Codes and Dual Codes ; Generator- and Check-matrices

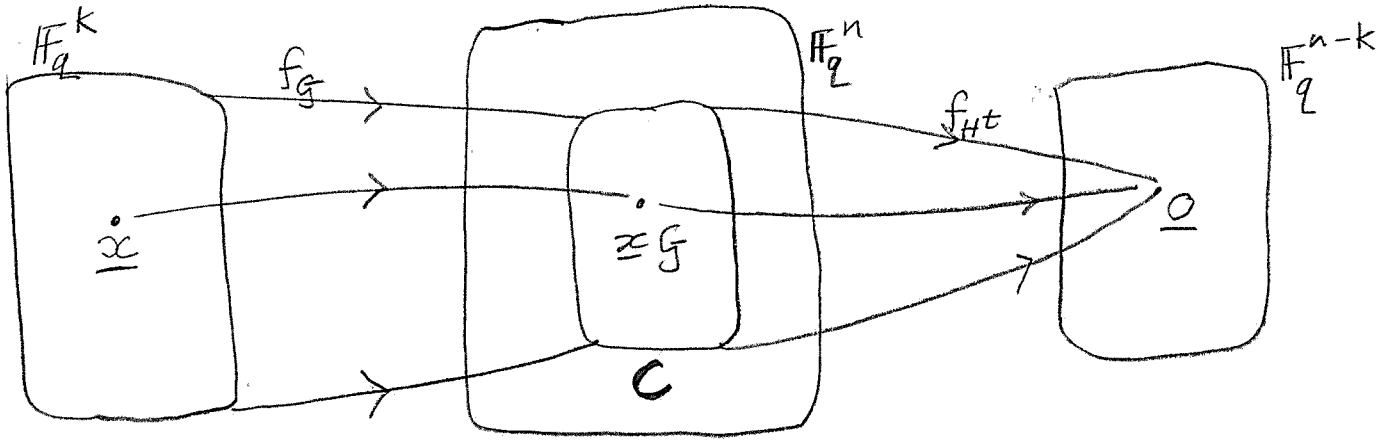
Codes/SKD/2019

	C	$C^\perp$
Generator-matrix	G	H
Check-matrix	H	G

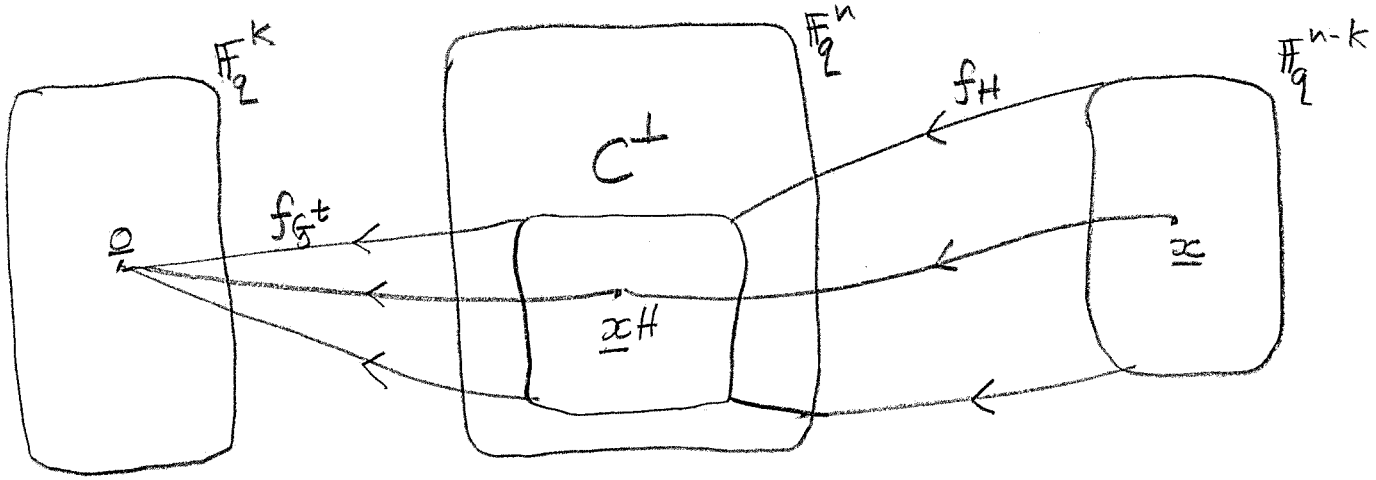
C is an $[n,k]$ code, $\quad C^\perp$ is an $[n,n-k]$ code.

$G \in M_{k,n}(\mathbb{F}_q)$ $\qquad f_G : \mathbb{F}_q^k \to \mathbb{F}_q^n$; $\qquad f_{G^t} : \mathbb{F}_q^n \to \mathbb{F}_q^k$

$H \in M_{n-k,n}(\mathbb{F}_q)$ $\qquad f_H : \mathbb{F}_q^{(n-k)} \to \mathbb{F}_q^n$ $\qquad f_{H^t} : \mathbb{F}_q^n \to \mathbb{F}_q^{(n-k)}$



$$\mathrm{Im}(f_G) = \{\mathbf{x}G \mid \mathbf{x} \in \mathbb{F}_q^k\} \;=\; C \;=\; \{\mathbf{x} \in \mathbb{F}_q^n \mid \mathbf{x}H^t = \mathbf{0}\} = \mathrm{Ker}(f_{H^t})$$



$$\mathrm{Ker}(f_{G^t}) = \{\mathbf{x} \in \mathbb{F}_q^n \mid \mathbf{x}G^t = \mathbf{0}\} = C^\perp = \{\mathbf{x}H \mid \mathbf{x} \in \mathbb{F}_q^{n-k}\} = \mathrm{Im}(f_H)$$