Calculus I, Tutorial Problem Sheet, Week 5

Differentiable functions

Q1. Use the limit definition of the derivative to calculate the derivative of $f(x) = \sqrt{x}$.

Q2. Show that if $g(x)$ is continuous at $x = 0$ then $g(x)\tan x$ is differentiable at $x = 0$.

Q3. Use L'Hopital's rule to calculate the limit as $x \to 0$ of the following
(a) $\frac{1-\cos 2x}{x}$, (b) $\frac{x^2}{1-\cos 2x}$.

Q4. Assuming that y is a differentiable function of x and satisfies $xy + y^2 - 3x - 3 = 0$, Calculate $\frac{dy}{dx}$ at the point $(-1, 1)$.

Extreme values

Q5. Find the global extreme values of $f(x) = \frac{1}{3}x^3 - 3x + |x^2 - 4|$ in $[-2, 4]$.

Q6. Either find the global maximum or justify that it does not exist for each of the following
(a) $f(x) = 1 - |1 - x^2|$ in $[0, \sqrt{2}]$, (b) $f(x) = x/(x^2 + 1)$ in $x \geq 0$,
(c) $f(x) = x\cos(\frac{1}{x})/(x + 1)$ in $x \geq 1$.

Partial derivatives

Q7. Given the function $f(x, y) = \log(1 + xy)$ calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial x}$.

Q8. Show that, for any constants A and B, the function $f(x, y) = A\cos x \sinh y + B\sin x \cosh y$ satisfies the equation $f_{xx} + f_{yy} = 0$.