QUESTION 1

a) Solve the following Bernoulli's differential equation using suitable substitution to make it linear.

$$\frac{dy}{dx} + \frac{1}{x} y = y^{-1/3}$$

Solution

$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{x} y = y^{-1/3} \Rightarrow y' + \frac{1}{x} y = y^{-1/3}$

divide the above equation by $y^{-1/3}$.

$\Rightarrow \quad \frac{1}{y^{-1/3}} y' + \frac{1}{x} \frac{y}{y^{-1/3}} = \frac{y^{-1/3}}{y^{-1/3}}$

$\Rightarrow \quad \frac{1}{y^{-1/3}} y' + \frac{1}{x} \frac{1}{y^{-4/3}} = 1. \longrightarrow (1)$

Let $\boxed{u = \frac{1}{y^{-4/3}}} \longrightarrow (a)$

$\Rightarrow \quad u' = \frac{4}{3} \frac{1}{y^{-1/3}} y'$

$\Rightarrow \quad \boxed{\frac{3}{4} u' = \frac{1}{y^{-1/3}} y'} \longrightarrow (b)$

Substitute (a) and (b) in eq (1)

$\Rightarrow \quad \frac{3}{4} u' + \frac{1}{x} u = 1$

$\Rightarrow \quad 3u' + \frac{4}{x} u = 4$

$\Rightarrow \quad u' + \frac{4}{3x} u = \frac{4}{3} \quad — (2)$

Equation (2) in now linear differential equation.

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$\therefore P(x) = 4/3x.$

Finding Integrating Factor $\mu(x)$.

$\because \mu(x) = e^{\int P(x)\,dx}$

$\Rightarrow \mu(x) = e^{\int 4/(3x)\,dx}$

$\Rightarrow \mu(x) = e^{\frac{4}{3}\int \frac{1}{x}\,dx}$

$\Rightarrow \mu(x) = e^{\frac{4}{3}\ln(x)}$

$\Rightarrow \mu(x) = e^{\ln(x^{4/3})}$

$\Rightarrow \boxed{\mu(x) = x^{4/3}}$

Multiply $\mu(x)$ with eq (2)

$\Rightarrow u' \cdot x^{4/3} + \frac{4}{3} \cdot \frac{x^{4/3}}{x} \cdot u = \frac{4}{3} x^{4/3}$

$\Rightarrow u' \cdot x^{4/3} + \frac{4}{3} \cdot x^{1/3} u = \frac{4}{3} x^{4/3}$

Left hand side of the equation is a total differential i.e. it can be enclosed in $d/dx(u \cdot v)$.

$\Rightarrow \frac{d}{dx}(x^{4/3} \cdot u) = \frac{4}{3} \cdot x^{4/3}$

$\Rightarrow d(x^{4/3} \cdot u) = \frac{4}{3} \cdot x^{4/3}\,dx$

Integrating both sides.

$\Rightarrow \int d(x^{4/3} \cdot u) = \int \frac{4}{3} \cdot x^{4/3}\,dx$

$\Rightarrow x^{4/3} \cdot u = \frac{4}{3}\int x^{4/3} \cdot dx.$

$\Rightarrow x^{4/3} \cdot u = \frac{4}{\cancel{3}}\left(\frac{\cancel{3}}{7} \cdot x^{7/3}\right) + c.$

$\Rightarrow x^{4/3} \cdot u = \frac{4}{7} x^{7/3} + c.$

divide the above eq. by $x^{4/3}$

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$$\Rightarrow \quad u = \frac{4}{7} \cdot \frac{x^{7/3}}{x^{4/3}} + \frac{c}{x^{4/3}}$$

$$\Rightarrow \quad u = \frac{4}{7} \cdot x + c \cdot x^{-4/3}$$

Since $u = 1/y^{-1/3} \Rightarrow y^{1/3}$.

$$\Rightarrow \quad y^{1/3} = \frac{4}{7} x + c x^{-4/3}.$$

$$\boxed{y^{1/3} = \frac{4}{7} x + c x^{-4/3}}$$ Answer.

b) Find the general solution of the second order differential equation.

$yy'' = (y')^2$

solution

$$\Rightarrow \quad y y'' = (y')^2. \longrightarrow ①$$

Let's reduce the 2^{nd} O.D.E to 1^{st} O.D.E.

