An Exact Method For The Multi-Vehicle Static Demand Responsive Transport Problem Based on Service Quality:

the case of One-to-One

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***Abstract*—Over the last years, various extensions of Vehicle Routing Problem, Considering the pick -up and delivery vehicle routing problem have been studied. In this work we tackle the pickup and delivery of passengers namely Demand Responsive Transport. Its idea is to group a number of passengers in the same vehicle with the respecting of the capacity of each vehicle in order to reduce operational cost with the maintaining of high quality service.**

This paper presents an exact Method which is the Branch and Bound Algorithm to solve a new variant of Demand Responsive Transport which is based on service quality but not as an option it is an obligation. The Multi-Vehicle Demand Responsive Transport Problem Based on Service Quality in the case of One-to-One(MVSDRTPBSQ1-t-1) under study presented the following characteristics: Users specify transportation requests between origins and destinations, each customer have his own origin and own destination, they may provide a time window on their desired departure or arrival time. All demands are knows before the pacification of vehicles tours and the transportation is supplied by a fleet of vehicle based at a common depot.
The aim of this paper is to find the best routing scheduled of all receipt requests with the respecting of hard Time Windows of each customer, its mean that the most important is the service quality of each customer.
The novelty of our paper is presented in the new manner of algorithm application. However, we will trying to take the general architecture of Branch & Bound algorithm and using valuation rules in a different way which can adapted for such problem to find an optimal solution.

Keywords—Demand Responsive Transport Problem; Exact Method; one-to-one transportation; Branch and Bound algorithm

I. INTRODUCTION

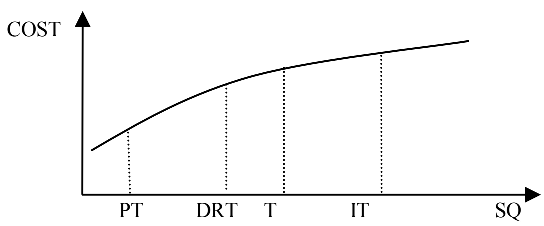
With development the technologies in the transportation systems, the distributed applications are more and more used. Because these systems needs to have Service Quality (SQ) expectations (delay, not comfortable travel , etc.),a set of SQ oriented system aimed at aiding passengers to be more satisfied in their daily transport.

Traditionally, SQ is measured by its inverse that is to say in terms of inconvenience.

Selecting the best routing with the best cost according to the application requirements and DRT contexts is a challenge if we want to provide enough QS to the customer.

If we look to the DRT we found that it can have a middle QS compared with other mode of transportation according to [5] when he defined the DRT as « services provide transport 'on demand' from passengers using fleets of vehicles scheduled to pick up and drop off people in accordance with their needs. DRT is an intermediate form of transport, somewhere between bus and taxi which covers a wide range of transport services ranging from less formal community transport through to area-wide service networks », the position of DRT against the other mode of transportation in term of quality and service can be presented in Fig.1. which presented a comparison between public transportation, DRT, tai and individual transport in term of cost and service quality :

Fig.1 The position of DRT in term of service quality and cost



PT: Public Transport
DRT: Demand Responsive Transport
T: Taxi
IT: Individual Transport

Each transportation user want feel comfortable when he makes all his daily transportation and the most important that he reached his destination at the time. It's our objective with the new formulation of the DRT based on Service Quality.

The remainder of this paper is organized as follows.in the next section we present a literature review on Multi-Vehicle Static DRT One to One Problem In section 3, the different types of static pickup and delivery problem is shown. In section 4, we define the problem .This followed by the description of an exact methods uses to solve this problem, section 5.In section 6 we discuss the computational results. Finally, we conclude this paper in section 7.

II. LITTERATURE REVIEW

Several studies on Dial a Ride Problem (DARP) have been done over these past years, we interested here on the static DRT with multiple vehicle.
The first consultancy study proposed by [20] in 1986, they used a strict time windows for the place of pickup and a soft time windows for the place of drop off, the proposed heuristic insert sequentially users into vehicle routes so as to yield the least possible increase in the objective function value.
This heuristic was applied to a real-life instance containing 28 vehicles and 2617 users.

