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Menu Engineering for Continuing Care Senior Living Facilities with Captive Dining Patrons

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Abstract. Continuing care facilities are a rapidly growing segment of senior living communities providing end-to-end solutions comprising independent living, assisted living, nursing home care, and ultimately hospice. All these establishments contain (in addition to other facilities associated with living, exercising, learning, activities, etc.) dining services managed by an interdisciplinary (finance, nutrition, dietitian, kitchen operations, hospitality, and procurement) team of executives, each with their own objective while cognizant of the overarching organizational, operational, and financial metrics. The residents of these facilities consume most of their meals at these dining facilities, necessitating that the food served meets the complete nutrition, dietary, cost, and operational requirements. Thus, the menu (often rotating every few weeks) of food items must be carefully chosen to be efficiently procured, processed, and served, all the while meeting the nutritional, dietary, and patron satisfaction constraints each put forth by the corresponding stakeholder. We address this complex, unwieldy, and large multiobjective optimization problem using mixed integer linear programming. We demonstrate how menu planners and chefs can analyze their decisions regarding menu structures and evaluate alternative menu interventions to improve menus' nutritional value while ensuring their residents' autonomy in making food choice decisions. Along the way, we interviewed various stakeholders, identified their objectives and constraints, gathered the necessary data, formulated and solved the resulting optimization problems, and produced demonstrably effective menus.

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Keywords: menu optimization • continuing care facilities • decision analysis • mixed integer linear programming • goal programming

Introduction

The total population of seniors in the United States continues to grow as the last baby boomers reach 65 years, with an expected 18 million increase between 2020 and 2030. This increase has been projected to result in one in five Americans being at least 65 years old by 2030, with approximately 90% of them having a chronic health condition (Colby and Ortman 2015). It is estimated that the size of the population in nursing homes will grow by 32% by the year 2050 (Agarwal et al. 2016). With this rise in the senior population, tending to the age group becomes necessary; continuing care facilities (CCFs), which provide end-to-end services of independent living, assisted living, nursing home care, and hospice, are becoming increasingly popular. These facilities consist of various (living, exercise, meeting, reading, activities, etc.) opportunities, and the dining services are central to them. The residents consume most of their meals at these dining facilities; therefore, by recognizing this importance, the dining services are managed by an interdisciplinary (finance, nutrition, dietitian, hospitality, information technology (IT), and operations) team of executives, each with his or her own objectives. For instance, finance managers want to minimize cost; nutritionists wish to make sure the nutritional needs are met; dietitians want to ensure those diet restrictions (medical or otherwise) are accounted for; customer service managers want to ensure resident satisfaction; and operations team members want to make sure the food can be procured and processed efficiently.

Furthermore, the targeted senior population is becoming increasingly diverse in their food preferences. Achieving all these conflicting objectives while providing food that makes the residents healthy and happy is not easy, especially with a menu that includes only a limited number of courses in each meal. The design of menus (often repeating every three or four weeks) to determine which food items (among the hundreds possible) are served at which meal (breakfast, lunch, dinner) on which day (position on the 21-day or 28-day cycle) is traditionally performed intuitively and manually, which often results in wasted efforts and ineffective menus. Existing decision support tools to design menus also lack systems that incorporate different diet types' conflicting preferences into the menu design process. We formulate this problem as a mixed integer, multiobjective linear optimization problem, demonstrating that the problem can be solved using off-the-shelf software. Furthermore, we present applications of the model to analyze menu construction decisions and menu improvement interventions based on the collected data from CCFs.

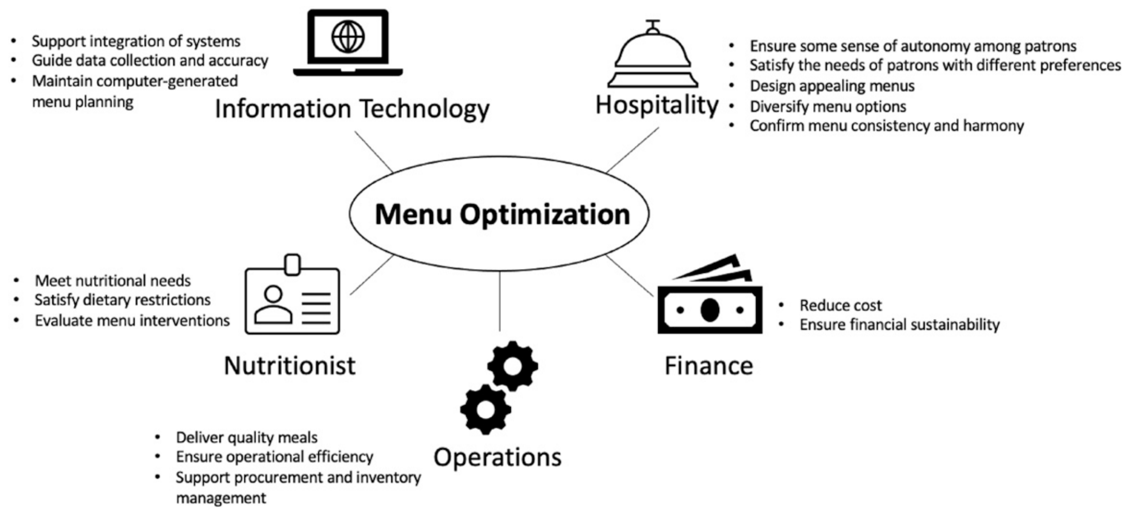
As shown in Figure 1, there are various menu planning and optimization perspectives. First, we present examples of these perspectives from the literature. Then, we describe how these different perspectives are incorporated into the model as objectives, satisficing strategies, or constraints with the justifications based on interviews with different stakeholders.

Nutritionist Perspective

The U.S. Department of Agriculture provides nutritional guidelines for older adults to prevent chronic health problems and malnutrition. Not meeting these guidelines contributed to an approximate 20% malnutrition rate among seniors in nursing homes in the United States (Bell et al. 2015). In many cases, seniors solely rely on daily meals in CCFs to meet their nutritional needs. Therefore, these daily meal menus are essential for the quality of life of their residents. The International Association of Gerontology and Geriatrics reports that eight countries, including the United States, have identified nutrition improvement as one of the top five research priorities for the elderly in nursing homes (Agarwal et al. 2016). The prevalence of nutritional deficits within the nursing home population can be determined based on the Minimum Data Set, Resident Assessment Instrument, or the Mini Nutritional Assessment (MNA) guidelines. For example, Suominen et al. (2005) used the MNA score and data from institutions in Helsinki, Finland, and found that 29% of residents were malnourished and 60% were at risk for being malnourished. Another study in southwest France found a malnourishment prevalence of 19.1% (Bourdel-Marchasson 2010). According to a UK–based study (Cowan et al. 2004), malnutrition is caused by organizational factors, such as the importance of nutrition not being realized, the absence of a dietitian, and monotonous unappetizing menus. In another study, continued malnutrition and subsequent weight loss over just six months had been shown to negatively impact mortality by increasing the likelihood of death twofold (Sullivan et al. 2002). Malnutrition only worsens as someone ages because of loss of muscle mass and subsequent body fat (Espín et al. 2016). Diet-optimization methods using linear integer programming were used to help determine diverse U.S. population subgroups, such as cancer prevention food plans (Masset et al. 2009) and menu scheduling for patients with high blood pressure (Hui et al. 2021).

The prevalence of malnutrition in CCF demonstrates how imperative it is that menus contain enough nutrition to allow residents to choose meals that meet their daily requirements. Dollahite et al. (1995) scrutinized 43 menus and found that even though professionals developed those menus, there was a lack of micronutrients, especially zinc. According to some studies in the United States, most nursing home residents have excessive amounts of fat, cholesterol, saturated fat, and sodium. In contrast, they have deficiencies of essential macronutrients and micronutrients; the lack of these

Figure 1. Perspectives in Menu Planning and Optimization



critical nutrients can lead to an increased risk of chronic disease (Mahadevan 2012, Morley et al. 2014), affect mobility, increase the risk of falls, and cause the inability to perform activities of daily life (Alexander et al. 2017). These findings indicate that CCFs currently have difficulty generating menus that meet recommended dietary allowances and satisfy the nutritional guidelines for Americans. Interestingly, a study in Canada (Lam et al. 2015) found that it is possible to manually generate super menus containing many, if not all, daily needed micronutrients recommended by Canada's Food Guide. Schaynová (2017) proposed a linear programming (LP) model to prevent mistakes when manually preparing diet plans by ensuring dietary recommendations. Ferguson et al. (2004) developed a four-phase robust approach based on LP to design, test, and refine the food-based dietary guidelines (FBDGs). Then, they illustrated the method using three- to six-year-old rural Malawian children. Anderson and Earle (1983) utilized a goal programming approach to meet specific nutrition requirements and suggest nutritional balance in selected diets. They concluded that improving the nutritional balance is costly and hard to justify.

Although a menu planner could efficiently learn how to create and modify menus, complexity arises when different diet types (e.g., vegetarian, low sodium, heart healthy, etc.) are considered simultaneously. Including more healthy options or trying to satisfy the needs of very different diet types may lower the overall satisfaction with a menu. Fortunately, it has been established that altering the portion sizes on menus by increasing the sizes of vegetables and decreasing the sizes of meat portions without letting the patrons know had no negative impact on consumer satisfaction (Reinders et al. 2017). In other words, including a slightly higher quantity of healthier items would not decrease patron satisfaction and feeling of autonomy. This paper presents a menu intervention demonstrating that overall menu satisfaction can be improved significantly under the same level of nutritional guideline requirements by making small but strategic changes in a subset of existing menu items.

Information Technology Perspective

Technology can aid CCFs in menu planning by providing them a baseline that targets different nutritional needs of seniors, such as vegan, heart healthy, and vegetarian. Alexander and Madsen (2017) found that out of 807 U.S. nursing homes, the average IT sophistication score was between 14.7 and 47.9 out of 100 in terms of nursing homes' IT capabilities, the extent of IT usage, and the degree of integration of IT in care, clinical support, and administration. Another study by Zhang et al. (2019) found that out of 32 nursing homes, 53.1% were technically efficient, which means many nursing homes rely on manual labor instead of IT to accomplish menu design tasks. Similarly, Ko et al. (2018) noted that nursing homes had deficient technological infrastructures, contributing to lower chances of utilizing and experiencing healthcare information technology benefits in business productivity and resident care. There is also evidence that although some nursing homes keep their IT systems up-to-date, their IT use is much lower, indicating that facilities are not using IT capabilities to the fullest extent possible (Alexander and Madsen 2017).

Computer-generated menu planning was used in studies from the late 1960s through the early 2000s. A multiobjective evolutionary algorithm was developed to find a feasible 21-day menu based on dietary constraints in equal or less time than a human trained to design menus (Seljak 2009). Although they found that their evolutionary method improved the quality of the menu, they noted that the human factor is still important while selecting a solution from the Pareto front in the last stage. In addition, Gazan et al. (2018) reviewed the approaches to optimize sustainable diets that are nutritionally adequate, economically affordable, culturally acceptable, and environmentally respectful. In general, to name a few, mixed integer programming (MIP) (Sklan and Dariel 1993), goal programming (Ferguson et al. 2006), and linear programming (Anderson and Earle 1983, Okubo et al. 2015) are applied in diet planning. Benvenuti and De Santis (2020) also used mathematical programming to optimize menus considering sustainability, cost, and cultural acceptance criteria. Their reported results showed a reduction in the environmental impact of the meal plans while ensuring an adequate nutritional intake at affordable prices. The model by Benvenuti and De Santis (2020) includes a subset of the decision variables used in this paper. The model defined in this paper includes menu diversification and dependency constraints over overlapping periods to ensure that all segments of a multiperiod menu are diverse and satisfy item dependencies defined by menu planners and the preferences of different patrons.

Hospitality Perspective

There are several procedures that menu planners and chefs can follow to design menus. For example, Mayerson and Thompson (2019) outline a six-step process to follow while designing a menu. These steps introduce the need for menu cycling, meal variations, diet restrictions, and resident choices and the importance of standardized recipes. In Mayerson and Thompson's (2019) approach, the first step involves determining resident menu choices and preferences, directly impacting resident autonomy and self-choice. Autonomy becomes difficult when nursing homes are modeled based on medical facilities where autonomy is based on familial consent (Sherwin and Winsby 2011). Many CCFs implement modifying autonomy, which is defined as altering self-choice to suit a resident's needs during conflict (Lützén and Nordin 1994). Considering residents' preferences and offering multiple meal choices on a menu is essential for maintaining some sense of autonomy.

