# BAN402

Project 1

15.09.22 - 23.09.22

NHH

Candidate numbers:

47, 146

Candidate numbers: 47, 146

# Part A

## Task 1

**Indexes and sets**
$f \in F$: Set of production facilities
$p \in P$: Set of pollutants

**Parameters**
$C_f$: Cost of processing one ton of fish at facility $f$
$R_{f,p}$: Tons of pollutant $p$ reduced per ton fish at facility $f$
$M_p$: Pollution reduction target of pollutant $p$

**Decision variables**
$x_f$: Tons of fish processed at location $f$

**Objective function**

$$Min: \sum_{f \in F} C_f \, x_f$$

**Constraints**

$$\sum_{f \in F} R_{f,p} x_f \geq M_p \quad \forall p \in P$$
$$x_f \geq 0 \quad \forall f \in F$$

**Solution**
When plotting this mathematical model into AMPL, we find that:
The *objective value* is equal to 2,730.77

And the optimal solution is achieved with the following processing scheme:

| Facility | Fish processed |
|---|---|
| $x_1$ | 11.54 |
| $x_2$ | 119.23 |
| $x_3$ | 0 |

The value of the pollutant reduction constraints in this configuration are

| Pollutant | Pollutant reduction |
|---|---|
| $\sum_{f \in F} R_{f,1}$ | 25 |
| $\sum_{f \in F} R_{f,2}$ | 35 |

# Task 2

We can analyse how sensitive the solution is to changes in the restrictions imposed by the government by calculating the shadow price for the constraints. This shows us how the optimal solution will change with a loosening or tightening of the constraint. The constraints are binding as the slack for both pollutants is equal to 0.

| | Shadow price | Lower bound | Current RHS | Upper bound | Slack |
|---|---|---|---|---|---|
| Pollutant 1 | 23.077 | 7.778 | 25 | 28 | 0 |
| Pollutant 2 | 61.539 | 31.25 | 35 | 112.5 | 0 |

# Task 3

We can use the shadow price in conjunction with the constraints lower bound, upper bound and current RHS in order to find new optimal values of the model if the constraints were to change.

### *Scenario 1: Pollutant 2 decreases to 25 tons*

By using the information in the previous table, we can determine that we cannot conclude on the effects of such a decrease without running the model again, as the Lower Bound on this particular solution is 31.25, which is a higher number than 25.

### *Scenario 2: Pollutant increases to 70 tons*

With a proposed increase of the minimum processing to 70, we can use the shadow price to conclude on the effect of such a change, as the increase is within the Upper Bound of the current model.

The change in the objective value can be determined by finding the product of the difference in RHS and the shadow price of Pollutant 2:

$$(70 - 35) \cdot 61.539 = 2{,}153.85$$

The new objective value is therefore:

$$2{,}730.77 + 2{,}153.85 = \ 4{,}884.62$$

# Task 4

In this scenario we feed the model in AMPL an altered data set in order to compare this scenario with the one specified in Task 1. With the price to process at Facility 1 ($C_1$) reduced to $20, we need to change the constraint in the AMPL data file.

The new *objective value* is $2{,}615.38$
And the new optimal solution is achieved with the following processing scheme:

| Facility | Fish processed |
|---|---|
| F1 | 11.54 |
| F2 | 119.23 |
| F3 | 0 |

The value of the pollutant reduction constraints in this configuration is still:

| Pollutant | Pollutant reduction |
| --- | --- |
| $\sum_{f \in F} R_{f,1}$ | 25 |
| $\sum_{f \in F} R_{f,2}$ | 35 |

This shows that our optimal decision remains the same even though the price changes. Even though the optimal decision stays the same, the objective value is reduced. This tells us that the optimal production distribution does not change with the reduction in price of the processing at Facility 1.

