DE & VC Unit IV

1) Find the directional derivative of

(i) $\phi = xy^2 + yz^2$ in the direction of $\bar{i} + 2\bar{j} + 2\bar{k}$ at the pt. $(2, -1, 1)$

(ii) $f = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2\bar{i} - \bar{j} - 2\bar{k}$

(iii) $\phi = x^4 + y^4 + z^6$ at the pt. $(-1, 2, 3)$ in the direction towards the pt $(1, -2, -1)$

(iv) $\bar{V} = x^2y^4 + z^2y^4 + x^2z^4$ at the pt $(2, 0, 3)$ in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 14$ at the pt $(3, 2, 1)$

2) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ & $x^2 + y^2 - z = 3$ at the pt $(2, -1, 2)$

3) Find $\operatorname{div} \bar{f}$, $\operatorname{curl} \bar{f}$ when $\bar{f} = \operatorname{grad}(x^3 + y^3 + z^3 - 3xyz)$

4) If $\bar{f} = xy^2\bar{i} + x^2yz\bar{j} + 3yz^2\bar{k}$ find $\operatorname{div} \bar{f}$, $\operatorname{curl} \bar{f}$ at $(1, 2, 3)$

5) If the given function $\bar{A} = (2x + 3y + az)\bar{i} + (bx + 2y + 3z)\bar{j} + (2x + cy + 3z)\bar{k}$ is irrotational, find the constants a, b, c. Also find ϕ such that $\bar{A} = \nabla\phi$

6) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential function.

7) Prove that $\nabla(r^n) = nr^{n-2}\bar{r}$

8) Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$

9) Prove that $\operatorname{div}(\operatorname{curl} \bar{F}) = 0$

10) Prove that $\operatorname{curl}(\operatorname{grad} \phi) = \bar{0}$