

$f(2y^2, 4y) = 2y^2 + \sqrt[3]{8}\cdot\sqrt[3]{y^3}$

$f(2y^2, 4y) = 2y^2 + (8)^{1/3}\cdot y$

$f(2y^2, 4y) = 2y^2 + (2^3)^{1/3}\cdot y$

$f(2y^2, 4y) = 2y^2 + 2y$ ✓

$f(2y^2, 4y) = 2y(y+1)$

3. Let $f(x,y) = xy + 3$
Find (a) $f(x+y, x-y)$.
$f(x+y, x-y) = (x+y)(x-y) + 3$.
$f(x+y, x-y) = x^2 - xy + xy - y^2 + 3$.
$f(x+y, x-y) = x^2 - y^2 + 3$

(b) $f(xy, 3x^2y^3)$
$f(xy, 3x^2y^3) = (xy)(3x^2y^3) + 3$
$f(xy, 3x^2y^3) = 3x^3y^4 + 3$.

4. Let $g(x) = x \sin x$
Find $g(x/y)$.
$g(x/y) = \frac{x}{y} \sin(x/y)$.

(b) $g(xy)$
$g(xy) = xy \cdot \sin(xy)$.

(c) $g(x-y)$
$g(x-y) = (x-y)(\sin(x-y)) \Rightarrow ?$
$g(x-y) = x \sin(x-y) - y \sin(x-y)$.

5. Find $F(g(x), h(y))$ if $F(x,y) = xe^{xy}$, $g(x) = x^3$, and $h(y) = 3y+1$.
$F(x,y) = xe^{xy}$ — ①
$F(g(x), h(y)) = ?$
$\because g(x) = x^3 \Rightarrow x$
& $h(y) = 3y+1 \Rightarrow y$.
putting in eqn ①
$F(x^3, 3y+1) = (x^3) \cdot e^{(x^3)(3y+1)} = ?$
$F(x^3, 3y+1) = x^3 e^{3x^3y + x^3}$

6. Find $g(u(x,y), v(x,y))$ if $g(x,y) = y \sin(x^2 y)$, $u(x,y) = x^2 y^3$ and $v(x,y) = \pi xy$.

$g(x,y) = y \sin(x^2 y)$ — ①

$g(u(x,y), v(x,y)) = ?$

$\therefore \quad u(x,y) = x^2 y^3 = x$ —

$v(x,y) = \pi xy = y$ —

put in eq ①.

$g(x^2 y^3, \pi xy) = y \sin(x^2 y)$.

$g(x^2 y^3, \pi xy) = \pi xy \cdot \sin((x^2 y^3)^2 \pi xy)$.

$g(x^2 y^3, \pi xy) = \pi xy \cdot \sin(\pi x^5 y^4)$.

7. Let $f(x,y) = x + 3x^2 y^2$, $x(t) = t^2$ and $y(t) = t^3$.

Find (a) $f(x(t), y(t))$.

$f(x(t), y(t)) = t^2 + 3(t^2)^2 (t^3)^2$

$f(x(t), y(t)) = t^2 + 3t^4 t^6$

$f(x(t), y(t)) = t^2 + 3t^{10}$

8. (b) $f(x(0), y(0))$.

first $x(0) = ?$ and $y(0) = ?$

$x(t) = t^2$ $\qquad$ $y(t) = t^3$

$x(0) = (0)^2$ $\qquad$ $y(0) = (0)^3$

$x(0) = 0$ $\qquad$ $y(0) = 0$.

$f(x(0), y(0)) = (0) + 3(0)^2 (0)^2$

$f(x(0), y(0)) = 0$.

(c) $f(x(2), y(2))$.

first $x(2) = ?$ and $y(2) = ?$

$x(t) = t^2$ $\qquad$ $y(t) = t^3$

$x(2) = (2)^2$ $\qquad$ $y(t) = (2)^3$

$x(2) = 4$ $\qquad$ $y(t) = 8$

$f(x(2), y(2)) = 4 + 3(4)^2 (8)^2$

$f(x(2), y(2)) = 4 + 3072$.

$f(x(2), y(2)) = 3076$.

