Answers to questions in

Lab 1: Filtering operations

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Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:

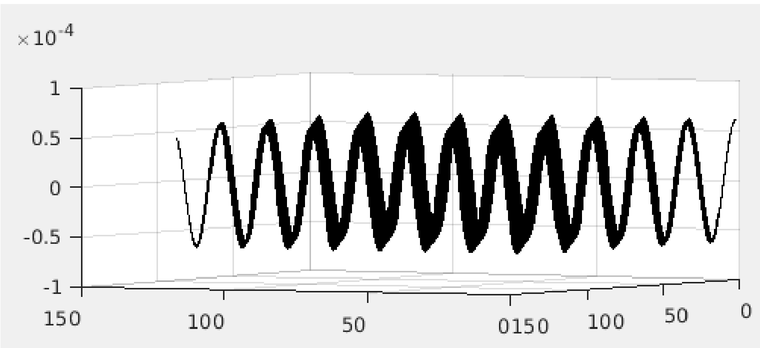
- amplitudes are constant
- frequency is directly proportional to the distance between (p,q) and the origin.

Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

a point in Fourier domain is projected in the spatial domain using inverse Fourier transform function, given by

$$f(m,n) = \frac{1}{\sqrt{MN}} \sum_{M=0}^{M-1} \sum_{N=0}^{N-1} \hat{f}(u,v) e^{2\pi i\left(\frac{mu}{M}+\frac{nv}{N}\right)}$$



Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

$$f(m,n) = \frac{1}{\sqrt{MN}} \sum_{M=0}^{M-1} \sum_{N=0}^{N-1} \hat{f}(u,v) e^{2\pi i\left(\frac{mu}{M}+\frac{nv}{N}\right)}$$

Amplitude :

$$amplitude = \frac{1}{\sqrt{MN}}$$

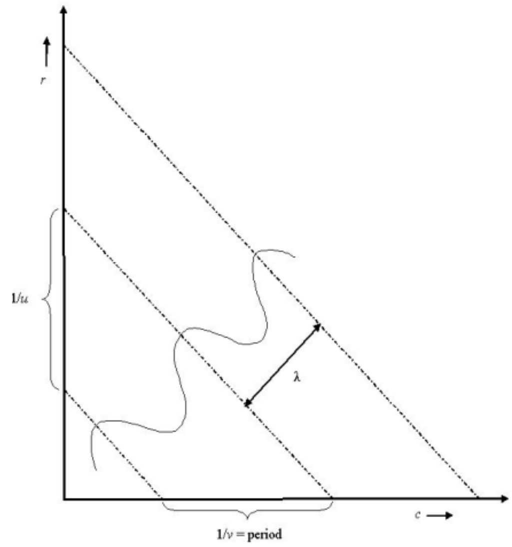
Code snippet:

```
amplitude = 1/sz;
```

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

Direction of travel depends on the relative position between the (p,q) and the origin.



- Directional dependency: The point P(p,q) is joined to the origin. The direction of the sine wave is the direction of the line connecting P to the origin.
- Wavelength dependency: (u, v) are the frequencies along (r, c) and the periods are 1/u and 1/v. Then wavelength of the sinusoid is

$$\lambda = \frac{1}{\sqrt{u^2+v^2}}$$

Mathematical Expression:

$$\omega_1 = 2\pi uc/M; \omega_2 = 2\pi nc/N$$
$$\lambda = 2\pi/\omega$$
$$|\omega| = \sqrt{\omega_1^2 + \omega_2^2}$$

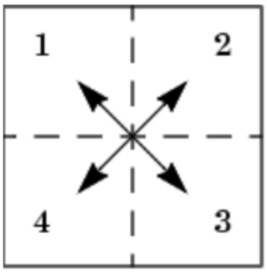
Code snippet:

```
w1 = 2 * pi * uc/sz; w2 = 2 * pi * nc/sz;
wavelength = 2 * pi / sqrt(w1^2+w2^2)
```

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:
A DFT is periodic & symmetrical complex conjugate. i.e. the spectrum repeats itself repeatedly, endlessly in both directions.
Periodicity: i.e. F(u,v) = F(u + kN, v + lN) where k, l □ [−∞, ..., −1, 0, 1, 2,..., ∞]. The N×N block of the Fourier coefficients F(u,v) computed from an N×N image with the 2D DFT is a single period from this infinite sequence.