In 1986,the idea developed by [3] was the construction of customers clusters then solve each cluster with a single vehicle used Sexton and Bodin algorithm [6] [1], this algorithm was tested on 85 instance from Baltimore data base. The amelioration of these approach was done by [14] in 1989, they divided it in two phases with the creation of small clusters of users, and then they used a colony generation to form the itinerary vehicle.

Reference [15] has ameliorate their single-vehicle exact algorithm in 1991 to the multiple-vehicle case and applied it to instances with n<55(n the number of passengers).

In 1996, [27] tackled a real case study when customers specify requests with a time window on their pickup and drop off places. Their objective was minimizing the total cost service for that, they developed a heuristic consisting of first assigning requests to routes by means of a parallel insertion procedure, and then performing intra-route and inter-route exchanges. Results were reported on instances involving between 276 and 312 requests.

In addition,[4] proposed a two phases approach which consisted first on the construction of users clusters, then they are grouped to form the vehicle possible tours [4].Results are reported for real-life instances involving between 859 and 1765 transportation requests per day.

Another study was done in 2003 by [11] based on tabu search algorithm .In this work; users provide a time window on their desired departure or arrival time. Besides, this approach provide a maximum ride time of any user.
The tabu search algorithm used based on deleting an application then reintegration into another itinerary.

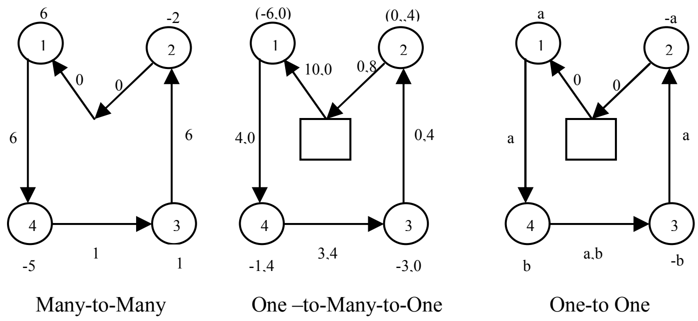
The DRT treated by [2] in 2003 is a hybrid DRT, it offers two type of transportation service. [2] Developed an insertion-reinsertion heuristic followed by tabu search. The aim of this approach is to minimizing the distance traveled by vehicle and also travels time of passengers. This approach was tested on real case study Lancasteren California comity in US.

In 2004, [12] applied a procedure to solve DRT; the objective is a weighted sum of the distance, travel time, inactivity of the vehicle. The result was variant between 500 and 1000 instance.

Reference [17] proposed four heuristic to solve DRT with multiple vehicle. The principle objective is minimizing the total vehicle used to serve passengers [17].They developed first a genetic algorithm for the grouping phase and insertion algorithm for the routing phase. The test was done on a real case study in Bruxelle.

III. DIFFRENT TYPES OF PICCK UP AND DELIVERY PROBLEM

Fig.2. Different types of static pickup and delivery problem



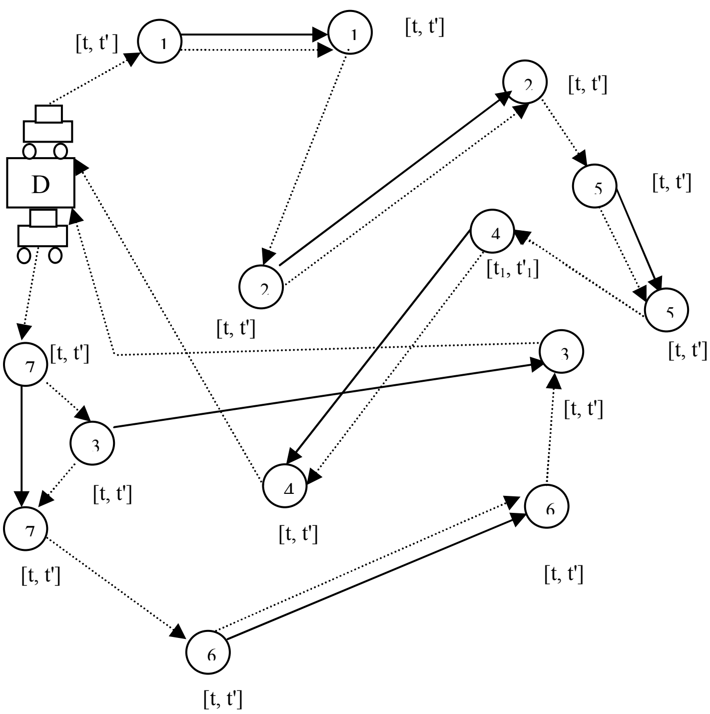
- Many-to-Many (MM): indicates that every vertex of the graph can be used as the origin or destination for all transported items.
- One-to-Many-to-One "(1-M-1): indicates that transported objects are first available at depot and then oriented to different destinations or transported objects are first available at different destination and then oriented to depot.
- "One-to-One" (1-1) :indicates that each transported object has its own precise origin and its own precise destination

