PRACTICE

DIRECTIONAL DERIVATIVES.

Directional derivatives allows us to compute the rate of change of a function w.r.t distance in any direction

$D_u f(x_0, y_0)$ → Direction derivative of a function at $P_0(x_0, y_0)$.

$f(x,y)$ and $u = u_1 i + u_2 j$ (unit vector), the directional derivate of f in direction of u at (x_0, y_0) is

Formula ①
$$D_u f(x_0, y_0) = \frac{d}{ds}\left[f(x_0 + su_1, y_0 + su_2)\right]_{s=0} \quad \text{2-space}$$

$$D_u f(x_0, y_0, z_0) = \frac{d}{ds}\left[f(x_0 + su_1, y_0 + su_2, z_0 + su_3)\right]_{s=0} \quad \text{3-space}$$

Ex.1. Let $f(x,y) = xy$. Find & interpret $D_u f(1,2)$ for unit vector. $u = \underbrace{(\sqrt{3}/2)}_{u_1} i + \underbrace{(1/2)}_{u_2} j$.

Sol
$$D_u f(1,2) = \frac{d}{ds}\left[f(1 + s\sqrt{3}/2, 2 + s\, 1/2)\right]_{s=0}$$

$$f(x,y) = xy$$

$$f\left(1 + \frac{\sqrt{3}}{2}s, 2 + \frac{s}{2}\right) = \left(1 + \frac{\sqrt{3}}{2}s\right)\left(2 + \frac{1}{2}s\right)$$

$$'' = 2 + \frac{1}{2}s + \sqrt{3}s + \frac{\sqrt{3}}{4}s^2$$

$$'' = 2 + \left(\frac{1}{2} + \sqrt{3}\right)s + \frac{\sqrt{3}}{4}s^2$$

$$D_u f(1,2) = \frac{d}{ds}\left[2 + \left(\frac{1}{2} + \sqrt{3}\right)s + \frac{\sqrt{3}}{4}s^2\right]_{s=0}$$

$$D_u f(1,2) = \left[0 + \frac{1}{2} + \sqrt{3} + \frac{\cancel{2}}{\cancel{4}_2}\sqrt{3}\, s\right]_{s=0}$$

$$D_u f(1,2) = \left[\frac{1}{2} + \sqrt{3} + \frac{\sqrt{3}}{2}s\right]_{s=0}$$

$$D_u f(1,2) = \frac{1}{2} + \sqrt{3} + \frac{\sqrt{3}}{2}(0)$$

$$\boxed{D_u f(1,2) = 1/2 + \sqrt{3}}$$

Since, $1/2 + \sqrt{3} \approx 2.23$, we conclude that if we move a small distance from point $(1,2)$ in direction of u, $f(x,y)$ will increase to 2.23 times the distance move.

Formula ②
$$\left(\frac{df}{ds}\right)_{u,P_0} = \lim_{s \to 0} \frac{f(x_0 + su_1, y_0 + su_2) - f(x_0, y_0)}{s}$$

Ex 2. Find the derivative of $f(x,y) = x^2 + xy$ at $P_0(1,2)$ in direction of $u = 1/\sqrt{2}\, i + 1/\sqrt{2}\, j$

Sol
$$\left(\frac{df}{ds}\right)_{u,P_0} = \lim_{s \to 0} \frac{f(1 + s\, 1/\sqrt{2}, 2 + s\, 1/\sqrt{2}) - f(1,2)}{s}$$

$f(x,y) = x^2 + xy$.
$f(1,2) = (1)^2 + (1)(2)$
$f(1,2) = 1 + 2$
$f(1,2) = 3$

$f(1 + s/\sqrt{2}, 2 + s/\sqrt{2}) = \left(1 + \frac{s}{\sqrt{2}}\right)^2 + \left(1 + s/\sqrt{2}\right)\left(2 + s/\sqrt{2}\right)$
$'' = 1 + \frac{s^2}{(\sqrt{2})^2} + 2(1)(s/\sqrt{2}) + 2 + s/\sqrt{2} + 2s/\sqrt{2} + \frac{s^2}{2}$
$'' = 1 + 2s/\sqrt{2} + s^2/2 + 2 + s/\sqrt{2} + 2s/\sqrt{2} + s^2/2$
$f(,) = 3 + 5s/\sqrt{2} + s^2$