Complex conjugation: |F(u,v)| = |F(-u, -v)|. Hence there are negative frequencies which are mirror images of the corresponding positive frequencies.



Windowing: The DFT is also an infinite data array, wrapping itself infinitely in both directions. The image is wrapped around itself, such that the left & the right sides, and the top & bottom coincide.

Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

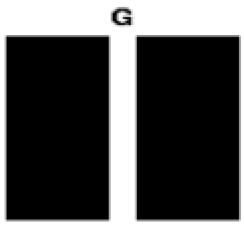
Answers:

Before the line, the image is visualized in the frequency domain in the interval [0, 2π].
After the line, when fftshift() function is computed, the (p,q) coordinates are centered. Then the image is visualized in the frequency domain in the interval [-π,π].

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

Consider an image like this:



Its pixels are zero everywhere else, except when □x & xy□ (57,73)

$$f(x,y) = \begin{cases} 1 & x \in (57,73) \\ 0 & elsewhere \end{cases}$$

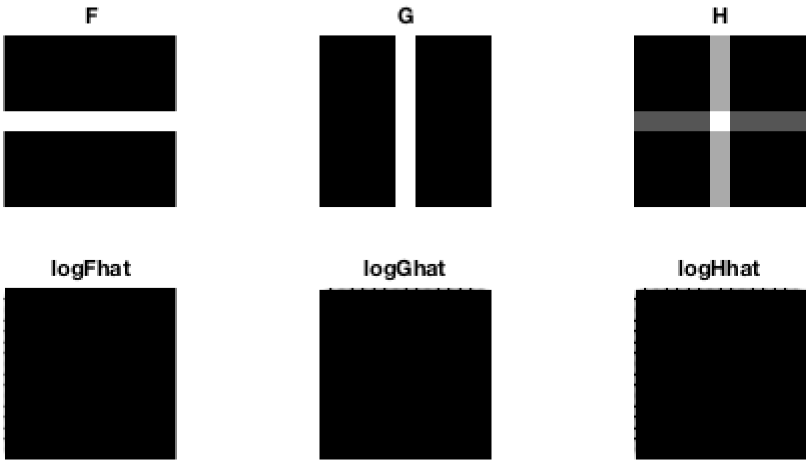
Question 8: Why is the logarithm function applied?

Answers:

- Pixel distribution is more focused on the lower range.
- Log transformation helps in expanding the range of low pixel values, and compress high pixel values
- This enhances the image contrast in the frequency domain.

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

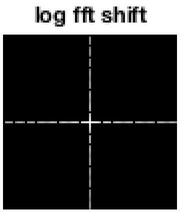
Answers:
In the images below:-

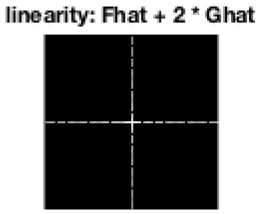


The spectrum of H consists of F, G spectra super-positioned. This implies linearity, given by following expression:-

$$\mathcal{F}\left[a\, f_1(m,n) + b\, f_2(m,n))\right] = a\, \hat{f_1}(u,v) + b\, \hat{f_2}(u,v)$$

a, b are scalars. F is Fourier transform operator on spatial images defined by f_1 and f_2.





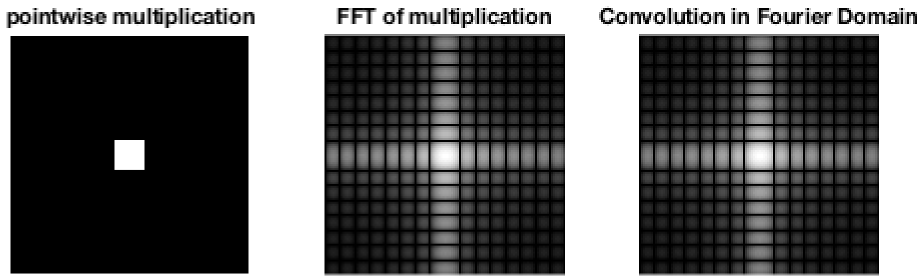
This principle also applies to the log transformed visualizations. The one to the left, represents log shifted H image, while the one to the right, represents the same operation on the individual frequency domain of F & G's.

Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

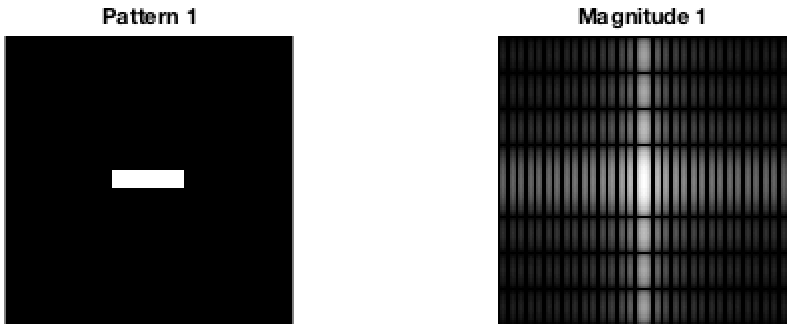
The same image can be displayed using convolution.

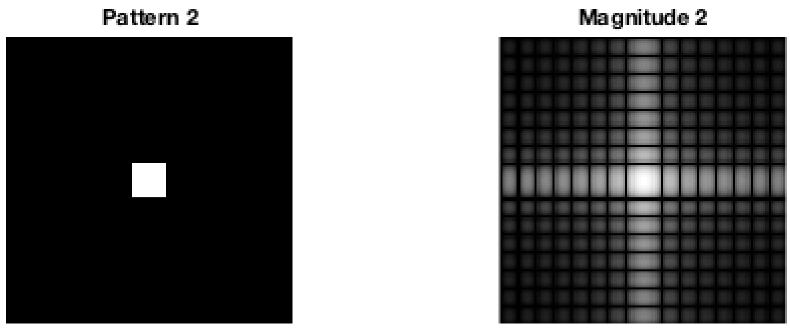
```
F = [ zeros(56, 128); ones(16, 128); zeros(56, 128)];
G = F';
subplot(2, 3, 1);
showgrey(F);
title('F')
subplot(2, 3, 2);
showgrey(G);
title('G')
subplot(2, 3, 4);
showgrey(F .* G);
title('pointwise multiplication')
subplot(2, 3, 5);
showfs(fft2(F .* G));%/128);
title('FFT of multiplication')

subplot(2, 3, 6)
Fhat = fft2(F)/128;
Ghat = fft2(G)/128;
Cs = conv2(fftshift(Fhat),fftshift(Ghat), 'same');
showfs(fftshift(Cs));
title('Convolution in Fourier Domain')
```



Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:





Pattern1 and 2 are spatial domain images. The length of the white region of Pattern 1 is halved in Pattern 2.
So, if Pattern1 := f(x,y) & Pattern2 := g(x,y).
The two are related as follows:-

$g(x, y) = f((x/2), (2y))$
i.e. in g(x, y), there's horizontal compression and vertical expansion.

By scaling property of Fourier transform, this translates to horizontal expansion and vertical compression. This is accordingly expressed by the magnitude spectra of the two images on the right.

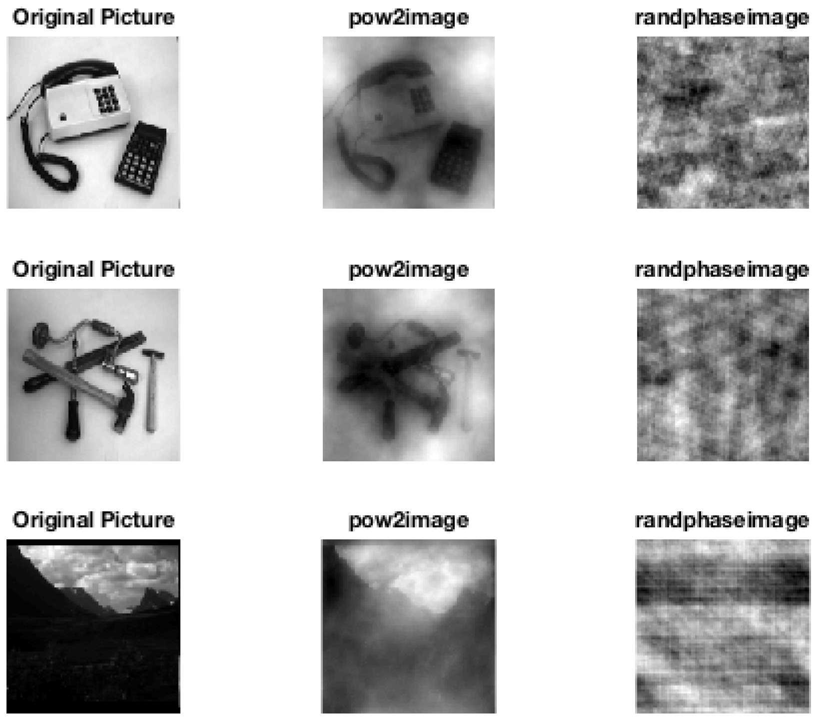
Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

An image rotated by angle Φ in spatial is also rotated by same angle Φ in the frequency domain. However, due to the rectangular orientation of pixels, the pixels lose their sharpness, when the angles are 30 degrees, or 60 degrees.

Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

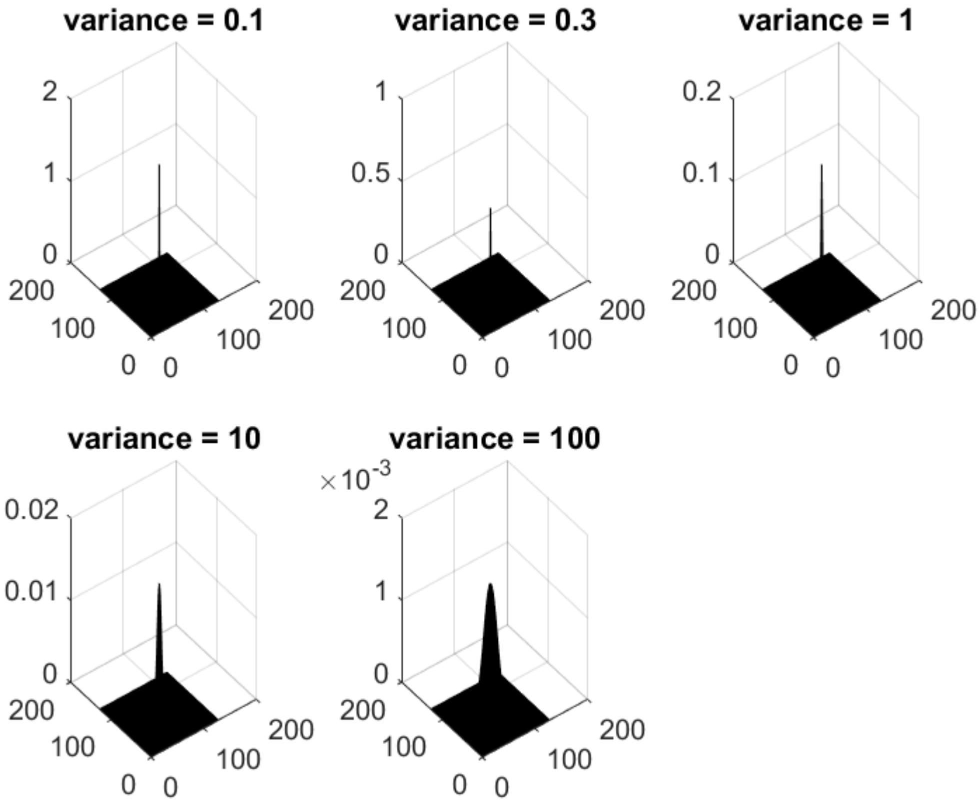


2nd column: Represents images obtained using the pow2image. The image's magnitude is the magnitude of the frequency components, and is reduced. The phase is retained. Hence it's discernible to the human brain.

3rd column: Here, the phases are randomly changed, but the magnitude is retained. Phases are the displacement of the basis functions from the origin. Phase contains much information about the position of the object in an image. The randphase function alters the position information, making it illegible to the human brain.

Question 14: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:

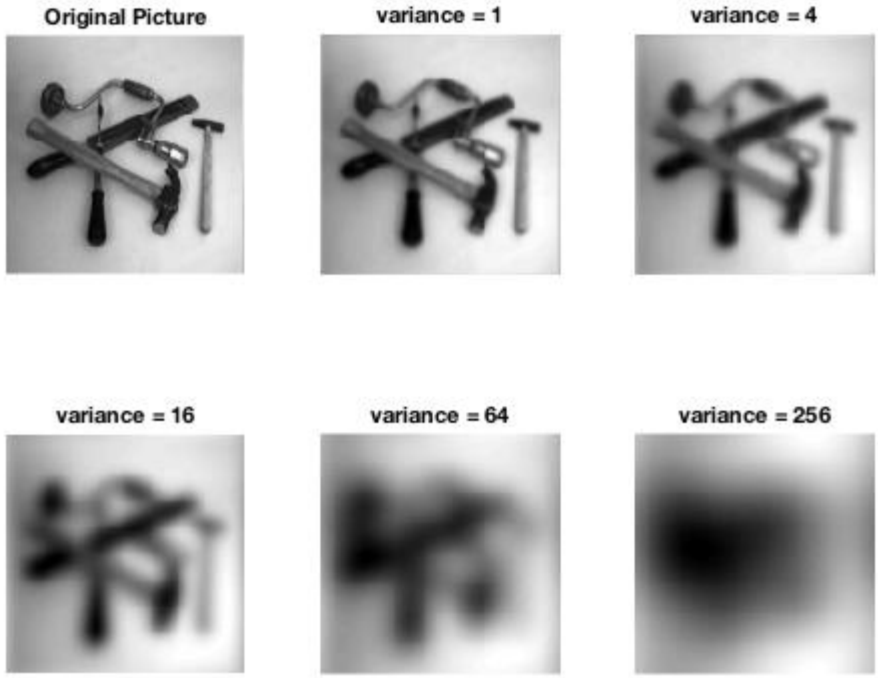


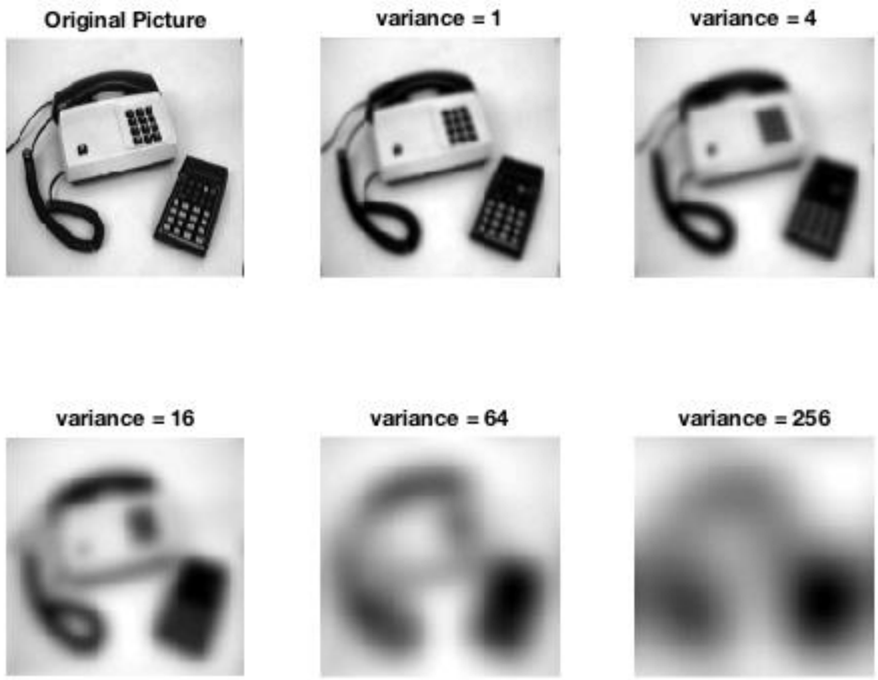
Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:
When t < 1, there's a noticeable difference between the covariance and the estimated value, because for variance < 1, the distribution becomes non-Gaussian. The no. of points sampled are directly proportional to the variance. So when t is small, lesser points are sampled from the region.

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:





Blurring is proportional to the variance, because variance also increases the lower cut-off frequency in the frequency domain of the Gaussian function. So as variance increases, the higher frequency components such as edges are lost.

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

Effect \ Filter type	Gaussian	Median	Ideal
Positive	Removal of noise.	Better at removing salt and pepper (SAP) noise, because neighboring values are used to calculation of the value. Extreme central values are thusly removed.	Simple to implement.
Negative	Important features such as edges also get blurred.	Extreme blurring of sudden changes in the images. For e.g. edges which are higher frequency components are blurred.	Ringing effect observed at boundaries since Ideal LPF is rectangular.

Question 18: What conclusions can you draw from comparing the results of the respective methods?