# Part B

## Task 1

**Indexes and sets**
$i \in I$: set of crude oils
$j \in J$: set of gasolines
$m \in M$: set of markets to sell the gasoline
$k \in K$: set of quality attributes

**Parameters**
$s_{j,m}$: Sales price of gasoline $j$ in market $m$
$p_i$: Purchase price of crude oil $i$
$c_j$: Cost of refining a barrel of gasoline $j$
$b_i$: Maximum barrels of crude oil $i$ available for purchase
$h_j$: Hours used to supervise a barrel of gasoline $j$
$t$: Maximum hours available
$d_{j,m}$: Minimum demand quantities of gasoline $j$ in market $m$
$a_{i,k}$: Units of attribute $k$ contributed per barrel of crude oil $i$
$o_{j,k}$: Limit of attribute $k$ per barrel of gasoline $j$

**Decision variables**
$X_{i,j}$ = barrels of crude $i$ used weekly to produce gas $j$
$Y_{j,m}$ = barrels of gas $j$ produced and sold in market $m$ weekly

**Objective function**

$$Max: \sum_{m \in M} \sum_{j \in J} Y_{j,m} s_{j,m} - \sum_{i \in I} \sum_{j \in J} p_i X_{i,j} - \sum_{j \in J} c_j Y_{j,m}$$

Candidate numbers: 47, 146

**Constraints**

Purchase limits of crude oil

$$\sum_{j \in J} X_{i,j} \leq b_i \quad \forall i \in I$$

Hours used must not exceed hours available

$$\sum_{m \in M} \sum_{j \in J} Y_{j,m} h_j \leq t$$

Minimum demand must be satisfied

$$Y_{j,m} \geq d_{j,m} \quad \forall j \in J, m \in M$$

Gasoline blends must satisfy minimum attribute requirements

$$\sum_{i \in I} X_{i,j} a_{i,Octane} \geq \sum_{m \in M} Y_{j,m} O_{j,Octane} \quad \forall j \in J$$

Gasoline blends must satisfy maximum sulphur rating

$$\sum_{i \in I} X_{i,j} a_{i,Sulphur} \leq \sum_{m \in M} y_{j,m} u_{j,Sulphur} \quad \forall j \in J$$

Non-negativity

$$X_{i,j}, Y_{j,m} \geq 0$$

Continuity constraints

$$\sum_{m \in M} Y_{j,m} = \sum_{i \in I} X_{i,j} \quad \forall j \in J$$

**Solution**
After implementing this model into AMPL, we get these results:
The *objective value* of the model, which measures the profit is $1,371,570
As the model consists of two decision variables, we can present the blending plan in two parts.

Part I: Optimal plan for $X_{i,j}$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_5$ |
|---|---|---|---|---|---|
| $C_1$ | 125 | 4,625 | 8,250 | 5,714.29 | 1,542.86 |
| $C_2$ | 10,375 | 4,625 | 0 | 0 | 0 |
| $C_3$ | 0 | 0 | 0 | 0 | 1,542.86 |
| $C_4$ | 0 | 0 | 0 | 2,285.71 | 914.286 |

Candidate numbers: 47, 146

Part II: Optimal plan for $Y_{j,m}$

| | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|
| $G_1$ | 3,000 | 2,500 | 5,000 |
| $G_2$ | 3,000 | 2,000 | 4,250 |
| $G_3$ | 1,500 | 1,000 | 5,750 |
| $G_4$ | 2,000 | 2,000 | 4,000 |
| $G_5$ | 1,000 | 1,000 | 2,000 |

# Task 2

*Scenario 1: $s_{1,1}$ increases to \$78, and $d_{1,1}$ decreases to 2700*

Profit: $1,384,470

Part I: Optimal plan for $X_{i,j}$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_5$ |
|---|---|---|---|---|---|
| $C_1$ | 0 | 4,625 | 8,375 | 5,714.29 | 1,542.86 |
| $C_2$ | 10,200 | 4,625 | 175 | 0 | 0 |
| $C_3$ | 0 | 0 | 0 | 0 | 1,542.86 |
| $C_4$ | 0 | 0 | 0 | 2,285.71 | 914.286 |

