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Chain Rule.

Ex. 1

Suppose that $z = x^2 y$, $x = t^2$, $y = t^3$.
Use the chain rule to find dz/dt, an check the result by expressing z as a function of t an differentiating directly.

Solution

$$z = x^2 y, \quad x = t^2, \quad y = t^3$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = 2xy \cdot 2t + x^2 \cdot 3t^2$$

$$\frac{dz}{dt} = 4\cancel{t}\,xy + 3t^2 x^2$$

$$\frac{dz}{dt} = 4t(t^2)(t^3) + 3t^2(t^2)^2$$

$$\frac{dz}{dt} = 4t^6 + 3t^6$$

$$\frac{dz}{dt} = 7t^6$$

Slide. 6 : Ex : 6.

Find dw/dt if $w = xy + z$, $x = \cos t$, $y = \sin t$ and $z = t$.
what is the derivative's value at $t = 0$?

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$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt}$$

$$\frac{dw}{dt} = -y \sin t + x \cos t + 1$$

$$\frac{dw}{dt} = -\sin t \cdot \sin t + \cos t \cdot \cos t + 1.$$

$$\frac{dw}{dt} = -\sin^2 t + \cos^2 t + 1.$$

$$\frac{dw}{dt} = \quad \because \cos^2 t - \sin^2 t = \cos 2t$$

$$\frac{dw}{dt} = \cos 2t + 1.$$

at $t = 0$

$$\left(\frac{dw}{dt}\right)_{t=0} = \cos 2(0) + 1$$

$$\left(\frac{dw}{dt}\right)_{t=0} = \cos(0) + 1$$

$$\left(\frac{dw}{dt}\right)_{t=0} = 1 + 1$$

$$\left(\frac{dw}{dt}\right)_{t=0} = 2$$

Ex. 1
Given that $z = e^{xy}$, $x = 2u + v$, $y = u/v$.
Find $\partial z/\partial u$ & $\partial z/\partial v$ using the chain rule.

Solution

$z = e^{xy}$, $x = 2u + v$ $\quad y = u/v$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial u} = e^{xy} \cdot y \cdot 2 + e^{xy} \cdot x \cdot \frac{1}{v}$$

$$\frac{\partial z}{\partial u} = 2ye^{xy} + \frac{x}{v} e^{xy}$$

$$\frac{\partial z}{\partial u} = e^{xy}\left(2y + \frac{x}{v}\right).$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$$\frac{\partial z}{\partial v} = e^{xy} \cdot \cancel{by} y \cdot (1) + e^{xy} \cdot x \cdot \left(-\frac{u}{v^2}\right)$$

$$\frac{\partial z}{\partial v} = ye^{xy} - \frac{xu}{v^2} e^{xy}$$

$$\frac{\partial z}{\partial v} = e^{xy}\left(y - \frac{xu}{v^2}\right).$$

$$\therefore \frac{\partial z}{\partial u} = e^{xy}\left(2y + \frac{x}{v}\right)$$

$$\frac{\partial z}{\partial u} = e^{(2u+v)\left(\frac{u}{v}\right)}\left(2\left(\frac{u}{v}\right) + \frac{2u+v}{v}\right)$$

$$\frac{\partial z}{\partial x} = e^{(2u+v)(u/v)}\left(\frac{2u}{v} + \frac{2u+v}{v}\right)$$

$$\frac{\partial z}{\partial x} = e^{(2u+v)(\frac{u}{v})}\left(\frac{2u+2u+v}{v}\right)$$

$$\frac{\partial z}{\partial x} = e^{(2u+v)(u/v)}\left(\frac{4u+v}{v}\right)$$

$$\frac{\partial z}{\partial y} = e^{xy}\left(y - \frac{xu}{v^2}\right)$$

$$\frac{\partial z}{\partial y} = e^{(2u+v)(\frac{u}{v})}\left(\frac{u}{v} - \frac{(2u+v)u}{v^2}\right)$$

$$\frac{\partial z}{\partial y} = e^{(2u+v)(\frac{u}{v})}\left(\frac{\cancel{uv} - 2u^2 - \cancel{uv}}{v^2}\right)$$

$$\frac{\partial z}{\partial y} = e^{(2u+v)(\frac{u}{v})}\left(\frac{\cancel{2uv} - 2u^2}{v^2}\right)$$

$$\frac{\partial z}{\partial y} = e^{(2u+v)(\frac{u}{v})}\left(-\frac{2u^2}{v^2}\right)$$

Ex. 2

Suppose that $w = e^{xyz}$, $x = 3u+v$, $y = 3u-v$, $z = u^2 v$. Use appropriate forms of the chain rule to find $\partial w/\partial u$ & $\partial w/\partial v$.