Let $\boxed{v = y'} \longrightarrow ⓐ$

$v' = y'' \Rightarrow \because y'' = \frac{dv}{dx}$ or $y'' = \frac{dv}{dy} \cdot \frac{dy}{dx}$.

$\because y' = dy/dx = v$.

Substitute ⓐ & ⓑ in eq ① $\boxed{y'' = v \cdot \frac{dv}{dy}.} \longrightarrow ⓑ$

$$\Rightarrow \quad y \cdot \cancel{v} \cdot \frac{dv}{dy} = (v)^{\cancel{2}}$$

$$\Rightarrow \quad y \cdot \frac{dv}{dy} = v$$ The equation is now separable.

$$\Rightarrow \quad \frac{1}{v} dv = \frac{1}{y} dy$$

Integrate above equation

$\Rightarrow \int \frac{1}{v}\, dv = \int \frac{1}{y}\, dy$

$\Rightarrow \ln(v) = \ln(y) + c$

$\Rightarrow e^{\cancel{\ln}(v)} = e^{(\ln(y)+c)}$

$\Rightarrow v = e^{\cancel{\ln}(y)} \cdot e^{c}$ $\boxed{e^c = c}$

$\Rightarrow v = y \cdot e^{c}$

$\Rightarrow v = y \cdot c$

Since $v = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = y \cdot c$

$\Rightarrow \frac{1}{y}\, dy = c\, dx$

Integrate above equation.

$\Rightarrow \int \frac{1}{y}\, dy = c \int dx$

$\Rightarrow \ln(y) = xc$

$\Rightarrow e^{\cancel{\ln}(y)} = e^{xc}$

$\Rightarrow \boxed{y = e^{cx}}$

Answer

QUESTION 2

Solve the following second order linear differential equations.

(i) $y'' - 6y' + 9y = xe^{3x}$

Solution

$\Rightarrow y'' - 6y' + 9y = xe^{3x}$

Finding homogeneous solution

$\Rightarrow y'' - 6y' + 9y = 0$

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Auxilary equation for D.E

$\Rightarrow$ $r^2 - 6r + 9 = 0$. $\because \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ $a = 1,\ b = -6,\ c = 9$.

$\Rightarrow$ $\dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$

$\Rightarrow$ $\dfrac{6 \pm \sqrt{36 - 36}}{2} = \dfrac{6 \pm \sqrt{0}}{2} = \dfrac{\cancel{6}^{3}}{\cancel{2}_{1}}$

$\Rightarrow$ $r_1 = r_2 = 3$.

roots are real and repeated.

$\Rightarrow$ $\boxed{y_h = c_1 e^{3x} + c_2 x e^{3x}}$ Homogenous Solution

Finding particular solution.

$\Rightarrow$ $y'' - 6y' + 9y = xe^{3x}$.

Substitute $y'' = D^2_y$ and $y' = Dy$.

$\Rightarrow$ $D^2_y - 6D_y + 9_y = xe^{3x}$.

$\Rightarrow$ $(D^2 - 6D + 9)y = xe^{3x}$.

$\Rightarrow$ $y_p = \dfrac{1}{D^2 - 6D + 9} \cdot xe^{3x}$.

$D \rightarrow D + a$; $a = 3$.

$D \rightarrow D + 3$.

$\Rightarrow$ $y_p = e^{3x} \cdot \dfrac{1}{(D+3)^2 - 6(D+3) + 9} \cdot x$.

$\Rightarrow$ $y_p = e^{3x} \cdot \dfrac{1}{D^2 + \cancel{6D} + \cancel{9} - \cancel{6D} - \cancel{18} + \cancel{9}} \cdot x$

$\Rightarrow$ $y_p = e^{3x} \cdot \dfrac{1}{D^2} \cdot x$.

$\because D = dy/dx$ or $D^2 = d^2y/dx^2$.

then $1/D^2 = \iint$

$\Rightarrow$ $y_p = e^{3x} \cdot \iint x \, dx \cdot dx$.

