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USING RANDOMIZATION TO SOLVE THE DETERMINISTIC SINGLE AND MULTIPLE VEHICLE ROUTING PROBLEM WITH SERVICE TIME CONSTRAINTS

Jairo R. Montoya-Torres

Escuela de Ciencias Económicas y Administrativas
Universidad de La Sabana
Autopista norte de Bogotá, D.C.
Chía (Cundinamarca), COLOMBIA

Edgar H. Alfonso-Lizarazo

Facultad de Ingeniería
Universidad de La Sabana
Autopista norte de Bogotá, D.C.
Chía (Cundinamarca), COLOMBIA

Edgar Gutierrez Franco

Facultad de Ingeniería
Universidad de La Sabana
Autopista norte de Bogotá, D.C.
Chía (Cundinamarca), COLOMBIA

Ana X. Halabi

Escuela de Ciencias Económicas y Administrativas
Universidad de La Sabana
Autopista norte de Bogotá, D.C.
Chía (Cundinamarca), COLOMBIA

ABSTRACT

This paper considers the deterministic vehicle routing problem with service time requirements for delivery. Service requests are also available at different times known at the initial time of route planning. This paper presents an approach based on the generation of service sequences (routes) using randomness. Both the single-vehicle and the multiple-vehicle cases are studied. Our approach is validated using random-generated data and compared against the optimal solution obtained by mathematical programming for small-sized instances, as well as against known lower bounds for medium to large-sized instances. Results show that our approach is competitive with reference to the value of the objective function, and requires less computational time in comparison with the exact resolution procedure.

1 INTRODUCTION

The vehicle routing problem (VRP) has received an immense attention from the scientific community during the last three to four decades. Since the first time the problem was presented in the scientific community by Dantzig and Ramser (1959), it plays a vital role in the design of distribution systems. Basically, the classical VRP consists of designing routes for a set of vehicles that are to requested to service at the lowest cost a set of geographically dispersed customers. In order to approach the basic version of the problem to more real-life contexts, several restrictions have been added: service time windows, pickup and deliveries, backhauls, etc. (Cordeau et al. 2005). The most widely studied vehicle routing problems are the Capacitated Vehicle Routing Problem (CVRP) and the Vehicle Routing Problem with Time Windows (VRPTW). These are surveyed by Laporte and Semet (2002), and Cordeau et al. (2002a).

This paper considers the classical Vehicle Routing Problem (VRP) with multiple uncapacitated homogenous vehicles (Laporte 1992, Toth and Vigo 2001, Golden, Raghavan and Wasil 2008). Formally, the classical VRP is defined on an undirected graph $G = (V, E)$ where $V = \{v_0, v_1, \dots, v_n\}$ is a vertex set and $E = \{(v_i, v_j): v_i, v_j \in V, i < j\}$ is an edge set. Vertex v_0 is a depot at which are based m identical (homogeneous) vehicles of capacity Q, while the remaining vertices represent customer or clients. A non-negative cost, distance or travel time matrix $C = (c_{ij})$ is defined on E. Each customer has a non-negative demand q_i and a non-negative service time t_i. The VRP consists on designing a set of m vehicle routes of least cost, each starting and ending at the depot, and such that each customer is visited exactly once by a vehicle, and the total demand of any route does not exceed the load capacity of vehicles. The VRP is a hard combinatorial optimization problem. Exact solution procedures have been proposed (Naddef and Rinaldi 2002, Baldacci, Hadjiconstantinou and Mingozzi 2004) but they can only solve relatively small instances and their computational time are highly variable. To this day, heuristics remain the only reliable approach for the solution of practical instances. Several literature surveys have been recently published on the use of heuristics algorithms to solve the VRP (Laporte and Semet 2002, Gendreau, Laporte and Potvin 2002, Cordeau et al. 2002b, Cordeau and Laporte 2004, Cordeau et al. 2005).

In this paper, we consider the version of the VRP in which vehicles remain inactive during a certain amount of time when a client is been serviced (i.e., there are service time requirements) and customers impose a lower bound on the service time (i.e., service at customer i cannot start before time r_i). We consider both the problem with a single vehicle and the problem with more than one vehicle to satisfy customer requests. Since the classical VRP is known to be NP-hard problem, our problem is at least that difficult. We propose an alternative resolution procedure based on randomness to solve the problem. Using an analogy with the problem of scheduling production tasks in homogeneous parallel machines with setup times, we design and test a solution procedure that randomly generates and compares several vehicle routes.

