

# THEORY OF AUTOMATA AND FORMAL LANGUAGES 13-03-23.

- valid characters of your language  $\rightarrow$  Alphabets.
  - Sigma set =  $\Sigma = \{\text{set of alphabets of your language}\}$
  - Formal Language = collection of operators by symbols.
  - multiple characters make a string/combination of one or more characters.
  - word = valid string of your language.
  - Rules to check validity.
  - Regular Expression (Reg Ex) : Rules will be written in the form of Reg Ex.  
match ("int", Reg Ex).
  - Identifier = variables (user defined).
  - Tokenization : a word belongs to which class as it is valid  $\leftarrow$  tokenization.
- Finite State machine.
- $\hookrightarrow$  an algo to check a given word applies Reg Ex.
- Decider Machine :
- $\hookrightarrow$  decider for regular languages (expressions).
- Characters + Rules (RegEx)  $\rightarrow$  Language.

Push down automaton.

14-03-23

## REGULAR EXPRESSION.

$$\Sigma = \{a\}$$

$$a^* = \{a, aa, aaa, aaaa, \dots\}$$

RegEx  
Kleen's Closure  
+ Empty Closure.

$L_1 = \{\text{All strings of } a \text{ of any length}\}$ .

|           |   |                          |                                |
|-----------|---|--------------------------|--------------------------------|
| $\lambda$ | 0 | $\Sigma^0 = \{\lambda\}$ | $\lambda$ - string of 0 length |
| a         | 1 | $\Sigma^1 = \{a\}$       | or                             |
| aa        | 2 | $\Sigma^2 = \{aa\}$      | $\epsilon$                     |
| aaa       | 3 | $\Sigma^3 = \{aaa\}$     |                                |

$$\Sigma^* = \{\lambda, a, aa, aaaa, \dots\}$$

$\hat{a} = \{a, aa, aaa, aaaa, \dots\}$ .

$a$  is not acceptable.

$L_2 = \{ \text{All strings of } a \text{ at least one length}\}$ .

- When we concatenate or take union of 2 RegEx, a new RegEx can be made.

### Concatenation

$$L_3 = \{a, b\}$$

$$L_4 = \{c, d\}$$

$$L_3 L_4 = \{ac, ad, bc, bd\}$$

ac is a word of  $L_3 L_4$   $L_3 L_4 \neq L_4 L_3$ .

$$W = ac \in L_3 L_4$$

$$ca \notin L_3 L_4$$

$$\begin{aligned} a^+ &= \{\lambda, a, aa, aaa, \dots\} \{a, aa, aaa, \dots\} \lambda \cdot a = a \\ &= \lambda \{a, aa, aaa, \dots\}. \quad a \cdot \lambda = a \\ &= a \{a, aa, aaa, \dots\} \\ &= \{a, aa, aaa, aaaa, \dots\}. \end{aligned}$$

$$a^+ a = a \quad \text{or} \quad a a^+ = a$$

concatenation is not always commutative  
but this example is!

### Union

$$\Sigma = \{a, b\}$$

$$L_5 = \{a\}$$

$$L_6 = \{b\}$$

$$L_7 = \{a, b\} \quad a + b$$

$$L_7 = L_5 \cup L_6$$

$$(a+b)^* = \{a, b\}^* = \{\lambda, a, aa, aaa, \dots, b, bb, bbb, \dots, ab, ba, abb, bba, \dots\}$$

abbba, abb, aaab, ...

$$(a+b)^* = \{\text{Any string of } a, b\}$$
  
match  $(b, (a+b)^*)$ .

$L_{10} = \{ \text{all strings of } a, b \text{ that starts with } a^q \cdot$   
 $a (a+b)^*$

$L_{11} = \{ \text{all strings of } a, b \text{ that start with } a \text{ an}$   
end at  $b^q \cdot$   
 $a (a+b)^* b \cdot$

Q2

20-03-2023

$$a^* b^* = \{ \lambda, aaaa, aaaa, \dots \} \cup \{ b, bb, bbb, \dots \}$$
$$= \{ \lambda, aaaa, aaaa, \dots, b, bb, bbb, \dots, ab, abb, \dots, aabb, \dots \}$$

Describe the language of the following regular expression.

$$a^* b^* = \{ \text{All strings of } a, b \text{ in which there is}$$

no a after  $b^q$

Books.

- ① Automata Theory by Cohen
- ② Introduction to Theory of Computation by Michael Sipser
- ③ Intro. By To TOC 3rd by Ullman

$$a^* b^+ = \{ \lambda, a, aa, aaa, \dots \} \cup \{ b, bb, bbb, \dots \}$$
$$= \{ b, bb, bbb, \dots, ab, abb, \dots, aab, aabb, \dots \}$$

$a^* b^+$  = { all strings of  $a, b$  in which there is  
no a after b and if a, it must follow  
by b }  $\cup$  string must end at at least one

$(b + ab)^* = \{ \text{all strings of } a, b \text{ in which either only}$   
single b is acceptable or all strings of  
 $a, b$  in which no consecutive a }

2023  
Natalie, Natalie

66

100

$(ba^*b)^*$  = {all strings of a,b in which 2 or

$(ba^*b)^*$  = { $\lambda$ , bb, bbbb, ..., bab, baabi, babbab, bbbbab,

- 1) Even no. of b's ✓
- 2) Start & end with b ✓
- 3) There should be no aba substring ✓
- 4) No single length string X atleast 2 length string

① Null is acceptable

② String of atleast two b's or even no. of.

③ All those strings in which there is no substring  
aba in string must end at b. with even no. of  
bs

④ Even b's are acceptable.

$(a+b)^*$  aa+  $(a+b)^*$  (HW).

## FINITE STATE MACHINE (FA/FSM)

21-03-2023

The decider of regular language.

Components

① Input Tape



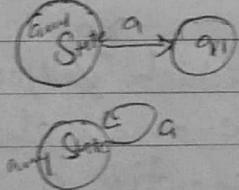
Types:

Deterministic

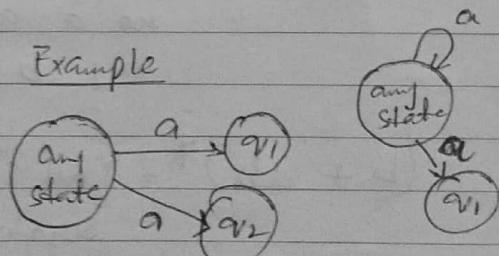
Non-deterministic.

One-way

Example machine



Example



can choose any one path

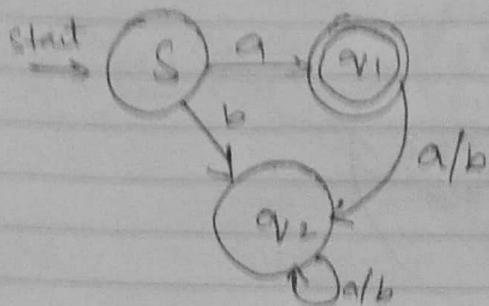
Acceptance State/Final state

$M(S, \Sigma, \delta, F)$

Rejection State/Non-final State.

$\delta(\text{current state}, a) = q_{V1}$   
more  
next state.

$$L = \{a^2\} \quad \Sigma = \{a, b\}$$



if(ch == a)  
 accept if(ch == a || ch == b)  
 else reject.  
 else  
 reject.

Input Tape

|   |   |   |    |
|---|---|---|----|
| a | b | b | 10 |
|---|---|---|----|

Transition Table

|       | a                      | b                      |
|-------|------------------------|------------------------|
| S     | $\delta(S, a) = q_1$   | $\delta(S, b) = q_2$   |
| $q_1$ | $\delta(q_1, a) = q_2$ | $\delta(q_1, b) = q_2$ |
| $q_2$ | $\delta(q_2, a) = q_1$ | $\delta(q_2, b) = q_2$ |

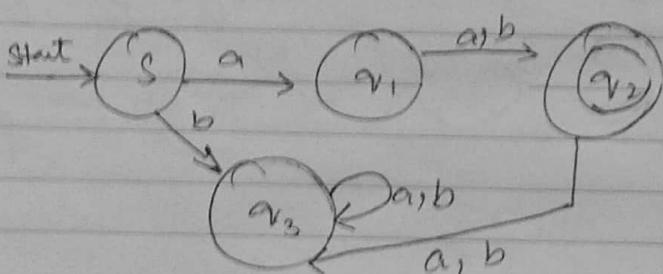
27-03-23

Finite State Machine

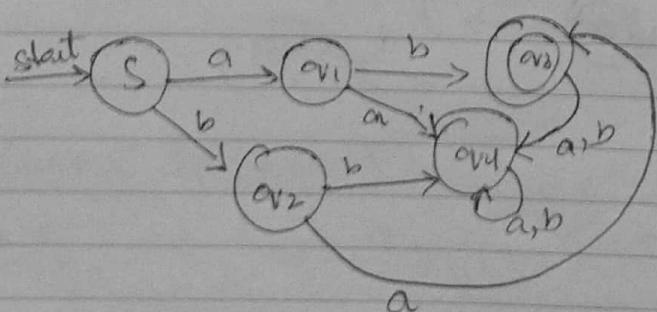
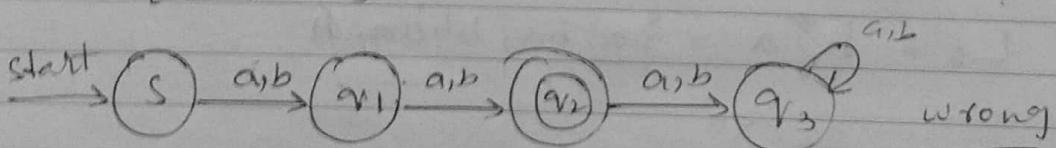
$$\Sigma = \{a, b\}$$

$$L_2 = \{aa, ab\}$$

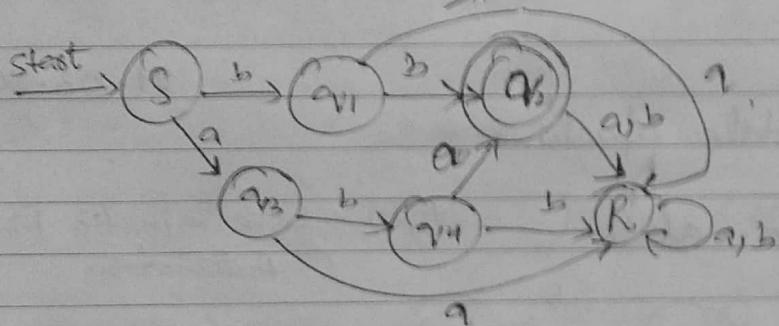
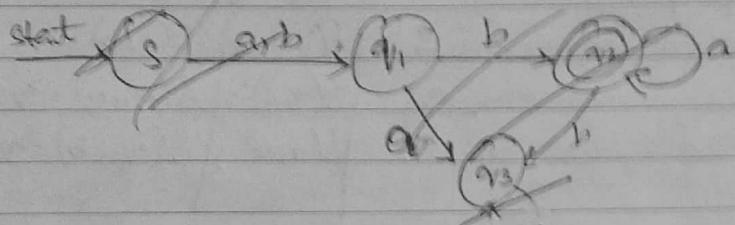
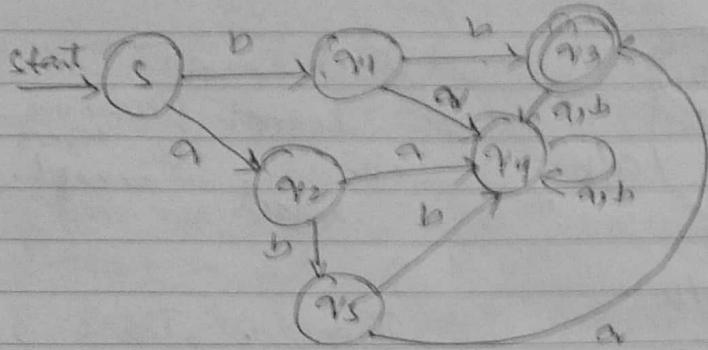
DFA  $\Rightarrow$  Deterministic Finite Automaton.



$$L_3 = \{ab, ba\}$$

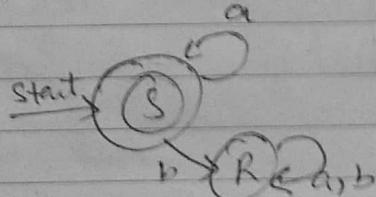


$L_4 = \{bbb, abab\}$ .

