BAN402

Project 3

01.11.22 - 09.11.22

NHH

Candidate numbers:

47, 146

Part A

Task 1

Sets
$i \in I$: Customer segment $i \in \{g, s\}$

Decision variables
Q_i: Demand quantity of segment i
p_i: Price of ticket charged to segment i in hundred NOK

Objective function
An objective function that maximizes the total revenue from ticket sales from all segments.

$$Max: \sum_{i \in I} p_i Q_i$$

Constraints
Demand of the general segment *g* is limited by the price set for segment *g*.

$$Q_g = 120{,}000 - 3{,}000 p_g \tag{1}$$

Demand of the student segment *s* is limited by the price set for segment *s*.

$$Q_s = 20{,}000 - 1{,}250 p_s \tag{2}$$

The venue's capacity sets an upper limit on the total ticket sales.

$$\sum_{i \in I} Q_i \leq 55{,}000 \tag{3}$$

Each segment *i* must be allocated at least 20% of the total capacity.

$$Q_i \geq 55{,}000 \cdot 20\% \tag{4}$$

The price of the general customer segment *g* and student customer segment *s* must be equal.

$$p_g = p_s \tag{5}$$

Non-negativity constraints.

$$Q_i \geq 0 \quad \forall i \in I \tag{6}$$
$$p_i \geq 0 \quad \forall i \in I \tag{7}$$

Results from Task 1:

	General	*Student*
Q_i	44 000	11 000
p_i	7.20	7.20
Revenue	316 800	79 200
Total		**396 000**

The optimal total revenue of the concert given the constraints presented in this alternative is 396 000. Ticket sales for each segment is given as $Q_g = 44\ 000$ and $Q_s = 11\ 000$.

Task 2

In this scenario, the model is loosened by removing the constraint of price equality between the different customer segments. We therefore remove constraint (5) from the model. The rest of the model is unchanged in this scenario.

Results from Task 2:

	General	*Student*
Q_i	44 000	11 000
p_i	25.33	7.20
Revenue	1 114 665	79 200
Total		**1 193 865**

The optimal total revenue of the concert given the constraints presented in this alternative is 1 193 865, which is significantly higher than in Alternative 1. Ticket sales for each segment is given as $Q_g = 44\ 000$ and $Q_s = 11\ 000$.

Task 3

a) Alternative 3a

In this task we will explain how we have changed the model in accordance with the task description on a per section basis.

Sets and indexes:

The set *I* is expanded to include the senior customer segment, *r*:

$i \in I$: Customer segment $i \in$ {g, s, r}

Constraints:

Demand of the senior segment *r* is limited by the price set for segment *r*.

$$Q_r = 15{,}000 - 1{,}400 p_r \tag{8}$$

Segment *g* has a minimum capacity constraint of 20% in this scenario.

$$Q_g \geq 55\ 000 \cdot 0.2 \tag{9}$$

Segment *s* and *r* has a combined minimum capacity constraint of 20% in this scenario.

$$Q_s + Q_r \geq 55\ 000 \cdot 0.2 \tag{10}$$

We assume the equality constraint between the general and student customer segment is removed, so we add a new equality constraint between the student and senior customer segments:

$$p_s = p_r \tag{11}$$

Results from Task 3a:

	General	*Student*	*Senior*
Q_i	44 000	8 679	2 321
p_i	25.33	9.06	9.06
Revenue	1 114 665	78 604	21 018
Total			**1 214 288**

b) Alternative 3b
In this alternative we keep all prices independent of each other, and only add three allocation constraints.

Constraints
Minimum segment allocation per customer segment group

$$Q_g \geq 55{,}000 \cdot 20\% \tag{12}$$
$$Q_s \geq 55{,}000 \cdot 10\% \tag{13}$$
$$Q_r \geq 55{,}000 \cdot 10\% \tag{14}$$

Results from Task 3b:

	General	*Student*	*Senior*
Q_i	44 000	5 500	5 500
p_i	25.33	11.60	6.79
Revenue	1 114 665	63 800	37 321
Total			**1 215 787**

c) Alternative 3c:
In this alternative the price can be set freely, except that the highest price cannot exceed the lowest times a factor of two.