Solution

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial u}$$

$$\frac{\partial w}{\partial u} = e^{xyz}\cdot yz\cdot 3 + e^{xyz}\cdot xz \cdot 3 + e^{xyz}\cdot xy\cdot 2u$$

$$\frac{\partial w}{\partial u} = 3yze^{xyz} + 3xze^{xyz} + 2uxye^{xyz}$$

$$\frac{\partial w}{\partial u} = e^{xyz}\left(3yz + 3xz + 2uxy\right).$$

$$\frac{\partial w}{\partial u} = e^{(3u+v)(3u-v)(u^2v)}\left\{3(3u-v)(u^2v) + 3(3u+v)(u^2v) + 2u(3u+v)(3u-v)\right\}$$

$$\frac{\partial w}{\partial u} \Rightarrow e^{(3u+v)(3u-v)(u^2v)}\left\{\cancel{3(3u}\ 9u^3v - \cancel{3u^2v^2} + 9u^3v + \cancel{3u^2v^2} + 2u(9u^2 - 3uv + 3uv - v^2)\right\}$$

$$\Rightarrow e^{(3u+v)(3u-v)(u^2v)}\left(18u^3v\ \cancel{-3u^2v^2+3u^2v^2}\ 18u^3 - 6u^2v + 6u^2v - 2v^2\right)$$

$$\frac{\partial w}{\partial u} \Rightarrow e^{(3u+v)(3u-v)(u^2v)}\left(18u^3v + 18u^3 - 2v^2\right)$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial v}.$$

$$\frac{\partial w}{\partial y} = e^{xyz}\cdot yz\cdot 1 + e^{xyz}\cdot xz\cdot(-1) + e^{xyz}\cdot u^2(xy)$$

$$\frac{\partial w}{\partial y} = yze^{xyz} - xze^{xyz} + u^2e^{xyz}$$

$$\frac{\partial w}{\partial y} = e^{xyz}\left(yz - xz + xyu^2\right).$$

Ex:3

Express $\partial w/\partial r$ and $\partial w/\partial s$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$ and $z = 2r$.

Solution

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial r}.$$

$$\frac{\partial w}{\partial r} = 1 \cdot \frac{1}{s} + 2 \cdot 2r + 2z \cdot 2.$$

$$\frac{\partial w}{\partial r} = \frac{1}{s} + 4r + 4z.$$

$$\frac{\partial w}{\partial r} = \frac{1}{s} + 4r + 4(2r)$$

$$\frac{\partial w}{\partial r} = \frac{1}{s} + 4r + 8r$$

$$\frac{\partial w}{\partial r} = \frac{1}{s} + 12r.$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}.$$

$$\frac{\partial w}{\partial s} = 1 \cdot \left(-\frac{r}{s^2}\right) + 2 \cdot \frac{1}{s} + 2z(0)$$

$$\frac{\partial w}{\partial s} = -\frac{r}{s^2} + \frac{2}{s}$$

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Ex: 4

Express $\partial w/\partial r$ and $\partial w/\partial s$ in terms of r & s if $w = x^2 + y^2$, $x = r - s$ and $y = r + s$.

solution

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial w}{\partial r} = 2x \cdot 1 + 2y \cdot 1$$

$$\frac{\partial w}{\partial r} = 2x + 2y$$

$$\frac{\partial w}{\partial r} = 2(r-s) + 2(r+s).$$

$$\frac{\partial w}{\partial r} = 2r - \cancel{2s} + 2r \cancel{+2s}$$

$$\frac{\partial w}{\partial r} = 4r.$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial s}.$$

$$\frac{\partial w}{\partial s} = 2x \cdot (-1) + 2y \cdot (1)$$

$$\frac{\partial w}{\partial s} = -2x + 2y$$

$$\frac{\partial w}{\partial s} = -2(r-s) + 2(r+s)$$

$$\frac{\partial w}{\partial s} = \cancel{-2r} + 2s + \cancel{2r} + 2s$$

$$\frac{\partial w}{\partial s} = 4s$$

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$w(x(\rho, \phi, \theta), y$ ...)

