Now, multiplying the I.F with eq ①.

$\Rightarrow$ $\cancel{e^{x}} x \dfrac{dy}{dx} + \cancel{x} \cdot \dfrac{y}{\cancel{x}} = x e^{x}$

$\Rightarrow$ $x \dfrac{dy}{dx} + y = x e^{x}$.

$\because \dfrac{d}{dx}[x\,y] = x \dfrac{dy}{dx} + y$.

$\Rightarrow$ $\dfrac{d}{dx}[x\,y] = x e^{x}$.

$\Rightarrow$ $d[x\,y] = x e^{x}\,dx$.

$\Rightarrow$ Applying integration

$\Rightarrow$ $\int d(x.y) = \int x e^{x}\,dx$.

$\Rightarrow$ $xy = \int x e^{x}\,dx$. — ②

Solve $\int x e^{x}\,dx$, using integration by parts.

Let $f = x$ $\quad g' = e^{x}$.

$f' = 1$ $\quad g = e^{x}$.

$\because \int f g' = fg - \int f' g$.

$\Rightarrow$ $\int x e^{x}\,dx = x e^{x} - \int e^{x}\,dx$.

$\Rightarrow$ $\int x e^{x}\,dx = x e^{x} - e^{x} + C$.

Substitute it back to our equation ②

$\Rightarrow$ $\boxed{xy = x e^{x} - e^{x} + c}$

Answer.

iii) $\dfrac{dy}{dx} = \dfrac{xy}{x^{2}+y^{2}}$.

Solve

$\dfrac{dy}{dx} = \dfrac{xy}{x^{2}+y^{2}}$.

Given equation is homogenous differential equatio