Answers to questions in Lab 1: Filtering operations

Name: Sai S. Kamat Program: TINVM

Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:

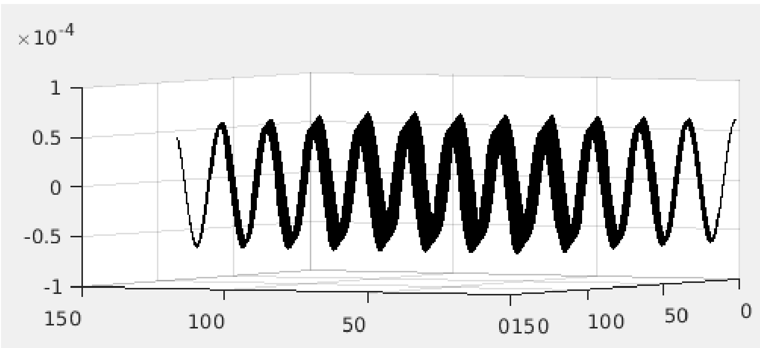
- amplitudes are constant
- frequency is directly proportional to the distance between (p,q) and the origin.

Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

a point in Fourier domain is projected in the spatial domain using inverse Fourier transform function, given by

$$f(m,n) = \frac{1}{\sqrt{MN}} \sum_{M=0}^{M-1} \sum_{N=0}^{N-1} \hat{f}(u,v) e^{2\pi i\left(\frac{mu}{M}+\frac{nv}{N}\right)}$$



Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

$$f(m,n) = \frac{1}{\sqrt{MN}} \sum_{M=0}^{M-1} \sum_{N=0}^{N-1} \hat{f}(u,v) e^{2\pi i\left(\frac{mu}{M}+\frac{nv}{N}\right)}$$

Amplitude :

$$amplitude = \frac{1}{\sqrt{MN}}$$

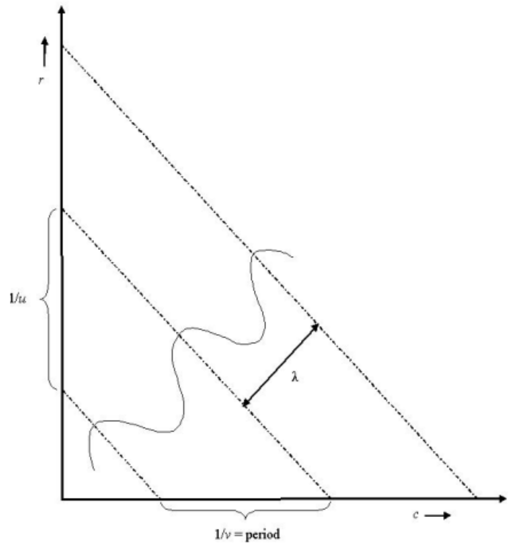
Code snippet:

```
amplitude = 1/sz;
```

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

Direction of travel depends on the relative position between the (p,q) and the origin.



- Directional dependency:The point P(p,q) is joined to the origin. The direction of the sine wave is the direction of the line connecting P to the origin.
- Wavelength dependency: (u, v) are the frequencies along (r, c) and the periods are 1/u and 1/v. Then wavelength of the sinusoid is

$$\lambda = \frac{1}{\sqrt{u^2+v^2}}$$

Mathematical Expression:

$$\omega_1 = 2\pi uc/M; \omega_2 = 2\pi nc/N$$
$$\lambda = 2\pi/\omega$$
$$|\omega| = \sqrt{\omega_1^2 + \omega_2^2}$$

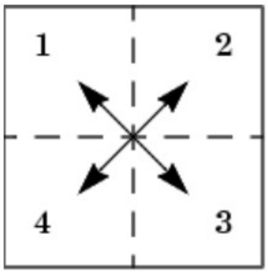
Code snippet:

```
w1 = 2 * pi * uc/sz; w2 = 2 * pi * nc/sz;
wavelength = 2 * pi / sqrt(w1^2+w2^2)
```

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:
A DFT is periodic & symmetrical complex conjugate. i.e. the spectrum repeats itself repeatedly, endlessly in both directions.
Periodicity: i.e. F(u,v) = F(u + kN, v + lN) where k, l ∈ [−∞, ..., −1, 0, 1, 2,..., ∞]. The N×N block of the Fourier coefficients F(u,v) computed from an N×N image with the 2D DFT is a single period from this infinite sequence.

