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Algorithm Study of Multiple-depot Vehicle Routing Problem Based on Fuzzy Simulation

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Abstract--**In this paper, the multiple-depot vehicle routing problem with fuzzy demands is considered, on the basis of uncertain demand of multiple-depot vehicle routing problem, a fuzzy chance constrained program is designed based on fuzzy credibility theory. Then the hybrid genetic algorithm based on fuzzy simulation is used to solve the vehicle routing model. In genetic algorithm, a new code is given, and introduce an evolution method that combining evolution of same depot vehicle and different depot vehicle, in order to avoid local constringency and get general optimization. The results of experiment indicated that the algorithm can effectively solve the fuzzy vehicle routing problem.**

Keywords- fuzzy vehicle routing problem; fuzzy credibility; fuzzy simulation; hybrid genetic algorithm

I. INTRODUCTION

Multiple-depot vehicle routing problem is the expansion of basic vehicle routing problem, which refers there are several depots to service multiple customers at the same time, and the demand of each customer is uncertain. Each depot can serve the customer and have multiple. The requirement is that at the objection of minimum total cost of transportation, to arrange the vehicle and the vehicle routing of each depot appropriately to meet the demand of all customers.

Vehicle Routing Problem belongs to classical problems of the complex combinatorial optimization, which has become front and hot problem in the field of operational research and combinatorial optimization since proposed. There have many study of vehicle routing problem under certain condition. However, in many practical applications, many information of vehicle routing problem are uncertain due to the existence uncertainties of the objective world, as well as the uncertainty of human observation and understanding. And there have correlative study of uncertain vehicle routing problem in home and world, but the theory and method are immature, we need further study. D. Teodorovic and G. Pavkovic [1] studied the vehicle routing problem with fuzzy demand using fuzzy inference algorithm on the basis of the introduction of the decision-maker's preference. Chen and Gen [2] studied the Vehicle Routing Problem with fuzzy appointment using genetic algorithm. Y. Zeng and B. Liu [3] studied vehicle routing problem with a time window using hybrid genetic algorithm. ZHANG Jian-yong etc. [4] studied vehicle routing problem with fuzzy demand using hybrid genetic algorithm based on fuzzy possibility theory. Cao Er-bao etc. [5] studied the Vehicle Routing Problem using differential evolution algorithm. Zhang Liping etc. [6-9] studied the certain demand multi-depot vehicle routing problem. However, all the study before based on the fuzzy possibility theory and study the basic vehicle routing problem. In this paper, a fuzzy chance constrained program is designed based on fuzzy credibility theory. Then the hybrid genetic algorithm based on fuzzy simulation is given to solve the vehicle routing model. In genetic algorithm, a new code is given, and introduce an evolution method that combining evolution of same depot vehicle and different depot vehicle, in order to avoid local constringency and get general optimization. The results indicated that the algorithm can effectively solve the fuzzy vehicle routing problem.

II. PROBLEM DESCRIPTIONS

Multi-depot FVRP with uncertain demand can be described that there are M depots and N service customers in a transport networks, each depot has Km（m=1,2,…,M）vehicles, its capacity is C, and the demand of each customer is a fuzzy variable, which represented by Di（i=1,2,…,N）, and the distance of customer(or depot) between i and j is c_{ij}. Each customer can be served by only one vehicle of the M depot, and each vehicle return it's depot after the service. The requirement is that at the objection of minimum total cost of transportation, to arrange the vehicle and the vehicle routing of each depot appropriately to meet the demand of all customers.

Because the demand is uncertain, it is difficult to accurately estimate, and the fact demand in one range, so the demands of customers are represented by triangular fuzzy variables. Suppose demands of customer is D, then $D=(d_1,d_2,d_3)$, which d_1,d_2,d_3 represent the left border, the point of membership degree and the right border of the fuzzy variables. The membership function of fuzzy variables [10] can be represented as follows:

$$\mu_D(x)=\begin{cases} 0 & x \le d_1 \\ (x-d_1)/(d_2-d_1) & d_1 \le x < d_2 \\ (d_3-x)/(d_3-d_2) & d_2 \le x < d_3 \\ 0 & x \ge d_3 \end{cases} \qquad (1)$$

Because the demand of each customer is fuzzy variable, the demand of a customer is $D_i=(d_{1i},d_{2i},d_{3i})$, after the service of k customers, its total capacity is $d_k^{'} = \sum_{i=1}^{k} d_i$, the deadweight is $Q_k = C - d_k^{'}$, it is also a triangular fuzzy Variable, and $Q_k = (C - \sum_{i=1}^{k} d_{3i}, C - \sum_{i=1}^{k} d_{2i}, C - \sum_{i=1}^{k} d_{1i}$. Whether to continue to serve next node by the comparison of Q_k and D_{k+1} (the demand of next cusomer) in the vehicles scheduling process. According to the credibility theory, the credibility that the demand of next node is less than the deadweight of vehicle can be represented as:

