

Calculus I, Background Material Problem Sheet

Example problems using assumed background material

- Q1. Let $f(x) = x^2 + ax + 1$, where $a \in \mathbb{R}$.
Derive the allowed values of a such that $\forall x \in \mathbb{R}, f(x) = |f(x)|$.
- Q2. Demonstrate graphically that $||x| - 1| \leq |x - 1|$, $\forall x \in \mathbb{R}$. Prove this inequality (the graphical demonstration should provide a hint).
- Q3. Graph the following functions
(a) $f(x) = |-x^2 + 3x|$, (b) $f(x) = e^{-(x+1)^2}$, (c) $f(x) = |-x^2 - x - 1|$
- Q4. For $x \in [0, 4\pi]$, on the same drawing, graph the following three functions
 $h(x) = \sin x$, $g(x) = 1 + \sin x$, $f(x) = 1/(1 + \sin x)$.
- Q5. For $x \in [-\pi, \pi]$, on the same drawing, graph the following three functions
 $h(x) = \cos x$, $g(x) = \cos(2x)$, $f(x) = 1/\cos(2x)$.
- Q6. For $x \in [-2, 2]$, on the same drawing, graph the following three functions
 $h(x) = e^{-x}$, $g(x) = e^{-x} - 1$, $f(x) = 1/(e^{-x} - 1)$
- Q7. State any vertical and horizontal asymptotes of the following functions
(a) $f(x) = -2x^2/(x^2 - 9)$
(b) $f(x) = 3x/(x^2 + 1)$
(c) $f(x) = x^3/(x^2 + 2)$
(d) $f(x) = \frac{3x^5 + 5x^2}{(x-1)(7x^4 + 9)}$
- Q8. A rational function is a ratio of two polynomials and is called proper if the degree of the numerator is less than the degree of the denominator. Write each of the following rational functions as the sum of a polynomial and a proper rational function

(a) $\frac{x^6 + x^4 - x^3 + 2x^2 + 5}{x^2 + 1},$

(b) $\frac{x^6 + 2x^4 + 3x^2 + 5}{x^4 + 2},$

(c) $\frac{2x^5 + 4x^3 - 1}{2x^3 - 1},$

(d) $\frac{x^7 + 6x^4 + 8x + 1}{x^3 + 4}.$

Q9. Evaluate each of the following expressions (without using a calculator). Here \log denotes the natural logarithm, \log_e , sometimes denoted \ln .

(a) $\log e$, (b) $\log_3 \frac{1}{9}$, (c) $\log_{\frac{1}{16}} 4$, (d) $\log_8 8^{-3}$, (e) $\log_5 625$

(f) $\log_{10} 10^n$, $n \in \mathbb{Z}$, (g) $\frac{\log(ne)}{m \log n + \log e^m}$, $n, m > 0$

Q10. Find the Cartesian equation for the tangent to the graph of $f : \mathbb{R} \mapsto \mathbb{R} : x \mapsto 5x^2 - 4x$ at the point $(1, 1)$.

Q11. Find the slope of the straight line which passes through the point $(-2, 0)$ and is also tangential to the graph of $f(x) = \sqrt{x}$ at some point.

Q12. Evaluate the following definite integrals

(a) $\int_{-4}^{-2} (x+3)^{14} dx$, (b) $\int_1^2 \frac{6-t}{t^3} dt$, (c) $\int_0^{\frac{1}{2}} \frac{8}{1+4q^2} dq$.

Q13. Calculate the following indefinite integrals

(a) $\int \frac{x}{\sqrt{2+3x^2}} dx$, (b) $\int \cot x dx$, (c) $\int \frac{1}{1+\sqrt{x}} dx$.

Q14. Calculate the following integrals

(a) $\int x\sqrt{1+x} dx$, (b) $\int x^2 \cos x dx$, (c) $\int x^n \log x dx$, where n is a positive integer,
(d) $\int e^x \sin(3x) dx$, (e) $\int e^{-x} \sinh x dx$.

Q15. Write the following in partial fraction form

(a) $\frac{7x^2 - x - 2}{(x^2 - 1)(2x - 1)}$, (b) $\frac{7 - 2x}{(x+1)(x-2)^2}$, (c) $\frac{8}{(x-1)^2(x+1)(x^2+1)}$.

Q16. Calculate the following integrals

(a) $\int \frac{1+x^4}{x^2-x} dx$, (b) $\int \frac{x^2+1}{x(x^2-1)} dx$, (c) $\int \frac{1}{x^3+x^5} dx$.

Q17. Show that $\cosh^2 x - \sinh^2 x = 1$.

Q18. Differentiate $\cosh x$.

Q19. Differentiate $\tanh x$.