

Calculus I, Gradescope Assignment, Week 5

Q1. Calculate

$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}.$$

2 marks

Solution:

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x} &= \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{x(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{x+2}{x} = 3.\end{aligned}$$

1 mark

1 mark

Q2. Calculate

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}.$$

3 marks

Solution:

$$\begin{aligned}\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9} &= \lim_{x \rightarrow 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{(x - 9)(\sqrt{x} + 3)} \\ &= \lim_{x \rightarrow 9} \frac{x - 9}{(x - 9)(\sqrt{x} + 3)} \\ &= \lim_{x \rightarrow 9} \frac{1}{\sqrt{x} + 3} = 1/6.\end{aligned}$$

1 mark

1 mark

1 mark

Q3. Calculate

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x}.$$

2 marks

Solution:

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x} &= \lim_{x \rightarrow 0} \frac{2(1 - \cos 2x)}{2x} \\ &= \lim_{u \rightarrow 0} \frac{2(1 - \cos u)}{u} = 0.\end{aligned}$$

1 mark

1 mark

Q4. Calculate $\lim_{x \rightarrow \pi/2} \{(x - \pi/2) \tan x\}$.

3 marks

Solution:

$$\begin{aligned}\text{Set } u &= x - \pi/2 \text{ then } \lim_{x \rightarrow \pi/2} (x - \pi/2) \tan x = \lim_{u \rightarrow 0} u \tan(u + \pi/2) \\ &= \lim_{u \rightarrow 0} \frac{u \sin(u + \pi/2)}{\cos(u + \pi/2)} = \lim_{u \rightarrow 0} \frac{-u \cos u}{\sin u} \\ &= -1.\end{aligned}$$

1 mark

1 mark

1 mark

Q5. Calculate

$$\lim_{x \rightarrow \infty} \frac{(x^2 + 1)^2 - (x^2 - 1)^2}{(x + 2)^3 - (x + 1)^3}.$$

3 marks

Solution:

$$\lim_{x \rightarrow \infty} \frac{(x^2+1)^2 - (x^2-1)^2}{(x+2)^3 - (x+1)^3} = \lim_{x \rightarrow \infty} \frac{4x^2}{3x^2+9x+7}$$

1 mark

$$= \lim_{x \rightarrow \infty} \frac{4}{3 + \frac{9}{x} + \frac{7}{x^2}}$$

1 mark

$$= 4/3.$$

1 mark

Q6. Calculate $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$.

2 marks

Solution:

$$\text{Setting } u = 1/x \text{ then } \lim_{x \rightarrow \infty} x \sin \frac{1}{x} = \lim_{u \rightarrow 0^+} \frac{\sin u}{u}$$

1 mark

$$= 1.$$

1 mark

Q7. Use the squeezing theorem to calculate $\lim_{x \rightarrow 0} \tan^2 \left(\frac{x}{2} \right) \cos^2 \left(\frac{x}{2} \right)$.

5 marks

Solution: As $0 \leq \cos^2 \left(\frac{x}{2} \right) \leq 1$, then $0 \leq \tan^2 \left(\frac{x}{2} \right) \cos^2 \left(\frac{x}{2} \right) \leq \tan^2 \left(\frac{x}{2} \right) \quad \forall x \neq 0 \in (-\pi, \pi)$.

2 marks

Now since $\lim_{x \rightarrow 0} 0 = 0$ and $\lim_{x \rightarrow 0} \tan \left(\frac{x}{2} \right) = 0$,

2 marks

then by the squeezing theorem, $\lim_{x \rightarrow 0} \tan^2 \left(\frac{x}{2} \right) \cos^2 \left(\frac{x}{2} \right) = 0$.

1 mark