Quiz 2

Date 18 - 1 20 24

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Class: BSCS - 5B.
Course: Differential Equations.

Solve the following equations.

① $y' + y = y^{-1/2}$.

$\because \frac{dy}{dx} + P(x)y = Q(x)y^n$; $n \neq 0$ or 1
is an non-linear D.E

Divide the equation by $y^{-1/2}$.

$$y' \cdot \frac{1}{y^{-1/2}} + y \cdot \frac{1}{y^{-1/2}} = \frac{\cancel{y^{-1/2}}}{\cancel{y^{-1/2}}}$$

$$\frac{1}{y^{-1/2}} y' + \frac{1}{y^{-1/2-1}} = 1$$

$$\frac{1}{y^{-1/2}} y' + \frac{1}{y^{-3/2}} = 1 \quad -①$$

Let $\boxed{u = \frac{1}{y^{-3/2}}}$ $-Ⓐ$

$$\frac{du}{dy} = \frac{3}{2} y^{1/2} y'$$

$$u' = \frac{3}{2} \frac{1}{y^{-1/2}} y'$$

$$\boxed{\frac{2}{3} u' = \frac{1}{y^{-1/2}} y'} \quad -Ⓑ$$

Substitute Ⓐ and Ⓑ in ①

$$\frac{2}{3} u' + u = 1$$

$$u' + \frac{3u}{2} = \frac{3}{2}.$$

equation is in linear form $\because \frac{dy}{dx} + P(x)y = Q(x)$

Finding integrating factor

$\because \mu(x) = e^{\int P(x)\,dx}$

$\Rightarrow \mu(x) = e^{\int 3/2\,dx} \Rightarrow \boxed{e^{3/2\,x}}$

multiply $\mu(x)$ with previous eqn.

$$e^{3/2\,x}\,u' + \left(\frac{3}{2}u\right)\left(e^{3/2\,x}\right) = \frac{3}{2}e^{\frac{3}{2}x}$$

$u \cdot \frac{3}{2} e^{3/2\,x} + e^{3/2\,x}\,u'$ ✓

$$\frac{d}{dx}\left(u \cdot e^{\frac{3}{2}x}\right) = \frac{3}{2}e^{3/2\,x}$$

$$d\left(u \cdot e^{3/2\,x}\right) = \frac{3}{2}e^{\frac{3}{2}x}\,dx.$$

integrating both sides.

$$\int d\left(u \cdot e^{\frac{3}{2}x}\right) = \frac{3}{2}\int e^{3/2\,x}\,dx.$$

$$u \cdot e^{\frac{3}{2}x} = \frac{\cancel{3}}{\cancel{2}}\left(\frac{\cancel{2}}{\cancel{3}}e^{\frac{3}{2}x}\right) + c.$$

$$u\,e^{\frac{3}{2}x} = e^{\frac{3}{2}x} + c.$$

$$u = \frac{\cancel{e^{3/2\,x}}}{\cancel{e^{3/2\,x}}} + \frac{c}{e^{3/2\,x}}$$

$$u = 1 + ce^{-\frac{3}{2}x}.$$

$$\because u = \frac{1}{y^{-3/2}}$$

$$\frac{1}{y^{-3/2}} = 1 + ce^{-3/2\,x}.$$

$$\boxed{y^{3/2} = 1 + ce^{-3/2\,x}}$$

Ans

(2) $y'' + 3y' + 2y = 2^x.$

First find homogenous solution.

$y'' + 3y' + 2y = 0$

$\because$ Auxillary equation $\Rightarrow r^2 + 3r + 2 = 0$

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$$\frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm \sqrt{1}}{2} = \frac{-3 \pm 1}{2}$$

$$r_1 = \frac{-3+1}{2} = \frac{-\cancel{2}}{\cancel{2}}, \quad r_2 = \frac{-3-1}{2} = \frac{-\cancel{4}\,2}{\cancel{2}}$$

$r_1 = -1$, $r_2 = -2$.

roots are real & distinct.

$\because y = \cancel{A}\, c_1 e^{r_1 x} + \cancel{B}\, c_2 e^{r_2 x}$.

$y_h = c_1 e^{-x} + c_2 e^{-2x}$.

Finding particular solution.

$y'' + 3y' + 2y = 2^x$.

case I : exponential, replace $D \to a$

$a = 1$

$\Rightarrow D^2 y + 3Dy + 2y = 2^x$.

$\Rightarrow (D^2 + 3D + 2)\, y = 2^x$.

$\Rightarrow y_p = \frac{1}{D^2 + 3D + 2} (2^x)$.

$\Rightarrow y_p = \frac{1}{(1)^2 + 3(1) + 2} (2^x)$

$\Rightarrow y_p = \frac{1}{6} (2^x)$

$\Rightarrow y_p = \frac{2^x}{6}$

$y = y_h + y_p$.

$$\boxed{y = c_1 e^{-x} + c_2 e^{-2x} + \frac{2^x}{6}}$$

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(3) $yy'' + (y')^2 = yy'$

Reduce equation to first order.

Let $y' = v$

$y'' = v' \Rightarrow v' = \frac{dv}{dx} \Rightarrow \frac{dv}{dy} \cdot \frac{dy}{dx}$

$\frac{dv}{dy} \cdot y' \quad \because y' = v$

$y v' + v^2 = y v$

$y'' \to \frac{dv}{dy} \times v$

Substitute $v = y'$ and

$y'' = \frac{dv}{dy} v$

$y \cdot \frac{dv}{dy} v + v^2 = y v$

$\cancel{v} \left(y \frac{dv}{dy} + v \right) = y \cancel{v}$

$y \frac{dv}{dy} + v = y$

$\div$ by y

$\frac{\cancel{y}}{\cancel{y}} \frac{dv}{dy} + \frac{v}{y} = \frac{\cancel{y}}{\cancel{y}}$

$\frac{dv}{dy} + \frac{v}{y} = 1 \quad —(1)$ linear form

find $\mu(y) = e^{\int P_{(y)} dy} = e^{\int \frac{1}{y} dy} = e^{\ln y} = \boxed{y}$

multiply $\mu(y) = y$ by eq (1)

$y \frac{dv}{dy} + \frac{v}{\cancel{y}} \cancel{y} = y$

$y \frac{dv}{dy} + v = y$

$\frac{d}{dy}(y v) = y$

$\Rightarrow \int d(yv) = \int y \, dy$

$\Rightarrow y v = \frac{y^2}{2} + c$

$= \boxed{v = \frac{y}{2} + c y^{-1}}$

next step ↑

$v = \frac{y}{2} + c y^{-1}$

$\because v = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y}{2} + c y^{-1}$

$\int \frac{1}{\frac{y}{2} + c y^{-1}} dy = \int dx$

$= \cancel{x + c}$

$\boxed{\int \frac{1}{\frac{y}{2} + c y^{-1}} dy = x + c}$