optimal combination of menu items to be included in each meal of a multiweek cycle menu. The system ensures that the dietary, nutrition, operational, financial, and customer experience constraints specified by the interdisciplinary group of executives are adequately accounted for. Such automated, comprehensive, versatile, data-driven, multiobjective decision support systems are essential for smooth dining operations of CCFs that are becoming increasingly popular as the population (here in the United States and abroad) becomes older and those aged 65 years and above are reliant on the CCF dining facilities for all their meals. The proposed system aims to assist menu planners not only in crafting their daily menus but also in analyzing their decisions about menu structures, evaluating menu improvement interventions, and identifying possible improvements in their menus or menu items. The applications demonstrated how menu planners could use the proposed modeling framework to analyze menu improvement decisions based on multiple objectives. The modeling framework utilizes various approaches to incorporate different stakeholders' perspectives in the decision processes, including a satisficing approach to represent the menu diversification goals of chefs and goal programming to prioritize nutritionists' point of view while ensuring autonomy of patrons. The proposed system could also be adapted to meet the needs of other dining operations with captive patrons, such as penitentiaries, schools and colleges, humanitarian operations, and remote operations with a residential workforce.

Appendix A. Description of the Modeling Framework

Model Parameters

T set of days in the planning horizon

W the number of days in an overlapping window pattern for menu diversification

I set of all menu items

L set of meal courses—for example, soup, salad, main dish, etc.

$l(i)$ the course type of item i

$I(l)$ set of items that can be served in a meal course $l \in L$

$S(l)$ set of subcategories of a meal course $l \in L$

$S'(l)$ a given subset of $S(l)$

$I(l, s)$ set of subcategory $s \in S(l)$ items that can be served in course l

M set of meals, that is, {"Breakfast," "Lunch," "Dinner"}

M' set of meals excluding breakfast—that is, {"Lunch," "Dinner"}

P set of individuals with different dietary requirements and/or preferences

$ns_{j,l}$ the number of menu items that should be included in a course l of a meal j

$np_{p,j,l}$ the number of items that a patron p can choose in a course l of a meal j

$mns_{j,S'(l)}$ the minimum number of items that should be included in meal j from $S'(l)$

$nsT_i^{\min}$ the minimum number of times that a menu item i is served within T

$nsT_i^{\max}$ the maximum number of times that a menu item i is served within T

$nsOWP_l$ number of times that an item in course l can repeat within each OWP

$nc_{l,s}^{\min}$ the minimum number of times that an item in subcategory s of course l should be served within an OWP

$nc_{l,s}^{\max}$ the maximum number of times that an item in subcategory s of course l should be served within an OWP

G_i the minimum gap between two servings of item i

nsd_l the number of times that an item can be served in the same day

K set of nutrients

$\omega_{k,i}$ nutritional value or amount of nutrient k in item i

c_i the relative cost of item i within its course

wc_l the relative weight of course l in cost calculations

$\pi_{p,i}$ the average normalized relative preference of item i by diet type p within its course

wp_l the weight of course l while calculating the total preference

$\alpha_{p,k}^{max}$ recommended upper limit for daily intake of nutrient k for diet type p

$\alpha_{p,k}^{min}$ recommended lower limit for daily intake of nutrient k for diet type p

$Q_p^{\max}$ maximum allowed deviation from the healthy eating guidelines for diet type p

Decision Variables

$x_{i,j,t}$ binary variable indicating whether item i is served in meal j on day t ($x_{i,j,t} = 1$) or not ($x_{i,j,t} = 0$)

$y_{p,i,j,t}$ binary variable indicating whether individual p picks item i in meal j on day t ($y_{p,i,j,t} = 1$) or not ($y_{p,i,j,t} = 0$)

$Q_{p,k,t}$ the violation of the constraints related to nutrient k for individual p on day t