Mayerson and Thompson's (2019) second step is concerned with designing an effective menu cycle. A CCF has two seasonal menus (summer and winter) and a rotation of menus every three to six weeks. Although this cycle of menus is consistent in many nursing homes, it may result in limited choices (Abbey et al. 2015). Therefore, Abbey et al. (2015) suggest that cyclic menus need to be rotated to promote choice through consistent change, which would positively affect meal consumption and satisfaction by avoiding restrictions on sugar and fat. The third step, variation, highlights the importance of diversifying main and side courses in the menu. Most CCFs offer full meals for breakfast, lunch, and dinner, including adequate sides and drinks. However, placing those items on a menu is critical to ensure that residents have various options over a menu cycle. The fourth step reinforces the need for meals for different dietary groups of residents, such as vegans, those who need pureed food, and those who need heart-healthy options. The number of common diets depends on nursing home sites, but there are commonalities within therapeutic diets. The fifth and sixth steps involve following regulations regarding standardized nutrition and serving sizes and selecting quality vendors. These steps and concerns, excluding the portion sizes, are incorporated into a mathematical modeling framework to assist menu planners in designing menus while considering conflicting objectives, many different types of constraints, and patrons with varying diet preferences.

Another concern in menu design is sensitivity to religious and cultural aspects of diets (Harper et al. 2014). Religion and culture impact residents' food choices and when they consume them. Our interviews with nursing homes in Pennsylvania showed that different cultures and holidays were celebrated via a special weekly or daily menu but not accounted for in the average menu cycle. Other studies indicated that most nursing homes account for cultural items (Chisholm et al. 2011). Maillot et al. (2010) developed and applied mathematical optimization for French adult diets to see whether low-cost healthy food plans are culturally and socially acceptable. They concluded that a minimum-cost nutritious diet also needs to consider social and cultural factors.

Low satisfaction with food items is a significant indicator of inadequate intake in nutrition in CCF, which directly relates to the 30%–65% of residents found to be malnourished in nursing homes (Wright et al. 2013). In a study focused on the perspectives of residents and staff on nursing home menus (Wang et al. 2020), the average satisfaction rating in meal choice was found to be between 4 and 6 on a scale out of 10. Failure to consider residents' food preferences when designing a menu leaves residents frustrated, physically unwell, and psychologically distressed and causes them not to achieve their nutritional goals. Staff are also negatively impacted because they have to deal with the complaints from residents, the effects of an unhealthy diet, and the inability to change the menus themselves because of corporate hierarchy (Wang et al. 2020). Because many residents depend solely on the establishment for food, CCFs must cater to their residents' preferences and nutritional needs (Milte et al. 2018); the nutritional requirements can only be achieved by serving healthy food that the residents prefer to have. Although mainly focused on the nutritional aspects of menu items to create daily menus with proper nutrition, the proposed modeling framework also considers residents' preferences toward different menu items.

Finance Perspective

Most of the time, CCFs operate as not-for-profit organizations, yet they are not immune to emphasis on margins because of the "No Margin, No Mission" mantra prevalent in these sectors. The finance managers at these facilities are focused on the cost of materials, investment in equipment and facilities, labor costs, and overhead. Although their objective would be to lower these costs as much as possible, they must acknowledge the sentiments expressed by the rest of the management team and manage the finances appropriately. Leung et al. (1995) introduced a different approach to diet planning by optimizing recipes rather than food items, that is, recipe-based optimization using integer programming, to meet nutritional requirements at minimum costs for institutions or individuals. Colombo et al. (2020) used linear programming with additional constraints to ensure the nutritional adequacy of school kids, reduce food-related greenhouse gas emissions, and suggest affordable menus. Compared with baseline menus, optimization resulted in environmentally friendly and cost-friendly menus that also meet all nutrient requirements.

Operations Perspective

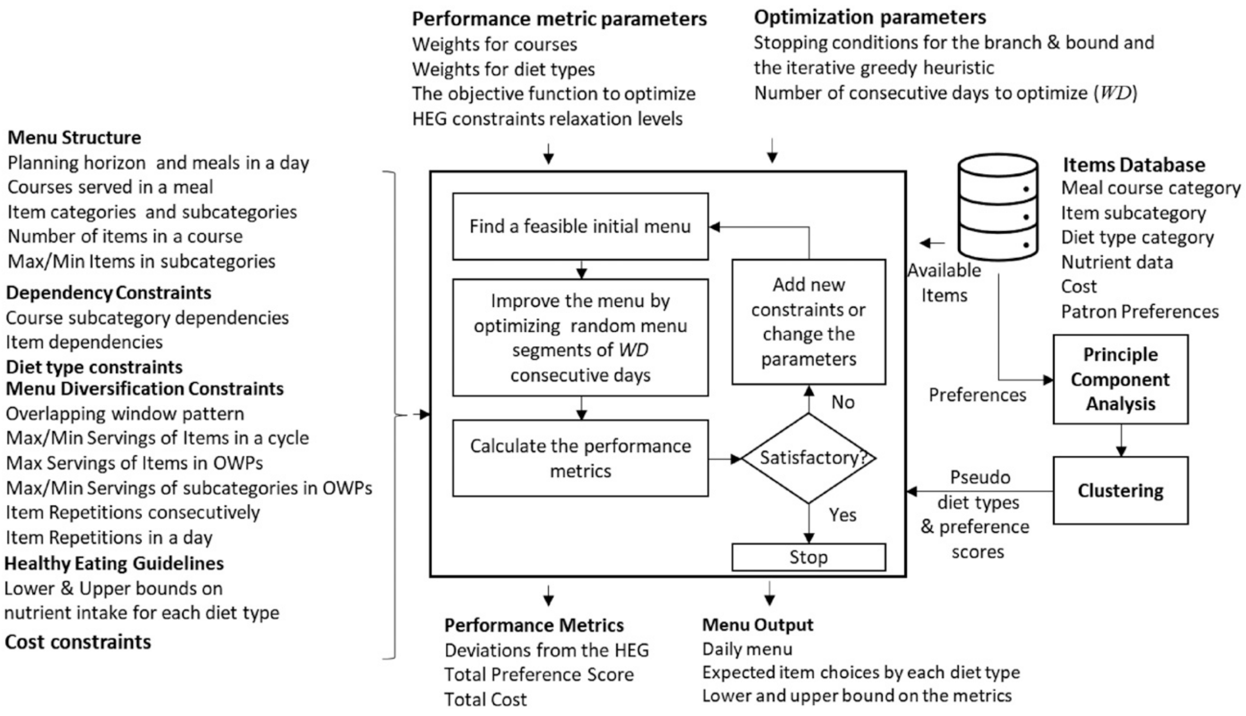
The operations team focuses on process efficiency, product quality, inventory management, procurement, workforce scheduling, and equipment utilization. These are challenging problems and are further complicated by the constraints imposed by the rest of the management team. The nutritionist wants to ensure the meals are healthy; the hospitality manager wants to make sure customers are satisfied (in terms of variety, dietary requirements, quality, and taste); the finance executive wants to minimize investments in labor, equipment, and material; and the IT manager wants all the information to be accurate and readily available. Given these conflicting objectives emphasized by an interdisciplinary team of executives, the operations manager must ensure that the menu is designed to achieve operational efficiency.

In this study, preexisting menus from several regional CCFs were collected, and a database of over 300 menu items and their nutritional facts were created. Several business leaders, head chefs, managers, and dietitians were interviewed to understand the processes and challenges of menu planning in CCF. Although these CCFs rely on a food information system for recipes and nutritional information, their menus are planned out by hand. All CCFs use cyclic menus with a three-to-five-week planning horizon and a six-month change (summer and winter menus). They celebrate cultural differences with festive weekly menus every couple of months and allow exceptions for residents of diverse backgrounds. Preferences for food items are calculated by tally sheets daily to mark the amount of each item ordered by every resident. The process described is tedious and requires extra staff to accommodate the manual labor. All data corresponding to food ordered are manually entered into an Excel file referencing the paper receipts. The second nursing home uses a food management system. Their top priority is the satisfaction of residents. A team consisting of head chefs, managers, and dietitians meets four times a year to plan the menus, which cycled every four weeks. They do not manually analyze their data; instead, they enter all data into a point-of-sale system that tracks various preferences for them. The CCF interviews have shown that every CCF is different in what they need for a menu optimization algorithm. Even though CCFs may operate under the same corporate structure, all CCFs see a place for it in their system. It is noted that individual CCFs usually design menus for themselves and determine the level of choice and variety on the menus they provide (Abbey et al. 2015).

Description of the Modeling Framework

This section presents a mathematical modeling framework to plan menus for CCFs. The proposed framework aims to incorporate the requirements of menu planners, healthy eating guidelines (HEGs), and choices of residents (patrons) with different diet preferences into the menu planning process (see Figure 2). Independently operated small CCFs have significant challenges in pleasing all patrons with a limited number of dish options served in a meal. Plus, menus should reduce food boredom and ensure a sense of autonomy while supporting a healthy diet for all. The modeling framework also puts forward a novel approach to the diversification of items in a cycle menu, one of the most challenging tasks in manual menu planning, according to the menu planners involved in developing the model. In addition to supporting the planning of menus, the

Figure 2. The Modeling Framework to Design Cyclic Menus for CCFs



proposed modeling framework can identify problems and deficiencies in the current menu options and items. The menu planning process varies significantly across CCFs, depending on their size, focus, and organizational structure. Considering all these factors and incorporating specific requirements of CCFs requires many different constraints and several conflicting objectives. Therefore, the modeling framework was conceptualized as flexible and generic to address the needs of many different CCFs, especially small, independent ones.

Figure 2 presents the input and output of the modeling framework. Next, the decision variables and constraints of the model are introduced, followed by an application highlighting a detailed set of the constraints available within the model. The equations of the mathematical model are provided in Appendix A.

Decision Variables

CCFs mostly use cycle menus, which are made of a pattern of meals that repeat regularly. Typically, a cycle menu is planned for a period of consecutive days (denoted by set T), and then it may repeat with minor or no changes after the last day. Although menus are structured differently across CCFs, a typical daily menu includes a set of meals that include several courses (e.g., soup, salad, main dish, etc.). Menu planners define the structure of a menu by specifying the set of meals (M) in a day, the course types, and the number of items served in each course. For the planning cycle, menu items are typically selected from the available items (denoted by set I) in the food management system. Depending on their availability or seasons, the set of items may change from cycle to cycle. Another critical input to the model is the classification of items in terms of the meal courses. Without loss of generality, all menu items can be classified into one of the meal courses as follows: L = {"breakfast," "breakfast side," "main," "salad," "side," "soup," "appetizer," "dessert"}. In the first phase of menu planning, menu planners usually think about categories of menu items instead of individual items. The subcategorization of the menu items allows menu planners to express their rough preferences while planning their menus as a set of constraints. Hence, menu items are also partitioned into subcategories according to their main ingredients or dietary attributes (set $S(l)$ denotes the subcategories of course l).

The model includes the following group of decision variables to represent the complex relationships described previously.

Item Inclusion. The first group of decision variables is for including items from the set of items to serve in daily meals. The binary decision variable $x_{i,j,t}$ indicates whether item i is served ($x_{i,j,t} = 1$) in meal j on day t or not ($x_{i,j,t} = 0$).

Patron Behavior. As stated previously, one of the objectives of the modeling framework is to support menu designers in designing menus appealing to different diet types (denoted by set P) using only a limited number of items. This requires representing the item-picking decisions of patrons in the model. The binary decision variable $y_{p,i,j,t}$ represents whether a diet type $p \in P$ chooses an item i in a meal j on a day t ($y_{p,i,j,t} = 1$) or not ($y_{p,i,j,t} = 0$).

HEGs. These decision variables capture the deviations from HEGs as a fraction of patrons' daily recommended nutrient intakes. The decision variable $Q_{p,k,t}$ represents the violation of the recommended upper or lower bound of nutrient k for individual p on day t.

Menu Structure Constraints

In CCFs, operational limitations and cost concerns mainly dictate the structure of a menu and the number of items included in a meal. For example, a facility may be able to prepare only two main dishes per meal. The cost is another factor limiting the number of items served in a meal. The following set of constraints defines the main structure of a menu.

Item Inclusion. These constraints ensure that a predefined number of items ($ns_{j,l}$) are served in course l of meal j.

Maximum Number of Items per Subcategory. When a meal includes multiple items of a course (e.g., two main dishes in a lunch), menu planners prefer to serve only one item from each subcategory of the specified course. In other words, these constraints are used to diversify items in a meal.

Minimum Number of Items in a Subcategory. A meal j may require including a minimum number of items ($mns_{j,S'(l)}$) from a given subset $S'(l)$ of a subcategory set $S(l)$ (e.g., lunch should include a dark green or red/orange side dish).

Dependency Constraints

This set of constraints represents menu planners' experiences and knowledge in designing menus in terms of which combinations of items/subcategories should be simultaneously included or avoided in a menu. Combined with the menu diversification constraints, menu dependency constraints allow menu planners to create menus that aim to please different patrons using a limited number of item options.