Part II: Optimal plan for $Y_{j,m}$

| | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|
| $G_1$ | 2,700 | 2,500 | 5,000 |
| $G_2$ | 3,000 | 2,000 | 4,250 |
| $G_3$ | 1,500 | 1,000 | 6,050 |
| $G_4$ | 2,000 | 2,000 | 4,000 |
| $G_5$ | 1,000 | 1,000 | 2,000 |

Even though the price of $G_1$ increases it is still more profitable to produce and sell more of $G_3$ and sell in $M_3$ as it has a higher price

*Scenario 2: $d_{3,3}$ increases to 4000, $d_{1,1}$ increases to 3100, $d_{1,2}$ decreases to 2400*

Profit: $1,371,570

Part I: Optimal plan for $X_{i,j}$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_5$ |
|---|---|---|---|---|---|
| $C_1$ | 125 | 4625 | 8250 | 5714.29 | 1542.86 |
| $C_2$ | 10375 | 4625 | 0 | 0 | 0 |
| $C_3$ | 0 | 0 | 0 | 0 | 1542.86 |
| $C_4$ | 0 | 0 | 0 | 2285.71 | 914.286 |

Part II: Optimal plan for $Y_{j,m}$

| | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|
| $G_1$ | 3100 | 2400 | 5000 |
| $G_2$ | 3000 | 2000 | 4250 |
| $G_3$ | 1500 | 1000 | 5750 |
| $G_4$ | 2000 | 2000 | 4000 |
| $G_5$ | 1000 | 1000 | 2000 |

The increase of $d_{3,3}$ does not matter as this constraint is not binding. $d_{1,1}$ and $d_{1,2}$ are however binding, and the increase of $d_{1,1}$ decreases the profit and the decrease of $d_{1,2}$ increases the profit, and the net effect is no change in profit.

### *Scenario 3: $b_3$ decreases to 1350, $b_4$ decreases to 2050*

Infeasible problem, no solution found. Even though the total number of barrels would still cover the demand, the composition of the barrels available would make it impossible to make enough high-quality gasoline to meet the demand.

### *Scenario 4: Alternative source of C2 (C5), with $b_2 = 1900$ and $p_2 = 41$*

Profit: $1,378,670

Part I: Optimal plan for $X_{i,j}$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_5$ |
|---|---|---|---|---|---|
| $C_1$ | 0 | 4,625 | 6,600 | 5,714.29 | 1,542.86 |
| $C_2$ | 8,725 | 4,625 | 1,650 | 0 | 0 |
| $C_3$ | 0 | 0 | 0 | 0 | 1,542.86 |
| $C_4$ | 0 | 0 | 0 | 2,285.71 | 914.286 |
| $C_5$ | 1,775 | 0 | 0 | 0 | 0 |

Part II: Optimal plan for $Y_{j,m}$

| | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|
| $G_1$ | 3000 | 2500 | 5000 |
| $G_2$ | 3000 | 2000 | 4250 |
| $G_3$ | 1500 | 1000 | 5750 |
| $G_4$ | 2000 | 2000 | 4000 |
| $G_5$ | 1000 | 1000 | 2000 |

We create a new variable $C_5$ to symbolise the new source of $C_2$ with a higher price. An extra 1,775 barrels of the new source of $C_2$ oil would be purchased to produce more $G_1$ and $G_3$.

# Part C

## Task 1

**Indexes and sets**
$r \in R$: Regions
$k \in K$: Markets
$p \in P$: Ports, where $p_1^*$ and $p_2^*$ is sending the product directly to the market, utilising no ports

**Parameters**
$n_r$: Available supply of apples in region $r$
$d_k$: Demand for metric tons of apples in each market
$c_{r,p}$: Cost for transportation between regions and ports
$t_{p,k}$: Cost of transportation between ports and markets

**Decision variable**
$x_{r,p}$: Metric tons of apples purchased from region $r$ through port $p$
$y_{p,k}$: Metric tons of apples shipped from port $p$ to market $k$

**Objective function**

$$Min: \sum_{p\in P}\sum_{r\in R} x_{r,p}c_{r,p} + \sum_{k\in K}\sum_{p\in P} y_{p,k}t_{p,k}$$