Performance Metrics

Deviations from the HEGs. For diet type p, the total deviation from the target given in the HEG requirements is shown as follows:

$$F_{Q,p} = \sum_{k \in K} \sum_{t \in T} Q_{p,k,t}. \tag{A.1}$$

Total Preference Score. The total preference score of a menu for diet type p is given as follows:

$$F_{\pi,p} = \sum_{j \in M} \sum_{i \in I} \sum_{t \in T} \pi_{p,i} y_{p,i,j,t}, \tag{A.2}$$

where $\pi_{p,i}$ is the weighted preference score of item i by diet type p,

$$\pi_{p,i} = wp_{l(i)} \pi'_{p,i} \qquad \forall p \in P, i \in I \tag{A.3}$$

such that $wp_{l(i)}$ is the weight of course l in a meal, and $l(i)$ denotes the course of item i. Finally, the total preference score is given as follows:

$$F_\pi = \sum_{p \in P} F_{\pi,p}. \tag{A.4}$$

Total Cost. The total cost within the planning horizon as follows:

$$F_c = \sum_{j \in M} \sum_{i \in I} \sum_{t \in T} c_i x_{i,j,t}. \tag{A.5}$$

Constraint Details

Menu Structure Constraints

Item Inclusion. In each day t, the daily menus are constructed by selecting a predefined number of items ($ns_{j,l}$) to include in course l of meal j as follows:

$$\sum_{i \in I(l)} x_{i,j,t} = ns_{j,l} \quad \forall j \in M, l \in L, t \in T, \tag{A.6}$$

where set $I(l)$ denotes the set of items that can be served in course l.

Maximum Number of Items per Subcategory. Let set $S'(l)$ be a given subset of $S(l)$ and $I(l, s)$ denote the set of subcategory s items that can be served in course l. The following constraint ensures that at most one item from each subcategory in $S'(l)$ will be included in a meal j on a day t as follows:

$$\sum_{i \in I(l,s)} x_{i,j,t} \le 1 \ \forall j \in M, l \in L, s \in S'(l), t \in T : ns_{j,l} \ge 1. \tag{A.7}$$

Minimum Number of Items in a Subcategory. A meal j may require including a minimum number of items ($mns_{j,S'(l)}$) from a given subset $S'(l)$ of subcategory set $S(l)$ as follows:

$$\sum_{s \in S'(l)} \sum_{i \in I(l,s)} x_{i,j,t} \ge mns_{j,S'(l)} \quad \forall j \in M, l \in L, t \in T. \tag{A.8}$$

Dependency Constraints

Course Subcategory Dependencies. The following constraints allow serving only one item for a given $S'(l) \subseteq S(l)$ when a meal j includes multiple items of a course l.

$$\sum_{s \in S'(l)} \sum_{i \in I(l,s)} x_{i,j,t} \le 1 \quad \forall j \in M, l \in L, t \in T : ns_{j,l} \ge 2 \tag{A.9}$$

Item Dependencies. Given subcategory subsets $S'(l')$ and $S''(l'')$ of courses l' and l'', the following constraints require serving an item from $S''(l'')$ if an item from $S'(l')$ is served.

$$\sum_{s \in S'(l')} \sum_{i \in I(l',s)} x_{i,j,t} \le \sum_{s \in S''(l'')} \sum_{i \in I(l'',s)} x_{i,j,t} \quad \forall j \in M, l \in L, t \in T \tag{A.10}$$

Menu Diversification Constraints

Maximum/Minimum Number of Servings of Items in a Menu Cycle. The following constraints set lower ($nsT_i^{\min}$) and upper ($nsT_i^{\max}$) bounds for the number of times that an item i is served within the planning horizon.

$$\sum_{j \in M, t \in T} x_{i,j,t} \ge nsT_i^{\min} \quad \forall i \in I \tag{A.11}$$

$$\sum_{j \in M, t \in T} x_{i,j,t} \le nsT_i^{\max} \quad \forall i \in I \tag{A.12}$$

Maximum Number of Servings of Items in OWPs. The following menu diversification constraints limit the number of times that an item can repeat in each OWP to an upper bound of $nsOWP_l$, which depends on the course type of the item, as follows:

$$\sum_{j \in M, c \in \{1,\dots,W\}} x_{i,j,nw(t,T,c-1)} \le nsOWP_l \ \forall l \in L, i \in I(l), t \in T, \tag{A.13}$$

where function $nw(t,T,c)$ returns the cth index after index t in T, wrapping around to the beginning of T. For example, given $T = \{1,\dots,21\}$, $nw(20, T, c-1)$ will return 20, 21, 1, and 2 for c = 1, 2, 3, and 4, respectively. Thereby, the constraint accounts for the variability of items at the beginning and end of the menu cycle and ensures the diversification of menu items over the entire planning cycle uniformly.