The remainder of this paper is arranged as follows. Section 2 presents in detail the proposed random-insertion algorithm. Section 3 is devoted to test the procedure on random-generated data taken from the literature for both the single and the multiple vehicle cases. This paper ends in section 4 by presenting some concluding remarks.

2 PROPOSED SOLUTION PROCEDURE

The advances in the development of pseudo-random number generators (RNGs) (L'Ecuyer 2006) might have opened new perspectives in the use of random-based solution approximation schemes for hard combinatorial optimization problems (Juan et al. 2009a). To test how random number can be used to solve such problems, we decided to implement such strategy to the VRP with service time requirements and constrained service time. This problem has been little studied in literature. The approach presented in this paper combines the design of random vehicle routes, since random techniques have proved to be extremely useful for obtaining numerical solutions to complex problems which cannot be efficiently solved by using analytical approaches (Juan et al. 2009b). Similar approaches have used in literature. Buxey (1979) was probably the first author to combine Monte Carlo Simulation with the Clarke-and-Wright algorithm to develop a procedure for the Capacitated Vehicle Routing Problem (CVRP). This method was revisited by Faulin and Juan (2008), who introduced an entropy function to guide the random selection of nodes. Other works include those proposed by Fernandez et al. (2000) and Juan et al. (2009a) to solve the basic CVRP. Juan et al. (2009a) were the first authors that used random-generation schemes to solve the VRP with service time requirements. Their study also considered capacity constraints and maximum loading of each vehicle.

In such context, the contribution of this paper is to introduce some random behavior so that random feasible solutions are obtained each time the randomized procedure is executed Then, just by considering a limited number of iterations, a set of different feasible solutions are generated. Each of these feasible solutions will consist of a set of roundtrip routes from the depot that, altogether, satisfy all problem constraints and node demands. Finally, the best solution is selected. Another highlight of our approach is that is can work without introducing not too many parameters in the algorithm design.

The solution approach presented in this paper is based on the work of Montoya-Torres et al. (2009), who proposed a random-insertion heuristic procedure for a parallel machine scheduling problem with setup times. Using an analogy to this production scheduling problem, the procedure proposed in this paper is based on the generation of random numbers for the balanced assignment of customers to service sequences of each uncapacitated vehicle. Figure 1 describes the procedure in detail.

3 EXPERIMENTS

3.1 Data sets

In order to analyze the performance of proposed solution procedure, experimental studies were conducted on a PC Pentium Dual-Core 1.73GHz bi-processor. Exact solution methods were programmed using X-press IVE while the proposed heuristic was programmed using Visual Basic for Applications (VBA) in MS Excel spreadsheets. Data was generated using the same data sets of Montoya-Torres et al. (2009). Integer travel costs/times were generated from a uniform distribution $[0, \min t_i]$, with t_i being integer service times generated from a uniform distribution [1, 100]. Integer available dates of service requests were generated using a uniform distribution $[0, \alpha \times n]$, where n is the number of clients to be serviced and α is a real number with values 0.6, 1.5 and 3.0. Five instances for each of value of α were generated. Problems with 10, 20, 50 or 100 clients were considered. Experiments were run with equal and unequal service request available dates. A full factorial experimental design gave a total of 120 testing scenarios. Because of the random behavior of the proposed solution procedure, 10 replications for each instance scenario were run and the best sequence (i.e., the sequence with minimum traveling cost/time) was registered and compared against the optimum value.

Initialization
Enter the number n of customers. Enter the number m of vehicles. For each customer, enter its service time t_i and its lower bound of service time r_i. Enter the matrix $C = (c_{ij})$ of travel times/costs for each pair of customers i and j, with $i \neq j$. Define the number of iterations ($niter$).
Algorithm 1. Compute the number of clients to be serviced by the vehicles. For the first $m-1$ vehicles, this bound is computed as $\lfloor n/m \rfloor$. The m-th vehicle has assigned the other customers. 2. Set $h = 1$, the first iteration. 3. Generate an integer random number R from an equilikely distribution between 1 and n. 4. Assign client R on the first vehicle with available positions in its route. If this client has already been assigned, repeat from step 3. 5. Repeat from step 3 until all customers have been scheduled in the routes. 6. Ensuring that service time constraints are respected, compute T^h, the maximum total travel duration for the routing plan of iteration h. 7. Do $h = h + 1$ and repeat from step 3 while $h \leq niter$ (that is, until the number of iterations is not reached). 8. Select the routing plan with $\min_h T^h$ (that is, select the route for each vehicle with minimum travel duration over all the iterations).