Constraints
Minimum segment allocation for each customer segment group *i*

$$Q_i \geq 55{,}000 \cdot 5\% \quad \forall i \in I \tag{15}$$

The minimum allowed price for each customer segment *i*

$$p_i \geq 6 \quad \forall i \in I \tag{16}$$

The maximum allowed difference between the highest and lowest price is by a factor of two.

$$p_i \leq 2p_j \quad \forall i, j \in I \tag{17}$$

Results from Task 3c:

	General	*Student*	*Senior*
Q_i	49 000	2 750	2 750
p_i	17.50	13.80	8.75
Revenue	866 250	37 950	24 063
Total			**928 263**

Results from all three scenarios in Task 3:
To address the question of which alternative we would suggest to the organizers, we must first base our recommendation upon preset criteria. We assume that the organizers want to maximize profit for this this event and given the uncertainty of the demand curves we deem alternative 3a and 3b as equivalent alternatives when it comes to profit. Therefore, we must take other factors into consideration. Given the high price difference between the lowest and highest price in alternative 3b, this might change the general audience's perception of value for the ticket, as the prices are public. When factoring in this human factor of perceived fairness and value, we would recommend the organizers to choose alternative 3a, given that there is uncertainty in the demand curves.

Part B

Task 1

In this task we will address the nutritional needs of diet type 1 by adding a constraint that adds a lower bound on the intake of *k1* over a three-day period.

Constraints

The total nutritional value of the chosen items *i* in meal *j* on day *t* through day *t+2* (where $\omega_{k1,i}$ is defined in units of the nutrient *k1* in item *i*) must be equal to or higher than 250.

$$\sum_{j\in M}\sum_{i\in I}\omega_{k1,i}\,y_{p1,i,j,t} + \sum_{j\in M}\sum_{i\in I}\omega_{k1,i}\,y_{p1,i,j,t+1} + \sum_{j\in M}\sum_{i\in I}\omega_{k1,i}\,y_{p1,i,j,t+2} \geq 250 \qquad \forall t \in T$$

Task 2

In line with the requirements of this new situation, we have made additions to the model in order to satisfy the goal of Romeo and Juliet of having at least two Happy Saturdays in the planning period.

We define 'vanilla ice cream' as *'vic'* and 'strawberry ice cream' as *'sic'* and 'pasta bologonese' as '*pb*'.

Parameters

θ_t: Binary, 1 if day *t* is a Saturday, 0 otherwise

Decision Variables

HS_t: Binary, 1 if day t is a happy Saturday, 0 otherwise.
$HM_{j,t}$: Binary, 1 if meal *j* on day t is defined as a happy meal, 0 otherwise

Constraints

The sum of all Happy Saturdays in the planning period *T* must be equal to or more than 2.

$$\sum_{t\in T} HS_t \geq 2 \tag{1}$$

Individual (or diet type) 1 cannot choose both types of ice cream in the same meal *j* on day *t*.

$$y_{\text{p1,vic},j,t} + y_{\text{p1,sic},j,t} \leq 1 \tag{2}$$

A happy meal is defined as a meal *j* on day *t* where both *pasta bologonese* and either *vanilla ice cream* or *strawberry ice cream* is chosen for Individual 1.

$$2 \cdot HM_{j,t} \leq y_{\text{p1},vic,j,t} + y_{\text{p1},sic,j,t} + y_{\text{p1},pb,j,t} \tag{3}$$

$$HM_{j,t} \geq \left(y_{\text{p1},vic,j,t} + y_{\text{p1},sic,j,t} + y_{\text{p1},pb,j,t}\right) - 1 \tag{4}$$

If a meal *j* on day *t* is defined as a happy meal, while day *t* is also a Saturday, day *t* is defined as a Happy Saturday.