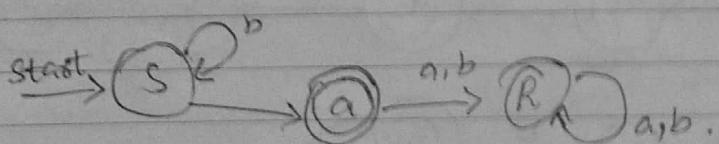


$\Sigma = \{a, b\}$

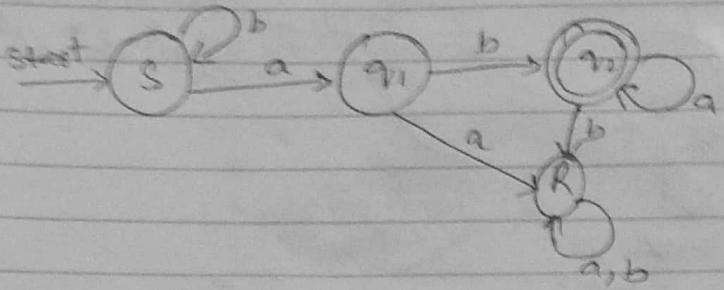
$L_5 = a^*$



$L_6 = b^*a = \{a, ba, bbb, \dots\}$

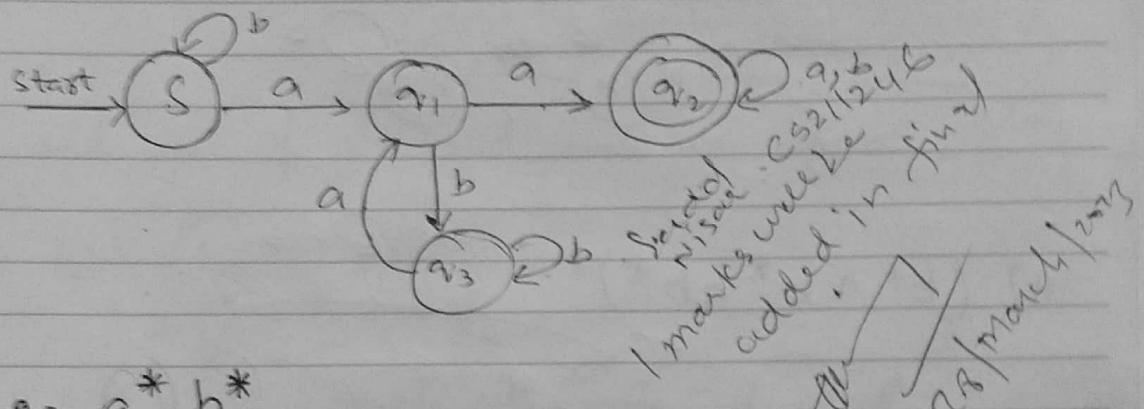


$$L_7 = b^* a b a^* \cdot \{ab, baba, \dots\}$$

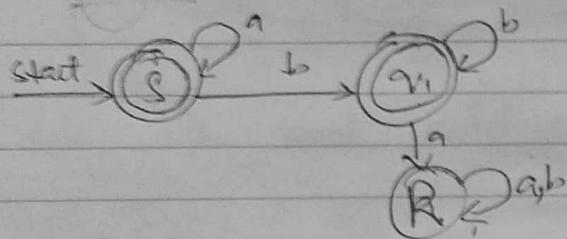


28-03-2023

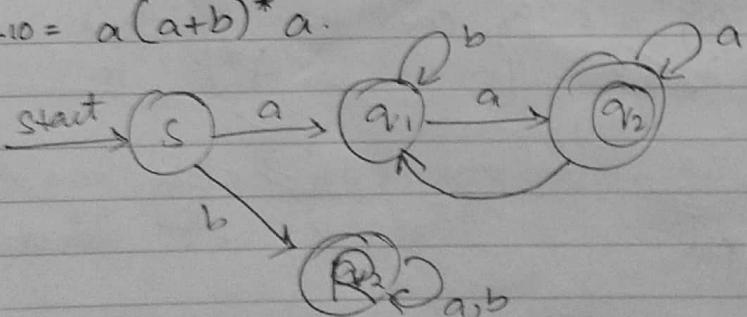
$$L_8 = (a+b)^* \underline{aa} (a+b)^*$$



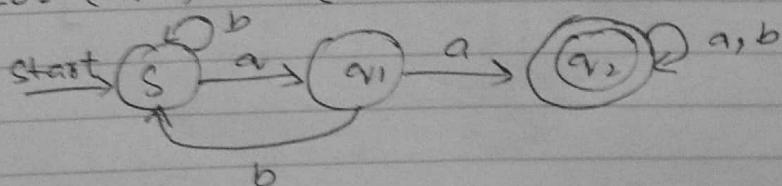
$$L_9 = a^* b^*$$

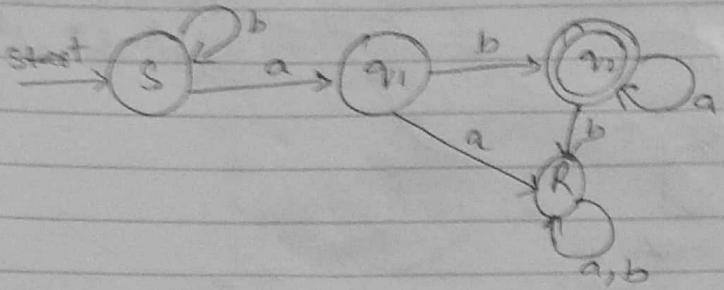


$$L_{10} = a(a+b)^* a$$

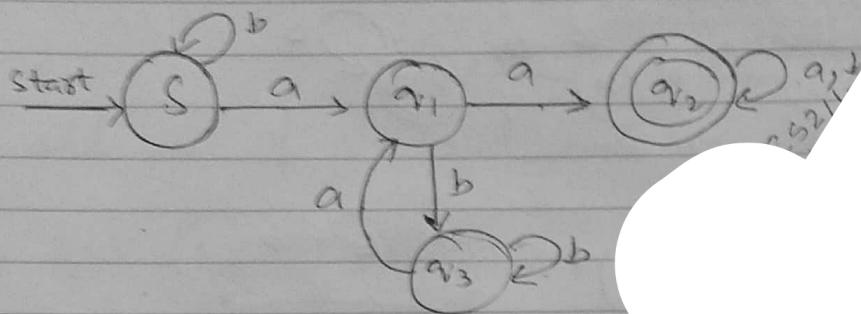
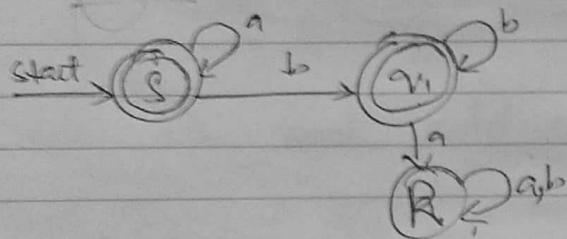
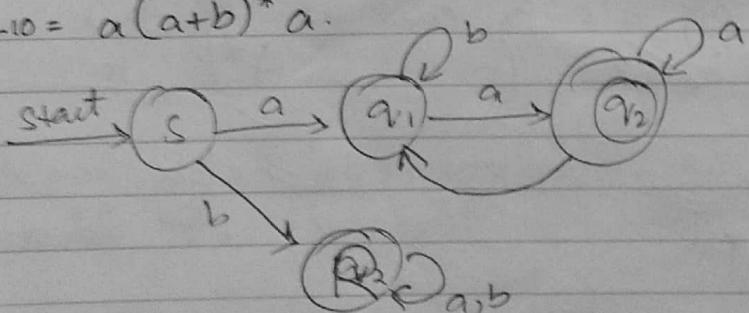
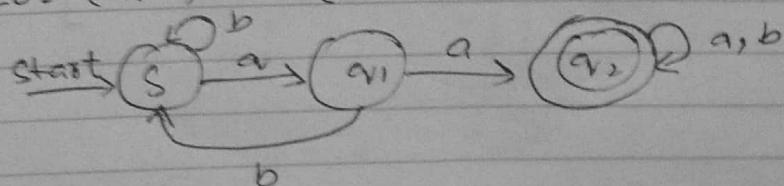


$$L_8 = (a+b)^* \underline{aa} (a+b)^*$$



$$L_7 = b^* a b a^* \cdot \{ab, baba, \dots\}$$


28-03-2023

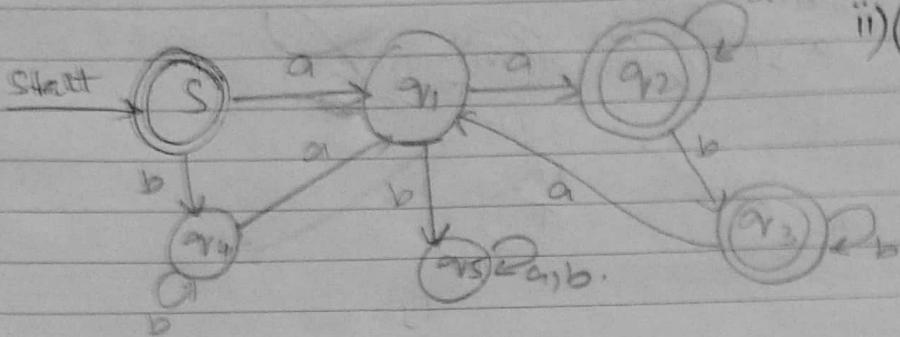
$$L_8 = (a+b)^* \underline{aa} (a+b)^*$$

$$L_9 = a^* b^*$$

$$L_{10} = a(a+b)^* a$$

$$L_8 = (a+b)^* \underline{aa} (a+b)^*$$


3/4/23.

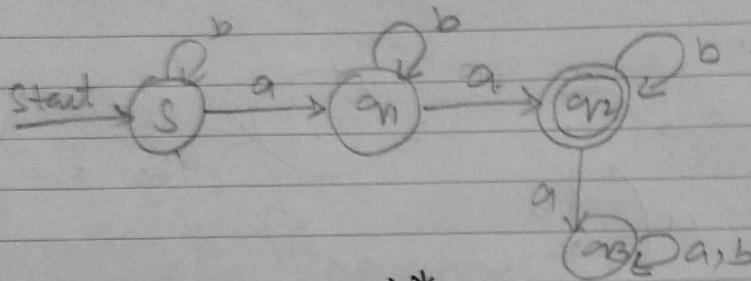
$$(b^*aa^+b^*)^* = \{ \lambda, aa, aaa, \dots, aabb \dots \}$$

baa, baabb, ...

