UNIT-I OPTICS

Interference:

Principle of Superposition of waves: When two or more waves travel simultaneously in a medium, the resultant displacement at any point is due to the algebraic sum of the displacements due to individual waves.

To understand this let us consider two waves travelling simultaneously in a medium. At any point let y1 be the displacement due to one wave at any instant in the absence of the other and y2 be the displacement of the other wave at same instant in the absence of the first wave. Then the resultant displacement due to the presence of both the waves is given by

$$Y = y_1 \pm y_2$$

+ve sign has to be taken when both the displacements y_1 and y_2 are in the same direction; -ve sign has to be taken when they are in the opposite direction.

Constructive interference Destructive interference

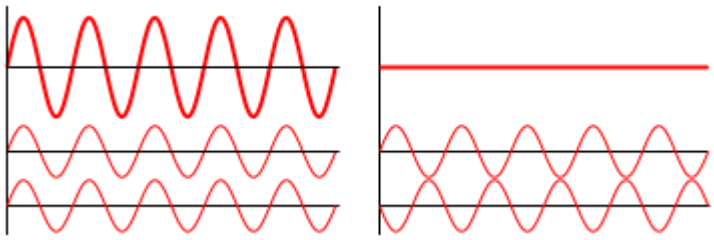


Fig.1.1

Interference: When two or more waves travel simultaneously in a medium superimpose with each other then the resultant intensity will be modified. In other words modification of intensity. They are superimpose the following ways

1. When two or more waves superimpose constructively , then the intensity will be maximum or bright fringes will appear.
2. When two or more waves superimpose distructively , then the intensity will be minimum or dark fringes will appear.

In interference the fringe width is equal.

Conditions for interference:

In order to get sustained interference pattern the following conditions are required.

1. The source must be monochromatic.

2. Two sources must be coherent. i.e they have same wavelength, amplitude and constant phase difference.
3. The distance between the coherent sources should be small.
4. The distance be tween the coherent sources and screen should be large.
5. The background should be dark.

Coherence: Two waves are said to be coherent if their waves have

i) Same wavelength
ii) Same amplitude and
iii) Constant phase difference.

Only such waves on superposition give rise to interference pattern, i.e bright and dark fringes.

i) If we choose two sources of a monochromatic light with same power, they emit waves of same wavelength with same amplitude but since the waves are emitted spontaneously by the two different sources, they will have randomly varying phase i.e., they will not have constant phase difference; hence two independent sources of light can never be coherent.

i) Now let us choose two sources derived from single monochromatic source of light as young did in his double slit experiment. He made use of Huygen's concept of wave front. He considered wave front as the locus of points having the same state of vibration. Each point on a given wave front is to be considered as the source of secondary source emitting secondary wavelets that are in phase each other. Thus they are coherent.

Coherence is of two types.

1. **Temporal coherence:** If it is possible to predict the phase relation at a point on the wave w.r.t. another point on the same wave then the wave has temporal coherence.
2. **Spatial coherence: :** If it is possible to predict the phase relation at a point on a wave w.r.t. another point on a second wave then the wave are said to be spatial coherence.

Young's double slit:

The phenomenon of interference of light was first discovered by Thomas young in 1801.Young allowed the sunlight to pass through a pinhole S and then at some distance through two sufficiently close pin holes S_1 and S_2 in an opaque screen. Finally the light was received on a screen on which he observed an uneven distribution of light intensity.

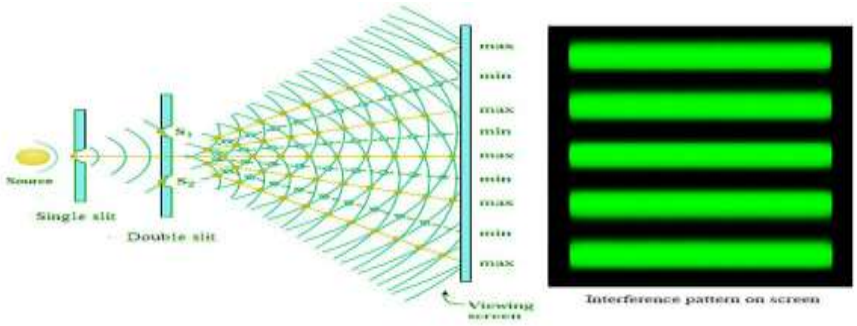


Fig.1.2

Let us consider the suoerposition of waves of same frequency having constant phase difference.Let a1 and a2 be the amplitudes of these waves.The displacement due to one wave at any instant t is given by,

$Y_1 = a_1 \sin(\omega t)$

And the displacement due to other wave at the same instant is given by,

$Y_2 = a_2 \sin(\omega t+\delta)$

where δ is the phase difference between two waves at an instant t.

By the principle of superposition of waves, the resultant displacement is equal to the sum of the displacements of all the waves. Hence the resultant displacement is

$$Y = Y_1 + Y_2$$

$$Y = a_1 \sin(\omega t) + a_2 \sin(\omega t+\delta)$$

$$= \sin(\omega t)[a_1 + a_2 \cos\delta] + \cos wt[a_2 \sin\delta]$$

Let $a_1 + a_2 \cos\delta = A\cos\phi$, $a_2 \sin\delta = A\sin\phi$

$$[a_1 + a_2 \cos\delta]^2 + [a_2 \sin\delta]^2 = A^2$$

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

The intensity at any point is proportional to the square of the amplitude i.e.

$$I = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

Condition for maximum intensity: The intensity will be maximum at points where the value of $\cos\delta = 1$,i.e.phase difference $\delta = 2n\pi$ where n=0,1,2,3,……

$$I = a_1^2 + a_2^2 + 2a_1a_2$$

$$I = (a_1 + a_2)^2$$

$$If\ a_1 = a_2 = a$$

$$I = 4a^2$$

Condition for minimum intensity: The intensity will be maximum at points where the value of $cos\delta = -1$,i.e.phase difference $\delta = (2n + 1)\pi$ where n=0,1,2,3,……

I = $a_1^2 + a_2^2 + 2a_1a_2$

$$I = (a_1 - a_2)^2$$

$$If\ a_1 = a_2 = a$$

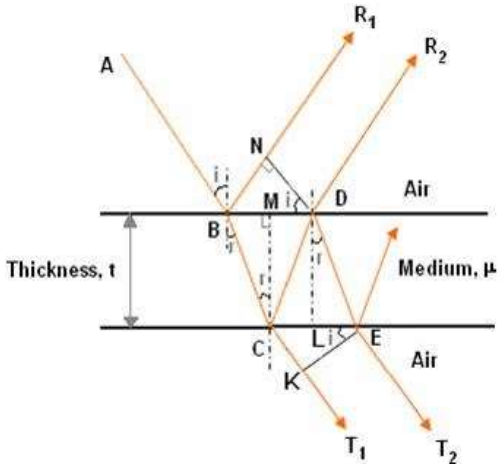
$$I = 0$$

Interference in thin films by Reflection:

When light is incident on upper surface of the thin film, a part of the light gets reflected from the upper surface while the remaining part is transmitted into the film. When the transmitted light is incident on the bottom surface of the film, a part of the light is reflected back towards the upper surface while the remaining part is transmitted into the bottom surface as shown in fig. Thus the two reflected light beams at the upper surface superimpose with each other and hence, produces interference patterns.

Consider a transparent thin film of thickness t with refractive index μ. The monochromatic light ray AB is incident on the upper surface of the film at an angle of incidence i.Let BR_1 and BC are reflected and transmitted rays respectively. The angle of refracted light is r. The ray CD will be reflected into the film and emerge through the film from the upper surface of film as alight ray DR_1'The two light rays $BR_1$and DR_1 superimpose each other depending upon path difference between them and hence ,produces the interference pattern.

In order to determine the path difference draw a normal DN to BR_1.The distance travelled by the light rays are equal from the point D
and N.When the light ray travels from B to N, at the same time the transmitted light ray travels from B to C and then C to D.



the path difference between light rays $BR_1$and DR_1 is

$$\Delta = \mu(BC + CD) - BN \ldots\ldots\ldots\ldots\ldots (1)$$

Consider ΔBCM

$$\cos r = \frac{MC}{BC}$$

$$BC = \frac{MC}{\cos r} = \frac{t}{\cos r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Similarly, from Δ DCM,

$$\cos r = \frac{MC}{CD}$$

$$CD = \frac{MC}{\cos r} = \frac{t}{\cos r} \quad \text{.....(3)}$$

Comparing Eqs. (2) and (3),we get,

$$BC = CD = \frac{t}{\cos r} \quad \text{.....(4)}$$

Consider that BD = BM + MD .....(5)

from Δ DCM,

$$\tan r = \frac{BM}{MC}$$

We get,

$$\text{BM} = t\ tan\ r \text{.....(6)}$$

Similarly,

$$\text{MD} = t\ tan\ r \text{ .....(7)}$$

Substituting the value of BM and MD in Eqn, (5),we get,

BD = BM + MD = 2t tan r .....(8)

From Δ NBD ,

$$\sin i = \frac{BN}{BD}$$

BN = BD sin i = 2t tan r sin i

According to Snell's law,we know that

Sin i =μ sin r

BN = 2t tan r (μ sin r) ..... (9)

Substituting the BN, BC, and CD values in Eqn. (1),we get ,

$$\Delta = \frac{2\mu t}{cos\ r} - 2\text{t tan r (} \mu \text{ sin r)}$$

$$= \frac{2\mu t}{cos\ r}(1 - sin^2 r)$$

$$\Delta = 2\mu\text{t cos r} \quad \text{..... (10)}$$

At the point B, reflection occurs from the upper surface of the thin film(denser medium).Light ray one undergoes an additional phase change of π or an additional path difference of (λ/2).