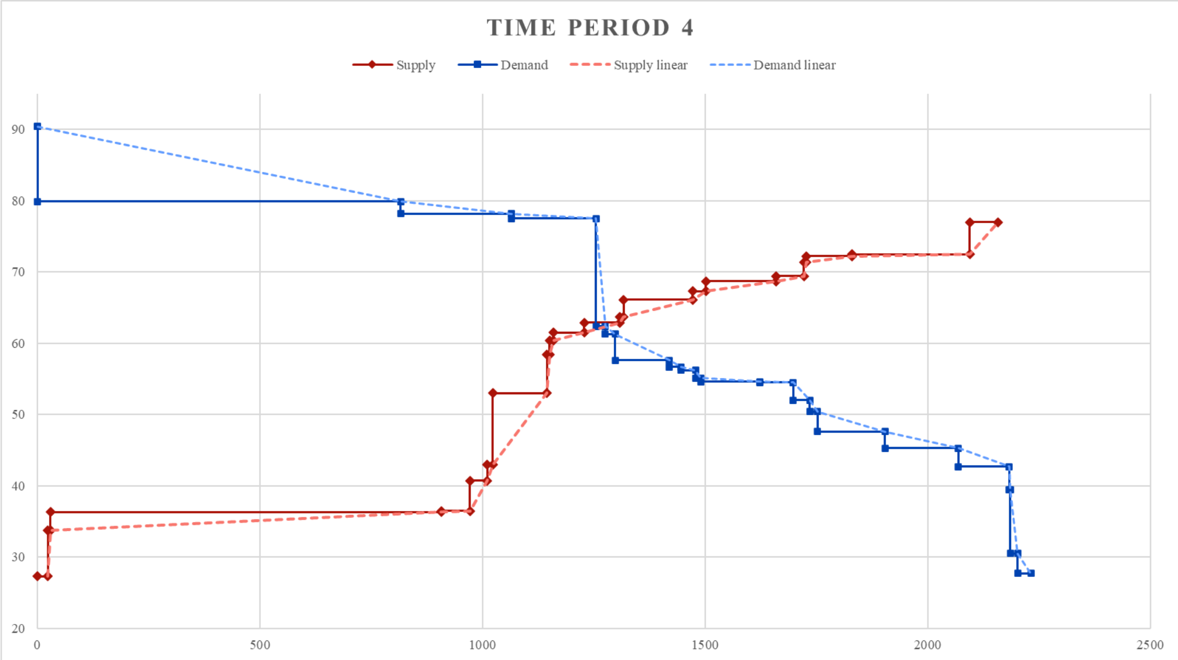
$$HM_{j,t} \cdot \theta_t = HS_t \quad \forall j \in M, t \in T \tag{5}$$

Maximum one happy meal *j* per day *t*, as we assume that it is not possible have the happy meal combination more than one time per day.

$$\sum_{j\in M} HM_{j,t} \leq 1 \quad \forall t \in T \tag{6}$$

Part C

Task 1



The system price is 62.35. There are five bids which are above this price and should be accepted, at 62.4, 77.5, 78.2, 86.6 and 90.4, respectively. In this example there would have been one unique intersection of the step function, with a system price of 59.2, which would not have changed which ask/bids got accepted.

Task 2a

Our strategy was to put in a bid slightly lower than the system price at a volume similar to the other bids and then vary the price and volume up and down slightly to see what direction to move in. This showed that decreasing prices while increasing volume was the direction that increased the profit the most, as the gain in volume compensated for the loss in system price.

The increase in revenue from volume outweighed the decrease in revenue from the lower system price. Increasing the volume while keeping the price low enough that all bids were accepted increased the profit all the way until we reached the production limit. Then we increased prices on all days until the profit on the specific day started decreasing. Note that the profit kept improving when increasing the price beyond the point of unsatisfied volume on day five.