Ex: 3

Suppose that $w = x^2 + y^2 - z^2$ and $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$ and $z = \rho\cos\phi$.
find $\partial w/\partial\rho$ an $\partial w/\partial\theta$.

$$\frac{\partial w}{\partial \rho} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \rho} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \rho} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial \rho}$$

$$\frac{\partial w}{\partial \rho} = 2x \cdot \sin\phi . \cos\theta + 2y . \sin\phi . \sin\theta - 2z . \cos\phi$$

$$\frac{\partial w}{\partial \rho} = 2(\rho\sin\phi\cos\theta) . \sin\phi\cos\theta + 2(\rho\sin\phi\sin\theta) . \sin\phi\sin\theta - 2(\rho\cos\theta)\cos\phi .$$

$$\frac{\partial w}{\partial \rho} = 2\rho\sin^2\phi\cos^2\theta + 2\rho\sin^2\phi\sin^2\theta - 2\rho\cos^2\phi .$$

$$\frac{\partial w}{\partial \rho} = 2\rho\sin^2\phi(\cos^2\theta + \sin^2\theta) - 2\rho\cos^2\phi$$

$\because \cos^2\theta + \sin^2\theta = 1$

$$\frac{\partial w}{\partial \rho} = 2\rho\sin^2\phi(1) - 2\rho\cos^2\phi .$$

$$\frac{\partial w}{\partial \rho} = 2\rho(\sin^2\phi - \cos^2\phi) .$$

$$\frac{\partial w}{\partial \rho} = -2\rho(\cos^2\phi - \sin^2\phi)$$

$$\frac{\partial w}{\partial \rho} = -2\rho\cos 2\phi$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial \theta}.$$

$$\frac{\partial w}{\partial \theta} = -2x\rho \sin\phi \sin\theta + 2y\rho \sin\phi \cos\theta + 2z \cancel{\rho \sin\phi}(0)$$

$$\frac{\partial w}{\partial \theta} = -2(\rho\sin\phi\cos\theta)\rho\sin\phi\sin\theta + 2(\rho\sin\phi\sin\theta)\rho\sin\phi\cos\theta + 0.$$

$$\frac{\partial w}{\partial \theta} = -2\rho^2 \cancel{\sin^2\phi \sin\theta\cos\theta} + 2\rho^2 \cancel{\sin^2\phi \sin\theta\cos\theta}$$

$$\frac{\partial w}{\partial \theta} = 0$$

Plane Tangent to a Surface $z = f(x,y)$ at $(x_0, y_0, f(x_0, y_0))$

The plane tangent to the surface $z = f(x,y)$ of a differentiable function f at the point $P_0(x_0, y_0, z_0) = (x_0, y_0, f(x_0, y_0))$ is.

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0.$$

Ex: 1

Find the plane tangent to the surface $z = x\cos y - ye^{x}$ at $(0,0,0)$.

Solution

$f_x(x,y) = x\cos y - ye^{x}$

$f_x(0,0) = \cos y - ye^{x}$

$f_x(0,0) = \cos(0) - \cancel{0}e^{0}$

$f_x(0,0) = 0$

$f_y(0,0) = -x\sin y - e^{x}$

$f_y(0,0) = -(0).\sin(0) - e^{0}.$

$f_y(0,0) = 0 - 1$

$f_y(0,0) = -1.$

$\therefore f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) - (z-z_0)$

$1(x-0) + (-1)(y-0) - (z-0) = 0$

$\boxed{x - y - z = 0}$

Ex: 2

Find an equation for the tangent plane and parametric equations for the normal line to the surface $z = x^2 y$ at point $(2,1,4)$.

