CYCLOIDAL CURVES

These curves are generated by a fixed point on the circumference of a circle, which rolls without slipping along a fixed straight line or a circle.

- The rolling circle is called as generating circle.
- The fixed straight line or a circle is called as directing line or directing circle.

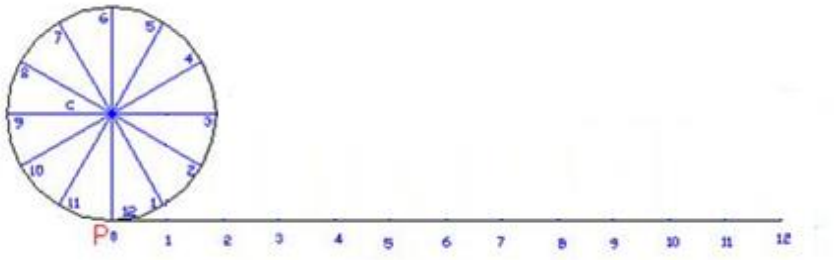
CYCLOID

Cycloid is a curve are generated by a fixed point on the circumference of a circle, which rolls without slipping along a fixed straight line.

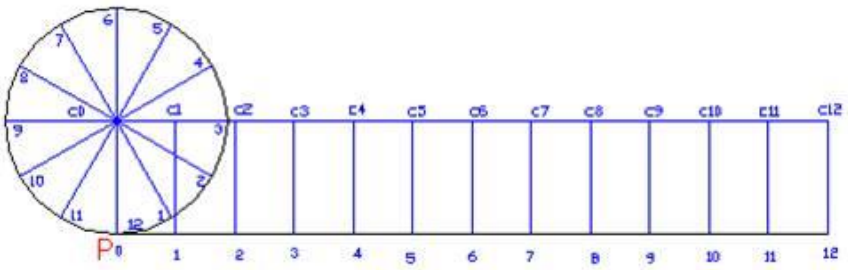
- **To construct a cycloid, given the diameter of the generating circle.**

1. With center C and given radius R, draw a circle.
2. Let P be the generating point. Draw a line PA tangential to and equal to the circumference of the circle.

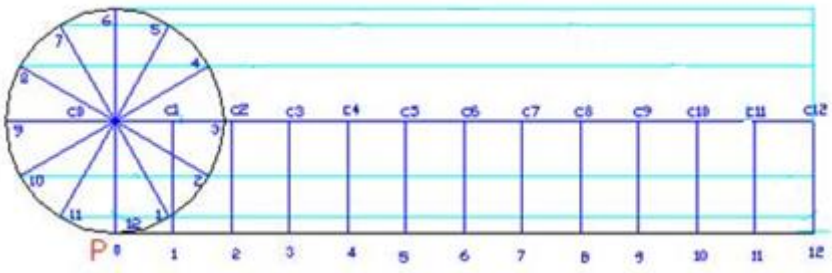
3. Divide the circle and the base line PA into the same number of equal parts (say 12). And mark the division points as shown.



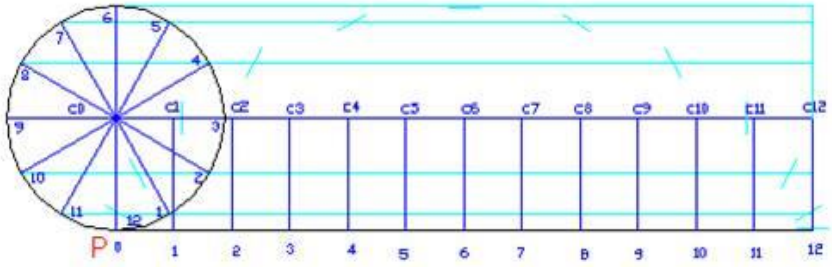
4. Through C, draw a line CC_{12} parallel and equal to P12. Draw perpendiculars at points 1,2 etc. cutting CC_{12} at points C_1, C_2 etc.



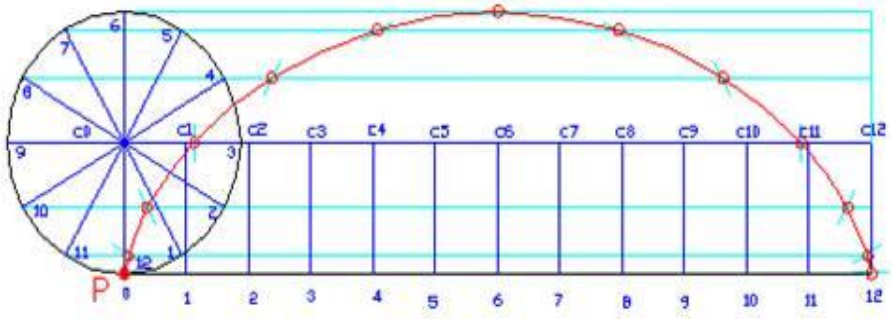
5. Through the points 1', 2' etc draw lines parallel to P12.



6. With your compass set to the radius of the circle and centers as C1, C2, C3, ... etc cut the arcs on the lines from circle through 1,2,3, .. etc.