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$\Rightarrow \quad y_p = e^{3x} \cdot \int \frac{x^2}{2} \, dx$

$\Rightarrow \quad y_p = e^{3x} \cdot \frac{x^3}{6}$

$\Rightarrow \quad \boxed{y_p = \frac{1}{6} x^3 e^{3x}}$ Particular Solution

$\therefore$ General solution : $y = y_h + y_p$

$\Rightarrow \quad \boxed{y = c_1 e^{3x} + c_2 x e^{3x} + \frac{1}{6} x^3 e^{3x}}$ Answer

(ii) $y'' + 9y = \sec(3x)$

Solution

$\Rightarrow \quad y'' + 9y = \sec(3x)$

Finding homogeneous solution.

$\Rightarrow \quad y'' + 9y = 0$

Auxillary equation of the D.E.

$\Rightarrow \quad r^2 + 9 = 0$

$\Rightarrow \quad \sqrt{r^2} = \sqrt{-9}$

$\Rightarrow \quad r = \pm 3i$

roots are complex. $\Rightarrow y_h = e^{\alpha x}[c_1\cos(\beta x) + c_2\sin(\beta x)]$.

$\alpha = 0, \quad \beta = 3$

$\Rightarrow \quad y_h = e^{(0)x}[c_1\cos(3x) + c_2\sin(3x)]$.

$\Rightarrow \quad \boxed{y_h = c_1\cos(3x) + c_2\sin(3x)}$ Homogeneous Solution

Finding particular solution using Wronksian method Variation of Parameters

$\Rightarrow \quad \because y_h = c_1 y_1 + c_2 y_2$.

$\Rightarrow \quad y_1 = \cos(3x), \quad y_2 = \sin(3x)$

$\because \; W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$\Rightarrow \; W(x) = \begin{vmatrix} \cos(3x) & \sin(3x) \\ -3\sin(3x) & 3\cos(3x) \end{vmatrix}$

$\Rightarrow \; W(x) = \cos(3x) \cdot 3\cos(3x) - \sin(3x)[-3\sin(3x)]$.

$\Rightarrow \; W(x) = 3\cos^2(3x) + 3\sin^2(3x)$.

$\Rightarrow \; W(x) = 3[\cos^2(3x) + \sin^2(3x)]$ $\quad \because \cos^2 x + \sin^2 x = 1$

$\Rightarrow \; \boxed{W(x) = 3}$

$$\because \; y_p = -y_1 \int \frac{f(x)\, y_2}{W(x)}\, dx + y_2 \int \frac{f(x)\, y_1}{W(x)}\, dx.$$

$\because \; W(x) = 3 \quad , \quad f(x) = \sec(x) = 1/\cos(x)$.

$$\Rightarrow \; y_p = -\cos(3x) \int \frac{\sin(3x)}{3\cos(3x)}\, dx + \sin(3x) \int \frac{\cancel{\cos(3x)}}{3\,\cancel{\cos(3x)}} \cdot dx$$

$$\Rightarrow \; y_p = -\cos(3x) \cdot \frac{1}{3} \int \tan(3x)\, dx + \sin(x) \cdot \frac{1}{3} \int dx.$$

$$\Rightarrow \; y_p = -\frac{1}{3} \cos(3x) \left(\frac{1}{3} \ln|\sec(3x)| \right) + \frac{1}{3} \sin(x) \cdot x$$

$$\Rightarrow \; \boxed{y_p = -\frac{1}{9} \cos(3x) \cdot \ln|\sec(3x)| + \frac{1}{3} x \cdot \sin(3x)}$$

Particular Solution

General Solution : $y = y_h + y_p$.

$$\Rightarrow \; \boxed{y = c_1 \cos(3x) + c_2 \sin(3x) - \frac{1}{9} \cos(3x) \cdot \ln|\sec(3x)| + \frac{1}{3} x \sin(3x)}$$

Answer.

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QUESTION 3

Find the general solution of give Cauchy-Euler differential equation.

$x^2 y'' + 4xy' + 2y = \sin(\ln x^2) + 2^{\ln x}$

<u>Solution</u>

$\Rightarrow \quad x^2 y'' + 4xy' + 2y = \sin(\ln x^2) + 2^{\ln x}$

Let $x = e^z \Rightarrow z = \ln x$.

then $x^2 y'' = Dy(D-1)y$ and $xy' = Dy$.

