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CLASS: BSCS-5B

COURSE: NUMERICAL COMPUTING

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ASSIGNMENT NO. 2

Date 29-November 2023

$83x - 11y - 4z = 95$

$7x + 52y + 13z = 104.$

$3x + 8y + 29z = 71$

Compare the two method and explain which method is better.

Solve

Checking diagonally dominant condition.

$83 > 11+4 \Rightarrow 83 > 15$

$52 > 7+13 \Rightarrow 52 > 20$

$29 > 3+8 \Rightarrow 29 > 11$ all conditions are true.

$x = \frac{95 + 11y + 4z}{83}$, $y = \frac{104 - 7x - 13z}{52}$, $z = \frac{71 - 3x - 8y}{29}.$

Lets suppose, $x_0 = 0$, $y_0 = 0$ and $z_0 = 0$

Guass-Jacobi Method.	Guass-Siede Method.
1st Iteration	1st Iteration
$x_1(y_0, z_0) = (0,0)$	$x_1(y_0, z_0) = (0,0).$
$x_1 = \frac{95 + 11(0) + 4(0)}{83} = \boxed{\frac{95}{83}}$	$x_1 = \frac{95 + 11(0) + 4(0)}{83} = \boxed{\frac{95}{83}}$
$y_1(x_0, z_0) = (0,0)$	$y_1(x_1, z_0) = \left(\frac{95}{83}, 0\right)$
$y_1 = \frac{104 - 7(0) - 13(0)}{52} = \frac{104}{52} = \boxed{2}$	$y_1 = \frac{104 - 7\left(\frac{95}{83}\right) - 13(0)}{52} = \boxed{1.845}$
$z_1(x_0, y_0) = (0,0)$	$z_1(x_1, y_1) = \left(\frac{95}{83}, 1.845\right)$
$z_1 = \frac{71 - 3(0) - 8(0)}{29} = \boxed{\frac{71}{29}}$	$z_1 = \frac{71 - 3\left(\frac{95}{83}\right) - 8(1.845)}{29} = \boxed{1.820}$
Relative Approximate Error $\lvert \epsilon_a \rvert$	Relative Approximate Error $\lvert \epsilon_a \rvert$
$\lvert \epsilon_{ax_1} \rvert = \left\lvert \frac{\frac{95}{83} - 0}{95/83} \right\rvert \times 100 = \boxed{100\%}$	$\lvert \epsilon_{ax_1} \rvert = \left\lvert \frac{\frac{95}{83} - 0}{95/83} \right\rvert \times 100 = \boxed{100\%}$
$\lvert \epsilon_{ay_1} \rvert = \left\lvert \frac{2 - 0}{2} \right\rvert \times 100 = \boxed{100\%}$	$\lvert \epsilon_{ay_1} \rvert = \left\lvert \frac{1.845 - 0}{1.845} \right\rvert \times 100 = \boxed{100\%}$

$$|\epsilon_{az_1}| = \left|\frac{\frac{71}{29} - 0}{\frac{71}{29}}\right| \times 100 = \boxed{100\%}$$

$\Rightarrow x_1 = \frac{95}{83}, y_1 = 2, z_1 = \frac{71}{29}$

2ND Iteration.

$x_2 : (y_1, z_1) = (2, \frac{71}{29})$.

$$x_2 = \frac{95 + 11(2) + 4(\frac{71}{29})}{83} = \boxed{1.527}$$

$y_2 (x_1, z_1) = (\frac{95}{83}, \frac{71}{29})$

$$y_2 = \frac{104 - 7(\frac{95}{83}) - 13(\frac{71}{29})}{52} = \boxed{1.233}$$

$z_2 (x_1, y_1) = (95/83, 2)$

$$z_2 = \frac{71 - 3(\frac{95}{83}) - 8(2)}{29} = \boxed{1.778}$$

Relative Approximate Error (ϵ_a)

$$|\epsilon_{ax_2}| = \left|\frac{1.527 - \frac{95}{83}}{1.527}\right| \times 100 = \boxed{25.04\%}$$

$$|\epsilon_{ay_2}| = \left|\frac{1.233 - 2}{1.233}\right| \times 100 = \boxed{62.20\%}$$

$$|\epsilon_{az_2}| = \left|\frac{1.778 - \frac{71}{29}}{1.778}\right| \times 100 = \boxed{37.61\%}$$

$\Rightarrow x_2 = 1.527, y_2 = 1.233, z_2 = 1.778$

3RD Iteration.