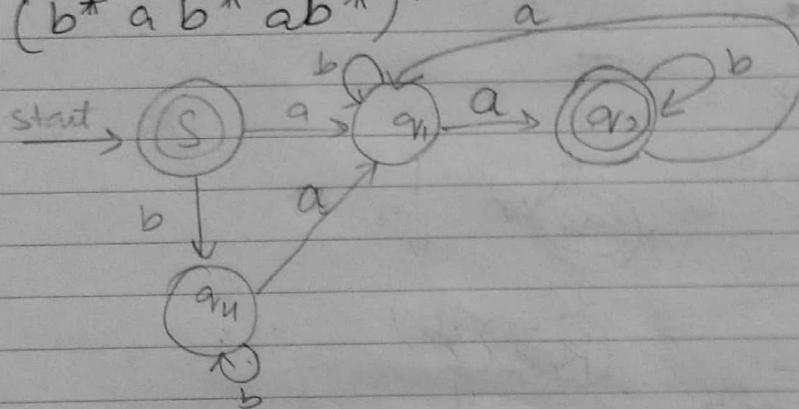
i)  $b^*ab^*ab^*$   
ii)  $(b^*ab^*ab^*)^*$



i)  $b^*ab^*ab^*$



ii)  $(b^*ab^*ab^*)^*$



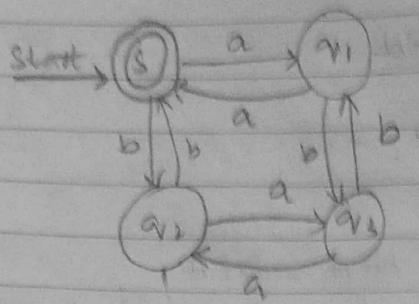
iii) even no. of a's in even no. of b's  
for exactly 2 a's:  $(\quad)^*$

$$= \{ \lambda, aa, bb, aabb, ab\underline{ba}, b\underline{ab}, aaabba, \\ \underline{aa} \underline{bb} \underline{ab} \underline{bb} \}$$

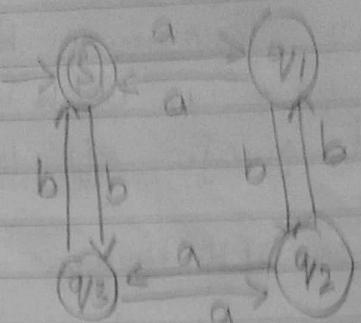
$(aa+bb)^*$

$$[aa + bb + (ba+ab)(aa+bb)^*(ab+ba)]^*$$

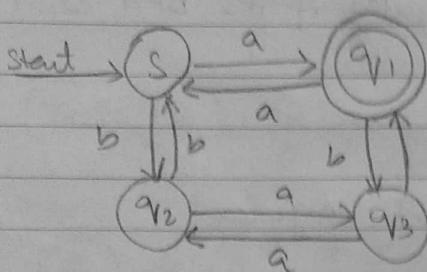
$$[aa + bb + (ba+ab)(aa+bb)^* (ab+ba)]^*$$



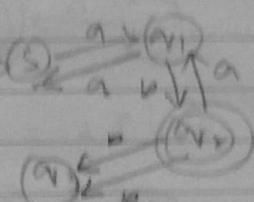
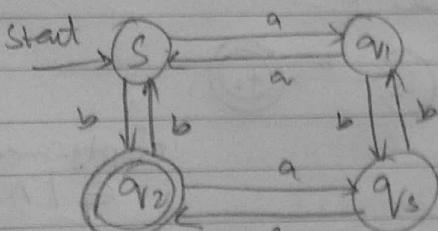
ba, baa, baab, ...



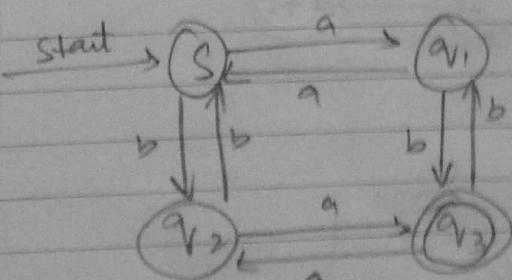
odd no. of a's and even no. of b's.



even a's and odd b's.

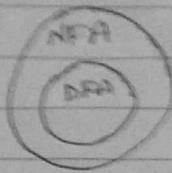


odd a's & odd b's.



10/04/23

## NFA



Every NFA is a DFA  
But not Vice Versa.

Kleene's Theorem.  $R.F = NFA = DFA$   
if  $\forall w \in L_1$ ,  $w$  is accepted by DFA " $M$ ".  
language of  $M$  is  $L$

$$M(L) = L_1$$

NFA or DFA kipasur bababai he.

Pumping Lemma:

Recognize when a given language is not regular  
→ a language is not regular if its DFA can't be made.

if  $\exists w \in L \ni$   
 $w \neq x^n y^m z$

Then language is not regular

## NFA

11/04/23

From Regular to NFA with empty moves.

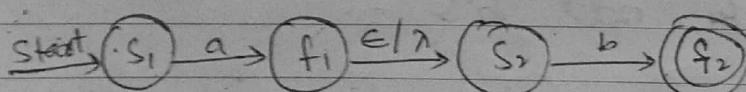
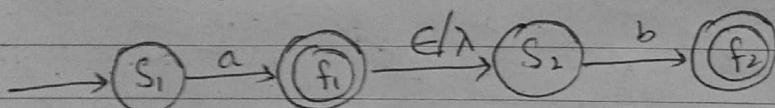
$$\gamma_1 = a$$

$$\gamma_2 = b$$

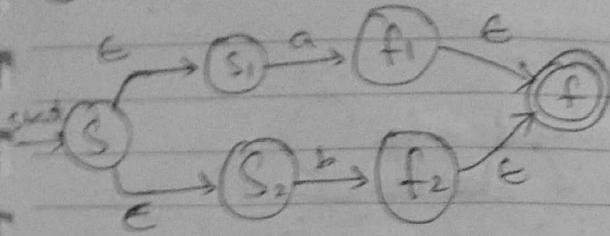


Empty-moves  
 $\in \{\lambda\}$

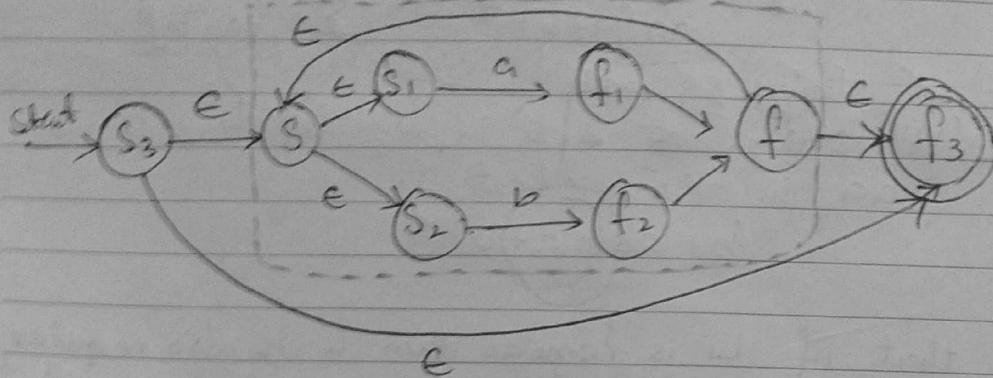
$$\gamma_1 \cdot \gamma_2 = ab.$$



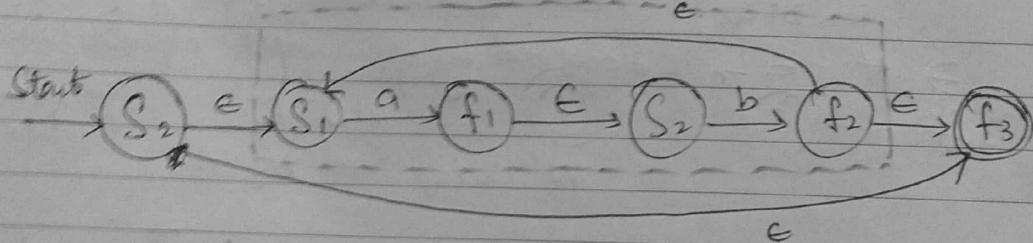
$$\textcircled{2} \quad \gamma_1 + \gamma_2 = a + b.$$



$$\textcircled{3} \quad (\gamma_1 + \gamma_2)^* = (a+b)^*$$



$$\textcircled{4} \quad (\gamma_1 \cdot \gamma_2)^* = (ab)^*$$



Transition function of NFA.  $\hat{\delta}$

$\hat{\delta}(\text{current state}, \text{alphabet}) = \text{set of states.}$   
 $\Rightarrow \hat{\delta}(S, a) = ?$

for example # 3.

$$\hat{\delta}(S_3, a) = \{S_3, f_1, f, f_3, S_1, S_2\}.$$

$$\hat{\delta}(S_3, b) = \{S_3, f_2, f, f_3, S, S_2\}.$$

## Closure Properties Of R.E.

If  $L_1, L_2$  are regular languages.

- i) Then  $L'$  is also regular
- ii) Then  $L_1 \cap L_2$  is also regular
- iii) Then  $L_1 \cup L_2$  is also regular.

$L_1$  = {All strings of  $a, b$  that start with  $aa$ }  
 $\Sigma = \{a, b\}$

$L_2$  = All strings of  $a, b$

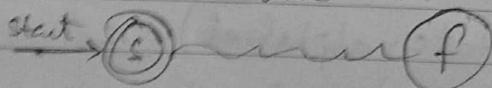
$L'_2$  = All strings of  $a, b$  in which there is no  
 $aa$  in the start.  $\Sigma = \{a, b\}$

$$(b + ab)(a + b)^*$$

Prove that if  $L_1$  is regular then  $L'$  is also regular.  
Solution this employs.

Given that  $L_1$  is regular  $\Rightarrow \exists$  a DFA  $M(S, \Sigma, \delta, Q, F)$   
 → for DFA.  
 $\Rightarrow$  If  $w \in L_1^\omega$  is accepted by  $M$ .  
 such that states  $(S)$  run  $(f)$  ie  $M(L) = L_1$ .  
 $w'$  is rejected by  $M$ .

Now, transform this DFA  $M$  into ' $M'$ ' such that  
 move all  $Q \in P$  into  $P'$  and,  
 all  $P'$  states into  $F$ .



This DFA  $M'$  will accept all  $w' \in L'$   
 Hence, proved that  $L'$  is also regular.

$$L_2 = (a+b)^* aa (a+b)^*$$

$$L_2' = (b+ab)^* + a \cdot$$

$$(b+ab)^* (a+b^*ba+a)$$

## SIMULATION OF DFA

18-04-23.

### ASSIGNMENT NO. 1.

Simulation:  
imitation of  
reality.

① Input number of states: Q

② Input number of alphabets: Σ

States [ ] = new StatesArray (no. of states)

Loop: Input states in states list.

Alphabets [ ] = new alphabet Array (no. of alphabets).

TransitionTable [ ][ ] =  
states × alphabets.

Set transition table.

initialState = "S"

currentState = initialState.

initialState = "S"

currentState = initialState.

## FINALS

$(aa)^* (bb)^* b$

$(aa)^* (a+b)^* (bb)^* b$ .

08-05-23.

