16 11 22

PARTIAL DERIVATIVES

ideas

1st principle method/rule..

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'_x(x,y) = \lim_{h\to 0} \frac{f(x+h, y) - f(x,y)}{h}$$

$$f'_y(x,y) = \lim_{h\to 0} \frac{f(x, y+h) - f(x,y)}{h}$$

$$f(x) \Rightarrow \frac{d\,f(x)}{dx} \Big/ \frac{dy}{dx} \quad \because f(x) = y.$$

For partial differentiation $\rightarrow$ $f(x,y) \Rightarrow \frac{\partial f(x,y)}{\partial x} \Big/ \frac{\partial z}{\partial x}$ $\quad \because f(x,y) = z$

∂ = dava

Partially diffe.. 1st time wrt x $\Rightarrow \frac{\partial z}{\partial x}$.

Partially diffe.. next time wrt x $\Rightarrow \frac{\partial z^2}{\partial x \partial x}$

partially diffe.. same function second time with respect to another variable $\Rightarrow \frac{\partial z^2}{\partial x \partial y}$

Q: Find $f_x(1,3)$, $f(x,y) = 2x^3y^2 + 2y + 4x$.

$$f(x,y) = 2x^3y^2 + 2y + 4x.$$

now P.D wrt "x."

$$\frac{\partial f(x,y)}{\partial x} = 6x^2y^2 + 0 + 4$$

$$\frac{\partial f(x,y)}{\partial x} = 6x^2y^2 + 4$$

$$\frac{\partial f(1,3)}{\partial x} = 6(1)^2(3)^2 + 4.$$

$$= 54 + 4$$

$$\boxed{\frac{\partial f(1,3)}{\partial x} = 58}$$

f_y

$$\frac{\partial(f(x,y))}{\partial y} = 4x^3y + 2 + 0$$
$$= 4x^3y + 2$$
$$= 4(1)^3(3) + 2$$

$$\boxed{\frac{\partial f(x,y)}{\partial y} = 14}$$

$\boxed{f_{xy}}$ $\because \partial f(x,y)/\partial x = 6x^2y^2 + 4$

$$\frac{\partial f(x,y)}{\partial x\,\partial y} = 12x^2y + 0$$

$$\frac{\partial f(1,3)}{\partial x\,\partial y} = 12(1)^2(3)$$

$$\boxed{\frac{\partial f(1,3)}{\partial x\,\partial y} = 36}$$

Assignment 1
2, 5, 7, 15,
20, 44

f_{yx}

$$\frac{\partial f(x,y)}{\partial y} = 4x^3y + 2$$

$$\frac{\partial f(x,y)}{\partial y\,\partial x} = 12x^2y$$

$$\frac{\partial f(1,3)}{\partial y\,\partial x} = 12(1)^2(3)$$

$$\boxed{\frac{\partial f(1,3)}{\partial y\,\partial x} = 36}$$

y

Ex: 1 :- Find $\partial f/\partial y$ as a function if $f(x,y) = y\sin xy$.

* $y \sin xy \rightarrow$ it repres ent a linear & trigonometric function if we partially diff. it wrt ~~to~~ "y"

$$f(x,y) = \underbrace{y}_{u}\ \underbrace{\sin xy}_{v} \qquad u \cdot v$$

$\because u \cdot v = v\,du + u\,dv$

f_y $$\frac{\partial f}{\partial y} = \sin(xy)(1) + y\cos(xy)(x)$$

$$\frac{\partial f}{\partial y} = \sin(xy) + xy\cos(xy).$$

f_x $$\frac{df}{dx} = y\cos(x \cdot y)(y)$$

$$\frac{df}{dx} = y^2\cos(xy)$$

17/11/22

Q $f(x,y) = 2x^2y^2 + \frac{2}{x^2}y + 3x - y + 2.$

Find $\frac{\partial f}{\partial x}$

$$\frac{\partial f}{\partial x} = 4xy^2 - \frac{4}{x^3}y + 3 - 0 + 0.$$

$$\frac{\partial f}{\partial x} = 4xy^2 - \frac{4}{x^3}y + 3.$$

Find $\frac{\partial f}{\partial y}.$

$$\frac{\partial f}{\partial y} = 4x^2y + \frac{2}{x^2} + 0 - 1 + 0$$

$$\frac{\partial f}{\partial y} = 4x^2y + \frac{2}{x^2} - 1.$$

Find $\cancel{\partial f}$ f_{yx}.

$$\frac{\partial f}{\partial y} = 4x^2y + \frac{2}{x^2} - 1$$

$$\frac{\partial f}{\partial y\, \partial x} = 8xy - \frac{4}{x^3}$$

Ex 0.8: 2

$$f(x,y) = \sin\left(\frac{x}{1+y}\right),$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left\{\sin\left(\frac{x}{1+y}\right)\right\}.$$

$$= \cos\left(\frac{x}{1+y}\right) \cdot \left(\frac{1}{1+y}\right) \cdot (1)$$

$$= \left(\frac{1}{1+y}\right)\cos\left(\frac{x}{1+y}\right).$$

$\because \frac{u}{v} = \frac{v\,du - u\,dv}{v^2}$.

