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CLASS: BSCS-4C

COURSE: DESIGN & ANALYSIS OF
ALGORITHMS

COURSE INSTRUCTOR:

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ASSIGNMENT # 02

QUESTION No. 1

Solve the following recurrence relations

1. $T(n) = T(n-1) + n.$

Recursive algorithm that loops through the input to eliminate one item.

$$\Rightarrow T(n) = T(n-1) + n. \quad \Rightarrow T(n-1) = T(n-1-1) + (n-1)$$

$$\Rightarrow T(n) = T(n-2) + (n-1) + n \quad T(n-1) = T(n-2) + (n-1)$$

$$\Rightarrow T(n) = T(n-3) + (n-2) + (n-1) + n \Rightarrow T(n-2) = T(n-2-1) + (n-2)$$

$$\Rightarrow \text{if it runs } k \text{ times.} \quad T(n-2) = T(n-3) + (n-2)$$

$$\Rightarrow T(n) = T(n-k) + (n-2) + (n-1) + n$$

$$\Rightarrow T(n) = (n-k) + (n-(k-1)) + (n-(k-2)) + \dots + (n-1) + n.$$

$$\text{Since } T(0) = 1.$$

$$\text{So, } n-k = 0$$

$$\text{or } n=k.$$

$$\Rightarrow T(n) = T(n-n) + (\cancel{n}-\cancel{n}+1) + (\cancel{n}-\cancel{n}+2) + \dots + (n-1) + n.$$

$$\Rightarrow T(n) = T(0) + 1 + 2 + \dots + (n-1) + n.$$

$$\Rightarrow T(n) = 1 + 1 + 2 + \dots + (n-1) + n.$$

$$\therefore 1 + 2 + \dots + (n-1) + n = \frac{n(n-1)}{2}$$

i.e. Sum of n natural numbers

$$\Rightarrow T(n) = 1 + \frac{n(n+1)}{2}$$

$$T(n) = 1 + \frac{n^2 + n}{2}$$

$$\Rightarrow \underline{O(n^2)}$$

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3. $T(n) = T(n/2) + n.$

Recursive algorithm that halves the input but must examine the every item in the input

$$\Rightarrow T(n) = T(n/2) + n. \quad \Rightarrow T(n/2) = T(n/4) + (n/2)$$

$$\Rightarrow T(n) = T(n/4) + n/2 + n. \quad T(n/2) = T(n/4) + n/2.$$

$$\Rightarrow T(n) = T(n/8) + n/4 + n/2 + n \Rightarrow T(n/4) = T(n/8) + (n/4)$$

and it runs k times

$$\therefore 2^3 = 8 \Rightarrow 2^k.$$

$$\Rightarrow T(n) = T(n/2^k) + n/2^{k-1} + n/2^{k-2} + \dots + n/2 + n$$

$$\text{Since } T(1) = 1$$

$$n/2^k = 1$$

$$\text{or } n = 2^k.$$

$$\Rightarrow T(n) = T(n/2^k) + n/2^{k-1} + n/2^{k-2} + \dots + n/2 + n$$

$$\Rightarrow T(n) = T(1) + n/2^{k-1} + n/2^{k-2} + \dots + n/2 + n.$$

$$\Rightarrow T(n) = 1 + n \left(\frac{1}{2^{k-1}} + \frac{1}{2^{k-2}} + \dots + \frac{1}{2} + 1 \right)$$

$$\text{Here } \Rightarrow \frac{1}{2^{k-1}} + \dots + \frac{1}{2} \approx 1$$

$$\Rightarrow T(n) = 1 + n(1+1).$$

$$\Rightarrow T(n) = 1 + 2n.$$

$$O(n)$$

u. $T(n) = 2T(n/2) + 1$

Recursive algorithm that splits the input into 2 halves and does a constant amount of other work.

$$\Rightarrow T(n) = 2T(n/2) + 1 \quad \Rightarrow T(n/2) = 2T(n/4) + 1$$

$$\Rightarrow T(n) = 2[2T(n/4) + 1] + 1 \quad T(n/2) = 2T(n/4) + 1$$

$$\Rightarrow T(n) = 4T(n/4) + 2 + 1 \quad \Rightarrow T(n/4) = 2(T(n/8) + 1) + 1$$

$$\Rightarrow T(n) = 4[2T(n/8) + 1] + 2 + 1 \quad T(n/4) = 2T(n/8) + 1$$

$$\Rightarrow T(n) = 8T(n/8) + 4 + 2 + 1$$

and it runs k times.