## CONTEXT FREE LANGUAGES (CFL):

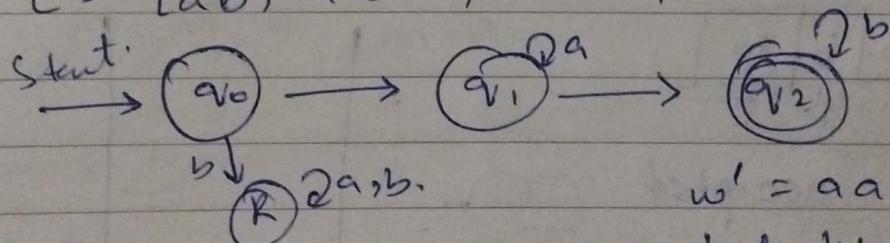
Claim:

There exist non-regular languages.

To prove this claim we will give an example.  
 $L = a^n b^n; n \geq 1$   
language that is not regular i.e. there does not exist any DFA which can accept all  $w \in L$  and reject all  $w' \notin L$ .

$$L = a^n b^n \quad n \geq 1$$

$L = \{ab, aabb, \dots\}$

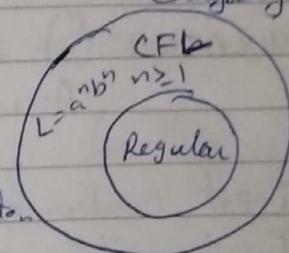


$$w' = aaab \notin L$$

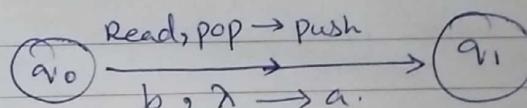
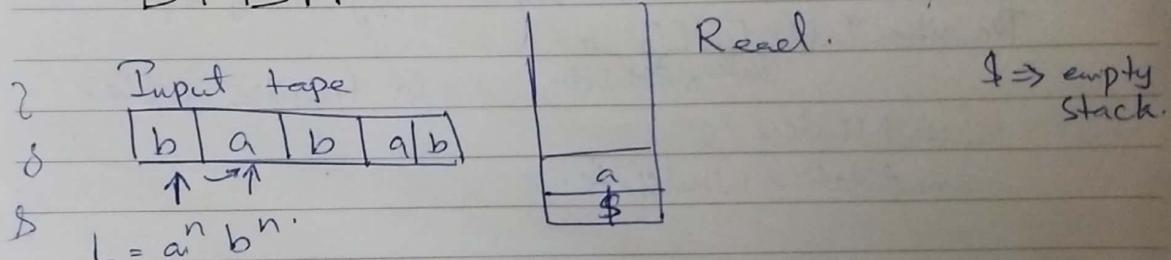
but this DFA accept this  $w'$ .

## Push Down Automata (PDA).

- Those free languages which decides by PDA are content-free languages.
  - All regular languages are content-free but not all content-free are regular.
  - PDA's have memory element.
  - Push Down Automaton is a decider of CFL.
- Deterministic Push Down Automata (DPDA).  
 → Non-deterministic Push Down Automaton (NPDA)

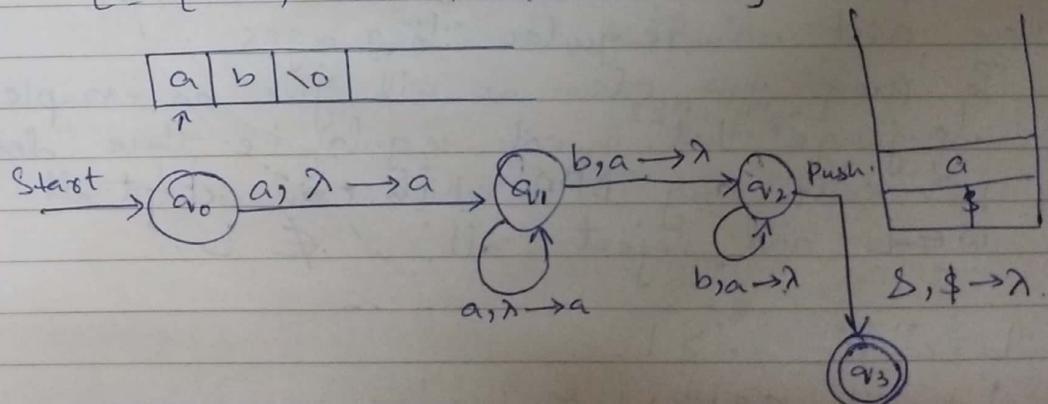


### DPDA



$$L = a^n b^n ; n \geq 1.$$

$$L = \{ab, aabb, aaabbb, \dots\}.$$



|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| a | a | a | b | b | b | b |
| ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

accepted

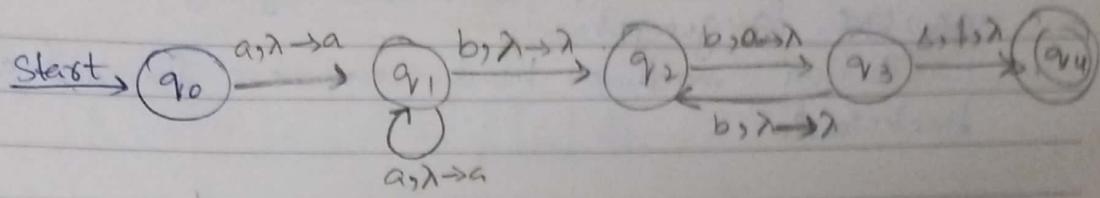
$q_2$  pop  
 $q_2$  pop  
 $q_2$  pop  
 $q_3$  pop

|           |           |           |
|-----------|-----------|-----------|
| $\lambda$ | $\lambda$ | $\lambda$ |
| a         |           |           |
| \$        |           |           |

$\text{push } q_1$   
 $\text{push } q_1$   
 $\text{push } q_{10}$

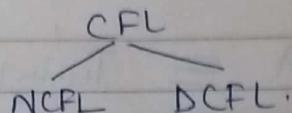
$$L = a^n b^{2n}$$

$$L = \{abb, aa\ bbbb, aaabb\ bbbb, \dots\}$$



NPDA  $\gg$  DPDA.

$$NPDA \neq DPDA$$



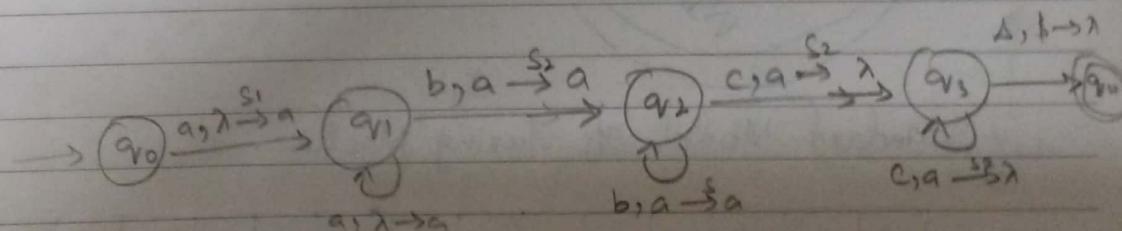
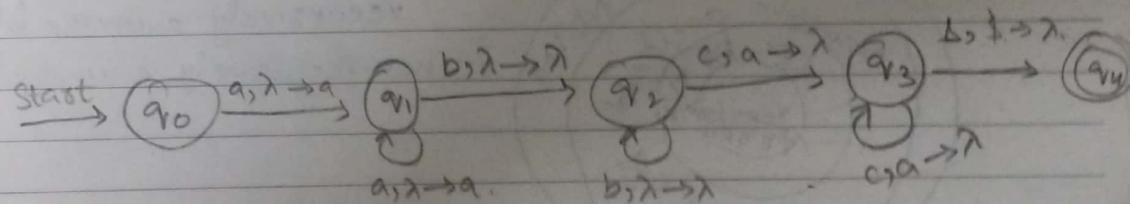
15-03-2023

. CFL are decidable by PDA and not by DFA.

$$L_1 = a^n b^n \quad n \geq 1$$

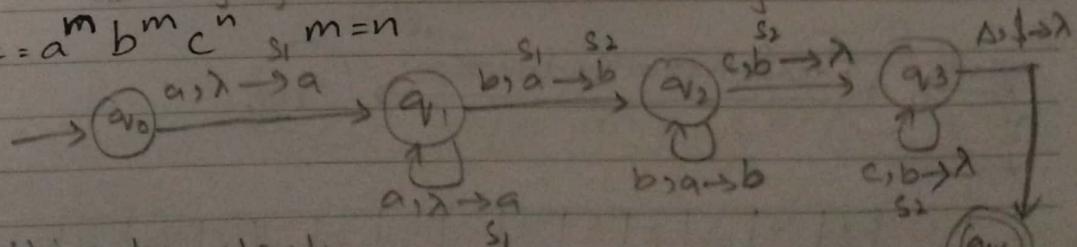
$$L_1 = \{ab, aabb, \dots\}.$$

$L_1 = a^n b^m c^n$      $m \neq n$ . (Non-context free languages)



This language can't be decided by PDA

$$L_2 = a^m b^m c^n \quad | \quad m=n$$



Using two stacks  
PDA can decide

## RECURSIVE LANGUAGES:

- Turing decidable languages are recursive languages.
- There exist non-context free languages which are recursive languages.
- In turing machine, head can move forward or backward.

Content free = Content free',  $CFL = CPL'$   
 Non-context free = Non-context free',  $NCFL = NCFL'$

- 2-Stack PDA  $\equiv$  Turing Machine

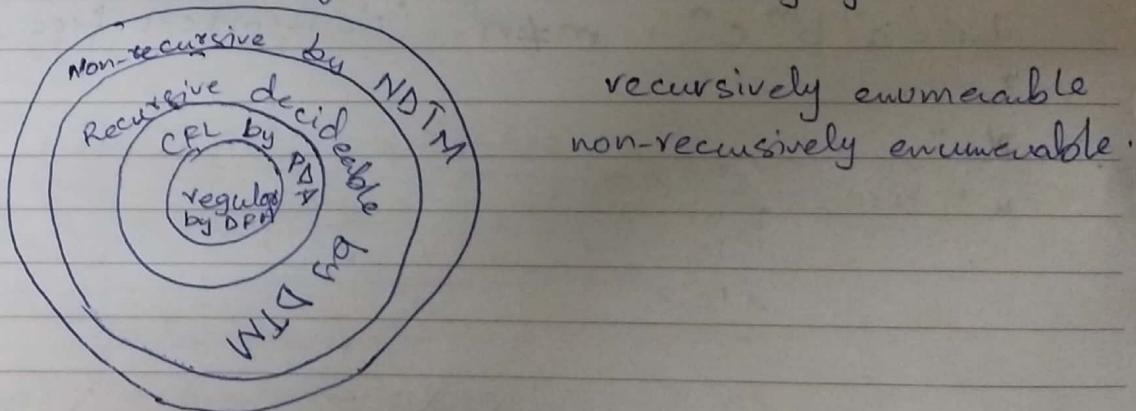
16-05-23.

Instantaneous Description / Computation.

