******The following questions are concerned with Chapter 2 of the notes - Linear Codes.******

17 Write out an addition table and a multiplication table for $\mathbb{Z}/5$ and $\mathbb{Z}/6$. Use your tables to show that $\mathbb{Z}/6$ is not a field.

18 Let $C = \{(0,0,2),(1,1,0),(2,2,1)\} \subseteq \mathbb{F}_3^3$. Is C a linear code? Find its span, $\langle C\rangle$, a ternary $[n,k,d]$ code. What are n, k, and d?

19 Show that a q-ary $[n,k,d]$ MDS code satisfies $d = n - k + 1$. Use this to check that the code $C = \{000, 111\} \subset \mathbb{F}_2^3$ is MDS.

20 Let $C = \langle\{(0,1,0,1,0),(1,0,1,1,0),(0,1,1,1,0),(1,0,0,1,0)\}\rangle \subseteq \mathbb{F}_2^5$. Find a basis for C, and its dimension.

21 Show that, for a prime p, the p-ary code $C = \{(0,1),(1,1),\ldots,(p-1,1)\} \subseteq F_p^2$, is a $(2,p,1)$ code but not a $[2,1,1]$ code.

22 Show that, in a decoding array constructed according to the algorithm given, every possible received word $\mathbf{y} \in \mathbb{F}_q^n$ appears exactly once. (Hint: Show first that the same $\mathbf{y}$ cannot appear twice in the same row, and then that the same $\mathbf{y}$ cannot appear twice in *different* rows. (This contradiction argument is similar to the proof of Proposition 2.10.))

23 Make a decoding array for the code $C_2 = \{(0,0,0),(1,1,1)\} \subseteq \mathbb{F}_2^3$. If C_2 is transmitted over a symmetric binary channel with symbol-error probability p, use Proposition 2.11 to find the probability that a codeword $\mathbf{c} \in C_2$ will be successfully decoded.

24 Make a decoding array for the code $C_3 = \{(0,0,0),(1,1,1),(2,2,2)\} \subseteq \mathbb{F}_3^3$. Use it to decode the words $(1,2,1)$ and $(1,2,0)$. If C_3 is transmitted over a symmetric ternary channel with symbol-error probability p, use Proposition 2.11 to find the probability that a codeword $c \in C_3$ will be successfully decoded.

25 In making your array for Q24, when did you have to make arbitrary choices? Which of these choices will affect decoding? Which words may be decoded differently by different arrays? Explain by considering a different (but still correct!) array for C_3. Is the situation the same for C_2 of Q23?

26 Suppose we have a decoding array for a q-ary $[n,k,2t+1]$ code C (t any integer). For $\mathbf{c} \in C$ and $r \le t$, where in the array would we find the the words of the sphere $S(\mathbf{c}, r)$? (Look at Q25, and/or draw a general, schematic array).

27 In terms of the definition of a perfect code ("there is some t such that...."), what words are in the first column of a decoding array for a perfect code? Explain why, for perfect codes, all arrays will decode identically.