Calculus I, Chapter 1 Problems

Functions, domain and range

Q1. By applying the vertical line test, or otherwise, determine whether each of the following equations gives a function $y(x)$

(a) $x^2 + (y-1)^2 = 1$

(b) $y = x^2 - 2x + 1$

(c) $x + |y| = 1$

(d) $|x| + y = 1$

(e) $y^2 = 4x^2$

Q2. State the domain and range of each of the following functions

(a) $f(x) = |x-1| - 7$

(b) $f(x) = 5 - \sqrt{2x}$

(c) $f(x) = 2\sqrt{x^2 - 3}$

(d) $f(x) = 2x^2/(x^2 + 4)$

(e) $f(x) = -x/(x^2 - 16)$

(f) $f(x) = e^{1/(x^2-4)}$

Even and odd functions

Q3. Are the following functions even, odd or neither? Justify your answers.

(a) $f(x) = (x-1)(x-2)$

(b) $f(x) = \sum_{k=0}^{n} x^{2k+1}$

(c) $f(x) = \sin(x^2)$

(d) $f(x) = \frac{x}{(x^2+1)\cos x}$

Q4. Are the following functions even, odd or neither? Justify your answers.

(a) $f(x) = e^x$

(b) $g(x) = \tan x$

(c) $h(x) = xe^{-\frac{1}{2}|\log x^2|}$

(d) $k(x) = \log|x|$

(e) $p(x) = (x^3 + x)/(x^3 - x)$

(f) $q(x) = \sin^2(4x)$

(g) $r(x) = x^2 - 4\sin x$

Q5. If $f : \mathbb{R} \mapsto \mathbb{R}$ is an even function and $g : \mathbb{R} \mapsto \mathbb{R}$ is an odd function then determine whether the following functions are even, odd or neither? Justify your answers.

(a) $f_1(x) = \begin{cases} f(x) & \text{if } x > 0 \\ -f(x) & \text{if } x < 0 \end{cases}$

(b) $f_2(x) = f(x) + |f(x)|$

(c) $f_3(x) = (g \circ f)(x)$

(d) $f_4(x) = (f \circ g)(x)$

(e) $f_5(x) = (g \circ g)(x)$

Q6. Write each of the following functions as the sum of an even function $f_{even}(x)$ and an odd function $f_{odd}(x)$.

(a) $f(x) = x^3 - 5x + 4$

(b) $f(x) = e^{x^3}$

(c) $f(x) = \log|x + 2|$

(d) $f(x) = x^3/(x^2 - 1)$

Composition, inverse and injective functions

Q7. Write a formula for the functions $f \circ g$ and $g \circ f$ and find the domain and range of each, where $f(x) = x^2$ and $g(x) = 1 - \sqrt{x}$.

Q8. Given $f(x) = x - 1$ and $g(x) = 1/(1 + x)$, find

(a) $(f \circ g)(\frac{1}{2})$

(b) $(f \circ f)(2)$

(c) $(g \circ f)(x)$

(d) $(g \circ g)(2)$

Q9. Given $u(x) = 2x - 3$, $v(x) = x^4$ and $f(x) = 1/x$, find $(f \circ (v \circ u))(x)$ and $(v \circ (f \circ u))(x)$

Q10. For each $f(x)$ given below, find the inverse function $f^{-1}(x)$ and identify its domain and range.

(a) $f(x) = x^5$

(b) $f(x) = x^4$, $x \geq 0$

(c) $f(x) = \frac{1}{2}x - \frac{7}{2}$

(d) $f(x) = 1/x^3$, $x \neq 0$

Q11. Which of the following functions are injective? Find the inverses of those which are and specify the domain of the inverse.

(a) $f(x) = (1 - x)^2$ in $[1, 2]$

(b) $f(x) = (x-1)/(x+2)$ in $\mathbb{R}\backslash\{-2\}$

(c) $f(x) = x^2 + 2x - 1$ in $[-1, 1]$

(d) $f(x) = x^2 + 2x - 1$ in $[-2, 2]$

Q12. Complete the table.

$g(x)$	$f(x)$	$(f \circ g)(x)$
$1/x$		x
$\frac{1}{x-1}$	$|x|$	
	$\frac{x-1}{x}$	$\frac{x}{x+1}$
	$\sqrt{x}$	$|x|$
$\sqrt{x}$		$|x|$

Q13. For $x \neq 0, 1$, define the following six functions

$$f_1(x) = x, \; f_2(x) = \frac{1}{x}, \; f_3(x) = 1-x, \; f_4(x) = \frac{1}{1-x}, \; f_5(x) = \frac{x-1}{x}, \; f_6(x) = \frac{x}{x-1}.$$

These have the property that the composition of any two of these functions is again one of these functions.

Complete the following table

$\circ$	f_1	f_2	f_3	f_4	f_5	f_6
f_1						
f_2			f_4			
f_3						
f_4						
f_5						
f_6						