Maximum/Minimum Number of Course Subcategories in OWPs. These constraints ensure that a minimum ($nc_{l,s}^{\min}$) and maximum ($nc_{l,s}^{\max}$) number of items from a subcategory s of a meal course l are served in each OWP. For a given l and $S'(l)$, such requirements can be expressed as follows:

$$\sum_{i \in I(l,s)} \sum_{c \in \{1,\dots,W\}} x_{i,j,nw(t,T,c-1)} \ge nc_{l,s}^{\min} \quad s \in S'(l), t \in T, \tag{A.14}$$

$$\sum_{i \in I(l,s)} \sum_{c \in \{1,\dots,W\}} x_{i,j,nw(t,T,c-1)} \le nc_{l,s}^{\max} \quad s \in S'(l), t \in T. \tag{A.15}$$

Limiting Consecutive Repetitions of Items. These constraints impose a gap of G_i days between two consecutive servings of item i as follows:

$$\sum_{j \in M} \sum_{c \in \{1,\dots,G_i\}} x_{i,j,nw(t+c-G_i,T,1)} \le 1 - \sum_{j \in M} x_{i,j,t-G_i} \quad \forall i \in I, t \in T : t > G_i > 0. \tag{A.16}$$

Item Repetition Constraints Within a Day. These constraints limit the number of times that an item can be served during a day as follows:

$$\sum_{j \in M} x_{i,j,t} \le nsd_l \quad \forall l \in L, i \in I(l), t \in T. \tag{A.17}$$

Diet Types. The following constraints indicate that a patron type p can choose only the items that are included in meal j on day t.

$$y_{p,i,j,t} \le x_{i,j,t} \quad \forall p \in P, j \in M, i \in I, t \in T \tag{A.18}$$

A patron type p is allowed to choose $np_{p,j,l}$ number of items of course l in meal j.

$$\sum_{i \in I(l)} y_{p,i,j,t} = np_{p,j,l} \quad \forall p \in P, j \in M, l \in L, t \in T : ns_{j,l} \ge 1 \tag{A.19}$$

HEG Constraints. The constraints regarding dietary guideline recommendations for the nutritional goals are given as follows:

$$\sum_{j \in M} \sum_{i \in I} \omega_{k,i} y_{p,i,j,t} \le \alpha_{p,k}^{\max} + \gamma_{p,k}^{\max} Q_{p,k,t} \quad \forall p \in P, k \in K, t \in T, \tag{A.20}$$

$$\sum_{j\in M}\sum_{i\in I}\omega_{k,i}y_{p,i,j,t} \geq \alpha^{\min}_{p,k} - \gamma^{\min}_{p,k}Q_{p,k,t} \qquad \forall p \in P, k \in K, t \in T. \tag{A.21}$$

In the aforementioned inequalities, the descriptions of parameters $\omega_{k,i}$, $\alpha^{\max}_{p,k}$, $\alpha^{\min}_{p,k}$, $\gamma^{\max}_{p,k}$, and $\gamma^{\min}_{p,k}$ depend on nutrient k as the healthy eating guidelines specify nutritional limits in terms of either absolute targets or percentage of the total calories or both. Therefore, in the constraints related to the limits on the percentage of total calories, parameters $\alpha^{\max}_{p,k}$, $\alpha^{\min}_{p,k}$, $\gamma^{\max}_{p,k}$, and $\gamma^{\min}_{p,k}$ represent the fraction of the total calories taken in a day, which is calculated as

$$TC_{p,t} = \sum_{j\in M}\sum_{i\in I}\omega_{1,i}y_{p,i,j,t}. \tag{A.22}$$

Finally, the deviations from the HEGs are capped by an upper bound as follows:

$$Q_{p,k,t} \leq Q^{\max}_{p} \qquad \forall p \in P, k \in K, t \in T. \tag{A.23}$$

Main dish constraints (l = "main" for all constraints):

• A maximum of one item from each main dish subcategory can be served in a meal (Constraint (A.7) with $S'(l) = S(l)$).

• Beef and pork main dishes cannot be served together in the same meal (Constraint (A.9) with $S'(l)$ = {"beef." "pork"}).

• Fish or shellfish main dishes cannot be served together in the same meal (Constraint (A.9) with $S'(l)$ = {"fish," "shellfish"}).

• Grain and pasta main dishes cannot be served together in the same meal (Constraint (A.9) with $S'(l)$ = {"grain," "pasta"}.

• A main dish can be served a maximum of two times in T (Constraint (A.12) with $nsT^{\max}_{i} = 2\ \forall i \in I(l)$).

• Each main dish subcategory must be served two or more times in an OWP (Constraint (A.14) with $nc^{\min}_{l,s} = 2$ and $S'(l) = S(l)$).