$$= 2^3 = 8 \Rightarrow 2^k$$

$$\Rightarrow T(n) = 2^k T(n/2^k) + 7$$

here, if $n = 8$ then $n-1 = 7$.

$$\Rightarrow T(n) = 2^k T(n/2^k) + n-1$$

$$\therefore T(1) = 1$$

$$\text{so } n/2^k = 1$$

$$\text{or } n = 2^k$$

$$\Rightarrow T(n) = n \cdot T(n/2^k) + n-1$$

$$\Rightarrow T(n) = n \cdot 1 + n-1$$

$$\Rightarrow T(n) = n + n-1$$

$$\Rightarrow T(n) = 2n-1$$

$$\Rightarrow O(n)$$

QUESTION No. 2

Given the merge sort Algorithm, dry-run. the complete algorithm and show each step for the given arrays.

a) 5, 2, 4, 7, 1, 3, 2, 6

1 2 3 4 5 6 7 8
A = [5 | 2 | 4 | 7 | 1 | 3 | 2 | 6]

MERGE-SORT(A, 1, 8)

1 2 3 4 5 6 7 8
[5 | 2 | 4 | 7 | 1 | 3 | 2 | 6]

MERGE-SORT(A, 1, 4)

[5 | 2 | 4 | 7]

MERGE-SORT(A, 5, 8)

[1 | 3 | 2 | 6]

MERGE-SORT(A, 1, 2)

[5 | 2]

MERGE-SORT(A, 3, 4)

[4 | 7]

MERGE-SORT(A, 5, 6)

[1 | 3]

MERGE-SORT(A, 7, 8)

[2 | 6]

MERGE-SORT(A, 1, 1)

[5]

MERGE-SORT(A, 2, 2)

[2]

MERGE-SORT(A, 3, 3)

[4]

MERGE-SORT(A, 4, 4)

[7]

MERGE-SORT(A, 5, 5)

[1]

MERGE-SORT(A, 6, 6)

[3]

MERGE-SORT(A, 7, 7)

[2]

MERGE-SORT(A, 8, 8)

[6]

MERGE(A, 1, 1, 2)

[5] [2]
i=1 j=1

[2] []
k=1

[5] [2]
i=1 j=1

[2] [5]
k=2

1 2
[2 | 5]

MERGE(A, 3, 3, 4)

[4] [7] [4] []
i=1 j=1 k=3

[4] [7] [4] [7]
i=1 j=1 k=4

3 4
[4 | 7]

MERGE(A, 5, 5, 6)

[1] [3] [1] []
i=1 j=1 k=5

[1] [3] [1] [3]
i=1 j=1 k=6

5 6
[1 | 3]

MERGE(A, 7, 7, 8)

[2] [6] [2] []
i=1 j=1 k=7

[2] [6] [2] [6]
i=1 j=1 k=8

7 8
[2 | 6]

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1	2	3	4	5	6	7	8
2	5	4	7	1	3	2	6

MERGE(A, 1, 2, 4)

2	5	4	7	2			
i=1	j=1	k=1					
2	5	4	7	2	4		
i=2	j=1	k=2					
2	5	4	7	2	4	5	
i=2	j=2	k=3					
2	5	4	7	2	4	5	7
	j=2	k=4					

MERGE(A, 5, 6, 8)

1	3	2	6	1			
i=1	j=1	k=5					
1	3	2	6	1	2		
i=2	j=1	k=6					
1	3	2	6	1	2	3	
i=2	j=2	k=7					
1	3	2	6	1	2	3	6
	j=2	k=8					
1	2	3	4	5	6	7	8
2	4	5	7	1	2	3	6

MERGE(A, 1, 4, 8)