Figure 1: Description of the proposed solution procedure

3.2 Experiments with one vehicle

The first sets of experiments were performed assuming that there is only one vehicle to service customers. This configuration is equivalent to solve well-known Traveling Salesman Problem (TSP). We first all considered that service requests are available at the same time (i.e., supposing that $r_i = 0$ for all clients). Afterwards, the same values of both servicing and travel times were taken but in addition considering unequal integer non negative lower bounds for the time a service can start (i.e., $r_i \geq 0$). The performance of proposed heuristic was computed as the deviation from the optimal solution as:

$$\%dev = \frac{T(H) - T(OPT)}{T(OPT)} \times 100\% \tag{1}$$

where $T(OPT)$ is the optimal total travel time and $T(H)$ is the travel time obtained using the proposed heuristic procedure.

Table 1 summarizes the results obtained from the experiments for the single-vehicle case when clients have released their service requests at the same time (i.e. when $r_i = 0$ for all clients) and when $r_i \geq 0$. In both tables, $T(OPT)$ represents the average values of the optimal travel cost and $T(H)$ represents the average value of the total travel cost applying the proposed heuristic. The last column of both tables corresponds to the average value of the deviation from the optimal solution for each set of clients. For the case of equal release dates, our algorithm the average deviation from the optimal solution is 3.4%. When $r_i \geq 0$, the average deviation is 4.4% of the optimal solution. Analyzing the individual instances, in 4% of the cases the heuristic obtained the optimal total cost, while in 29% of the cases the value of the travel total cost was within a 2% of the optimal value.

Finally, it is important to note that the running time of the proposed procedure for small instances (10-client and 20-client instances) was less than 3 seconds, while the time required to run the experiments for large instances was between 20 and 30 seconds for 50-client instances and about 55 seconds for instances with 100 clients.

Table 1: Average total travel cost for experiments with $r_i = 0$ and $r_i \geq 0$ with one vehicle

Number of clients	Cases with $r_i = 0$			Cases with $r_i \geq 0$		
	Avg. $T(OPT)$	Avg. $T(H)$	Avg. $\%dev$	Avg. $T(OPT)$	Avg. $T(H)$	Avg. $\%dev$
10	535.1	552.0	3.2%	535.1	564.2	5.4%
20	1137.3	1195.9	5.2%	1137.3	1206.7	6.1%
50	2600.6	2685.2	3.3%	2600.6	2691.1	3.5%
100	5241.5	5346.0	2.0%	5227.5	5350.3	2.3%
	Average deviation over all tests		3.4%	Average deviation over all tests		4.4%

3.3 Experiments with multiple vehicles

A computational study was also performed using the same random-generated data as described previously but considering configurations with 3 and 5 homogenous uncapacitated vehicles. As for the single-vehicle case, because of the random behavior of the proposed algorithm, 10 replications for each instance scenario were run and the best sequence (i.e., the route with minimum total travel cost) was registered and compared against the optimum route.

As explained previously, the NP-completeness of this problem unable us to obtain optimal solutions without excessive computational costs even for small instances (Cordeau et al. 2005). A lower bound on the travel cost can be found, as shown by equation (2). Results of our experimental study are hence compared against a lower bound derived from the one proposed by Kurz and Askin (2001) for the identical parallel machine scheduling problem.

$$LB(T) = \max\{LB1, LB2\} \tag{2}$$

Where $LB1$ and $LB2$ are respectively:

$$LB1 = \frac{1}{m}\left\{\sum_{j=1}^{n}\left[t_i + \min_{i\in\{1,\ldots,n\}} c_{ij}\right]\right\} \tag{3}$$

$$LB21 = \max_i\left\{r_i + t_i + \min_{j\in\{1,\ldots,n\}} c_{ij}\right\} \tag{4}$$

The performance of the proposed heuristic was computed as the deviation from such lower bound as:

$$\%dev = \frac{T(H) - T(LB)}{T(LB)} \times 100\% \tag{5}$$

where $T(LB)$ is the lower bound of the total travel time and $T(H)$ is the travel time obtained using the proposed heuristic procedure.

Tables 2 and 3 present the summary of results, respectively, with equal and unequal jobs release dates. From these results, we can observe that the algorithm performs well, with reference to the percentage deviation from the lower bound of the travel cost: the average deviation, regardless of the number of clients, is 9.9% with equal release dates and 12.2% for the case with unequal release dates. In terms of the computational costs, the higher the number of jobs, the higher the time to find a solution. For the large instance in our tests (100 clients), the computational time was never higher than 12 seconds. For 10-client and 20-client instances, the CPU time was less than 1 second.

