A Genetic Clustering-Based TCNN Algorithm for Capacity Vehicle Routing Problem

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***Abstract*—A novel genetic clustering-based transiently chaotic neural network (GCTCNN) algorithm for Capacity Vehicle Routing Problem (CVRP) is proposed. CVRP can be partitioned into two kinds of decisions: the selection of vehicles among the available vehicles and the routing of the selected fleet. Using the clustering algorithm the customers are grouped into clusters and each cluster is served by one vehicle. Then transiently chaotic neural network solves the routes to optimality. Computation on benchmark problems and comparison with other known algorithm show that the proposed algorithm produces excellent solutions in short computing times.**

I. INTRODUCTION

The vehicle routing problem (VRP) is one of the well-known NP-hard optimization problems [1] occurring in many transport logistics and distribution systems of considerable economic significance. It involves minimizing cost vehicle schedules for a fleet of vehicles originating and terminating from a central depot. The vehicles serve a set of nodes with vehicle capacity and travel time constraints. All nodes must be assigned to vehicles and each node is served exactly once. Efficient scheduling of vehicles can save a significant amount of logistics cost and improve the customer service level.

The vehicle routing problem with vehicle capacity constraints is considered in this paper. The resulting problem is called capacity vehicle routing problem (CVRP). The CVRP is similar to the TSP, however, there are several constraints, such as multiple vehicles and maximum vehicle capacity, so the problem is more complicated than the TSP. In recent years many approaches for the optimization problems have been proposed and there have been important advances in the development of exact and approximate algorithms for solving the CVRP. As far as the exact approaches are concerned, the most significant progress has been made in the design of branch and bound or Lagrangian decomposition algorithms. Laporte and Nobert [2], Laporte, Mercure and Nobert [3] and Fisher [4] among the ones who have developed various branch and bound approaches. Christofides, et al. [5] study the dynamic programming approach as applied to the CVRP by introducing a state-space relaxation procedure. As for the approximate algorithms, there exist many studies dating back to Clarke and Wright [6] and some among many can be listed as Gillett and miller [7], Christofides et al. [8], and Desrochers and Verhoog [9].

In this paper we presents a new approach to the vehicle routing problem. The method is based on genetic clustering and transiently chaotic neural network (TCNN). A fundamental aspect of this approach is the possibility of adjusting the parameters of TCNN throughout the process. The proposed algorithm requires the following inputs: number of vehicles; load capacity of each vehicle; customer coordinates, demand related to the customers, and coordinates of the depot. CVRP is divided into two basic problems: the choice of vehicles using the clustering and the routing of fleet using TCNN. Correspondingly, the customers with close Euclidean distance are grouped in the same vehicle. A new computational energy function expression is developed for the corresponding TCNN approach. The computational results on four benchmark problems show that the proposed method can obtain a good solution and the time is reasonable.

II. PROBLEM FORMULATION

The CVRP discussed in this paper may be described as follows: N customers geographically dispersed in a planar region must be served from a unique depot. Each customer asks for a quantity $q_i(i=1,2,\text{L},n)$ of goods. L vehicles with a fixed capacity Q are available to deliver the goods stored in the depot. Each customer must be visited just once by only one vehicle. The objective of the problem is minimizing the total cost of all routes without violating the individual capacity of each vehicle. The depot is denoted by $i=0$. The objective can be written as follows:

$$\text{minimize} \sum_{k=1}^{L}\sum_{i}\sum_{j} C_{ijk}\cdot x_{ijk} \tag{1}$$

III. GCTCNN ALGORITHM FOR THE PROBLEM

The method involves two phases. In the first phase, clustering procedure based on genetic algorithm for solving the transportation problem is used to group customers into regionally bounded clusters based on Euclidean Distance. In the route optimization phase, TCNN connects the produced point clusters to form and optimize the routes.