Course Subcategory Dependencies. Menu planners wish to avoid serving some course subcategories together when a meal includes multiple items of the same course. For example, a meal should not include beef and pork main dishes together.

Item Dependencies. Menu planners also pay attention to dependencies between item types. For example, a starchy side should be served if a meal includes a main dish type of beef, pork, or poultry.

Menu Diversification Constraints

A well-designed cycle menu should include a pattern of items from different course subcategories based on patrons' needs and preferences while ensuring that menu items do not repeat frequently. Menu planners and dietitians know that repeating even favorite dishes often reduces overall satisfaction with food services in CCFs. Therefore, avoiding the repetition of menu items can make a difference in patrons' perception of menu quality. Placement of popular items on the menu is also important. Another concern is to ensure item diversification uniformly over the whole menu cycle. Because a cycle menu usually repeats after the planning period, it is also essential to consider the repetition of items served at the beginning and end of the planning cycle. Considering these factors while creating a menu that includes a diverse set of menu items to satisfy the needs and preferences of different diet types under strict nutritional requirements is a challenging process, according to our interviews.

In the proposed modeling framework, desired item patterns and menu assortment requirements are imposed using upper and lower bounds on the number of times the course subcategories and dishes are served during a period of W consecutive days, referred to as an overlapping window pattern (OWP). The planning cycle includes $|T|$ OWPs such that the tth OWP starts on day t and ends on day $t+W$. Depending on the length of W, the OWPs overlap to ensure that all consecutive segments of a cycle menu will follow item diversification and pattern requirements defined by the menu planners, including the beginning and end of the menu. An example for OWPs with $|T| = 8$ and $W = 2$ is given in Figure 3.

The menu planners can use the OWP concept with the following constraints to achieve a uniformly diverse menu.

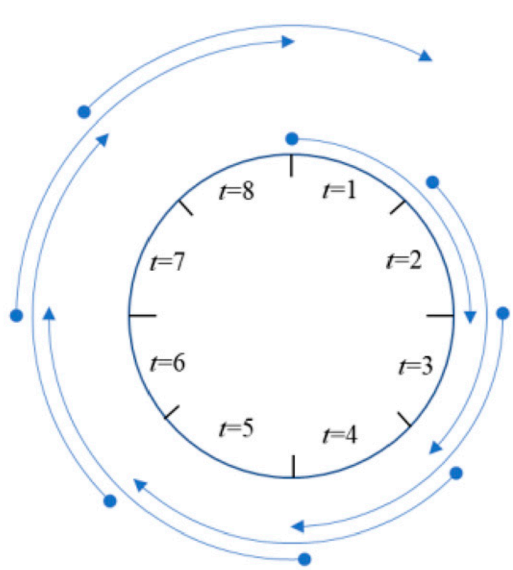
Maximum/Minimum Number of Servings of Items in a Menu Cycle. The first group of menu diversification constraints sets lower ($nsT_i^{\min}$) and upper ($nsT_i^{\max}$) bounds on the number of times an item i can be served within the entire cycle menu. A lower bound constraint is only needed if an item is planned to be served within the planning horizon because of operational reasons, the item's popularity, or special events.

Maximum Number of Servings of Items in OWPs. The second group of menu diversification constraints limits the number of times that an item can repeat within each OWP to an upper bound ($nsOWP_l$), which usually depends on the course category of the item. For example, main dishes may be allowed to be served only once, whereas a side dish may be served multiple times within an OWP. Using the concept of OWP, these constraints account for the variability of items at the beginning and end of the menu cycle and ensure the uniform diversification of menu items over the entire planning cycle.

Maximum/Minimum Number of Course Subcategories in OWPs. This group of menu diversification constraints is concerned with the variety of course subcategories. They ensure that a specified minimum ($nc_{l,s}^{\min}$) and maximum ($nc_{l,s}^{\max}$) number of items from a subcategory s of a meal course l are served within each OWP. The lower bounds on subcategory servings are used to create a pattern of menu items. For example, menu planners may wish to serve a poultry dish once within each OWP.

Limiting Consecutive Repetitions of Items. Menu planners also pay attention to not serving the same items consecutively, even if some items are allowed to repeat within an OWP. These constraints impose a gap of G_i days between two consecutive servings of item i. In other words, if an item i is served on day t, it cannot be served in the following G_i days.

Figure 3. (Color online) An Example of the OWP to Ensure Menu Diversity in All Segments of the Menu ($|T| = 8$, $W = 2$)



Item Repetition Constraints Within a Day. This set of menu diversity constraints concerns the variability of items within a day by limiting the number of times (nsd_l) that an item of subcategory l can be served during a day.

Diet Type Constraints. Another classification of the menu items is based on diet types, such as L('diet') = {"heart-healthy," "vegetarian," "vegan," "gluten free"}. The similar types of constraints given previously can be used to construct menus considering diet types. However, because of operational concerns, none of the partner CCFs in this study offer standalone menus specific to diet types. Instead, they include items to satisfy the needs of patrons with some dietary restrictions.

The constraints briefly described previously represent the perspective of chefs and menu planners who are concerned about designing a menu that is appealing to the patrons and makes sense in terms of consistency and harmony among the items served.

Diet Types

A unique aspect of the proposed modeling framework is considering patrons with different eating patterns to address the challenges of catering to multiple groups of patrons with conflicting food preferences using a menu with limited options. We propose a pseudo diet type concept, representing multiple distinct eating patterns in the same diet type. As discussed in the applications given in the next section, patron types are extracted from historical data using a data-driven approach. A patron type p chooses $np_{p,j,l}$ number of the items served in course l of meal j, and it is assumed that the patron will consume all picked items. This assumption does not accurately reflect the real-life behaviors of patrons because they can skip items available on the menu, do not consume all items, or pick different items than the model suggests. Because the exact behavior of each patron cannot be predicted and modeled as it is, a better modeling approach is to treat the dietary requirements as a target to achieve. The modeling framework considers healthy eating guidelines as targets using a satisficing strategy. It aims to design menus that minimize the total deviation from those targets or satisfy them within a given margin of error. From the point of view of menu engineering, the assumption that patrons consume all picked items will lead to menus satisfying the needs of patrons with various dietary patterns and minimize the deviation from the healthy eating targets. This assumption also prevents the model from generating an artificially feasible menu to satisfy nutritional requirements; but in practice, patrons' actual choices can make the menu infeasible. The model output represents patrons' maximum nutrient intake considering their preferences and provides a realistic target to plan menus.

HEG Constraints

The requirements of HEGs depend on age group, gender, diet restrictions, and nutrient type. In this study, the U.S. Department of Health and Human Services and the U.S. Department of Agriculture's requirements for the Healthy U.S.-Style Eating Pattern are used to model dietitians' perspectives based on the recommendation of the CCF partners. The constraints regarding HEG recommendations for the nutritional goals are expressed as the recommended upper and lower bounds on the nutrient intake. An example of the Healthy U.S.-Style Eating Pattern for men and women 51 years and older is given in Table 1, where parameter $\omega_{k,i}$ represents the amount of nutrient k in item i and $\alpha^{\max}_{p,k}$ and $\alpha^{\min}_{p,k}$ are the recommended upper and lower limits for the intake of nutrient k for patron type p. The unit of the parameter $\omega_{k,i}$ is either kilocalories (kcal) or weight (grams or

Table 1. Healthy Eating Guidelines Published by the U.S. Department of Health and Human Services and U.S. Department of Agriculture (USDA 2015) for Men (m) and Women (f) over 51 Years Old

K	Description (unit)	Unit of $\omega_{k,i}$	P	(Lower limit) $\alpha^{min}_{p,k}$	(Upper limit) $\alpha^{max}_{p,k}$	$\gamma^{min}_{p,k}$	$\gamma^{max}_{p,k}$
1	Calories (kcal)	kcal	m	2,000	2,000	2,000	2,000
			f	1,600	1,600	1,600	1,600
2	Calories from fat (% kcal)	kcal or kcal = 9 g	(m,f)	$0.2TC_{p,t}$	$0.35TC_{p,t}$	$0.2\gamma^{min}_{p,1}$	$0.35\gamma^{max}_{p,1}$
3	Saturated fat (% kcal)	kcal = 9 g	(m,f)	0	$0.1TC_{p,t}$	0	$0.1\gamma^{max}_{p,1}$
4	Trans fat (g)	g	(m,f)	0	0.1	0	0.1
5	Cholesterol (mg)[a]	mg	(m,f)	0	300	0	300
6	Sodium (mg)	mg	(m,f)	0	2,300	0	2,300
7	Calories from carb (% kcal)	kcal or kcal = 4.0 g	(m,f)	$0.45TC_{p,t}$	$0.65TC_{p,t}$	$0.45\gamma^{min}_{p,1}$	$0.65\gamma^{max}_{p,1}$
8	Dietary fiber (g)	g	m	28	∞	28	∞
			f	22.4	∞	22.4	∞
9	Added sugar (% kcal)	kcal = 3.86 g	(m,f)	0	$0.1\ TC_{p,t}$	0	$0.1\gamma^{max}_{p,1}$
10	Calories from protein (% kcal)	kcal = 4.0 g	(m,f)	$0.1TC_{p,t}$	$0.35TC_{p,t}$	$0.1\gamma^{min}_{p,1}$	$0.35\gamma^{max}_{p,1}$

Note. The most recent version of the guidelines was not available when the application data were collected.

[a]The key recommendation from the 2010 dietary guidelines to limit consumption of dietary cholesterol to 300 mg per day was not included in the 2015 edition. However, this limit was adopted in this study.

milligrams) of the macronutrients of item i. The HEGs also specify nutritional limits in terms of either absolute targets (e.g., consume less than 2,300 mg per day of sodium) or percentage of the total calories (e.g., consume less than 10% of calories per day from saturated fats) or both. In Table 1, $TC_{p,t}$ represents the total calories taken in a day t by patron p to model the HEG constraints based on the percentage of total calories.

As stated earlier, designing a menu that satisfies the HEGs for all diet types on each planning day is difficult. Therefore, the HEG recommendations are considered soft constraints in the model such that the variable $Q_{p,k,t}$ measures violations in the HEG constraint related to nutrient k by diet type p on day t. In the model, the variable $Q_{p,k,t}$ is bounded by a parameter $Q_p^{\max}$ indicating the maximum allowed deviation from the HEG recommendations. Parameters $\gamma_{p,k}^{\max}$ and $\gamma_{p,k}^{\min}$ are used to express $Q_{p,k,t}$ as a fraction of the nutrient limits. For the constraints defined as the percentage of the total calorie intake, the upper and lower limits depend on $TC_{p,t}$ (e.g., $k = 2$ in the table). The normalization parameters $\gamma_{p,k}^{\max}$ and $\gamma_{p,k}^{\min}$ are set to the upper and lower limits of $TC_{p,t}$, respectively, to have a linear model.

When constructing the menu item database in this study, the calories of some items were calculated based on the macronutrient weight if an item lacked calorie information. The calculations of calories (in kilocalories) from nutrients are also given in Table 1. In the table, the lower limit of zero indicates no lower limit constraints for that macronutrient; similarly, the upper limits of infinity indicate no upper limit constraints. An identical value of the upper and lower limits indicates a target value (i.e., an "=" type constraint).

Cost Constraints

The cost constraints impose the budget on the daily menus or balance the cost of the meals (e.g., a dinner meal is expected to be more expensive than a lunch meal). The cost of item i (c_i) can be actual monetary value if available or relative cost value (high-value to low-value item) as some menu planners may prefer to use.

Objective Function and Performance Metrics

In addition to the constraints given previously, menu planning also involves considering several conflicting objectives representing the perspective of the different stakeholders described previously. These objectives can be used as the basis for the objective function (Appendix A) or some criteria to satisfy during the optimization process. The final menus are evaluated based on the following performance metrics.

Total Preference Score. Besides the diversity of items, menu planners aim to include items patrons like. The total preference score represents patrons' overall desirability of a menu. The preference score $\pi_{p,i}$ of item i by diet type p can be obtained from historical data, through questionnaires, or based on menu planners' experiences. In the lack of exact preference data, chefs usually rely on their experience. This study obtained preference scores using a survey and normalized them based on the importance of items in a course. For each diet type p, the total preference score ($F_{\pi,p}$) of a menu by diet type p is calculated by summing the weighted preference scores of items picked by the patron. Then, the total preference score is calculated as the weighted average of the total preference scores of the diet types. In the applications presented in the next section, the diet type weights are based on the ratios of diet types in the target population, and the total preference score is used as the primary objective to optimize the model for given decision alternatives or menu designs (e.g., different menu structures decisions). The primary justification for using the total preference score as the primary objective is that patrons will likely pick items based on personal preferences but not nutritional concerns. For example, the model output may recommend that patrons pick certain items in a menu optimized using nutritional objectives. However, in reality, they will pick items that they prefer the most. In other words, the preference score accurately represents patrons' actual daily item selection decisions. Thereby, for a given menu decision, the calculated performance metrics will be closer to what might happen in real life. Because of these reasons, a satisficing decision-making strategy is used to incorporate other perspectives into the model. At the termination of optimization, the total preference scores of the diet types are returned for evaluating alternative decisions about menu structure and menu enhancement strategies.