IV. PROBLEM DESCRIPTION AND FORMULATION

A. Description

The main idea of MVSDRTPBSQ1-1 is to regroup a set of passengers in the same vehicle based on transportation request sanded by those passengers with the respecting of each used vehicle capacity. Each passenger has his own origin and his own destination with precise time of pickup and delivery; he can dispose for his two places of origin and destination time windows. This problem is NP hard [8]. It enriched the vehicle routing problem pickup and delivery (VRPPD) in many ways. Its functioning system can be presented in this following figure.

Fig.3. Multi-Vehicle Static DRT



In this section we will present our formulation for the MVSDRTPBSQ1-1; it consists of determining the best routing schedule for the vehicles. The challenge is to combine the contradictory factors: low cost of transportation versus a high level of service.

We will define all parameters and constraints used for this problem. Our objective is to minimize the travel time between different places, whether pick up points or delivery points with the maintaining of the high quality of service.

B. Mathematical formulation

Let n be the number of customers of our problem and let G = (N, A) be a directed complete graph where N =P∪D∪{0,2n+1} and A={(i,j)\i,j∈N,i≠j}.

P= {1,…,n} ; contains all starting nodes.

D= {n+1,…,2n}; contains all arrival nodes.

The node 0 and 2n+1 represents the depot.

For each customer there are a starting node i and an arrival node i+n.

This problem is characterized by the following data:

- K: Set of vehicle
- C_K: Capacity of vehicle k
- T_K: Total duration of a tour of vehicle k
- q_j: Quantity to treat at node j such as q_0 =$q_{(2n+1)}$=0
- [e_j,l_j]: Time windows of node j
- t_{ij}: Travel time from node i to node j
- d_{ij}: Distance between two visited node
- α: Delay coefficient ; it is a constant fixed by the customers
- ε: Penalty percentage determined by the dispatch center of transportation to ensure the good quality of service
- β: Penalty coefficient related to the cost such as $\beta = \varepsilon C_{ij}$. The penalty will be applying for each delay α of H_{ik} exceeds the time windows $[e_i, l_i]$, exactly when $H_{jk} > l_i$
- C_{ij}: Cost associated for each arc (i,j)∈A ,

The variables are as follows:

- H_{ik}: Real time of begging of vehicle k its service at node i
- ta_i: Arrival time in node i
- x_{ij}^k: Assumes 1 if vehicle k move from node i to j and 0 if not.
- Q_{ik}: The quantity existing at vehicle k after completing its service at node i

The problem addressed in this paper is to minimize the total travel time between different nodes. This is referring to minimize the distance between nodes and of course minimize the total cost, with the maintaining of height quality of service expressed in a function of penalty.

The objective function is to minimize the cost and maximize the quality of service (minimize the not satisfaction of customers)

Then it can be presented as two functions:

➢ Function related to the cost presented as follow:

$$f_1 = \sum_{k\in K} \sum_{i\in N} \sum_{j\in N} C_{ij}\, x_{ij}^k \quad (1)$$

➢ Function related to the quality of service expressed as a function of penalty in term of cost, can be presented as follows:

$$f_2 = \sum_{i\in N} max\left(0, \beta\left(\frac{H_i^k - l_i}{\alpha}\right)\right) \quad (2)$$

We have two function related to cost can be added in one global objective function presented as follows:

$Min\ F =$

$$\sum_{k\in K}\sum_{i\in N}\sum_{j\in N} C_{ij}\, x_{ij}^k + \sum_{i\in N} max\left(0, \beta\left(\frac{H_i^k - l_i}{\alpha}\right)\right) \quad (3)$$

