ASSIGNMENT No. 1

Date ______ 20__

TOPIC: FUNCTIONS OF TWO OR MORE VARIABLES

QUESTION

Let $f(x,y) = x + \sqrt[3]{xy}$.

(a) Find $f(t, t^2)$.

Solve

$f(t,t^2) = t + \sqrt[3]{(t)(t^2)}$

$f(t,t^2) = t + \sqrt[\cancel{3}]{t^{\cancel{3}}}$

$f(t,t^2) = t + t$

$f(t,t^2) = 2t$

Ans

(b) Find $f(x, x^2)$

Solve

$f(x,x^2) = x + \sqrt[3]{(x)(x^2)}$

$f(x,x^2) = x + \sqrt[\cancel{3}]{x^{\cancel{3}}}$

$f(x,x^2) = x + x$

$f(x,x^2) = 2x$.

Ans

(c) Find $f(2y^2, 4y)$.

Solve

$f(2y^2, 4y) = 2y^2 + \sqrt[3]{(2y^2)(4y)}$

$f(2y^2, 4y) = 2y^2 + \sqrt[3]{8y^3}$

$f(2y^2, 4y) = 2y^2 + \sqrt[3]{8}\ \sqrt[\cancel{3}]{y^{\cancel{3}}}$

$f(2y^2, 4y) = 2y^2 + (2^{\cancel{3}})^{1/\cancel{3}} \cdot y$

$f(2y^2, 4y) = 2y^2 + 2y$

$f(2y^2, 4y) = 2y(y+1)$.

Ans

QUESTION

Find $F(g(x), h(y))$ if $F(x,y) = x e^{xy}$, $g(x) = x^3$ and $h(y) = 3y + 1$.

Solve

$F(x,y) = x e^{xy}$.

$F(g(x), h(y)) = x e^{xy}$.

$g(x) = x^3$, $h(y) = 3y + 1$

$F(x^3, 3y+1) = (x^3) \cdot e^{(x^3)(3y+1)}$.

$F(x^3, 3y+1) = x^3 e^{3x^3 y + x^3}$.

or $F(g(x), h(y)) = x^3 e^{3x^3 y + x^3}$.

Ans

QUESTION

Let $f(x,y) = x + 3x^2 y^2$, $x(t) = t^2$ and $y(t) = t^3$. Find

a) $f(x(t), y(t))$

Solve

$f(x,y) = x + 3x^2 y^2$.

$f(x(t), y(t)) = t^2 + 3(t^2)^2 (t^3)^2$

$f(x(t), y(t)) = t^2 + 3(t^4)(t^6)$

$f(x(t), y(t)) = t^2 + 3t^{10}$.

Ans

b) $f(x(0), y(0))$

Solve

$x(t) = t^2$, $y(t) = t^3$

$x(0) = (0)^2$, $y(0) = (0)^3$

$x(0) = 0$, $y(0) = 0$.

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$f(x(0), y(0)) = (0) + 3(0)^2(0)^2$

$f(x(0), y(0)) = 0.$

Ans

c) $f(x(2), y(2))$

solve

$x(t) = t^2$, $y(t) = t^3$

$x(2) = 2^2$, $y(2) = 2^3$

$x(2) = 4$, $y(2) = 8$

$f(x(2), y(2)) = 4 + 3(4)^2(8)^2$

$f(x(2), y(2)) = 3076.$

Ans

QUESTION

One method for determining relative humidity is to wet the bulb of thermometer, whirl it through the air, and then compare the thermometer reading with the actual air temperature. If the relative humidity is less than 100%, the reading on the thermometer will be less than the temperature of the air. The difference in temperature is known as wet-bulb depression. The accompanying table gives the relative humidity as a function of the air temperature and the wet-bulb depression. Uset the table to complete part (a—c).

Air Temperature (°C)

Wet-Bulb Depression (°C)	15	20	25	30
3	71	74	77	79
4	62	66	70	73
5	53	59	63	67.

(a) What is the relative humidity if the air temperature is 20°C and the wet-bulb temperature thermometer reads 16°C ?

Solve

∵ wet-bulb depression is difference between air temperature and wet-bulb thermometer's temperature.

Wet-bulb Depression = Air temperature − wet-bulb Depr. Temp.

Wet-bulb Depression = 20 − 16

Wet-bulb Depression = 4°C.