There is a chance that this is a local maximum, and that the optimal solution is a high price and low volume so we performed some random tests and could not find any signs of performance improvements. Thus, the maximum profit we found was €206 513.25

Hour	*Price*	*Volume*	*PS*	*PD*	*s*	*d*	*Bid*	*Cost*	*Volume*	*Accepted volume*	*Profit*
1	32.05	1279	31.2	34.1	8	2	31	11	1200	1200	25 260.00
2	47.28	1790	46	47.5	1200	83	46	11	1200	1200	43 536.00
3	54.55	1471	54.3	58.9	9	130	54	11	1200	1200	52 260.00
4	40.24	2172	40	42.7	1200	103	40	11	1200	1200	35 088.00
5	53.15	1798	53	53.3	1195	32	53	11	1200	1195	50 369.25
										Total	206 513.25

Task 2b

This task offers two relaxations of the problem. Firstly, we are no longer required to place any bids. Secondly, the volume of the bids no longer needs to be the same. Since there are only relaxations in this modification of the model, the solution cannot be any worse than the one we found in task 2a. Not selling any power will not improve the profit since there are no periods where the system price is less than the cost of production. Since we found in the previous task that the best solution was to send the maximum size bid every period, the relaxation of not requiring equal sized bids means we can only increase the profit by decreasing the volume and increasing the price for some days.

Thus, we started systematically lowering the volume and increasing the price for the various days, starting with lowering the volume by some small amount and increasing the price until the full volume of the bid would not be filled, and going back to the price just below. This process was iterated with lower volumes until we found the maximum profit per day. The best solution we were able to find was slightly higher than in task 2a, with a 2.2% improvement. This shows that since this was a relaxation of the constraints in task 2a, the results necessarily could not get any worse and in this case, there was in addition also an improvement.

Hour	*Price*	*Volume*	*PS*	*PD*	*s*	*d*	*Bid*	*Cost*	*Volume*	*Accepted volume*	*Profit*
1	39.3	1261	38.6	39.5	13	98	31	11	1020	1020	28 866.00
2	47.28	1790	46	47.5	1200	83	46	11	1200	1200	43 536.00
3	54.55	1471	54.3	58.9	9	130	54	11	1200	1200	52 260.00
4	43.84	2069	40	45.3	1097	165	40	11	1100	1097	36 025.48
5	53.15	1798	53	53.3	1195	32	53	11	1200	1195	50 369.25
										Total	211 056.73

Task 3

Our initial thought of what strategy to use in this problem was to use the maximum available volume and underbid the previous highest accepted block bid. We chose to use all four possible days as it did not seem realistic that we would experience system prices below our production costs on any day. We chose the last four days of the week as these had a higher system price than the first day. Underbidding the highest bid gave a profit of €136 992. Then to see if we could achieve a higher system price, we tested a bid of the same volume but slightly above the previously highest accepted block bid and got a worse result.

The opposite direction, going below the second highest block bid, improved the result slightly to €137 508, but the profit would not change any further the lower we set the price, all the way to the limit of one euro. The only two remaining cases to evaluate are if lowering the volume would improve the profit and if using the first four days would. Neither of these alterations improved the profit. Thus, the best solution we were able to find was:

hour	*Price*	*Volume*	*PS*	*PD*	*s*	*d*	*Bid*	*Cost*	*Volume*	*Accepted volume*	*Profit*
1	41.85	820	40.6	39.5	138	226	43.9	11	0	0	-
2	31.21	2172	30.4	34.6	64	103	43.9	11	1200	1200	24 252.00
3	46.35	1592	46.2	50.8	9	251	43.9	11	1200	1200	42 420.00
4	36.48	1691	35.5	39.4	65	84	43.9	11	1200	1200	30 576.00
5	44.55	1898	41.4	45.2	81	32	43.9	11	1200	1200	40 260.00
										Total	137 508.00

