A New Algorithm for Vehicle Routing Problems with Capacity Limited Based on Minimum Spanning Tree

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***Abstract*—An optimized strategy is proposed to investigate the vehicle routing problems. We build here a Conversion of the Complex road network with minimum spanning trees and decision to search the nearest approach. An algorithm is developed to obtain min-spanning from spanning trees. By tagging the min-spanning tree, we obtain the optimized route such that the cycle capacity keeps minimum value. The corresponding algorithm is realized with the Development tool of Jbuilder 9, which shows that our algorithm proposed here is valid and concise, and simplifies the complexity of previous results.**

Keywords- distribution delivers; the minimum spanning tree; distribution node; distribution routing; vehicle routing problem (VRP)

I. INTRODUCTION

Modern logistics, as an advanced organization and management of technology, is widely considered as a major source of profit in addition to reducing consumption of goods and improving labor productivity[1]. So it has important theoretical value and practical significance to either study the high-performance storage and flow of goods, services and its correlative information from the original places of production to consumptive destinations, or plan effective implementation and control by computer applicable technology in the existing logistics system.

Logistics research includes Location Problem(LP), Vehicle Routing Problem(VRP)and the Inventory Control Problem(ICP). The VRP is investigated in this paper which was first proposed by G. Dantzig[2] in 1959. The problem can be stated that given the location of distribution center, customer demand as well as location of the road network, schedule and arrange vehicle's running path such that costs of the whole system of running is minimums, while service qualities are all satisfied. VRP is an NP problem. The solutions avail to the VRP composite of precise algorithm [3,4], heuristic algorithm[5,6] and meta-heuristic algorithm. Precise algorithm includes the branch demarcation algorithm and dynamic programming algorithm etc. Precise algorithm can be only suitable to small-scale VPR. The solution to the VRP by the meta-heuristic algorithm is now a hot research, which includes mainly genetic algorithm [7,8],ant colony algorithm [9,10]. The genetic algorithm is not relying on the low level of field knowledge or the restrictions to the search space, and no necessary assumptions of the continuous derivable or a single peak objective function are needed. Furthermore, the approach is also of ability of self-organization, self-adaptive and high parallelism in the process of solution, as well as strong commonality and robustness. Ant colony algorithm using positive feedback parallel catalytic mechanism, has an excellent distributed computation mechanism, and is easily to combine with the other method. Other representatives research, such as Simulate Anneal Arithmetic(SAA), Tabu Search Algorithm(TSA), see the reference of [11,12]. In the above methods to solve large-scale VRP problem, there are still some problems such as slow convergence speed or converge at a local optimal point easily. At present, the research with the minimum spanning tree to solve the VRP is not reported yet.

In this paper, to seek the optimal path, we propose a transfer strategy, construct feasible solution, update the information elements and turn traffic link map into a least tree or sub-tree generation. Finally, we combine these trees into a sub-minimum spanning tree which leads to determine the best distribution lines. The result shows that the proposed algorithm here is effective, and simplifies the complexity of the algorithm greatly.

II. DESCRIPTION OF THE PROBLEM

It has been interesting in the field of the transportation and distribution logistics how to arrange vehicles to carry from one or more distribution centers to geographically dispersed customers, such that the total cost gets the minimum value. We discuss here the VRP with vehicles capacity limited subject to customers' demands are met by one deliver center (SDVRP). General assumption of VRP is that vehicles will return to the distribution center from the distribution center after providing services for customers, which is referred as Closed VRP (CVRP), if the vehicles do not need to return to service centers, it is said to be Open VRP(OVRP), We consider the former closed SDVRP(CSDVRP).