Air temperature = 20°C

Wet-bulb depression = 4°C.

According to function table.

Relative humidity = 66%

Ans

(b) Estimate the relative humidity if the air temperature is 25°C and the Wet-bulb depression is 3.5°C.

Solve

Air temperature = 25°C

Wet-bulb depression = 3.5°C.

According to function table

Relative humidity = 73.5% => $77 - \left(\frac{1}{2}\right) \times 7 = 73.5$

(c) Estimate the relative humidity if the air temperature is 22°C and the wet-bulb depression is 5°C.

Solve

Air temperature = 22°C

Wet-bulb depression = 5°C

According to function table

Relative humidity = 60.6% => $59 + \left(\frac{2}{5}\right) \times 4 = 60.6$.

QUESTION

Find $g(u(x,y,z), v(x,y,z), w(x,y,z))$ if $g(x,y,z) = z\sin xy$, $u(x,y,z) = x^2z^3$, $v(x,y,z) = \pi xyz$ and $w(x,y,z) = \frac{xy}{z}$

Solve

$$g(x,y,z) = z\sin(xy)$$

$$g(u(x,y,z), v(x,y,z), w(x,y,z)) = \frac{xy}{z}\sin\left((x^2z^3)(\pi xyz)\right)$$

$$g(u(x,y,z), v(x,y,z), w(x,y,z)) = \frac{xy}{z}\cdot\sin(x^3yz^4\pi)$$

QUESTION

In each part, select the term that best describes the level curves of the function f, choose from the terms lines, circles, non circular ellipses, parabolas or hyperbolas

a) $f(x,y) = x^2 - 2xy + y^2$

Solve

Level curves represents the set of points (x,y) in the plane for which the function f has constant value e.g. $f(x,y)=c$. Therefore, to describe level curves of two variable function, we'll choose a constant value c.

$$f(x,y) = x^2 - 2xy + y^2$$

Let $f(x,y) = c$, where $c = 4$.

$$x^2 - 2xy + y^2 = 4.$$

$$(x-y)^2 = 4 \qquad \because a^2 - 2ab + b^2 = (a-b)^2$$

$$x - y = 2 \qquad \text{square root on both sides}$$

$$x = y + 2.$$

$x = y + 2$

The above equation describes the linear relationship between x and y.
Therefore, term lines decribes the level curves of this function.

b) $f(x,y) = 2x^2 + 2y^2$.

Solve

The standard equation of a circle

$$(x-a)^2 + (y-b)^2 = r^2.$$

$$f(x,y) = 2x^2 + 2y^2.$$

Let $f(x,y) = c$, where $c = 4$.

$$2x^2 + 2y^2 = 4.$$

$$2(x^2 + y^2) = 4$$

$$x^2 + y^2 = 2.$$

comparing above equation with the standard equation of a circle, it satisfies the standard equation of a circle for $a=0$, $b=0$ and $r = \sqrt{2}$.
Therefore, term circles describes the level curves of this function.

d) $f(x,y) = 2y^2 - x$

Solve

The equation of parabola can be represented as.

$$x = a(y-h)^2 + k.$$

$$f(x,y) = 2y^2 - x$$

Let $f(x,y) = c$, where $c = 5$

$$2y^2 - x = 5.$$

$$x = 2y^2 - 5.$$

$$x = 2y^2 - 5$$

comparing above equation with the presented equation of parabola, we can conclude that it satisfies the equation of parabola for $a=2, h=0$ and $k=5$.
Therefore, the term describe the level curves of the given function is parabola.

c) $f(x, y) = x^2 - 2x - y^2$

Solve

The standard equation of a hyperbola is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

we have, $f(x,y) = x^2 - 2x - y^2$

Let $f(x,y) = c$, where $c = 0$

$$x^2 - 2x - y^2 = 0$$
$$x^2 - 2x + 1 - 1 - y^2 = 0$$
$$(x-1)^2 - y^2 - 1 = 0$$
$$(x-1)^2 - y^2 = 0 + 1$$
$$(x-1)^2 - y^2 = 1$$

comparing above equation with the presented equation of hyperbola, we can conclude that it satisfies the equation of hyperbola for $h=1, k=0, a=1$ and $b=1$.
Therefore, the term describe the level curves of the given function is hyperbola.