7 *Compute the gradient,* $\underline{\nabla} f$*, for the following functions:*

(a) $f(x,y,z) = \sqrt{x^2+y^2+z^2}$,

(b) $f(x,y,z) = xy+yz+xz$,

(c) $f(x,y,z) = 1/(x^2+y^2+z^2)$.

8 *Show that* $\underline{h}(s) = \left(s/\sqrt{2}, \cos(s/\sqrt{2}), \sin(s/\sqrt{2})\right)$ *is the arc-length parameterisation of a helix, that is that* $|\frac{dh}{ds}| = 1 \quad \forall s$.

Calculate the directional derivative of the scalar field $f(\underline{x}) = (\log(x^2+y^2+z^2))$ *along* $\underline{h}(s)$ *at* $s = \sqrt{2}\pi$.

9 *Draw a sketch of the contour plot of the scalar field on* $\mathbb{R}^2$ $f(\underline{x}) = xy$*, as well as the gradient of* f*. What do you notice?*

10 *Let* $f, g : \mathbb{R}^3 \to \mathbb{R}$ *be scalar fields on* $\mathbb{R}^3$*,* $h : \mathbb{R} \to \mathbb{R}$ *be a function on* $\mathbb{R}$ *and* a *be a constant in* $\mathbb{R}$*. Show (using the definition of* $\underline{\nabla}$*) that*

$$\underline{\nabla}(af(\underline{x})g(\underline{x}) + h(f(\underline{x}))) = a(\underline{\nabla} f)g + af\underline{\nabla} g + \underline{\nabla} f \frac{dh}{df}.$$

11 *Exam question June 2001 (Section B): You are given the following family of scalar functions labelled by a real parameter* λ *:* $\Phi_\lambda(x,y,z) = (y-\lambda)\cos x + zxy$.

(a) *What are their derivatives in the direction* $\mathbf{V} = \mathbf{e_1} + 2(\mathbf{e_2} + \mathbf{e_3})$*?*

(b) *Which member of the family has its gradient at the point* $(\frac{\pi}{2}, 1, 1)$ *equal to* $\frac{\pi}{2}(\mathbf{e_1} + \mathbf{e_2} + \mathbf{e_3})$*?*

(c) *Calling this particular member of the family* Φ_{λ_0}*, in which direction is* Φ_{λ_0} *decreasing most rapidly when starting at the point* $(\frac{\pi}{2}, 1, 1)$*?*

12 *Exam question June 2002 (Section A): Give the unit vector normal to the surface of equation* $x^2/a^2 + y^2/b^2 + z^2/c^2 = 4$ *where* a, b, c *are three real constants.*
What is the unit vector normal to a sphere of radius 2 at the point $(x,y,z) = (\sqrt{2}, 0, \sqrt{2})$*?*

13 *Find the vector equations of tangent and normal lines in* $\mathbb{R}^2$ *to the following curves at the given points*

(a) $x^2 + 2y^2 = 3$ *at* $(1,1)$,

(b) $xy = 1$ *at* $(2, 1/2)$,

(c) $x^2 - y^3 = 3$ *at* $(2,1)$.

14 *Exam question June 2003 (Section A): Find the directional derivative of the function* $\phi(x,y,z) = xy^2z^3$ *at the point* $P = (1,1,1)$ *in the direction from P towards* $Q = (3,1,-1)$*. Starting from P, in which direction is the directional derivative maximum and what is the value of this maximum?*

15 *Exam question June 2002 (Section A): What is the derivative of the scalar function* $\phi(x,y,z) = x\cos z - y$ *in the direction* $\mathbf{V} = \mathbf{e_1} + \mathbf{e_2} + \mathbf{e_3}$*? What is the gradient at the point* $(x,y,z) = (0, 1, \pi/2)$*? In which direction is* ϕ *increasing the most when moving away from this point?*

16 *A marble is released from the point* $(1, 1, c - a - b)$ *on the elliptic paraboloid defined by* $z = c - ax^2 - by^2$*, where* a, b, c *are positive real numbers and the* z*-coordinate is vertical. In which direction in the* (x, y) *plane does the marble begin to roll?*

17 *In which direction does the function* $f(x, y) = x^2 - y^2$ *increase fastest at the points (a)* $(1, 0)$*, (b)* $(-1, 0)$*, (c)* $(2, 1)$ *? Illustrate with a sketch.*

18 *Let* $f(x, y) = (x^2 - y^2)/(x^2 + y^2)$.

- *(a) In which direction is the directional derivative of* f *at* $(1, 1)$ *equal to zero?*
- *(b) What about at an arbitrary point* (x_0, y_0) *in the first quadrant?*
- *(c) Describe the level curves of* f *and discuss them in the light of the result in (b).*