• A main dish subcategory can be served a maximum of 10 times in an OWP (Constraint (A.15) with $nc^{\max}_{l,s} = 10$ and $S'(l) = S(l)$).

• There must be a day gap between two servings of the same main dish (Constraint (A.16) with $G_i = 1\ \forall i \in I(l)$).

Side dish constraints:

• A maximum of one same subcategory of side dish can be served in a meal ((Constraint (A.7) with l = "side" and $S'(l) = S(l)$).

• At least one dark green, red/orange, or starchy vegetable side dish must be served in a meal (Constraint (A.8) with l = "side" and $S'(l)$ = {"dark green," "red orange," "starch"}, $mns_{j,S'(l)} = 1$).

• If a main dish of type beef, poultry, and pork is served, then a starch side dish must be served (Constraint (A.10) with l' = {"main"}, $S'(l')$ = {"beef," "poultry," "pork"}, l'' = {"side"}, $S''(l'')$ = {"starch"}).

• If a vegetable main dish is served, then a grain side must be served (Constraint (A.10) with l' = {"main"}, $S'(l')$ = {"vegetable"}, l'' = {"side"}, $S''(l'')$ = {"grain"}).

Soup:

• A maximum of one same subcategory of soups be served in a meal (Constraint (A.7) with l = "soup" and $S'(l) = S(l)$).

• An OWP must include one item from each soup subcategory (Constraint (A.14) with l = "soup" $nc^{\min}_{l,s} = 1$, and $S'(l) = S(l)$).

Dessert (l = "dessert"):

• An OWP must include a pie and a cheesecake (Constraint (A.14) with $nc^{\min}_{l,s} = 1$ for $S'(l)$ = {"cake," "cheesecake"}, and $nc^{\min}_{l,s} = 0$, otherwise).

Breakfast/breakfast side:

• At most one item from each subcategory of breakfast items can be served in a breakfast (Constraint (A.7) with l = "breakfast" and $S'(l) = S(l)$).

• At most one item from each subcategory of breakfast side items can be served (Constraint (A.7) with l = "breakfast side" and $S'(l) = S(l)$).

• If a batter or cereal item is served, then a fruit breakfast side must be served (Constraint (A.10) with l' = {"breakfast"}, $S'(l')$ = {"batter," "cereal"}, l'' = {"breakfast side"}, $S''(l'')$ = {"fruit"}).

• If an egg or meat item is served, then a starch breakfast side must be served (Constraint (A.10) with l' = {"breakfast"}, $S'(l')$ = {"egg", "breakfast"}, l'' = {"breakfast side"}, $S''(l'')$ = {"starch"}).

Cost:

• The total cost of breakfast must be less than 20% of the daily menu cost.

• The total cost of lunch must be less than 40% of the daily menu cost.

• The total cost of dinner must be between 40% and 70% of the daily menu cost.

Appendix B. Solving the MIP Model Using an Iterative Greedy Heuristic

Objective function

$$\max F_{\pi} = \sum_{p\in P} dw_p F_{\pi,p}, \tag{B.1}$$

where dw_p represents the relative weight of diet type p, which can be calculated based on the percentage of patrons with diet type p.

Procedure Iterative Greedy Heuristic (IGH)(){
Use CPLEX to find an initial feasible solution within a given CPU time
Let $x^{**}_{i,j,t}$ and $y^{**}_{p,i,j,t}$ represent the values of the decision variables in the feasible solution
Let F^{**} be the objective function value of the feasible solution
While (F^{**} is improved) {
Let RD be a random list of planning days
For $d = RD[1], \ldots, RD[T]$ {
Fix $x_{i,j,t} = x^{**}_{i,j,t}\ \ \forall i \in I,\ j \in M, t \in T$
Fix $y_{p,i,j,t} = y^{**}_{p,i,j,t}\ \ \forall p \in P,\ i \in I,\ j \in M, t \in T$
For $c = 1, \ldots, WD${
Unfix $x_{i,j,nw(d,T,c-1)}\ \ \forall i \in I,\ j \in M$
Unfix $y_{p,i,j,nw(d,T,c-1)}\ \ \forall p \in P,\ i \in I,\ j \in M$
}
Solve the model optimality to find a new solution
Let F^{*}, $x^{*}_{i,j,t}$, and $y^{*}_{p,i,j,t}$ be the objective and variable values of the new solution
If $F^{*} < F^{**}$ **Then** {
For $c = 1, \ldots, WD${