Assignment No. 02

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Discrete Structures

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Topic: Counting Techniques

Question # 01 (a)

Find the number of different ways that the 13 letters in the word ACCOMMODATION can be arranged in a line if all the vowels (A, I, O) are next to each other.

Solution

ACCOMMODATION

Total characters = 13

Total vowels = 6.

According to condition, all vowels should be next to each other, so combining all 6 vowels from total characters as an object, we have

Total = 8

No. of ways to arrange 7 character and all vowels next to each other (in an object)

$\Rightarrow 8!$ ways.

As we know that all vowels in an object can arrange themselves as well

$\Rightarrow 6!$ ways

So, it'll count as

$\Rightarrow 8! * 6!$

There are characters repeating, A two times, C two times, O 3 times and M two times. There counts will be divided by total count, so they'll not be counted unnecessarily.

$$\Rightarrow \frac{8! * 6!}{2! * 2! \times 2! * 3!}$$

$\Rightarrow 604800$

There are 604800 possible ways that the 13 letters in the word ACCOMMODATION can be arranged in a line if all vowels are next to each other.

(b)

There are 7 Chineese, 6 European and 4 Americans students at an international conference. Four of the students are to be choosen to take part in a television broadcast. Find the number of different ways the students can be chosen if at least one chineese and at least one European students are included.

Solution

Since, there is no specific order to arrange all students, so this is problem of combination.

Choosing one chinese, one european and two american students from

7 Chinese	6 European	4 Americans	
1	1	2	= 4

now, for all possible combinations.

2	1	1	= 4
1	2	1	= 4
3	1	0	= 4
1	3	0	= 4
2	2	0	= 4

4/12/22

These are all the ways to choose four students acc. to the condition.

∵ nC_r, where n is the total objects and r is objects to be chosen

For each combination, we multiply the count.

$\Rightarrow 7C_1 \times 6C_1 \times 4C_2 = 252$

$\Rightarrow 7C_2 \times 6C_1 \times 4C_1 = 504$

$\Rightarrow 7C_1 \times 6C_2 \times 4C_1 = 420$

$\Rightarrow 7C_3 \times 6C_1 \times 4C_0 = 210$

$\Rightarrow 7C_1 \times 6C_3 \times 4C_0 = 140$

$\Rightarrow 7C_2 \times 6C_2 \times 4C_0 = 315$.

Now, adding ways for each combination.

$\Rightarrow 252 + 504 + 420 + 210 + 140 + 315$.

$\Rightarrow 1841$

There are 1841 possible ways to chose four students if atleast one chinese and one european student must be included.

QUESTION # 02 (a)

(i) Find how many numbers there are between 100 and 999 in which all three digits are different?

Solution

H	T	U
9 choice	9 choices	8 choices

Arrangement will be in a way that no digit would be repeated

⇒ Starting from hundred's place, making a 3 digit number we can't have 0 and we have 0–9 numbers to arrange, so it'll be 9 ways out of 10 to arrange digits on hundred's place.

⇒ On tens place, we can have 0 and we've chosen a digit for hundred's, so for ten it'll be 9

4/12/22

=> For units place, since 2 numbers are already chosen for hundered's and tens place, we have 8 choices

Multiplying all the choices.

=> 9 x 9 x 8

=> 648.

There are 648 three digit numbers between 100 to 999 without repeating any digit.

(ii)

Find how many of the numbers in part (i) are odd numbers greater then 700.

Solution

We have 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

We have to make a threed digit number greater than 700 and odd without repeating any digit.

H	T	U

if, odd numbers from 0 → 9 are fixed on units place, the number will be odd always and these are 1, 3, 5, 7, 9 → total 5.

There will be two counting solutions,

=> ∵ number will be greater than 700, so on hundereds place, we have choices for 7, 8 and 9, if we arrange 7 or 9 on hundereds place, so on units place, there are 4 odd number choices left and on tens place, there can be any number from 0 to 9 except those of hundereds and units place, hence

H	T	U
2 choices	8 choices	4 choices

multiplying this count

$2 \times 8 \times 4 = 64$

4/12/22

2) Now on hundreds place number 8 can be placed, so only one choice here. On unit place, all odd numbers from 0 to 9 can be placed, so 5 choices and f tens place, other than 2 numbers chosen for hundreds and units place, 8 choices left, hence.