Deviations from the HEGs. Nutritionists' objective is to make sure that a menu follows the HEG requirements. Strictly enforcing the HEG requirements is almost impossible and leads to infeasible or unpopular menus, especially considering different diet types. Menu planners are more concerned with minimizing the total deviation from the targets given in the HEGs for the macronutrient types. Therefore, a satisficing approach is used to incorporate the nutritionists' objective in the model. In the modeling framework, the sum of daily deviations from the HEG requirements for each diet type p and macronutrient type k (i.e., $Q_{p,k,t}$ decision variables) is capped by parameter $Q_p^{\max}$, representing the level of HEG violations. Then, the model can be solved to optimize the total preference score for various levels of $Q_p^{\max}$ to discover the trade-offs between the menu's healthiness and desirability. In other words, this satisficing decision-making strategy from the perspective of the nutritionists is used as a multicriteria decision analysis tool without assigning relative objective weights.

Total Cost. Another metric to evaluate a menu is the total cost, obtained by summing the cost of all items included in a menu. The cost is not only dictated by the types of items included but also the structure of the menu. If necessary, a satisficing approach described previously can be used to incorporate the cost objective in the model.

Implementation Details and Applications

This section presents applications of the proposed modeling framework in CCFs. The initial constraints and parameters of the model were established based on the input collected through interviews with facility managers, chefs, and dietitians from several CCFs. In the first round of interviews, the menu planners responded to the research team's open-ended questions and described their processes of constructing menus, objectives, and challenges. The stakeholders also provided their needs and deficiencies in crafting their menus. The menu construction and diversification constraints were defined based on the common themes that emerged in these initial interviews. The stakeholders reviewed these constraints and objectives in the second round, and we revised them accordingly. In this round, the stakeholders also discussed how to evaluate the performance metrics, such as total preference score and cost, because the data were not available to quantify these metrics. After creating the database of items and setting initial input parameters, the model was run to generate several menus. The stakeholders reviewed these initial menus and provided more detailed feedback, such as menus not having color or some items appearing together. Then, the model constraints, parameters, and menu items were updated based on their feedback. The target CCF used a three-week cycle menu ($T = \{1, \ldots ,21\}$), and we used $W = 5$ after final consultations with several menu planners.

Creating a Database of Menu Items

A comprehensive database of 381 menu items, most of which were obtained from the menu booklets of the target CCF, was compiled. The database included the macronutrient values, patron preferences, relative costs, item classifications, and subcategories for the items. Additional menu items from Nutritionix (www.nutritionix.com) were also included in the database to increase the variety of the items. The values of the macronutrients given in Table 1 were collected from the menu booklets or Nutritionix. Table 2 summarizes the breakdown of the items in terms of meal courses and the course subcategories.

Predicting Patron Preferences and Establishing Pseudo Diet Types

The target CCF did not have historical data on patron preferences for the menu items. Our interviews with several CCF administrators and menu planners also revealed that item preferences data are not readily available in most CCFs. Chefs and kitchen administrators plan menus based on their previous experience and availability of ingredients. Therefore, we established the preference scores of the items through a survey in which 28 patrons rated the menu items on the Likert scale as "Prefer a great deal (3)," "Neutral (2)," and "Do not prefer (1)." From the perspective of CCFs, this scoring system is a straightforward way to collect feedback on their patrons' food preferences.

The next step is identifying different diet types (set P) from the collected preferences and assigning patrons to these diet types to calculate the average preference values. In this study, diet types refer to the

Table 2. The Breakdown of the Items in Terms of Meal Courses and the Course Subcategories

Course (l)	Subcategories of course ($S(l)$) and number of items in each subcategory									
Main dish	Beef	Fish	Poultry	Grain	Pasta	Pork	Shell	Vegetable		
$\lvert S(l)\rvert$	31	7	33	6	25	13	8	33		
Side	Dark green	Fried	Fruit	Fungus	Grain	Legume	Protein	Red orange	Starch	Vegetable—other
$\lvert S(l)\rvert$	17	4	1	4	11	4	1	15	25	10
Soup	Clear	Cream	Legume	Noodle	Potage					
$\lvert S(l)\rvert$	5	15	4	8	2					
Dessert	Cake	Cheesecake	Cobbler	Cookie	Ice cream	Pie	Pudding			
$\lvert S(l)\rvert$	2	6	2	2	7	4	5			
Breakfast	Batter	Cereals	Egg	Meat	Toast					
$\lvert S(l)\rvert$	11	2	9	5	1					
Breakfast side	Fruit	Protein	Starch							
$\lvert S(l)\rvert$	4	9	3							
Appetizer	Fried	Protein	Vegetable							
$\lvert S(l)\rvert$	3	7	6							
Salad ($\lvert S(l)\rvert$)	8									

Table 3. The Model Parameters Used in the Sample Applications

			Course (l)							
Constraint	Parameter	Meal (j)	Breakfast	Breakfast side	Soup	Salad	Main	Side	Appetizer	Dessert
Meal constraints	$ns_{j,l}$	Breakfast	2	2	0	0	0	0	0	0
Meal constraints	$ns_{j,l}$	Lunch	0	0	2	1	2	3	0	1
Meal constraints	$ns_{j,l}$	Dinner	0	0	2	1	2	3	0	1
Diet types	$np_{p,j,l}$*	Breakfast	1	1	0	0	0	0	0	0
Diet types	$np_{p,j,l}$*	Lunch	0	0	1	1	1	2	0	1
Diet types	$np_{p,j,l}$*	Dinner	0	0	1	1	1	2	0	1
Maximum servings of items in OWPs	$nsOWP_l$		2	3	2	4	1	3	0	2
Day item repetition constraints	nsd_l		1	1	1	1	1			1
	wp_l		0.5	0.2	0.5	0.5	1.0	0.2	0.4	0.5
	wc_l		0.4	0.2	0.3	0.6	1.0	0.2	0.6	0.4

*Identical for all diet type p.

general population of patrons with different dietary preferences. Patrons with special diet requirements because of chronic health conditions were excluded from diet types because they directly work with dietitians to set their diets, and their needs can be added to a menu. Overall, diet types can be determined based on gender, age, or dietary preferences. In our interviews, the CCF administrators and menu planners indicated that they do not prefer crafting menus for specific diets (e.g., Mediterranean-style diet, low-carb diet, etc.). Instead, menu planners try to include items that satisfy some diet types (e.g., vegetarian), which can be included in the model as constraints. The partner CCF's decision of not offering different menus catered toward specific diet types is mainly dictated by their operational limitations and cost concerns of segmenting their menus within the limits of the number of dishes served in a meal. Therefore, the modeling framework utilizes a data-driven approach to extract pseudo diet types, which represent the preferences of many diet types using a few groups. Such a higher-level grouping of diet types aims to address the challenges of satisfying the tastes of different groups using only a limited number of items served in each meal.

In the first step of identifying pseudo diet types, principal components analysis (PCA) was used to reduce the dimensions of the collected data. Using PCA, the preference ratings of all items by 28 patrons were analyzed and five factors were identified. Then, these five factors were used as clustering dimensions in a clustering algorithm to assign patrons to the three clusters (pseudo diet types). Finally, the average preference score $\pi'_{p,i}$ of item i by diet type p was calculated by averaging preference scores over patrons assigned to diet type p. The final preference score $\pi_{p,i}$ was calculated as $\pi_{p,i} = wp_{l(i)}\pi'_{p,i}$, where $wp_{l(i)}$ is the weight of course l in a meal and $l(i)$ denotes the course of item i to account for the importance of a course in a meal (e.g., a main dish is weighed more than a side dish).

We did not have access to the cost data for most of the menu items. Hence, the menu items were ranked on a continuum from three (the most expensive) to one (the most economical) by two experts within each course category and then multiplied by a scaling weight as $c_i = wc_{l(i)}c'_i$, where c'_i is the average cost rating of item i by the experts and $wc_{l(i)}$ is the cost weight of course l. The values of parameters wp_l and wc_l are given in Table 3, which summarizes the parameters of the menu construction and diversification constraints used in the following applications. The daily menu structure is based on the current menu of a target CCF. The detailed constraints are available in Appendix A.

Solving the MIP Model

With 381 menu items and three different diet types, the resulting MIP model was quite large to be optimally solved. In practice, the menu planners will use the menus suggested by the MIP model as a starting point. Therefore, the model's primary goals are to generate menus that satisfy the HEG constraints, incorporate menu planners' menu structure and diversity requirements, follow guidelines and regulations as much as possible, and maximize different patron preferences. Although incorporating all these factors and requirements in the model results in an extensive MIP model that is difficult to optimize, generating menus that satisfy all the constraints takes precedence over finding an optimal one. The modeling framework has been motivated by the lack of a toolbox that can consider all these factors. When the MIP model was solved using CPLEX, feasible solutions within a 10% optimality gap were found quickly; but the branch and bound search stalled after several feasible solutions. Therefore, as introduced in Appendix B, an iterative greedy heuristic (IGH) was developed as follows.

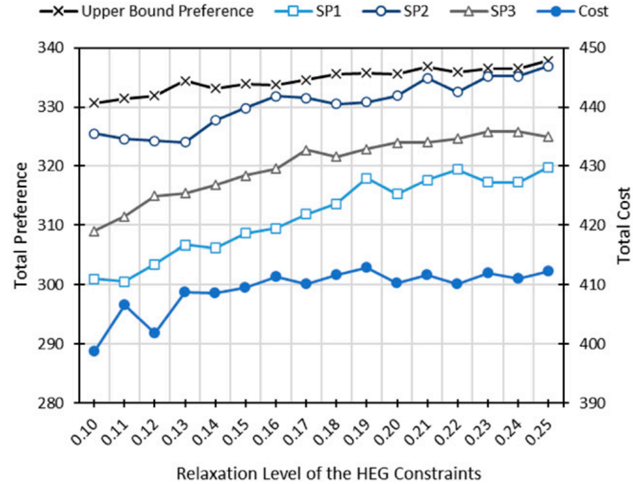
First, an initial feasible solution is found using the branch and bound in CPLEX within an allowed CPU time. Then, the feasible solution is improved by solving the MIP model only for randomly selected two successive days ($WD = 2$) after fixing all binary variables related to other days. In other words, the IGH aims to improve the best feasible solution by solving smaller, more manageable problems.

Application: Balancing Patron Preferences and Nutritional Requirements

According to the Food and Agriculture Organization of the United Nations, whereas the average American consumes more than 3,700 calories daily (FAO 2018), it is challenging for menu planners to craft menus that satisfy the preferences of patrons and follow the HEGs at the same time. Typically, the most popular menu items tend to have a high level of calories and fat or sugar content, causing a trade-off between these objectives. Therefore, menu planners should balance the satisfaction of patrons and the HEGs while crafting their menu. This analysis presents a scenario in which menu planners investigate the trade-offs between patrons' total preference scores and the level of violations in nutritional requirements specified in the HEGs of the U.S. Food and Drug Administration (Table 1). Discovering such relationships provides menu designers with valuable information on how to improve their current menu structures (e.g., the number of items to be served in a course) and how to modify individual menu items (e.g., reducing portion sizes or recipes). The trade-offs between the total preference scores and the HEG violations were discovered by setting the upper bound of the $Q_{p,k,t}$ variables to values ranging from 0.1–0.25 and solving the MIP model to maximize the weighted sum of the total preference scores of the three pseudo diet types, which were extracted from the patron preferences ($P = \{SP1, SP2, SP3\}$ with respective weights {0.21, 0.29, 0.50}). For SP2, the HEGs suggested for women were adopted because this group had relatively more women than men, and the other two groups used the HEGs for men. As discussed earlier, maximizing the total preference score was selected as the objective to mimic the actual behavior of patrons because they would be more likely to pick their favorite items on the menu rather than considering their nutritional intake. The details of the IGH and the objective function and performance metrics are given in Appendix B.

