Deeper Insight into the iOS App

Edward A. Silkworth
Teachers College, Columbia University
eas2156@tc.columbia.edu

Abstract

This document consists of more context, a more in-depth overview, starting terminology and a summary of formulae.

1 Context

Assumed:

- students have already entered repayment
- any interest that would be capitalized has already been capitalized

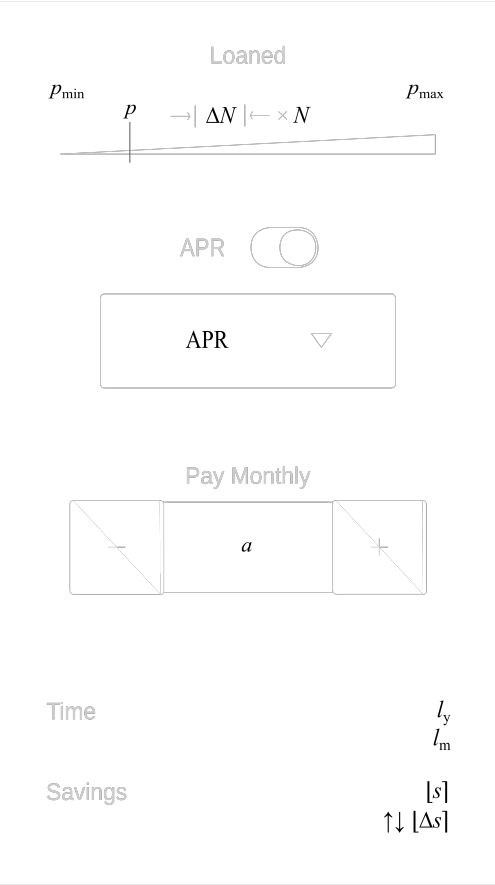
Disregarded:

- loan origination fees
- variable interest rates
- variable payments
- multiple loans
- true number of days in each month
- prepayment penalties
- defaulting on loans
- loan consolidation

Caution:

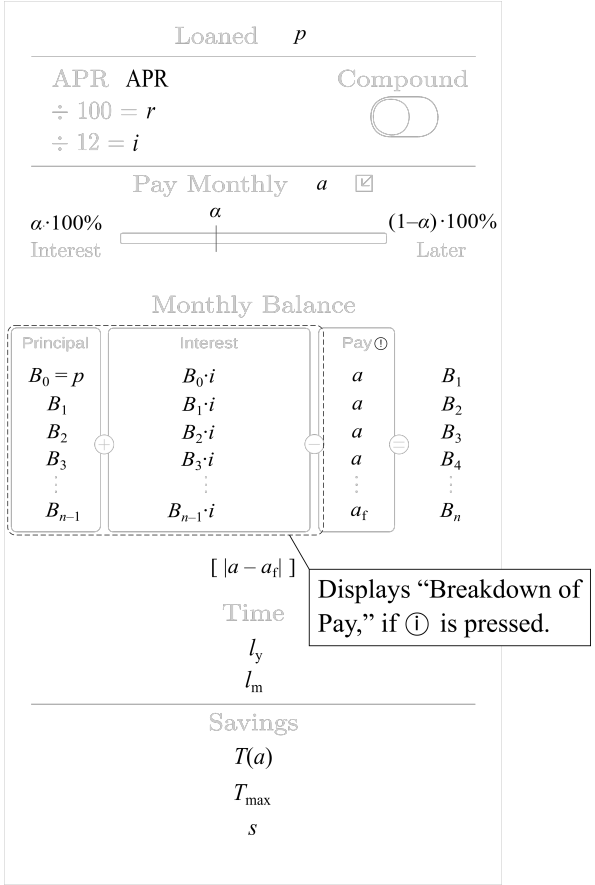
- Loan servicers may require a higher, minimum monthly payment

Main Screen



Not displayed: increment timers

Mathematics Screen



Caution: a and α are different quantities.
“$[\, |a - a_f| \,]$” means that the difference may be displayed.

