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Multi-Objective Joint Optimization of Loading and Capacity Vehicle Routing Problem

Chao Wang, Chun Jin, and Jim Han

***Abstract*—To solve the capacity vehicle routing problem (CVRP) more effectively and save related resources, this paper proposes a multi-objective joint optimization problem of loading problem and CVRP (LCVRP), and builds the appropriate mathematical model. We design a multistage algorithm to solve it. In the first stage of the algorithm, we give a novel loading algorithm to work out the minimum number of transport vehicles. Numerical experiments manifest that we can get the minimum vehicles in LCVRP, and the satisfactory solutions of LCVRP are better than those of CVRP in some instances of VRPLIB. The experiment part of this paper shows the testing for E022 instance in detail.**

***Index Terms*— Vehicle Routing Problem, Traveling Salesman Problem, Loading Problem, Multi-objective Joint Optimization**

I. Introduction

VEHICLE Routing Problem (VRP) is a famous combinatorial optimization problem in operations research. It occupies an important part of the application of modern logistics. From the perspective of supply chain structure, transportation links between all levels of enterprises and their customers, large storage and terminal operation. Distribution of raw materials and finished products in manufacturing enterprises are also practical applications of various types of VRP. Since VRP was proposed in 1959 by Dantzig and Ramzer [1], many extension and changing patterns have arisen in academic research and practical application, such as the capacity vehicle routing problem (CVRP) , vehicle routing problem with time window (VRPTW), vehicle routing problem with backhauls (VRPB) and so on. CVRP is the most common problem, it is defined as: Suppose there are n clients with the demand $d_i<Q$ (i=1,2, $\cdots$,n), and a depot that owns m vehicles with the same rated capacity Q. Each client is exactly served once. Both the beginning and the end of each vehicle are at the depot. How can we arrange the path to minimize the total distance of all routes?

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Chao Wang is with Institute of Systems Engineering, Faculty of Management and Economics, Dalian University of Technology, and School of Software, Dalian Jiaotong University, Dalian, China (e-mail: 13130001974@163.com). She was born in 1974, Ph.D. candidate, associate professor. Her research interests include algorithms design, logistics management and system optimization.
Chun Jin is with Institute of Systems Engineering, Dalian University of Technology, Dalian, China (e-mail: jinchun@dlut.edu.cn). He was born in 1963, ph.D., professor, Ph.D. supervisor. His main research interests include logistics management and system optimization, logistics simulation.
Jim Han is with Department of Information Technology and Operations Management, College of Business, Florida Atlantic University, Boca Raton, USA(e-mail: han@fau.edu). He was born in 1957, Ph.D., professor, Ph.D. supervisor. His main research interests include logistics simulation, logistics management and system optimization.

The methods in the literature on the VRP can be classed into two classes: exact methods and approximate methods. The exact methods [2] can find the optimal solution within a limited number of clients, because the time complexity of exact methods grows exponentially with the number of clients increasing. The most effective algorithms in this class are cutting-plane and branch and bound. However, approximate methods have the advantage of finding a near optimal solution in reasonable time, such as saving algorithm (C-W) proposed by Clarke and Wright[3], sweep algorithm [4], simulated annealing algorithm (SA) [5], tabu search algorithm (TS) [6,7],genetic algorithm (GA) [8,9], ant colony algorithm (ACA) [10], and particle swarm optimization algorithm (PSO) [11], these heuristic algorithms mainly focus on path optimization. Recently, joint optimization of loading problem and CVRP has been more concerned, Bortfeldt [12], Leung [13,14], Duhamel [15], Zhu [16], Zachariadis [17] etc. have studied and attempted in this filed.

Based on literature study and practical application, this paper proposes a multi-objective joint optimization problem of loading problem and CVRP (LCVRP) that saves related resources, facing the current status of overcrowding transportation and squeezing the fixed costs of logistics enterprises in China. We design a multistage algorithm for this specific problem, in the first stage of which we propose a novel loading algorithm-Largest Demand Fit Largest Residual Capacity (LDFLRC) to work out the minimum number of vehicles for transportation. In fact, the model of LCVRP and the multistage algorithm are helpful to solve the CVRP more effectively.

The remainder of this paper is organized as follows. The description of LCVRP and modeling is described in Section 2. Section 3 is dedicated to introduce the multistage algorithm. Section 4 reports on numerical experiments while the paper is summarized in Section 5.

II. Description and Mathematical Model of LCVRP

The objective function of CVRP is just to minimize the total distance, and the optimization in loading process hasn't been considered. In the classical model of CVRP, clients' demands are loaded in turn, when loading a client's demand d_i to a vehicle results in exceeding Q , an empty vehicle is added, and the whole d_i is loaded into the added vehicle. This process continues, until all clients' demands have been loaded. On the basis of CVRP, we take into account the loading process in another way to minimize the distribution vehicles, which is the multi-objective joint optimization of LCVRP proposed by this paper.

LCVRP can be described as follows: Given a graph G (V, E) , where $V=\{v_i \mid i = 0,1,2,\cdots,n\}$ is the set of vertices of a depot and n clients, $E=\{v_i \rightarrow v_j \mid i,j = 0,1,2,\cdots,n\}$ is the set of all edges. The demands' set is $D=\{d_i \mid d_i \geq 0,\ i=0,1,2,\ \cdots,n\}$, where d_i is the demand of the vertex i. The set of distribution vehicles is $Veh=\{1,2,\ \cdots, K\}$, the rated capacity of each vehicle is denoted by Q.

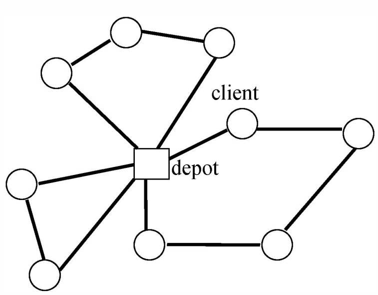
Compare the demand d_i with the rated capacity Q, we can get the partition of the set V : $V_0=\{v_0 \mid d_0=0\}$, $V_S=\{ v_i \mid 0<d_i \leq Q\}$, $V_L=\{ v_i \mid d_i >Q \}$, where V_0 is the depot, V_S is the set of client vertices with little demands, and V_L is the set of client vertices with large demands. The demand of each vertex in V_L must be split for transport. As a result, these vertices were served more than one time by different vehicles. While the demand segmentation is not allowed for transport to each vertex in V_S, in this case, the client is served exactly once by a vehicle. The total clients' demands of each route can not exceed Q. The transport vehicles start from and return to the depot to meet each client's demand.

p_{ik} denotes the ratio of the quantity of cargo that belongs to vertex i transported by vehicle k to d_i, $0 \leq p_{ik} \leq 1$.

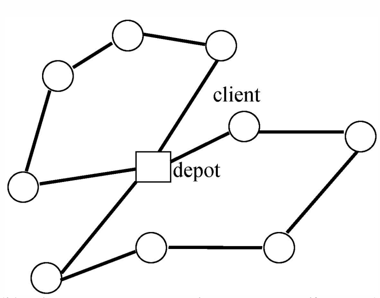
c_{ij} represents the distance between vertex i and vertex j.

$x_{ij}^{k}=1$ denotes that vehicle k leaves from vertex i to vertex j, otherwise $x_{ij}^{k}=0$, so $x_{ij}^{k} \in \{0,1\}$.

The target of LCVRP is to minimize the number of distribution vehicles and optimize routes on the premise of meeting all clients' demands. The figure 1 shows the difference between CVRP and LCVRP, there is an example of CVRP with 9 clients and 3 routes in Figure 1 (a). If we use the loading algorithm, the number of routes would be reduced sometimes, as shown in Figure 1 (b), the corresponding example of LCVRP with 9 clients and 2 routes.



(a) An example of CVRP



(b) The LCVRP example corresponding to (a)

Fig. 1. The comparison between CVRP and LCVRP

The mathematical model of LCVRP is as follows :

$$\min \ |Veh| \tag{1}$$

$$\min \sum_{k\in Veh}\sum_{j\in V}\sum_{i\in V} c_{ij}x_{ij}^{k} \tag{2}$$

subject to

$$\sum_{k\in Veh}\ \sum_{j\in V/\{v_0\}} x_{0j}^{k} = K \tag{3}$$

$$\sum_{i\in V/\{v_0\}} x_{0i}^{k} = 1 \qquad k\in Veh \tag{4}$$

$$\sum_{i\in V} x_{ih}^{k} - \sum_{j\in V} x_{hj}^{k} = 0 \qquad h\in V/\{v_0\}, k\in Veh \tag{5}$$

$$\sum_{k\in Veh} p_{ik} = 1 \qquad i\in V_L \tag{6}$$

$$p_{ik} = \begin{cases} 1, & d_i \text{ transported by vehicle } k \\ 0, & \text{otherwise} \end{cases} \qquad i\in V_s, k\in Veh \tag{7}$$

$$\sum_{i\in V/\{v_0\}} d_i p_{ik} \leq Q \qquad k\in Veh \tag{8}$$

$$\sum_{j\in V} x_{ji}^{k} \geq p_{ik} \qquad k\in Veh, i\in V/\{v_0\} \tag{9}$$

where formula (1) is the first objective function to minimize distribution vehicles $K= |Veh|$, in the case that d_i less than Q doesn't be split, the second objective function is to minimize the total distance of all routes, which is denoted by formula (2), two objective functions have a lexicographic order (1) $\prec$ (2) in LCVRP, i.e., path optimization is on the basis of minimizing the distribution vehicles, formula (3) to (9) are constraints, (3) denotes that K vehicles depart from the depot, (4) restricts only one next node for each vehicle leaving from the depot, formula (5) is about connectivity constraint, for each client vertex, the vehicles entering into must leave , formula (6) represents the demand of the vertex in V_L must be met, while the demand of the vertex in V_S is wholly transported by just a vehicle, which is denoted by formula (7), formula (8) denotes that the total cargo quantity transported by each vehicle does not exceed the rated capacity, and formula (9) denotes that each client is only served by the vehicles which visit it.

III. THE MULTISTAGE ALGORITHM FOR LCVRP

As VRP is a NP hard problem, various heuristic algorithms and their improvement algorithms have been applied to it, but their targets mainly focus on path optimization. The multistage algorithm is custom-designed for LCVRP in this paper, and also applies to CVRP.

A. Basic idea of the multistage algorithm

The multistage algorithm is a decomposition method, and is roughly divided into three phases. A novel loading algorithm assigns all the clients' demands to the minimum vehicles in the first stage. The second stage, the parallel iterated 2-Opt, solves traveling salesman problem (TSP) and schedules clients inner routes. The fist two stages generate a better initial solution. Further optimization to the initial solution by TS is the task of the third stage. The relationship of the three stages is shown in figure 2.

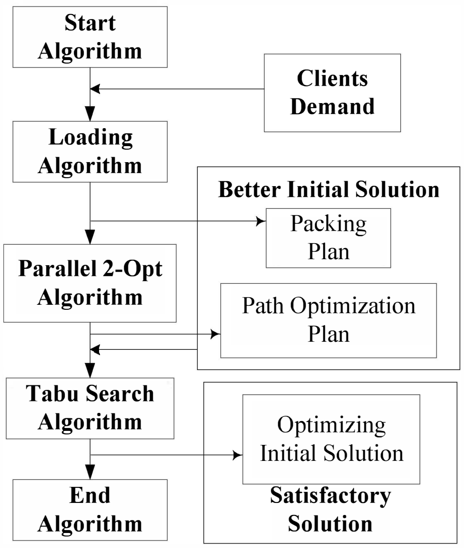


Fig. 2. The flowchart of the multistage algorithm

B. The design of LDFLRC

The first stage of the multistage algorithm assigns routes. We adopt the strategy that largest demand fits the largest residual capacity of transport vehicles. In real life, we have a common sense that we can save space if we first pack items in large size, then put small items in the remaining space. The idea of LDFLRC is described as follows:

1) Initialize the cardinality of *Veh*, $K=\left\lceil (\sum_{i\in V} d_i)/Q \right\rceil$, $\forall V_i \in V_L$, split its d_i into two parts $\lfloor d_i/Q \rfloor Q$ and $d_i - \lfloor d_i/Q \rfloor Q$, send $\lfloor d_i/Q \rfloor$ vehicles from the depot to V_i directly, and determine their routes $V_0 \rightarrow V_i \rightarrow V_0$, set $d_i^{'} = d_i - \lfloor d_i/Q \rfloor Q$, $0 \le d_i^{'} \le Q$, then set $d_i = d_i^{'}$, now $\forall d_i \in D$, $0 \le d_i \le Q$.

2) Load vehicles by the strategy that the largest demand fits the largest residual capacity. Sort all members of D in non-ascending order, the result is denoted by

$$D^{'} = \left\{ d_{1C_1}^{'}, d_{2C_2}^{'}, \cdots d_{iC_i}^{'} \cdots, d_{nC_n}^{'} \middle| C_i \in V/\{v_0\}, i = 1,2,\cdots,n \right\}.$$

Load $d_{iC_i}^{'}$ into the vehicle with the largest residual capacity q_t in turn, if $q_t < d_{iC_i}^{'}$, add a vehicle to *Veh*, K=K+1, and load the $d_{iC_i}^{'}$ wholly to the new added vehicle. Repeat this loading process until all demands are loaded.

The remarkable advantage of LDFLRC is that if the resources of the depot are limited, especially the number of vehicles is not enough to meet the total demands, at the condition $K \ge \left\lceil \sum_{i\in V} d_i / Q \right\rceil$, the vehicles sent can be reused.

The steps of LDFLRC are as follows:

Step 1: Initialize $K = \left\lceil (\sum_{i\in V} d_i)/Q \right\rceil$, q_j=Q (j=1,2,⋯,K), D={d_i|i=1,2, ⋯,n}.

Step 2: For each client in V_L, assign $\lfloor d_i/Q \rfloor$ vehicles to transport directly, set $d_i = d_i - \lfloor d_i/Q \rfloor Q$.

Step 3: Sort all members of D in non-ascending order→ the set D'.

Step 4: Take the maximum $d_{iC_i}^{'}$ from D', then D'= D'-{ $d_{iC_i}^{'}$ }, find the maximum residual capacity q_t and the corresponding vehicle t. If $d_{iC_i}^{'} \le q_t$, then load $d_{iC_i}^{'}$ into vehicle t, set q_t=q_t- $d_{iC_i}^{'}$, otherwise add an empty vehicle, K=K+1, load $d_{iC_i}^{'}$ to the empty vehicle, set q_K= Q- $d_{iC_i}^{'}$.

Step 5: Repeat step 4 until D'=Φ, the assignment process is over, and an initial solution of LCVRP appears.

If we adopt heapsort algorithm in LDFLRC, then the time complexity is about $O(n\log n)$.

C. The parallel iterated 2-Opt algorithm

The second stage of the multistage algorithm is path optimization inner routes, which is to schedule clients in each route. We solve TSP for each route by the parallel iterated 2-Opt algorithm, a better initial solution of LCVRP is generated.

The specific steps are as follows:

Step 1: Construct m initial solutions X_i (randomly or by heuristic algorithm, i=1,2,…,m), set k =0.

Step 2: Calculate the evaluation function for each solution, which is denoted by $f(X_i)$, and record the current best solution P_g and $f(P_g)$.

Step 3: If $k > k_{\max}$, the algorithm is over, output the best solution P_g and $f(P_g)$, otherwise go to step 4.

Step 4: Do 2-Opt [18] operation for each current solution to generate the corresponding new solution $\overline{X_i^k}$, if $f(\overline{X_i^k}) < f(X_i^k)$, then $X_i^k \leftarrow \overline{X_i^k}$, and update P_g and $f(P_g)$.

Step 5: k =k +1, turn to step3.