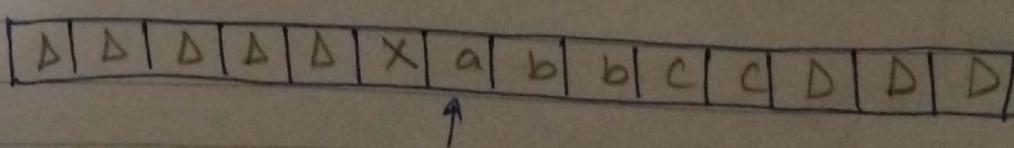
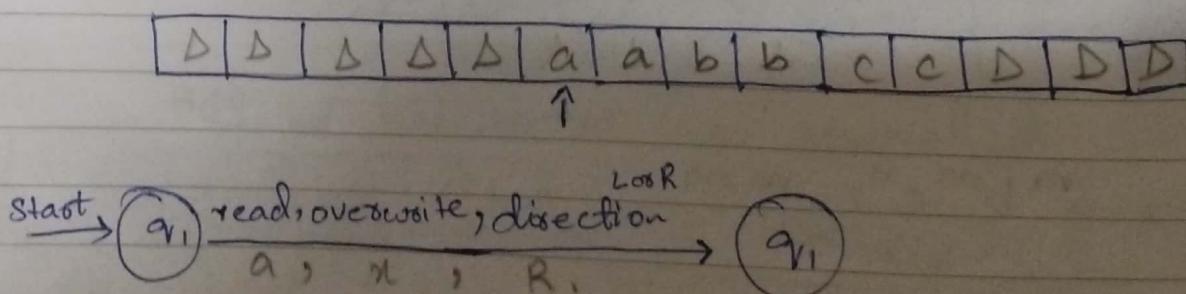
$L = a^n b^n c^n \quad n \geq 1$ . (can't be decided by PDA).

## TURING MACHINE:

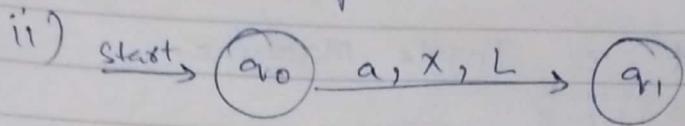
The decider of all recursive languages.



Standard Model of Turing Machine.



$\boxed{D \ D \ a \ a \ b \ b \ c \ c \ D}$



$\boxed{D \ D \ x \ a \ b \ b \ c \ c \ D}$

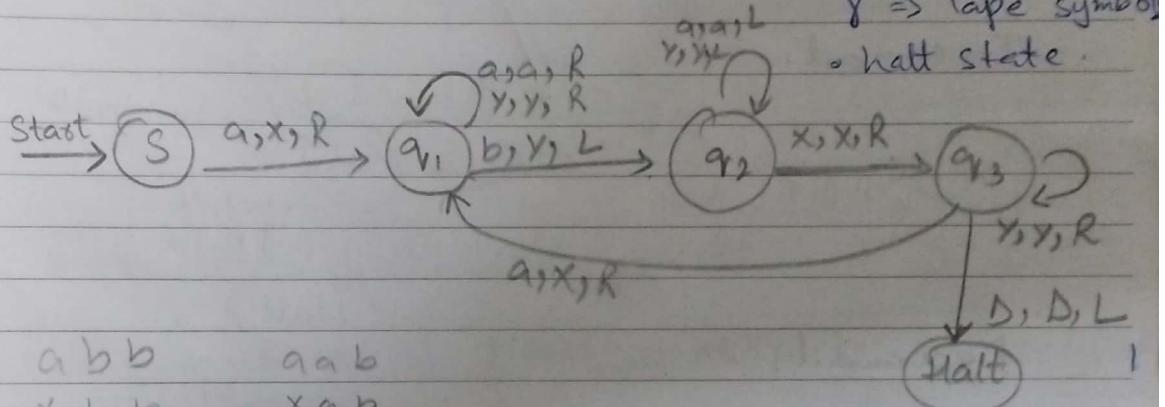
\* Every algo has Turing machine & vice versa.

$$L = a^n b^n$$

$$L = \{ab, aabb, aaabbb, \dots\}$$

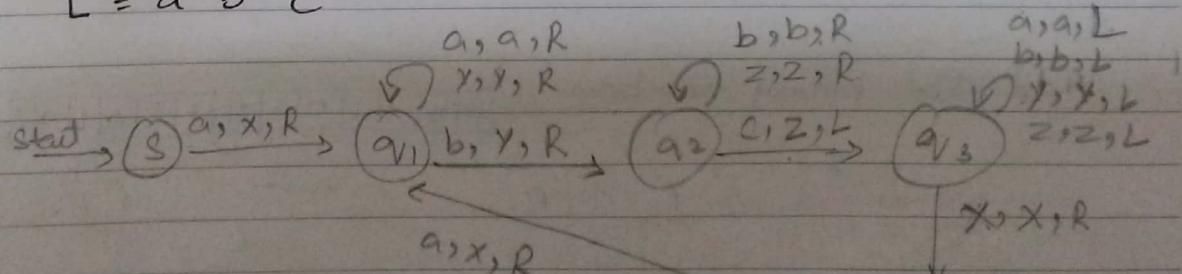
$\Sigma \Rightarrow$  Sigma set  
valid choice of  
language.

$Y \Rightarrow$  Tape symbol  
• halt state.

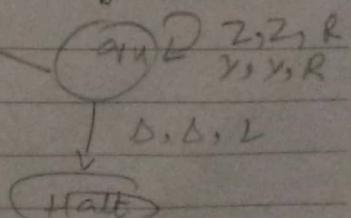


|     |     |
|-----|-----|
| abb | aab |
| xbb | xab |
| xyb | xab |
| xyb | xay |
| .   | xay |
| xay |     |
| xx  | xy  |

$$L = a^n b^n c^n$$

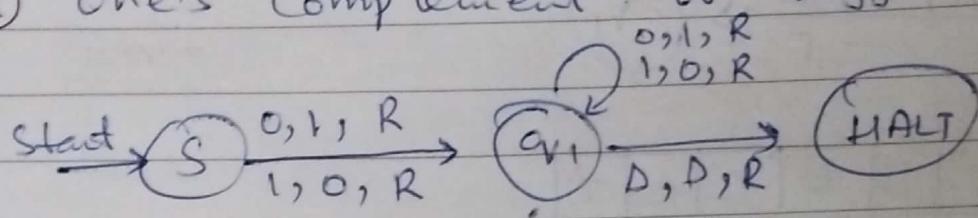


|         |  |
|---------|--|
| aabbcc  |  |
| xaabbcc |  |
| xabbc   |  |
| xybcc   |  |
| xybccc  |  |
| xybzcc  |  |
| xybz    |  |
| xybz    |  |
| xxyyzz  |  |



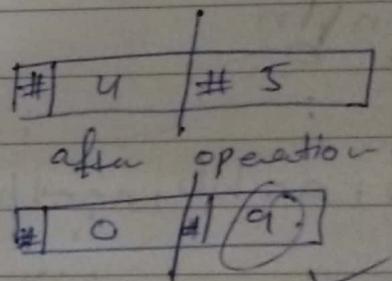
# Turing Machine.

① One's Complement or Toggle Machine.

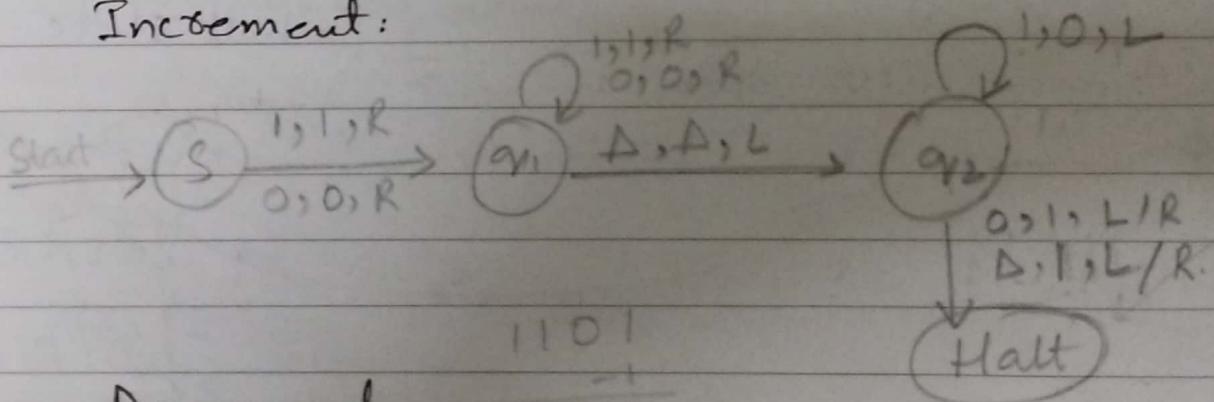


② Add two binary numbers.

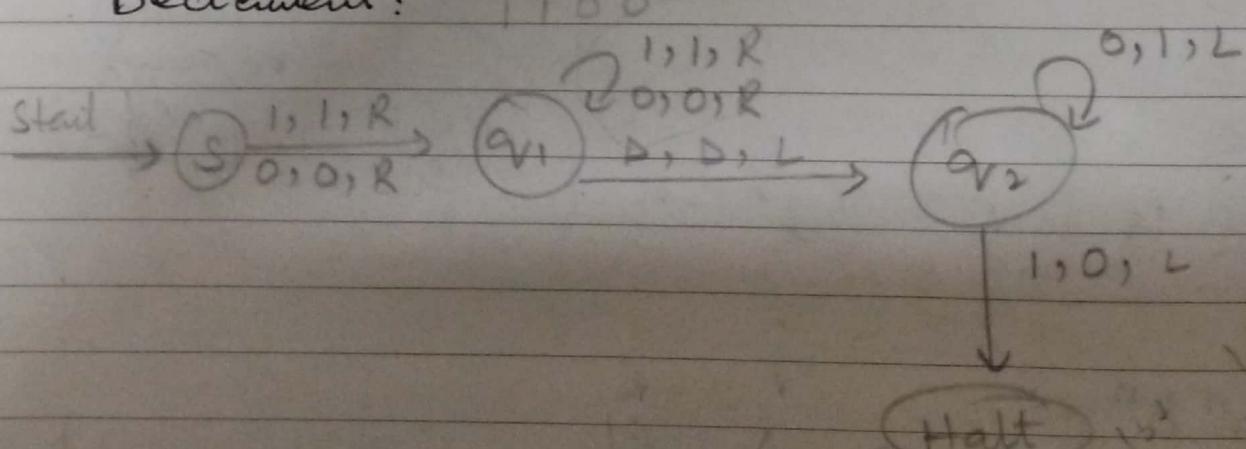
$$\begin{array}{r}
 4 + 5 = 9 \\
 \text{Decrement} \quad \text{Increment} \\
 \begin{array}{r}
 3 \\
 2 \\
 1 \\
 0
 \end{array}
 \quad \begin{array}{r}
 6 \\
 7 \\
 8 \\
 \boxed{9}
 \end{array}
 \end{array}$$



Increment:



Decrement:



Add two binary numbers.

NTM

$$\begin{array}{r}
 \#01101\#11010
 \end{array}$$

0101R

0,0,L  
1,1,L

1,0,L

23-05-23

Insert.

