4.6 Summary: Integration

You should have a good understanding of the definitions of indefinite and definite integrals, and how they are related via the Fundamental Theorem of Calculus. You should also know how to use methods to calculate integrals such as integration by parts, recurrence relations and simplifications by considering even and odd (parts of) functions integrated over an interval symmetric about $x = 0$. Here are some key points:

- A function F is an *indefinite integral* or *antiderivative* of a function f if the derivative of F is f, i.e. if $F'(x) = f(x)$. Note that for a given f, F is not unique, but is fixed up to an *integration constant*.

- The *definite integral* of f over an interval $[a, b]$ is the area under the graph of f over this interval and can be defined as a limit of *Riemann sums*. When this limit exists we say f is *integrable* on $[a, b]$ (and if f is continuous on $[a, b]$ then it is integrable on $[a, b]$) and we denote the integral by $\int_a^b f(x)dx$.

- The *Fundamental theorem of calculus* says that if we define $F(x) = \int_a^x f(t)dt$ then F is an antiderivative of f, i.e. $F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$.

- You should know the definition of *even* and *odd functions* and that any function f can be uniquely split into its even and odd parts $f = f_{even} + f_{odd}$. The integral of any odd function over $[-a, a]$ is zero and the integral of any even function over $[-a, a]$ is twice the integral of that function over $[0, a]$.

- Sometimes we have a family of definite or indefinite integrals parametrised by n which we can denote I_n. If we can find a *recurrence relation* relating the integrals for different values of n, we can use this to calculate I_n. E.g. if we can determine I_{n+1} in terms of I_n and we can calculate I_1 then the recurrence relation determines I_n for all $n \in \mathbb{N}$.

- This section also contained some results for limits which are summarised as "*exponentials beat powers beat logs*" as $x \to \infty$. We also saw that $\lim_{y\to\infty}\left(1 + \frac{x}{y}\right)^y = e^x$.