Date ______ 20__

NAME: SAAD NISAR BUTT

REG. NO.: CS211246

CLASS: BSCS-4C

COURSE LINEAR ALGEBRA

COURSE INSTRUCTOR:
DR. ARIF HUSSAIN

ASSIGNMENT NO. 3

Teacher's Signature ______ RAZA Paper Products Page No. ____

QUESTION No. 1

Find eigen-values and vectors using following matrices.

$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

$Ax = \lambda x$
$Ax = \lambda I x$
$Ax - \lambda I x = 0$
$(A - \lambda I)x = 0$
$Bx = 0$
$|B| = 0$
↳ characteristic equation

Solution

Finding eigen values:
According to the characteristic equation:

$|B| = 0 \Rightarrow |A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3 - \lambda & 0 \\ 8 & -1 - \lambda \end{vmatrix} = 0$$

$\Rightarrow \{(3-\lambda)(-1-\lambda)\} - (0)(8) = 0$

$\Rightarrow -3 - 3\lambda + \lambda + \lambda^2 - 0 = 0$

$\Rightarrow -3 - 2\lambda + \lambda^2 = 0$

$\Rightarrow \lambda^2 - 2\lambda - 3 = 0$

$\Rightarrow \lambda^2 - 3\lambda + \lambda - 3 = 0$

$\Rightarrow \lambda(\lambda - 3) + 1(\lambda - 3) = 0$

$\Rightarrow (\lambda + 1)(\lambda - 3) = 0$

$\Rightarrow \lambda + 1 = 0 \quad , \quad \lambda - 3 = 0$

$\Rightarrow \lambda = -1 \quad , \quad \lambda = -3$

Eigen values $\lambda_1 = -1$, $\lambda_2 = -3$.

Now, to find eigen vectors:

$\because Ax = \lambda x$

$Ax = \lambda I x$

$Ax - \lambda I x = 0$

or $(Ax - \lambda I)x = 0$

for $\lambda_1 = -1$

$(A - \lambda I)x = 0$

$\Rightarrow \left(\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - (-1)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\Rightarrow \left(\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 4 & 0 \\ 8 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 4x_1 + 0 = 0. \Rightarrow 4x_1 = 0 \Rightarrow x_1 = 0$

$8x_1 + 0 = 0 \Rightarrow 8x_1 = 0 \Rightarrow x_1 = 0$.

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

at origin.

for $\lambda = 3$.

$(A - \lambda I)x = 0$.

$\Rightarrow \left(\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - (3)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \left(\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 0 \\ 8 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow$ $0x_1 + 0x_2 = 0 \Rightarrow 0 = 0$

$8x_1 - 4x_2 = 0 \Rightarrow 8x_1 = 4x_2$

$\Rightarrow$ $8x_1 = 4x_2$

if $x_1 = 4$ and $x_2 = 8$

then equation will be satisfied.

$x_1 = 4$ or 1

$x_2 = 8$ or 2.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Original Matrix $\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$.

Eigen vectors. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Eigen values $(-1, 3)$.

2. $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

<u>Solution</u>

Finding eigen values.

According to characteristic equation.

$|B| = 0 \Rightarrow |A - \lambda I| = 0$.

$$\Rightarrow \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0.$$

$$\Rightarrow \begin{vmatrix} 1 - \lambda & 2 \\ 3 & 2 - \lambda \end{vmatrix} = 0.$$

$\Rightarrow \{(1-\lambda)(2-\lambda)\} - (2)(3) = 0.$

$\Rightarrow 2 - \lambda - 2\lambda + \lambda^2 - 6 = 0.$

$\Rightarrow -4 - 3\lambda + \lambda^2 = 0$

P.T.O

Date ____ 20__

$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$

$\Rightarrow \lambda^2 - 4\lambda + \lambda - 4 = 0$

$\Rightarrow \lambda(\lambda - 4) + 1(\lambda - 4) = 0$

$\Rightarrow (\lambda + 1)(\lambda - 4) = 0$

$\Rightarrow \lambda + 1 = 0 \quad , \quad \lambda - 4 = 0$

$\Rightarrow \lambda = -1 \quad , \quad \lambda = 4$

Eigen values: $\lambda_1 = -1$, $\lambda_2 = 4$.

Now, to find eigen vectors.

$\because Ax = \lambda x$.

and $(A - \lambda I)x = 0$.

for $\lambda = -1$.

$\Rightarrow (A + I)x = 0$.

$\Rightarrow \left(\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x_1 + 2x_2 = 0$

$3x_1 + 3x_2 = 0$.

$2x_1 + 2x_2 = 0$

$2(x_1 + x_2) = 0$.

$x_1 + x_2 = 0$.

$x_1 = -x_2$

$3x_1 + 3x_2 = 0$.

$3(x_1 + x_2) = 0$.

$x_1 + x_2 = 0$.

$x_1 = -x_2$.

it implies if $x_1 = t$ then $x_2 = -t$.

here t is a parameter.

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ -t \end{bmatrix}$. or the simplest form $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

for $\lambda = 4$.