Let G = (V, E) be a directed graph with n + 1 vertexes, V is set of vertex nodes of the graph, while E is sets of the edges, and $E = (<v_i,v_j> (v_i,v_j \in V, i \neq j)$. Denote $V = (v_1,v_2, \ldots\ldots,v_n)$, where v_0 is the distribution center, $v_1 \sim v_n$ are n distribution vertex nodes respectively, whose the amount of goods demand are q_i $(1 \leq i \leq n)$. We presume that the supply capacity of distribution centers is C,there are m transport vehicles availed in the distribution center, the greatest distance and the load of each vehicle are denoted as L_i, $W_i(i = 1, 2, \ldots\ldots, m)$ respectively. the R_k, $(1 \leq k \leq m)$ denotes transport vehicle's running path, whereas N_k is the total customers to service on the R_k. R^i_k is the ith customer of in the route R_k. Set vector (X_i,Y_i) to be the coordinates of the

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vertex v_i, define the distance between v_i, v_j as $d_{ij}= \| v_{ij} \|$, $\|\bullet\|$ is (•)Euclide norm. The model with vehicles capacity limited of the CSDVRP can be described as follow:

$$\min\ F = \sum_{k=1}^{m}\sum_{i=1}^{N_{k-1}}(d_{r_k^i r_k^{i+1}} + d_{r_k^{Nk}0}+d_{0r_k^i}) \tag{1}$$

$$\text{subject to} \quad \sum_{i=1}^{n} q_i \le C \tag{2}$$

$$\sum_{i=1}^{N_{k-1}}(d_{r_k^i r_k^{j+1}} + d_{r^{N_{k0k}}} + d_{0r_k^j}) \le L_k \tag{3}$$

$$\sum_{i=1}^{N_k} q_i \le W_k, (k = 1,2,..., m) \tag{4}$$

$$\sum_{i=1}^{n} N_k = n \tag{5}$$

$$(R_i \setminus v_0) \cap (R_j \setminus v_0) = \Phi \tag{6}$$

Where the equation (5) and (6) is used to ensure that each customer to be served by and only by one car.

III. OPTIMIZATION STRATEGY AND ITS ALGORITHM

A. Optimization strategy

See an example shown in Figure1, where v_0 is the distribution center, from which goods delivers to v_i and v_j ($i \neq j$). Denote the distances between v_0 and v_i or v_j as d_{0i} and d_{0j}, and the distance between v_i and v_j is $\| v_{ij} \|$. We develop two delivery strategy as shown in Figure 1 (a) and Figure 1 (b):

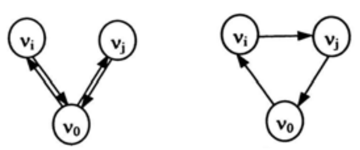

Figure 1. (a) (b)

In Figure1(a), the delivers route distance is $2^* \| v_{0i} \| +2^* \| v_{0j} \|$,while it is $\| v_{0i} \| + \| v_{0j} \| + \| v_{ij} \|$ in Figure1(b). It is obvious that the Figure1 (b) is lower cost than the Figure1 (a) since $\| v_{0i} \| + \| v_{0j} \| - \| v_{ij} \| > 0$. In other words, in permit of the vehicle's capacity, the more the customers exist in the route, the less mileages costs, which imply the responding deliver route is more reasonable.

B. An Algorithm

As an illustrative example as shown as in Figure 2 (a) , the transport networks consist of vertex nodes labeled v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8,v_9. The numbers along the edge denote distances between nodes. Provided distribution center that may be arbitrary to appointed, the algorithm can be constructed as follow:

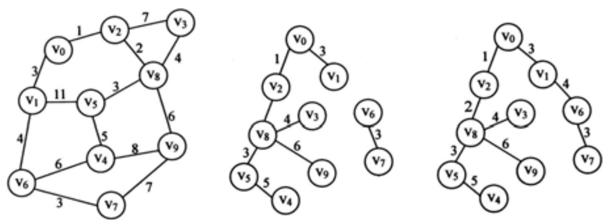

Figure 2. (a) (b) (c)

1) The construction algorithm of the least spanning sub-tree

(1) Set node v_0 (the distribution center) to be the root of tree, in brief, it is called TRN ;

(2) Set the distribution center node as the current node, it is called CN for short, CN: $\Leftarrow$ TRN;

(3) Count the numbers (named as NNN)of the adjacency nodes to CN, if NNN> 0 goto (4) else goto (6);