7. Draw a smooth curve through cutting points. This curve is the required cycloid.



Construction of Tangent and Normal to a point on Cycloid

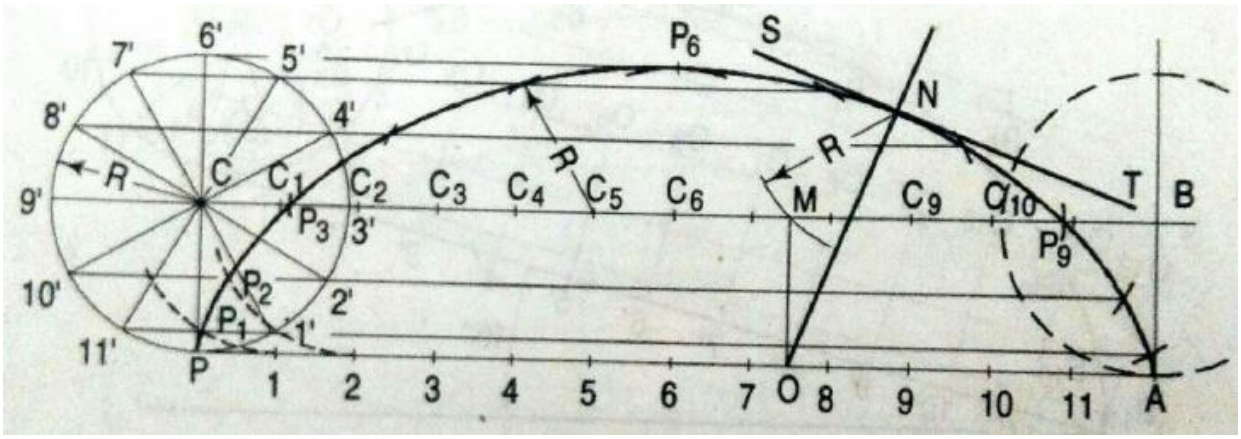
(i) With centre *N* and radius equal to *R*, draw an arc cutting *CB* at *M*.

(ii) Through *M*, draw a line *MO* perpendicular to the directing line *PA* and cutting it at *O*.

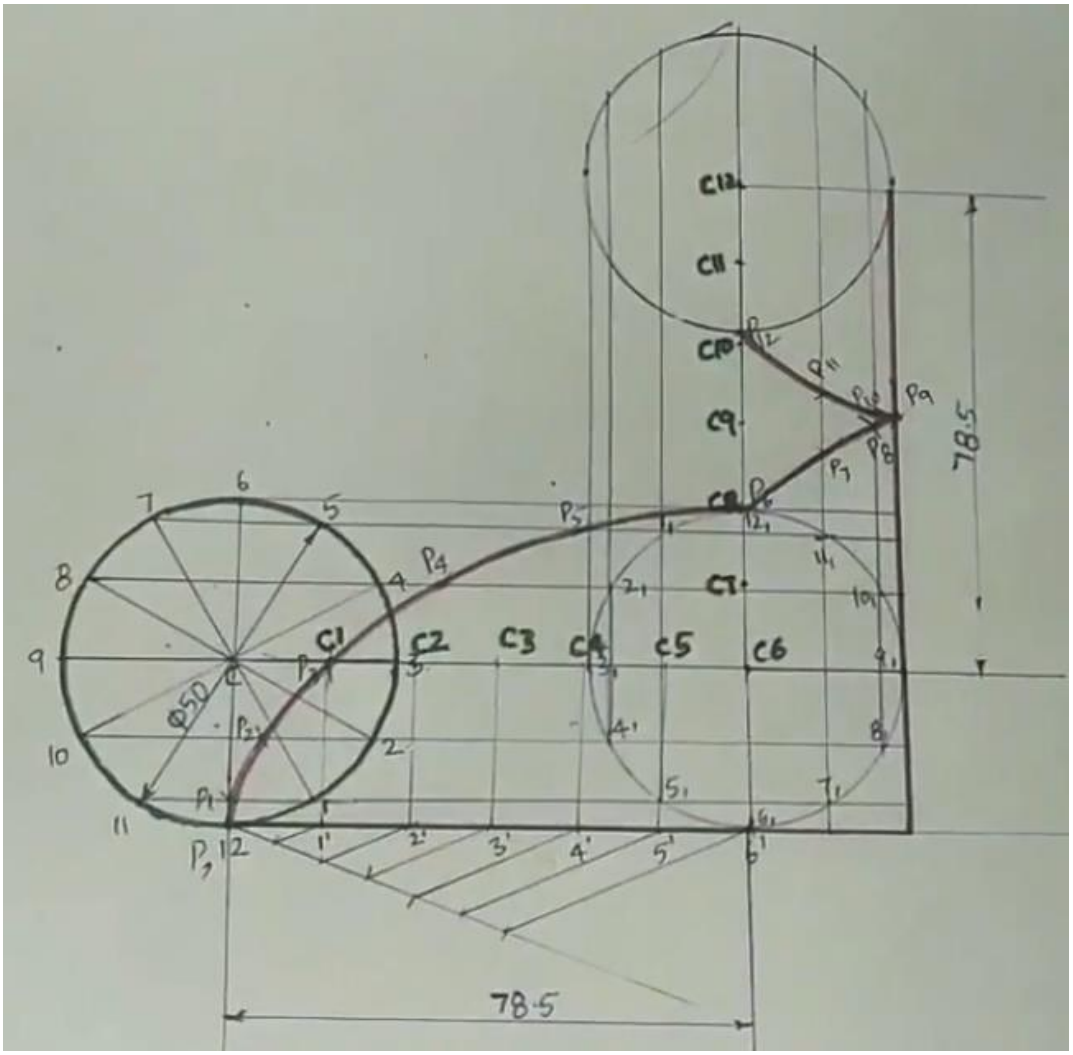
O is the point of contact and *M* is the position of the centre of the generating circle, when the generating point *P* is at *N*.

(iii) Draw a line through *N* and *O*. This line is the required normal.