A. Generation of the Routes (GR)

Although clustering is a powerful set partitioning tool, which can group the given set of data into several mutually exclusive clusters in light of similar features of the data. It suffers from local minima when the objective function is not convex. Chiou [10] proposed a genetic clustering algorithm to overcome the disadvantage. Herein an improved algorithm based on genetic clustering method is presented to satisfy the vehicle capacity constraints and group the customers into the vehicles.

$$\sum_i x_{ik} \cdot q_i < Q \quad \forall k \quad .$$

$$x_{ik} = \begin{cases} 1 & \text{if vehicle } k \text{ visit node } i \text{ in routing path} \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

In this algorithm integer encoding is used to replace the binary encoding to curtail the length of chromosomes. Each gene of the chromosome represents the cluster to which each customer is assign. The objective function is defined as follows.

$$F = \max \sum_K \min_{v_m \in S_i, v_n \in S_j} \| v_m - v_n \| \qquad (3)$$

where m and n are the customers in clusters S_i and S_j , respectively. $\|v_m - v_n\|$ is the Euclidean distance between m and n . K is the number of the clusters.

The number of the clusters is set as $K = \frac{\sum_n q_i}{Q} + h$, initially. Where h is an empirical integer. The population size $pop_{size} \in [20,100]$ is generally recommended.

Another difference between the proposed algorithm and the genetic clustering algorithm is the capacity constraint must be judged after operator is finished. Once the constraint is violated, the generated individual is regarded as invalid and discarded.

$$\sum_{m \in S_i} q_m < Q \qquad (4)$$

The process is completed when all points are distributed to the vehicles and CVRP is divided into several single vehicle routing problems (SVRPs). The total minimum of all partitions can be gotten by routing the divided SVRPs:

$$\begin{aligned} \text{minimize} \sum_k C_k &= \text{minimize}(C_1 + C_2 + \mathrm{L} + C_k) \\ &= \text{minimize} C_1 + \text{minimize} C_2 + \mathrm{L} + \text{minimize} C_k \quad (5) \\ &= \sum_k (\text{minimize} \sum_i \sum_j C_{ijk} \cdot x_{ij}) \end{aligned}$$

B. Route Scheduling (RS)

SVRP is a close route problem similar to traveling salesman problem (TSP). The difference between them is that each node has a demand and the route must satisfy the vehicle capacity. Herein the depot is considered as one virtual demand node having zero demand $q = 0$. All routes have satisfied the vehicle capacity, so it can be seen as TSP whose objective is to minimize the total cost:

$$\text{minimize} \sum_i \sum_{\substack{j \\ j \neq i}} C_{ijk} \cdot x_{ij} \, .$$

$$x_{ij} = \begin{cases} 1 & \text{if link from node } i \text{ to } j \text{ exists in the routing path} \\ 0 & \text{otherwise} \end{cases} \qquad (6)$$

Since Hopfield and Tank applied Hopfield Neural Network (HNN) to optimization problems [11], it has been recognized as a powerful tool for optimization. However, HNN suffer from the local minimum problems whenever applied to optimization problems [12]. Compared with HNN, TCNN [13] can overcome the shortcomings by introducing chaos which is generated by negative self-feedback into HNN. With a time-variant parameter to control the chaos, TCNN goes through an inverse bifurcation process and gradually approaches to HNN with converging to a stable equilibrium point.

To apply TCNN model to the optimization problem of vehicle routing, the most important step is mapping the objectives and constraints of the problem onto the energy function of the network. The Lyapunov energy function is defined as follows:

$$E = \frac{A}{2} \sum_{i=0}^{n} \sum_{\substack{j=0 \\ j \neq i}}^{n} C_{ij} \cdot x_{ij} + \frac{B}{2} \sum_{i=0}^{n} (\sum_{\substack{j=0 \\ j \neq i}}^{n} x_{ij} - 1)^2 + \frac{C}{2} \sum_{j=0}^{n} (\sum_{\substack{i=0 \\ j \neq i}}^{n} x_{ij} - 1)^2 \quad (7)$$

$$\frac{A}{2} c_{ij} + \frac{B}{2} (2 \sum_{a \neq j}^{n} x_{ia} - 1) + \frac{C}{2} (2 \sum_{l \neq i}^{n} x_{lj} - 1)] = -\partial E / \partial x_i \quad (8)$$

In Eq. (7), A, B, C are all arbitrary and positive constants. The first term minimizes the total link cost of a routing path by taking into account the cost of all existing links; the second and the third terms derive the neurons towards convergence to a valid route consisting of connected nodes.