**Constraints**
Maximum supply available

$$\sum_{p\in P} x_{r,p} \leq n_r \quad \forall r \in R$$

Demand constraint

$$\sum_{p\in P} y_{p,k} = d_k \quad \forall k \in K$$

Continuity constraint

$$\sum_{r\in R} x_{r,p} = \sum_{k\in K} y_{p,k} \quad \forall p \in P$$

Non-negativity

$$x_{r,p}, y_{p,k} \geq 0 \quad \forall r \in R, p \in P, k \in K$$

**Optimal solution**

*Objective value:* A total transport cost of $10,680

Optimal plan for $x_{r,p}$:

Candidate numbers: 47, 146

| | $p_1^*$ | $p_2^*$ | $p_1$ | $p_2$ |
|---|---|---|---|---|
| $r_1$ | 0 | 24 | 126 | 0 |
| $r_2$ | 40 | 11 | 0 | 109 |

Optimal plan for $y_{p,k}$:

| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | $k_7$ | $k_8$ | $k_9$ | $k_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $p_1^*$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 40 |
| $p_1^*$ | 0 | 0 | 0 | 35 | 0 | 0 | 0 | 0 | 0 | 0 |
| $p_1$ | 24 | 30 | 0 | 0 | 0 | 52 | 0 | 0 | 20 | 0 |
| $p_2$ | 0 | 0 | 40 | 0 | 15 | 0 | 42 | 12 | 0 | 0 |

This solution provides us with the shipping plan that provides us with the values of the decision variables that minimized transport costs between the two regions and the markets.

# Task 2

*Objective value:* A total cost of $18,412

| | $p_1^*$ | $p_2^*$ | $p_2$ |
|---|---|---|---|
| $r_1$ | 84 | 35 | 0 |
| $r_2$ | 0 | 0 | 191 |

| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | $k_7$ | $k_8$ | $k_9$ | $k_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $p_1^*$ | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 40 |
| $p_1^*$ | 0 | 0 | 0 | 35 | 0 | 0 | 0 | 0 | 0 | 0 |
| $p_2$ | 0 | 30 | 40 | 0 | 15 | 52 | 42 | 12 | 0 | 0 |

With the closing of Port 1, the model still fulfils the demand of the markets, but in this case *FreshFruits* only have one port at their disposal. As the ports significantly reduced shipping costs in the previous scenario, one expects an increase in shipping costs, which is the case when comparing the objective values of the two solutions.

# Task 3

The change in weekly supply causes the problem to become infeasible because the total supply of apples (300) becomes less than the total demand of apples (310).

Allowing for some unsatisfied demand makes the problem solvable and returns the following solution. We can change the demand constraint to consider the possibility of some unsatisfied demand.

First of all, we need to add a parameter:
$u_k$: Maximum unsatisfied demand of market $k$

Then we can modify our demand constraint in order to consider the possibility of having some unsatisfied demand:

Candidate numbers: 47, 146

$$\sum_{p \in P} y_{p,k} = d_k \cdot (1 - u_k) \quad \forall k \in K$$

Now we can solve the model with these modifications in place:

*Objective value:* A total cost of $9,612

| | $p_1^*$ | $p_2^*$ | $p_1$ | $p_2$ |
|---|---|---|---|---|
| $r_1$ | 0 | 11.6 | 113.4 | 0 |
| $r_2$ | 36 | 19.9 | 0 | 98.1 |

| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | $k_7$ | $k_8$ | $k_9$ | $k_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $p_1^*$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 36 |
| $p_1^*$ | 0 | 0 | 0 | 31.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| $p_1$ | 21.6 | 27 | 0 | 0 | 0 | 46.8 | 0 | 0 | 18 | 0 |
| $p_2$ | 0 | 0 | 36 | 0 | 13.5 | 0 | 37.8 | 10.8 | 0 | 0 |

This scenario produced an objective value that is lower than the original model. Within the framework of this model this seems probable, as a cost minimizing function will improve when demand goes down and they have to transport less. Therefore, the proposal of the manager might seem like a good solution to the problem at hand.