(iv) Through *N*, draw a line *ST* at right angles to *NO*. *ST* is the tangent to the cycloid.



- **A circle of 60 mm diameter rolls on a horizontal line for a half revolution and then on a vertical line for another half revolution. Draw the curve traced out by a point P on the circumference of the circle.**



EPICYCLOID

The curve generated by a point on the circumference of a circle, which rolls without slipping along another circle outside it, is called an epicycloid.

- **To construct a epicycloid, given the diameter of the generating circle r and directing circle R.**

1. With centre O and radius R, draw the directing circle (only a part of it may be drawn). Draw a radius OP and produce it to C, so that CP = r.
2. With C as centre, draw the generating circle. Let P be the generating point.
3. In one revolution of the generating circle, the point P will move to a point A, so that the arc PA is equal to the circumference of the generating circle.
4. The position of A may be located by calculating the angle subtended by the arc PA at centre O, by the formula,

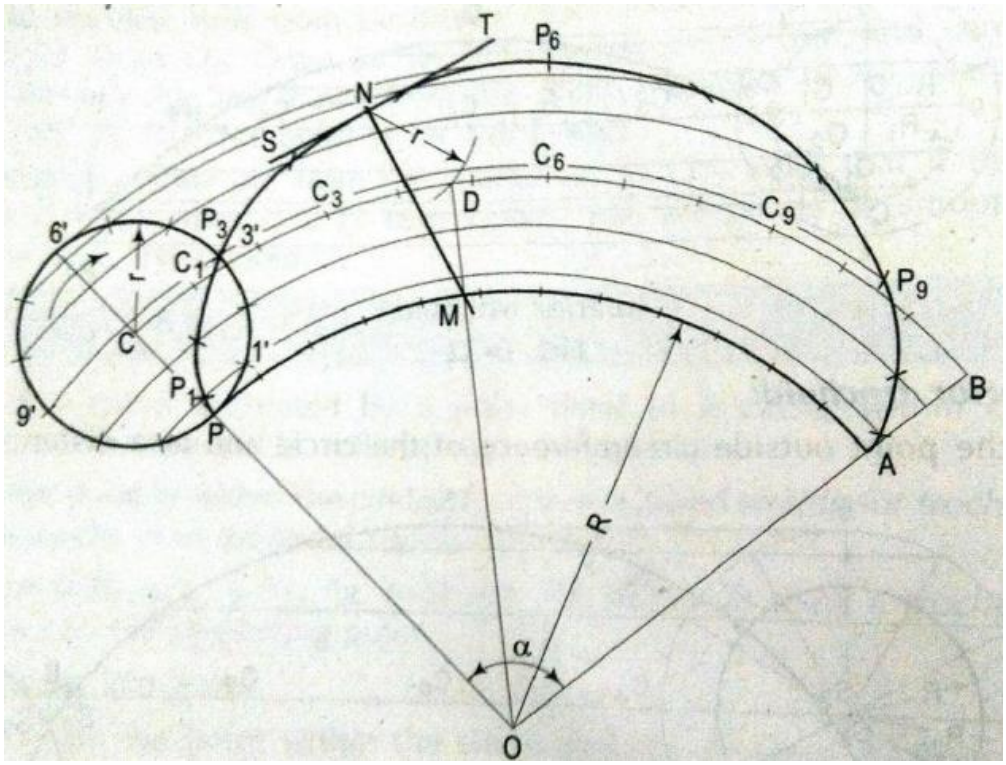
$$\angle POA = 360^\circ \times \frac{r}{R}$$

   i. Set- off this angle and obtain the position of A.
   ii. With centre O and radius equal to OC, draw an arc intersecting OA- produced at B. This arc CB is the locus of the centre C.
   iii. Divide CB and generating circle into 12 equal parts.
   iv. With centre O, describe arcs through points 1', 2', 3' etc.
   v. With centres C_1, C_2 etc. And radius equal to r, draw arcs cutting the arcs through 1', 2' etc. at points P_1, P_2 etc.
   vi. Draw the required epicycloid through the points P, P_1, P_2.....A.

Tangent and Normal

Draw the tangent and normal at point N.

1. With centre N and radius equal r, draw an arc cutting the locus of the centre C at point D.
2. Draw a line through O and D, cutting the directing circle at M.
3. Draw a line through N and M. This line is the normal. Draw a line ST through N and at right angles to NM. ST is the tangent.



