19/10/22

Multivariate Calculus

$f(x) = y$

$f(x)$ → input → domain

y → output → range

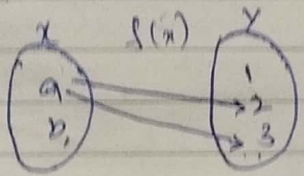
every function is a relation but not every relation is a function.

constant function. $f(x) = x - x + 10$ $x = 0, 1, 2, 3, \ldots$

① In function, one domain can't have multiple ranges, but ~~on~~ multiple domains can have one range.

$R = \{(a, 2), (a, 3)\}$. $x = \{a, b\}$

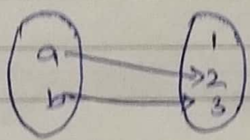
$y = \{1, 2, 3\}$.



not a function.

Maths → Num → Sets → Groups → Rings → Fields

② no elements in domain should be empty.



function.

Differentiation :-

Derivative → change

$\frac{dy}{dx}$ change in y with respect to x.

slope $= \frac{y_2 - y_1}{x_2 - x_1}$.

$f(x) = x^2 + 2x$

$f(x, y) = x^2 + y^2 + 3$ (partial diff).

$\frac{\partial x}{\partial y}$ ∂ = dava

① point (change)

② max / min

③ value (max point / min point)

Integration. → Sum Σ

Area under the curve $\int$

PAPERWORK

M.C Lecture # 1

FUNCTIONS OF SEVERAL VARIABLES

n-tuples
no. of x-variables

$f(x) = 2x.$ single variable

$W = f(-1) = -2.$

two different variables 2-axis

$z = f(x, y) = x^2 + 2x + y$

$f(0, -2) = (0)^2 + 2(0) + (-2)$

$f(0, -2) = -2.$

$z = -2$

3-axis.

$f(x, y, z) = x^2 + y^2 + z^2.$

$x = 0, 1, 2$

$y = 1, 2, 3$ n-tuples

$z = -1, -2, -3$

Contour
a bounded region which can be represented in lines

Ex. 2

$\text{cost} = f(d, m) = 40d + (15/100)\, m$

days = 5 miles 300

$\text{cost} = f(5, 300) = 40(5) + \left(\frac{15}{100}\right) 300$

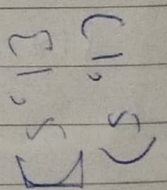
$\quad " \quad = 200 + 45$

$\text{cost} = 245\ \$$

Ex. 4

$\text{ATM} = P(r, t) = (950 + 2t) * e^{(-r/7)}$

$A.P = P(2, 12) = 731.938\ \text{mb}$



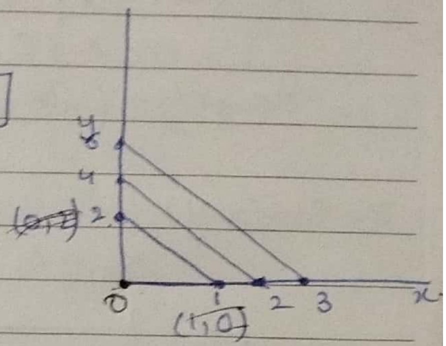
MULTIVARIATE CALCULUS

26/10/22

contour represents a bounded region.

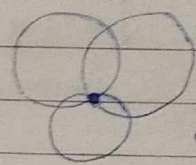
$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-0}{0-1} = \frac{2}{-1} = \boxed{-2}$$



Ex: 06

(a) loan = \$3000
interest = 7%
monthly payment = \$60



satellite get coordinates

Distance formula.

$$\text{Distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Ex: 3

for sphere distance from origin is 1

$$\sqrt{x^2+y^2+z^2} = 1 \xrightarrow{\text{Sq b/s}} x^2+y^2+z^2 = 1$$

all values satisfies the inequality

$$x^2+y^2+z^2 \leq 1$$

$x^2+y^2 < 1$ → interior points included.
$x^2+y^2 = 1$ → boundary
$x^2+y^2 \leq 1$ → boundary or interior points.