Therefore, $\text{path difference} = 2\mu t \cos r + \frac{\lambda}{2}$ ..... (11)

Condition for bright fringe:

When the path difference is equal to an integral multiple of λ ,then the rays one and two meet in phase and undergoes constructive interference. Therefore,

$$n\lambda = 2\mu t \cos r + \frac{\lambda}{2}$$

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \text{.....(12)}$$

Condition for dark fringe:

When the path difference is equal to an half integral multiple of λ ,then the rays one and two meet out of phase and undergoes distructive interference. Therefore,

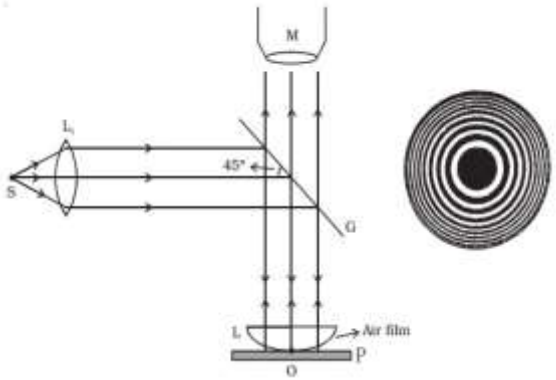
$$(2n+1)\frac{\lambda}{2} = 2\mu t \cos r + \frac{\lambda}{2}$$

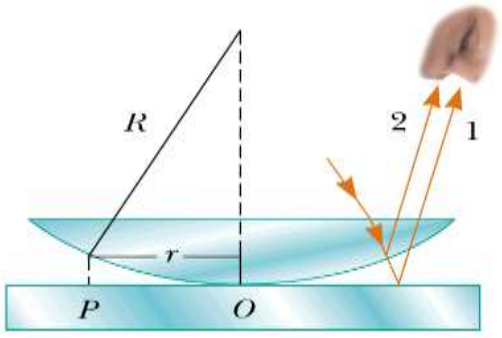
$$n\lambda = 2\mu t \cos r \text{ ..... (13)} \qquad n = 0,1,2,3, \ldots . etc.$$

Depending on the above conditions, the interference pattern consists of bright and dark fringes.

:Newton's Rings

When a plano-convex lens with its convex surface is placed on a plane glass sheet, an air film of gradually increasing thickness is formed between the lens and the glass plate. The thickness of film at the point of contact is zero. If monochromatic light is allowed to fall normally on the lens, and the film is viewed in reflected light, alternate bright and dark concentric rings are seen around the point of contact. These rings were first discovered by Newton, that's why they are called Newton's rings. Newton's rings are formed due to interference between the light waves reflected from the top and bottom surfaces of the air film formed between the lens and glass sheet.





Let the radius of curvature of the convex lens is **R** and the radius of ring is '**r**'. Consider light of wave length '**λ**' falls on the lens. After refraction and reflection two rays **1** and **2** are obtained. These rays interfere each other producing alternate bright and dark rings. At the point of contact the thickness of air film is zero, the path difference is (λ/2) and as a 180^O phase change occurs. Hence dark ring is obtained at the centre.

Path difference for bright ring $2\mu t \cos r = (2n-1)\frac{\lambda}{2}$

Path difference for dark ring $n\lambda = 2\mu t \cos r$

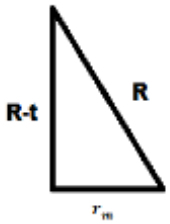
<u>To find the radius of curvature of plano convex lens</u>

Let's consider a dark ring with radius *r* at a point and thickness is *t*. From the triangle

$$R^2 = (R-t)^2 + r^2$$

$$R^2 = R^2 - 2Rt + t^2 + r^2$$

$$2Rt = t^2 + r^2$$



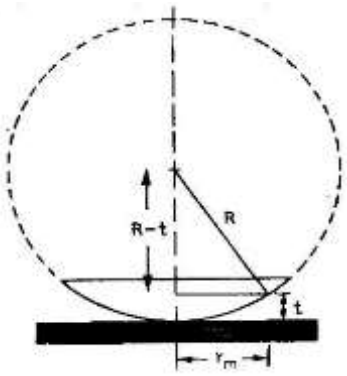


Fig.1.7

Here, *t* is several wavelenths or so, while the ring radius *r* is usually some millimetres, so we can neglect the t^2 term. Further,

$$2Rt = r^2$$

$$t = \frac{r2}{2R}$$

for a dark ring, we have seen above that $t = n\lambda/2$, where *n* is a positive integer. Substituting gives

$$Rn\lambda = r^2 \quad \text{or} \quad r = (Rn\lambda)^{1/2}$$

diameter $of\, n^{th}$ dark ring is $D_n^2 = 4Rn\lambda$

For a bright ring, we have seen that $t = (2n-1)\lambda/4$ where n is positive integer. Substituting gives

$$\frac{r^2}{2R} = (2n-1)\lambda/4$$

Diameter of n^{th} bright ring is

$$D_n^2 = 2\lambda R(2n-1)$$

diameter of n^{th} dark ring is $D_n^2 = 4Rn\lambda$

Difference between the above two rings is

$$D_m^2 - D_n^2 = 4R\lambda(m-n)$$

$$\text{R} = \frac{D_m^2 - D_n^2}{4\lambda(m-n)}$$

To find the refractive index of given liquid medium:

The experiment is performed when there is an air film between the glass plate and plano convex lens. The diameters of m^{th} and n^{th} dark rigs are determined with the help of a travelling microscope.

$$D_m^2 - D_n^2 = 4R\lambda(m-n) \quad \text{..... (1)}$$

Now, the liquid is introduced between the glass plate and plano convex lens, liquid film is formed.Again, the diameters of the same m^{th} and n^{th} dark rings are to be obtained. Then we have,

$$D_m'^2 - D_n'^2 = \frac{4R\lambda(m-n)}{\mu} \quad \text{..... (2)}$$

From equations (1) and (2),we get

$$\mu = \frac{4R\lambda(m-n)}{D_m'^2 - D_n'^2} = \frac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2}$$

Using the above formula, refractive index of liquid(μ) can be calculated.

Applications of thin film interference :

1. It is used in antireflection coatings and optical filters.They are used to control the amount of light reflected or transmitted at a certain wavelength at a surface.
2. Thin film interference is used to allow only specific wavelengths of light to pass through the device. Deposition processes are used to create these films.
3. Used for very accurate determination of the wavelength of light.
4. Used for testing the flatness of a surface.
5. Used for measuring small distance such ascompression and elongation in metal rodsor crystals.
6. Used for finding the thickness of small objects such as thin films.
7. Used for determining refractive index of liquids and gases.

Colours in thin films :

When a thin film isexposed to white light such as sunlight,beautiful colours appear in the reflected light.

Ex: Soap buuble,Oil slicks

In the case of soap bubble, let us assume that thickness of the film t is a constant. Then in the formula $2\mu tcor,\ \mu$, r are variables. Since white light has varying wavelength value , μ also varies with wavelength .Also due to curved nature of bubble. Even if parallel rays are incident, angle of incidence varies for different points on the bubble and hence accordingly angle of refraction varies.Varying values of μ, r can satisfy the condition for constructive interference for a particular wavelength only.Accordingly that point will appear bright in that particular colour.In a similar way different points satisfy the condition for constructive interference for different colours and hence appear multi-coloured.

Oil film:

In this case since the film is perfectly flat, when parallel rays such as sun light is incident , the angle of incidence and hence angle of refraction will remains constant. But for different wavelengths μ values varies and also thickness of film may not be constant throughout the film. Hence different points on the film satisfy the condition for constructive interference for different colours depending on the values of refractive index and thickness and hence appear multi-coloured.

In the case of thin films, interference pattern is multi-coloured.No dark band is seen because if a particular point satisfy condition for destructive interference for particular wavelength ,the same point may satisfy condition for constructive interference for different wavelength and hence throught the film we see multicolours.