Answers:

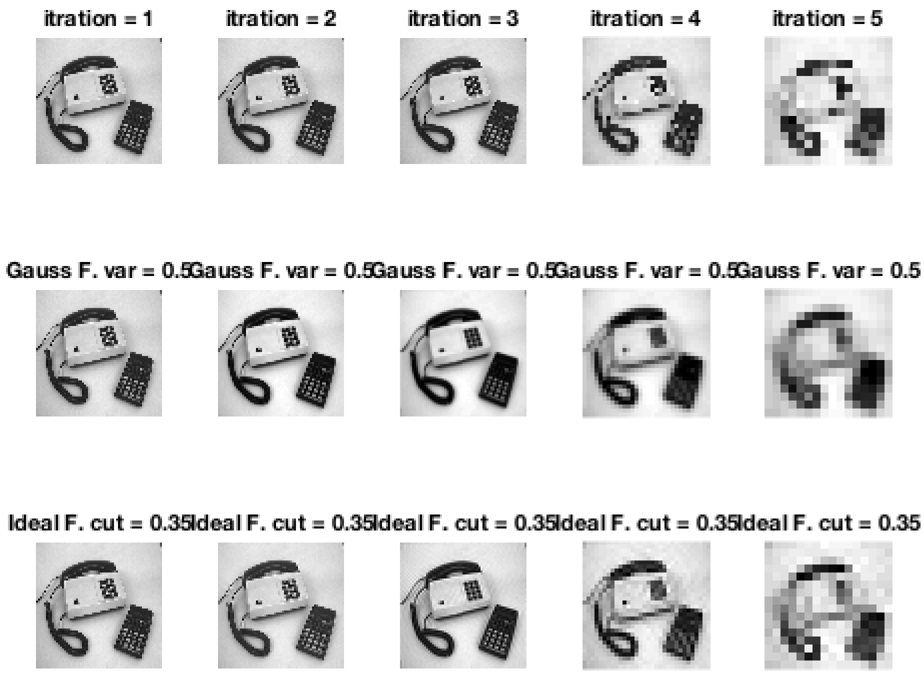
In a Gaussian filter, the variance is directly proportional to the blurring because variance in spatial domain of Gaussian distribution is inversely proportional to the variance of the same distribution in the frequency domain.

The ideal lowpass filter is a perfect disc with radius r = D0 in the frequency domain. In the spatial domain, this filter is represented however by two components. An intense component in the origin and a component comprised by concentric circles around the first component. The first component is responsible for the blurring and the second one for the ringing effect. The lower the cutoff frequency, the higher the ringing effect is spread, i.e. the ringing effect of each pixel reaches longer distances. Since a multiplication of the Fourier transform of two signals in the frequency domain is equivalent of a convolution in the spatial domain, the above latter component affects the filtered image in a way such that noise is not only maintained but also transformed. For instance, salt and pepper noise in the SAP image is enhanced and magnified, resulting in large freckle-like shapes in the image.

In contrast to the two afore-mentioned lowpass filters, the median filter is a nonlinear filter whose operations are centered only on a neighborhood. As the filter works with the median of a neighborhood of pixels, and not the mean, it can directly remove the effect that outliers have in images, that is, small regions of pixels affected by noise. The above two reasons are the reasons why this filter is so successful in removing salt and pepper noise.

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:



Subsampling results in pixels of bigger size, hence a neighborhood of pixels is compacted into one, and the different values of all those pixels are lost. Hence, information is lost, and shapes become coarser. If the subsampling occurs at a lower frequency than the Nyquist frequency, then the image's characteristics are distorted irrevocably.

The first thing noticed in this exercise is that there is a qualitative information loss balance: it is possible to smooth the subsampled images to a higher degree than the original images.

Smoothing the original images in that degree results in information loss, just as the one introduced when subsampling. Since the two filters used here are lowpass filters, we can see that the outline in the image remains fairly accurate, even higher variances, or lower cutoff frequencies. About smoothing the subsampled versions of the original images, the same effects as given above the Gaussian filter introduces blurring and the ideal lowpass filter introduces ringing. The higher the resolution of the image, the higher the level of details preserved about blurring, and the coarser the ringing effect.

Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

From the above question we could conclude that smoothing before subsampling could prevent from losing information of the image. This can be explained by the sampling theorem. To lower the information loss during the process of the subsampling, the sampling frequency should be above the Nyquist frequency, which is half the maximum frequency. If we first smooth the image by Gaussian filter or lowpass filter, the maximum frequency will decrease, which causes a reduction of the Nyquist frequency. This means that the relatively low sampling frequency could also satisfy the requirement of the sampling theorem. In this way, the information loss will decrease.

Laboration 1 in DD2423 Image Analysis and Computer Vision

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Approved on (date) Course assistant