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$$(df/ds)_{u_1 P_0} = \lim_{s\to 0} \frac{(\cancel{3} + 5s/\sqrt{2} + s^2) - \cancel{3}}{s}$$

$$= \lim_{s\to 0} s^2 + 5s/\sqrt{2} \,/\, s$$

$$= \lim_{s\to 0} \cancel{s}(s + 5/\sqrt{2}) / \cancel{s} \quad \text{applying limit}$$

$$\boxed{(df/ds)_{u_1 P_0} = 5/\sqrt{2}}$$

Since, $5/\sqrt{2} \approx 3.53$, we conclude that if we move small distance from $P_0(1,2)$ in direction of u_1, the function $f(x,y)$ will increase 3.53 times the distance move.

Rate of change of $f(x,y) = x^2 + xy$ at $P_0(1,2)$ in direction u is $5/\sqrt{2}$

Unit vector u in xy-plane can be expressed as $u = \cos\phi\, i + \sin\phi\, j$, where ϕ is angle from +ve x-axis to u. Thus, $D_u f(x_0,y_0) = f_x(x_0,y_0)\cos\phi + f_y(x_0,y_0)\sin\phi$.

Formula 3

Ex 3. Find directional derivative of $f(x,y) = e^{x\cdot y}$ at $(-2,0)$ in direction of u making an angle of $\pi/3$ with +ve x-axis.

Sol

$D_u f(-2,0) = f_x(-2,0)\cos\phi + f_y(-2,0)\sin\phi$.

$f_x(-2,0) = e^{x\cdot y}\cdot(y) = e^{(-2)(0)}(0) = 0$

$f_y(-2,0) = e^{x\cdot y}(x) = e^{(-2)(0)}(-2) = -2$.

$\therefore u = \cos\phi\, i + \sin\phi\, j$.

$u = \cos(\pi/3)\, i + \sin(\pi/3)\, j$

$u = 1/2\, i + \sqrt{3}/2\, j$.

$D_u f(-2,0) = 0\cdot\cos(\pi/3) + (-2)\sin(\pi/3)$

$D_u f(-2,0) = 0 - 2(\sqrt{3}/2)$

$D_u f(-2,0) = -\sqrt{3}$

Ex. 4. Find D.D of $f(x,y,z) = x^2 y - yz^3 + z$ at point $(1,-2,0)$ in the direction of vector $a = 2i + j - 2k$.

Solve

$f_x(1,-2,0) = 2xy = 2(1)(-2) = -4$.

$f_y(1,-2,0) = x^2 - z^3 = (1)^2 - (0)^3 = 1$

$f_z(1,-2,0) = -3yz^2 + 1 = -3(-2)(0)^2 + 1 = 1$

finding unit vector a

$$u = \frac{a}{|a|} = \frac{2i + j - 2k}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{2i + j - 2k}{3} = \frac{2}{3}i + \frac{1}{3}j - \frac{2}{3}k$$

$u_1 = \frac{2}{3}, u_2 = \frac{1}{3}, u_3 = -\frac{2}{3}$

Formula (4)

$\because$ $D_uf(x_0,y_0,z_0) = f_x(x_0,y_0,z_0)u_1 + f_y(x_0,y_0,z_0)u_2 + f_z(x_0,y_0,z_0)u_3$

$D_uf(1,-2,0) = -4\left(2/3\right) + (1)(1/3) + (1)(-2/3)$

$D_uf(1,-2,0) = -8/3 + 1/3 - 2/3$

$D_uf(1,-2,0) = -3.$

Gradient:

If $f(x,y)$, then gradient of f is

Formula (5) $\nabla f(x,y) = f_x(x,y)i + f_y(x,y)j.$

$\nabla f(x,y,z) = f_x(x,y,z)i + f_y(x,y,z)j + f_z(x,y,z)k$

Rules (1) $\nabla(f+g) = \nabla f + \nabla g$ (2) $\nabla(f-g) = \nabla f - \nabla g.$

