

1.7 Summary: Functions

You should have a good and precise mathematical understanding of functions and various definitions, particularly focussing on real-valued functions of a single real variable. Here are some key points:

- A function f is a mapping from its *domain*, $\text{Dom } f$, to a *codomain*. Its *range*, $\text{Ran } f$, is the image of $\text{Dom } f$, i.e. the set of all values that $f(x)$ can actually take for $x \in \text{Dom } f$.
- Functions are single-valued, i.e. $f(x)$ must have a unique value for all $x \in \text{Dom } f$. This can be checked graphically by the *vertical line test*.
- Functions can be *even* – symmetric under reflection in the y -axis.
- Functions can be *odd* – symmetric under rotation by 180° around the origin, or equivalently by two reflections, in the x - and y -axes (in either order).
- Typical functions are neither even nor odd, but can be uniquely written as a sum of an even and an odd function, i.e. $f(x) = f_{\text{even}}(x) + f_{\text{odd}}(x)$. You should know how to define f_{even} and f_{odd} in terms of f .
- We can define *piecewise* functions using different expressions for different parts of the domain. At boundaries of intervals we use filled or empty circles to indicate that a point is or is not included.
- You should be familiar with the operations on functions: *linear combinations*, *product*, *ratio* and *composition*, and know what the domain of the resulting functions is in terms of the domains of the original functions.
- You should know the conditions for the *inverse* of a function to exist. You should know the definitions of *surjective*, *injective* and *bijective*.