Assignment No. 2

Date ________ 20____

Topic :- Partial Derivatives

Question

Evaluate the indicated partial derivatives.
$f(x,y) = 10x^2y^4 - 6xy^2 + 10x^2$; $f_x(x,y)$, $f_y(x,y)$.

Solve

$$f(x,y) = 10x^2y^4 - 6xy^2 + 10x^2$$

$$f_x(x,y) = 20xy^4 - 6y^2 + 20x^1.$$

Ans

$$f_y(x,y) = 40x^2y^3 - 12xy.$$

Ans

Question

$f(x,y) = \dfrac{1}{xy^2 - x^2y}$; $f_x(x,y)$, $f_y(x,y)$.

Solve

$$f(x,y) = \frac{1}{xy^2 - x^2y}$$

$$\because \frac{u}{v} = \frac{v\,du - u\,dv}{v^2}$$

$$f_x(x,y) = \frac{(xy^2 - x^2y)\,f_x(1) - (1).f_x(xy^2 - x^2y)}{(xy^2 - x^2y)^2}$$

$$f_x(x,y) = \frac{(xy^2 - x^2y)(0) - (1)(y^2 - 2xy)}{(xy^2 - x^2y)^2}$$

$$f_x(x,y) = \frac{0 - y^2 + 2xy}{(xy^2 - x^2y)^2}$$

$$f_x(x,y) = \frac{-y^2 + 2xy}{(xy^2 - x^2y)^2}$$

Ans

$$f(x,y) = \frac{1}{xy^2 - x^2y}$$

$$\frac{u}{v} = \frac{v\,\partial u - u\,\partial v}{v^2}$$

$$f_y(x,y) = \frac{(xy^2 - x^2y)\,f_y(1) - (1)\,f_y(xy^2 - x^2y)}{(xy^2 - x^2y)^2}$$

$$f_y(x,y) = \frac{(x^2y - xy^2)(0) - (1)(2xy - x^2)}{(xy^2 - x^2y)^2}$$

$$f_y(x,y) = \frac{0 - 2xy + x^2}{(xy^2 - x^2y)^2}$$

$$f_y(x,y) = \frac{x^2 - 2xy}{(xy^2 - x^2y)^2}$$

Ans

QUESTION No.

$$\frac{\partial}{\partial p}\left(e^{-7p/q}\right),\ \frac{\partial}{\partial q}\left(e^{-7p/q}\right).$$

Solve

$$\Rightarrow \frac{\partial}{\partial p}\left(e^{-7p/q}\right).$$

$$\Rightarrow e^{-7p/q} \cdot \frac{\partial}{\partial p}\left(-7\frac{p}{q}\right)$$

$$\Rightarrow e^{-7p/q} \cdot -\frac{7}{q}.$$

$$\Rightarrow -\frac{7}{q}\,e^{-7p/q}$$

$$\frac{\partial}{\partial p}\left(e^{-7p/q}\right) = -\frac{7}{q}\,e^{-7p/q}.$$

Ans

$$\Rightarrow \frac{\partial}{\partial v} e^{(-7p/v)}$$

$$\Rightarrow e^{-7p/v} \cdot \frac{\partial}{\partial v}\left(-\frac{7p}{v}\right).$$

$$\Rightarrow e^{-7p/v} \cdot (-7)\left(\frac{v\cdot(0) - p(1)}{v^2}\right)$$

$$\Rightarrow -7\, e^{-7p/v} \left(\frac{-p}{v^2}\right).$$

$$\Rightarrow \frac{7p}{v^2}\, e^{-7p/v}.$$

$$\frac{\partial}{\partial v}\left(e^{-7p/v}\right) = \frac{7p}{v^2}\, e^{-7p/v}.$$

Ans

QUESTION

Let $f(x,y) = xe^{-y} + 5y$.

a) Find the slope of the surface $z = f(x,y)$ in the x-direction at the point $(3,0)$.

Solve

$$f_x(3,0) = ?$$

$$f_x(x,y) = e^{-y}$$

$$f_x(3,0) = e^{-0}$$

$$f_x(3,0) = 1$$ Ans

b) Find the slope of the surface $z = f(x,y)$ in the y-direction at point $(3,0)$.