The differential equations describing the network dynamics of TCNN for the SVRP can be written as follows:

$$x_{ij}(t) = \frac{1}{1 + e^{-y_{ij}(t)/\varepsilon}}, \qquad i, j = 0, 1, \mathrm{L}\ , n \qquad (9)$$

$$\begin{aligned} y_{ij}(t+1) = k y_{ij}(t) - \alpha [\frac{A}{2} C_{ij} + \frac{B}{2} (2 \sum_{a \neq j}^{n} x_{ia} - 1) \\ + \frac{C}{2} (2 \sum_{l \neq i}^{n} x_{lj} - 1)] - z(t) [x_{ij}(t) - I_0] \end{aligned} \qquad (10)$$

$$z(t+1) = (1 - \beta) z(t) \qquad (11)$$

where x_{ij} is assumed to be the neuron output which represents to visit city i in visiting order j ; $y_{ij}(t)$ is the internal state of neuron; α is positive scaling parameter for neuronal inputs; k is damping factor of nerve membrane($0<k<1$); β is damping factor of $z(t)$ ($0<\beta<1$); I_0 is positive parameter; ε is steepness parameter of the output function ($\varepsilon>0$); $z(t)$ is self-feedback connection weight or refractory strength ($z(t)\geq 0$), it corresponds to the temperature in the simulated annealing and controls the speed of convergence and inverse divaricating. If $z(t)$ is a positive constant, TCNN will become CNN [14]. When $z(t)=0$, TCNN will degenerate into HNN.

The output of TCNN cannot be convergent $\{0,1\}^{n\times n}$ absolutely. It can be $0<x_{ij}<1$. To improve the computation performance, we use the method of CHEN as follows.

$$x_{ij}^D(t)=\begin{cases}1 & \text{iff } x_{ij}(t)>\sum_k\sum_l x_{kl}(t)\Big/n\times n\\ 0 & \text{otherwise}\end{cases} \tag{12}$$

C. GCTCNN Algorithm

The proposed algorithm generates all possible routes that can be visited by several vehicles and selects the optimal or near optimal routes that have the minimum total cost. In detail, the algorithm is composed of the following steps:

Step 1. Data initialization. n, H_i, q_i, C_{ijk}, Q, K, pop_{size} are given.

Step 2. Initialization Judgment. Judge if the initializations of the chromosome satisfy the constraint of the vehicle capacity. If the constraint is violated, the generated individual is regarded as invalid and discarded. Then the generation process is repeated until enough individuals are gained.

Step 3. Route partition. The roulette wheel selection, two points crossover and gene mutation operator at a certain rate b and g are executed. The constraint of the vehicle capacity must be judged after every operator is finished. The process continues to evolve until there are over 80% chromosomes with the same fitness in an epoch.

Step 4. Route Scheduling. Set a virtual demand $q=0$ for the depot and add it to the vertex sets, then schedule the virtual k TSPs by applying TCNN procedure in "RS". If the process is finished, store the cost $C_{total,l}=\sum C_k$ and the closed routes R_l are written.

Step 5. Rewriting of the optimal solution. Adjust the orders of the routes R_l to use the depot as the start point, which must not change the order among points.