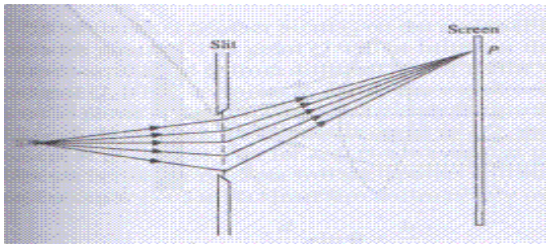
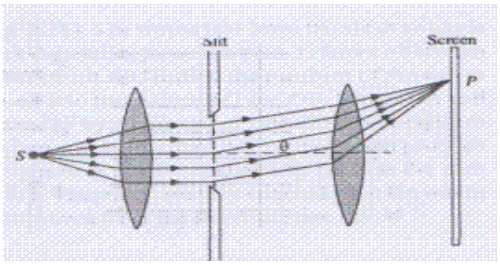
UNIT I Diffraction

Diffraction: The phenomenon of bending of light round the corners of obstacles and spreading of light wave into the geometrical shadow of an obstacle placed in the path of light is called diffraction.

Differences between Interference and diffraction:

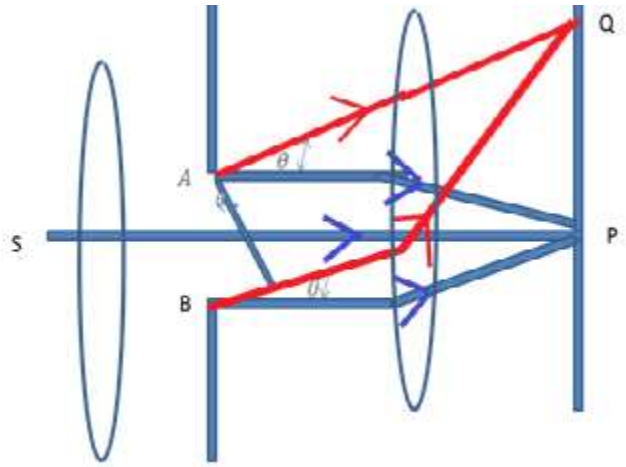
Interference	Diffraction
1. It is due to the superposition of two different wave fronts originating from two coherent sources. 2. Interference fringes are of equal width. 3. All bright fringes are of the same intensity. 4. All the dark fringes have zero intensity.	1. It is due to the superposition of secondary wavelets originating from the different parts of the same wave front. 2. Diffraction bands decreases in their widths as the order increases. 3. The bright fringes are of varying intensity 4. The intensity of dark fringes is not zero.

Types of diffraction:

Fresnel diffraction	Fraunhofer diffraction
1. Either a point source or an illuminated narrow slit is used. 2. The incident wave front is either spherical or cylindrical. 3. The source and screen are finite distances from the obstacle producing diffraction. 4. No lens is used to focus the rays. 	1. Extended source at infinite distance is used. 2. The incident wave front is a plane wave front. 3. The source and screen are infinite distances from the obstacle producing diffraction. 4. Converging lens is used to focus parallel rays. 

Fraunhofer diffraction – Single slit

Consider a slit width a .Let a plane wave front of monochromatic light of wavelength propagating normally towards the slit is incident on it. The diffracted light through the slit is focused by the convex lens on a screen placed in the focal plane of the lens. According to Huygens's principle, every point on the wave front in the plane of the slit is a source of secondary wavelets, which spread out to the right in all directions. These wavelets travelling normal to the slit OP are brought to focus at P by the lens. Thus, P is a bright central image. The secondary wavelets travelling at an angle θ with the normal are focused at a point Q on the screen. Depening on path difference, the point Q may have maximum or minimum intensities.



To find the intensity at Q ,let us draw the normal AK from A to the light ray B. The path difference between the two waves from A and B in a direction θ is

$$sin\theta = \frac{BK}{AB}$$

Path difference (BK) = **AB** $sin\theta$

$$= a\ sin\theta$$

Phase difference $(\emptyset) = \frac{2\pi}{\lambda} a\ sin\theta$

Intensity in Single-Slit Diffraction

To find the intensity at any point on the screen ,the slit can be divided in to n number of slits. The phase difference between any two successive waves from these parts would be

$$d = \frac{1}{n}(phase\ \ difference)$$

$$d = \frac{1}{n}(\frac{2\pi}{\lambda}\ a\ sin\theta)$$

According to vector polygon method, the resultant amplitude (R) is

$$R = a\ \frac{\sin(nd/2)}{\sin(\frac{d}{2})}$$

$$R = a\ \frac{\sin(n\frac{1}{n}(\frac{2\pi}{\lambda}\ \text{a}\ sin\theta)/2)}{\sin(\frac{\frac{1}{n}(\frac{2\pi}{\lambda}\ \text{a}\ sin\theta)}{2})}$$

$$R = \frac{a\ sin(\pi/\lambda)\ \ a\ sin\theta}{sin(\pi/n\lambda)\ \ a\ sin\theta}$$

$$R = \frac{asin\propto}{\frac{sin\propto}{n}}$$

Where $\propto = (\pi/\lambda)\ \ e\ sin\theta$ $\qquad$ $R = \frac{na\ sin\propto}{\propto}$

$$R = \frac{A\ sin\propto}{\propto}$$

$R^2 = \left(\frac{A\ sin\propto}{\propto}\right)^2$ = I= Intensity

Intensity distribution: The intensity at any point on the screen will be

$$I = A^2\ \frac{Sin^2\alpha}{\alpha^2}$$

1. **Principal maxim** It is obtained ,when the angle of diffraction is zero or no path difference

$$\propto = \left(\frac{\pi}{\lambda}\right)\ e\ sin\theta = 0$$

$$\alpha = 0$$

$$\theta = 0$$

2. **Minimum intensity:** The intensity will be minimum when

$$sin\alpha = 0$$

$$\alpha = \frac{\emptyset}{2} = \pm\pi, \pm 2\pi, \pm 3\pi \ldots\ldots\ldots\ldots\ldots, \pm m\pi$$

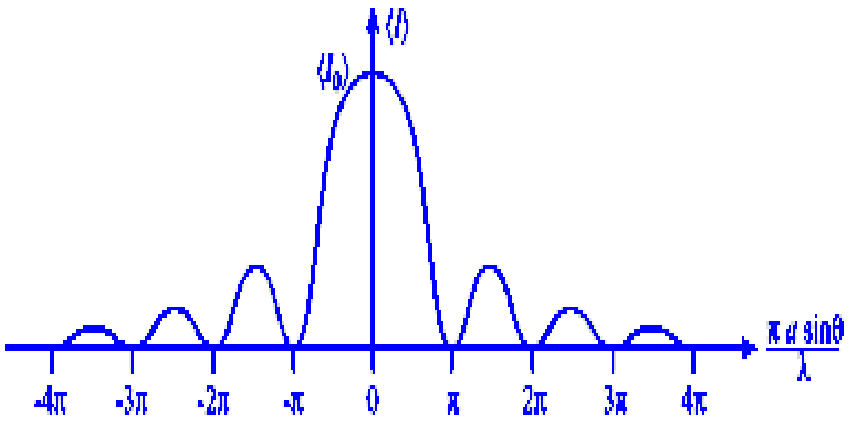
$$\propto = (\pi/\lambda)\ asin\theta = \pm\, m\pi$$

$$a\ sin\theta = \pm m\lambda \quad \textbf{(m = 1,2,3,4.....etc).}$$

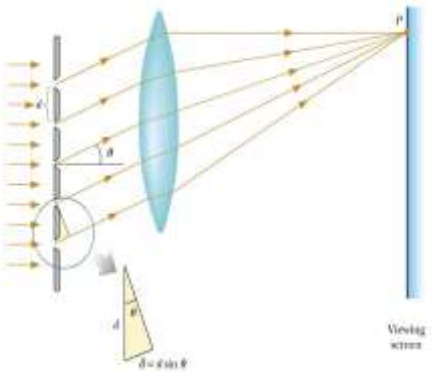
3. **Secondary maximum**

It is in between two minima. It can be obtained when the path difference is

$$a\ sin\theta = \pm(2m+1)\lambda/2 \quad \textbf{(m = 1,2,3,4.....etc).}$$



<u>Diffraction -N-Slits or Grating :</u>



An arrangement consisting of a large number of equidistant parallel narrow slits of equal width separated by equal opaque portions is known as a diffraction grating. The rulings act as obstacles having a definite width 'b' and the transparent space between the rulings act as slit of width 'a'. The combined width of a ruling and a slit is called grating element ($e = a + b$).

Let a plane monochromatic light of wavelength incident

normally on a plane transmission grating. As the width of each slit is of the order of λ , the wave spreading all directions beyond the grating. In other words , the N slits acts as N coherent sources. Consequently ,the waves interfere producing intensity maxima and minima.