Solve

$$f_y(3,0) = ?$$

$f_y(x,y) = -xe^{-y} + 5$
$f_y(3,0) = -(3)e^{-0} + 5$
$f_y(3,0) = -3(1) + 5$
$f_y(3,0) = 2$ Ans

QUESTION

Confirm that the mixed second-order partial derivatives of f are the same.
$f(x,y) = 4x^2 - 8xy^4 + 7y^5 - 3$

Solve

$f_{xy}(x,y) = f_{yx}(x,y)$
$f(x,y) = 4x^2 - 8xy^4 + 7y^5 - 3$
$f_x(x,y) = 8x - 8y^4$
$f_{xy}(x,y) = -32y^3$

$f_y(x,y) = -32xy^3 + 35y^4$
$f_{yx}(x,y) = -32y^3$

$f_{xy}(x,y) = -32y^3$
and $f_{yx}(x,y) = -32y^3$

Hence, proved mixed second order partial derivative of f are the same i.e.
$f_{xy}(x,y) = f_{yx}(x,y)$
$-32y^3 = -32y^3$

QUESTION

$f(x,y) = \ln(x^2+y^2)$

Solve

$f_{xy}(x,y) = f_{yx}(x,y)$

$f(x,y) = \ln(x^2+y^2)$.

$f_x(x,y) = \dfrac{1}{x^2+y^2} \cdot 2x = \dfrac{2x}{x^2+y^2}$

$f_{xy}(x,y) = \dfrac{(x^2+y^2) \cdot f_{xy}(2x) - (2x) f_{xy}(x^2+y^2)}{(x^2+y^2)^2}$

$f_{xy}(x,y) = \dfrac{(x^2+y^2)(0) - (2x)(2y)}{(x^2+y^2)^2}$

$f_{xy}(x,y) = \dfrac{-4xy}{(x^2+y^2)^2}$

$f_y(x,y) = \dfrac{1}{x^2+y^2} \cdot 2y = \dfrac{2y}{x^2+y^2}$

$f_{yx}(x,y) = \dfrac{(x^2+y^2) f_{yx}(2y) - (2y) f_{yx}(x^2+y^2)}{(x^2+y^2)^2}$

$f_{yx}(x,y) = \dfrac{(x^2+y^2)(0) - (2y)(2x)}{(x^2+y)^2}$

$f_{yx}(x,y) = \dfrac{-4xy}{(x^2+y^2)^2}$

Hence, it is proved that mixed second-order partial derivative of f are same

$f_{xy}(x,y) = f_y(x,y)$

$\dfrac{-4xy}{(x^2+y^2)^2} = \dfrac{-4xy}{(x^2+y^2)^2}$

Question

Show that the function $u(x,t) = \sin c\omega t \sin \omega x$ satisfies the wave equation for all real values of ω.

Solve

Wave equation formation.

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

a is any constant, t is time.

c is constant

$\boxed{a = c}$

$$u(x,t) = \sin(c\omega t) \sin(\omega x)$$

$$\frac{\partial u}{\partial t} = \cos(c\omega t) \sin(\omega x) \cdot c\omega.$$

$$\frac{\partial^2 u}{\partial t^2} = -\sin(c\omega t) \cdot \sin(\omega x) \cdot c\omega \cdot c\omega$$

$$\frac{\partial^2 u}{\partial t^2} = -\sin(c\omega t) \cdot \sin(\omega x) \cdot c^2 \omega^2.$$

$$\frac{\partial^2 u}{\partial t^2} = -c^2 \omega^2 \cdot \sin(c\omega t) \cdot \sin(\omega x).$$

$$\frac{\partial u}{\partial x} = \sin(c\omega t) \cdot \cos(\omega x) \cdot \omega$$

$$\frac{\partial^2 u}{\partial x^2} = \sin(c\omega t) \cdot (-\sin(\omega x) \cdot \omega \cdot \omega)$$

$$\frac{\partial^2 u}{\partial x^2} = -\omega^2 \sin(c\omega t) \cdot \sin(\omega x),$$

$$\because \frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

$$\Rightarrow -c^2\omega^2 \sin(c\omega t) \cdot \sin(\omega x) = -c^2 \omega^2 \sin(c\omega t) \cdot \sin(\omega x)$$

Hence, proved that function $u(x,t)$ satisfies wave equation