Part D

Sets and indexes
I: Set of candidate locations of bike racks
N: Set of inhabitants in the city
B: Type of bike ∈ {'electrical', 'conventional'}

Parameters
f_i: Fixed cost of installing a bike rack at location i
v_i: Variable cost per bike initially allocated to location i
c_b: Cost of acquiring bike b
$d_{i,n}$: Distance from the house of inhabitant n to bike rack i
t: Maximum distance for an inhabitant to become a potential user

Decision variables
$x_{i,n}$: Binary, 1 if inhabitant n is a potential user of bike rack i, 0 otherwise
$y_{b,i}$: Amount of bikes b acquired at location i
g_i: Binary, 1 if location i is chosen as a bike rack, 0 otherwise
$r_{i.n}$: Binary, 1 if location i is in range of inhabitant n

Objective function
Minimize the sum of fixed costs, variable costs, and acquisition costs for each chosen bike rack location.

$$Min: \sum_{i\in I} f_i \cdot g_i + \sum_{b\in B}\sum_{i\in I} v_i \cdot y_{b,i} + \sum_{b\in B}\sum_{i\in I} y_{b,i} \cdot c_b$$

Constraints
For every location i chosen as a bike rack location the number of electrical bikes acquired must be exactly 2.

$$y_{electrical,i} = 2 \cdot g_i \quad \forall i \in I \tag{1}$$

The total amount of potential users N connected to all bike racks I must be at least 50% of the population of the city.

$$\sum_{i\in I}\sum_{n\in N} x_{i,n} \geq 0.5 \cdot |N| \tag{2}$$

For all chosen bike rack locations i the amount of bike b acquired must be more than or equal to 5% of the total number of potential customers n. M is defined as a sufficiently large number.

$$\sum_{b\in B} y_{b,i} \geq 0.05 \cdot \sum_{n\in N} x_{i,n} \quad \forall i \in I \tag{3}$$

$$\sum_{b\in B} y_{b,i} \leq M \cdot g_i \quad \forall i \in I \tag{4}$$

An individual n can only be assigned to a potential bike rack location i if the distance d between the two locations is within the set maximum range t.

$$r_{i,n} \geq x_{i,n} \quad \forall i \in I, n \in N \tag{5}$$

Defining *r* decision variable within the model as a binary variable that is dependent on the relationship between the distance *d* between location *i* and individual *n* and the maximum set range *t*. Note that this decision variable can be defined as a parameter with data set where *t* is defined.

$$t \geq d_{i,n} - M\left(1 - r_{i,n}\right) \qquad \forall i \in I, n \in N \tag{6}$$

$$t \leq d_{i,n} + M \cdot r_{i.n} \qquad \forall i \in I, n \in N \tag{7}$$

For any given individual *n* the number of bike racks *i* they can become a potential customer of must not exceed one.

$$\sum_{i \in I} x_{i,n} \leq 1 \quad \forall n \in N \tag{8}$$

Following constraint (8), if a given individual *n* is within range of two active bike rack locations *i*, then they can only become a potential customer of the closest bike rack in terms of distance *d*.

$$x_{i,n} \cdot d_{i,n} \leq d_{j,n} \quad \forall i, j \in I, n \in N \tag{9}$$

There cannot be an individual *n* which is connected to a location *i* which is not chosen as a bike rack location. M is defined as a sufficiently large number.

$$\sum_{n \in N} x_{i,n} \leq M \cdot g_i \quad \forall i \in I \tag{10}$$

Non-negativity and binary constraints.

$$x_{i,n} \in \{0,1\} \quad \forall i \in I, n \in N \tag{11}$$
$$r_{i,n} \in \{0,1\} \quad \forall i \in I, n \in N \tag{12}$$
$$g_i \in \{0,1\} \quad \forall i \in I \tag{13}$$
$$y_{b,i} \geq 0 \quad \forall b \in B, i \in I \tag{14}$$