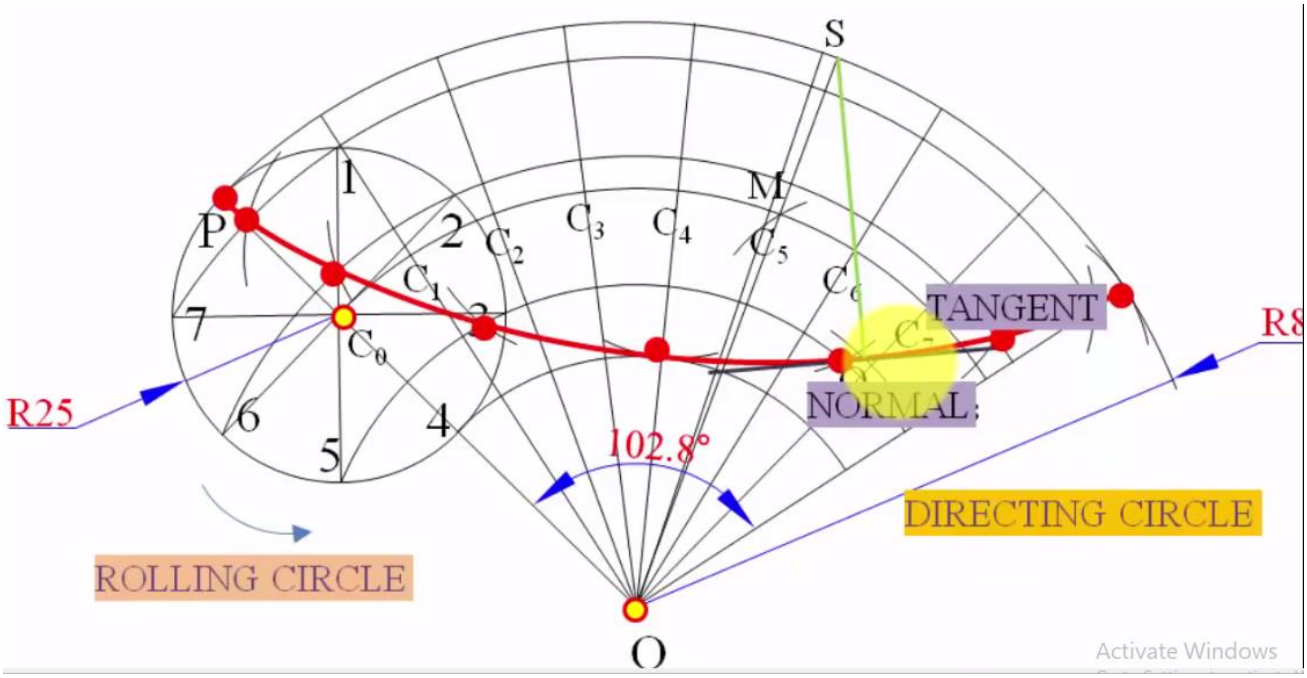
HYPOCYCLOID

The curve generated by a point on the circumference of a circle, which rolls without slipping along another circle inside it, is called hypocycloid.

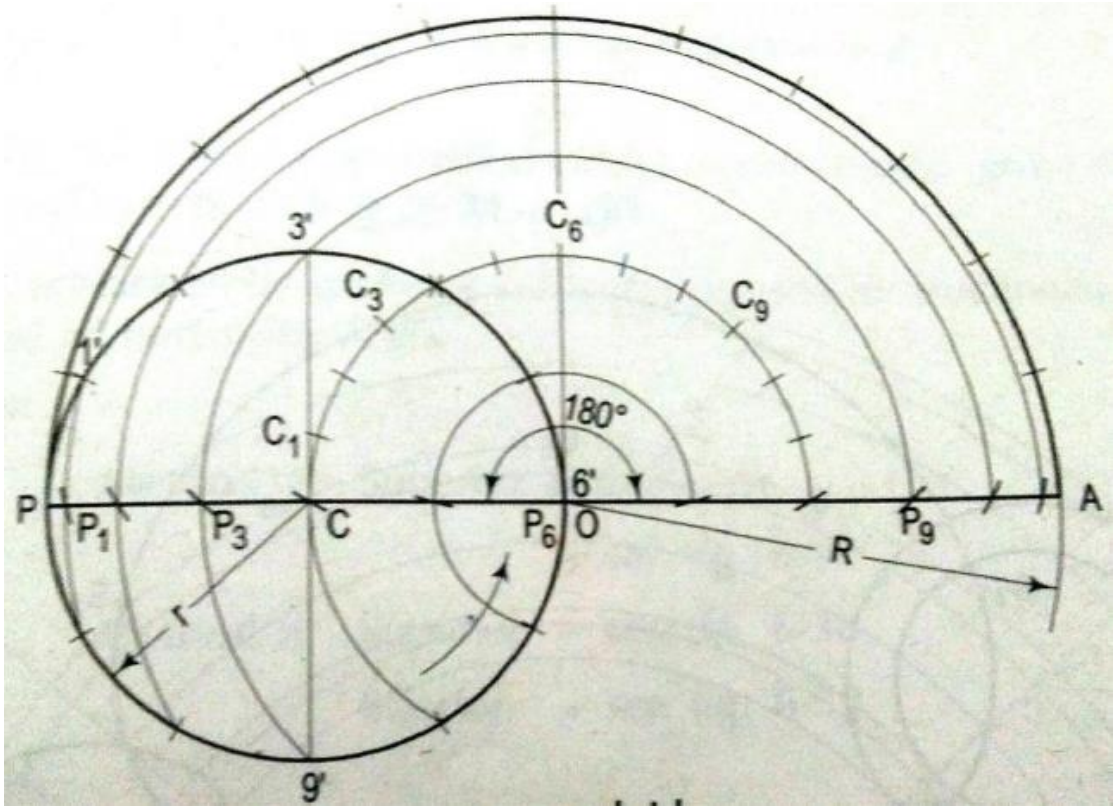
- **To construct a hypocycloid, given the diameter of the generating circle r and directing circle R.**

The method for drawing the hypocycloid is same as for epicycloid. **Note that the centre C of the generating circle is inside the directing circle.**

- **A circle of 50 mm diameter rolls on the circumference of another circle of 175 mm diameter and inside it. Trace the locus of the point on the circumference of the rolling circle for one complete revolution. Name the curve. Draw tangent and normal to the curve at a point 50 mm from the centre of the directing circle.**



- **Show by means of a drawing when the diameter of the rolling circle is twice that of the generating circle, the hypocycloid is a straight line. Take the diameter of the generating circle equal to 60 mm.**



INVOLUTE

The involute is a curve traced out by an end of a piece of thread unwound from a circle or a polygon, the thread being kept tight.