(4) Find out the shortest distance adjacency node(named as DSNN) to the current node CN, record the shortest distance(named as SD) and connect DSNN with the CN into a side expressed by ternary vector groups < CN, DSNN, SD>. Check whether the vector < CN, DSNN, SD> exists in sets of sub-tree or not, if it is true then goto (5); when the sub-tree only contains the vector <CN,DSNN>, then add the vector < CN, DSNN, SD> to this sub-tree set, and goto (5); otherwise produce a new sub-tree sets named DSNN to which add the vector < CN, DSNN, SD>; set DSNN to CN, then goto (3);

(5) Choose any node not marked in the nets, set it to CN, goto (3); if all nodes are marked in the traffic nets, then turn to (7);

(6) Label the chosen isolated node in step 5, and insert the vector <CN,CN,∞> into the new sub-tree, goto (5);

(7) Find out all isolated nodes that is similar to the vector <CN,CN, ∞>from all sub-trees, and delete all the isolate nodes that can not be arrived to from the distribution center. Now we get all sub-trees generation shown in Figure 2 (b).

2) Construction from the least spanning sub-tree to the minimum spanning tree

In section 3.2.1, we obtain the least spanning sub-tree, in this section, we will modify the algrithm to get the minimum spanning tree. The details steps can be summed up as follow:

(1) Calculate the number of sub-tree , named as SubTreeNumber in Figure 2 (b);

(2) If SubTreeNumber <2, then goto (9) else goto (3)

(3) For (I = 1; I <(SubTreeNumber-1); I + +) do

(4) Take out the sub-tree I;

(5) For (Num = I +1; Num <SubTreeNumber; Num + +) do

(6) choose the sub-tree numbered Num, check if it is directly linked to the sub-tree I , if it is false then goto (5) or calculate to determine the adjacency node named k that is nearest to node I, forthermore record the results with the vector <I.node[k],Num.node[m],SD> into the array of inter-tree, then goto (5);

(7)End the loop Num, compare with the sub-tree numbered I and other sub-trees to find out the sub-tree that is nearest to node I, record this sub-tree and its neighbor nodes to the shortest array of inter-tree, goto (3);

(8)End the loop I , find out the two shortest distance sub-trees among the shortest array of inter-tree, connect the shortest directly adjacent nodes to form a sub-tree, set SubTreeNumber $\Leftarrow$ SubTreeNumber-1,goto (2)

(9) The minimum spanning tree obtained is shown in Figure 2 (c)

C. Optimization of the minimum spanning tree and Tagging node

The approach by transferring the actual traffic map into a minimum spanning tree, based on which to determine deliver route, often results in mistakes. See an example shown in Figure 2 (c) , it is needed to carry goods to node v_4 and node v_7 that are located at different sub-trees that merge with the same root node v_0. According to the definition of the minimum spanning tree(MST), if one first delivers goods to the node v_4 (or v_7), returns to the root node v_0, and then from the root node to another node v_7 (or v_4), at last back to the root v_0, by simple calculation with MST , one gets the running distance equals to $2* \| v_{04} \| +2* \| v_{07} \| = 2*3$ (1 +2 +2 +5) +2*3 (3 +4 +3) = 40. Instead, if one first goes to node v_4 then to node v_6,but not return to the root node v_0, from node v_6 to node v_7, and then return to the root node v_0, the running distance is $2* \| v_{04} \| +2* \| v_{07} \| +6 +3 =$ (1 +2 +2 +5) +6 +3 + (3 +4 +3) = 29, it is clearly that the distance is shorter than the former method. So, it is necessary to modifying algorithm for the minimum spanning tree :

(1) For the minimum spanning tree shown in Figure 2 (c), reinstate the tree braches between sub-trees below root node v_0, dot them. As shown in Figure 3 (a) there are only three braches: connected graph edges $<v_1,v_5>$, $<v_9,v_7>$, $<v_4,v_6>$.