On the other hand, if we look at the problem from a business perspective instead of a cost minimizing perspective it is possible to point out some flaws in the model. First of all, lower costs do not inherently produce better results for the business. The cost minimizing solution to all business cases is (without constraints) to shut down all operations. This is not a flaw of the model per say, but we can gain insight by discussing how we might alter the model in order for it to better guide our decisions.

In this case we would advise the management to switch the model from a cost minimizing model to a profit maximizing model. This would allow the transportation model to better allocate the resources away from the less profitable markets to the more profitable markets. This alteration is better adapted to comparing scenarios where supply is reduced.

# Part D

## Task 1

### a) Could this model be infeasible?

A model is designated as infeasible if there exists no solution that satisfies all the constraints of the model.

The model will be infeasible if supply is lower than the demand:

$$\sum_{i \in I} \sum_{k \in K_l} s_{i,k,t} < d_{j,l,t} \quad \forall j \in J, l \in L, t \in T$$

The flow for all combinations of supply points, assortment and time periods must be less or equal to the supply available. The flow to each demand point for each assortment group, each period must be equal to the demand. If the supply available of the assortments that make up an assortment group is not enough to make up for the demand of that group in a demand point, in a period, then there will be no feasible solution.

### b) Could this model be unbounded?

A model is designated as unbounded when the objective function value can be improved without any limit. As the objective function is a minimizing problem, and there is a non-negativity stipulation in the model (the decision variables cannot have a lower value than zero) the only way for this model to be unbounded is if the $c_{ijkt}$ has a negative value. If the assumption of the task holds, then $c_{ijkt}$ is always positive, thus the model cannot be unbounded within these constraints and assumptions.

## Task 2

*New assumptions:* *Objective value* $= 234{,}560$

$c_{ijkt} = 1 \quad \forall\, i \in I, j \in J, k \in K, t \in T$

### a) Total demand quantity

Given these new assumptions, we can explore whether it is possible for us to determine the aggregated demand quantity over the course of the set time frame. It is important to note in this task, and the tasks that follow that we do not possess the full data set, only the aggregate flow. With an aggregated figure, we lose information inherent in the original calculations, therefore our arguments must be rooted in the model itself.

With this in mind, we can first establish that the model itself has been solved, therefore the constraints must hold. For example, $\sum_{i \in I} \sum_{k \in K_l} x_{i,j,k,t} = d_{j,l,t}$ as a constraint must not be confused with a mathematical formula (the distinction lies within the causality of the equation). In this case however, we know that the model is solved, therefore we can assume that the equation holds.

Flow is equal to demand for all $j$, $l$ and $t$

(1)

$$\sum_{i\in I}\sum_{k\in K_l} x_{i,j,k,t} = d_{j,l,t} \quad \forall j \in J, l \in L, t \in T$$

Therefore, aggregate flow must be equal to aggregate demand

(2)

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K_l}\sum_{t\in T} x_{i,j,k,t} = \sum_{j\in J}\sum_{l\in L}\sum_{t\in T} d_{j,l,t}$$

According to our new assumptions, $c_{i,j,k,t}$ for all $i$, $j$, $k$ and $t$ is equal to 1. The objective function must therefore be equal to:

(3)

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K_l}\sum_{t\in T} c_{i,j,k,t}x_{i,j,k,t} = 234{,}560$$

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K_l}\sum_{t\in T} 1\cdot x_{i,j,k,t} = 234{,}560$$

Therefore, the aggregate flow must be equal to the *objective value* which is assumed to be 234,560:

(4)

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K_l}\sum_{t\in T} x_{i,j,k,t} = 234{,}560$$

With the value for flow determined, we can use the first equation (1) in order to determine the total demand quantity.

(5)

$$\sum_{j\in J}\sum_{l\in L}\sum_{t\in T} d_{j,l,t} = 234{,}560$$

The total demand quantity must be equal to the optimal objective value when the cost is 1.