Complex conjugation: $|F(u,v)| = |F(-u, -v)|$. Hence there are negative frequencies which are mirror images of the corresponding positive frequencies.



Windowing: The DFT is also an infinite data array, wrapping itself infinitely in both directions. The image is wrapped around itself, such that the left & the right sides, and the top & bottom coincide.

Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

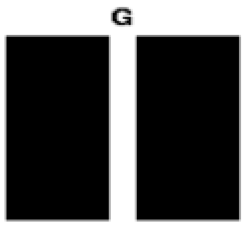
Answers:

Before the line, the image is visualized in the frequency domain in the interval $[0, 2\pi]$.
After the line, when fftshift() function is computed, the (p,q) coordinates are centered. Then the image is visualized in the frequency domain in the interval $[-\pi,\pi]$.

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

Consider an image like this:
Its pixels are zero everywhere else, except when
$\forall x$ & $xy \in (57,73)$



$$f(x,y) = \begin{cases} 1 & x \in (57,73) \\ 0 & elsewhere \end{cases}$$

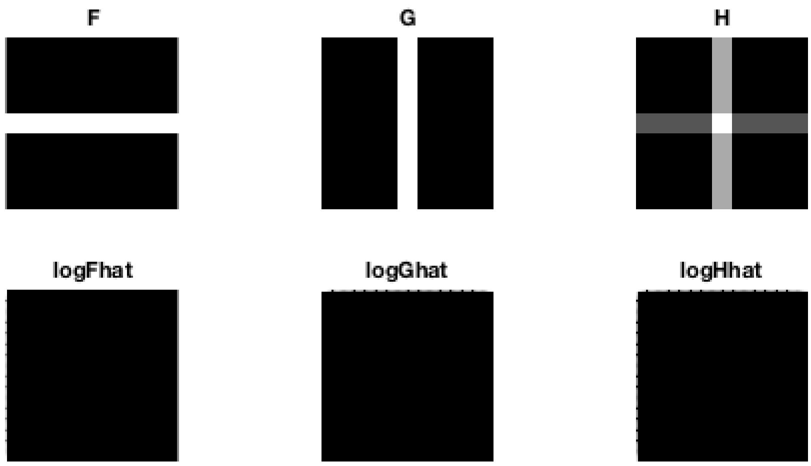
Question 8: Why is the logarithm function applied?

Answers:

- Pixel distribution is more focused on the lower range.
- Log transformation helps in expanding the range of low pixel values, and compress high pixel values
- This enhances the image contrast in the frequency domain.

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

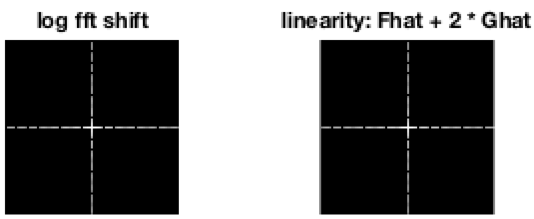
Answers:
In the images below:-



The spectrum of H consists of F, G spectra super-positioned. This implies linearity, given by following expression:-

$$\mathcal{F}\,[a\, f_1(m,n) + b\, f_2(m,n))] = a\,\hat{f}_1(u,v) + b\,\hat{f}_2(u,v)$$

a, b are scalars. F is Fourier transform operator on spatial images defined by f_1 and f_2.



This principle also applies to the log transformed visualizations. The one to the left, represents log shifted H image, while the one to the right, represents the same operation on the individual frequency domain of F & G's.