$$C = C_r\{D_{k+1} \le Q_k\}$$
$$= \begin{cases} 1 & d_{3,k+1} \le q_{1k} \\ \dfrac{d_{1,k+1} - q_{3k}}{2[(d_{1,k+1} - q_{3k}) - (d_{2,k+1} - q_{2k})]} & d_{1,k+1} \le q_{3k}, d_{2,k+1} \ge q_{2k} \\ \dfrac{2(d_{2,k+1} - q_{2k}) - (d_{3,k+1} - q_{1k})}{2[(d_{2,k+1} - q_{2k}) - (d_{3,k+1} - q_{1k})]} & d_{2,k+1} \le q_{2k}, d_{3,k+1} \ge q_{1k} \\ 0 & d_{1,k+1} \ge q_{3k} \end{cases} \quad (2)$$

Clearly, the credibility of serving next node increases with the C. Suppose β is the critical value(select the appropriate value by experiment) that decide whether continue to serve next customer. If $C \ge \beta$, continue to serve next customer, if $C < \beta$, a new vehicle to complete the remaining transport tasks.

III. Problem Model

Suppose the number of customer is 1,2,..,N, and the number of depot is N+1,N+2,…,N+M.

Variable definition:

$$x_{ij}^{mk} = \begin{cases} 1, \text{vehicle k of depot m from customer i to j} \\ 0, \text{otherwise} \end{cases}$$

The fuzzy chance constrained program of MDFVRP based on fuzzy credibility theory [11,12] is:

$$\min(\sum_{i=1}^{N+M}\sum_{j=1}^{N+M}\sum_{m=1}^{M}\sum_{k=1}^{K_m} c_{ij} x_{ij}^{mk} + d) \quad (3)$$

subject to:

$$C_r\{d_{i_{l+1}}^m \le Q_{i_l}^{mk}\} \ge \beta \quad \mathrm{m} \in \{1,2,...,\mathrm{M}\} \quad k \in \{1,2,...,K_m\} \quad (4)$$

$$\sum_{j=1}^{N}\sum_{k=1}^{K_m} x_{ij}^{mk} \le K_m \quad \mathrm{i = m} \in \{\mathrm{N+1, N+2,..., N+M}\} \quad (5)$$

$$\sum_{j=1}^{N} x_{ij}^{mk} = \sum_{j=1}^{N} x_{ji}^{mk} \le 1 \quad \mathrm{i = m} \in \{\mathrm{N+1, N=2,..., N+M}\} \quad (6)$$

$$\sum_{j=1}^{N+M}\sum_{m=1}^{M}\sum_{k=1}^{K_m} x_{ij}^{mk} = 1 \quad \mathrm{i} \in \{1,2,...,\mathrm{N}\} \quad (7)$$

$$\sum_{i=1}^{N+M}\sum_{m=1}^{M}\sum_{k=1}^{K_m} x_{ij}^{mk} = 1 \quad \mathrm{j} \in \{1,2,...,\mathrm{N}\} \quad (8)$$

$$\sum_{j=1}^{N+M} x_{ij}^{mk} = \sum_{j=1}^{N+M} x_{ji}^{mk} \quad \mathrm{i = m} \in \{\mathrm{N+1, N+2,..., N+M}\} \quad (9)$$

$$x_{ij}^{mk} = 0 \text{ or } 1 \quad \mathrm{i,j} \in \{1,2,..N+M\} \; \mathrm{m} \in \{1,2,..M\} \; \mathrm{k} \in \{1,2,..K_m\} \quad (10)$$

In the above expression, formula (3) is the objective function that minimize the total routing distance, d is the extra routing distance, formula (4) represents that the credibility that the capacity of customer il+1 less than the deadweight of vehicle k of depot m should be a confidence interval, formula (5) represents that the number of serve customer less or equal that total number of each depot, formula (6) shows that each vehicle start form one depot and return its depot, formula (7) and formula (8) shows that each customer only be serve one time by a vehicle, formula (9) represents that vehicle does not form depot to depot, formula (10) are the constraints of variable value.

IV. Hybrid Genetic Algorithms Based on Fuzzy Simulation

Because there are multiple depots, we commonly translate into single-depot vehicle routing problem. First, assign each customer to the depots according nearest neighbor rule, then optimize the problem of single-depot vehicle routing for each depot. We get the optimal solution for each depot by this method, but it is not the general optimal solution. As this reason, a new code is given in this paper, and introduce an evolution method that combining evolution of same depot vehicle and different depot vehicle, in order to avoid local constringency and get general optimization.

A．Genetic

I）Coding: For multi-depot vehicle routing problem, there are many different code method in literature[6-9], and get good results. But these methods are to certain vehicle routing problem and can not apply to fuzzy demand vehicle routing problem, because the demand vehicle number is uncertain for each depot. In this paper, a new code method is given to solve this problem. For each chromosome, it is represented as follows:

（(Cus_1,Dep_1）（Cus_2,Dep_2）…（Cus_N,Dep_N））

Cus_i is the number of customer, Dep_i is the number of depot, the served order of customer of each depot is the order in chromosome.