8. Let $g(x,y) = ye^{-3x}$, $x(t) = \ln(t^2+1)$ and $y(t) = \sqrt{t}$. Find $g(x(t), y(t))$.

$e^{\ln x} = x$.
$e^{-\ln x} = \frac{1}{x}$

$g(x(t), y(t)) = ?$

$x(t) = \ln(t^2+1)$

$y(t) = \sqrt{t}$

$g(\ln(t^2+1), \sqrt{t}) = \sqrt{t} \cdot e^{-3[\ln(t^2+1)]}$

$g(\ln(t^2+1), \sqrt{t}) = \sqrt{t} \cdot e^{-3(\ln t^2 + \ln)}$

$g(\ln(t^2+1), \sqrt{t}) = \sqrt{t} \cdot e^{-3\ln t^2 - 3\ln}$

$= \sqrt{t} \cdot \frac{1}{(t^2+1)^3}$

$g(\ln(t^2+1), \sqrt{t}) = \sqrt{t} \cdot e^{-3[\ln(t^2+1)]}$.

$g(\ln(t^2+1), \sqrt{t}) = \sqrt{t} \cdot \frac{1}{(t^2+1)^3}$.

$g(\ln(t^2+1), \sqrt{t}) = \frac{\sqrt{t}}{(t^2+1)^3}$.

9-10. Suppose that the concentration C in mg/L of medication in a patient's bloodstream is modeled by the function $C(x,t) = 0.2x(e^{-0.2t} - e^{-t})$, where x is the dosage of the medication in mg & t is the number of hours since the beginning of administration of the medication.

9(a) Estimate the value of $C(25,3)$ to two decimal places. Include appropriate units & interpret your answer in a physical context.

$C(x,t) = 0.2x(e^{-0.2t} - e^{-t})$

$C(25,3) = 0.2(25)(e^{-0.2(3)} - e^{-3})$

$C(25,3) = 5(e^{-0.6} - e^{-3})$

$C(25,3) = 5(0.49)$

$C(25,3) = 2.49$ mg/L

(b) If the dosage is 100 mg, give a formula for the concentration as a function of time t.

$C(100,t) = 0.2(100)(e^{-0.2t} - e^{-t})$

$C(100,t) = 20(e^{-0.2t} - e^{-t})$

(c) Give a formula that describes the concentration after 1 hour in terms of dosage x.

$$C(x, 1) = 0.2x\left(e^{-0.2(1)} - e^{-(1)}\right)$$

$$C(x, 1) = 0.2x\left(e^{-0.2} - e^{-1}\right).$$

10. (a) Suppose that the medication in the bloodstream reaches an effective level after a half hour. Estimate how much longer the medication remains effective.

$$C(x, t) = 0.2x\left(e^{-0.2t} - e^{-t}\right).$$

15 One method for determining relative humidity is to wet the bulb of thermometer, while it's through the air, & then compare the thermometer reading with the actual air temperature. If the relative humidity is less than 100%, the reading on the thermometer will be less than the temperature of the air. This difference in temperature is known as the wet-bulb depression. The accompanying table gives the relative humidity as a function of the air temperature and the wet-bulb depression. ~~The accompanying~~ Use the table to complete parts (a)–(c).

Wet Bulb Depression (°C)	Air Temperature (°C)			
	15	20	25	30
3	71	74	77	79
4	62	66	70	73
5	53	59	63	67

(a) What is the relative humidity if the air temperature is 20°C & the wet-bulb thermometer reads 16°C?

as given in question, reading on thermometer will be less than temperature of the air, if relative humidity is less than 100% & the difference b/w temperatures is wet-bulb depression.

if W = wet bulb depression & A = Air Temperature.

W = Temperature of air − Temperature on thermometer (for wet bulb)

W = 20 − 16

W = 4°C

from function table

relative humidity = 66%.

(b) Estimate the relative humidity if the air temperature is 25°C & wet-bulb depression is 3.5°C

A = 25°C

W = 3.5°C.

= 77 − (1/2)7 → diff. b/w R.H (1st value.)

= 73.5%. → bcz of 3 − 4 (half) 3.5

relative humidity = 74%

(c) Estimate the relative humidity if the air temperature is 22°C & the wet-bulb depression is 5°C.

59 + (2/5) 4

57.4%

relative humidity = 60.6%.

