A Hybrid Intelligent Algorithm for Multiple Capacitated Vehicle Routing Problem

Hua-li Sun[1], Wen-Juan Li[1], Yao-feng Xue[2*]
1. Management School, Shanghai University, Shanghai 200444, China
2. Distance Education College & e-Educational System Engineering Research Center, East China Normal University, Shanghai 200062, China.
*Email: yaofeng.xue@163.com

***Abstract*—A new hybrid particle swarm optimization （HPSO） algorithm which is based on particle swarm optimization algorithm and simulated annealing algorithm is proposed in this paper for solving multiple capacitated vehicle routing problem. The basic scheme consists in Particle swarm optimization with time-varying parameters and simulated annealing with memory and tempering features. Results from the computational experiment shows that the hybrid particle swarm algorithm is effective.**

I. INTRODUCTION

The capacitated vehicle routing problem (CVRP) is a problem introduced by Dantzig and Ramser in 1959[1], it is NP-hard as it is a natural generalization of the Traveling Salesman Problem (TSP), and even the simplest model has a combination of explosion problem as the dimension increases. So the evolutionary computing methods have been applied for CVRP to find a near optimal solution in a reasonable amount of time, there are genetic algorithm, Tabu Search, Simulated annealing algorithm, ant colony optimization, particle swarm optimization and Dynamic Evolutionary Algorithm and so on. Hybrid algorithm is a combination of two or more heuristics which is a fast, efficient method, and now is one of the most widely common algorithms. Particle Swarm Optimization(PSO) is proposed by Kennedy and Eberhart in1995[2], it is a population based evolutionary method that mimics the behavior of flock of birds or fish. Simulated Algorithm(SA) is proposed by Kirkpatrick in 1993[3] to solve optimization problems. Simulated annealing which is to search the solution space through simulating annealing procedure in metallurgical operations is a class of local search algorithms which known as threshold algorithms. In this paper, particle swarm algorithm convergence is fast but poor local search ability, while the simulated annealing local search capability, but easy to fall into local optimum characteristics, combine the two, proposed a new hybrid heuristic algorithm and that the CVRP algorithm is used to solve the problem.

II. PROBLEM DESCRIPTION

The vehicle routing problem is often described as the problem in which vehicles based on a central depot are required to visit geographically dispersed customers in order to fulfill known customer demands. The problem is to construct a low cost, feasible set of routes – one for each vehicle. A route is a sequence of locations that a vehicle must visit along with the indication of the serve it provides[4], The vehicle must start and finish its tour at depot.

To facilitate the mathematical description, we define a variable as follows:

$$x_{ij}^k = \begin{cases} 1 & \text{If client j is serviced next to client i by vehicle k.} \\ 0 & \text{others} \end{cases}$$

So the mathematical model can be created as follows [5]

$$\min \sum_{k=1}^{m} \sum_{j=0, j\neq i}^{n} \sum_{i=0}^{n} c_{ij} x_{ij}^k \quad (1)$$

$$\sum_{k=1}^{m} \sum_{j=1}^{n} x_{ij}^k \leq m \quad i = 0 \quad (2)$$

$$\sum_{j=1}^{n} x_{ij}^k = \sum_{j=1}^{n} x_{ji}^k \leq 1 \quad i = 0, \ k \in \{1, 2, ..., m\} \quad (3)$$

$$\sum_{k=1}^{m} \sum_{j=0, j\neq i}^{n} x_{ij}^k = 1 \quad i \in \{1, 2, ..., n\} \quad (4)$$

$$\sum_{k=1}^{m} \sum_{i=0, i\neq j}^{n} x_{ij}^k = 1 \quad j \in \{1, 2, ..., n\} \quad (5)$$

$$\sum_{i=1}^{n} d_i \sum_{j=0, j\neq i}^{n} x_{ij}^k \leq q_k \quad k \in \{1, 2, ..., m\} \quad (6)$$

where i ,($i \in \{1, 2, ..., n\}$)presents customer, d_i is client i 's demand, c_{ij} $(i, j \in \{1, 2, ..., n\}, \ i \neq j)$ is the cost between client i and client j , q_k $(k \in \{1, 2, ...m\})$ is the maximum capacity of the kth vehicle.

In the models, formula(1) is the objective function, it is represented by the sum of the minimum cost of transportation of vehicles, formula(2) is to say the maximum number of vehicles from the depot is m; formula(3) is to say vehicles must go out from the depot and go back to the depot after completing the mission; formula(4) and (5) are to say each client must be visited once; formula (6) is each vehicle's capacity restriction.