(2) Calculate the distances from each node belong to the dotted line to the root node respectively, for example, for the vector edge $<v_1,v_5>$; as to $\| v_{01} \| = 3$; $\| v_{05} \| = 6$; one gets $\| v_{01} \| + \| v_{05} \| = 9$, while one gets also $\| v_{15} \| =11$.assume that $\| v_{15} \| < \| v_{01} \| + \| v_{05} \|$,which implies that the distance between node v_1 and node v_5 is short than the distance calculated from the search by the minimum spanning tree, reserve this side, or delete the vector edge $<v_1,v_5>$ permanently. Complete all the calculations for the dotted sides respectively, one observes that vector edge $<v_1,v_5>$ has been deleted, the tree becomes that as shown as in Fig.3 (b);

(3)Mark each node exception for root node v_0, in Figure 3 (b), tag the distance number between each node and root node v_0. The minimum spanning tree obtained as shown in Figure 3 (c).

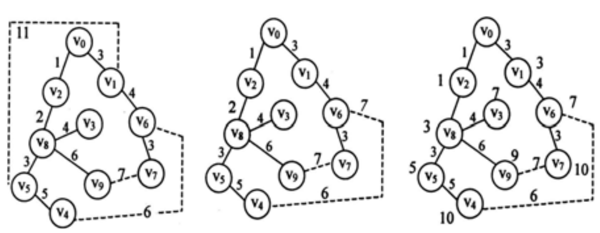

Figure 3. (a) (b) (c)

D. Determination of the deliver route

It is the best delivery route to distribute goods from center (root nodes)to any given place numbered vi, the path from center to customer and back along the same way is easily fund out by the tree depth-first traversal algorithm of a single point of delivery, It is obvious that the path ensure that a single point of distribution is optimized. It is worthy mentioned that A*-algorithm, Floyd and Dijkstra algorithm are avaible to find best route.

The above algorithm, after being modified, can also be used to solve OSDVR, the detail algorithm for OSDVR can be described as follows:

Step1: locate nodes on the minimum spanning tree

(1) Define a set that includes all child nodes of the root node; name the set with child's name;

(2) Gather all nodes to deliver as a set, which is the initial set named InitSet;

(3) Take out any node in InitSet , assign whose name to Name, delete the node in InitSet;

(4) Use the algorithm of depth-first traversal to search the node Name, and justify which child of the root it belongs to and super add this node to the corresponding child set;

(5) Determine whether the set is empty or not, if it is not empty then goto (3), else goto Step2;

Step2: Seek out the nodes that are in the same path;

By the depth-first traversal algorithm, traverse the minimum spanning tree to find out those nodes that are on the same path from each node of each child set, (for example in figure 3(c), v_2,v_8,v_3 are in the same path as shown), then incorporate these nodes into a new subset;

Step3: Determine the optimized route for multi-point deliver

Judge each child set is empty or not, if it is true, then remove this set. The algorithm for multi-point distribution can be described as follows:

(1)Count the number n of child set, case n> 1,goto (4), case n = 1, goto (2), case n = 0, goto (6);

(2)Traversal every distribution node by the algorithm of depth-first traversal, then back to the root along the same way, which forms a path, which is the optimal deliver route;

(3)Search whether there exist direct path among the sets, and if not, goto (2);

(4)Check all sets, remove the nodes and subsets which have not direct interpenetrating path;

(5)Restore each son set into a transport links map in form as shown as in Figure 2 (a), and then repeat the steps of 3.1, 3.2 and 3.3 until a triangle or direct line appears in form as shown as in Figure 1 (b);

(6)Determine the distribution path between the sub-trees as well as that of each sub-tree, the optimized deliver path is thus obtained for multi-point distribution.

IV. CONCLUSIONS

A new approach to searching optimized distribution route is presented, which turns the complicated network into a visual spanning tree that is used to find out the best deliver path. Our method greatly reduces the complexity of the previous algorithm, and characterizes in visual. The decision stratagem depends on only shortest distance, so it is convenient to be used to achieve the optimized deliver path armed at minimum cost. The operation on the running environment of JBuilder 9 shows that cargo needed can not only be delivered in the shortest time which reduces the cost, but also the important information such as the average running time through each road, the average cost .

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