The cost metric was not considered an objective in this application; but the cost constraints discussed previously were included in the model, and the model output included the total cost metric. Figure 4 presents the upper bound of the total preference score of the best initial solution found by CPLEX and the total preference scores of pseudo diet types, SP1, SP2, and SP3, found by the IGH for different levels of relaxation in the HEG constraints. These final results had a 4% optimality gap on average. The results indicate that the preference values were improved for all diet types as the HEG constraints were relaxed, indicating a strict trade-off between the total preference scores and HEG constraint violations. A feasible menu was not found for less than 10% relaxation of the HEG constraints. Although the results given in Figure 4 were expected from an optimization perspective, they provided significant insight for improving the structure of menus or individual items from a decision analysis perspective. For example, the total preference scores of the three diet types were optimized simultaneously, although the menu included only two main dishes per meal. On the other hand, the improvement in the preference objective continued linearly even after 20% relaxation despite our expectation that the improvement in the total preference score would be marginal after a certain level of relaxation in the HEG constraints. These observations suggested that the generated menus did not satisfy patrons' expectations even after considerable relaxation of the HEG constraints, indicating that more fundamental changes were needed, such as modifying the current menu structure or the items. Figure 4 also provides the total cost of the final menus. Allowing higher violations in the HEG requirements enabled the inclusion of more expensive items with higher calories on the menu. However, the total cost levels stabilized after the relaxation level of 15%.

Figure 4. (Color online) The Upper Bound on the Total Preference Score of the Best Initial Solution, the Total Preference Scores of Pseudo Diet Types, SP1, SP2, and SP3, and the Total Cost for Different Levels of Relaxation in the HEG Constraints



Note. $dw_p = \{0.21, 0.29, 0.50\}$ for SP1, SP2, and SP3, respectively.

Application: Analyzing Menu Structure Decisions and Menu Improvement Interventions

The proposed modeling framework can be used to analyze alternative menu improvement decisions from different perspectives. For example, increasing the number of main dishes in a meal could improve the overall menu satisfaction without compromising the HEG requirements. Before implementing this change, however, the menu planners would like to investigate whether the expected improvements in overall menu satisfaction are justified considering current operational and cost limitations. In addition, the model's output can provide a target for how to modify meals or individual items. For example, the model could not find any feasible solution for less than 10% relaxation of the HEG constraints. This outcome suggested that menu improvement strategies, such as portion sizing or slight changes in item recipes, should be considered without significantly reducing patrons' overall satisfaction. The detailed analysis of the violations in the HEG constraints showed that sodium intake and calories from added sugar were the strictest constraints of the model, reaching their upper relaxation bounds every day during the planning horizon. For example, Figure 5 illustrates the sodium intake and calories from added sugar of diet type SP1 for the case with 10% relaxation. As seen in the figure, the deviations from the HEG requirement are very close to the 10% relaxation upper bound almost daily. Based on these results, another menu enhancement decision could be to revise items to address high sodium and sugar content levels. Therefore, we tested two strategies to demonstrate the use of the modeling framework in the analysis of menu enhancement interventions: (i) adding a third main dish at dinner and (ii) reducing the sodium content of items with more than 300 mg by 25% (red. sodium) and portion sizes of desserts with more than 250 kcal per serving by 20%. It was assumed that these two strategies would not change the cost and preference of the individual items.

Figure 6 compares these two menu enhancement interventions with the menus optimized previously. Reducing sodium levels of high-sodium items and portion size of some heavy desserts provided significant improvements, considering that the baseline preference score was about 300. In addition, the improvement in the preference scores became marginal with the increasing level of relaxation in the HEG constraints, suggesting that patrons were becoming satisfied with the items included in the menu, unlike the results in Figure 4. In fact, such a level of improvement was not expected because the changes in the items were minor and only a small subset of the items was modified. This result suggested that the HEG constraints were difficult to satisfy with the existing menu items of the target CCF. The proposed modifications to a subset of items were straightforward to implement and enabled designing menus to increase the satisfaction of the different diet types and incorporate the requirements of menu planners, chefs, and dietitians under strict HEG constraints. Adding a third main dish also improved the total preference score by reducing the level of preference conflicts among the diet types (see Figure 6). However, this strategy also increased the cost, and the improvements were insignificant compared with small changes in the menu items.

While interpreting the aforementioned results, the underlying assumptions and limitations of the modeling framework should be considered. Most importantly, the model assumed that the patrons would pick and consume all available items, and the other sources of nutrients were not considered. In addition, the menu planners' requirements and menu diversification constraints were implemented as hard constraints, which made the problem very constrained and increased its size. Another limitation of the model

Figure 5. (Color online) Final Decision Variable Values Related to the Relaxation of the HEG Sodium Intake and Calories from Added Sugars Constraints for SP1 Under a 10% Relaxation

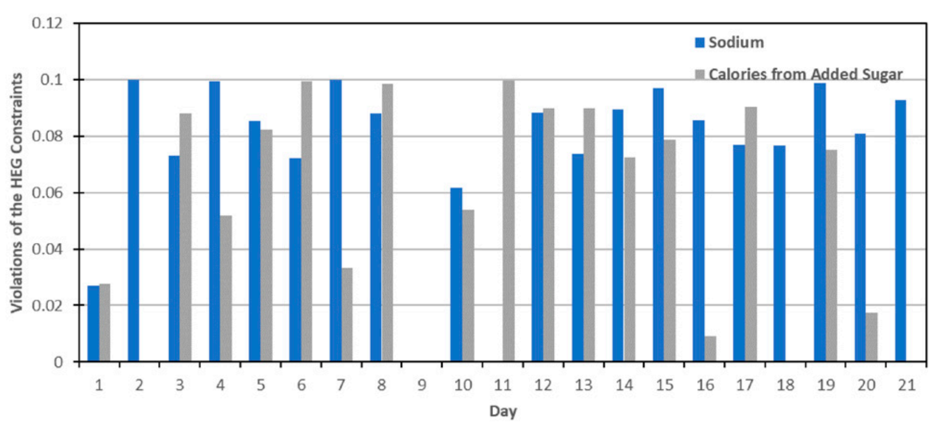
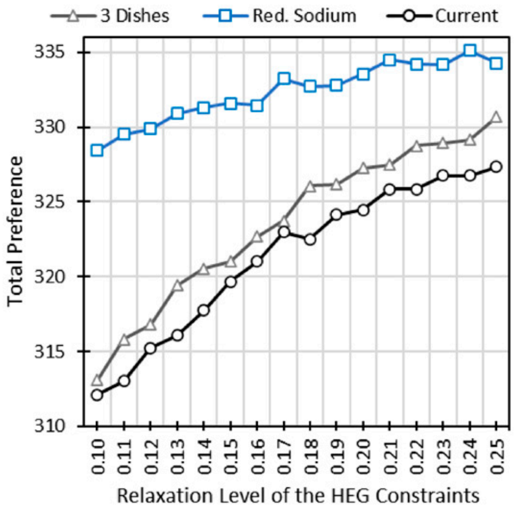


Figure 6. (Color online) Comparison of Menu Improvement Interventions of Three Dishes (Serving Three Main Dishes in Dinners) and Red. Sodium (i.e., Reduced Sodium and 20% Smaller Portions of Heavy Desserts) over the Current Menu Structure



is the computational complexity. The IGH took about an average of 32,522 and 9,414 seconds to find the final menu for the Current and Reduced Sodium strategies, respectively, on the Pennsylvania State University's Institute for Computational and Data Sciences' Roar supercomputer. The final solutions had optimality gaps between 3% and 5%. Despite these limitations, the model's output was helpful in several ways. For example, menu planners were able to analyze the complex trade-offs between HEG requirements and conflicting preference scores of various diet types. In addition, the model's output provided insight on how to improve individual items by identifying targets and menu items to modify through interventions, such as portion sizing and slight changes in recipes, without significantly reducing the preference scores of diet types.

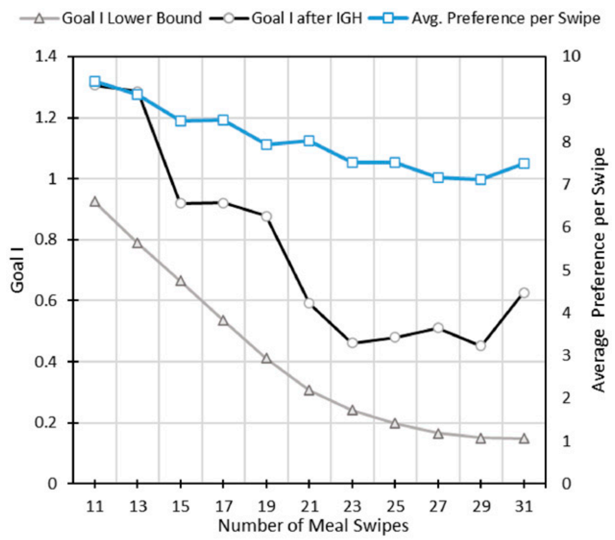
Application: Analyzing the Number of Meal Swipe Decisions and Trade-offs

Some CCF have meal packages (with associated prices) that limit the number of meals patrons can have during the planning cycle. These meal packages usually include breakfast and a certain number of lunches or dinners. The proposed modeling framework can be revised to design menus considering a meal package in which patrons can have up to a certain number of meals within a planning period. Let nm_p be the number of lunch and dinner swipes for diet type p. Determining the best number of meal swipes is a challenging task because of the irregular times patrons choose to eat; thus, even after producing optimal menus, there might still be significant nutrient deficiencies. Therefore, menu planners are concerned with determining the value of nm_p along with daily menu items to ensure that patrons can get their required nutrients as much as possible without relying on other sources. A strategy for addressing this concern is to design a menu such that the nutrient intakes of patrons deviate from the healthy eating guidelines at a minimum possible level at any point during the planning cycle. For a given nm_p value for each patron type p, this strategy can be implemented by placing patrons' favorite items on the menu in a way that patrons are enticed to use their meal swipes without a large gap between swipes. Such a strategy will also maintain the feeling of autonomy compared with restricting patrons from using their swipes on certain meals. Planning a menu with limited swipes requires analyzing trade-offs between the number of meal swipes, violations in the HEG constraints, and considering preferences of the different diet types. The proposed modeling framework can be used to plan menus considering such trade-offs by including the decision variable $u_{p,j,t}$ to indicate whether patron p uses a swipe for lunch or dinner meal j on day t or not and with minor modifications in the constraints (Appendix A).

We used a preemptive goal programming approach with two stages (Appendix C) to answer the aforementioned questions and discover the trade-off. In Stage I, the problem was solved to minimize only the maximum total normalized violations of the HEG constraints in any period of three consecutive days during the planning cycle (Goal I). The target in this stage was zero violation. The output of this stage provided a feasible bound for Goal I and roughly determined when patrons should use their meal swipes. In Stage II, the normalized deviation from the target preferences (Goal II) was minimized without degrading the objective value found in Stage I. The output of this stage aimed to place items that patrons prefer to have on the menu according to the bound determined in Stage I. Because meal swipes were only used for lunches and dinners, to account for the nutrients taken in breakfast, it was assumed that individuals could pick one from essential breakfast items (oatmeal, pancakes, or omelets and vanilla yogurt, hash browns, fruit cup, and potatoes); all breakfast diversification constraints and budget constraints were removed from the model in the analyses. In addition, a smaller database of 182 items, which appeared in the menus optimized in the previous application, was used in this analysis.

Figure 7 illustrates the attainment of Goal I after the IGH, the theoretical lower bound of Goal I for different numbers of meal swipes, and the average preference score of the pseudo diet types per swipe. The IGH improved Goal I significantly with the increasing meal swipes from 11 to 23 but failed to do so after 24 swipes. The average preference scores per used swipe

Figure 7. (Color online) Goal I Lower Bound, Attainment of Goal I After the Greedy Iterative Search (Goal I After IGH), and the Average Preference of the Pseudo Diet Types per Meal Swipe



were reduced steadily from 11–25 swipes and became stable after 25 swipes for all diet types. These results indicate that in Stage II, the MIP model prioritized the items the patrons preferred the most when the patrons had a limited number of swipes and the patrons selected those items. With the increasing number of swipes, the items' marginal contributions to the total preference score were also diminished as items with lower preferences were included in the menu because of the menu diversification constraints limiting the items' repetition. In summary, this application demonstrates the challenges in planning menus for meal plans that allow a limited number of meal swipes. Menus prepared by the proposed two-stage goal programming approach may motivate patrons to use their swipes voluntarily without restricting them to specific days or meals. The proposed modeling framework can also help menu planners to discover complex trade-offs in menu design and answer what-if questions regarding how to structure the menus under a meal plan.

Lessons Learned

Although populations have become more diverse in their food preferences, small-sized CCFs lack the resources to satisfy the needs of many different diet types. This study showed the challenges of incorporating the points of view of different stakeholders in the menu design for CCFs. Menu planners and chefs are concerned that menus should include some design patterns, which they mastered through training or experience while ensuring that the resulting menus are diverse and satisfy the needs of patrons with various dietary preferences. They focus on dependencies among the items served and aim to provide exciting offerings uniformly throughout a menu cycle. Therefore, it is essential to include this satisficing approach to decision making in optimization models. In particular, the chefs and menu planners found the proposed OWP approach, which was used to create uniformly diverse menus over the planning cycle, helpful in evaluating the assortment quality of their menus. Combined with the OWP approach, the concept of pseudo diet types aims to design nutritionally balanced menus that address the needs of increasingly diverse patrons using a limited number of menu options, which is an increasing challenge that these CCFs face. The concept of pseudo diet types is particularly important for organizations that offer limited meal options, such as K–12 cafeterias.

