Calculus I, Gradescope Assignment, Week 3

Q1. Write a formula for the functions $f \circ g$ and $g \circ f$ and find the domain and range of each, where $f(x) = \sqrt{x+2}$ and $g(x) = 1/x$. 6 marks

Solution:

$(f \circ g)(x) = f(g(x)) = f(1/x) = \sqrt{\frac{1}{x} + 2}$ 1 mark

Dom $(f \circ g) = (-\infty, -\frac{1}{2}] \cup (0, \infty)$ 1 mark

Ran $(f \circ g) = [0, \infty) \backslash \{\sqrt{2}\}$ 1 mark

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+2}) = 1/\sqrt{x+2}$ 1 mark

Dom $(g \circ f) = (-2, \infty)$ 1 mark

Ran $(g \circ f) = (0, \infty)$ 1 mark

Q2. Given $u(x) = 2x - 3,\ v(x) = x^4$ and $f(x) = 1/x,$ find $(u \circ (v \circ f))(x)$ and $(v \circ (u \circ f))(x)$ 2 marks

Solution:

$(u \circ (v \circ f))(x) = u(v(1/x)) = u(1/x^4) = 2/x^4 - 3$ 1 mark

$(v \circ (u \circ f))(x) = v(u(1/x)) = v(2/x - 3) = (2/x - 3)^4$ 1 mark

Q3. Find the inverse of the function $f(x) = x^3 + 1$ and identify the domain and range of this inverse function. 3 marks

Solution:

Write $y = f^{-1}(x)$ and use $f(y) = x$.

$f(y) = y^3 + 1 = x$ hence $y = (x-1)^{\frac{1}{3}} = f^{-1}(x)$. 1 mark

Dom $f^{-1} =$ Ran $f = \mathbb{R}$. 1 mark

Ran $f^{-1} =$ Dom $f = \mathbb{R}$. 1 mark

Q4. Find the inverse of the function $f(x) = 1/x^2,\ \forall\ x > 0$, and identify the domain and range of this inverse function. 3 marks

Solution:

Write $y = f^{-1}(x)$ and use $f(y) = x$.

$f(y) = 1/y^2 = x$ hence $y = 1/\sqrt{x} = f^{-1}(x)$. 1 mark

Dom $f^{-1} =$ Ran $f = (0, \infty)$. 1 mark

Ran $f^{-1} =$ Dom $f = (0, \infty)$. 1 mark

Q5. Show that the function $f(x) = (1+3x)^3$ in $\mathbb{R}$ is injective and find its inverse. Specify the domain of the inverse. 3 marks

Solution:

Apply the horizontal line test (a statement is enough, the graph is not required) or
$f(x_1) = f(x_2)$ iff $(1+3x_1)^3 = (1+3x_2)^3$ iff $(1+3x_1) = (1+3x_2)$ iff $x_1 = x_2$. 1 mark
Write $y = f^{-1}(x)$ and use $f(y) = x$.
So $f(y) = (1+3y)^3 = x$ hence $y = \frac{1}{3}(x^{\frac{1}{3}} - 1) = f^{-1}(x)$. 1 mark
Dom f^{-1} = Ran $f = \mathbb{R}$. 1 mark

Q6. Show that the function $f(x) = (1-x)^2$ in $[-1, 2]$ is not injective. 1 mark

Solution:

Apply the horizontal line test (a statement is enough, the graph is not required)
or eg. $f(2) = 1 = f(0)$. 1 mark

Q7. Complete the table 5 marks

$g(x)$	$f(x)$	$(f \circ g)(x)$
$x-7$	$\sqrt{x}$	
$x+2$	$3x$	
	$\sqrt{x-5}$	$\sqrt{x^2-5}$
$\frac{x}{x-1}$	$\frac{x}{x-1}$	
	$1+\frac{1}{x}$	x

Solution: One mark per correct entry 5 marks

$g(x)$	$f(x)$	$(f \circ g)(x)$
$x-7$	$\sqrt{x}$	$\mathbf{\sqrt{x-7}}$
$x+2$	$3x$	$\mathbf{3x+6}$
$\mathbf{x^2}$	$\sqrt{x-5}$	$\sqrt{x^2-5}$
$\frac{x}{x-1}$	$\frac{x}{x-1}$	$\mathbf{x}$
$\mathbf{\frac{1}{x-1}}$	$1+\frac{1}{x}$	x