Table 2: Average total travel cost for the multiple-vehicle case with $r_i = 0$

Number vehicles	3 vehicles				5 vehicles			
Number of clients	10	20	50	100	10	20	50	100
Avg. $T(LB)$	178.3	379.1	866.9	1747.2	105.0	198.8	520.1	1048.3
Avg. $T(H)$	192.1	406.3	908.9	1792.0	122.5	252.8	560.1	1096.9
Avg. $\%dev$	7.7%	7.2%	4.9%	2.9%	16.7%	27.2%	7.7%	5.0%

Table 3: Average total travel cost for the multiple-vehicle case with $r_i \geq 0$

Number vehicles	**3 vehicles**				**5 vehicles**			
Number of clients	**10**	**20**	**50**	**100**	**10**	**20**	**50**	**100**
Avg. $T(LB)$	217.0	433.5	866.9	1747.2	160.5	258.4	520.1	1048.3
Avg. $T(H)$	218.4	512.9	916.3	1796.2	172.2	314.2	596.5	1114.2
Avg. $\%dev$	0.7%	18.3%	5.7%	3.2%	7.3%	21.6%	9.5%	6.7%

4 CONCLUDING REMARKS

This paper considered the vehicle routing problem with service time and a lower bound on the time start service. Because this combinatorial problem is NP-complete, this paper proposed a heuristic algorithm. The strategy of the algorithm is based on a random behavior to design vehicle routes. Computational experiments were performed in order to analyze the performance of the proposed heuristic. Data from literature were considered. The first test was performed with customer service demands with unconstrained starting time of service. The second set of tests considered a lower bound on the service starting time. For the single-vehicle case, compared against the optimal solution, the proposed heuristic performed very well giving routes with a total duration value no greater than the 10% of the optimum. In average, the proposed procedure was between about 2% and 6% of the optimal solution. The computational time was less than 2 seconds for small instances, and never higher than 1 minute for large instances (100 clients). The case with multiple vehicles was also studied. Results of the computational experiments showed that our algorithm performs well in comparison with the lower bound of the total travel duration. The average deviation from this bound was 9.9% and 12.2% respectively for unconstrained and constrained cases with service starting time, regardless of the number of clients.

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AUTHOR BIOGRAPHIES

JAIRO R. MONTOYA-TORRES is an Associate Professor, Director of the Master in Operations Management program and Director of Logistics Management and International Marketing undergraduate program at the School of Economics and Management Sciences, Universidad de La Sabana, Chía, Colombia. He holds a Ph.D. degree from École des Mines de Saint-Etienne, France. His research interests are on modeling and simulation of production and logistics systems, production scheduling and supply chain management and design. His work has appeared in international journals and proceedings. He has acted as Guest-Editor for international journals. In 2009 he has been invited research within the Chair in Supply Chain of Ecole Centrale Paris, France, and the Center for Transportation and Logistics (CTL) of the Massachusetts Institute of Technology, Cambridge, USA. He can be contacted by e-mail at <jairo.montoya@unisabana.edu.co>. His personal webpage is <http://jrmontoya.wordpress.com>.

EDGAR ALFONSO LIZARAZO is full time professor at the Faculty of Engineering of Universidad de La Sabana, Chia, Colombia. He holds a Bachelor of Science in Industrial Engineering from Universidad Distrital Fransisco José de Caldas, Bogotá, Colombia, and a Master in Industrial Engineering (Operations Research and Statistics) from Universidad de los Andes, Bogotá, Colombia. His research interests include optimization of logistics systems and Data Envelopment Analysis (DEA). His email is <edgar.alfonso@unisabana.edu.co>.

EDGAR GUTIERREZ FRANCO is full time professor at the Faculty of Engineering of Universidad de La Sabana, Chía, Colombia. He holds a Bachelor of Science in Industrial Engineering from Universidad de La Sabana and a Master in Industrial Engineering (Operations Research and Statistics) from Universidad de los Andes, Bogotá, Colombia. His research interests include optimization of logistics systems and sustainable supply chain management. He has been invited research at the Center for Transportation and Logistics of the Massachusetts Institute of Technology, Cambridge, USA. He can be contacted by email at <edgar.gutierrez@unisabana.edu.co>.

ANA X. HALABI is Assistant Professor and Head of Logistics at the School of Economics and Management Sciences, Universidad de La Sabana, Chía, Colombia. She holds a Bachelor of Science in Industrial Engineering and a Master of Science in Hydro-systems both from Pontificia Universidad Javeriana, Bogotá, Colombia. She is currently enrolled within the doctoral program in Economics and Administrative Sciences at Macquarie University, Australia. Her research interests include expert systems for decision-making and analysis of logistics systems. Her e-mail is <ana.halabi@unisabana.edu.co>.