Subject to:

$$\sum_{k\in K}\sum_{j\in N} x_{ij}^k = 1 \quad \forall i \in N \quad (4)$$

$$\sum_{j\in N} x_{ij}^k = \sum_{j\in N} x_{i+n,j}^k \quad \forall i \in N, k \in K \quad (5)$$

$$\sum_{j\in P} x_{0j}^k = 1 \quad \forall k \in K \quad (6)$$

$$\sum_{j\in N} x_{ij}^k = \sum_{j\in N} x_{ji}^k \quad \forall i \in N, k \in K \quad (7)$$

$$\sum_{j\in P} x_{j;2n+1}^k = 1 \ \forall k \in K \quad (8)$$

$$Q_j^k \geq Q_i^k + q_j x_{ij}^k - M\left(1 - x_{ij}^k\right) \forall j \in N, \forall i \in N, \forall k \in K \quad (9)$$

$$C_k \geq Q_j^k \quad \forall j \in N, \forall k \in K \quad (10)$$

$$H_i^k = 1_{\{ta_i > e_i\}} ta_i + 1_{\{ta_i \leq e_i\}} e_i \quad \forall i \in N, \forall k \in K \quad (11)$$

$$e_i \leq H_i^k \quad \forall i \in N, \forall k \in K \quad (12)$$

$$H_i^k \leq H_{i+n}^k \ \forall i \in P, \forall k \in K \quad (13)$$

$$H_{2n+1}^k - H_0^k \leq T_k \ \forall\ i \in P, \forall k \in K \quad (14)$$

$$x_{ij}^k \in \{0,1\} \forall\ i \in N, \forall j \in N, \forall k \in K \quad (15)$$

C. Discussion

Constraint (4) impose that each customer is assigned to exactly one vehicle.

Constraint (5) ensures that the starting node i and the arrival node are $n + i$ served by the same vehicle.

Constraint (6) and (7) and (8) characterize the path to be followed by a vehicle k, it must leave the depot only once (6), if it served a customer it must leave him (7), and finally return to the depot also only once (8).

The non-violation of the capacity limits of each vehicle is handled in Constraints (9), (10). M is a very big quantity ($+\infty$), so if the vehicle reach the node j ($x_{ij}^k = 1$) the quantity Q_{jk} existing at vehicle k after completing its service at node j is equal to the demand at this node q_j add to the quantity existing at vehicle k after completing its service at node i (9). Now the capacity of vehicle k should be greater than or equal the quantity Q_{jk} to treat at node j (10).

Constraint (11) it is an indicator function for give us the exact value of real time service beginning H_i^k, its equal to ta_i if $ta_i > e_i$ or its equal to e_i if $ta_i \leq e_i$

In constraint (12) the real time service H_{ik} in node i must not exceed the lower bound of time windows e_i.

Constraint (13) enforces the respect of precedence; the real time of beginning service H_{ij} at starting node i must be before the real time of beginning service $H_{i+n,j}$ at his arrival node $i + n$.

Each tour must be limited to the total duration of tours T_K reserved to the vehicle k mentioned in constraint (14).

Finally, constraint (15) restricts the variables to binary values.

V. SOLUTION METHODOLOGY

In literature many studies have tackled the DRTP which several approaches are proposed to solve this variant.

In this paper, we propose a Branch and Bound algorithm to solve this new variant of DRT Based on SQ. we will try to use the general algorithm adding a new methodology in the way of application of B&B.

The Branch and bound is a technique for solving optimization problems that uses divide and conquer strategy to partition the solution space into sub-problems and then solves each sub-problem recursively, [16].

This method was usefully used to solve many Vehicles Routing Problem, like the work of [16]; they have applied this method to solve large scale integer quadratic multi-knapsack problems. Reference [12] used the Branch and Bound method to solve a capacitated vehicles routing problem.

In 2010, [19] used this method to solve a single vehicle routing problem.

For us, our objective is to reduce the total cost of vehicle tours with the maintaining of a high quality service i.e. reduce the number of tours, using the best planning of vehicle routing of pickup and delivery passenger and do not exceed the delays time desired by the passenger. The proposed methods must give us a best result to solve the MVSDRTPBSQ1-1.