The results also show that using hard constraints to model dietary requirements may not be the best approach in the absence of accurate and complete data. The slight relaxation of these constraints or minor changes in menu items can lead to significantly different menus. Therefore, stakeholders can use the optimization models as scenario analysis tools to study the structure of their menu or items, and chefs can use the generated menus as a starting point of their planning process rather than implementing the menus as they are. In our interviews, menu planners also suggested iteratively using the model to revise the optimized menus by adding new parameters or constraints and quickly calculating the performance metrics of the reoptimized menus. Hence, the constraints were parameterized to facilitate the process of the running model with different constraints and menu structures. This enables embedding the model into an easy-to-use decision support system. Considering the limited resources of CCFs, this decision support system can help establish one uniform and effective system across multiple CCFs at the corporate level.

The computational results in this study showed that minimizing the HEG constraint violations and maximizing the preference scores of patrons with different dietary patterns is a challenging problem for the data set collected from the target CCF. It was shown that the menu items in their current state were a reason for this contradiction. Although this result is specific to the data set used in this study, there seems to be a need to make individual menu items healthier in CCFs to create more appealing menus that can better follow the HEG suggestions.

Conclusion

We proposed, designed, implemented, and evaluated a menu engineering system that could determine the optimal combination of menu items to be included in each meal of a multiweek cycle menu. The system ensures that the dietary, nutrition, operational, financial, and customer experience constraints specified by the interdisciplinary group of executives are adequately accounted for. Such automated, comprehensive, versatile, data-driven, multiobjective decision support systems are essential for smooth dining operations of CCFs that are becoming increasingly popular as the population (here in the United States and abroad) becomes older and those aged 65 years and above are reliant on the CCF dining facilities for all their meals. The proposed system aims to assist menu planners not only in crafting their daily menus but also in analyzing their decisions about menu structures, evaluating menu improvement interventions, and identifying possible improvements in their menus or menu items. The applications demonstrated how menu planners could use the proposed modeling framework to analyze menu improvement decisions based on multiple objectives. The modeling framework utilizes various approaches to incorporate different stakeholders' perspectives in the decision processes, including a satisficing approach to represent the menu diversification goals of chefs and goal programming to prioritize nutritionists' point of view while ensuring autonomy of patrons. The proposed system could also be adapted to meet the needs of other dining operations with captive patrons, such as penitentiaries, schools and colleges, humanitarian operations, and remote operations with a residential workforce.

Appendix A. Description of the Modeling Framework

Model Parameters

T set of days in the planning horizon

W the number of days in an overlapping window pattern for menu diversification

I set of all menu items

L set of meal courses—for example, soup, salad, main dish, etc.

$l(i)$ the course type of item i

$I(l)$ set of items that can be served in a meal course $l \in L$

$S(l)$ set of subcategories of a meal course $l \in L$

$S'(l)$ a given subset of $S(l)$

$I(l, s)$ set of subcategory $s \in S(l)$ items that can be served in course l

M set of meals, that is, {"Breakfast," "Lunch," "Dinner"}

M' set of meals excluding breakfast—that is, {"Lunch," "Dinner"}

P set of individuals with different dietary requirements and/or preferences

$ns_{j,l}$ the number of menu items that should be included in a course l of a meal j

$np_{p,j,l}$ the number of items that a patron p can choose in a course l of a meal j

$mns_{j,S'(l)}$ the minimum number of items that should be included in meal j from $S'(l)$

$nsT_i^{\min}$ the minimum number of times that a menu item i is served within T

$nsT_i^{\max}$ the maximum number of times that a menu item i is served within T

$nsOWP_l$ number of times that an item in course l can repeat within each OWP

$nc_{l,s}^{\min}$ the minimum number of times that an item in subcategory s of course l should be served within an OWP

$nc_{l,s}^{\max}$ the maximum number of times that an item in subcategory s of course l should be served within an OWP

G_i the minimum gap between two servings of item i

nsd_l the number of times that an item can be served in the same day

K set of nutrients

$\omega_{k,i}$ nutritional value or amount of nutrient k in item i

c_i the relative cost of item i within its course

wc_l the relative weight of course l in cost calculations

$\pi_{p,i}$ the average normalized relative preference of item i by diet type p within its course

wp_l the weight of course l while calculating the total preference

$\alpha_{p,k}^{max}$ recommended upper limit for daily intake of nutrient k for diet type p

$\alpha_{p,k}^{min}$ recommended lower limit for daily intake of nutrient k for diet type p

$Q_p^{\max}$ maximum allowed deviation from the healthy eating guidelines for diet type p

Decision Variables

$x_{i,j,t}$ binary variable indicating whether item i is served in meal j on day t ($x_{i,j,t} = 1$) or not ($x_{i,j,t} = 0$)

$y_{p,i,j,t}$ binary variable indicating whether individual p picks item i in meal j on day t ($y_{p,i,j,t} = 1$) or not ($y_{p,i,j,t} = 0$)

$Q_{p,k,t}$ the violation of the constraints related to nutrient k for individual p on day t

Performance Metrics

Deviations from the HEGs. For diet type p, the total deviation from the target given in the HEG requirements is shown as follows:

$$F_{Q,p} = \sum_{k \in K} \sum_{t \in T} Q_{p,k,t}. \tag{A.1}$$

Total Preference Score. The total preference score of a menu for diet type p is given as follows:

$$F_{\pi,p} = \sum_{j \in M} \sum_{i \in I} \sum_{t \in T} \pi_{p,i} y_{p,i,j,t}, \tag{A.2}$$

where $\pi_{p,i}$ is the weighted preference score of item i by diet type p,

$$\pi_{p,i} = wp_{l(i)} \pi'_{p,i} \quad \forall p \in P, i \in I \tag{A.3}$$

such that $wp_{l(i)}$ is the weight of course l in a meal, and $l(i)$ denotes the course of item i. Finally, the total preference score is given as follows:

$$F_\pi = \sum_{p \in P} F_{\pi,p}. \tag{A.4}$$

Total Cost. The total cost within the planning horizon as follows:

$$F_c = \sum_{j \in M} \sum_{i \in I} \sum_{t \in T} c_i x_{i,j,t}. \tag{A.5}$$

Constraint Details

Menu Structure Constraints

Item Inclusion. In each day t, the daily menus are constructed by selecting a predefined number of items ($ns_{j,l}$) to include in course l of meal j as follows:

$$\sum_{i \in I(l)} x_{i,j,t} = ns_{j,l} \quad \forall j \in M, l \in L, t \in T, \tag{A.6}$$

where set $I(l)$ denotes the set of items that can be served in course l.

Maximum Number of Items per Subcategory. Let set $S'(l)$ be a given subset of $S(l)$ and $I(l,s)$ denote the set of subcategory s items that can be served in course l. The following constraint ensures that at most one item from each subcategory in $S'(l)$ will be included in a meal j on a day t as follows:

$$\sum_{i \in I(l,s)} x_{i,j,t} \le 1 \; \forall j \in M, l \in L, s \in S'(l), t \in T : ns_{j,l} \ge 1. \tag{A.7}$$

Minimum Number of Items in a Subcategory. A meal j may require including a minimum number of items ($mns_{j,S'(l)}$) from a given subset $S'(l)$ of subcategory set $S(l)$ as follows:

$$\sum_{s \in S'(l)} \sum_{i \in I(l,s)} x_{i,j,t} \ge mns_{j,S'(l)} \quad \forall j \in M, l \in L, t \in T. \tag{A.8}$$

Dependency Constraints

Course Subcategory Dependencies. The following constraints allow serving only one item for a given $S'(l) \subseteq S(l)$ when a meal j includes multiple items of a course l.

$$\sum_{s \in S'(l)} \sum_{i \in I(l,s)} x_{i,j,t} \le 1 \quad \forall j \in M, l \in L, t \in T : ns_{j,l} \ge 2 \tag{A.9}$$

Item Dependencies. Given subcategory subsets $S'(l')$ and $S''(l'')$ of courses l' and l'', the following constraints require serving an item from $S''(l'')$ if an item from $S'(l')$ is served.

$$\sum_{s \in S'(l')} \sum_{i \in I(l',s)} x_{i,j,t} \le \sum_{s \in S''(l'')} \sum_{i \in I(l'',s)} x_{i,j,t} \quad \forall j \in M, l \in L, t \in T \tag{A.10}$$

Menu Diversification Constraints

Maximum/Minimum Number of Servings of Items in a Menu Cycle. The following constraints set lower ($nsT_i^{\min}$) and upper ($nsT_i^{\max}$) bounds for the number of times that an item i is served within the planning horizon.

$$\sum_{j \in M, t \in T} x_{i,j,t} \ge nsT_i^{\min} \quad \forall i \in I \tag{A.11}$$

$$\sum_{j \in M, t \in T} x_{i,j,t} \le nsT_i^{\max} \quad \forall i \in I \tag{A.12}$$

Maximum Number of Servings of Items in OWPs. The following menu diversification constraints limit the number of times that an item can repeat in each OWP to an upper bound of $nsOWP_l$, which depends on the course type of the item, as follows:

$$\sum_{j \in M, c \in \{1, \ldots, W\}} x_{i,j,nw(t,T,c-1)} \le nsOWP_l \; \forall l \in L, i \in I(l), t \in T, \tag{A.13}$$

where function $nw(t,T,c)$ returns the cth index after index t in T, wrapping around to the beginning of T. For example, given $T = \{1, \ldots ,21\}$, $nw(20, T, c-1)$ will return 20, 21, 1, and 2 for $c = 1$, 2, 3, and 4, respectively. Thereby, the constraint accounts for the variability of items at the beginning and end of the menu cycle and ensures the diversification of menu items over the entire planning cycle uniformly.

Maximum/Minimum Number of Course Subcategories in OWPs. These constraints ensure that a minimum ($nc_{l,s}^{\min}$) and maximum ($nc_{l,s}^{\max}$) number of items from a subcategory s of a meal course l are served in each OWP. For a given l and $S'(l)$, such requirements can be expressed as follows:

$$\sum_{i \in I(l,s)} \sum_{c \in \{1, \ldots, W\}} x_{i,j,nw(t,T,c-1)} \ge nc_{l,s}^{\min} \quad s \in S'(l), t \in T, \tag{A.14}$$

$$\sum_{i \in I(l,s)} \sum_{c \in \{1, \ldots, W\}} x_{i,j,nw(t,T,c-1)} \le nc_{l,s}^{\max} \quad s \in S'(l), t \in T. \tag{A.15}$$

Limiting Consecutive Repetitions of Items. These constraints impose a gap of G_i days between two consecutive servings of item i as follows:

$$\sum_{j \in M} \sum_{c \in \{1, \ldots, G_i\}} x_{i,j,nw(t+c-G_i,T,1)} \le 1 - \sum_{j \in M} x_{i,j,t-G_i} \quad \forall i \in I, t \in T : t > G_i > 0. \tag{A.16}$$

Item Repetition Constraints Within a Day. These constraints limit the number of times that an item can be served during a day as follows:

$$\sum_{j \in M} x_{i,j,t} \le nsd_l \quad \forall l \in L, i \in I(l), t \in T. \tag{A.17}$$

Diet Types. The following constraints indicate that a patron type p can choose only the items that are included in meal j on day t.

$$y_{p,i,j,t} \le x_{i,j,t} \quad \forall p \in P, j \in M, i \in I, t \in T \tag{A.18}$$

A patron type p is allowed to choose $np_{p,j,l}$ number of items of course l in meal j.

$$\sum_{i \in I(l)} y_{p,i,j,t} = np_{p,j,l} \quad \forall p \in P, j \in M, l \in L, t \in T : ns_{j,l} \ge 1 \tag{A.19}$$

HEG Constraints. The constraints regarding dietary guideline recommendations for the nutritional goals are given as follows:

$$\sum_{j \in M} \sum_{i \in I} \omega_{k,i} y_{p,i,j,t} \le \alpha_{p,k}^{\max} + \gamma_{p,k}^{\max} Q_{p,k,t} \quad \forall p \in P, k \in K, t \in T, \tag{A.20}$$

$$\sum_{j\in M}\sum_{i\in I}\omega_{k,i}y_{p,i,j,t} \geq \alpha^{\min}_{p,k} - \gamma^{\min}_{p,k}Q_{p,k,t} \qquad \forall p\in P, k\in K, t\in T. \tag{A.21}$$

In the aforementioned inequalities, the descriptions of parameters $\omega_{k,i}$, $\alpha^{\max}_{p,k}$, $\alpha^{\min}_{p,k}$, $\gamma^{\max}_{p,k}$, and $\gamma^{\min}_{p,k}$ depend on nutrient k as the healthy eating guidelines specify nutritional limits in terms of either absolute targets or percentage of the total calories or both. Therefore, in the constraints related to the limits on the percentage of total calories, parameters $\alpha^{\max}_{p,k}$, $\alpha^{\min}_{p,k}$, $\gamma^{\max}_{p,k}$, and $\gamma^{\min}_{p,k}$ represent the fraction of the total calories taken in a day, which is calculated as

$$TC_{p,t} = \sum_{j\in M}\sum_{i\in I}\omega_{1,i}y_{p,i,j,t}. \tag{A.22}$$

Finally, the deviations from the HEGs are capped by an upper bound as follows:

$$Q_{p,k,t} \leq Q^{\max}_{p} \qquad \forall p\in P, k\in K, t\in T. \tag{A.23}$$

Main dish constraints (l = "main" for all constraints):

• A maximum of one item from each main dish subcategory can be served in a meal (Constraint (A.7) with $S'(l) = S(l)$).