2	4	5	7	1	2	3	6	1						
i=1	j=1	k=1												
2	4	5	7	1	2	3	6	1	2					
i=1	j=2	k=2												
2	4	5	7	1	2	3	6	1	2	2				
i=1	j=2	k=3												
2	4	5	7	1	2	3	6	1	2	2	3			
i=1	j=3	k=6												
2	4	5	7	1	2	3	6	1	2	2	3	4		
i=1	j=4	k=5												

Date _____ 20__

2	4	5	7
---	---	---	---

$i=3$

1	2	3	6
---	---	---	---

$j=4$

1	2	2	3	4	5		
---	---	---	---	---	---	--	--

$k=5$

2	4	5	7
---	---	---	---

$i=4$

1	2	3	6
---	---	---	---

$j=4$

1	2	2	3	4	5	6
---	---	---	---	---	---	---

$k=7$

2	4	5	7
---	---	---	---

$i=4$

1	2	3	6
---	---	---	---

1	2	2	3	4	5	6	7
---	---	---	---	---	---	---	---

$k=8$

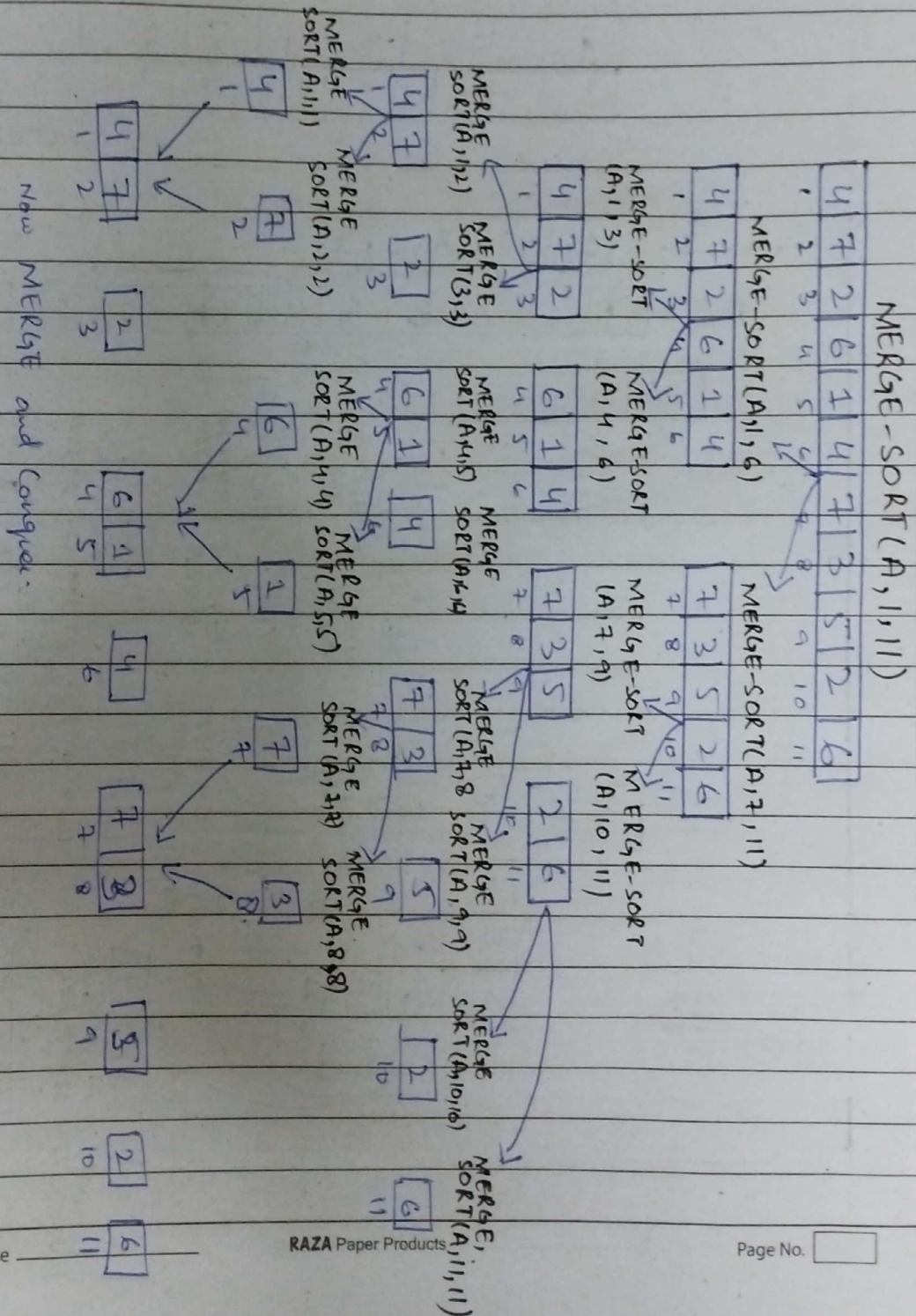
1	2	2	3	4	5	6	7
---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8