Resultant amplitudes due to N-slits is

$$R = a\,\frac{\sin(nd/2)}{\sin(\frac{d}{2})}$$

where R = amplitudes due to N-slits

n = number of slits (N)

$$d = phase\ difference\ (\emptyset)$$

a = amplitude of any single slit

$$a = \frac{A\ sin \propto}{\propto}$$

Phase difference between any two slits is

$$\emptyset = \frac{2\pi}{\lambda}(a+b)sin\theta$$

Now the amplitude is

$$R = \frac{A\ sin \propto}{\propto}\ \ \frac{\sin(N\ \frac{2\pi}{2\lambda}(a+b)sin\theta)}{\sin(\frac{2\pi}{2\lambda}\ (a+b)sin\theta)}$$

$$R = \left(\frac{A\ sin \propto}{\propto}\ \right)\left(\frac{\sin N\beta}{\sin\beta}\right)$$

$$Where\ \beta = \frac{\pi}{\lambda}(a+b)sin\theta$$

$$I = R^2\ = \left(\frac{A\ sin \propto}{\propto}\ \right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

Intensity distribution

1. **Principal maximum can be obtained, when** $\sin\beta = 0,\ \beta \neq 0$

$$\beta = m\pi$$

$$(a+b)sin\theta = \pm m\lambda$$

Where m=0,1,2,3,....etc

2. **Minimum intensity can be obtained ,when** $\sin N\beta = 0$

$$N\beta = n\pi$$

$$N(a+b)sin\theta = \pm n\lambda$$

Where n = 1,2,3,.....(N-1)

Applications - diffraction Grating:

1. **Maximum number of orders possible with a grating:** When light is incident normally on a grating , in the transmitted light on either side of central maximum , we get diffracted rays. But there is a limit on the maximum number of orders available with a grating.

$$m = \frac{sin\theta}{N\lambda}$$

Maximum angle of diffraction (θ) = 90^0

$$m = \frac{1}{N\lambda}$$ (m is order of the diffraction grating)

2. **Wavelength of diffraction grating :**

We know that,Principal maximum obtained

$$(a+b)sin\theta = \pm m\lambda$$

$$(a+b) = grating\ element = \frac{1}{N}$$

Where N is lines per unit width

$$sin\theta = Nm\lambda$$

$$\lambda = \frac{sin\theta}{Nm}$$

3. Resolving power of grating:

The capacity of an optical instrument to show separate images of very closely placed two objects is called resolving power.

The resolving power of a diffraction grating is defined as its ability to form separate diffraction maxima of two closely separated wave lengths.

If λ is the mean value of the two wavelengths and dλ is the difference between two , then resolving power may be defined as

$$Resolving\ power\ = \ \frac{\boldsymbol{\lambda}}{\boldsymbol{d\lambda}}$$

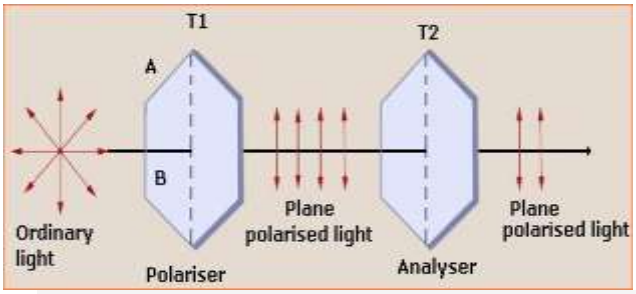
POLARIZATION

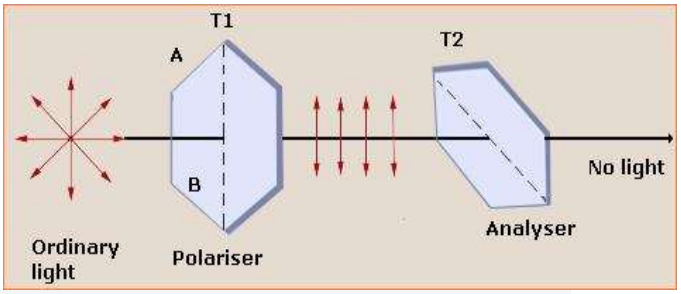
POLARIZATION :

The process of making the vibrations of unpolarized light confined to a single plane is known as polarization.

Explanation :

Consider an ordinary light is passed through a tourmaline crystal as shown in fig. below. If the tourmaline crystal is rotated by taking the incident beam as an axis, there is no variation in the intensity of the emergent beam. The tourmaline crystal has the property of allowing the components of light that are vibrating parallel to its axis to pass through it, and it absorbs all other components of the light that are not parallel to its axis of the crystal. The emergent beam from the crystal vibrates in only one direction, i.e ., it vibrates parallel to the axis of the crystal (Fig.1). The device that produces the polarized light is called polarizer. Consider that the emergent beam from the first crystal is passed through another tourmaline crystal kept perpendicular to the first crystal. No light is emerging from the second crystal (Fig.2).Thus, the first crystal produces polarized light is known as **Polarizer (T1)** and the second crystal analyses the polarized light, hence it is known as **Analyser (T2).**

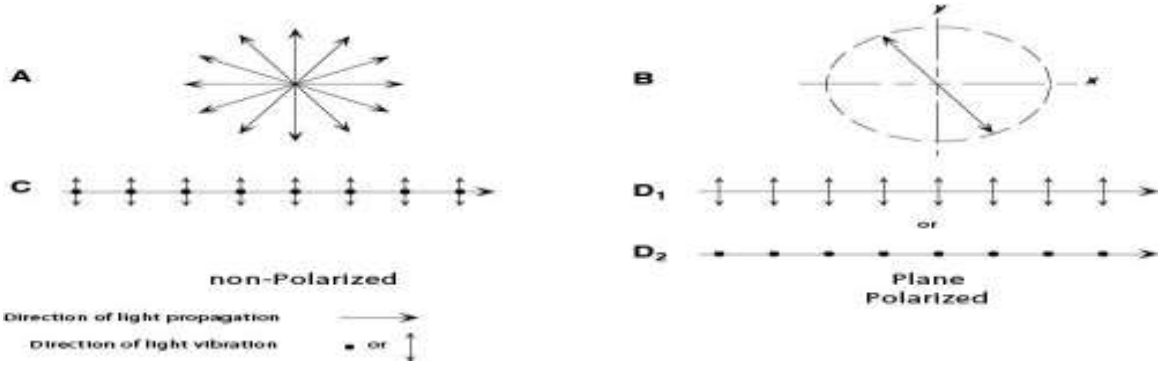




Pictorial Representation of light:

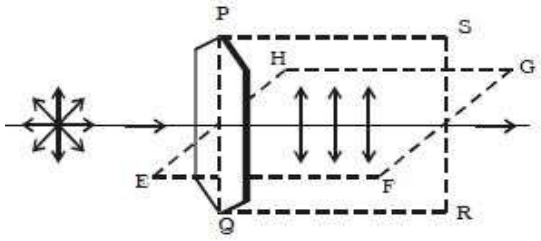
The plane polarized light, vibrating parallel to the plane of this paper and vibrating perpendicular to the plane of this paper , is represented as shown in fig. below. The unpolarized light or ordinary light has vibrations that are both parallel and perpendicular to the plane of the paper and it is represented as shown in fig.

Pla ne



of vibration and polarization:

The plane in which the light is vibrating is known as the plane of vibration and the plane perpendicular the plane of vibration is known as plane of polarization. The emergent beam vibrates parallel to the axis of the polarizer. The plane PQRS is parallel to the axis of the



polarizer and hence, it is known as plane of vibration. The plane EFGH perpendicular to the plane of vibration is known as plane of polarization. It should be noted that no vibrations takes place in this plane EFGH.

UNPOLARIZED AND POLARIZED LIGHT

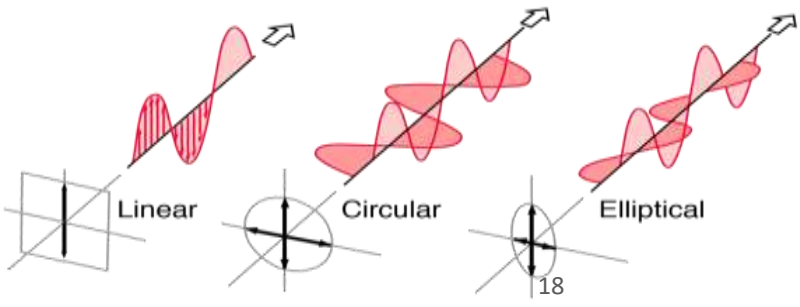
Un polarized light	polarized light
• It consists of an infinite number of waves, each having its own direction of vibration. • It is symmetrical about the direction of propagation.	• The vibrations in polarized light are confined to one particular direction only.i.e.one-sideness property. • It is Unsymmetrical about the direction of propagation

Types of polarized lights:

Plane polarized light /linearly polarized light:

The vibrations are confined to a single plane then it is known as plane polarized light or linearly polarized light.