D. Further optimization by TS

The third stage of the multistage algorithm further optimizes the initial solution by TS. The TS is a global local search algorithm with memory feature. It avoids circuitous search by Tabu criteria, releases excellent state by breaking mechanism, explores the diversity of solutions effectively, and attempts to realize global optimization.

The steps of further optimization are as follows:

Step 1: Initialize Tabu list T=Φ, k =0.

Step 2: Take the result generated from the first two stages as the initial solution X.

Step 3: If $k > k_{\max}$, the algorithm is over, output X, otherwise go to step 4.

Step 4: Choose a move s from the neighborhood move set $S(X)$, which can improve the current X, $s \in S \wedge s \notin T$, to generate a new solution X'.

Step 5: X←X', T←s, turn to step 3.

Tabu list stores just |T| (the length of Tabu list) neighborhood moves. The current move is forbidden in the next $|T|$-loop to avoid back to the old solution, and is released after $|T|$ moves.

IV. NUMERICAL EXPERIMENTS

Now we calculate the instance E022.dat of symmetric CVRP instances in VRPLIB to compare the model of CVRP with that of LCVRP. There are 22 vertices in the E022 instance, Q=6000, and other data is shown in Table I .

TABLE I
THE COORDINATES AND DEMANDS OF VERTICES IN E022.DAT

i	X	Y	d_i	i	X	Y	d_i
1	145	215	0	12	128	231	1200
2	151	264	1100	13	156	217	1300
3	159	261	700	14	129	214	1300
4	130	254	800	15	146	208	300
5	128	252	1400	16	164	208	900
6	163	247	2100	17	141	206	2100
7	146	246	400	18	147	193	1000
8	161	242	800	19	164	193	900
9	142	239	100	20	129	189	2500
10	163	236	500	21	155	185	1800
11	148	232	600	22	139	182	700

A. Solution for CVRP

The initial solution of E022 for CVRP is shown in Table II.

TABLE II
THE INITIAL SOLUTION OF E022 FOR CVRP

Route	Clients	Loading	Clients' vertices
1	4	4000	2, 3, 4, 5
2	7	5700	6, 7, 8, 9, 10, 11, 12
3	5	5900	13, 14, 15, 16, 17
4	3	4400	18, 19, 20
5	2	2500	21, 22

After the initial solution is further optimized by TS, we get a satisfactory solution as shown in TABLE III.

TABLE III
THE SATISFACTORY SOLUTION OF E022 FOR CVRP

Route	Distance	Loading	Sequence
1	102.58	5400	1→11→9→4→5→12→14→1
2	83.67	5900	1→13→16→19→21→18→1
3	112.17	5600	1→7→2→3→6→8→10→1
4	14.14	300	1→15→1
5	76.41	5300	1→22→20→17→1
Total	388.97		

From table III, we can see that there are 5 vehicles in the satisfactory solution of E022 instance for CVRP model. The path synthesis of the satisfactory solution for CVRP is shown in figure 3.

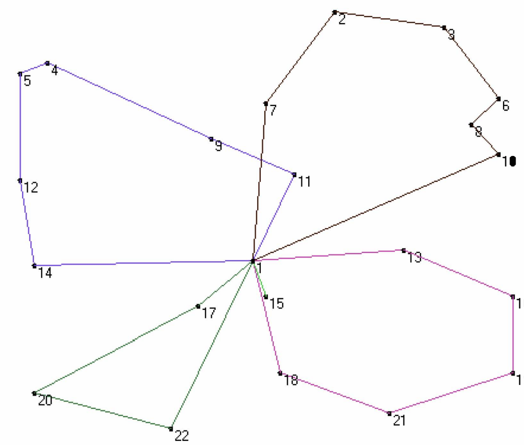


Fig. 3. The effect of synthesis routes for E022's CVRP model

B. Solution for LCVRP

We compute E022.dat again to test and analyze the performance of the multistage algorithm for LCVRP model. The result of E022 computed by LDFLRC is shown in Table IV .