Sorted!

b) 4, 7, 2, 6, 1, 4, 7, 3, 5, 2, 6

	1	2	3	4	5	6	7	8	9	10	11
A =	4	7	2	6	1	4	7	3	5	2	6

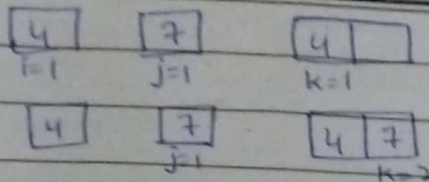


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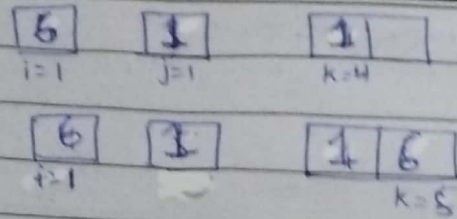
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MERGE(A, 1, 1, 2)

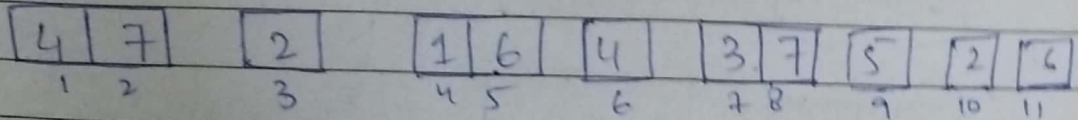
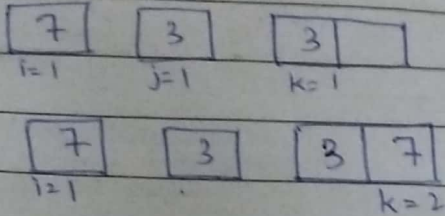


MERGE(A, 4, 4, 5) Date _____

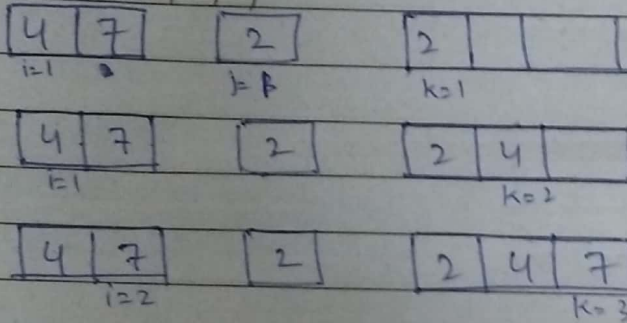
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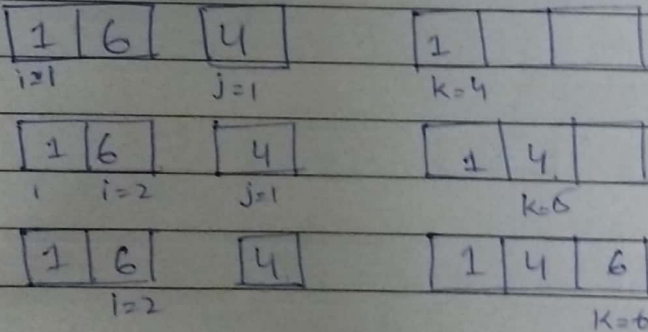
MERGE(A, 7, 7, 8)