## b) Total demand quantity in T6

In accordance with the previous task, it is difficult to disaggregate a figure without the data set used to calculate it. Therefore, we must once again determine whether or not we can conclude what the total demand quantity is for period 6 ($t_6$).

Even though we possess the value for $y_{ijk0}$ (100), we still don't have the $s_{ikt}$ and $x_{ijkt}$ for all i, j, k for each time-period (only the aggregated flow). Therefore, we cannot conclude what the specific demand of T6 is without making further assumptions in the model.

Based on these findings, it would be reasonable to assume that we cannot find the exact value of $\sum_{j\in J}\sum_{l\in L} d_{j,l,6}$ with the information we have at hand. One the other hand, we can try to approximate the total demand quantity for this time period using the information we currently possess. This would not be the actual value in period 6 since we do not know how the demand is distributed across the periods. The average demand across the 12 periods is:

(6)

$$\frac{\sum_{j\in J}\sum_{l\in L}\sum_{t\in T} d_{j,l,t}}{12} = \frac{234{,}560}{12} = 19{,}546.6\bar{6}$$

In summary, we cannot conclude what is the total demand quantity for period 6 with the information we currently possess, although we can find a rough approximation of what the mean total demand quantity each month would have to be in order to produce the observed *objective value*.

## c) Total supply availability

In this task we will elaborate on the question of whether or not we can conclude what the total supply availability is within the same framework as task is a) and b).

Based on the constraints in the model that is presented, we can only create an area of possibility for $s_{ikt}$ for all i, j, k. We cannot determine an exact value of aggregated supply availability.

With this in mind, we can conclude that it is not possible to determine the exact total supply availability using only the information we have been given.

We can however create an estimate of the total supply availability by using the constraints in the model, as they must hold in order for the model to be solved. We can take equation (4) from task a) and apply it to constraint (2) in the model to approximate $\sum_{i\in I}\sum_{k\in K}\sum_{t\in T} s_{i,k,t}$.

(7)

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K_l}\sum_{t\in T} x_{i,j,k,t} = 234{,}560$$

$$\sum_{j\in J} x_{i,j,k,t} \geq s_{i,k,t} \quad \forall i \in I, k \in K, t \in T$$

For the constraint to be upheld we can therefore assume that the total supply availability is higher or equal to the aggregate flow.

(8)

$$\sum_{i\in I}\sum_{k\in K}\sum_{t\in T} s_{i,k,t} \geq 234{,}560$$

This gives a possible area where the total supply availability must lie, which is higher or equal to 234,560.

## d) Total supply availability in T6

No, we cannot. Supply availability is given outside of our system of equations, and we cannot de-aggregate the supply for all the periods since we do not know how it is distributed.

Task 3

For all supply points, the total flow $\sum_{j \in J} x_{i,j,k,t}$ cannot exceed a difference of 10% from the previous period's total flow $\sum_{j \in J} x_{i,j,k,t-1}$.

$$\left| \sum_{j \in J} \frac{x_{i,j,k,t} - x_{i,j,k,t-1}}{x_{i,j,k,t}} \right| \leq 0.10 \qquad \forall i \in I, k \in K, t \in T \qquad \textit{NOT LINEAR}$$

The constraint above fulfils the requirements. The absolute value is used to get the absolute difference and reducing the need to only one equation. This would normally break linearity, but since the decision variables has a non-negativity constraint, linearity is maintained. This equation is still not linear though since it divides a decision variable by a decision variable.

$$\left| \sum_{j \in J} (x_{i,j,k,t} - x_{i,j,k,t-1}) \right| \leq \sum_{j \in J} 0.10 x_{i,j,k,t} \qquad \forall i \in I, k \in K, t \in T$$

Thus, we multiply both sides by $x_{i,j,k,t}$ and linearity is maintained.

Since this is a restriction with a less than or equal sign, it will necessarily make the objective function worse or the same, never better, as all the values $x_{i,j,k,t}$ can take now, it could also take before. Before, all the flow could be placed in the months where cost is low, if allowed by the demand. And now it must be somewhat evenly distributed across the periods.