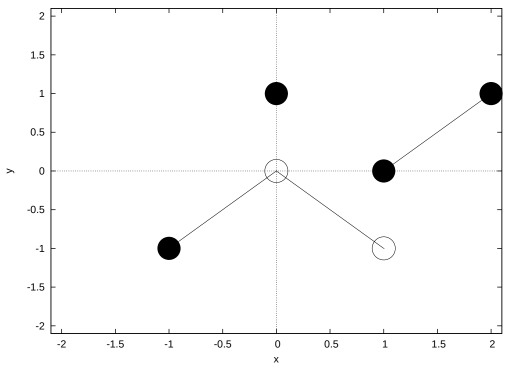
Calculus I, Chapter 2 Problems

Limits

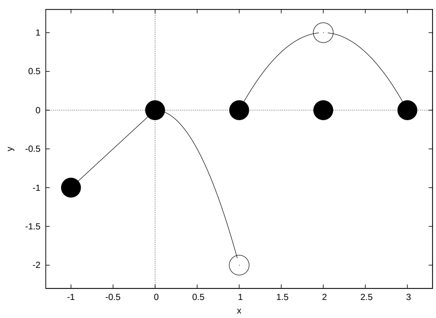
Q1. Consider the given graph of the function $f(x)$. Are the following statements true or false?



$f(x)$ for Q1

(a) $\lim_{x\to 0} f(x)$ exists, (b) $\lim_{x\to 0} f(x) = 0$, (c) $\lim_{x\to 0} f(x) = 1$
(d) $\lim_{x\to 1} f(x) = 1$, (e) $\lim_{x\to 1} f(x) = 0$, (f) $\lim_{x\to a} f(x)$ exists $\forall\ a \in (-1, 1)$.

Q2. Consider the given graph of the function $f(x)$. Are the following statements true or false?



$f(x)$ for Q2

(a) $\lim_{x\to 2} f(x)$ does not exist, (b) $\lim_{x\to 2} f(x) = 1$, (c) $\lim_{x\to 1} f(x)$ does not exist,
(d) $\lim_{x\to a} f(x)$ exists $\forall\ a \in (-1, 1)$ (e) $\lim_{x\to a} f(x)$ exists $\forall\ a \in (1, 3)$.

Q3. If $f(x) > 0\ \forall\ x \neq a$ and $\lim_{x\to a} f(x) = L$, can we conclude that $L > 0$? Justify your answer.

Q4. Justify whether the following statement is true or false.

If $\lim_{x\to a} f(x)$ exists then so does $\lim_{x\to a} \sqrt{f(x)}$.

Q5. Calculate the following limits

(a) $\lim_{x\to 0}(2-x)$, (b) $\lim_{x\to -1} \frac{3x^2}{2x-1}$, (c) $\lim_{x\to \pi/2} x\sin x$, (d) $\lim_{x\to\pi} \frac{\cos x}{1-\pi}$.

Q6. Calculate the following limits

(a) $\lim_{x\to 1} \frac{x^4-1}{x^3-1}$, (b) $\lim_{x\to 2} \frac{x^3-8}{x^4-16}$, (c) $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$, (d) $\lim_{x\to 4} \frac{4x-x^2}{2-\sqrt{x}}$.

Q7. Calculate the limit as $x \to 0$ of the following

(a) $\frac{1-\cos x}{x^2}$, (b) $\frac{x^2}{1-\cos 2x}$, (c) $\frac{x^2}{1-\cos 4x}$.

Q8. Does $\lim_{x\to 0} \frac{\sin(x+|x|)}{x}$ exist?

If the limit exists then find it.

Q9. In each case either evaluate the limit or state that no limit exists

(a) $\lim_{x\to 3} \frac{x^2+x+12}{x-3}$, (b) $\lim_{x\to 3} \frac{x^2+x-12}{x-3}$, (c) $\lim_{x\to 3} \frac{(x^2+x-12)^2}{x-3}$, (d) $\lim_{x\to 3} \frac{(x^2+x-12)}{(x-3)^2}$,

(e) $\lim_{h\to 0} \frac{1-1/h^2}{1+1/h^2}$, (f) $\lim_{h\to 0} \frac{1+1/h}{1+1/h^2}$.

Q10. Calculate the limit as $x \to \infty$ of the following

(a) $\frac{6x+7}{1-2x}$, (b) $\frac{x^2}{x^2+\sin^2 x}$.

Q11. Calculate the following limits

(a) $\lim_{x\to\infty} \frac{\cos(1/x)}{1+(1/x)}$, (b) $\lim_{x\to\infty} \left(\frac{1}{x}\right)^{1/x}$,

(c) $\lim_{x\to\infty}(3+\frac{2}{x})\cos(1/x)$, (d) $\lim_{x\to\infty}\{(\frac{3}{x^2}-\cos(1/x))(1+\sin(1/x))\}$.

Q12. For each of the following statements, either give a proof that it is true or a counter example to show that it is false:

(a) If $g(x) > 0\ \forall\ x > 0$ and $\lim_{x\to\infty}(f(x)-g(x)) = 0$ then $\lim_{x\to\infty}(f(x)/g(x)) = 1$.

(b) If $g(x) > 0\ \forall\ x > 0$ and $\lim_{x\to\infty}(f(x)/g(x)) = 1$ then $\lim_{x\to\infty}(f(x)-g(x)) = 0$.

Q13. In each case either evaluate the limit or state that no limit exists

(a) $\lim_{u\to -5} \frac{u^2}{5-u}$, (b) $\lim_{y\to 0}(2y-8)^{1/3}$, (c) $\lim_{x\to 0} \frac{(x-2)(1-\cos 3x)}{2x}$, (d) $\lim_{t\to 5} \frac{t-5}{t^2-25}$,

(e) $\lim_{x\to -2} \frac{x+2}{\sqrt{x^2+5}-3}$, (f) $\lim_{x\to\infty} \frac{-3x^4+x^2+1}{-5x^4-1}$, (g) $\lim_{t\to 0} \frac{5t^3+8t^2}{3t^2-16t^4}$, (h) $\lim_{x\to 3} \frac{\tan(2(x-3))}{x-3}$,

(i) $\lim_{x\to -3} \frac{x+3}{x^2+4x+3}$, (j) $\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2}$, (k) $\lim_{t\to 1} \frac{t^2+t-2}{t^2-1}$, (l) $\lim_{t\to -\infty} \frac{t^3+1}{t^2+1}$.