(3) $\nabla(kf) = k\nabla f$ (4) $\nabla(fg) = \cancel{\nabla f \nabla g}\ f\nabla g + g\nabla f.$

(5) $\nabla(f/g) = g\nabla f - f\nabla g / g^2$

Let $f(x,y) = x-y$ and $g(x,y) = 3y$

$\nabla f = i - j$ $\nabla g = 3j.$

(2) $\nabla(f-g) = \nabla f - \nabla g = i - j - 3j = \boxed{i-4j}.$

(4) $\nabla(f\cdot g) = f\nabla g + g\nabla f \Rightarrow (x-y)(3j) + 3y(i-j).$

$\nabla(f\cdot g) = 3xj - 3yj + 3yi - 3yj$

$\nabla(f\cdot g) = 3yi + (3x-6y)j$

$\nabla(f\cdot g) = \nabla((x-y)(3y)) = \nabla(3xy - 3y^2)$

Directional Derivative is a dot product of Gradient f at P_0 and u.

$D_uf(x_0,y_0) = \nabla f(x_0,y_0)\cdot u$

$D_uf(x_0,y_0,z_0) = \nabla f(x_0,y_0,z_0)\cdot u.$

D.D is a dot product, if $f(x,y)$ is differentiable in an open region containing $P_0(x_0,y_0)$, then

Formula (6) $\left(df/ds\right)_{u,P_0} = (\nabla f)_{P_0}\cdot u.$

Ex. Find the derivative of $f(x,y) = xe^y + \cos(xy)$ at $(2,0)$ in direction of $v = 3i - 4j$

Sol $f_x(2,0) = e^y - \sin(xy)\cdot y \Rightarrow e^0 - \sin(2\cdot 0)\cdot(0) = 1$

$f_y(2,0) = xe^y - \sin(x\cdot y)\cdot x = 2(e^0) - \sin(2\cdot 0)\cdot(2) = 2$

$u = \dfrac{v}{|v|} = \dfrac{3i-4j}{\sqrt{3^2+(-4)^2}} = \dfrac{3i-4j}{5} = \dfrac{3}{5}i - \dfrac{4}{5}j$

$u_1 = \dfrac{3}{5}$, $u_2 = -\dfrac{4}{5}$

$\because \nabla f(x,y) = f_x(x,y)\, i + f_y(x,y)\, j$

$\nabla f(2,0) = i + 2j$

$\because \left(\frac{df}{ds}\right)_{u P_0} = \nabla f \cdot u$

$\quad\quad = (i+2j)\left(\frac{3}{5} i - \frac{4}{5} j\right)$ $\quad \because i \cdot i = j \cdot j = k \cdot k = 1$ all others $= 0$

$\quad\quad = \frac{3}{5}(i \cdot i) - \frac{8}{5}(j \cdot j)$

$\quad\quad = \frac{3}{5} - \frac{8}{5}$

$\quad\quad = -5/5$

$\left(\frac{df}{ds}\right)_{u P_0} = -1$ ✓

Properties of D.D $D_u f = \nabla f \cdot u = |\nabla f| \cos\theta$.

① f increase most rapidly when $\cos\theta = 1$ or when $\theta = 0$ & u is the direction of ∇f.

$$D_u f = |\nabla f| \cos(0) = |\nabla f|.$$

② f decreases most rapidly ~~when~~ in direction of $-\nabla f$

$$D_u f = |\nabla f| \cos(\pi) = -|\nabla f| \cdot \{\pi = 180, \cos 180 = -1\}.$$

③ any direction u orthogonal $u_\perp$ to a gradient $\nabla f \neq 0$ is a direction of zero change in f because θ then equals to $\pi/2 = 90°$.

$$D_u f = |\nabla f| \cos(\pi/2) = |\nabla f| \cdot 0 = 0.$$

__Ex. 1__ Let $f(x,y) = x^2 e^y$. find max. value of D.D at $(-2,0)$. & find u in direction where max. value occurs.

Sol

$\nabla f(x,y) = f_x(x,y)\, i + f_y(x,y)\, j$.