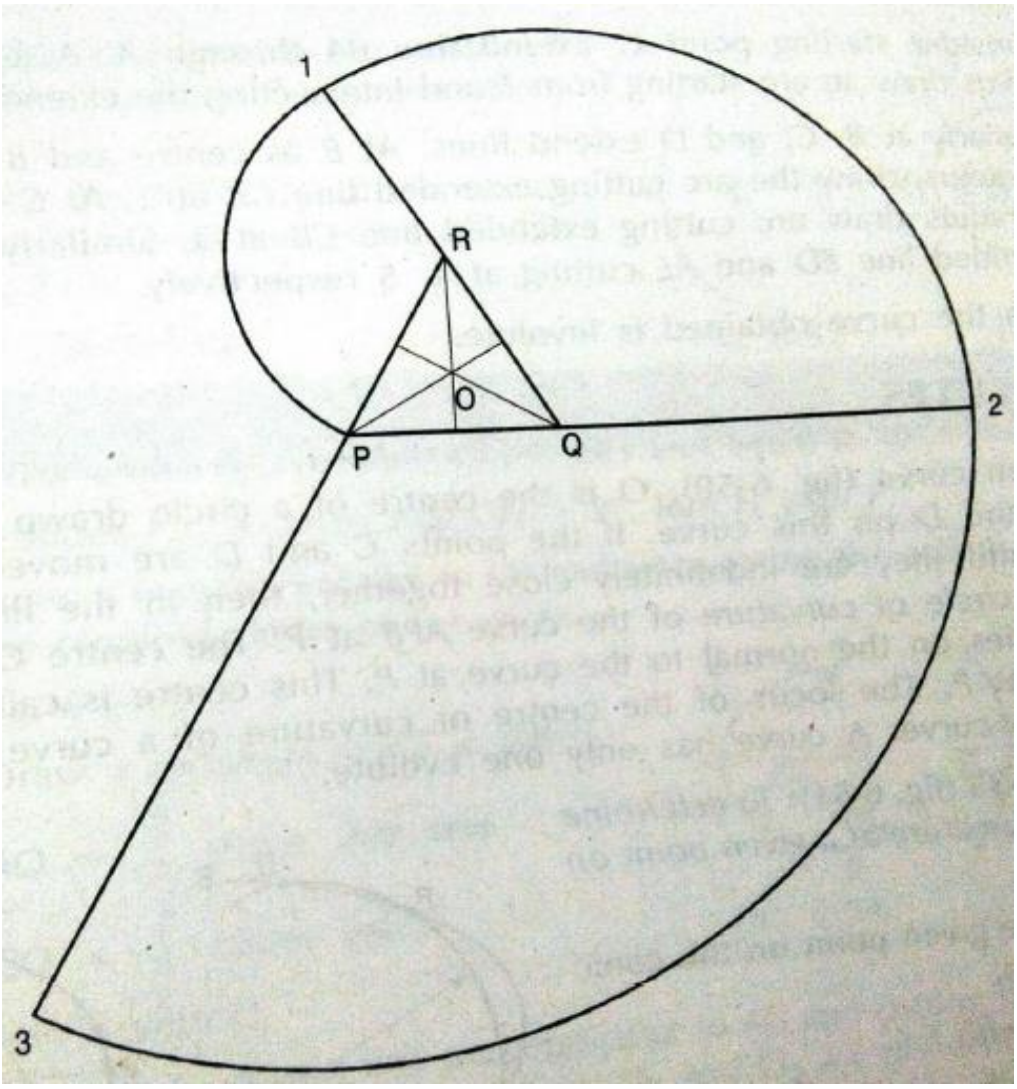
(or)

It is also defined as a curve traced out by a point in a straight line which rolls without slipping along a circle or polygon.

Draw the involute of a given triangle

- **A thin triangular equilateral plate of 20 mm side is pinned at its centriod O. An inelastic string circumscribes complete perimeter of the plate. One end of the string is attached to one of the apex of the plate. Draw the curve traced out by other end of the string keeping it tight, when the string is unwounded.**

1. Draw an equilateral triangle of 20mm sides. Determine its centroid by drawing perpendicular at midpoint of each side.
2. Consider starting point P. Taking PR=20 mm as radius and R as centre, draw the arc to intersect the extended line QR at point 1, of 40 mm as radius.
3. At the point Q, extend the line PQ equal to 2 QR = 40 mm. Q as centre draw the arc to cut at the point 2.
4. Extend line RP equal to 3 PQ = 60 mm P as centre, 60 mm as radius draw the arc to intersect the extended line RP at 3.
5. Thus the curve obtained is involute.