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left\{ \sin\left(\frac{x}{1+y}\right)\right\}.$$

$$= \cos\left(\frac{x}{1+y}\right) \cdot (x) \cdot \left\{\frac{\partial}{\partial y}\left(\frac{1}{1+y}\right)\right\}.$$

$$= -\cos\left(\frac{x}{1+y}\right) \cdot x \cdot (1+y)^{-1}$$

$$= -\cos\left(\frac{x}{1+y}\right) \cdot x\,(1+y)^{-2} \cdot (0+1)$$

$$\frac{\partial f}{\partial y} = -\cos\left(\frac{x}{1+y}\right) \cdot \frac{x}{(1+y)^2}$$

Partial Derivatives viewed as rate of change & slopes

Ex:1. $W = 35.74 + 0.6215T + (0.4275T + 35.75)\,v^{0.16}$

points $(T, v) = (25, 10)$

P.D wrt to v = ?

$$\frac{fw}{fv} = 0 + 0 + 0.16(0.4275T + 35.75)\,v^{0.16-1}$$

$$\frac{fW}{fv} = (0.16)(0.4275T + 35.75)\,v^{-0.84}$$

put $T = 25$ & $v = 10$.

$$\frac{fw}{fv} = (0.16)[\underbrace{0.4275(25) + 35.75}_{46.4375}](10)^{.84}$$

$$\frac{fw}{fv} = \cancel{6.712}. \; -0.5796.$$

$y = x^2 + 2$ $\qquad$ $z = 2x - 3$.

$$\frac{dz}{dx} = \frac{dz}{\boxed{dy}} \cdot \frac{\boxed{dy}}{dx} \rightarrow \text{chain rule}$$

23/11/22

Implicit Partial Derivatives / Differentiation:

Ex: 01

$$x^2+y^2+z^2=1.$$

$$\frac{\partial}{\partial y}\cdot(x^2+y^2+z^2)=\frac{\partial}{\partial y}(1)$$

$$\left(0+2y\left(\frac{\partial y}{\partial y}\right)+2z\cdot\frac{\partial z}{\partial y}\right)=0.$$

$$2y+2z\left(\frac{\partial z}{\partial y}\right)=0.$$

$$\cancel{2}z\cdot\frac{\partial z}{\partial y}=-\cancel{2}y.$$

$$\boxed{\frac{\partial z}{\partial y}=\frac{-\cancel{2}y}{\cancel{2}z}=\frac{-y}{z}}$$

$\because\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$ or $\left(\frac{2}{3},\frac{1}{3},-\frac{2}{3}\right)$.

put both points in answer

$$=\frac{-1/\cancel{3}}{2/\cancel{3}}=\boxed{-1/2}$$

$$=\frac{+1/\cancel{3}}{+2/\cancel{3}}=\boxed{1/2}$$

Ex. 02

$$x^3+y^3+z^3+6xyz=1$$

find $\frac{\partial z}{\partial x}$.

$$f_x=\frac{\partial z}{\partial x}=\frac{\partial}{\partial x}(x^3+y^3+z^3+6xyz)=\frac{\partial(1)}{\partial x}.$$

$$\Rightarrow 3x^2\frac{\partial x}{\partial x}+0+3z^2\frac{\partial z}{\partial x}++6y\frac{\partial}{\partial x}(xz)=0$$

$$\Rightarrow 3x^2+3z^2\frac{\partial z}{\partial x}+6y\left(z\frac{\partial x}{\partial x}+x\frac{\partial z}{\partial x}\right)=0.$$

$$\Rightarrow 3x^2+3z^2\,\partial z/\partial x+6yz\,\cancel{\partial x/\partial x}+6xy\,\partial z/\partial x=0$$

$$\Rightarrow 3z^2\,\partial z/\partial x+6xy\,\partial z/\partial x=-3x^2-6yz.$$

$$\Rightarrow \frac{\partial z}{\partial x}(3z^2+6xy)=-3x^2-6yz$$

$$\Rightarrow \boxed{\frac{\partial z}{\partial x}=\frac{-3x^2-6yz}{3z^2+6xy}}\rightarrow\frac{-\cancel{3}(x^2+2yz)}{\cancel{3}(z^2+2xy)}$$

$$\Rightarrow \boxed{-\frac{x^2+2yz}{z^2+2xy}}$$

find $\frac{\partial z}{\partial y}$

$$f_y = \frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(x^3+y^3+z^3+6xyz\right) = \frac{\partial}{\partial y}(1).$$

$$\Rightarrow 0 + 3y^2 \cdot \frac{\partial y}{\partial y} + 3z^2\frac{\partial z}{\partial y} + 6x\frac{\partial}{\partial y}(yz) = 0$$