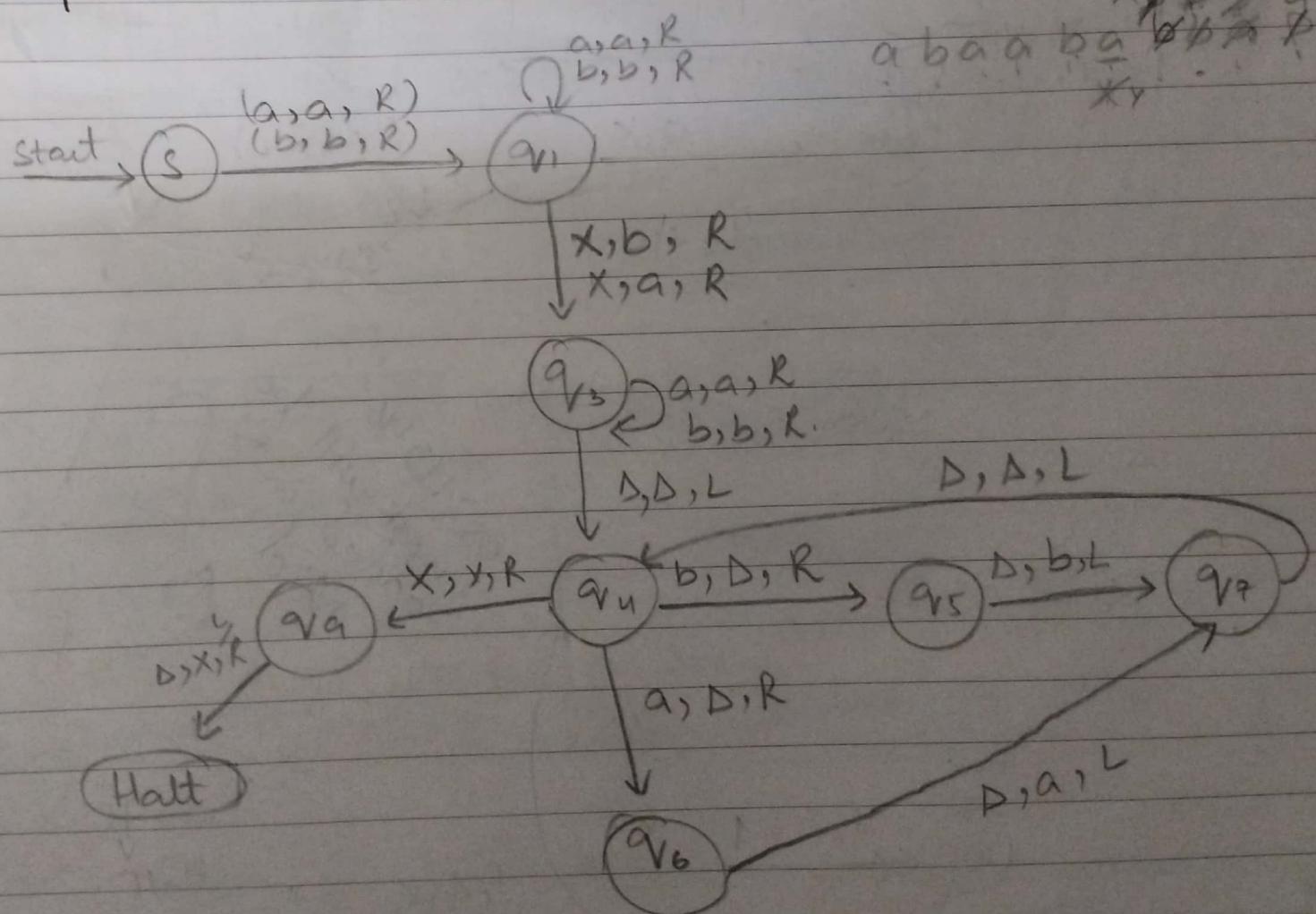
|   |   |   |   |   |   |   |     |
|---|---|---|---|---|---|---|-----|
|   | 0 | 1 | 2 | 3 | 4 | 5 |     |
| D | D | D | a | b | a | b | D D |

|   |   |   |   |   |   |   |     |
|---|---|---|---|---|---|---|-----|
|   | 0 | 1 | 2 | 3 | 4 | 5 | 6   |
| D | D | D | a | b | a | b | D D |

Copy-Paste, Cut-Paste, Delete

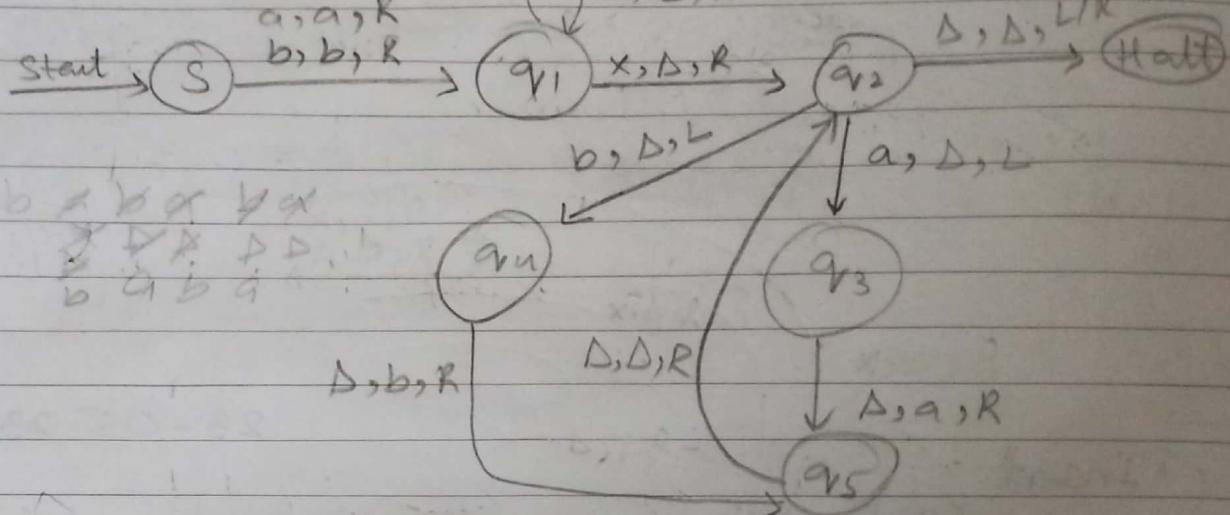
- \* position
- \* shift all characters to the right
- \* Then place the new one at pos. -

Turing machine to Insert a character at specific position.

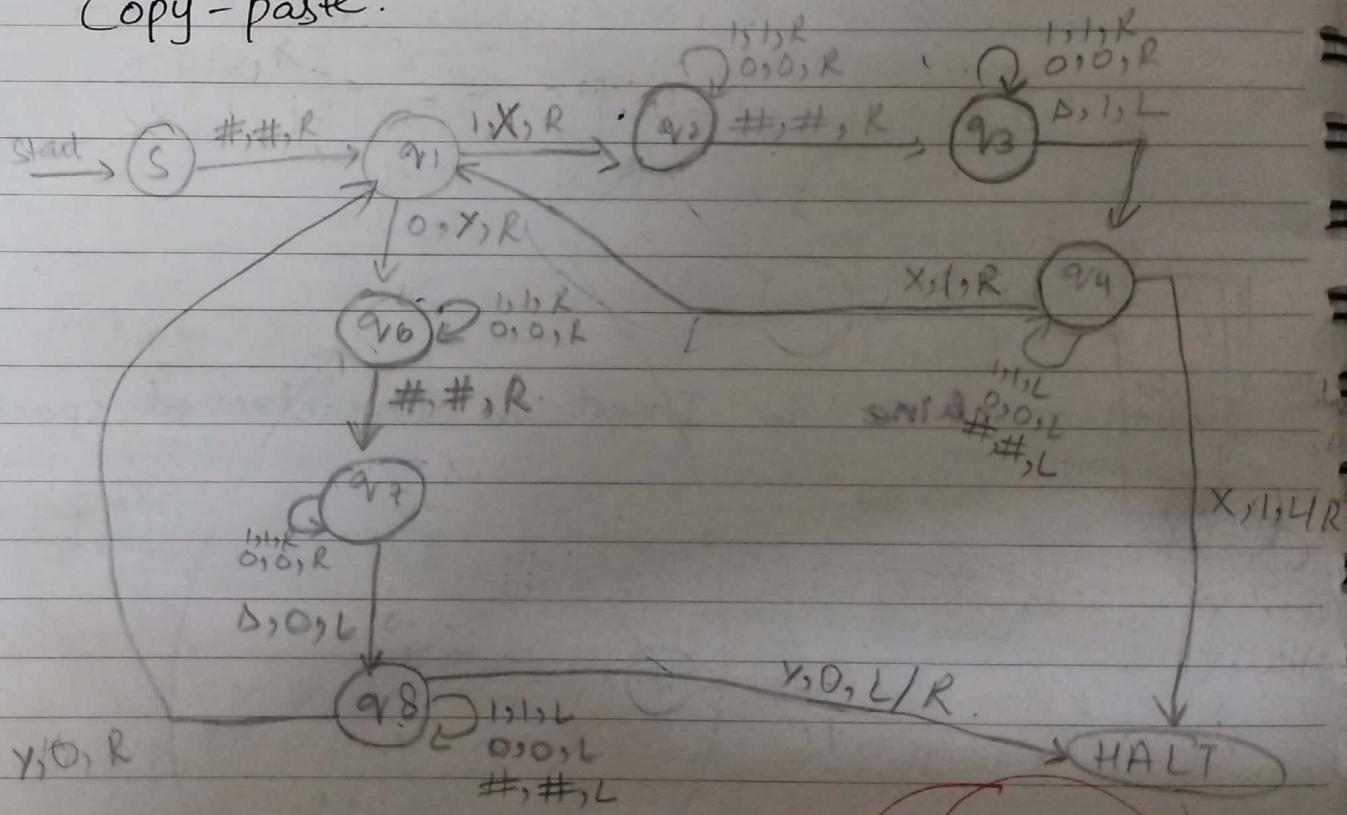


Delete: abab

abbabb  
abbaab



Copy-paste.



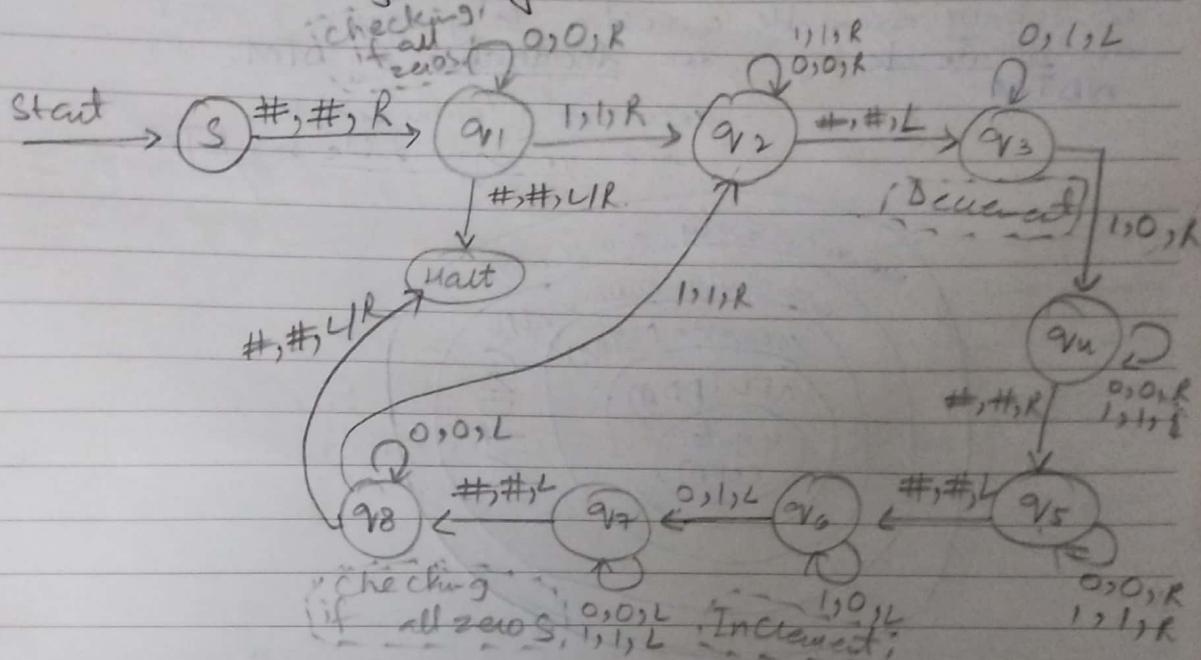
#1010#1010#

#abbabb#DDDD#

CS2124  
marked will be  
added in  
Mic.