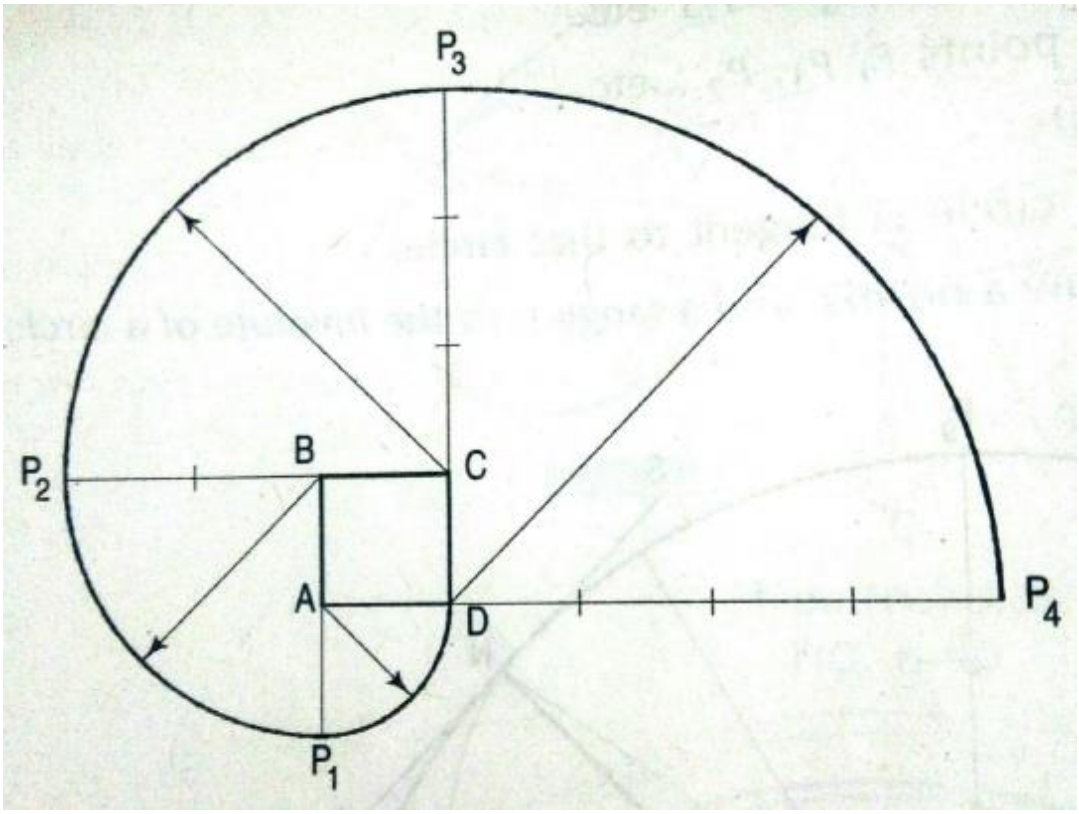
Draw the involute of a given square

Let *ABCD* be the given square.

(i) With centre *A* and radius *AD*, draw an arc to cut the line *BA*-produced at a point P_1.

(ii) With centre *B* and radius BP_1 (i.e. $BA + AD$) draw an arc to cut the line *CB*-produced at a point P_2. Similarly, with centres C and *D* and radii CP_2 (i.e. $CB + BA + AD$) and DP_3 (i.e. $DC + CB + BA + AD$ = perimeter) respectively, draw arcs to cut *DC*-produced at a point P_3 and *AD*-produced at a point P_4.

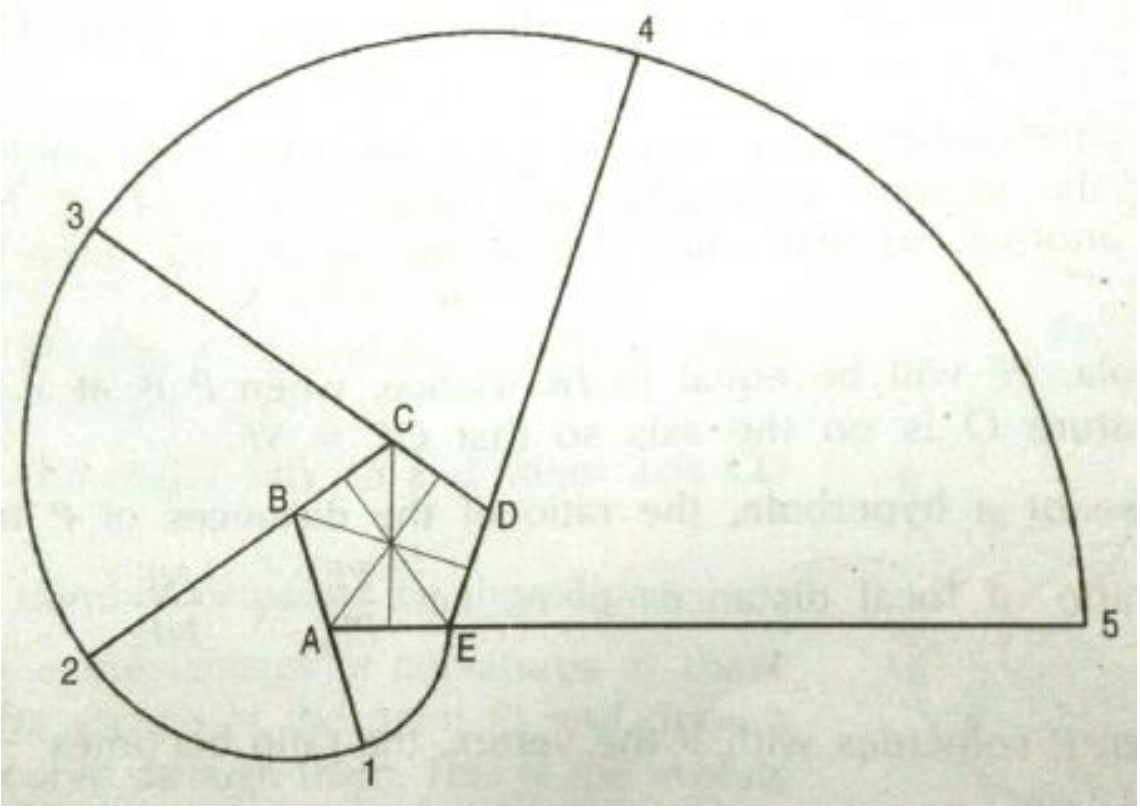
The curve thus obtained is the involute of the square.



