Calculus I, Tutorial Problem Sheet, Week 3

Functions: Even, odd and inverse functions

Q1. Are the following functions even, odd or neither? Justify your answers.

(a) $f(x) = (x-1)(x-2)$

(b) $f(x) = \sum_{k=0}^{n} x^{2k+1}$

(c) $f(x) = \frac{x}{(x^2+1)\cos x}$

Q2. If $f : \mathbb{R} \mapsto \mathbb{R}$ is an even function and $g : \mathbb{R} \mapsto \mathbb{R}$ is an odd function then determine whether the following functions are even, odd or neither? Justify your answers.

(a) $f_1(x) = \begin{cases} f(x) & \text{if } x > 0 \\ -f(x) & \text{if } x < 0 \end{cases}$

(d) $f_2(x) = (g \circ g)(x)$

Q3. Which of the following functions are injective? Find the inverses of those which are and specify the domain of the inverse.

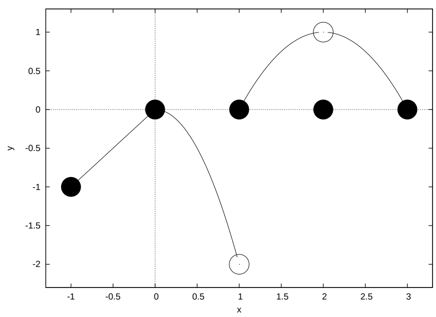
(a) $f(x) = (1-x)^2$ in $[1, 2]$

(b) $f(x) = (x-1)/(x+2)$ in $\mathbb{R}\backslash\{-2\}$

(c) $f(x) = x^2 + 2x - 1$ in $[-2, 2]$

Limits

Q4. Consider the given graph of the function $f(x)$. Are the following statements true or false?



$f(x)$ for Q4

(a) $\lim_{x\to 2} f(x)$ does not exist, (b) $\lim_{x\to 2} f(x) = 1$, (c) $\lim_{x\to 1} f(x)$ does not exist,

(d) $\lim_{x\to a} f(x)$ exists $\forall\ a \in (-1, 1)$ (e) $\lim_{x\to a} f(x)$ exists $\forall\ a \in (1, 3)$.

Q5. In each case either evaluate the limit, or state that no limit exists

(a) $\lim_{x\to\pi/2} x\sin x$, (b) $\lim_{x\to 1}\frac{x^4-1}{x^3-1}$, (c) $\lim_{x\to\pi}\frac{\cos x}{1-\pi}$. (d) $\lim_{x\to 1}\frac{x-1}{\sqrt{x+3}-2}$,

(e) $\lim_{x\to 0}\frac{x^2}{1-\cos 2x}$ (f) $\lim_{x\to 3}\frac{(x^2+x-12)}{(x-3)^2}$, (g) $\lim_{h\to 0}\frac{1+1/h}{1+1/h^2}$.

Q6. If $f(x) > 0\ \forall\ x \neq a$ and $\lim_{x\to a} f(x) = L$, can we conclude that $L > 0$? Justify your answer.

Q7. Does $\lim_{x\to 0}\frac{\sin(x+|x|)}{x}$ exist?

If the limit exists then find it.

Q8. Calculate the limit as $x\to\infty$ of the following

(a) $\frac{6x+7}{1-2x}$, (b) $\frac{x^2}{x^2+\sin^2 x}$.