IV. COMPUTATIONAL RESULTS

To illustrate the effectiveness and performance of the proposed algorithm in this paper, we present the computational results on common benchmark problems from the public library TSPLIB [15]. All instances are planar and we have used the distance proposed in the TSPLIB, that truncates the Euclidean distance to its nearest integer. The program for the CVRP is coded in Matlab and run on Intel Celeron 1000 with 258M RAM. The parameters of the proposed algorithm, $pop_{size}=40$, $b=1.00$, $g=0.01$. $\varepsilon=1e-4$; $m'=2$; $B=C=1$; $k=0.9$; $\varepsilon=1/250$; $\alpha=0.015$; $I_0=0.65$; $z(0)=0.08$ β and A are various in different problem. Each instance is randomly run 15 times. Instances are referred to using the initial of the author name, the number of nodes (customers plus the depot) and the number of available vehicles. Table I shows the computational results for the four problems. The ratio r of total demand to total vehicle capacity provides a measure of the tightness of vehicle capacity constraints. GCTCNN represents the objective value of genetic clustering TCNN algorithm found over 15. Running time is shown in column four. BSK of Table I shows the best known solution so far.

TABLE I

COMPUTATIONAL RESULTS FOR BENCHMARK PROBLEMS

Problem	r	GCTCNN	Time	BSK
En22k4	0.94	375	6	375
En30k3	0.94	567	40	534
En51k5	0.97	575	537	521
En76k8	0.95	754	1269	735

TABLE II

COMPARISON WITH OTHER ALGORITHMS ON FOUR PROBLEMS

Problem	GCTCNN	Heuristic1	Heuristic2
En22k4	375	376	375
En30k3	567	587	554
En51k5	575	581	547
En76k8	754	776	766

Table II shows the comparison of GCTCNN with well-known algorithms from the literature [16] on the 4 problems. The column labeled Heuristic1 refers to the heuristic 1, while the column labeled Heuristic2 refers to the heuristic 2 in literature [16]. GCTCNN is better than the heuristic1 for the selected problems and equivalent to the heuristic2.

The hybrid TCNN system features a significant number of parameters, but the parameters are robust when the number of the nodes is same. Table III lists the routes for En22k4 using HTCNN algorithm, the parameters of β and A, and the last results.

TABLE III

OPTIMAL ROUTES FOR EN22K4 USING CTCNN ALGORITHM

Routes	Optimal routes	β	A	Written routes
Route 1	17-20-18-15-12-0	0.005	0.01	0-17-20-18-15-12-0
Route 2	21-14-0-16-19	0.005	0.1	0-16-19-21-14-0
Route 3	3-8-10-0-13-11-4	0.007	0.01	0-13-11-4-3-8-10-0
Route 4	6-0-9-7-5-2-1	0.005	0.01	0-9-7-5-2-1-6-0

Fig. 1 shows the time evolution of some neuron outputs with $\alpha = 0.015, \beta = 0.003$. TCNN behaves erratically and unpredictably during the first 250 iterations and eventually converges to a stable fixed point around iteration 380 through the reversed period-doubling bifurcation.

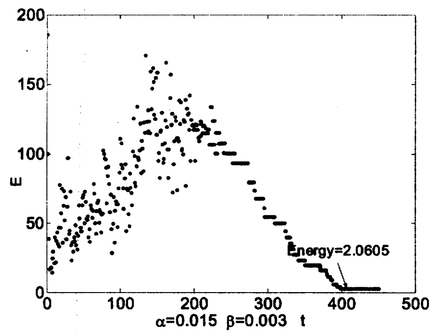

Fig. 1. Time evolution of energy function for 9 demand points

V. CONCLUSIONS

A new genetic clustering-based TCNN algorithm for solving multi-vehicle routing problems has been presented. The method converts a CVRP into several SVRPs by deciding the fleet size. An improved genetic clustering method is used to assign the problem effectively. SVRP is solved by setting the depot as a virtual demand node. TCNN is utilized for global searching and converging quickly to a stable equilibrium point. The computation result shown the proposed hybrid genetic approach is competitive with the known-published methods. The proposed method can obtain approximately optimal solutions in the fixed iterational steps. Extension of this approach to more complex cases like stochastic scheduling problems is a promising subject for further research.

Future work will be conducted to further improve the proposed algorithm. Existing alternate metaheuristic features and insertion procedures including techniques explicitly designed for the capacitated VRP will be examined to enhance genetic operators while reducing computational cost.

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