2.8 Summary: Limits and continuity

You should have a good understanding of the concept of a limit as well as knowing the precise definition of a limit, and how continuity is defined in terms of limits. You should be familiar with methods to calculate limits. Here are some key points:

- You should know the ε-δ definition of a limit $\lim_{x\to a} f(x)$ and understand the concept of taking x closer and closer to a but not reaching a (so the limit does not depend in any way on $f(a)$).
- A function $f(x)$ is *continuous* at $x = a$ if $\lim_{x\to a} f(x) = f(a)$.
- Any algebraic combination of continuous functions gives a continuous function assuming the combination does to involve division by zero since limits of algebraic combinations of functions can be evaluated as the algebraic combinations of the limits (assuming they exist and don't involve dividing by zero).
- Indeterminate limits of the form $0/0$ or ∞/∞ may be well-defined, and if so can be calculated by cancelling common factors in the numerator and denominator.
- You should know how to use the *Pinching (Squeezing) theorem*.
- $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$.
- You should know the definitions of *left-sided* and *right-sided limits* and how these are used to classify *discontinuities* at $x = a$.
- You should understand the concept of limits as $x \to \pm\infty$ and that they are equivalent to one-sided limits $u \to 0^{\pm}$ by changing variable to $u = 1/x$.
- You should know what the *Intermediate Value Theorem (IVT)* is and how it can be used to prove the existence of, and numerically approximate, zeros of continuous functions.