Solution

~~let~~ ~~$w = xy$~~

let $w = F(x,y,z) = z - x^2 y$

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$$\frac{\partial w}{\partial x} = -2x \qquad \frac{\partial w}{\partial y} = -x^2 \qquad \frac{\partial w}{\partial z} = 1.$$

$P_0 \to (x, y, z) = (2, 1, 4).$

$$\frac{\partial w}{\partial x} = -4 \qquad \frac{\partial w}{\partial y} = -4 \qquad \frac{\partial w}{\partial z} = 1.$$

Gradiant of F at $(2, 1, 4)$

$$\nabla F(x, y, z) = -4i - 4j + k.$$

equation ~~for~~ plane tangent to surface

$\therefore \cancel{f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + f_z(z - z_0)}$

$$\therefore f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$$

$$-4(x - 2) + (-4)(y - 1) + 1(z - 4) = 0.$$

$$-4x + 8 - 4y + 4 + z - 4 = 0.$$

$$-4x - 4y + z = -8$$

eq. for normal line

$x = x_0 + f_x(P_0)t \qquad y = y_0 + f_y(P_0)t$

$x = 2 + (-4)t) \qquad y = 1 + (-4)t$

$x = 2 - 4t \qquad y = 1 - 4t.$

$z = z_0 + f_z(P_0)t$

$z = 4 + (1)t$

$z = 4t$

Ex.1

Find the tangent plane an normal line of the surface.

$$f(x,y,z) = x^2 + y^2 + z - 9 = 0.$$

a circular paraboloid

at point $P_0(1,2,4)$.

Solution

$$w = f(x,y,z) = x^2 + y^2 + z - 9.$$

$$\frac{\partial w}{\partial x} = 2x \qquad \frac{\partial w}{\partial y} = 2y \qquad \frac{\partial w}{\partial z} = 1.$$

$$P_0(x_0, y_0, z_0) = (1,2,4).$$

$$\frac{\partial w}{\partial x} = 2 \qquad \frac{\partial w}{\partial y} = 4 \qquad \frac{\partial w}{\partial y} = 1.$$

Gradiant of f at P_0.

$$\nabla f\Big|_{P_0} = 2i + 4j + k.$$

Eq. for plane tangent to surface.

$$f_x(P_0)(x-x_0) + f_y(P_0)(y-y_0) + f_z(P_0)(z-z_0) = 0.$$

$$2(x-1) + 4(y-2) + 1(z-4) = 0.$$

$$2x - 2 + 4y - 8 + z - 4 = 0.$$

$$2x + 4y + z = 14$$

Eq. for normal line.

$$x = x_0 + f_x(P_0)t$$

$$x = 1 + 2t.$$

$y = y_0 + f_y(P_0)t$ $\quad$ $z = z_0 + f_z(P_0)t$

$y = 2 + 4t$ $\quad$ $z = 4 + t.$

Ex. 2

Consider the ellipsoid $x^2 + 4y^2 + z^2 = 18$

(a) Find an equation of the tangent plan to the ellipsoid at the point $(1, 2, 1)$.

Solution

$w = f(x,y,z) = x^2 + 4y^2 + z^2 - 18.$

$\frac{\partial w}{\partial x} = 2x$ $\quad$ $\frac{\partial w}{\partial y} = 8y$ $\quad$ $\partial z = 2z$

at P_0 $(1, 2, 1)$.

$\frac{\partial w}{\partial x} = 2(1)$ $\quad$ $\frac{\partial w}{\partial y} = 8(2)$ $\quad$ $\frac{\partial w}{\partial z} = 2(1)$

$\frac{\partial w}{\partial x} = 2$ $\quad$ $\frac{\partial w}{\partial y} = 16$ $\quad$ $\frac{\partial w}{\partial z} = 2.$

Gradient of f at P_0

$\therefore f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$

$2(x - 1) + 16(y - 2) + 2(z - 1) = 0$

$2x - 2 + 16y - 32 + 2z - 2 = 0$

$2x + 16y + 2z = 36$

$\cancel{2}(x + 8y + z) = \cancel{36}^{18} / \cancel{2}$

$x + 8y + z = 18$

(b) Find parametric equation of the line that is normal to the ellipsoid at the point (1, 2, 1).

