Calculus I, Gradescope Assignment, Week 3

Q1. Calculate
$$\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}.$$
2 marks

Solution.

$\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}=\lim_{x\to 1}\frac{(x+2)(x-1)}{x(x-1)}$ 1 mark

$=\lim_{x\to 1}\frac{x+2}{x}=3.$ 1 mark

Q2. Calculate
$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}.$$
3 marks

Solution.

$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)}$ 1 mark

$=\lim_{x\to 9}\frac{x-9}{(x-9)(\sqrt{x}+3)}$ 1 mark

$=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=1/6.$ 1 mark

Q3. Calculate
$$\lim_{x\to 0}\frac{1-\cos 2x}{x}.$$
2 marks

Solution.

$\lim_{x\to 0}\frac{1-\cos 2x}{x}=\lim_{x\to 0}\frac{2(1-\cos 2x)}{2x}$ 1 mark

$=\lim_{u\to 0}\frac{2(1-\cos u)}{u}=0.$ 1 mark

Q4. Calculate $\lim_{x\to\pi/2}\{(x-\pi/2)\tan x\}$. 3 marks

Solution.

Set $u=x-\pi/2$ *then* $\lim_{x\to\pi/2}(x-\pi/2)\tan x=\lim_{u\to 0}u\tan(u+\pi/2)$ 1 mark

$=\lim_{u\to 0}\frac{u\sin(u+\pi/2)}{\cos(u+\pi/2)}=\lim_{u\to 0}\frac{-u\cos u}{\sin u}$ 1 mark

$=-1$, *where in the last step we used one of our standard trigonometric identities.* 1 mark

Q5. Calculate
$$\lim_{x\to\infty}\frac{(x^2+1)^2-(x^2-1)^2}{(x+2)^3-(x+1)^3}.$$
3 marks

Solution.

$\lim_{x\to\infty} \frac{(x^2+1)^2-(x^2-1)^2}{(x+2)^3-(x+1)^3} = \lim_{x\to\infty} \frac{4x^2}{3x^2+9x+7}$ 1 mark

$= \lim_{x\to\infty} \frac{4}{3+\frac{9}{x}+\frac{7}{x^2}}$ 1 mark

$= 4/3.$ 1 mark

Q6. Calculate $\lim_{x\to\infty} x\sin\frac{1}{x}$. 2 marks

Solution.

Setting $u = 1/x$ *then* $\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{u\to 0^+} \frac{\sin u}{u}$ 1 mark

$= 1$, *where in the last step we used one of our standard trigonometric identities.* 1 mark

Q7. Use the squeezing theorem to calculate $\lim_{x\to 0} \tan^2\left(\frac{x}{2}\right)\cos^2\left(\frac{2}{x}\right)$. 5 marks

Solution. *As* $0 \leq \cos^2\left(\frac{2}{x}\right) \leq 1$, *then* $0 \leq \tan^2\left(\frac{x}{2}\right)\cos^2\left(\frac{2}{x}\right) \leq \tan^2\left(\frac{x}{2}\right) \quad \forall x \neq 0 \in (-\pi,\pi)$. 2 marks

Now since $\lim_{x\to 0} 0 = 0$ *and* $\lim_{x\to 0} \tan(\frac{x}{2}) = 0$, 2 marks

then by the squeezing theorem, $\lim_{x\to 0} \tan^2\left(\frac{x}{2}\right)\cos^2\left(\frac{2}{x}\right) = 0$. 1 mark