3 Terminology

Principal/initial balance ($):

$$\{\, p \mid p \in \mathbb{W} \,\}$$

Total increments (increments):[1]

$$\{\, N \mid N \in \mathbb{N} \,\}$$

Annual interest rate (/year):

$$\left\{ r \;\middle|\; r \in \frac{\mathbb{W}}{10{,}000} \right\}$$

Monthly interest rate (/month):

$$\{\, i \mid i \in \mathbb{R}^{+} \cup \{0\} \,\}$$

Monthly principal balance ($):

$$\left\{ B \;\middle|\; B \in \frac{\mathbb{W}}{100}, B \leq p \right\}$$

Number of months (months):

$$\{\, m \mid m \in \mathbb{W} \,\}$$

Monthly payment amount ($):[2]

$$\left\{ a \;\middle|\; a \in \frac{\mathbb{N}}{100}, a \geq a_{\min} \right\}$$

Proportion of interest that is paid (part):

$$\left\{ \alpha \;\middle|\; \alpha \in \frac{\mathbb{W}}{100}, \alpha \leq 1 \right\}$$

Monthly outstanding interest ($):

$$\left\{ O \;\middle|\; O \in \frac{\mathbb{W}}{100} \right\}$$

Length of repayment (years):

$$\{\, l_{\mathrm{y}} \mid l_{\mathrm{y}} \in \mathbb{W} \,\}$$

Length of repayment (months):

$$\{\, l_{\mathrm{m}} \mid l_{\mathrm{m}} \in \mathbb{N} \,\}$$

Total payments ($):

$$\left\{ T \;\middle|\; T \in \frac{\mathbb{N}}{100} \right\}$$

Savings ($):

$$\left\{ s \;\middle|\; s \in \frac{\mathbb{W}}{100} \right\}$$

[1] The researcher opted to capitalize variables that primarily involve totals.

[2] See section 4 for more information on $a_{\min}$, more specifically $a_{\min_n}$ and $a_{\min_{120}}$.

4 Formulae

Increment size ($/increment):[3]

$$\Delta N = \frac{p_{\max} - p_{\min}}{N}, \text{ for } N \neq 0$$

Annual interest rate (/year):[4]

$$r = \text{APR}(\%) \div 100$$

Monthly interest rate (/month):[5]

$$i = \frac{r}{12} \qquad \text{if interest is } \textit{not} \text{ compounded (default)}$$

$$i \approx \left(1 + \frac{r}{365.25}\right)^{\frac{365.25}{12}} - 1 \qquad \text{if interest is compounded}$$

Monthly principal balance ($):

$$B_m = B_{m-1} - \left[a - \alpha\left(B_{m-1} \cdot i\right)\right]$$

$$\text{interest paid}$$

$$B_m = B_{m-1} - \left[a - \alpha\left(B_{m-1} \cdot i\right)\right]$$

Monthly outstanding interest ($):

$$O_m = O_{m-1} + \left(B_{m-1} \cdot i\right) - \alpha\left(B_{m-1} \cdot i\right)$$

$$\text{interest owed}$$

$$O_m = O_{m-1} + \left(B_{m-1} \cdot i\right) - \alpha\left(B_{m-1} \cdot i\right)$$

Remark 4.1. "Interest paid" and "interest owed" require an adjustment, however. Each of their terms may span more than two decimal places, and cents span only two, so round each quantity to the nearest two decimal places.

Monthly principal balance (adjusted):

$$B_m = B_{m-1} - \left\{a - \left\lfloor \alpha\left(B_{m-1} \cdot i\right) \times\ 100 \right\rceil \div 100\right\}$$

Monthly outstanding interest (adjusted):

$$O_m = O_{m-1} + \left\lfloor \left(B_{m-1} \cdot i\right) \times 100 \right\rceil \div 100 - \left\lfloor \alpha\left(B_{m-1} \cdot i\right) \times 100 \right\rceil \div 100$$

Remark 4.2. For rounding $\alpha\left(B_{m-1} \cdot i\right)$, why not round $B_{m-1} \cdot i$ and then $\alpha\left(B_{m-1} \cdot i\right)$? This is because doing so would make computations less precise than by rounding once.

[3] See section 1 of "Extra Insight into the iOS App" for examples of all formulae.

[4] Technically, $r = \text{APR}$, but some may not grasp the relationship between percents and decimals.