Partially polarized light: The plane polarized light contain small additional components of unpolarized light is known as partially polarized light.



Circularly polarized light: The amplitude of the electric vector is a constant but rotates in the form of a helix.

Elliptical polarized light: The amplitude of the electric vector is not a constant but varies periodically then it results in elliptically polarized light.

Production of polarized light:

There are different methods to produce plane polarized light . They are

I. **Reflection** (**Brewster's law**) II. **Refraction** III.**Double refraction**

Brewster's law:

The refractive index of the material medium μ is equal to the tangent of the angle of polarization (p).

$$\mu = tanp$$

Applying this law ,it can be proved that the reflected and refracted rays are at right angles.

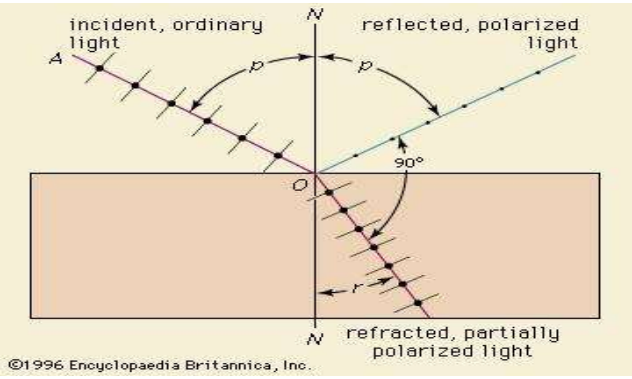


FIGURE 2.4

From snell's law

$\mu = \frac{sini}{sinr}$ .....(1)

From Brewster's law

$\boldsymbol{\mu = tanp}$**.....(2)**

From eqn.(1) &(2)

$tanp = \frac{sini}{sinr}$

$\frac{sinp}{cosp} = \frac{sini}{sinr}$ .....(3)

From the above Fig. i = p

Equation (3) becomes cos P = sin r

cos P = cos (90-r)

P = 90 - r

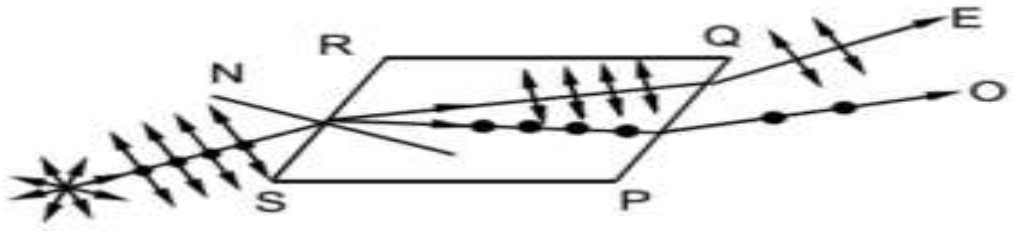
$$P + r = 90 \text{ .....(4)}$$

Thus, the reflected and refracted rays are perpendicular to each other.

Double refraction:

When unpolarized light passes through a calcite crystal, it is split into two plane polarized refracted lights. The one which obeys laws of refraction and plane perpendicular to the principal section is known as ordinary ray. The other which does not obey the laws of refraction and has vibrations in the principal section is called extraordinary ray. This phenomena is known as double refraction and was discovered by Bartholinus. From Fig. The refractive indices of O-ray and E-ray for calcite crystal are given by

$\mu_o = \frac{sini}{sinr_1}$ and $\mu_E = \frac{sini}{sinr_2}$ Since $r_1 < r_2$, $\mu_o > \mu_E$



Thus, the velocity of light for O-ray inside the crystal will be less than the E-ray. It is clear that μ_o is same for all angles of incidence while μ_E varies with angles of incidence.Hence, the O-ray travels with same velocity and it is represented by a spherical wavefront, whereas the E-ray travels with different velocities in different directions and is given by elliptical wave front. Both the rays travel with same velocities along optic axis.

Characteristics of O-ray and E-ray

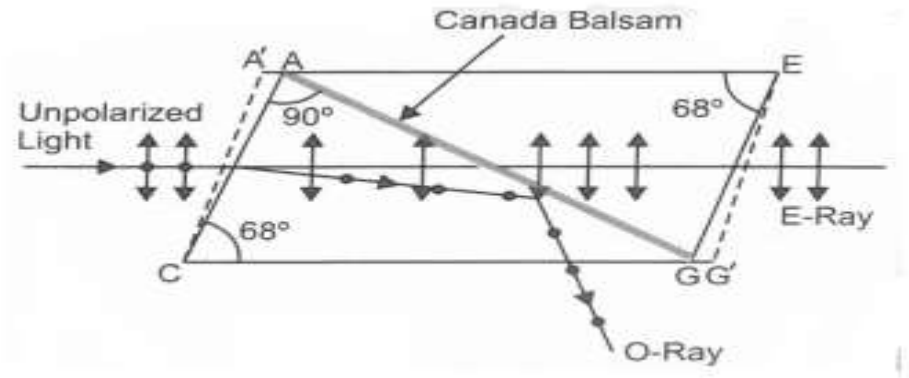
O-ray	**E-ray**
• It obeys the convention laws of refraction. • It travels with the same velocity in all directions in the crystal. • Refractive index is constant.	• It doesn't obeys the convention laws of refraction. • It travels with different velocity in different directions in the crystal. • Refractive index is varies.
But both having same velocity along optic axis. Both the rays are plane polarized.	

Nicol's Prism:

It is a device used to produce plane polarised light.This was invented by William Nicol in 1828.

Principle: Calcite crystal is modified such that it eliminates one of the two refracted rays by total internal reflection.

Construction: A calcite crystal whose legth is three times its bredth is taken.The two ends A'C and EG' of the crystal are cut ,so that the angles A'CG' reduces from71 to 68.Then the crystal is cut into two halvesalong the plane AG which passes through the blunt corners and perpendicular to both the principle section and end faces.The two cut faces are well polished and cemented together using a thin layer of Canada balsam,a transparent material.It has refractive index 1.55 for sodium wavelength.



Working:

When unpolarised light sodium light enters Nicls prism,it splits into O-ray and E ray.O-ray has refractive index1.6584 while E-ray has 1.48.Inside the crystal, O-ray travel from denser medium to rarer medium(canadabalsan).then the O-ray is totally reflected using the phenomena of total internal reflection.

WAVE PLATES or Retarders

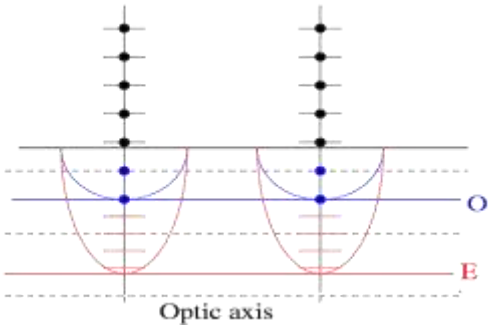
A retardation plate resolves a polarized light beam into two orthogonal components, retards the phase of one component relative to the other, and then recombines the two to form a single beam with new polarization characteristics.

- Half Wave Plate

- Quarter Wave Plate

Quarter Wave Plate

A quarter wave plate is a device from a double refracting crystal whose refracting faces are cut parallel to the direction of the optic axis. When a plane polarized light is incident on calcite crystal, the light split up into O-ray and E-ray. They travel along the same path with different velocities. Velocity of E-ray is greater than the velocity of O-ray. As a result, a phase difference is introduced between them. The thickness of this device is cut in such a way that it can produce a path difference of (λ/4) or a phase difference of (π/2) between O-ray and E-ray.



Let $\boldsymbol{\mu_E}$ and $\boldsymbol{\mu_O}$ be the refractive indices of the E-ray and O-ray respectively. Let t be the thickness of the crystal. The path difference between these rays is

$$\Delta = (\boldsymbol{\mu_O} - \boldsymbol{\mu_E})\, t \quad \text{..... (1)}$$

The QWP is cut in such a way thatit can produce a path difference = λ/4 .....(2)

From (1) and (2)

$$t = \frac{\lambda}{4(\boldsymbol{\mu_O} - \boldsymbol{\mu_E})}$$

Applications:

1. **A** QWP is produced circularly polarized light, if the optic axis of the crystal makes an angle of 45^0 with the incident beam of light.
2. **A** QWP is produced elliptical polarized light, if the optic axis of the crystal makes an angle other than 45^0 with the incident beam of light.

Half Wave Plate

A Half wave plate is a device from a double refracting crystal whose refracting faces are cut parallel to the direction of the optic axis. When a plane polarized light is incident on calcite crystal, the light split up into O-ray and E-ray. They travel along the same path with different velocities. Velocity of E-ray is greater than the velocity of O-ray. As a result ,a phase difference is introduced between them. The thickness of this device is cut in such a way that it can produce a path difference of (λ/2) or a phase difference of (π) between O-ray and E-ray.