TABLE IV
THE ASSIGNMENT COMPUTED BY LDFLRC

Route	Clients	Loading	Clients' vertices
1	5	5600	20, 12, 19, 22, 15
2	5	5700	6, 13, 18, 3, 11
3	5	5600	17, 14, 16, 4, 10
4	6	5600	21, 5, 2, 8, 7, 9

From table IV, we can see that 4 vehicles are the minimum vehicles sent for transport to meet all clients' demands in E022 instance, which is less than the model of CVRP. In fact, the distribution vehicles of LCVRP is not more than that of CVRP.

Each route is scheduled by the parallel iterated 2-Opt algorithm, TABLE V records the result.

TABLE V
THE INITIAL SOLUTION OF E022 FOR LCVRP

Route	Distance	Sequence
1	135.378510	1→12→20→22→19→15→1
2	141.367599	1→11→3→6→13→18→1
3	150.139755	1→17→14→4→10→16→1
4	190.268723	1→9→7→5→2→8→21→1
Total	617.155	

The effect of synthesis routes after the first two stages of the multistage algorithm is shown in Figure 4, in which the vertices are marked by two items, the first item is the serial number, and the second is the demand.

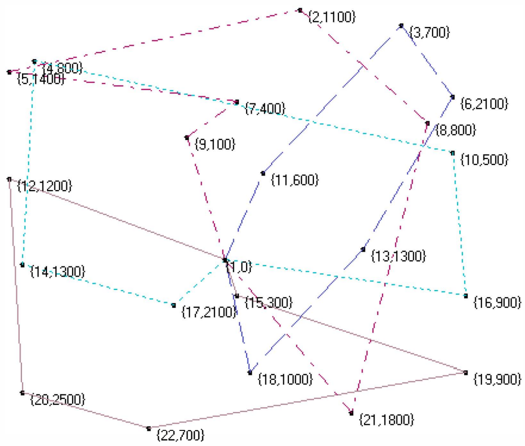


Fig. 4. The effect of synthesis routes after the second stage

Finally, the satisfactory solution of E022 instance is got by TS, the last stage of the multistage algorithm, the result is shown in Table VI.

TABLE VI
THE SOLUTION OF E022 COMPUTED BY THE MULTISTAGE ALGORITHM

Route	Distance	Loading	Sequence
1	112.56	5800	1→10→8→6→3→2→7→11→1
2	76.86	5600	1→17→20→22→15→1
3	100.29	4800	1→14→12→5→4→9→1
4	83.67	5900	1→18→21→19→16→13→1
Total	373.37		

The effect of synthesis routes after the last stage of the multistage algorithm is shown in Figure 5, we see that the result is better than figure 4.

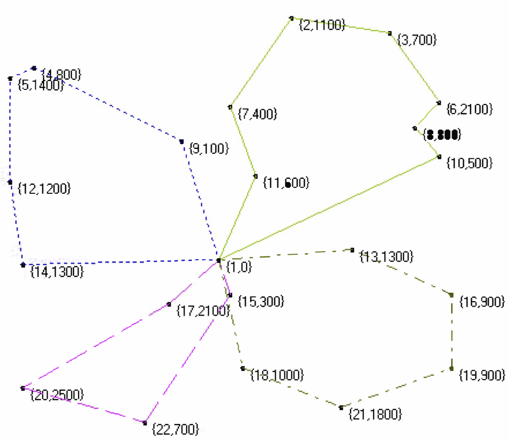


Fig. 5. The synthesis routes of satisfactory solution

Compare the satisfactory solution of LCVRP with that of CVRP, we know the distribution vehicles of LCVRP is less than that of CVRP, and the satisfactory solution of LCVRP, 373.37, is better than that of CVRP, 388.97, in E022 instance.

V. CONCLUSION

The paper has proposed a multi-objective joint optimization problem of LCVRP, and built the corresponding mathematical model. We specially design the multistage algorithm to solve this problem. LDFLRC, the first stage of the multistage, can work out the minimum vehicles. From numerical examples, we can see that the solution of LCVRP is better than that of CVRP in E022 instance of VRPLIB.

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