III. A HYBRID INTELLIGENT ALGORITHM FOR MULTIPLE CAPACITATED VEHICLE ROUTING PROBLEM

In this paper, a new hybrid optimization algorithm which is based on particle swarm optimization algorithm and simulated annealing algorithm is proposed. The basic

978-1-4244-5265-1/10/$26.00 ©2010 IEEE

principles of the hybrid optimization algorithm is relies on particle swarm optimization algorithm and simulated annealing algorithm. Initial particles are produced by distributed algorithm [6] for particle swarm optimization algorithm, and then every generation's particles are optimized by simulated annealing algorithm. We refer to the solution as solution individuals.

A. *Global particle swarm optimization algorithm*

In this paper, Global PSO algorithm [7] is adopted, specific form is as follows:

$$V_i = w \cdot V_{i-1} + c_1 \cdot r_1 \cdot (p_i - x_i) + c_2 \cdot r_2 \cdot (p_g - x_i) \quad (7)$$

$$x_i = x_{i-1} + V_{i-1} \quad (8)$$

where w is Weight which is to adjust the capacity of global search and local exploration of the particle swarm, Vi is particle i's velocity, c1, c2 are learning factors, Usually c1 = c2 = 2, r1, r2 are random numbers between (0, 1), Pi is the best solution (fitness) particle i has achieved so far, Pg is the best solution (fitness) all particles have achieved so far, xi is the location of particle i.

1) The initial particle

Usually the positions and velocities of particles in initial generation are generated randomly, in this paper, the initial particles are produced by distribution-type heuristic algorithm which is improved from cheapest insertion [7] to improve the efficiency. Firstly, every vehicle is assigned a customer randomly, and then the rest customers are assigned by the principle that the customer is assigned in the same vehicle with the nearest customer which has been assigned.

2) Particle code

Two-dimensional codes are used to express particle information in this paper, the first dimension represents clients, and the second dimension represents vehicles. customers and vehicles are presented by a serial number according to natural number, for example, the code{（1,2）（2,1）（3,1）} is to say the first customer is traveled by the second vehicle, the second and the third tasks are traveled by the first vehicle.

3) Preferences

The initial Wmax is set 0.9[8], and reduces to Wmin lineally which is set 0.4 according to formula (9). c1 and c2 are set 2.0, because low values allow particles being pulled back before wandering away from the target area, while the high ones lead particles suddenly rushed or rushed across the target area. Vi and the position x_i may fly to the outside of the target space, so the values of Vi and x_i should be limited within a certain range. As Vi may be positive or negative, it is set in [-s, s]; as the position must be positive, it is set in [1,m], they are both integer.

$$W = W\max - iter \cdot \frac{W\max - W\min}{iter\max} \quad (9)$$

4) Fitness function

Fitness function is the performance evaluation for particles, it indicates the relative superiority of the particles mapping the objective function value to. In this paper, the fitness function is presented by the total route length, so the fitness function (fit (c)) is defined as the objective function (F (c)). The smaller particle is superior and will be retained.

5) Revision of solutions

The particle's velocity which is generated according to Equation (7) is a rational number, and the particle's location is a rational number too. Because the location which is presented by a vehicle number is a integer, the new hybrid particle swarm optimization algorithm rounds off velocity to a integer.

B. *Improved SA algorithm*

Because basic simulated annealing algorithm has strong local searching abilities but is easy to fall into local trap, we add a memory function and tempering function in the hybrid algorithm [9]. It operates as follows.

1) Generation of the neighborhood solutions

The integral part of an annealing algorithm is its neighborhood generation scheme, on the basis of which different annealing algorithms are developed. In the simulated annealing algorithm, a small neighborhood can't complete global search, while a large neighborhood costs a good chunk of time. Considering all these factors, the neighborhood is generated randomly in this paper. Generate r1, r2 which are integers in [0, m] randomly, and change the customers which are at the position of r1, r2, then do evaluation through fitness for the results of exchange.

2) Cooling schedule

Annealing process is controlled by the cooling schedule in simulated annealing algorithm. Usually, perform a complete neighborhood search using the λ –interchange generation mechanism without, however, implementing any move.

The cooling schedule is commonly decreased continually, a linear declining is adapted and the decline rate is set 0.96 in this paper. In this paper, the initial temperature is set 20, the termination of the annealing temperature is set 0.1. Termination temperature has a direct impact on search granularity of SA, the smaller termination can make more careful search in the solution space. Length of the inner loop is made 1 according to the method of Neighborhood formation.