Draw the involute of a given pentagon

- **A regular pentagonal plate of 20 mm side is fixed at its centre. An inelastic rope is circumscribed along the perimeter of the pentagonal. Draw the path of free end of the rope when it is unwounded keeping, tight for one complete turn.**

1. Construct pentagon of 20 mm side as shown in fig.
2. Name the corners as A, B, C, D and E.
3. Consider starting point E. Extend line BA through A. A as centre, AE as radius draw an arc starting from E and intersecting the extended line BA at 1.
4. Similarly at B, C and D extend lines. At B as centre and B1 as radius, draw arc cutting extended line CB at 2. At C and C3 as radius, draw arc cutting extended line CB at 3. Similarly draw arcs for extended line ED and AE cutting at 4, 5 respectively.
5. Thus the curve obtained is involute.



Draw an involute of a given circle.

- With centre C, draw the given circle. Let P be the starting point, i.e the end of the thread.
- Suppose the thread to be partly unwound, say upto a point 1. P will move to a position P_1 such that $1P_1$ is tangent to the circle and is equal to the arc 1P. P_1 will be a point on the involute.

Construction

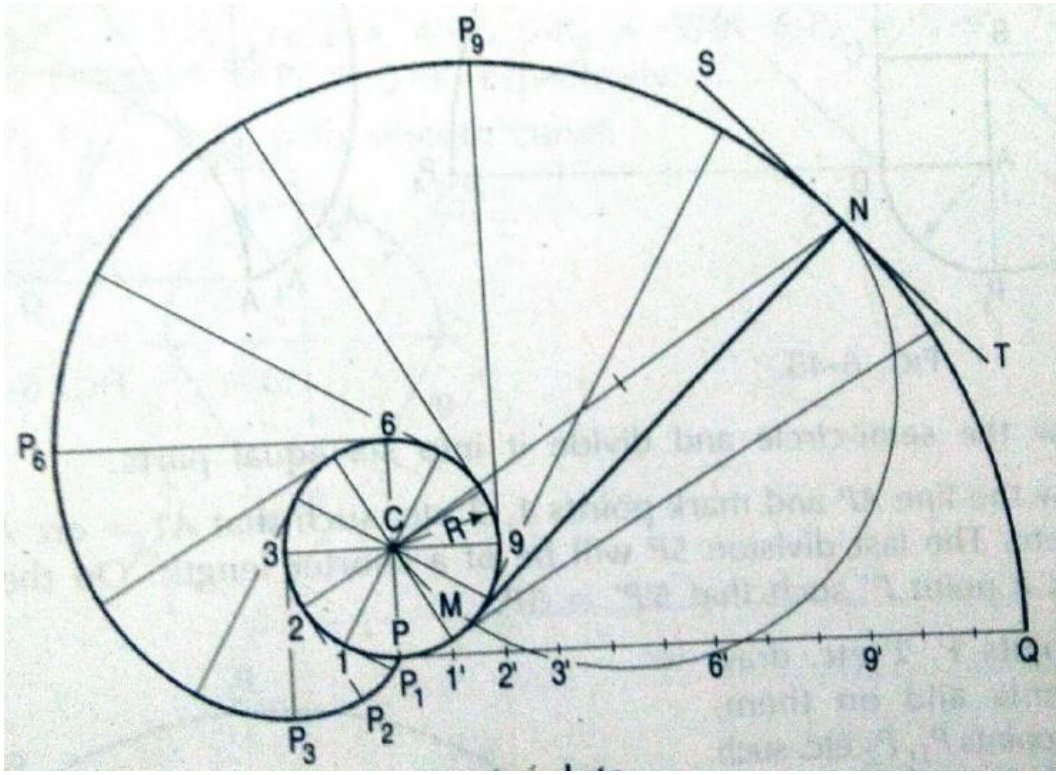
1. Draw a line PQ, tangent to the circle and equal to the circumference of the circle.
2. Divide PQ and the circle into 12 equal parts.
3. Draw tangents at points 1, 2, 3 etc. and mark on them points P_1, P_2, P_3 etc. such that $1P_1$=P1', $2P_2$=P2', $3P_3$=P3' etc.

Draw the involute through the points P_1, P_2, P_3 etc.