$\quad\quad = 2x e^y\, i + x^2 e^y\, j$.

$\quad\quad = 2(-2) e^0\, i + (-2)^2 e^0\, j$

$\nabla f(-2,0) = -4i + 4j$

max value occurs when f increases most rapidly.

hence, $D_u f = \nabla f = |\nabla f|$

$\quad\quad = \left|\sqrt{(-4)^2 + (4)^2}\right| = \sqrt{16+16} = 4\sqrt{2}$

$\|\nabla f\| = 4\sqrt{2}$.

unit vector of $\nabla f = \nabla f / |\nabla f|$.

$$\frac{-4i + 4j}{4\sqrt{2}} = \boxed{-\frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j = u}$$

$\frac{\nabla f}{|\nabla f|}$

Ex: 2. Find the direction in which $f(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$.

(a) increases most rapidly at $(1,1)$.

$$\nabla f(x,y) = f_x(x,y)i + f_y(x,y)j$$
$$\nabla f(x,y) = xi + yj$$
$$\nabla f(1,1) = i + j \checkmark$$

increases most rapidly.

$$D_u f = \nabla f = |\nabla f|.$$

finding direction. $u = \frac{\nabla f}{|\nabla f|} = \frac{i+j}{\sqrt{1^2+1^2}} = \frac{1}{2}i + \frac{1}{2}j$.

(2) decrease most rapidly at $(1,1)$.

$$\nabla f(1,1) = i + j.$$

decreases most rapidly.

$$D_u f = \nabla f = -|\nabla f| = -i - j;$$

finding direction. $u = \frac{-i-j}{\sqrt{(-1)^2+(-1)^2}} = -\frac{1}{2}i - \frac{1}{2}j$.

(c) what are the directions of zero change in f at $(1,1)$?

$$\nabla f(1,1) = i + j.$$

zero change

$$D_u f = \nabla f = 0.$$

direction of zero change at $(1,1)$ are the directions orthogonal to ∇f

$n = -1/\sqrt{2}\, i + 1/\sqrt{2}\, j$ & $-n = 1/\sqrt{2}\, i - 1/\sqrt{2}\, j$

Q Find the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P_0(1,1,0)$ in direction of $2i - 3j + 6k$.

$$f_x(1,1,0) = 3x^2 - y^2 = 3(1)^2 - (1)^2 = 3 - 1 = 2.$$
$$f_y(1,1,0) = 0 - 2xy - 0 = -2(1)(1) = -2.$$
$$f_z(1,1,0) = 0 - 0 - 1 = -1$$

direction of v

$$u = \frac{v}{|v|} = \frac{2i - 3j + 6k}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{2i - 3j + 6k}{7} = \frac{2}{7}i - \frac{3}{7}j + \frac{6}{7}k$$

$$\left(\frac{df}{ds}\right)_{u,P_0} = \nabla f \cdot u \qquad \nabla f(1,1,0) = 2i - 2j - k.$$
$$= (2i - 2j - k) \cdot \left(\frac{2}{7}i - \frac{3}{7}j + \frac{6}{7}k\right)$$
$$= \frac{4}{7} + \frac{6}{7} - \frac{6}{7}$$

$\because i \cdot i = j \cdot j = k \cdot k = 1$

$$\left(\frac{df}{ds}\right)_{u(1,1,0)} = \frac{4}{7}$$

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(b) In what direction f change most rapidly at P_0, & what are the rates of change in these directions?

f increases most rapidly in direction of $|\nabla f|$ & decreases in $-|\nabla f|$

$|\nabla f| = \sqrt{2^2+(-2)^2+(-1)^2} = \sqrt{4+4+1} = 3$

increases $= |\nabla f|$ $\quad$ decrease $-|\nabla f|$

3 $\quad$ -3

Q. Find the D.D of $f(x,y) = xye^{2xy} + \sin xy$. at $(-3,2)$ in direction of $a = -3i + 4j$

(1) ⇒ $f(x,y) = x\,ye^{2xy} + \sin xy$.