Solution

Eq. of normal line

$x = x_0 + f_x(P_0)\,t$

$x = 1 + 2t$

$y = y_0 + f_y(P_0)\,t$

$y = 2 + 16t$

$z = z_0 + f_z(P_0)\,t$

$z = 1 + 2t.$

(c) Find the acute angle that the tangent plane at point (1, 2, 1) makes with xy-plane

Solution

Since $n_1 = (2, 16, 2)$

$n_2 = (0, 0, 1).$

$\because \cos\theta = \dfrac{|n_1 \cdot n_2|}{\|n_1\| \times \|n_2\|}.$

$\cos\theta = \dfrac{|(2,16,2)\cdot(0,0,1)|}{\|(2,16,2)\|\ \|(0,0,1)\|}.$

$\cos\theta = \dfrac{2}{2\sqrt{66} \times 1} = \dfrac{1}{\sqrt{66}}$

$\theta = \cos^{-1}(1/\sqrt{66}).$

$\theta = 82.92^\circ \approx 83^\circ.$

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Ex. 1

The Surfaces $f(x,y,z) = x^2 + y^2 - z = 0$. and $g(x,y,z) = x + z - 4 = 0$. meet in ~~the~~ an ellipse E. Find parametric equation for the line tangent to E at point $(1,1,3)$.

Solution

$w = f(x,y,z) = x^2 + y^2 - z$.

$\frac{\partial w}{\partial x} = 2x \qquad \frac{\partial w}{\partial y} = 2y$.

at $P_0(1,1,3)$.

$\frac{\partial w}{\partial x} = 2 \qquad \frac{\partial w}{\partial y} = 2$.

So,

$$\nabla f \Big|_{(1,1,3)} = 2i + 2j.$$

$b = g(x,y,z) = x + z - 4 = 0$.

$\frac{\partial b}{\partial x} = 1 \qquad \frac{\partial b}{\partial z} = 1$.

at $P_0(1,1,3)$.

$\frac{\partial b}{\partial x} = 1 \qquad \frac{\partial b}{\partial z} = 1$.

So

$$\nabla g \Big|_{(1,1,3)} = i + k.$$

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for cross product

$v = \nabla f \times \nabla g$

as tangent line is pallel to it.

$v = \nabla f \times \nabla g$.

$v = (2i + 2j) \times (i + k)$.

$$v = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$v = i(2*1) - j(2*0) + k(0-2)$

$v = \underline{2i} - \underline{2j} - \underline{2k}$.

Eqs of tangent line.

$x = x_0 + (f_x \times g_x)(P_0)t$

$x = 1 + 2t$

$y = y_0 + (f_y \times f_y)(P_0)t$.

$y = 1 - 2t$.

$z = z_0 + (f_z \times f_z)(P_0)t$.

$z = 3 - 2t$.

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Ex. 2

Find Parametric equations of the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point $(1,1,2)$.

Solution

paraboloid $x^2 + y^2 - z = 0$.

$w = f(x,y,z) = x^2 + y^2 - z$

ellipsoid $3x^2 + 2y^2 + z^2 = 9$.

$b = g(x,y,z) = 3x^2 + 2y^2 + z^2 - 9 = 0$.

for w

$\frac{\partial w}{\partial x} = 2x \qquad \frac{\partial w}{\partial y} = 2y \qquad \frac{\partial w}{\partial z} = -1.$

at point $(1,1,2)$.

$\frac{\partial w}{\partial x} = 2 \qquad \frac{\partial w}{\partial y} = 2 \qquad \frac{\partial w}{\partial z} = -1.$

for B

$\frac{\partial b}{\partial x} = 6x \qquad \frac{\partial b}{\partial y} = 4y \qquad \frac{\partial b}{\partial z} = 2z$

at point $(1,1,2)$.

$\frac{\partial b}{\partial x} = 6 \qquad \frac{\partial b}{\partial y} = 4 \qquad \frac{\partial b}{\partial z} = 4.$

now, the gradiants of both functions are

$\nabla f(x,y,z) = 2i + 2j - k$

$\nabla g(x,y,z) = 6i + 4j + 4k.$

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a tangent vector to the curve of intersection will be.

$\nabla f \times \nabla g$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 2 & 2 & -1 \\ 6 & 4 & 4 \end{vmatrix}$$

$= i(8+4) - j(8+6) + k(8-12).$

$= 12i - 14j - 4k.$

Eqs of tangent line.

$x = x_0 + (f_x \times g_x)(P_0)t.$

$x = 1 + 12t.$

$y = y_0 + (f_y \times g_y)(P_0)t$

$y = 1 + (-14)t.$

$y = 1 - 14t.$

$z = z_0 + (f_z \times g_z)(P_0)t.$

$z = 2 + (-4)t$

$z = 2 - 4t.$