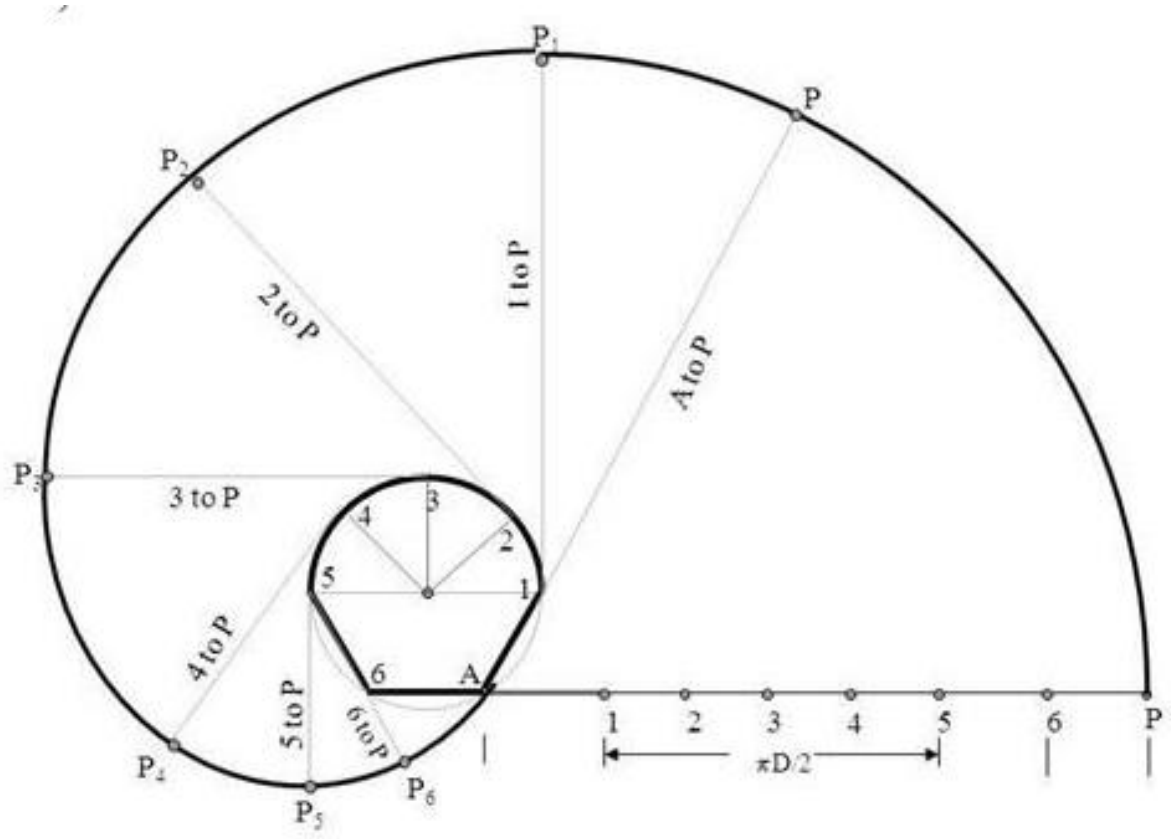
Tangent and Normal

1. Draw a line joining C with N.
2. With CN as diameter describe a semi-circle cutting the circle at M.

3. Draw a line through N and M. This line is the normal. Draw a line ST, perpendicular to NM and passing through N. ST is the tangent to the involute.



- **Apole is of a shape of half hexabon and semicircle. Astring is to be wound having lengt h equal to the pole perimeter. Draw path of free end p of string when wound completely. (Take hex 30 mm sides and semicircle of 60 mm diameter).**



Problems on Conics

1. Construct an ellipse when the distance between the focus and the directrix is 30mm and the eccentricity is 3/4. Draw the tangent and normal at any point P on the curve using directrix.
2. The major axis of an ellipse is l50 mm long and the minor axis is 100 mm long. Find the foci and draw the ellipse by Arcs of Circles method. Draw a tangent to the ellipseat appoint on it 25 mm above the major axis?
3. Construct an ellipse when the major axis is l20mm and the distance between the foci is 108 mm. determine the length of the minor axis.
4. The foci of an ellipse are 90mm apart and the minor axis is 72mm long. Determine the length of the major axis. Construct the ellipse.
5. The distance between two fixed points is equal to 75mm. A point P moves such that the sum of its distances from the two fixed points is always a constant and is equal to 90inm. Draw the locus of P and determine the minor axis.
6. The sum of the distances of a point P from two fixed points is 120 mm and the distance between the fixed points is 80 mm. Draw the locus of the Point P.

7. The major and minor axes of an ellipse are l20mm and 80mm. Construct an ellipse using arcs of circles method?
8. Construct an ellipse of 120 mm major axis and 80 mm minor axis using concentric circle methods?
9. The major and minor axes of an ellipse are 120mm and 80mm. Construct an ellipse using oblong method.
10. Draw an ellipse at major axis 100 mm and minor axis 65 mm using oblong method. Draw a tangent and normal at a point 25 mm above the major axis.
11. Draw an ellipse of major and minor axes of 140mm and 85 mm respectively, using oblong method.
12. Inscribe an ellipse in a parallelogram having sides l50mm and 100mm long and an included angle of 120^0.
13. A plot of ground is in the shape of rectangle 110 m x 50 m. Inscribe an elliptical lawn in it. Take a scale of 1: 1000.
14. The foci of an ellipse are 90mm apart and the minor axis is 65mm long. (i) Determine the length of the major axis and draw half the ellipse by concentric circles method and the other half by oblong method. (ii) Draw tangent and normal to the curve drawn.
15. A fixed point F is 7.5cm from a fixed straight line. Draw the locus of a point P moving in such a way that its distance from the fixed straight line is equal to its distance from F. Name the curve. Draw normal and tangent at a point 6cm from F.
16. A cricket ball thrown from the ground level reaches the wicketkeeper's gloves. Maximum height reached by the ball is 5 m. The ball travels a horizontal distance of 11 m from the point of projection. Trace the path of the ball.
17. The vertex of a hyperbola is 65mm from its focus. Draw the curve if the eccentricity is 3/2. Draw a normal and a tangent at a point on the curve, 75mm from the Directrix.
18. Sketch the section planes of a cone which produces parabola and hyperbola.
19. Construct a parabola when the distance of the focus from the directrix is 50 mm.
20. Construct a epicycloid, rolling circle 60 mm diameter and directing circle 60 mm diameter. Draw a tangent to it at a point 45 mm from the center of the directing circle.
21. Draw epicycloids and hypocycloid of a rolling circle of 60 mm diameter and directing circle of 120 mm diameter.
22. Draw an involute of half circle of 60 mm diameter which is compounded by half square of 60 mm side.