Let $\boldsymbol{\mu_E}$ and $\boldsymbol{\mu_O}$ be the refractive indices of the E-ray and O-ray respectively. Let t be the thickness of the crystal. The path difference between these rays is

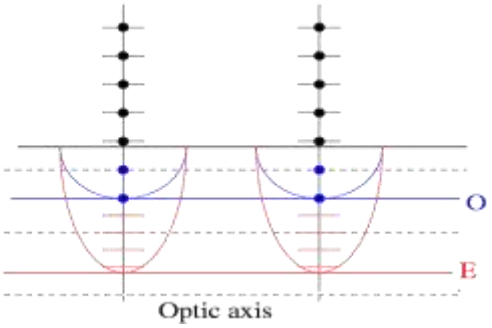
$$\Delta = (\boldsymbol{\mu_O} - \boldsymbol{\mu_E})\, \mathbf{t} \quad \textbf{..... (1)}$$

The HWP is cut in such a way that

it can produce a path difference = λ/2 .....(2)

From (1) and (2)

$$t = \frac{\lambda}{2(\boldsymbol{\mu_O} - \boldsymbol{\mu_E})}$$

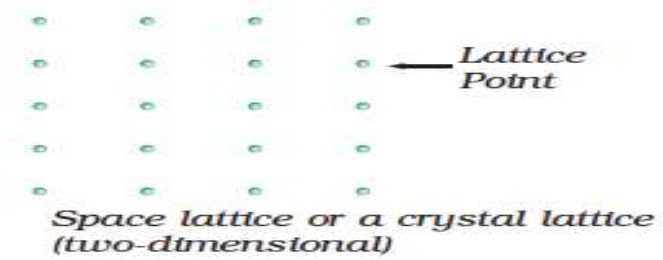


Applications :

1. **A** HWP is produced plane polarized light,

UNIT II CRYSTAL STRUCTURE AND X-RAY DIFFRACTION

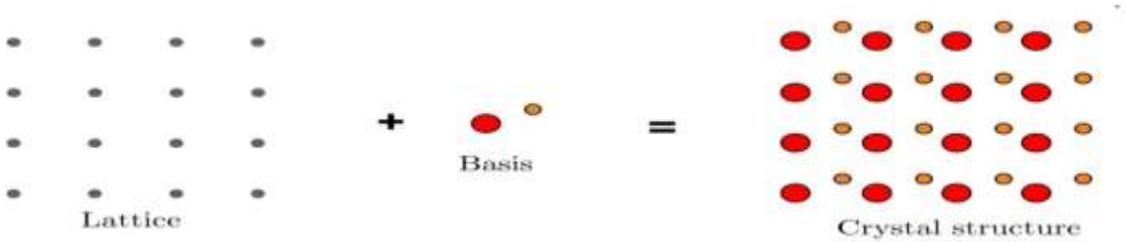
a) Space lattice: It is defined as an infinite array of points in three dimensions in which every point has surroundings identical to that of every other point in the array.



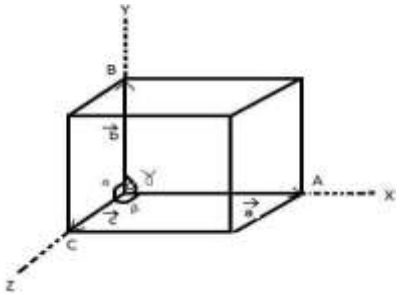
b) Basis: Group of atoms or molecules identical in composition.

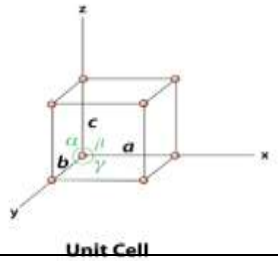
c) Crystal structure: periodic arrangement of atoms or molecules in 3D space.

Lattice + basis = crystal structure



d) Lattice parameters: the primitives (a,b,c) and interfacial angles (α,β,γ,) are the basic lattice parameters which determine the actual size of unit cell.





Unit Cell

e) Unit cell: is a minimum volume cell which on repetition gives actual crystal structure.

Atomic radius (r) – The atomic radius is defined as half the distance between neighboring atoms in a crystal of pure element.

Co-ordination number: The no of equidistant neighbors that an atom has in the given structure .Greater the co-ordination no, the atoms are said to be closely packed.

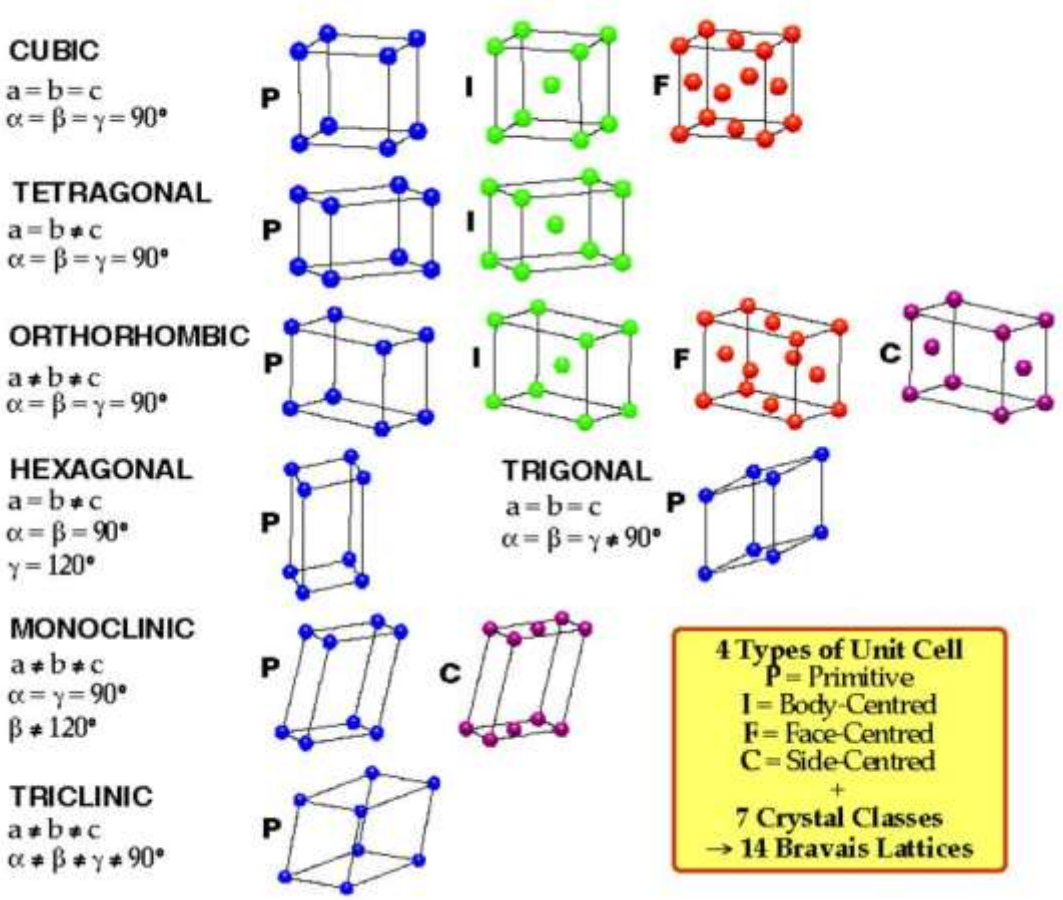
For Simple Cubic: 6, BCC: 8, FCC: 12

d) Packing factor (PF): It is the ratio of volume occupied by the atoms or molecule in unit cell to the total volume of the unit cell.

Atomic Packing Factor (APF) = $\frac{(Volume\ of\ all\ the\ atoms\ in\ Unit\ cell/}{Total\ Volume\ of\ the\ Unit\ cell}$

For Simple Cubic: 52%, BCC: 68%, FCC: 74%

Bravais lattices



Calculate the Packing factor of SC, BCC, FCC (or) Show that FCC is the closest packing of all the three cubic structures.

Simple cubic:

There are 8 atoms at 8 corners of the cube. The corner atoms touch with each other. If we take a corner atom as a reference, this atom is surrounded by 6 equidistant nearest neighbors.

Co-ordination number: - (N) = 6:- is defined as number of equidistant nearest neighbors that an atom has in the given structure.