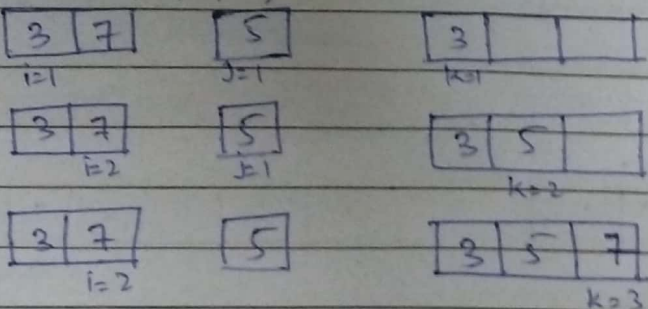
MERGE(A, 1, 2, 3)



MERGE(A, 4, 5, 6)



MERGE(A, 7, 8, 9)



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MERGE(A, 10, 10, 11)

2	6	2	
i=1	j=1	k=1	

2	6	2	6
	j=1	k=2	

2	4	7	1	4	6	3	5	7	2	6
1	2	3	4	5	6	7	8	9	10	11

MERGE(A, 1, 3, 6)

2	4	7	1	4	6	1				
i=1	j=1	k=1								

2	4	7	1	4	6	1	2			
i=1	j=2	k=2								

2	4	7	1	4	6	1	2	4		
i=2	j=1	k=3								

2	4	7	1	4	6	1	2	4	4	
i=3	j=2	k=4								

2	4	7	1	4	6	1	2	4	4	6
i=3	j=3	k=5								

2	4	7	1	4	6	1	2	4	4	6	7
i=3		k=6									

MERGE(A, 7, 9, 11)

3	5	7	2	6	2				
i=1	j=1	k=1							

3	5	7	2	6	2	3			
i=1	j=2	k=2							

3	5	7	2	6	2	3	5		
i=2	j=2	k=3							

3	5	7	2	6	2	3	5	6	
i=3	j=2	k=4							

3	5	7	2	6	2	3	5	6	7
i=3		k=5							

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1	2	4	4	6	7
1	2	3	4	5	6

2	3	5	6	7
7	8	9	10	11

MERGE(A, 1, 6, 11)

1	2	4	4	6	7
i=1					

2	3	5	6	7
j=1				

1										
k=1										

1	2	4	4	6	7
i=2					

2	3	5	6	7
j=1				

1	2									
k=2										

1	2	4	4	6	7
i=3					

2	3	5	6	7
j=1				

1	2	2								
k=3										

1	2	4	4	6	7
i=3					

2	3	5	6	7
j=2				

1	2	2	3							
k=4										

1	2	4	4	6	7
i=3					

2	3	5	6	7
j=2				

1	2	2	3	4						
k=5										

1	2	4	4	6	7
i=4					

2	3	5	6	7
j=2				

1	2	2	3	4	4					
k=6										

1	2	4	4	6	7
i=5					

2	3	5	6	7
j=2				

1	2	2	3	4	4	5				
k=7										

A[7,8]

MERGE
SORT
(A, 8)

6

✓₂

6

(A, 3, 7, 8)

6

j=1

2

k=7

2	6
---	---

18 k=8

6

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1	2	4	4	6	7
---	---	---	---	---	---

$i=5$

2	3	5	6	7
---	---	---	---	---

$j=3$

1	2	2	3	4	4	5	6			
---	---	---	---	---	---	---	---	--	--	--

$k=8$

1	2	4	4	6	7
---	---	---	---	---	---

$i=5$

2	3	5	6	7
---	---	---	---	---

$j=4$

1	2	2	3	4	4	5	6	6		
---	---	---	---	---	---	---	---	---	--	--

$k=9$

1	2	4	4	6	7
---	---	---	---	---	---

$i=5$

2	3	5	6	7
---	---	---	---	---

$j=5$

1	2	2	3	4	4	5	6	6	7	
---	---	---	---	---	---	---	---	---	---	--

$k=10$

1	2	4	4	6	7
---	---	---	---	---	---

2	3	5	6	7
---	---	---	---	---

$j=5$

1	2	2	3	4	4	5	6	6	7	7
---	---	---	---	---	---	---	---	---	---	---

$k=11$

1	2	2	3	4	4	5	6	6	7	7
---	---	---	---	---	---	---	---	---	---	---

1 2 3 4 5 6 7 8 9 10 11

sorted