$\Rightarrow \quad Dy(D-1)y + 4Dy + 2y = \sin(2z) + 2^z$.

$\Rightarrow \quad D^2y - Dy + 4Dy + 2y = \sin(2z) + 2^z$.

$\Rightarrow \quad D^2y + 3Dy + 2y = \sin(2z) + 2^z$

Finding homogenous solution

$\Rightarrow \quad x^2y'' + 4xy' + 2y = 0$

Auxillary equation for the D.E.

$\Rightarrow \quad r^2 + 3r + 2 = 0. \qquad \because -b \pm \sqrt{b^2 - 4ac} / 2a$

$\Rightarrow \quad \dfrac{-3 \pm \sqrt{(3)^2 - 4(1)(2)}}{2(1)} \Rightarrow \dfrac{-3 \pm \sqrt{9-8}}{2} \Rightarrow \dfrac{-3 \pm 1}{2}.$

$\Rightarrow \quad r_1 = \dfrac{-3+1}{2} = \dfrac{-2}{2}, \quad r_2 = \dfrac{-3-1}{2} = \dfrac{-4}{2}$

$\Rightarrow \quad r_1 = -1, \quad r_2 = -2.$

roots are real and distinct

$\Rightarrow \quad y_h = c_1 e^{r_1 z} + c_2 e^{r_2 z},$

$\Rightarrow \quad y_h = c_1 e^{(-1)\ln x} + c_2 e^{(-2)\ln x}$

$\Rightarrow \quad y_h = \cancel{c_1} \cancel{e}^{\ln x^{-1}} + c_2 \cancel{e}^{\ln x^{-2}}$

$\Rightarrow \quad \boxed{y_h = c_1 x^{-1} + c_2 x^{-2}}$

Homogenous solution

Finding particular solution. $\therefore 2^z \Rightarrow e^{\ln(2^z)}$

$\Rightarrow e^{z \ln 2}$

$\Rightarrow e^{\ln 2 \cdot z}$

$\Rightarrow D^2y + 3Dy + 2y = \sin(2z) + 2^z$

$\Rightarrow (D^2 + 3D + 2)y = \sin(2z) + e^{(\ln 2) z}$

$\Rightarrow y_p = \dfrac{1}{D^2+3D+2} \cdot [\sin(2z) + e^{(\ln 2) \cdot z}]$

$\Rightarrow y_p = \dfrac{1}{D^2+3D+2} \cdot \sin(2z) + \dfrac{1}{D^2-3D+2} \cdot e^{(\ln(2))z}$

$D^2 \Rightarrow -(a^2)$; $a = 2$ $\quad$ $D \to a$; $a = \ln(2)$

$D^2 = -4$ $\quad$ $D = \ln(2)$.

$\Rightarrow y_p = \dfrac{1}{-4+3D+2} \cdot \sin(2z) + \dfrac{1}{\{\ln(2)\}^2 - 3 \cdot \ln(2) + 2} e^{\ln(2) z}$

$\Rightarrow y_p = \dfrac{1}{+3D-2} \cdot \sin(2z) + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2) + 2} e^{\ln(2) z}$

$\Rightarrow y_p = \dfrac{1}{3} \cdot \dfrac{1}{D-2/3} \cdot \sin(2z) + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2)+2} e^{\ln(2) \cdot z}$

$\Rightarrow y_p = \dfrac{1}{3} \cdot \dfrac{1}{D-2/3} \times \dfrac{D+2/3}{D+2/3} \cdot \sin(2z) + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2)+2} e^{\ln(2) \cdot z}$

$\Rightarrow y_p = \dfrac{1}{3} \cdot \dfrac{D+2/3}{D^2 - 4/9} \sin(2z) + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2)+2} e^{\ln(2) \cdot z}$

$\Rightarrow D^2 \to -(a)^2 = -4$

$\Rightarrow y_p = \dfrac{1}{3} \cdot \dfrac{D+2/3}{-4-4/9} \cdot \sin(2z) + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2)+2} e^{\ln(2) \cdot z}$

$\Rightarrow y_p = \dfrac{1}{\cancel{3}1} \cdot \dfrac{D+2/3}{-40/\cancel{9}3} \cdot \sin(2z) + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2)+2} e^{\ln(2) \cdot z}$