Total number of atoms :- (n) =1:- each corner atom is shared by 8 unit cells, the share of each corner atom to a unit cell is 1/8 th of an atom (8×1/8 =1)

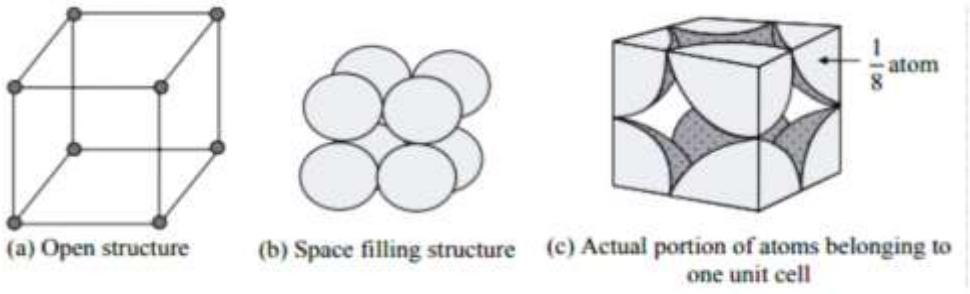
Nearest neighbor distance (2r):- the distance between centers of two nearest neighbor atoms will be 2r if 'r' is the radius of the atom.

Atomic radius: - (r) = 2r:- is defined as the distance between nearest neighbors in a crystal.

Lattice constant: - a =2r

Atomic Packing Factor (APF) = $\frac{\text{Volume of all the atoms in Unit cell}}{Total\ volume\ of\ the\ Unit\ cell}$

$= 1\times(4/3)\pi r^3/(a^3) = \mathbf{52}\%$

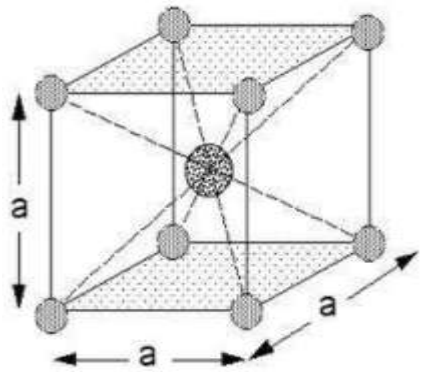


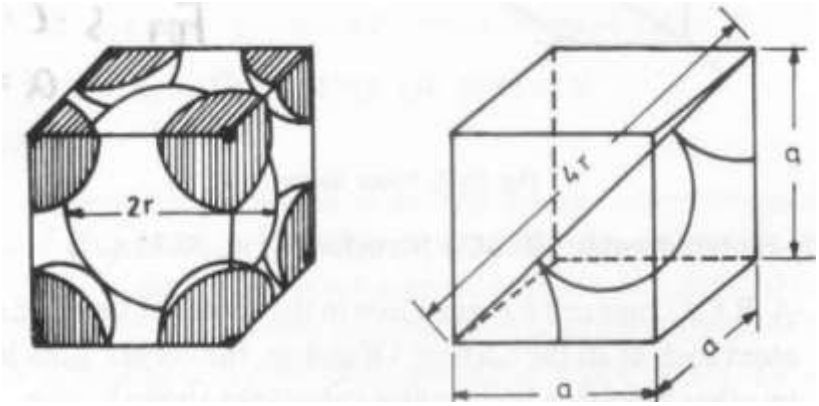
(a) Open structure (b) Space filling structure (c) Actual portion of atoms belonging to one unit cell

Ex:-polonium at room temperature.

Body centered

In a unit cell there are 8 atoms at 8 corners and another 1 atom at the body center. The 8 corner atoms are shared by 8 unit cells, and as the center atom is entirely within the unit cell, it is not shared by any surrounding unit cell.





Co-ordination number =8

Nearest neighbor distance = $\frac{a\sqrt{3}}{2}$

Lattice constant = a= $\frac{4r}{\sqrt{3}}$

Number of atoms per unit cell = 1 +1= 2

Volume of all atoms in unit cell = v = 2 × (4/3) πr^3

Volume of unit cell =V= a^3= $(\frac{4r}{\sqrt{3}})^3$

Atomic Packing Factor is $\frac{2 \times (4/3)\, \pi\, r^3}{(\frac{4r}{\sqrt{3}})^3}$= 0.68 = 68%

FACE CENTERED

In FCC there are 8 atoms at 8 corners of the unit cell and 6 atoms at 6 faces. Considering the atoms at the face center as origin, it can be observed that this face is common to 2 unit cells and there are 12 points surrounding it situated at a distance equal to half the face diagonal of the unit cell.

Co- ordination number = N= 12

Number of atoms in unit cell = 8 ×1/8+ 6×1/2=4

Lattice constant =a=2r = ($\sqrt{2}a/2$)

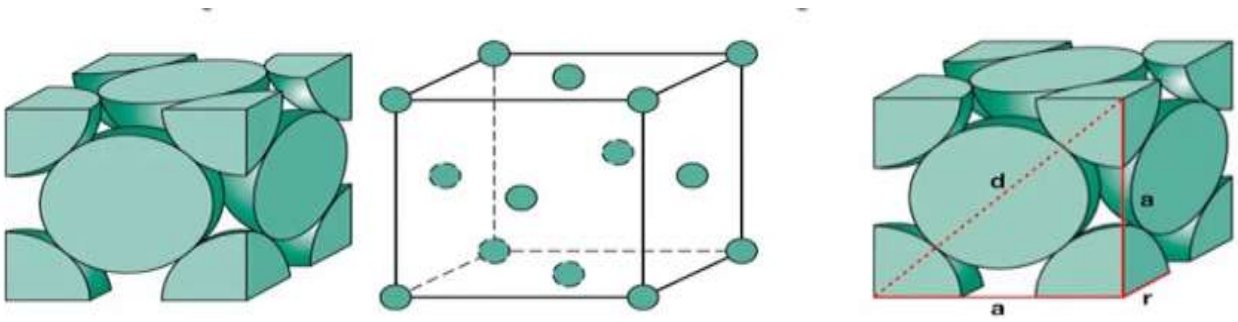
Volume of the unit cell =V= a^3=$(4r/\sqrt{2})^3$

Volume of all atoms in unit cell = v= $4\times(4/3)\pi r^3$

Atomic Packing Factor = v/V =$\frac{4\times(4/3)\pi r^3}{(4r/\sqrt{2})^3}$ =0.74=**74**%

Ex:- Cu, Al, Pb, and Ag.

By the above values of Atomic packing factors we can say that FCC is the closest packed structure of all the three cubic structures.



Miller indices: Miller indices are the reciprocals of intercepts made by the crystal planes on the crystallographic axes when reduced to smallest integers.

- **Rules for Miller Indices**

i. Determine the intercepts (a,b,c) of the face along the crystallographic axes, *in terms of unit cell dimensions.*
ii. Take the reciprocals
iii. Clear fractions
iv. Reduce to lowest terms
v. If a plane has negative intercept, the negative number is denoted by a bar ($\bar{}$) above the number.

Never alter negative numbers. **For example, do not divide -1, -1, -1 by -1 to get 1,1,1.**

vi. If plane is parallel to an axis, its intercept is zero and meets at infinity.
vii. The three indices are enclosed in parenthesis, *(hkl)*. A family of planes is represented by *{hkl}*.

Important features of Miller indices:

- Miller indices represent a set of parallel equidistant planes.
- All the parallel equidistant planes have the same Miller indices.
- If a plane is parallel to any axis, then the plane intersects that axis at infinity and Miller indices along that direction is zero.
- If the miller indices of the two planes have the same ratio (844,422,211), then the planes are parallel to each other.
- If a plane cuts an axis on the –ve side of the origin, then the corresponding index is –ve, and is indicated by placing a minus sign above the index.

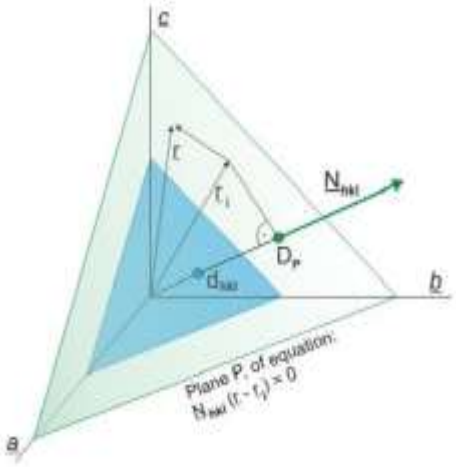
Ex: if a plane cuts –ve y-axis, then the miller index of the plane is (h $\bar{k}$ l)

Separation between successive planes :

- Consider a crystal in which the three axes are orthogonal and the intercepts are same. Take ‘o’ as origin, and the reference plane passes through the origin i.e entirely lies on the axis.
- The next plane ABC is to be compared with the reference plane which makes the intercepts $ah,,$ on x,y,z axes respectively.
- Let (h k l) be the miller indices.

- Let ON=d be a normal drawn to the plane ABC from origin 'o' which gives the distance of separation between adjacent planes.
- Let the normal ON makes an angles α,β,γ with x,y,z axes respectively

Angle α= NOA, angle β=NOB, angle c = NOC.