II) Fitness value calculation: For each chromosome of population v_k (k = 1,2, ..., n), the total routing distance is

$$f(v_k) = d_k + d_k' \quad (11)$$

which d_k is the expected routing distance of the chromosome v_k, d_k' is the extra routing distance.

Sort the chromosome according to $f(v_k)$, suppose α is known, the fitness calculation based on forward sorting as follows:

$$eval(v_k) = \alpha(1-\alpha)^{k-1} \quad (12)$$

III）Selection operation: Making roulette wheel selection as the selection mechanism, and retain the best chromosome in order to the survival of the best individual to the next generation.

Ⅳ) *Crossover operation:* In every generation, the selected individuals are crossed by probability p_c. In this paper, single-point cross is used, produce cross-point random for the two chromosomes A and B, then the new individual A' and B' are got after removing the same item.

Ⅴ) *Variation operation:* Variation the selected individuals for every generation by the probability p_m.

B. Hybrid genetic algorithms

First, assign each customer to the depots according nearest neighbor rule, then optimize the problem, the steps are as follows:

Ⅰ) Construct chromosome and produce an initial feasible population for all customer.

a) Producing generated customers randomly.

b) Arranging customers of each depot according to formula(4) and producing feasible chromosome, and calculating the expected distance.

c) Repeat *a)* and *b)* until to get n feasible chromosomes.

Ⅱ) Calculate the extra distance because of service "failure".

a) Producing the actual demand by simulation.

b) Calculating the extra distance of each depot due to service "failure" in the condition of actual demand.

c) Repeat *a)* and *b)* M times.

d) Calculating the average of M times, which is the estimated value of extra distance.

Ⅲ) Determine the termination conditions of evolution.

Ⅳ) Execute genetic operation N times according *A* for the routing of each depot.

Ⅴ) Arrange the customers for every chromosome of a new generation by the method of Ⅰ), and calculate the expected routing distance.

Ⅵ) For the better chromosomes of population, exchange partial customer of different depot according the distance between customer and depot. Then calculate the expected routing distance, if the distance less than the original chromosome, preserve the new chromosome, otherwise preserve the original.

Ⅶ) Return to Ⅲ) until to satisfy the termination conditions.

V. EXPERIMENT AND RESULT ANALYSIS

In experiment, the numbers of customer are 30, and the coordinate of customers are produced randomly. The maximum carrying capacity of each vehicle is 20, the demand of customers between 0 and 10. The numbers of population are 30, the evolution generation is 500, crossover probability and mutation probability are 0.3 and 0.2.

Because the influence of β value for expected distance and extra distance, the value of β between 0 and 1 were tested in this paper. And the relationship between expected distance and extra distance was got, which shows in TABLE II.

In order to certificate the effect of this algorithm, we give the comparison of total distance between the improve algorithm and basic algorithm, shows in TABLE II.

From the comparison of the TABLE II., we can see that the total distance of improve algorithm less than the basic algorithm in all β value. So we can get the conclusion that the improved genetic algorithm is better than the basic genetic algorithm, and the experiment indicated that the algorithm can effectively solve the fuzzy vehicle routing problem.

TABLE I. EXPERIMENT RESULTS FOR DIFFERENT β

β	**Expected Distance**	**Extra Distance**	**Total Distance**
0	1078.273	326.367	1404.64
0.1	1152.008	218.831	1370.839
0.2	1159.083	178.206	1337.289
0.3	1192.422	134.448	1326.87
0.4	1203.942	119.608	1323.55
0.5	1210.373	105.247	1315.62
0.6	1239.922	100.22	1340.142
0.7	1276.862	95.087	1371.949
0.8	1300.437	92.164	1392.601
0.9	1312.606	86.65	1399.256
1	1346.69	55.759	1402.449

TABLE II. COMPARISON OF TWO ALGORITHMS

β	0	0.1	0.2	0.3	0.4	0.5
Improve Algorithm	1435.6	1388.8	1347.3	1335.9	1331.6	1329.6
Basic Algorithm	1404.4	1370.5	1337.3	1326.8	1323.5	1315.6
β	0.6	0.7	0.8	0.9	1	
Improve Algorithm	1350.1	1383.9	1384.6	1400.3	1410.4	
Basic Algorithm	1340.3	1371.4	1392.6	1399.2	1402.9	

VI. CONCLUSIONS

On the basis of traditional vehicle routing problem, multiple-depot vehicle routing problem under condition of fuzzy demand is analyzed in this paper. A fuzzy chance constrained program is designed based on fuzzy credibility theory in view of vehicle routing problem of multi-vehicle single cart yard has uncertain demand. Then the hybrid genetic algorithm based on fuzzy simulation is given to solve the vehicle routing model. In genetic algorithm, a new code is given, and introduce an evolution method that combining evolution of same depot vehicle and different depot vehicle, in order to avoid local constringency and get general optimization. The results of experiment show that the algorithm is effective.

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