$$\Rightarrow 3y^2 + 3z^2\frac{\partial z}{\partial y} + 6x\left(z\frac{\partial y}{\partial y} + y\frac{\partial z}{\partial y}\right) = 0.$$

$$\Rightarrow 3y^2 + 3z^2\frac{\partial z}{\partial y} + 6xz\cancel{\frac{\partial y}{\partial y}} + 6xy\frac{\partial z}{\partial y} = 0.$$

$$\Rightarrow 3z^2\frac{\partial z}{\partial y} + 6xy\frac{\partial z}{\partial y} = -3y^2 - 6xz.$$

$$\Rightarrow \frac{\partial z}{\partial y}\left(3z^2 + 6xy\right) = -3y^2 - 6xz.$$

$$\Rightarrow \boxed{\frac{\partial z}{\partial y} = \frac{-3y^2 - 6xz}{3z^2 + 6xy}}$$

$$\frac{\partial z}{\partial y} = \frac{-\cancel{3}(y^2+2xz)}{3(z^2+6xy)}$$

$$\boxed{\frac{\partial z}{\partial y} = -\frac{y^2+2xz}{z^2+6xy}}$$

P.D & Continuity.

Do it yourself.

Mixed 2^{ND} Order of P.D.

$f_x = \frac{\partial f}{\partial x}$

$f_{xx} = \frac{\partial^2 f}{\partial x^2}$

$f_{yx} = \frac{\partial^2 f}{\partial y \partial x}$

$f_{yxy} = \frac{\partial^3 f}{\partial y \partial x \partial y}$

Ex: 01

$f(x,y) = x\cos y + ye^x$

find second-order derivative:

find $\frac{\partial^2 f}{\partial x \partial y}$

$f_x(x,y) = \frac{\partial}{\partial x}(x\cos y + ye^x)$

$f_x(x,y) = (\cos y (1) \cancel{\frac{\partial x}{\partial x}} + y \cdot e^x)$

$\boxed{f_{xy}(x,y) = -\sin y + e^x}$

find f_{yx} / $\frac{\partial^2 f}{\partial y \partial x}$

$f_y'(x,y) = x(-\sin(y)) + e^x$

$f_y(x,y) = -x\sin(y) + e^x$

$f_{yx}(x,y) = -1 \cdot \sin(y) + e^x$

$\boxed{f_{yx}(x,y) = -\sin y + e^x}$

- Clairaut's Theorem.

- Higher Order P.D.

P.D with more than three variables.

Ex. 2 if $f(\rho, \theta, \phi) = \rho^2 \cos\phi \sin\theta$, then

$f_\rho = 2\rho\cos\phi\sin\theta$

$f_\theta = \rho^2\cos\phi\cos\theta$

$f_\phi = -\rho^2\sin\phi\sin\theta$

Q. if $f(\rho,\theta,\phi) = \rho^2\cos\phi\sin\theta + \sin\theta + \cos\phi$

1. $\partial f/\partial\rho = 2\rho\cos\phi\sin\theta + 0 + 0.$
2. $\partial f/\partial\theta = \rho^2\cos\phi\cos\theta + \cos\theta + 0$
3. $\partial f/\partial\phi = -\rho^2\sin\phi\cos\theta + \sin\theta - \sin\phi$

$\frac{\partial^2 f}{\partial\theta \cdot \partial\phi} = -\rho^2\sin\phi\cos\theta + 0$

$= -\rho^2\cos\theta\sin\phi$

Laplace Formation.

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

if it follow, then it is follows Laplace transform-ation.

Q. $u(x,y) = e^x \sin y$

$$\frac{\partial f}{\partial x} = e^x . \sin y \qquad \frac{\partial f}{\partial y} = e^x \cos y$$

$$\frac{\partial^2 f}{\partial x^2} = e^x \sin y \qquad \frac{\partial^2 f}{\partial y^2} = -e^x \sin y$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

$$e^x \sin y + (-e^x \sin y) = 0.$$

$$e^x \sin y - e^x \sin y = 0$$

$$\boxed{0 = 0}$$

Hence, proved it follows Laplace transformation

Formation of wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

a is any constant
t is time.

24/11/22

Q. verify that the function $u(x,t) = \sin(x - at)$ satisfy the wave equation.

$$\frac{\partial u}{\partial t} = \cos(x-at).(0-a)$$

$$\frac{\partial u}{\partial t} = -a\cos(x-at)$$

$$\frac{\partial^2 u}{\partial t^2} = -a(-\sin(x-at).(0-a))$$

$$\frac{\partial^2 u}{\partial t^2} = -a^2 \sin(x-at)$$

$$\frac{\partial^2 u}{\partial x} = \cos(x-at)\cdot(1-0)$$

$$\frac{\partial u}{\partial x} = \cos(x-at)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x-at)\cdot(1-0).$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x-at).$$

$$\because \frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

$$a^2 \cdot -\sin(x-at) = -\sin(x-at)\ a^2$$

or $$-\sin(x-at)\cdot a^2 = -\sin(x-at)\cdot a^2.$$

Hence, proved it follows wave equation formation