• Beef and pork main dishes cannot be served together in the same meal (Constraint (A.9) with $S'(l)$ = {"beef." "pork"}).

• Fish or shellfish main dishes cannot be served together in the same meal (Constraint (A.9) with $S'(l)$ = {"fish," "shellfish"}).

• Grain and pasta main dishes cannot be served together in the same meal (Constraint (A.9) with $S'(l)$ = {"grain," "pasta"}.

• A main dish can be served a maximum of two times in T (Constraint (A.12) with $nsT^{\max}_i = 2\ \forall i \in I(l)$).

• Each main dish subcategory must be served two or more times in an OWP (Constraint (A.14) with $nc^{\min}_{l,s} = 2$ and $S'(l) = S(l)$).

• A main dish subcategory can be served a maximum of 10 times in an OWP (Constraint (A.15) with $nc^{\max}_{l,s} = 10$ and $S'(l) = S(l)$).

• There must be a day gap between two servings of the same main dish (Constraint (A.16) with $G_i = 1\ \forall i \in I(l)$).

Side dish constraints:

• A maximum of one same subcategory of side dish can be served in a meal ((Constraint (A.7) with l = "side" and $S'(l) = S(l)$).

• At least one dark green, red/orange, or starchy vegetable side dish must be served in a meal (Constraint (A.8) with l = "side" and $S'(l)$ = {"dark green," "red orange," "starch"}, $mns_{j,S'(l)} = 1$).

• If a main dish of type beef, poultry, and pork is served, then a starch side dish must be served (Constraint (A.10) with l' = {"main"}, $S'(l')$ = {"beef," "poultry," "pork"}, l'' = {"side"}, $S''(l'')$ = {"starch"}).

• If a vegetable main dish is served, then a grain side must be served (Constraint (A.10) with l' = {"main"}, $S'(l')$ = {"vegetable"}, l'' = {"side"}, $S''(l'')$ = {"grain"}).

Soup:

• A maximum of one same subcategory of soups be served in a meal (Constraint (A.7) with l = "soup" and $S'(l) = S(l)$).

• An OWP must include one item from each soup subcategory (Constraint (A.14) with l = "soup" $nc^{\min}_{l,s} = 1$, and $S'(l) = S(l)$).

Dessert (l = "dessert"):

• An OWP must include a pie and a cheesecake (Constraint (A.14) with $nc^{\min}_{l,s} = 1$ for $S'(l)$ = {"cake," "cheesecake"}, and $nc^{\min}_{l,s} = 0$, otherwise).

Breakfast/breakfast side:

• At most one item from each subcategory of breakfast items can be served in a breakfast (Constraint (A.7) with l = "breakfast" and $S'(l) = S(l)$).

• At most one item from each subcategory of breakfast side items can be served (Constraint (A.7) with l = "breakfast side" and $S'(l) = S(l)$).

• If a batter or cereal item is served, then a fruit breakfast side must be served (Constraint (A.10) with l' = {"breakfast"}, $S'(l')$ = {"batter," "cereal"}, l'' = {"breakfast side"}, $S''(l'')$ = {"fruit"}).

• If an egg or meat item is served, then a starch breakfast side must be served (Constraint (A.10) with l' = {"breakfast"}, $S'(l')$ = {"egg", "breakfast"}, l'' = {"breakfast side"}, $S''(l'')$ = {"starch"}).

Cost:

• The total cost of breakfast must be less than 20% of the daily menu cost.

• The total cost of lunch must be less than 40% of the daily menu cost.

• The total cost of dinner must be between 40% and 70% of the daily menu cost.

Appendix B. Solving the MIP Model Using an Iterative Greedy Heuristic

Objective function

$$\max F_{\pi} = \sum_{p\in P} dw_p F_{\pi,p}, \tag{B.1}$$

where dw_p represents the relative weight of diet type p, which can be calculated based on the percentage of patrons with diet type p.

Procedure Iterative Greedy Heuristic (IGH)(){
Use CPLEX to find an initial feasible solution within a given CPU time
Let $x^{**}_{i,j,t}$ and $y^{**}_{p,i,j,t}$ represent the values of the decision variables in the feasible solution
Let F^{**} be the objective function value of the feasible solution
While (F^{**} is improved) {
  Let RD be a random list of planning days
  For $d = RD[1], \ldots, RD[T]$ {
    Fix $x_{i,j,t} = x^{**}_{i,j,t}\ \ \forall i\in I,\ j\in M, t\in T$
    Fix $y_{p,i,j,t} = y^{**}_{p,i,j,t}\ \ \forall p\in P,\ i\in I,\ j\in M, t\in T$
    For $c = 1, \ldots, WD${
      Unfix $x_{i,j,nw(d,T,c-1)}\ \ \forall i\in I,\ j\in M$
      Unfix $y_{p,i,j,nw(d,T,c-1)}\ \ \forall p\in P,\ i\in I,\ j\in M$
    }
    Solve the model optimality to find a new solution
    Let F^{*}, $x^{*}_{i,j,t}$, and $y^{*}_{p,i,j,t}$ be the objective and variable values of the new solution
    If $F^{*} < F^{**}$ **Then** {
      For $c = 1, \ldots, WD${

Set $x^{**}_{i,j,x_{i,j,nw(d,T,c-1)}} = x^{*}_{i,j,x_{i,j,nw(d,T,c-1)}}$
$\forall i \in I,\ j \in M$
Set $y^{**}_{p,i,j,nw(d,T,c-1)} = y^{*}_{p,i,j,nw(d,T,c-1)}$
$\forall p \in P, i \in I,\ j \in M$
Set $F^{**} = F^{*}$
}
}
}
}
}
}

Appendix C. Goal Programming Approach to Discover Trade-offs Among the Number of Meal Swipes, Violations in the HEG Constraints, and Patron Preferences

Additional Notation

nm_p the number of lunch and dinner swipes for diet type p

$u_{p,j,t}$ binary decision variable to indicate whether patron p uses a swipe for lunch or dinner meal j on day t or not

$Q^{\max}_p$ the maximum total weighted violation of the constraints related to the HEG constraints in any period of consecutive WN days during the planning cycle

The following two constraints are added to the model to represent the number of meals that patrons can have in the planning horizon:

$$\sum_{i \in I} y_{p,i,j,t} \le M \times u_{p,j,t} \quad \forall p \in P, j \in M', l \in L, t \in T, \tag{C.1}$$

$$\sum_{j \in M'} \sum_{t \in T} u_{p,j,t} = nm_p \quad \forall p \in P. \tag{C.2}$$

The maximum violation $Q^{\max}_p$ can be expressed as an upper bound constraint in the model as follows:

$$\sum_{k \in K} \sum_{c \in \{1,\ldots,WN\}} wp_{p,k} Q_{p,k,nw(t,T,c-1)} \le Q^{\max}_p \quad \forall p \in P, t \in T, \tag{C.3}$$

where $wp_{p,k}$ is the weight of the penalty of the nutrient k for individual p. If the MIP model with additions of Constraints (C.1), (C.2), and (C.3) is solved for minimizing $\sum_{p \in P} Q^{\max}_p$ for a given nm_p and WN, the resulting $u_{p,j,t}$ variables will indicate when patrons should use their meal swipes to avoid extended periods of nutrient deficiencies. However, a menu planned only based on this objective assumes that patrons will use their swipes to prevent nutrient deficiency, which is unrealistic in practice because patrons are free to use their meal swipes at their will, and they are unlikely to use them to count their nutrient intakes. In reality, patrons make their decisions based on their preferences of the items available on the menu. Therefore, items should be strategically placed on a menu by considering the preferences of the patrons. To optimize these two objectives, a goal programming approach can be used with an objective function as follows:

$$\min\ \underbrace{w_Q \sum_{p \in P} \frac{Q^{\max}_p}{Q^G_p}}_{\text{Goal I}} + \underbrace{w_\pi \sum_{p \in P} \frac{(F^G_{\pi,p} - F_{\pi,p})}{F^G_{\pi,p}}}_{\text{Goal II}}, \tag{C.4}$$

where $F^G_{\pi,p}$ is the target value (300 in this study) of the total preference score for patron type p (note that the target value of the maximum total weighted violation of the HEG constraints is zero); Q^G_p is a coefficient to normalize the maximum total weighted violation between zero and one (two in this study); and w_Q and w_π are the positive weights that reflect the relative importance of these two objectives.

Instead of solving the problem for different values of w_Q and w_π, a preemptive approach with two stages is adopted as follows: In the first stage, the problem is solved to minimize only the maximum total weighted deviation from the HEG constraints (Goal I). The output of this stage provides a feasible bound for Goal I and roughly determines when patrons should use their meal swipes. In the second stage, the deviation from the target preferences (Goal II) is minimized without degrading the objective value found in the first stage. The brief procedure of the preemptive solution approach is as follows:

Procedure Preemptive Goal Programming () {
Add Constraints (C.1), (C.2), and (C.3) to the model
Use Equation (C.4) as the objective function
Stage I:
Set $w_Q = 1.0$ and $w_\pi = 0.0$
Solve the problem using the IGH
Let f^{Q^*} be the best objective function of Stage I
Stage II:
Add the following constraint to the model

$$\sum_{p \in P} \frac{Q^{\max}_p}{Q^G_p} \le f^{Q^*} \tag{C.5}$$

Set $w_Q = 0.0$ and $w_\pi = 1.0$
Solve the problem using CPLEX
}

References

Abbey KL, Wright OR, Capra S (2015) Menu planning in residential aged care—The level of choice and quality of planning of meals available to residents. *Nutrients* 7(9):7580–7592.

Agarwal E, Marshall S, Miller M, Isenring E (2016) Optimising nutrition in residential aged care: A narrative review. *Maturitas* 92:70–78.

Alexander GL, Madsen RW (2017) A report of information technology and health deficiencies in US nursing homes. *J. Patient Safety* 17(6):e483–e489.

Alexander GL, Madsen RW, Miller EL, Schaumberg MK, Holm AE, Alexander RL, Wise KK, Dougherty ML, Gugerty B (2017) A national report of nursing home information technology: Year 1 results. *J. Amer. Medical Inform. Assoc.* 24(1):67–73.

Anderson A, Earle M (1983) Diet planning in the third world by linear and goal programming. *J. Oper. Res. Soc.* 34(1):9–16.

Bell CL, Lee AS, Tamura BK (2015) Malnutrition in the nursing home. *Current Opinion Clinical Nutrition Metabolic Care* 18(1):17–23.

Benvenuti L, De Santis A (2020) Making a sustainable diet acceptable: An emerging programming model with applications to schools and nursing homes menus. *Frontiers Nutrition* 7:1–16.

Bourdel-Marchasson I (2010) How to improve nutritional support in geriatric institutions. *J. Amer. Medical Directors Assoc.* 11(1):13–20.

Chisholm A, Jensen J, Field P (2011) Eating environment in the aged-care residential setting in New Zealand: Promoters and barriers to achieving optimum nutrition. Observations of the foodservice, menu and meals. *Nutrition Dietetics* 68(2):161–166.

Colby S, Ortman JM (2015) Projections of the size and composition of the US population: 2014 to 2060. Accessed September 1, 2022, https://mronline.org/wp-content/uploads/2019/08/p25-1143.pdf.

Colombo PE, Patterson E, Lindroos AK, Parlesak A, Elinder LS (2020) Sustainable and acceptable school meals through optimization analysis: An intervention study. *Nutrition J.* 19(1):1–15.

Cowan DT, Roberts JD, Fitzpatrick JM, While AE, Baldwin J (2004) Nutritional status of older people in long term care settings: Current status and future directions. *Internat. J. Nursing Stud.* 41(3):225–237.

Dollahite J, Franklimns D, McNew R (1995) Problems encountered in meeting the recommended dietary allowances for menus designed according to the dietary guidelines for Americans. *J. Amer. Dietetic Assoc.* 95(3):341–347.