$\Rightarrow y_p = -\dfrac{3}{40} \cdot \left(D + \dfrac{2}{3}\right) \cdot \{\sin(2z)\} + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2)+2} e^{\ln(2) \cdot z}$

$\Rightarrow y_p = \dfrac{-3}{40} \left\{ 2\cos(2z) + \dfrac{2}{3} \cdot \sin(2z) \right\} + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2)+2} e^{\ln(2) \cdot z}$

$\Rightarrow y_p = -\dfrac{3}{20} \cos(2z) - \dfrac{1}{20} \sin(2z) + \dfrac{1}{\{\ln(2)\}^2 - 3\ln(2)+2} e^{\ln(2) \cdot z}$

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$\because z = \ln x$ and $2z = \ln x^2$

$\Rightarrow$ $y_p = -\frac{8}{20}\cos(\ln x^2) - \frac{1}{20}\sin(\ln x^2) + \frac{e^{\ln(2)\ln(x)}}{\{\ln(2)\}^2 - 3\ln(2) + 2} \cdot e$

General solution : $y = y_h + y_p$

$$\boxed{y = c_1 x^{-1} + c_2 x^{-2} + -\frac{3}{20}\cos(\ln x^2) - \frac{1}{20}\sin(\ln x^2) + \frac{e^{\ln(2)\cdot\ln(x)}}{\{\ln(2)\}^2 - 3\ln(2) + 2}}$$

Answer

QUESTION 4

Find the general ~~solution~~ power series solution near $x=0$ of the equation

$$y'' - xy' = 0.$$

Solution

$\Rightarrow$ $y'' - xy' = 0 \rightarrow (1)$

according to Special case of Mclaurin series, a function can be expanded near $x=0$ in this way

$\Rightarrow$ $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$

$\Rightarrow$ $y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + \dots (a)$

$\Rightarrow$ $y' = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + 6c_6 x^5 + \dots$

$\Rightarrow$ $y'' = 2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + 30c_6 x^4 + \dots$

Substitute above D.E's in eq (1)

$\Rightarrow$ $(2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + 30c_6 x^4 \dots) - x(c_1 + 2c_2 + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + 6c_6 x^5 + \dots) = 0$

$\Rightarrow$ $(2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + 30c_6 x^4 + \dots) - (c_1 x + 2c_2 x^2 - 3c_3 x^3 - 4c_4 x^4 - 5c_5 x^5 - 6c_6 x^6 + \dots) = 0$

$\Rightarrow$ $2c_2 + (6c_3 - c_1)x + (12c_4 - 2c_2)x^2 + (20c_5 - 3c_3)x^3 + (30c_6 - 4c_4)x^4 = 0 \rightarrow (2)$

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$\Rightarrow$ Computing each term.

$\Rightarrow$ $2c_2 = 0 \Rightarrow \boxed{c_2 = 0}$

$\Rightarrow$ $6c_3 - c_1 = 0 \Rightarrow 6c_3 = c_1 \Rightarrow \boxed{c_3 = \frac{1}{6}c_1}$

$\Rightarrow$ $12c_4 - 2c_2 = 0 \Rightarrow 12c_4 - 2(0) \Rightarrow \boxed{c_4 = 0}$,

$\Rightarrow$ $20c_5 - 3c_3 = 0 \Rightarrow 20c_5 - \cancel{3}\left(\frac{1}{\cancel{6}_2}c_1\right) = 0 \Rightarrow \boxed{c_5 = \frac{1}{40}c_1}$,

$\Rightarrow$ $30c_6 - 4c_4 = 0 \Rightarrow 30c_6 - 4(0) = 0 \Rightarrow \boxed{c_6 = 0}$

Now, substitute all c's values in eq ⓐ'

$\Rightarrow$ $y = c_0 + c_1x + (0)x^2 + \frac{1}{6}c_1x^3 + (0)x^4 + \frac{1}{40}c_1x^5 + (0)x^6 + \cdots$

$\Rightarrow$ $y = c_0 + c_1x + \frac{1}{6}c_1x^3 + \frac{1}{40}c_1x^5 + \cdots$

$\Rightarrow$ $$\boxed{y = c_0 + c_1\left(x + \frac{1}{6}x^3 + \frac{1}{40}x^5 + \cdots\right)}$$

Answer.