$f_x(x,y) = y\left[x \cdot e^{2xy}(2y) + e^{2xy}(1)\right] + \cos(xy)(y)\}$.

$f_x(-3,2) = 2\left[(-3)\,e^{2(-3)(2)}(2\cdot2) + e^{2(-3)(2)}\right] + \cos(-3\cdot2)(2)\}$.

$f_x(-3,2) = 2\left(-12\,e^{-12} + e^{-12}\right) + 2\cos(-6)$.

$f_x(-3,2) = 2(-11e^{-12}) + 2(0.9945)$

$f_x(-3,2) = -22e^{-12} + 2(0.9945)$

$f_x(-3,2) = 1.988$.

(2) ⇒ $f_y(x,y) = xye^{2xy} + \sin xy$.

$f_y(x,y) = x\left[y \cdot e^{2xy}(2x) + e^{2xy}\cdot(1)\right] + \cos(xy)(x)$.

$f_y(-3,2) = -3\left[2\cdot e^{2(-3)(2)}(2(-3)) + e^{2(-3)(2)}\right] + \cos(-3\cdot2)(-3)$.

$f_y(-3,2) = -3(-12e^{-12} + e^{-12}) - 3\cos(-6)$.

$f_y(-3,2) = -3(11e^{-12}) - 3\cos(-6)$.

$f_y(-3,2) = -33e^{-12} - 3\cos(-6)$

$f_y(-3,2) = -2.983$.

$\nabla f(-3,2) = 1.988i - 2.983j$.

direction of a.

$u = \frac{a}{|a|} = \frac{-3i+4j}{\sqrt{(-3)^2+(4)^2}} = -\frac{3}{5}i + \frac{4}{5}j$

$\left(\frac{df}{ds}\right)_{uP_0} = (1.988i - 2.983j)\left(-\frac{3}{5}i + \frac{4}{5}j\right)$.

$= -1.1928 - 2.3864$.

$\left(\frac{df}{ds}\right)_{uP_0} = -3.5792$

1-8 Find $D_u f$ at P. $D_u f = f_x(x,y)u_1 + f_y(x,y)u_2$

① $f(x,y) = (1+xy)^{3/2}$; $P(3,1)$;
$u = 1/\sqrt{2}\, i + 1/\sqrt{2}\, j$

$f_x(x,y) = \frac{3}{2}(1+xy)^{1/2}(0+y)$
$f_x(3,1) = \frac{3}{2}(1+(3)(1))^{1/2}(1)$
$f_x(3,1) = 3$
$f_y(x,y) = \frac{3}{2}(1+xy)^{1/2}(0+x)$
$f_y(3,1) = \frac{3}{2}(1+(3)(1))^{1/2}(3)$
$f_y(3,1) = 9$
$u_1 = 1/\sqrt{2}$, $u_2 = 1/\sqrt{2}$
$D_u f(x,y) = 3 \times 1/\sqrt{2} + 9 \times 1/\sqrt{2}$
$D_u f(3,1) = 3+9/\sqrt{2}$
$D_u f(3,1) = 12/\sqrt{2} \times \sqrt{2}/\sqrt{2}$
$D_u f(3,1) = 12\sqrt{2}/2$
$D_u f(3,1) = 6\sqrt{2}$ A

② $f(x,y) = \sin(5x-3y)$; $P(3,5)$
$u = 3/5\, i - 4/5\, j$

$f_x(x,y) = \cos(5x-3y).(5-0)$
$f_x(3,5) = \cos(5\times3-3\times5).5$
$f_x(3,5) = \cos(0).5$
$f_x(3,5) = 5$
$f_y(x,y) = \cos(5x-3y).(0-3)$
$f_y(3,5) = \cos(5\times3-3\times5).(-3)$
$f_y(3,5) = -3\cos(0)$
$f_y(3,5) = -3$
$u_1 = 3/5$ $u_2 = -4/5$
$D_u f(3,5) = 5\times\frac{3}{5} + (-3)(-4/5)$
$D_u f(3,5) = 3 + 12/5$
$D_u f(3,5) = 27/5$ A