Espín V, Hurtado MV, Noguera M (2016) Nutrition for elder care: A nutritional semantic recommender system for the elderly. *Expert Systems* 33(2):201–210.

FAO (2018) *Food Balances* (Food and Agriculture Organization of the United Nations, Rome).

Ferguson EL, Darmon N, Briend A, Premachandra IM (2004) Food-based dietary guidelines can be developed and tested using linear programming analysis. *J. Nutrition* 134(4):951–957.

Ferguson EL, Darmon N, Fahmida U, Fitriyanti S, Harper TB, Premachandra IM (2006) Design of optimal food-based complementary feeding recommendations and identification of key "problem nutrients" using goal programming. *J. Nutrition* 136(9):2399–2404.

Gazan R, Brouzes CM, Vieux F, Maillot M, Lluch A, Darmon N (2018) Mathematical optimization to explore tomorrow's sustainable diets: A narrative review. *Adv. Nutrition* 9(5):602–616.

Harper C, Cullen P, Fodey K (2014) Nutritional guidelines and menu checklist for residential and nursing homes. Accessed September 1, 2022, https://www.efad.org/wp-content/uploads/2021/11/nutritional_guidlines_and_menu_checklist_march_2014.pdf.

Hui LS, Sufahani SF, Khalid K, Abd Wahab MH, Idrus SZS, Ahmad A, Subramaniam TS (2021) Menu scheduling for high blood pressure patient with optimization method through integer programming. *J. Phys. Conf. Ser.* 1874(1):1–8.

Ko M, Wagner L, Spetz J (2018) Nursing home implementation of health information technology: Review of the literature finds inadequate investment in preparation, infrastructure, and training. *INQUIRY: J. Healthcare Organ. Provision Financing* 55:0046958018778902.

Lam IT, Keller HH, Duizer L, Stark K (2015) Micronutrients on the menu: Enhancing the quality of food in long-term care for regular, nontherapeutic menus. *Canadian J. Dietetic Practice Res.* 76(2):86–92.

Leung P, Wanitprapha K, Quinn LA (1995) A recipe-based, diet-planning modelling system. *British J. Nutrition* 74(2):151–162.

Lützén K, Nordin C (1994) Modifying autonomy—A concept grounded in nurses' experiences of moral decision-making in psychiatric practice. *J. Medical Ethics* 20(2):101–107.

Mahadevan M (2012) Menu engineering for assisted-living elderly: A public health imperative. *Perspectives Public Health* 132(5):211–212.

Maillot M, Darmon N, Drewnowski A (2010) Are the lowest-cost healthful food plans culturally and socially acceptable? *Public Health Nutrition* 13(8):1178–1185.

Masset G, Monsivais P, Maillot M, Darmon N, Drewnowski A (2009) Diet optimization methods can help translate dietary guidelines into a cancer prevention food plan. *J. Nutrition* 139(8):1541–1548.

Mayerson D, Thompson K (2019) Menu planning in long-term care. Accessed September 1, 2022, hospitalnews.com/menu-planning-in-long-term-care/.

Milte R, Ratcliffe J, Chen G, Miller M, Crotty M (2018) Taste, choice and timing: Investigating resident and carer preferences for meals in aged care homes. *Nursing Health Sci.* 20(1):116–124.

Morley JE, Caplan G, Cesari M, Dong B, Flaherty JH, Grossberg GT, Holmerova I, Katz PR, Koopmans R, Little MO (2014) International survey of nursing home research priorities. *J. Amer. Medical Directors Assoc.* 15(5):309–312.

Okubo H, Sasaki S, Murakami K, Yokoyama T, Hirota N, Notsu A, Fukui M, Date C (2015) Designing optimal food intake patterns to achieve nutritional goals for Japanese adults through the use of linear programming optimization models. *Nutrition* 14(1):1–10.

Reinders MJ, Huitink M, Dijkstra SC, Maaskant AJ, Heijnen J (2017) Menu-engineering in restaurants-adapting portion sizes on plates to enhance vegetable consumption: A real-life experiment. *Internat. J. Behav. Nutrition Physical Activity* 14(1):1–11.

Schaynová L (2017) A nutrition adviser's menu planning for a client using a linear optimization model. *Acta Polytechnica. Hungarica* 14(5):121–137.

Seljak BK (2009) Computer-based dietary menu planning. *J. Food Composition Anal.* 22(5):414–420.

Sherwin S, Winsby M (2011) A relational perspective on autonomy for older adults residing in nursing homes. *Health Expectations* 14(2):182–190.

Sklan D, Dariel I (1993) Diet planning for humans using mixed-integer linear programming. *British J. Nutrition* 70(1):27–35.

Sullivan D, Morley J, Johnson L, Barber A, Olson J, Stevens M, Yamashita B, Reinhart S, Trotter J, Olave X (2002) The GAIN (geriatric anorexia nutrition) registry: The impact of appetite and weight on mortality in a long-term care population. *J. Nutrition Health Aging* 6(4):275–281.

Suominen M, Muurinen S, Routasalo P, Soini H, Suur-Uski I, Peiponen A, Finne-Soveri H, Pitkala K (2005) Malnutrition and associated factors among aged residents in all nursing homes in Helsinki. *Eur. J. Clinical Nutrition* 59(4):578–583.

USDA (2015) 2015–2020 Dietary Guidelines for Americans, U.S. Department of Health and Human Services and U.S. Department of Agriculture, 8th ed. http://health.gov/dietaryguidelines/2015/guidelines/.

Wang D, Everett B, Brunero S, Northall T, Villarosa AR, Salamonson Y (2020) Perspectives of residents and staff regarding food choice in residential aged care: A qualitative study. *J. Clinical Nursing* 29(3-4):626–637.

Wright OR, Connelly LB, Capra S, Hendrikz J (2013) Determinants of foodservice satisfaction for patients in geriatrics/rehabilitation and residents in residential aged care. *Health Expectations* 16(3):251–265.

Zhang L, Zeng Y, Fang Y (2019) Evaluating the technical efficiency of care among long-term care facilities in Xiamen, China: Based on data envelopment analysis and Tobit model. *BMC Public Health* 19(1):1–9.

Verification Letters

Richard K. Chambers III, Director of Dining Services, Brethren Village, 3001 Lititz Pike, Lititz, Pennsylvania 17543 writes:

"I am very happy to provide this support letter for Dr. Sadan Kulturel-Konak and her research team who developed a menu optimization tool for Continued Care Senior Living Facilities. I, Richard K. Chambers III, work as Director of Dining Services at Brethren Village, and I have been involved in food industry for over 40 years.

"Throughout my interviews with Sadan, we talked about objectives and constraints and concluded that her idea proposed system is, in fact, a very important tool to produce demonstrably effective menus.

"The research team captured different perspectives of menu optimization through their comprehensive review of what has been done. They devised a survey to collect different preferences of older resident's food selections. The research team shared the preliminary results that they received with us, and we provided them some feedback to consider. We were very happy that the research team incorporated input from us and others while developing their model.

"Our patrons consume most of their meals at our dining facilities necessitating that the food served meets the entire nutrition, dietary, cost, and operational requirements. Thus, the variety of our rotating menu of food items are chosen carefully while meeting the nutritional, dietary, and patron satisfaction constraints each put forth by the corresponding stakeholder.

"Currently we do the menu design seasonally, and each time it takes countless hours. The menu design is a very complex and entails a very labor-intensive process. Therefore, I think that the multiobjective menu optimization model that this research team has developed will be an asset to many different types of facilities to assist this process of creating healthy and effective menus. With this new suggested model, the labor might be reduced by at least half of the usual time. We had to postpone trying this suggested model because of the pandemic but we look forward to trying it soon after the pandemic slows down.

"I can be reached at 717-581-4286 should you need further information about this letter of support."

Brinda Vijaya Sankar, Independent Registered Dietitian and Certified Dietitian–Nutritionist, 9 Murfield Dr., Ithaca, New York 14850 writes:

"I am enthusiastically writing this letter to support the research project of Mr. Gavirneni. I have over 15 years of experience in clinical & food service nutrition. During my tenure at Kendal, I was working as a Dietitian & Health Center Dining Manager and had the wonderful opportunity to collaborate with Mr. Gavirneni on his project—"Menu engineering for continued care senior living facilities". Kendal (https://www.kendal.org/) is a CCRC and our dining department catered to different client groups such as independent residents, assisted living, short term rehab and long-term nursing care residents. With the managements support I was able to share our menus, recipes & nutritional data with the research team.

"Mr. Gavirneni has been in contact with me since the inception of this project, we have discussed this in detail for over a couple years now. Along the way, there was a number of issues that were discussed such as menu popularity, cost, nutritional information, production, purchasing, etc. As I had the dual role of Health Center Dining Manager and Registered Dietitian, it was good to see the challenges of both the clinical and operational side of things. Whenever Mr. Gavirneni, shared data and ideas from the project for me to review, it helped me identify what the drawbacks or constraints were on the menus. Bringing meals to the table was a complex process especially client satisfaction. We spend a lot of time choosing menus, implementing it in the menu cycle and removing it from the cycle if it does not work. Most of the menu decisions are trial and error and historical data, when Mr. Gavirneni showed that all this can be done with the ease of the program in a timely fashion, I was very excited. Unfortunately, we did not get a chance to try it in our CCRC since I went on maternity leave and did not join back due to personal reasons. In the future if I have an opportunity to work in a senior setting, will look forward to implementing the modeling framework that was developed by Mr. Gavirneni and his team.

"Please do not hesitate to contact me if I can help any further."

Joseph A. Ertel, Managing Chef, Pennsylvania State University, Reading, Pennsylvania 19610 writes:

"I am very happy to provide this support letter for Dr. Sadan Kulturel-Konak and her research team who developed a menu optimization tool for Continued Care Senior Living Facilities.

"I, Joseph A. Ertel, work as the managing chef for Pennsylvania State University, and I have been involved in every aspect of the food and hospitality industry for over twenty-five years. Through interviews with this research team, we have talked about our objectives and constraints, and how this is in fact an especially important tool to produce demonstrably effective menus. The research team captured different perspectives of menu optimization through their comprehensive review of what has been done and devised a survey to collect different preferences of food selections tailored to the specifics of this clientele. The research team shared the preliminary results they received with us, and we provided them with feedback to consider.

"We were very happy that the research team incorporated input from us and others while developing their model. While our patrons fall into a different demographic, they consume most of their meals at our dining facilities that incorporate a similar cycle menu. This structure necessitates that the food we serve meets a complete nutrition, dietary, cost, and operational requirement. Thus, the variety of this cycle menu of food items is chosen carefully while meeting the nutritional, dietary, and patron satisfaction constraints that are each put forth by the corresponding stakeholder.

"Currently I do menu design twice a year. While it only takes me about 6 hours biannually per location to create the initial draft, the total menu design process is a very complex and labor heavy process that requires many revisions along with interdepartmental communications with our dieticians and nutritionists and totals many person-hours by the end of the process. Therefore, I think that the multiobjective menu optimization model that this research team has developed will be an asset to many different types of facilities to assist in this process of creating healthy and effective menus.

"I particularly see this as a great asset in sports hospitality, one of the fastest growing segments with tremendous market share and the need for highly tailored and adaptable nutritive selections. With this new suggested model, the effort to craft specific menus for many individual athletes would greatly minimize errors and miscommunications between trainers, nutritionists, dieticians, and food preparers, effectively adding a liaison to the process. We had to postpone trying this suggested model because of the pandemic but we look forward to trying it soon after the pandemic slows down.

"I can be reached at 814-3214847 or jae210@psu.edu should you need further information about this letter of support."

Sadan Kulturel-Konak is a professor of management information systems at Pennsylvania State University, Berks. Her research focuses on modeling and optimizing complex systems using hybrid approaches combining heuristic methods and exact techniques from operations research. The primary application areas of her research include designing and redesigning facilities to provide significant economic benefits for industries.

Abdullah Konak is a Distinguished Professor of information sciences and technology at Pennsylvania State University, Berks. His research interests are modeling, analyzing, and optimizing complex systems using computational intelligence combined with data sciences and operations research. He has published papers on a broad range of topics, such as network design, system reliability, sustainability, green logistics, cybersecurity, facilities design, production management, and predictive analytics.

Lily Jakielaszek is a cloud computing and big data analysis consultant for PricewaterhouseCoopers. She graduated with a BSc in applied data science in May 2022 from Penn State, where she performed undergraduate research. Her main research interests include machine learning, graph databases, automation, and big data analysis.

Nagesh Gavirneni is a professor of operations management at the Johnson Graduate School of Management of Cornell University. His research interests are in the areas of supply chain management, inventory control, production scheduling, simulation, and optimization. He is now using these models and methodologies to solve problems in healthcare, agriculture, and humanitarian logistics in developing countries.