$x_3 (y_2, z_2) = (1.233, 1.778)$.

$$x_3 = \frac{95 + 11(1.233) + 4(1.788)}{83} = \boxed{1.394}$$

$y_3 (x_2, z_2) = (1.527, 1.778)$

$$y_3 = \frac{104 - 7(1.527) - 13(1.778)}{52}$$

$y_3 = \boxed{1.349}$

$$|\epsilon_{ax_1}| = \left|\frac{1.820 - 0}{1.820}\right| \times 100 = \boxed{100\%}$$

$\Rightarrow x_1 = \frac{95}{83}, y_1 = 1.845, z_1 = 1.820$

2ND Iteration

$x_2 (y_1, z_1) = (1.845, 1.820)$

$$x_2 = \frac{95 + 11(1.845) + 4(1.820)}{83} = \boxed{1.476}$$

$y_2 (x_2, z_1) = (1.476, 1.820)$

$$y_2 = \frac{104 - 7(1.476) - 13(1.820)}{52} = \boxed{1.346}$$

$z_2 (x_2, y_2) = (1.476, 1.346)$

$$z_2 = \frac{71 - 3(1.476) - 8(1.346)}{29} = \boxed{1.924}$$

Relative Approximate Error (ϵ_a)

$$|\epsilon_{ax_2}| = \left|\frac{1.476 - \frac{95}{83}}{1.476}\right| \times 100 = \boxed{22.45}$$

$$|\epsilon_{ay_2}| = \left|\frac{1.346 - 1.845}{1.346}\right| \times 100 = \boxed{37.07}$$

$$|\epsilon_{az_2}| = \left|\frac{1.924 - 1.820}{1.924}\right| \times 100 = \boxed{5.40\%}$$

$\Rightarrow x_2 = 1.476, y_2 = 1.346, z_2 = 1.924$.

3RD Iteration.

$x_3 (y_2, z_2) = (1.346, 1.924)$.

$$x_3 = \frac{95 + 11(1.346) + 4(1.924)}{83} = \boxed{1.415}$$

$y_3 (x_3, z_2) = (1.415, 1.924)$

$$y_3 = \frac{104 - 7(1.415) - 13(1.924)}{52}.$$

$\boxed{y_3 = 1.328}$

$z_3(x_2, y_2) = (1.527, 1.233)$

$$z_3 = \frac{71 - 3(1.527) - 8(1.233)}{29} = \boxed{1.950}$$

Relative Approximate Error (ϵ_a)

$$|\epsilon_{ax_3}| = \left|\frac{1.394 - 1.527}{1.394}\right| \times 100 = \boxed{9.54\%}$$

$$|\epsilon_{ay_3}| = \left|\frac{1.349 - 1.233}{1.349}\right| \times 100 = \boxed{8.59\%}$$

$$|\epsilon_{az_3}| = \left|\frac{1.950 - 1.778}{1.950}\right| \times 100 = \boxed{8.82\%}$$

$\Rightarrow x_3 = 1.394,\ y_3 = 1.349,\ z_3 = 1.950$

$z_3(x_3, y_3) = (1.415, 1.328)$

$$z_3 = \frac{71 - 3(1.415) - 8(1.328)}{29} = \boxed{1.93}$$

Relative Approximate Error (ϵ_a)

$$|\epsilon_{ax_3}| = \left|\frac{1.415 - 1.476}{1.415}\right| \times 100 = \boxed{4.31\%}$$

$$|\epsilon_{ay_3}| = \left|\frac{1.328 - 1.346}{1.328}\right| \times 100 = \boxed{1.35}$$

$$|\epsilon_{az_3}| = \left|\frac{1.935 - 1.924}{1.935}\right| \times 100 = \boxed{0.56}$$

$\Rightarrow x_3 = 1.415,\ y_3 = 1.328,\ z_3 = 1.935$

Guass-Siedel method is more efficient than Guass Jacobi method as Guass-Siedel method need less number of iterations to converge to the actual solution and the rate of error is less in it, error decreases at faster rate.