Domains & Ranges

function

$\sqrt{2-2^2}$
$\sqrt{2-4}$
$\sqrt{-2}$

natural domains
all real values
never imaginary values.
① $\sqrt{-1}$ ✗
② $\frac{x}{0} = \infty$ ✗

$$z = \sqrt{y-x^2}$$

domain: $y - x^2 \geq 0$ ✓
$y \geq x^2$ ✓

range: $[0, \infty)$

$f(x) = x^2$
domain: all $\mathbb{R}$
range: $[0, \infty)$

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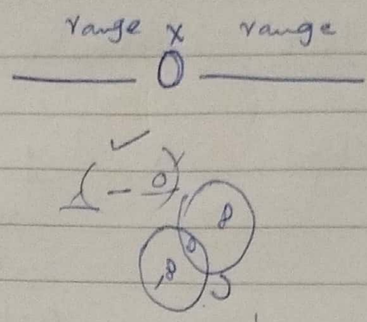
$z = \dfrac{1}{xy}$



domain: $x \cdot y \neq 0$ ✓

or $x \neq 0, y \neq 0$

range $(-\infty, 0) \cup (0, \infty)$

$\{-\infty, 0\} \cup \{0, \infty\}$

$z = \sin(xy)$ domain = entire plane (2-D)

range = $[-1, 1]$.

$i = \sqrt{-1}$

$i^2 = -1$

$w = \sqrt{x^2 + y^2 + z^2}$

domain → entire space

$x^2 + y^2 + z^2 \geq 0$.

range → $[0, \infty)$.

$w = 1/x^2 + y^2 + z^2$

domain → $(x, y, z) \neq (0, 0, 0)$ (no zero at same time).

range → $(0, \infty)$

$w = xy \ln z$.

$\ln(-)$

domain: $z > 0$. (half space) ⟶ all -ve values excluded thats why half-space

range: $(-\infty, \infty)$.

Slide (16)

Ex: 1

(a) $f(x, y) = \dfrac{\sqrt{x+y+1}}{x-1}$

Domain $= \{(x, y) \mid x+y+1 \geq 0, x \neq 1\}$

domain

$x + y + 1 \geq 0$, $x - 1 \neq 0$

$\boxed{y \geq -x-1}$, $\boxed{x \neq 1}$

(1) let $x = 0$

$y \geq -0-1$

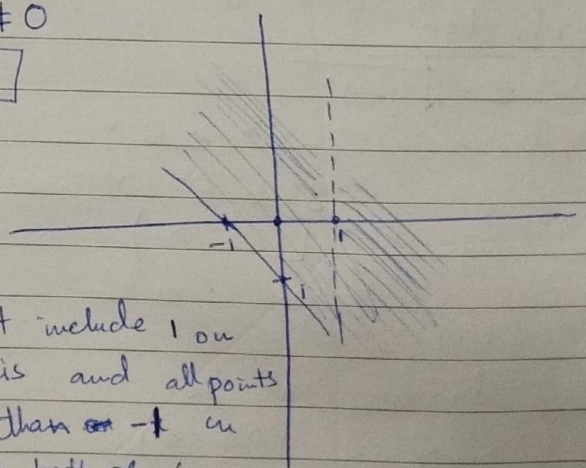
$\boxed{y \geq -1}$

(2) $x + y \geq -1$

let $y = 0$

$x + 0 \geq -1$

$\boxed{x \geq -1}$



doesn't include 1 on x axis and all points less than ~~-1~~ on -1 on both x y axes

(2) $f(x,y) = x \ln(y^2 - x)$

domain: $y^2 - x > 0$

$y^2 > x.$

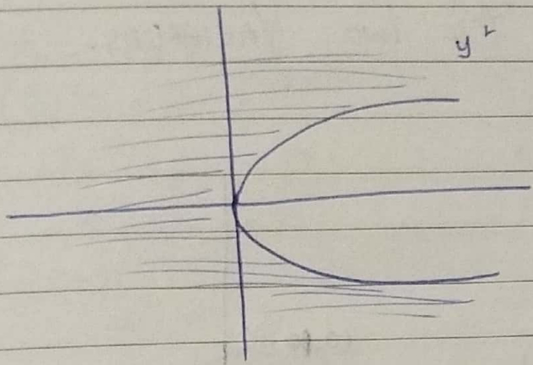
$y = x^2 + 2x + 3$

if coefficient of x^2 is +ve, ∪

if -ve, ∩

if coefficient of y^2 is +ve, ⊂

if -ve, ⊃.



$x^2 + y^2 = (r)^2$

Ex: 2 H.W.