- Then form Δ le NOA

  $\text{Cos } \alpha = \frac{ON}{OA} = \frac{d}{a/h} = \frac{dh}{a}$
- Similarly $\cos \beta = \frac{ON}{OB} = \frac{d}{b/k} = \frac{dk}{b}$
- $\text{Cos } \gamma = \frac{ON}{OC} = \frac{d}{c/l} = \frac{dl}{c}$
- According to cosine law of directions, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma =$
- Therefore $\left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{b}\right)^2 + \left(\frac{dl}{c}\right)^2 = 1$
- $d^2\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right] = 1$
- In a cubic crystal a = b = c.
- Therefore

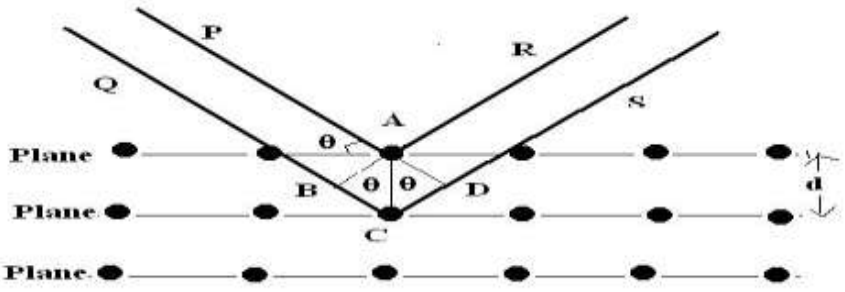
  $d^2\left[\frac{h^2}{a^2} + \frac{k^2}{a^2} + \frac{l^2}{a^2}\right] = 1$

- Therefore $d^2 = \frac{a^2}{h^2+k^2+l^2}$

  i.e $d = \frac{a}{\sqrt{h^2+k^2+l^2}}$

Bragg's law

Let us consider a set of planes in a crystal spaced by inter planar distance d.Consider a narrow monochromatic X-ray beam of wavelength λ , incident on the first plane, at a glancing angle θ as shown in fig.The incident beam undergoes multiple reflections between the parallel planes of the crystal.



Consider a ray PA,incident on the first plane and it reflected in the direction AR from that plane by the atom A. Let the incident beam PA making glancing angle θ to thr first plane.Similarly ,consider a parallelee ray QC is reflected in the direction CR by another atom C,in the second plane..

To determine the path difference between the two rays PAR and QCS, draw normals from the point A to the lines QC and CS.Let the normals AB and AD .Therefore,the path difference between these two rays is equal to BC+CD.

In the Δ ACB ,sinθ =$\frac{BC}{AC}$

Therfore,BC = AC sinθ = d sinθ

Similarly, CD = d sinθ

Therfore, the path difference is given as

∇=BC+CD = d sinθ + d sinθ = 2 d sinθ ……(1)

The two rays AR and CS interfere constructively and produce a maximum intensity ,the the path difference is nλ

∇= nλ ……(2)

From eqn.(1) and (2)

2 d sinθ = nλ (n= 1,2,3, etc.)

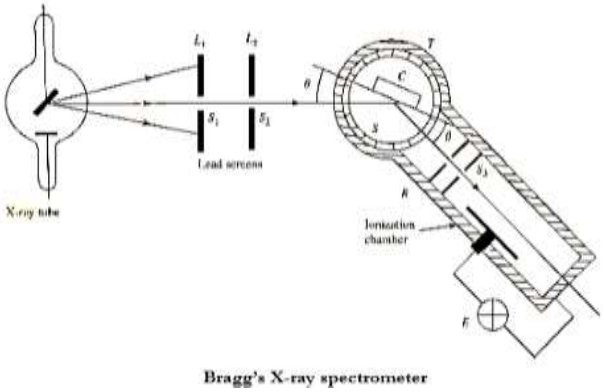
This is known as Bragg's law.

Bragg's Spectrometer:

Bragg's spectrometer is used to measure lattice constant and inter planar distance using Bragg's law.

Construction:

X-ray spectrometer consists of a turn table on which a crystal is mounted. It is a capable of rotating through any desired position by a vertical axis.It also consisting of ionization chamber to collect the intensity of reflected beam.



Bragg's X-ray spectrometer

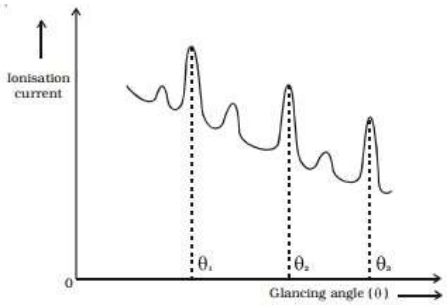


Fig Plot of glancing angle and ionisation current

Working:
X-ray from X-ray tube passes through two narrow slits S1 and S2 and strikes the single crystal mounted on the turntable. The intensity of reflected beam is measured at different angles using ionization chamber. The Bragg angle i.e., the angle at which reflection is maximum is deduced from this. The glancing angle is varied and the corresponding values of the intensity of the reflected beam are noted. A graph is drawn between the glancing angle and intensity as shown in fig.The graph is known as the X-ray spectrum.The angles corresponding to the prominent peaks are taken as $sin\theta_1$, $sin\theta_2$, $sin\theta_3$ respectively.This ahows that the peaks refer that first,second and third order respectively,of the same wavelength.

Using Bragg's equation

$$\mathbf{2\ d\ \sin\theta = n\lambda}$$

Inter planar spacing and lattice constant can be calculated.

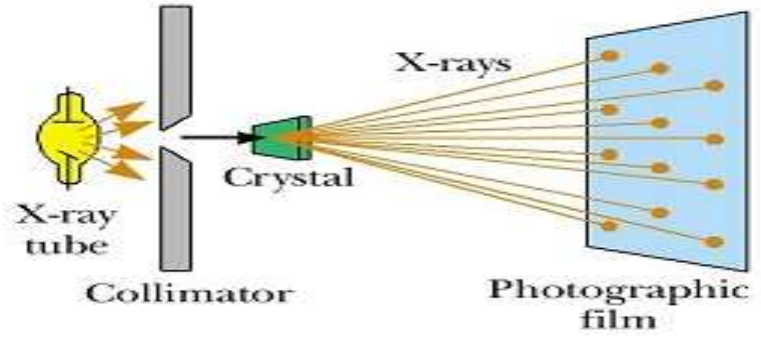
$$d \propto \frac{\lambda}{\sin\theta} \qquad d_{100} : d_{110} : d_{111} = \frac{1}{\sin\theta_1} : \frac{1}{\sin\theta_2} : \frac{1}{\sin\theta_3}$$

From the Bragg angle it is possible to determine the interplanar spacing of a crystal from which crystal type can be deduced.

Laue method

This method is used to study the orientation of the crystal and crystal symmetry.In this method a single crystal specimen is held fixed and it is irradiated with white X-ray radiation.The rays diffracted through the crystal is made to fall on a photographic film placed in its front as shown in fig.

Since the crystal is fixed in position,the angles of diffraction are also fixed.An interpretation of the reflection spot in the photographic film leads to know about the crystal structure of the specimen.



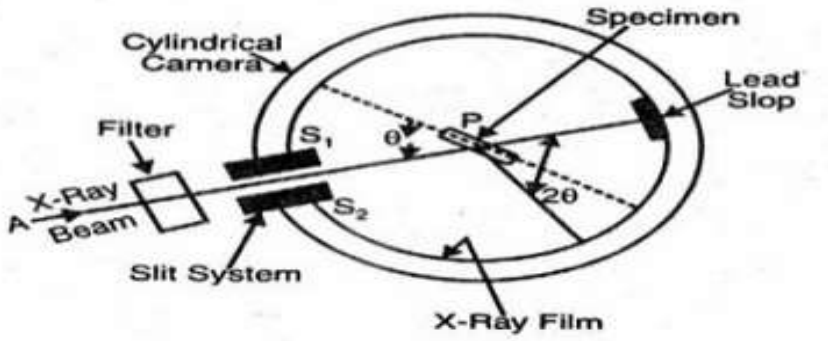
Powder method

- In powder method, a specimen is finely powdered and taken in a thin walled capillary tube.
- The specimen consists of tiny crystals or crystallites which are oriented randomly.
- When a narrow beam of X-ray incident on the specimen, it comes across few crystallites with planes at glancing angle θ so as to satisfy Bragg's Law.
- Since all the orientations are equally likely, the diffracted rays will form a cone with the line of incident beam as the axis and the semi cone angle θ.
- These diffracted beams are detected by placing a photographic film along the circumference of the circle with specimen at the center.
- If "L" is the circumferential distance from the two extreme edges of cone formed by the diffracted X-rays and "R" is the radius of the circle along which film was placed, then;

$$\frac{L}{2\pi R} = \frac{4\theta}{360}$$

$$\therefore \theta = \frac{45L}{\pi R}$$

- Using θ, we can find the inter-planar spacing using Bragg's Law.



Powder crystal method.