16. Use the table in Q. 15 to complete parts (a) – (c).

(a) What is the wet-bulb depression if the air temperature is 30°C & the relative humidity is 73%.

according to the table.

wet-bulb depression = 4°C.

(b) Estimate the relative humidity if the air temperature is 15°C & the wet-bulb depression is 4.25°C.

A = 15°C W = 4.25°C

62 – (0.25) 4

relative humidity = 59.75%

(c) Estimate the relative humidity if the air temperature is 26°C & the wet-bulb depression is 3°C.

A = 26°C W = 3°C

77 + (2/5) 2

relative humidity = 77.4%

17-20 These exercises include functions of three variables.

17. Let $f(x,y,z) = xy^2z^3 + 3$. Find.

(a) $f(2,1,2)$.

$f(2,1,2) = (2)(1)^2(2)^3 + 3$

$f(2,1,2) = 16 + 3$

$f(2,1,2) = 19$.

(b) $f(-3,2,1)$.

$f(-3,2,1) = (-3)(2)^2(1)^3 + 3$

$f(-3,2,1) = -12 + 3$

$f(-3,2,1) = -9$.

(c) $f(0,0,0)$

$f(0,0,0) = (0)(0)^2(0)^3 + 3$

$f(0,0,0) = 0 + 3$

$f(0,0,0) = 3$.

(d) $f(a,a,a)$

$f(a,a,a) = (a)(a)^2(a)^3 + 3$

$f(a,a,a) = a^6 + 3$.

(e) $f(t, t^2, -t)$

$f(t, t^2, -t) = (t)(t^2)^2(-t)^3 + 3$

$f(t, t^2, -t) = -t^8 + 3$

$f(t, t^2, -t) = 3 - t^8$.

(f) $f(a+b, a-b, b)$

$f(a+b, a-b, b) = (a+b)(a-b)^2(b)^3 + 3$

$f(a+b, a-b, b) = (a+b)(a^2 - 2ab + b^2)b^3 + 3$

$f(a+b, a-b, b) = (a+b)(a^2b^3 - 2ab^4 + b^5) + 3$

$f(a+b, a-b, b) = a^3b^3 - 2a^2b^4 + ab^5 + a^2b^4 - 2ab^5 - b^6 + 3$

$f(a+b, a-b, b) = a^3b^3 - a^2b^4 - ab^5 - b^6 + 3$

18. Let $f(x, y, z) = zxy + x$. Find

(a) $f(x+y, x-y, x^2)$

$f(x+y, x-y, x^2) = (x^2)(x+y)(x-y) + x + y$

$f(x+y, x-y, x^2) = x^2(x^2 - xy + xy - y^2) + x + y$

$f(x+y, x-y, x^2) = x^2(x^2 - y^2) + x + y$

$f(x+y, x-y, x^2) = x^4 - x^2y^2 + x + y$.

(b) $f(xy, y/x, xz)$.

$f(xy, y/x, xz) = (xz)(xy)(y/x) + xy$

$f(xy, y/x, xz) = xy^2z + xy$.

19. Find $F(f(x), g(y), h(z))$ if $F(x, y, z) = ye^{xyz}$, $f(x) = x^2$, $g(y) = y + 1$, and $h(z) = z^2$.

$F(x, y, z) = y \cdot e^{xyz}$.

$F(f(x), g(x), h(z)) = (y+1) \cdot e^{(x^2)(y+1)(z^2)}$.

$F(f(x), g(y), h(z)) = (y+1)\, e^{x^2yz^2 + x^2z^2}$.

20. Find $g(u(x, y, z), v(x, y, z), w(x, y, z))$ if $g(x, y, z) = z\sin(xy)$, $u(x, y, z) = x^2z^3$, $v(x, y, z) = \pi xyz$ and $w(x, y, z) = xy/z$

$g(x, y, z) = z\sin(xy)$.

$g(u(x, y, z), v(x, y, z), w(x, y, z)) = \frac{xy}{z} \cdot \sin(x^2z^3 \cdot \pi xyz)$

$g(u(x, y, z), v(x, y, z), w(x, y, z)) = \left(\frac{xy}{z}\right) \cdot \sin(\pi x^3y^2z^4)$