Ex: 3 $f(x,y) = \sqrt{9 - x^2 - y^2}$

domain: $9 - x^2 - y^2 \geq 0$

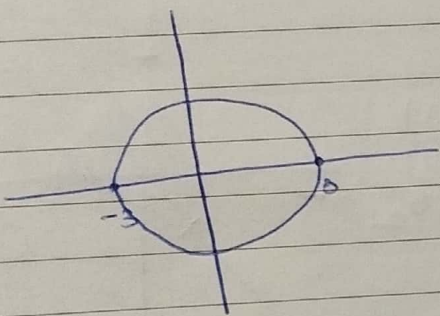
$9 - x^2 \geq y^2$

if

$9 \geq x^2 + y^2$

$(3)^2 \geq x^2 + y^2$

(circle).



Ex: 1 $f(x,y) = \sqrt{y+1} + \ln(x^2 - y)$.

domain $y + 1 \geq 0$ $x^2 - y \geq 0$.

$y \geq -1$ $x^2 \geq y$.

$-1 \leq y \leq x^2$

$f(e, 0) = \sqrt{0+1} + \ln(e^2 - 0)$.

$f(e, 0) = 3$

$f(x,y,z) = \sqrt{1 - x^2 - y^2 - z^2}$

$f(0, \tfrac{1}{2}, \tfrac{1}{2}) = \sqrt{1 - 0^2 - (\tfrac{1}{2})^2 - (\tfrac{1}{2})^2}$

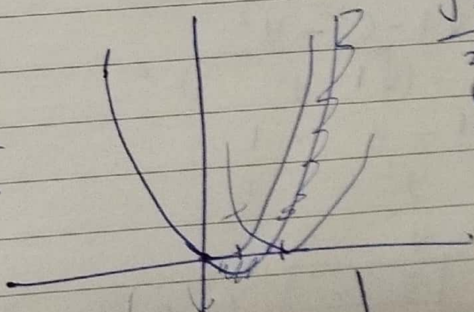
x y z

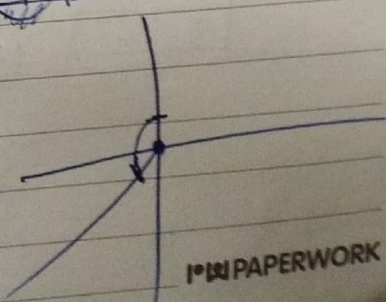
$= \sqrt{1 - 0 - \tfrac{1}{4} - \tfrac{1}{4}}$

$= \dfrac{\sqrt{2}}{2}$

$\dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$\dfrac{(\sqrt{2})^2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$





domain $1 - x^2 - y^2 - z^2 \geq 0$

$1 \geq x^2 + y^2 + z^2$

$x^2 + y^2 + z^2 \leq 1$

PAPERWORK

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$f(x) = 3 + \sqrt{x^2 - 2}$

domain $x^2 - 2 \geq 0$ range $[3, \infty)$

$x^2 \geq 2$

GRAPH OF FUNCTIONS OF TWO VARIABLES.

Ex:3

(a) $f(x,y) = 1 - x - \frac{1}{2}y$.