30-05-23

Turing Machine to Add two Binary Numbers.  
without checking overflow condition

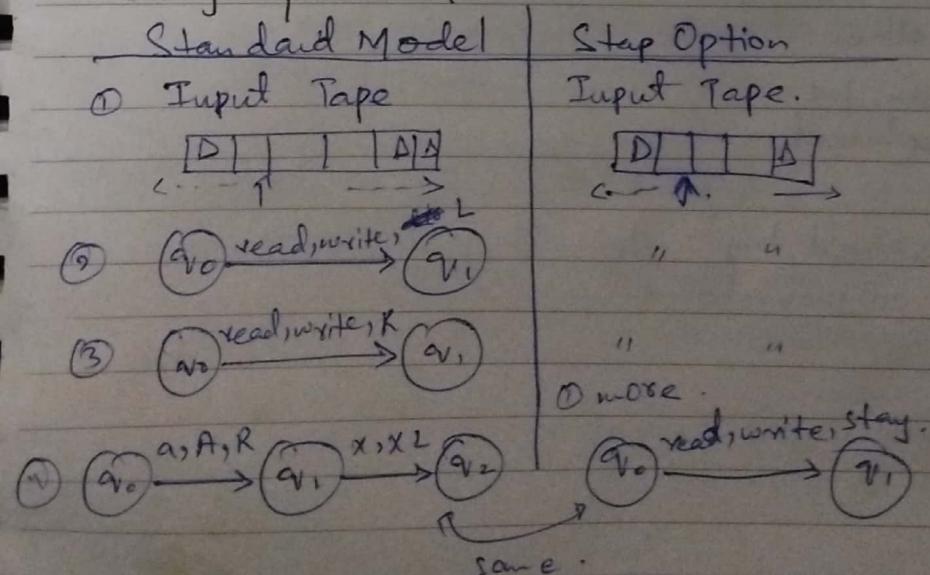


05-06-23

## VARIATIONS OF TURING MACHINE

- |                                 |                                |
|---------------------------------|--------------------------------|
| ① Standard T.M. (Already seen). | Information Retrieval          |
| ② Stay option T.M.              | (flow, search)<br>engine works |
| ③ Two/Multitape Tape T.M.       | ④ Semi-infinite T.M            |
| ⑤ Multi-track T.M.              | ⑥ Off-line T.M.                |
|                                 | ⑦ non-deterministic T.M.       |

### Stay Option Machine



$\text{NDTM} \neq \text{DTM}$  (not same power).

$\downarrow$        $\downarrow$   
recursively enumerable recursive languages  
are decidable by NDTM. are decidable by DTM.

recursively enumerable  
are recognizable by DTM.



Solvable

NP Problem

Solvable by  
NDTM in a  
polynomial time.

NP Non-Deterministic  
Polynomial Time.

Unsolvable

NP Hard

NP-Complete

The problem onto  
which all other  
problems can be  
mapped/reduced

NP-hard

can't be solved by  
NDTM in polynomial time.

How to check the valid sequence of a language?

int i ; → valid  
i int ; → invalid

How to check which one is valid?

→ Write rules to validate it.

→ How to write those rules formally.  
We make grammar

Grammar = Set of rules.

rules = variables / constants.

✓ ↗  
non-terminal terminal.  
(capital letters) (small letters).

• rules are called productions

### Productions

Exactly one non-terminal → terminal / non-terminal combination of both.

e.g.  $\xrightarrow{\text{read as}} \text{can be}$

$$\textcircled{1} \quad X \xrightarrow{\text{ }} aB$$

$$\textcircled{2} \quad B \xrightarrow{\text{ }} a$$

$$\textcircled{3} \quad C \xrightarrow{\text{ }} D$$

$$\textcircled{4} \quad D \xrightarrow{\text{ }} b$$

$S \Rightarrow$  Starting rule.

• In ~~one~~ grammar, there can be more than one starting rule.

Example Grammar  $G$   $\Sigma = \{a, b\}$ .

$$\textcircled{1} \quad S \xrightarrow{\text{ }} aS$$

$$\textcircled{2} \quad S \xrightarrow{\text{ }} bS$$

$$\textcircled{3} \quad S \xrightarrow{\text{ }} \lambda$$

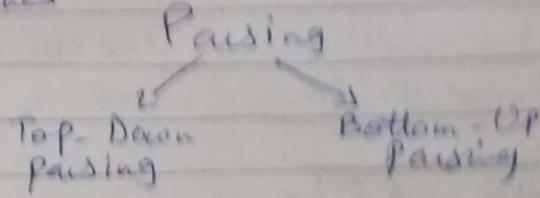
Q) Find whether the string  $w$   $w = abab \in L(G)$ ?

To check this,  $w$  is parsed by Grammar  $G$ , if  $G$  generate  $w$  then we say  $w \in L(G)$ .



Now, how to parse?

2 approaches



### Captains Log

- ④ C → aS
  - ⑤ S → bS
  - ⑥ S → A
  - ⑦ S → aX
  - ⑧ X → bXA
  - ⑨ A → ?

## ① Top-Down Parsing

↳ left most derivation

↳ parse tree

## Left-most derivation

$$\boxed{a} \boxed{b} \boxed{a} \boxed{b} \rightarrow$$

$\downarrow K^{\alpha}$

$$S \Rightarrow aS \quad (\text{rule 1})$$

$\Rightarrow \text{abs}$  (rule 2)

$\Rightarrow abas$  (rule 1)

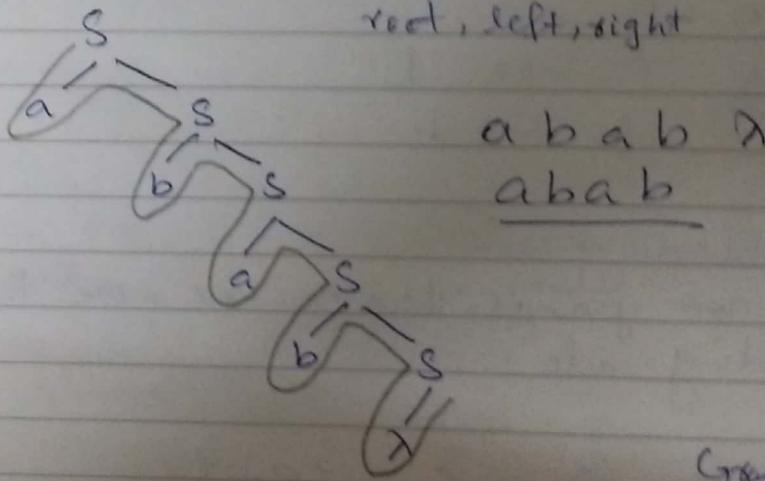
$\Rightarrow ababS$  (rule 1)

$\Rightarrow abab \succ (rule\ 3)$

$$\Rightarrow abab$$

## Parse - Tree

root, left, right



Back-tracking      a b a b.

General

$s \rightarrow a$

$s \rightarrow b$

8

$$S \rightarrow a/b.$$

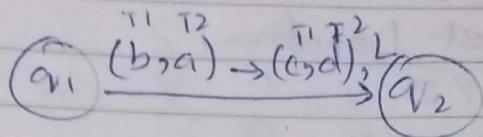
$a \overset{s}{\sim} x$  rule u.

rule 4.

b x

its wrong  
backtracking

## Multi Track



|   |   |   |    |   |   |   |         |
|---|---|---|----|---|---|---|---------|
| D | D | a | b  | a | b | D | tracks  |
| D | D | b | dd | c | d | D | track 2 |

$\overbrace{P \quad P}$

6 - 6 - 23

## Grammar:

### Regular Grammar

$$a^* = \{ \lambda, a, aa, aaa, aaaa, \dots \}$$

Grammar:

1.  $S \rightarrow a$  X not necessary rule.

2.  $S \rightarrow \lambda$  ✓

3.  $S \rightarrow aS$  ✓

$$(ab)^* = \{ \lambda, ab, abab, \dots \}$$

Left most derivation.

for aaa.

$S \Rightarrow aS$

$\Rightarrow aaaS$

$\Rightarrow aaaS$ .

$\Rightarrow aaa\lambda$

$\Rightarrow aaa$ . ✓

Grammar.

①  $S \rightarrow abS$

②  $S \rightarrow \lambda$ .

L.M.D : abab.

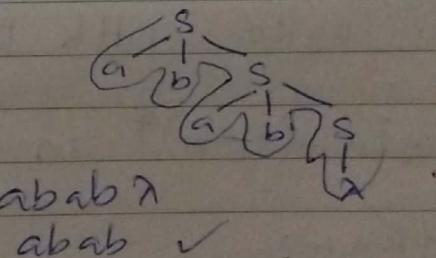
$S \Rightarrow abS$

$\Rightarrow ababS$

$\Rightarrow abab\lambda$

$\Rightarrow abab$ . ✓

Parse tree / derivation tree.



$G_2 \quad \Sigma = \{a, b\}$ .

①  $S \rightarrow XYZ$

②  $X \rightarrow abX$

③  $X \rightarrow \lambda$

④  $Y \rightarrow bY$

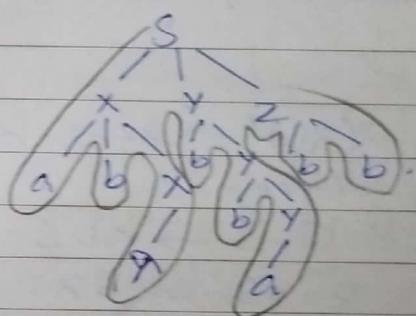
⑤  $Y \rightarrow a$

⑥  $Z \rightarrow bb$

⑦  $X \rightarrow aaY$

Recent-decent Parsing

Parse Tree:-



\* Every regular expression can be converted into grammar but not every grammar can be converted into regular expression.

Foma (a tool)  $\Rightarrow$  input: RegEx output: DFA  
In python: nltk library.

RegEx:  $(a+b)^* aa (a+b)^*$

Grammar

- ①  $S \rightarrow XaaX$
- ②  $X \rightarrow aX$
- ③  $X \rightarrow bX$
- ④  $X \rightarrow \lambda$

RegEx:  $(a+b)^+ aa (a+b)^+$

Grammar

- ①  $S \rightarrow XaaX$
- ②  $X \rightarrow aX$
- ③  $X \rightarrow bX$
- ④  $X \rightarrow a$
- ⑤  $X \rightarrow b$

or  $X \rightarrow aX \mid bX \mid a \mid b$

↓ short form

No. of rules are same.