③ $f(x,y) = \ln(1+x^2+y)$; $P(0,0)$;
$u = -1/\sqrt{10}\, i - 3/\sqrt{10}\, j$

$f_x(x,y) = (1/1+x^2+y)(0+2x+0)$
$f_x(0,0) = (1/1+0^2+0)(2(0))$
$f_x(0,0) = 0$
$f_y(x,y) = (1/1+x^2+y)(0+0+1)$
$f_y(0,0) = (1/1+0^2+0)(1)$
$f_y(0,0) = 1$
$u_1 = -1/\sqrt{10}$, $u_2 = -3/\sqrt{10}$
$D_u f(0,0) = 0(-\frac{1}{\sqrt{10}}) + 1(-3/\sqrt{10})$
$D_u f(0,0) = -3/\sqrt{10}$
A

④ $f(x,y) = \frac{cx+dy}{x-y}$; $P(3,4)$;
$u = \frac{4}{5}i + \frac{3}{5}j$

$f_x(x,y) = \frac{(x-y)(c+0) - (cx+dy)(1-0)}{(x-y)^2}$
$f_x(x,y) = \frac{c(x-y) - (cx+dy)}{(x-y)^2}$
$f_x(3,4) = \frac{c(3-4) - (c3+4d)}{(3-4)^2}$
$f_x(3,4) = -c - 3c - 4d/1$
$f_x(3,4) = -4c - 4d$.
$f_y(x,y) = \frac{(x-y)(d) - (cx+dy)(0-1)}{(x-y)^2}$
$f_y(x,y) = \frac{d(x-y) + (cx+dy)}{(x-y)^2}$
$f_y(3,4) = \frac{d(3-4) + (3c+4d)}{(3-4)^2}$
$f_y(3,4) = -d + 3c + 4d/1$
$f_y(3,4) = +3c + 3d$.
$u_1 = 4/5$, $u_2 = 3/5$
$D_u f(3,4) = (-4c-4d)\frac{4}{5} + (+3c+3d)(\frac{3}{5})$
$= \frac{-16c-16d}{5} + (\frac{+9c+9d}{5})$
$= -16c-16d+9c+9d/5$
$= -7c-7d/5$.
$D_u f(3,4) = -\frac{7}{5}(c+d)$
A

5. $f(x,y,z) = 4x^5y^2z^3$; $P(2,-1,1)$;

$u = \frac{1}{3}i + \frac{2}{3}j - \frac{2}{3}k$

Sol $f_x(x,y,z) = 20x^4y^2z^3$

$f_x(2,-1,1) = 20(2)^4(-1)^2(1)^3$

$f_x(2,-1,1) = 320.$

$f_y(x,y,z) = 8x^5yz^3$

$f_y(2,-1,1) = 8(2)^5(-1)(1)^3$

$f_y(2,-1,1) = -256.$

$f_z(x,y,z) = 12x^5y^2z^2$

$f_z(2,-1,1) = 384.$

$u_1 = 1/3,\ u_2 = 2/3,\ u_3 = -2/3.$

$D_uf(x,y,z) = 320(1/3) + (-256)(2/3) + (384)(-2/3)$

$D_uf(2,-1,1) = -320$ Ans.

6. $f(x,y,z) = ye^{xz} + z^2$; $P(0,2,3)$

$u = 2/7\, i - 3/7\, j + 6/7\, k$

Sol $f_x(x,y,z) = ye^{xz}.z = 2\times3\times e^{0\times3} = 6$

$f_y(x,y,z) = e^{xz} = e^{0\times3} = 1$

$f_z(x,y,z) = ye^{xz}.x + 2z = 2\times0\times e^{0\times3} + 2(3) = 6$

$u_1 = 2/7,\ u_2 = -3/7,\ u_3 = 6/7.$

$D_uf(0,2,3) = 6\times\frac{2}{7} + 1\times(\frac{-3}{7}) + 6\times(6/7).$

$D_uf(0,2,3) = 45/7$ Ans

7. $f(x,y,z) = \ln(x^2+2y^2+3z^2)$; $P(-1,2,4)$;