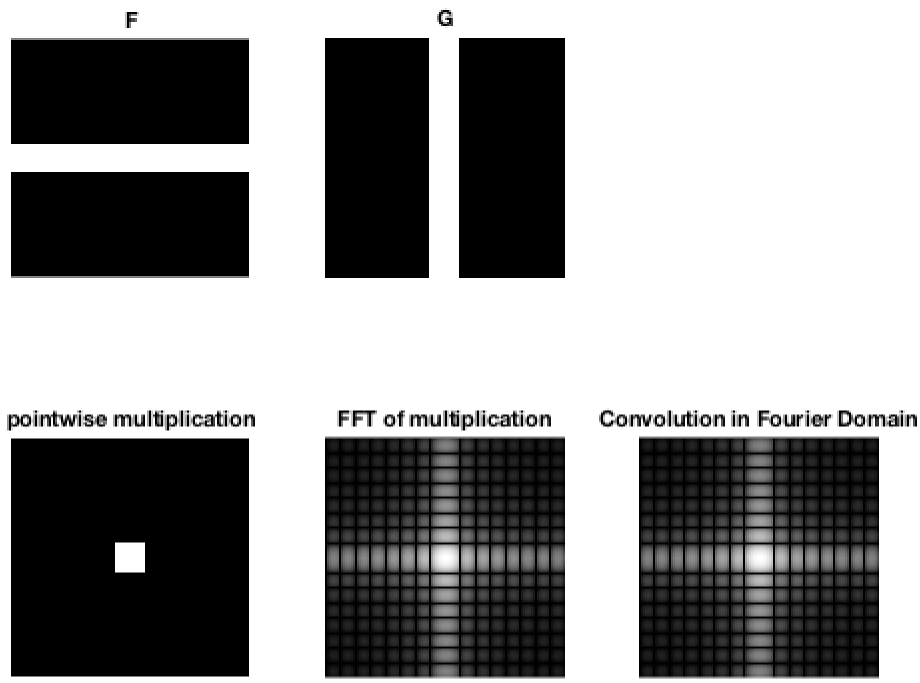
Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

The same image can be displayed using convolution.

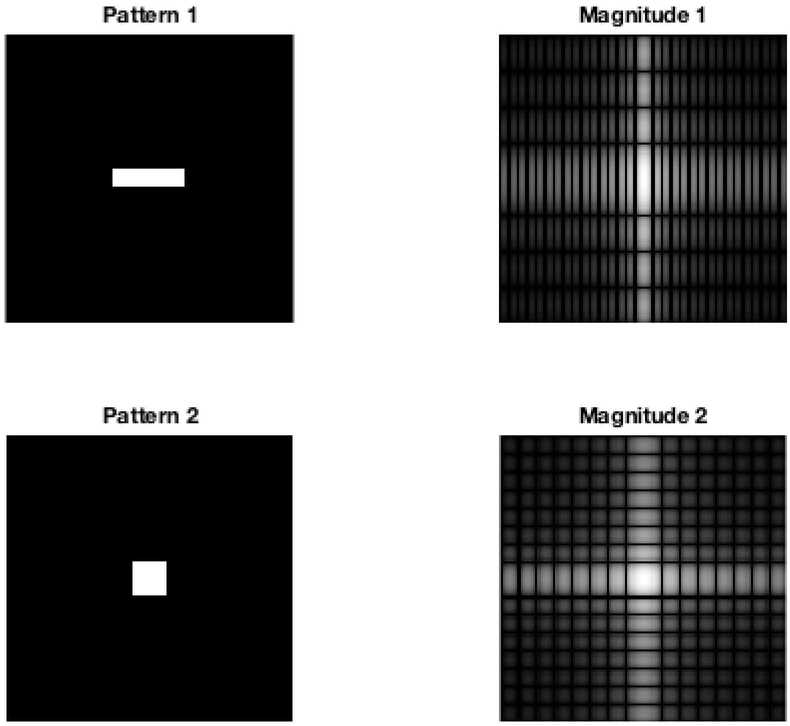
```
F = [ zeros(56, 128); ones(16, 128); zeros(56, 128)];
G = F';
subplot(2, 3, 1);
showgrey(F);
title('F')
subplot(2, 3, 2);
showgrey(G);
title('G')
subplot(2, 3, 4);
showgrey(F .* G);
title('pointwise multiplication')
subplot(2, 3, 5);
showfs(fft2(F .* G));%/128);
title('FFT of multiplication')

subplot(2, 3, 6)
Fhat = fft2(F)/128;
Ghat = fft2(G)/128;
Cs = conv2(fftshift(Fhat),fftshift(Ghat), 'same');
showfs(fftshift(Cs));
title('Convolution in Fourier Domain')
```



Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:



Pattern1 and 2 are spatial domain images. The length of the white region of Pattern 1 is halved in Pattern 2.
So, if Pattern1 := f(x,y) & Pattern2 := g(x,y).
The two are related as follows:-

$$g(x, y) = f((x/2), (2y))$$

i.e. in g(x, y), there's horizontal compression and vertical expansion.

By scaling property of Fourier transform, this translates to horizontal expansion and vertical compression. This is accordingly expressed by the magnitude spectra of the two images on the right.

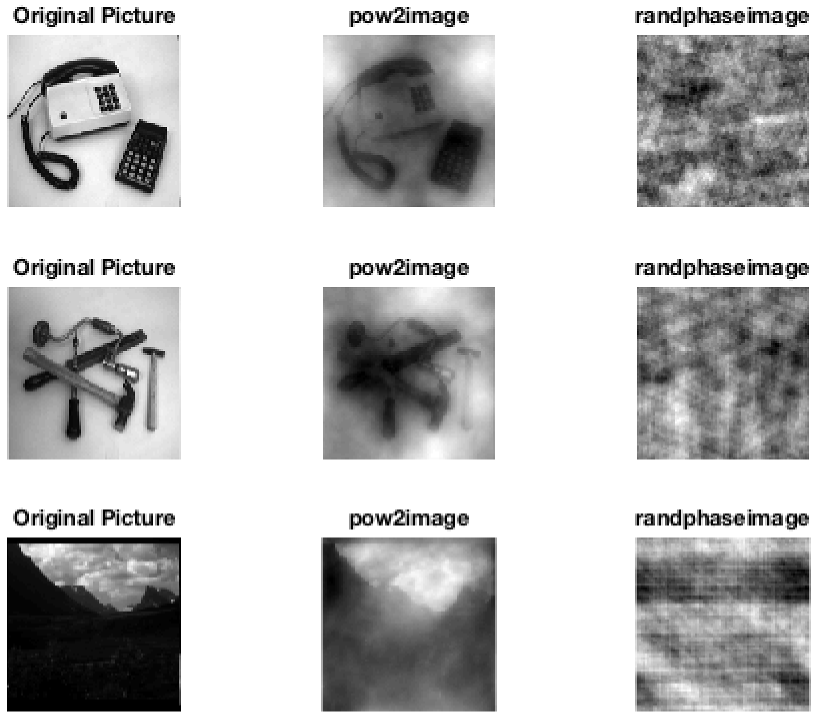
Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

An image rotated by angle Φ in spatial is also rotated by same angle Φ in the frequency domain. However, due to the rectangular orientation of pixels, the pixels lose their sharpness, when the angles are 30 degrees, or 60 degrees.

Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:



2nd column: Represents images obtained using the pow2image. The image's magnitude is the magnitude of the frequency components, and is reduced. The phase is retained. Hence it's discernible to the human brain.

3rd column: Here, the phases are randomly changed, but the magnitude is retained. Phases are the displacement of the basis functions from the origin. Phase contains much information about the position of the object in an image. The randphase function alters the position information, making it illegible to the human brain.

Question 14: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:

Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

Question 18: What conclusions can you draw from comparing the results of the respective methods?

Answers:

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:
