Calculus I, Tutorial Problem Sheet, Week 7

The fundamental theorem of calculus

Q1. Let $F(x) = \int_\pi^x t \sin t \, dt$. Calculate $F(\pi)$, $F'(x)$ and $F'(\pi/2)$.

Q2. Let

$$F(x) = -\int_0^{x^2} \frac{2}{3+e^t} \, dt.$$

Find all critical points of $F(x)$ and determine whether they are local minima, maxima or points of inflection. Prove that $F(300) > F(310)$.

Q3. Calculate the derivatives of the following functions:

$$(a) \; F(x) = \int_{x^2}^1 (t - \sin^2 t) \, dt,$$

$$(b) \; G(t) = \int_{t^2}^{t^4} \sqrt{u} \, du.$$

Integration using a recurrence relation

Q4. For integer $n \geq 0$ define

$$I_n = \int_0^{\pi/4} \cos^{n+1} x \, dx.$$

Find a recurrence relation between I_n and I_{n-2} and hence evaluate I_2 and I_4.

Double integrals

Q5. Calculate $\iint_D x^3 y \, dxdy$, where D is the triangle with vertices $(0,0), (1,0), (1,1)$.

Q6. Calculate $\iint_D \sqrt{xy} \, dxdy$,

where D is the finite region between the curves $y = x$ and $y = x^2$.

Q7. Calculate

$$\int_0^{\pi/2} \left(\int_x^{\pi/2} \frac{\sin y}{y} \, dy \right) dx.$$

Q8. Use polar coordinates to calculate $\iint_D e^{-(x^2+y^2)} \, dxdy$, where D is the unit disc centred at the origin.