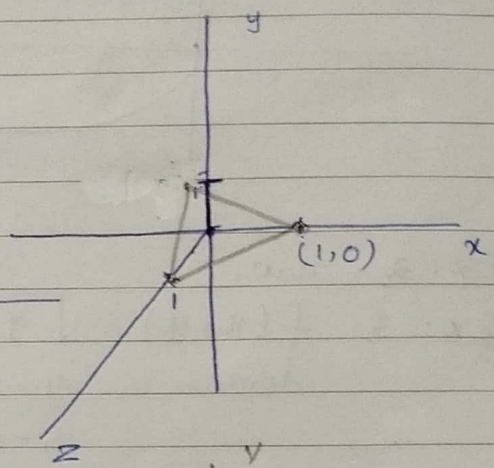
$z = 1 - x - \frac{1}{2}y$

$f(0,\phi) = 1 - 1/2$

$f(0,1) = 1/2$

$f(1,0) = 1 - 1 - \frac{1}{2}(0)$

$f(1,0) = 0$



$f(0,0) = 1 - 0 - \frac{1}{2}(0)$

$f(0,0) = 1$

$(x,y,z) = (0,1,\frac{1}{2})$

$(x,y,z) = (1,0,0)$

$(x,y,z) = (0,0,1)$

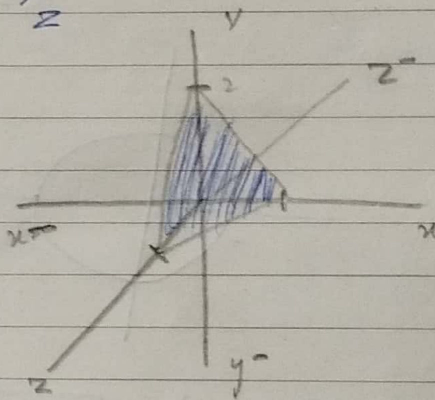
(b) $f(x,y) = \sqrt{1-x^2-y^2}$

(0,0,1)
(1,0,0)
(0,2,0)

$f(0,0) = \sqrt{1-0^2-0^2}$

$z = \sqrt{1}$

$\boxed{z = \pm 1}$ $(0,0,\pm 1)$



z=0

$0 = \sqrt{1-x^2-\phi^2}$

$(0)^2 = (\sqrt{1-x^2-1})^2$

$\cancel{1} - x^2 - \cancel{1} = 0$

$x^2 = 0$

$\boxed{x = 0}$ $(0,1,0)$

$1 = \sqrt{1-0^2-y^2}$

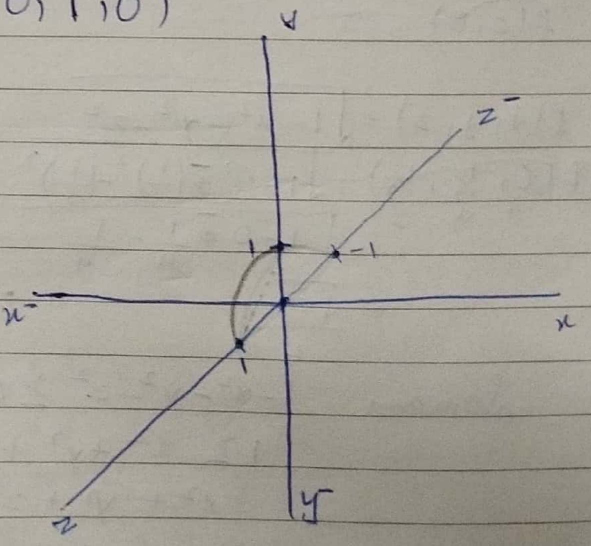
$(1)^2 = (\sqrt{1-y^2})^2$

$1 - y^2 = 1$

$y^2 = 1-1$

$y^2 = 0$

$\boxed{y = 0}$ $(0,0,\phi)$



$z = -\sqrt{x^2 + y^2} \quad (x, y) = (0, 0)$

$z = -\sqrt{0^2 + 0^2}$

$z = 0 \quad (x, y, z) = (0, 0, 0)$

$0 = +\sqrt{x^2 + 1^2}$

$(0)^2 = +(\sqrt{x^2 + 1^2})^2$

$+(x^2 + 1) = 0$

$-x^2 - 1 = 0$

$-x^2 = 1$

$x^2 = \sqrt{-1}$

$(\sqrt{x^2+1})^2 = (0)^2$

$x^2 + 1 = 0$

$x^2 = -1$

$-\sqrt{x^2 + 1^2} = 0$

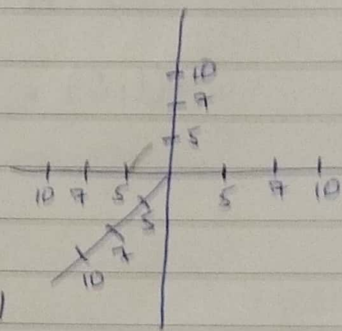
Level- Curves ⟶ multiple curves/circles!

Ex:3 $f(x, y) = 100 - x^2 - y^2$

① $f(x, y) = 0$

② $f(x, y) = 51$

③ $f(x, y) = 75$



$z = 100 - x^2 - y^2$ for $z = 0$

$\boxed{x^2 + y^2 = 100}$ $\boxed{r = 10}$

$51 = 100 - x^2 - y^2$ for $z = 51$

$x^2 + y^2 = 100 - 51$

$\boxed{x^2 + y^2 = 49}$ $\boxed{r = 7}$

$75 = 100 - x^2 - y^2$ for $z = 75$

$x^2 - y^2 = 100 - 75$

$\boxed{x^2 - y^2 = 25}$ / $\boxed{r = 5}$