3) The memory function

As the simulated annealing procedure is random, when t reaches to a large value, some inferior solution will be accepted, the final result may be not the deepest extreme point. So a memory device to hold each of the minimum extreme points to make sure the final output is the minimum extreme point of history.

4) The back fire function

When t reaches to a very small value, the acceptance rate will be small, and the inferior solution is difficult to be accepted, so it is easy to fall into local optimum. To make the solution out of local "traps", we increase the temperature to a large value again when it reaches to the termination in order to escape from the "trap" of high wall. We have backfire temperature Tstart1 is 10, and the termination is 0.01.

C. *Specific steps of hybrid particle swarm optimization algorithm*

In this section, we provide a short description of the hybrid particle swarm optimization algorithm

Step1: Set parameters for hybrid particle swarm optimization algorithm;

Step2: Assign all customers for vehicles;

Step3: Allocate customers by particle swarm optimization with time-varying parameters;

Step4: Optimize routs by simulated annealing with memory and tempering features;

Step5：If the terminal condition of PSO is satisfied, then stop, else go to step3.

IV. COMPUTATIONAL RESULTS

The computational experiment has been performed on a Core Solo Processor personal computer at 1.83 GHz equipped with 1 Gb of RAM and run under Windows XP. The algorithm is implemented in C language.

TABLE I. RESULTS COMPARISON

Problem	A-n32-K5	A-n33-K5	B-n35-k5	P-n16-k8
Customers	31	32	34	15
Demands	100	100	100	35
Vehicle Capacity	5	5	5	8
r	0.82	0.89	0.87	0.88
HA Value	781	661	983	396
HA Time(s)	10.69	9.87	11.02	7.38
Hvalue	784	661	955	450

We have considered four classes of benchmark problems for test problems. The data of all instances are available at www.branchandcut.org/VRP/. For all problems reported in Table 1, the edge cij is the Euclidean distance between customers i and j. All the instances which truncate the Euclidean distance to its nearest integer are planar [10]. In the table 1, the ratio r provides a measure of the tightness of vehicle capacity constraints, HA value shows the hybrid algorithm's value which is described by the total route length and HA time shows computing time, the column Hvalue shows the best feasible solution value.

V. CONCLUSION

In this paper, A new hybrid particle swarm optimization （HPSO） algorithm which is based on particle swarm optimization algorithm and simulated annealing algorithm is proposed. Focusing on total traveled distance minimization, Particle swarm optimization with time-varying parameters is to allocate customers and simulated annealing with memory and tempering features is to optimize routs. Results from the computational experiment shows that the hybrid particle swarm algorithm is effective.

ACKNOWLEDGMENTS

This work was supported by the Innovation Program of Shanghai Municipal Education Commission (Grant No. 08YS21) and the Planning Program of Shanghai Philosophy and Social Science (Grant No. 2009EJB001).

REFERENCES

[1] Dantzig B, Ramser H. The truck dispatching problem. Management Science, 1959,6(1):80-91.

[2] Kennedy J, Eberhart R, Particle swarm optimization. Proceedings of the IEEE International Conference on Neural Network, Perth. 1995: 1942-1948.

[3] Kirkpatrick S, Gelatt C, Vecchi M. Optimization by Simulated Annealing. Science. 1983, 220 (4598):671-680.

[4] Bodin L, Golden B, Assad A, Michael O. Routing and scheduling of vehicles and crews. Computers and Operations Research. 1983,10(2):63-211.

[5] Golden B, Wasil E, Kelly J, Chao I. The impact of metaheuristics on solving the vehicle routing problem: algorithms, problem sets, and computational results. Boston: Kluwer. 1998:33–56.

[6] Christofides N, Eilon S, An algorithm for the vehicle dispatching problem, Operations Research Quarterly. 1969, 20(3):309-318.

[7] Jun L. Assignment heuristic Algorithm for Vehicle Scheduling Problem. Systems Engineering - Theory & Practice[J] .1999,1(1):27-33.

[8] Eberhart R, Shi Y, Particle Swarm Optimization, Applications and resources. Proceedings of IEEE Congress on Evolutionary Computation, Piscataway. 2001(1):81-86.

[9] Lishan K, etal. Non-numerical parallel algorithms (Volume I)[M], Science Press. 1994.4.

[10] Reinelt G. TSPLIB - A traveling salesman problem library, ORSA Journal of Computing, 1993, 3:376-384.