This section introduces the basic concepts of Branch and Bound procedure. We describe the processing steps of algorithm including details of branching scheme, Bounding scheme.

The B&B methods is a constructive tree search method that uses implicit enumeration based on the notion of bound to avoid enumeration of broad classes of bad solutions. It is based on two concepts:

- Branching which is to partition the solution spaces into sub problems for each individual optimize;
- Bounding consist of determining an optimum for set of feasible solutions associated with selected node.

The idea here is to apply the principle of B&B at this problem which is multivehicle. We will first divide our problem to a set of problems then we will apply the B&B in each problem aside from other problems which mean that we will have a multi brunch and bound algorithms. We must indicate here that we talk about an exact method so we must give all possible cases. The B&B algorithm in this paper presented some modifications in the algorithmic kernels. Our algorithm can be presented into seven steps.

Step 1: Assign starting nods for all existing vehicles, all possible cases must be mentioned uses a matrix.

Step2: Check the reappearance of nodes already appeared in the previous assignment for a given vehicle and eliminates redundant nodes

Step 3: Depending on the number of existing cases, our problem is divided on set problems

Step 4: Apply t the B&B on each case separately.

Step 5: Each case divided also into others sub problem depending on the number of vehicle and nodes belong to each vehicle in each case

Step 6: Apply the B&B on each sub problem.

Step7: Determine the lower bound LB for each B&B algorithm, LB= F_0^*

Fig.4. Different cases of affectations starting nodes to vehicles from n!

Cases

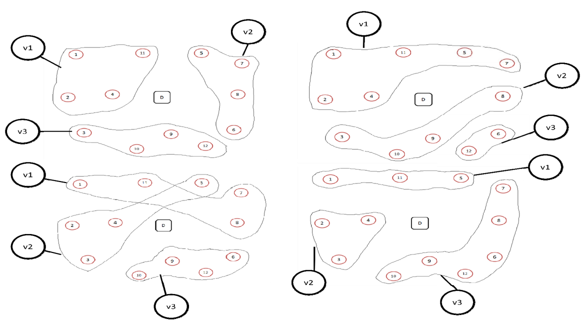


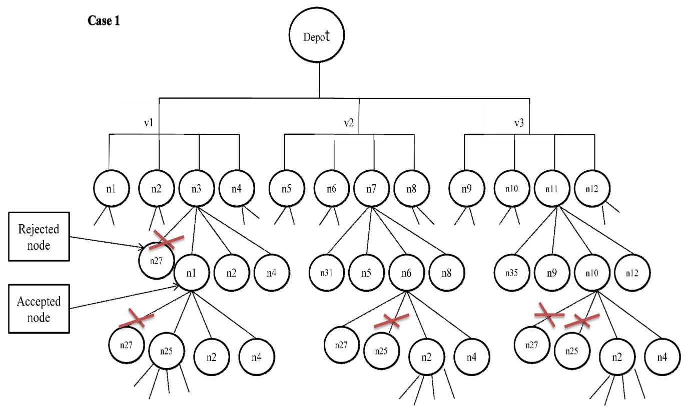
Fig.5. Matrix of all possibles cases

	v1	v2	v3	
Case 1	n1,n2,n3,n4	n5,n6,n7,n8	n9,n10,n11,n12	⇨ B&Bs
Case 2	n3,n6,n11	n4,n5,n7,n8,n9 ,n10	n1,n2,n6,n11, n12	⇨ B&Bs
Case 3	~~n9,n10,n11, n12~~	~~n1,n2,n3,n4~~	~~n5,n6,n7,n8~~	
Case 4	n12,n8,n4,n9, n6	n10,n11,n1,n2	n3,n5,n6,,n7	⇨ B&Bs
	. . .	. . .	. . .	
Case n!	n7,n3,n2,n12	N9,n1,n4,n8	N5,n6,n10,n11	⇨ B&Bs

In this stage, we try to build our B&B tree specific for each case, case-by-case. The head of tree is the depot and its first descendants must be set of nodes belong to each vehicle.
Now the branching is made according nodes exist in the list of vehicle in each case wish are belong to D. Once we are testing selected nodes from D, we choose nodes which satisfy our constraints. After, we move to the next level of each branch when we will test level by level at the beginning the arrival nodes of nodes already selected and in the same time the remaining unselected nodes.
In each level we calculate solution wish will be compared with the previous found F^*, if new $F > F^*$, F will be rejected, otherwise $F^*=F$. Branching is stopped when there is one broken constraint.
This iteration will be repeating for all the branches of tree and for all existing cases.