RegEx:  $a^+ b^*$

Grammar:

- ①  $S \rightarrow aX$     or     $S \rightarrow XY$
  - ②  $X \rightarrow aX$
  - ③  $X \rightarrow bY$
  - ④  $Y \rightarrow bY$
  - ⑤  $Y \rightarrow \lambda$
  - ⑥  $X \rightarrow \lambda$
- $\begin{array}{ll} X \rightarrow aX & \text{if } a^+ \\ X \rightarrow a & \end{array}$
- $\begin{array}{ll} Y \rightarrow \lambda & \text{if } b^* \\ Y \rightarrow bY & \end{array}$

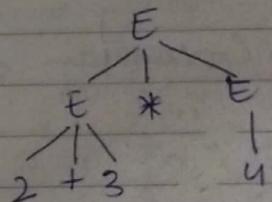
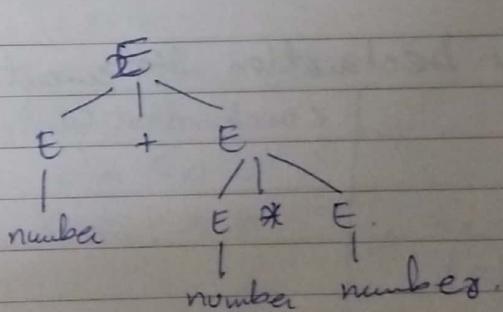
12-06-23

## Ambiguous Grammar

Consider the given grammar.

$$\begin{aligned} E &\rightarrow \text{number} & \text{number} \in \mathbb{R} \\ E &\rightarrow E+E \\ E &\rightarrow E * E \end{aligned}$$

$2 + 3 * 5$ .



• a string of a language, if it can be parsed by more than one ways using grammar then this is ambiguous grammar.

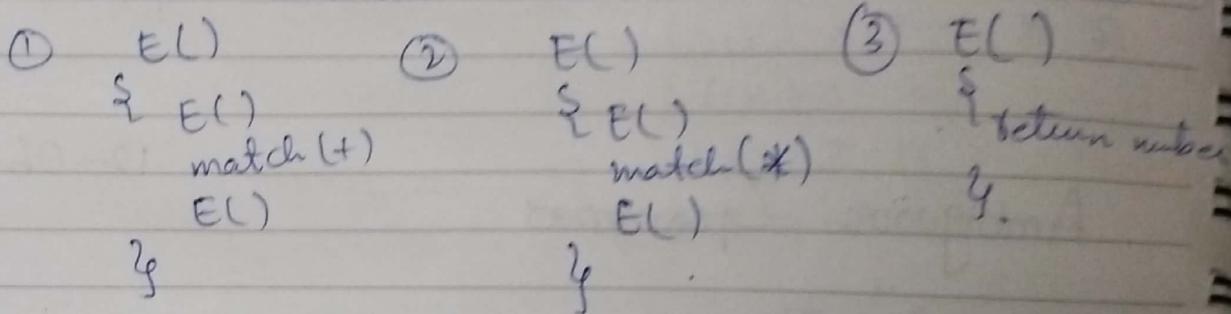
• Ambiguity of every grammar is not always resolvable.

Convert into postfix to end ambiguity.

## LL Parser

- ↳ Top down parsing techniques.
- ↳ Input to left to right scan like hair.
- ↳ Left most derivation.

## Recent - Decent Parser.



Postfix Solution of Grammar of Previous Question.

$$E \rightarrow \text{number}$$

$$E \rightarrow E E +$$

$$E \rightarrow E E * .$$

Grammar for Declaration Statements of P.L.s.

|   |                               |   |
|---|-------------------------------|---|
| ① | $\text{int } i;$              | $\langle \text{Declaration Statement} \rangle.$ |
| ② | $\text{int } i = 0;$          |   |
| ③ | $\text{int } i, j, \dots, k;$ |   |
| ④ | $\text{int } i = 0, j = 1;$   |   |

①  $\langle \text{Declaration Statement} \rangle \rightarrow \langle \text{Datatype} \rangle \langle \text{space} \rangle \langle \text{ID} \rangle ;$

②  $\langle \text{Declaration Statement} \rangle \rightarrow \langle \text{Datatype} \rangle \langle \text{space} \rangle \stackrel{\langle \text{ID} \rangle}{=} \langle \text{value} \rangle ;$

## Palindromes.

$w = \text{reverse}(w)$ .

$\Sigma = \{a, b\}$ . Ex: abbaabba, babab

## Grammar:

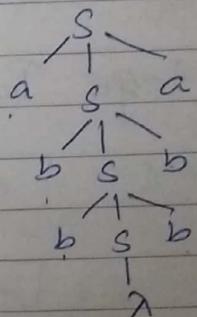
$$\begin{aligned} S &\rightarrow a \ S \ a \\ S &\rightarrow b \ S \ b \\ S &\rightarrow a \\ S &\rightarrow b \\ S &\rightarrow \lambda \end{aligned}$$

① abbbba.

Left-most derivation.

$$\begin{aligned} S &\Rightarrow a \ S \ a. \\ S &\Rightarrow a \ b \ S \ b \ a \\ S &\Rightarrow a \ b \ b \ S \ b \ b \ a. \\ S &\Rightarrow a \ b \ b \lambda \ b \ b \ a. \\ S &\Rightarrow a \ b \ b \ b \ b \ a. \end{aligned}$$

## Parse-Tree.

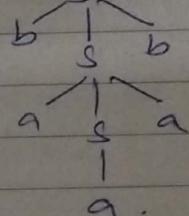


② babab.

Left-most derivation.

$$\begin{aligned} S &\Rightarrow b \ S \ b. \\ S &\Rightarrow b \ a \ S \ a \ b. \\ S &\Rightarrow b \ a \ b \ a \ b. \end{aligned}$$

## Parse-Tree



Equal Equal.

$a^n b^n$

## Grammar.

$$\begin{aligned} S &\rightarrow a \ S \ b. \\ S &\rightarrow \lambda. \quad \rightarrow \text{if } n \geq 0. \\ S &\rightarrow ab. \quad \rightarrow \text{if } n > 0. \end{aligned}$$

aa bb.

(n=2)

$$\begin{aligned} S &\Rightarrow a \ S \ b \\ a \ S \ b &\Rightarrow aa \ S \ bb \\ aa \ S \ bb &\Rightarrow aa bb \end{aligned}$$

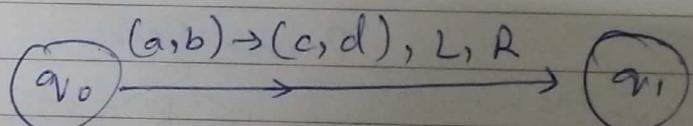
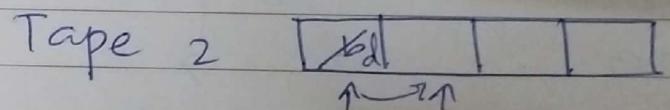
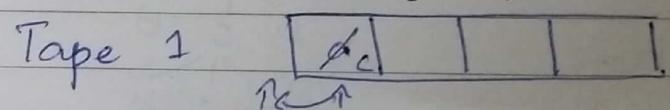
a a b b.      S  $\Rightarrow$  ab

## Equal Equal Grammar.

1.  $S \rightarrow aB$ .
2.  $S \rightarrow bA$
3.  $B \rightarrow aBB$
4.  $B \rightarrow bA$
5.  $A \rightarrow aB$

if else statement ki grammar ambiguous kya  
hai?

Two-tape Turing Machine.



## Pumping Lemma

- Is a given language regular or not?
- Pumping lemma is an algorithm that recognizes that a given language is not regular.
- 3 properties to know (regular or not), for all infinite strings of a language
- Hence, if it is not then it'll return not a regular language.

If a language is regular, then there must exist strings  $\xrightarrow{L} S \in L$ .

$$L = \{S_1, S_2, \dots, S_n\}$$

pumping length 'P' (after a unique length)  
 $S = xyz$  ( $x$  can be written as  $\kappa$ ).

$$P = 3; |S| \geq P. \quad \text{E.g.: } a^* = \{\lambda, a, aa, aaa, aaaa, \dots\}$$

①  $S = x^{(i)} z. \quad (i > 0)$

②  $|y| > 0. \quad (y \Rightarrow \text{loop}). \quad \text{Diagram: } S \xrightarrow{y} \underbrace{y \dots y}_{(\kappa \text{ before } y \text{ and } \kappa \text{ after } y)}$

③  $|xy| \leq P \quad (\kappa \text{ and } y \text{ are concatenated}).$

$L$  is regular if it holds above three conditions.

### Examples

①  $a^* = \{\lambda, a, aa, aaa, \dots\}$ .

$$S = \lambda \quad P = 0$$

$$S = xyz \quad \lambda \cdot \lambda \cdot \lambda$$

②  $S = a. \quad S = xyz. \quad P = 1$

case no. 1

$$x = a.$$

$$y = \lambda$$

$$z = \lambda$$

①  $S = a \cdot \lambda \cdot \lambda$

②  $|xy| = |a \cdot \lambda| = |a|$

③  $|y| \Rightarrow |\lambda|; y > 0 \quad \checkmark \quad 1 \leq P(\text{False})$

case no. 2

$$x = \lambda$$

$$y = a$$

$$z = \lambda$$

case no. 3

$$x = \lambda$$

$$y = \lambda$$

$$z = a$$

(2)  $L = a^n b^n$  ( $n \geq 0$ ) This is not regular.

$$L = a^7 b^7$$

$S = \underbrace{aa}_{x} \underbrace{aaaaa}_{y} \underbrace{aabbb}_{z} bbbb$ ,  $\Rightarrow L$ .

$S \in L \quad x y^i z \quad P = 7$

if  $S = x y^2 z$ .

$S = \underbrace{aa}_{x} \underbrace{aaaaa}_{y} \underbrace{aaaaa}_{y} \underbrace{bbbbbb}_{z}$ .

- ek string se generate hame wale strings bhi language a part hame chahiye.

(3)  $L = a^* b^* \Rightarrow \{\lambda, a, b, ab, aabb, aaabb, aabb\}$

$S = aaabb$

$x = aa$ ,  $y = ab$ ,  $z = b$ . wrong way

$\frac{aa}{x} \frac{ab}{y} \frac{b}{z}$

$aaababb \notin L$  if  $i=2$

we can't take  $y$  as different alphabets like  $aabb$ .  
 $y$  should be loop wala either  $a$  or  $b$ .

so,  $S = \underbrace{aaa}_{x} \underbrace{ab}_{y} \underbrace{b}_{z}$ .

$aaaaabb \in L \quad i=2$

19-06-28

Insertion, Deletion, Addition

Palindrome

Context free grammar.

Equal Equal.

Most appropriate machine of a language.

Identify the grammar of a language.

Deriving a string through Parse tree or left-most derivation

Equal a's in b's.

Grammar

abbbba

$$\textcircled{1} \quad S \rightarrow S A S B S$$

$$\textcircled{2} \quad S \rightarrow S B S A S$$

$$\textcircled{3} \quad S \rightarrow \lambda$$

Halting Problem (Undecidable Problem)

f(m, w)

M → machine

{}

w → string

{ m( )  
return 1/N }

Question: Does M halt on w?

Universal Turing Machine

→ It can take a machine as an input.

- Every T.M can be represented as a binary number but not vice versa.

- Binary numbers set = B (superset)
- Turing machines set = T. (subset.)

$$B \supset T$$

Countably Infinite Set.

Is a set, its ke corresponding ek bijective function likh Sakta.

- Will take an element as input & assign a unique integer to every element