H	T	U
1ch.	8ch.	5ch.

multiplying this count

$1 \times 8 \times 5 = 40$

Adding both counts.

$64 + 40 = 104$.

There are 104 different numbes of three digits which can be made greater than 700 and odd.

(b)

A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be atleast 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers.

Solution

We have to count number of bunches of in a way that every bunch contain at least two of each type of flowers making 7 flowers per bunch.

Different ways would be

6 Roses	5 Tulips	4 Daffodils	
2	2	3	= 7
2	3	2	= 7
3	2	2	= 7.

PAPERWORK

4/12/22

These are the ways to make bunches.
Since we don't have any specific order to arrange the flowers, ~~its~~ so it's a problem of combination

∵ nC_r => where n are total objects &
r are object to be choosen.

$$nC_r = \frac{n!}{(n-r)! * r!}$$

for each combination of every possible way, we multiply them

$$\Rightarrow 6C_2 \times 5C_2 \times 4C_3 = 600$$
$$\Rightarrow 6C_2 \times 5C_3 \times 4C_2 = 900$$
$$\Rightarrow 6C_3 \times 5C_2 \times 4C_2 = 1200$$

Now, Adding ~~every~~ all counts of all combinations.

$$\Rightarrow 600 + 900 + 1200$$
$$\Rightarrow 2700$$

There are 2700 bunches which can be made containg atleast 2 of each type of flowers.

QUESTION # 03 (a).

Find the number of different arrangements which can be made of all 10 letters of the word WALLFLOWER if.

(i) there are no restictions.

W A L L F L O W E R

These 10 letters can arrange themselves in 10! ways

$$\Rightarrow 10!$$

Butt, letter W and L repeating themselves 2 and 3 times respectively, so it is unnecessary to count them so divide it

$$\Rightarrow \frac{10!}{2! \times 3!}$$

4 /12/ 22

$\Rightarrow 302400$

There are 302400 ways to arrange letters in WALLFLOWER without any restictions.

(ii) there are exactly six letters between two W's.

W A L L F L O W E R

There are only 3 ways to arrange six letters between two W's because.

① W __ __ __ __ __ __ W __ __

② __ W __ __ __ __ __ __ W __

③ __ __ W __ __ __ __ __ __ W

If W is fixed now, so we have 8 letters left which can be arranged in 8! ways. But there is letter L repeating 3 times so it'll be divided.

$$\Rightarrow \frac{8!}{3!}$$

for every two W count, 8!/3! are counted, so

$$\Rightarrow \left(\frac{8!}{3!}\right) \times 3$$

$\Rightarrow 20160.$

There are total 20160 arrangements, so that six letters will be between two W's.

4/12/22

(b)

A team of 6 people is to be chosen from 5 swimmers, 7 athletes and 4 cyclists. There must be atleast 1 from each actiity and there must be more athletes than cyclists. Find the number of different ways in which the team can be made/chosen.

Solution

We have to make team of 6 people which must include at'least one of swimmer, cyslist and athletes and athletest should be more than cyslist

We, have different ways to choose.

5 Swimmers	7 Athletes	4 cyclists.	
1	3	2	= 6
3	2	1	= 6
2	3	1	= 6
1	4	1	= 6

These are the teams to made

Since, we don't have any order to arrange teams members, that is why this is combination problem.

$$\therefore \; nCr = \frac{n!}{(n-r)! * r!}$$

For each combination of every possible count, it'll be multiplied.

$\Rightarrow 5C_1 \times 7C_3 \times 4C_2 = 1050$

$\Rightarrow 5C_3 \times 7C_2 \times 4C_1 = 840$

$\Rightarrow 5C_2 \times 7C_3 \times 4C_1 = 1400$

$\Rightarrow 5C_2 \times 7C_4 \times 4C_1 = 700$

Adding all counts of each combination.

$\Rightarrow 1050 + 840 + 1400 + 700$

$\Rightarrow 3990$.

There are 3990 way to make different teams acc. to the condition given.

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