Fig. 6 represents the main paradigm of our tree solution with the procedure of Branch and Bound algorithm applied on MVSDRTPBSQ1-1.

Fig.6. Main paradigm of Branch and Bound algorithm tree for a given case



VI. EXPERIMENTAL RESULTS

At the begging we must mentioned that no test instances are has been made in the literature for the version of the MVSDRTPBSQ1-1 studied in this paper. To analyze the conduct of the B&B algorithm, we have thus generated a set of 20 instances according to the data presented in the benchmarking of [8]. Information regarding time window widths, vehicle capacity and maximum ride route duration was provided in generated instances.
The randomly generated instances contain between 16 and 28 requests. For i=1,…,n, vertex v_i is the origin of request i while vertex v_{n+i} is its destination. For each vertex the load q_i is equal either to 1 or -1 depending on whether the vertex corresponds to the origin or the destination of a request.
The location of the depot is equal to the 0 at the departure and 2n+1 when vehicles have completed their routing. . For every arc (i,j) ∈ A, the routing cost C_{ij} and travel time t_{ij} are equal to the Euclidean distance between the two vertices, the percentage penalty $\varepsilon = 5\%$ and the penalty coefficient $\beta = 5\%C_{ij}$.A time windows $[e_i,l_i]$ is also associated to each vertex, for each tour there is a total duration $T_K=8\times60=480$.
Two groups of instances were created by using different parameters for generating time windows. The number of used vehicles to serve transportation requests varies between 2 and 4. In all instances the capacity of a vehicle is equal to 6.
Tables 1 presents results obtained by our algorithm applied to instances mentioned previously.

TABLE I. RESULTS OF B&B ALGORITHM

Instances	n	K	Q	F	CPU(min)
p2-16	16	2	3	325.521	0.07
p2-20	20	2	3	372.928	0.20
p2-24	24	2	3	408.702	3.32
p2-28	28	2	3	449.153	9.02

p3-16	16	3	3	*297.265	0.11
p3-20	20	3	3	*305.445	1.50
p3-24	24	3	3	*401.728	1.22
p3-28	28	3	3	506.404	170.03
p4-16	16	4	3	316.339	1.87
p4-20	20	4	3	383.349	315.80
p4-24	24	4	3	376.551	711.30
p4-28	28	4	3	*380.940	742.00

Table 1 provides the number of requests n and the number of vehicles K in each of the 28 randomly generated instances. For instances p2-16 to p2-28 the number of vehicles is limited to 2. We observed that our objective function takes value between 326 and 450 with a CPU wish does not exceed 10 with 28 requests. For the second group with 3 existing vehicles we observed a decrease in the F due to the respect of the arrival time indicated in the time windows so we can conclude that the penalty coefficient was decrease implying a decrease in F. Finally with third group uses 4 vehicles we found a disparate value of F compared with the previously group.

For the instances p2-16, p3-16 and p4-16 we found successively F2=325.521, F3= *297.265 and F4=316.339 so we can conclude that with 16 instance it's most effective to use 3 vehicle and not 2 or 4. The same with 20 and 24 instances, so in our case bases on SQ it is not always profitable the reduction of number of vehicle because we have here another important factor wish is the SQ. The most important is the satisfaction of customer mainly with the presence of penalty function. On the other side the uses of 4 vehicle is effective only in the presence of 28 instance, F4*=380.940.

VII. CONCLUSION

In this paper we have discussed the static case of new variant of DRT problem based on SQ with hard time windows called MVSDRTPBSQ1-1.We have tried to solve this new variant used an exact methods. We have described a Branch and Bound algorithm in this paper and we have proposed some modifications. The solution methodology is flexible and can be easily adapted for others VRP problems; however, we have succeeded to found a very interested result after applied B&B algorithm. In the future research we will try to solve MVSDRTPBSQ1-1 with metaheuristics methods and compare results

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