[5] See section 2 of "Extra Insight into the iOS App" for more information on the compound interest rate.

Basic algorithm for computing the principal balance and outstanding interest:

$$
\begin{array}{ll}
\texttt{set} & B_0 = p\ ; \\
& O_0 = 0\ ; \\
& m = 1
\end{array}
$$

$$
\begin{array}{ll}
\texttt{while} & B_{m-1} - \left\{ a - \left\lfloor \alpha\left(B_{m-1}\cdot i\right) \times 100 \right\rceil \div 100 \right\} > 0 \quad \texttt{do} \\
\quad & B_m = B_{m-1} - \left\{ a - \left\lfloor \alpha\left(B_{m-1}\cdot i\right) \times 100 \right\rceil \div 100 \right\}\ ; \\
\quad & O_m = O_{m-1} + \left\lfloor \left(B_{m-1}\cdot i\right) \times 100 \right\rceil \div 100 - \left\lfloor \alpha\left(B_{m-1}\cdot i\right) \times 100 \right\rceil \div 100\ ; \\
\quad & m = m+1
\end{array}
$$

$$
\begin{array}{ll}
\texttt{set} & n = m \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{paid now} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad | \\
\texttt{set} & a_{\mathrm{f}} = B_{n-1} + \left\lfloor \left(B_{n-1}\cdot i\right) \times 100 \right\rceil \div 100 \ + \ O_{n-1}\ ; \\
& B_n = B_{n-1} - \left\{ a_{\mathrm{f}} - \left\lfloor \left(B_{n-1}\cdot i\right) \times 100 \right\rceil \div 100 - O_{n-1} \right\} \quad (=0) \\
& (O_n = 0)
\end{array}
$$

Remark 4.3. Making outstanding interest due the final month is arbitrary. However, by doing so students may see the impact of interest on loans more clearly. Not only may final month's payment be higher, but also it may break equation 4.1—as in *break the bank*.

Remark 4.4. If one is programming the `while` loop in Swift, round B_m and O_m to the nearest two decimal places after each iteration of the loop. Otherwise, the values of B_m and O_m may drift by one cent or more over time. Simultaneously, inform Swift to round a quantity upward two places, if it has a hundredth remainder between, say, 0.499999 and 0.5 exclusive; otherwise, Swift will round downward. The researcher checked and rounded proportion of interest that is paid, interest owed, interest paid, B_m, O_m and the absolute minimum monthly payment. He checked and rounded other quantities, as well, but doing so was not as crucial. Checking and rounding was performed succinctly with the custom function `CR()`.

Monthly Balance,

$$
\begin{array}{rcrcrcl}
B_0 & + & \left\lfloor \left(B_0\cdot i\right) \times 100 \right\rceil \div 100 & - & a & = & B_1 \\
B_1 & + & \left\lfloor \left(B_1\cdot i\right) \times 100 \right\rceil \div 100 & - & a & = & B_2 \\
B_2 & + & \left\lfloor \left(B_2\cdot i\right) \times 100 \right\rceil \div 100 & - & a & = & B_3 \\
B_3 & + & \left\lfloor \left(B_3\cdot i\right) \times 100 \right\rceil \div 100 & - & a & = & B_4 \\
\vdots & & \vdots & & \vdots & & \vdots \\
B_{n-1} & + & \left\lfloor \left(B_{n-1}\cdot i\right) \times 100 \right\rceil \div 100 & - & a_{\mathrm{f}} & = & B_n
\end{array}
\tag{4.1}
$$