$\Rightarrow (A - 4I)x = 0$.

$\Rightarrow \left(\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\Rightarrow \left(\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \right) \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\Rightarrow -3x_1 + 2x_2 = 0. \Rightarrow 3x_1 = 2x_2$

$3x_1 - 2x_2 = 0. \Rightarrow 3x_1 = 2x_2.$

here, if $x_1 = 2$ and $x_2 = 3$, both sides will be equal and the values will be satisfied.

hence, $x_1 = 2$, $x_2 = 3$.

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Original Matrix. $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

Eigen vectors $\begin{bmatrix} t \\ -t \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Eigen values $(-1, 4)$.

$$\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$$

Solution

Finding eigen value.

According to the characteristic equation

$|B| = 0 \Rightarrow |A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 5-\lambda & -1 \\ 1 & 3-\lambda \end{vmatrix} = 0.$

$\Rightarrow \{(5-\lambda)(3-\lambda)\} - (-1)(1) = 0.$

$\Rightarrow 15 - 5\lambda - 3\lambda + \lambda^2 + 1 = 0.$

$\Rightarrow 16 - 8\lambda + \lambda^2 = 0.$

$\Rightarrow \lambda^2 - 8\lambda + 16 = 0.$

$\Rightarrow \lambda^2 - 4\lambda - 4\lambda + 16 = 0.$

$\Rightarrow \lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$\Rightarrow (\lambda - 4)(\lambda - 4) = 0.$

$\Rightarrow \lambda - 4 = 0 \quad , \quad \lambda - 4 = 0.$

$\Rightarrow \lambda = 4 \quad , \quad \lambda = 4$

or $\lambda = 4.$

Eigen value: $\lambda = 4.$

Now to find eigen vectors.

$\because Ax = \lambda x.$

and $(A - \lambda I)x = 0$

for $\lambda = 4.$

$\Rightarrow (A - 4I)x = 0$

$\Rightarrow \left(\begin{bmatrix} 5 & -1 \\ +1 & 3 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \left(\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow x_1 - x_2 = 0$

$x_1 - x_2 = 0$

or

$x_1 = x_2.$

$x_1 = t$ or $x_2 = t$; t is a paramete.

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\Rightarrow$ in its simplest form.

Original Matrix. $\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$

Eigen Vectors $\begin{bmatrix} t \\ t \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Eigen values (4).

Date ______ 20__

QUESTION No. 2

Find the inverse of matrix using row operations.

$$A^{-1} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -2/3 \\ 1 & 0 & 1/3 \end{bmatrix}.$$

Solution

We'll find it's inverse using matrix operation
The matrix will be.

$$\Rightarrow \left[\begin{array}{ccc|ccc} -3 & 2 & 0 & 1 & 0 & 0 \\ -4 & 1 & -2/3 & 0 & 1 & 0 \\ 1 & 0 & 1/3 & 0 & 0 & 1 \end{array}\right] \quad (-1/3)R_1 \rightarrow R_1$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -2/3 & 0 & -1/3 & 0 & 0 \\ -4 & 1 & -2/3 & 0 & 1 & 0 \\ 1 & 0 & 1/3 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} (4)R_1 + R_2 \rightarrow R_2 \\ (-1)R_1 + R_3 \rightarrow R_3 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -2/3 & 0 & -1/3 & 0 & 0 \\ 0 & -5/3 & -2/3 & -4/3 & 1 & 0 \\ 0 & 2/3 & 1/3 & 1/3 & 0 & 1 \end{array}\right] \quad (-3/5)R_2 \rightarrow R_2$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -2/3 & 0 & -1/3 & 0 & 0 \\ 0 & 1 & 2/5 & 4/5 & -3/5 & 0 \\ 0 & 2/3 & 1/3 & 1/3 & 0 & 1 \end{array}\right] \quad \begin{array}{l} (-2/3)R_2 + R_3 \rightarrow R_3 \\ (2/3)R_2 + R_1 \rightarrow R_1 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 4/15 & 1/5 & -2/5 & 0 \\ 0 & 1 & 2/5 & 4/5 & -3/5 & 0 \\ 0 & 0 & 1/15 & -1/5 & 2/5 & 1 \end{array}\right] \quad (15)R_3 \rightarrow R_3$$

Date ______ 20____

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 4/15 & 1/15 & -2/15 & 0 \\ 0 & 1 & 2/5 & 4/5 & -3/5 & 0 \\ 0 & 0 & 1 & -3 & 6 & 15 \end{array}\right] \begin{array}{l} (-2/5)R_3 + R_2 \to R_2 \\ (-4/15)R_3 + R_1 \to R_1 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -4 \\ 0 & 1 & 0 & 2 & -3 & -6 \\ 0 & 0 & 1 & -3 & 6 & 15 \end{array}\right]$$

The right side is the inverse of A^{-1}
ie. $(A^{-1})^{-1}$
and it is equals to $A \Rightarrow$ the original matrix
we've concluded that $(A^{-1})^{-1} = A$.