QUESTION 5

Reduce the order of given differential equation if one of the solution is $y = x$ and find the general solution.

$$x^2y'' - x(x+2)y' + (x+2)y = 0.$$

Solution

$\Rightarrow$ $x^2y'' - x(x+2)y' + (x+2)y = 0 \longrightarrow ①$.

it is given that $y_1 = x$.

Let $y_1 \neq u(x)\, y_2$ or $\frac{y_1}{y_2} \neq u(x)$.

Since one solution is known then.

$\Rightarrow$ $y_2 = y_1 \cdot u$.

Formula:

$\Rightarrow$ $$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{(y_1)^2} \cdot dx.$$

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divide eq ① by x^2.

$$\Rightarrow \frac{\cancel{x^2} y''}{\cancel{x^2}} - \frac{x(x+2)y'}{x^2} + \frac{(x+2)}{x^2} y = 0.$$

$$\Rightarrow y'' - \frac{x+2}{x} y' + \frac{x+2}{x^2} y = 0.$$

$$P(x) = -\frac{x+2}{x}, \quad y_1 = x.$$

Substitite in formula

$$\Rightarrow y_2 = x \int \frac{e^{-\int -\frac{x+2}{x} dx}}{(x)^2} dx$$

$$\Rightarrow y_2 = x \int \frac{e^{\int 1 + \frac{2}{x} dx}}{(x)^2} dx.$$

$$\Rightarrow y_2 = x \int \frac{e^{x + 2\ln x}}{x^2} dx.$$

$$\Rightarrow y_2 = x \int \frac{e^x \cdot \cancel{e}^{\cancel{\ln} x^2}}{x^2} dx.$$

$$\Rightarrow y_2 = x \int \frac{e^x \cdot \cancel{x^2}}{\cancel{x^2}} dx$$

$$\Rightarrow y_2 = x \int e^x dx.$$

$$\Rightarrow \boxed{y_2 = x \cdot e^x + c.}$$

$$\therefore y = c_1 y_1 + c_2 y_2.$$

$$\boxed{y = c_1 x + c_2 x e^x}$$

Answer

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$\because z = \ln x$ and $2z = \ln x^2$

$\Rightarrow y_p = -\frac{8}{20}\cos(\ln x^2) - \frac{1}{20}\sin(\ln x^2) + \frac{e^{\ln(2)\cdot\ln(x)}}{\{\ln(2)\}^2 + 3\ln(2) + 2}$

General solution : $y = y_h + y_p$

$\because e^{\ln(2)\cdot\ln(x)} = 2^{\ln(x)}$

$$\boxed{y = c_1 x^{-1} + c_2 x^{-2} - \frac{3}{20}\cos(\ln x^2) - \frac{1}{20}\sin(\ln x^2) + \frac{2^{\ln(x)}}{\{\ln(2)\}^2 + 3\ln(2) + 2}}$$

Answer

QUESTION 4

Find the general ~~solution~~ power series solution near $x=0$ of the equation

$$y'' - xy' = 0.$$

Solution

$\Rightarrow y'' - xy' = 0 \rightarrow (1)$

according to Special case of Mclaurin series, a function can be expanded near $x=0$ in this way

$\Rightarrow f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$

$\Rightarrow y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + \cdots (a)$

$\Rightarrow y' = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + 6c_6 x^5 + \cdots$

$\Rightarrow y'' = 2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + 30c_6 x^4 + \cdots$

Substitute above D.E's in eq (1)

$\Rightarrow (2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + 30c_6 x^4 \cdots) - x(c_1 + 2c_2 + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + 6c_6 x^5 + \cdots) = 0$

$\Rightarrow (2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + 30c_6 x^4 + \cdots) - (c_1 x + 2c_2 x^2 - 3c_3 x^3 - 4c_4 x^4 - 5c_5 x^5 - 6c_6 x^6 + \cdots) = 0$

$\Rightarrow 2c_2 + (6c_3 - c_1)x + (12c_4 - 2c_2)x^2 + (20c_5 - 3c_3)x^3 + (30c_6 - 4c_4)x^4 = 0 \rightarrow (2)$