Breakdown of Pay,

$$\begin{array}{rcrcl}
a - \left\lfloor \alpha\,(B_0 \cdot i) \times 100 \right\rceil \div 100 \text{ Prin.} & + & \left\lfloor \alpha\,(B_0 \cdot i) \times 100 \right\rceil \div 100 \text{ Int.} & = & a \\
a - \left\lfloor \alpha\,(B_1 \cdot i) \times 100 \right\rceil \div 100 \text{ Prin.} & + & \left\lfloor \alpha\,(B_1 \cdot i) \times 100 \right\rceil \div 100 \text{ Int.} & = & a \\
a - \left\lfloor \alpha\,(B_2 \cdot i) \times 100 \right\rceil \div 100 \text{ Prin.} & + & \left\lfloor \alpha\,(B_2 \cdot i) \times 100 \right\rceil \div 100 \text{ Int.} & = & a \\
a - \left\lfloor \alpha\,(B_3 \cdot i) \times 100 \right\rceil \div 100 \text{ Prin.} & + & \left\lfloor \alpha\,(B_3 \cdot i) \times 100 \right\rceil \div 100 \text{ Int.} & = & a \\
 & & & & \vdots \\
B_{n-1} \text{ Prin.} & + & \left\lfloor (B_{n-1} \cdot i) \times 100 \right\rceil \div 100 + O_{n-1} \text{ Int.} & = & a_{\mathrm{f}}
\end{array}$$

Length of repayment (years):

$$l_{\mathrm{y}} = \left\lfloor \frac{n}{12} \right\rfloor$$

Length of repayment (months):

$$l_{\mathrm{m}} = n - 12 \cdot l_{\mathrm{y}}$$

Special cases of algorithm:[6]

```
if  n = 1  then
    if  a − a_f > 0  then
        display  "Refunded $[ |a − a_f| ]"
    else if  a − a_f < 0  then
        display  "Pay Extra $[ |a − a_f| ]"
```

Remark 4.5. Be mindful of the differences between $\lfloor\ \rceil$, $\lfloor\ \rfloor$ and $|\ |$. The former brackets correspond to the nearest integer (i.e., `nint`) function, middle ones to the nearest lowest integer (i.e., `floor`) function, and latter the absolute value (i.e., `abs`) function.

Absolute minimum monthly payment ($):

$$a_{\min_n} = \left\lfloor \alpha\,(p \cdot i) \times 100 \right\rceil \div 100 + 0.01$$

$$a_{\min_n} = \underbrace{\left\lfloor \alpha\,(p \cdot i) \times 100 \right\rceil \div 100}_{\text{applied to interest}} + \underbrace{0.01}_{\text{applied to principal}}$$

[6]For simplicity, let us assume the refund is sent, or extra pay is made, immediately thereafter.

Ten-year minimum monthly payment ($):[7]

$$a_{\min_{120}} = \begin{cases} \left\lceil \frac{p}{120} \times 100 \right\rceil \div 100 & \text{if } i > 0 \text{ and } \alpha = 0 \\ \left\lceil \frac{p}{120} \times 100 \right\rceil \div 100 & \text{if } i = 0 \\ \left\lceil \frac{\alpha\,(p \cdot i)\,(1+\alpha \cdot i)^{120}}{(1+\alpha \cdot i)^{120}-1} \times 100 \right\rceil \div 100, \text{ for } \alpha \cdot i \neq 0 & \text{if } i > 0 \text{ and } 0 < \alpha \leq 1 \text{ (default)} \end{cases}$$

Remark 4.6. Ten-year minimum monthly payment is for repaying student loans within ten years, *not* for repaying student loans for at least ten years.

Total payments ($):

$$T(a) = (n-1)a + a_{\mathrm{f}}$$
$$T_{\max} = T(a_{\min,\,\alpha=1})$$

Savings ($):

$$s = T_{\max} - T(a)$$

Remark 4.7. Each total payment still accounts for α. As α increases, n increases and O_{n-1} decreases; this is because every month less money is applied to principal and more is applied to interest. As α decreases, the opposite effect occurs because more money is applied to principal and less to interest.

Remark 4.8. Savings will be rounded to the nearest integer in the app's main screen but not in its mathematics screen.

Change in savings ($):

$$s_1 = T_{\max} - T(a_1), \text{ where } a_1 \geq a_{\min}$$
$$s_2 = T_{\max} - T(a_2), \text{ where } a_2 \geq a_{\min}$$
$$\Delta s = |s_2 - s_1|$$

Remark 4.9. Change in savings is displayed only in the main screen and will be rounded to the nearest integer. Also, the direction of change will be indicated with an up arrow, if $s_2 - s_1 > 0$, or down arrow, if $s_2 - s_1 < 0$.

[7] See section 3 of "Extra Insight into the iOS App" for more information on the ten-year payment.