$u = -\frac{3}{13}i - \frac{4}{13}j - \frac{12}{13}k$

Sol $f_x(x,y,z) = \left(\frac{1}{x^2+2y^2+3z^2}\right)(2x).$

$f_x(-1,2,4) = \left(\frac{1}{(-1)^2+2(2)^2+3(4)^2}\right)(2(-1))$

$f_x(-1,2,4) = -2/57.$

$f_y(x,y,z) = \left(\frac{1}{x^2+2y^2+3z^2}\right)(4y).$

$f_y(-1,2,4) = \left(\frac{1}{(-1)^2+2(2)^2+3(4)^2}\right)(4(2))$

$f_y(-1,2,4) = 8/57$

$f_z(x,y,z) = \left(\frac{1}{x^2+2y^2+3z^2}\right)(6z).$

$f_z(-1,2,4) = \left(\frac{1}{(-1)^2+2(2)^2+3(4)^2}\right)(6(4)).$

$f_z(-1,2,4) = 8/19$

$u_1 = -3/13,\ u_2 = -4/13,\ u_3 = -12/13$

$D_uf(-1,2,4) = (-2/57)(-3/13) + (8/57)(-4/13) + (8/19)(-12/13)$

$D_uf(-1,2,4) = -\frac{314}{741}$ Ans

8. $f(x,y,z) = \sin xyz$; $P(\frac{1}{2}, \frac{1}{3}, \pi)$

; $u = \frac{1}{\sqrt{3}}i - \frac{1}{\sqrt{3}}j + \frac{1}{\sqrt{3}}k$

Sol $f_x(x,y,z) = \cos xyz \cdot yz$

$f_x(\frac{1}{2},\frac{1}{3},\pi) = \cos(\frac{1}{2}\times\frac{1}{3}\times180)(\frac{\pi}{3})$

$f_x(\frac{1}{2},\frac{1}{3},\pi) = \frac{\sqrt{3}}{2}\times\frac{\pi}{3}$

$f_x(\frac{1}{2},\frac{1}{3},\pi) = \pi\sqrt{3}/6.$

$f_y(x,y,z) = \cos xyz \cdot xz$

$f_y(\frac{1}{2},\frac{1}{3},\pi) = \cos(\frac{1}{2}\times\frac{1}{3}\times180)(\frac{\pi}{2})$

$f_y(\frac{1}{2},\frac{1}{3},\pi) = \frac{\sqrt{3}}{2}\times\frac{\pi}{2}$

$f_y(1/2, 1/3, \pi) = \pi\sqrt{3}/4.$

$f_z(x,y,z) = \cos xyz \cdot xy.$

$f_z(\frac{1}{2},\frac{1}{3},\pi) = \cos(\frac{1}{2}\times\frac{1}{3}\times180)(\frac{1}{2}\times\frac{1}{3})$

$f_z(\frac{1}{2},\frac{1}{3},\pi) = \frac{\sqrt{3}}{2}\times\frac{1}{6}$

$f_z(\frac{1}{2},\frac{1}{3},\pi) = \frac{\sqrt{3}}{12}$

$u_1 = \frac{1}{\sqrt{3}},\ u_2 = -\frac{1}{\sqrt{3}},\ u_3 = \frac{1}{\sqrt{3}}$

$D_uf(\frac{1}{2},\frac{1}{3},\pi) = (\pi\sqrt{3}/6)(1/\sqrt{3}) + (\pi\sqrt{3}/4)(-1/\sqrt{3}) + (\sqrt{3}/12)(1/\sqrt{3})$

$D_uf(\frac{1}{2},\frac{1}{3},\pi) = \frac{\pi}{6} - \frac{\pi}{4} + \frac{1}{12}$

$D_uf(\frac{1}{2},\frac{1}{3},\pi) = \frac{2\pi - 3\pi + 1}{12}$

$D_uf(\frac{1}{2},\frac{1}{3},\pi) = -\pi + 1/12$

$D_uf(\frac{1}{2},\frac{1}{3},\pi) = \boxed{\frac{1-\pi}{12}}$