EXAMPLE
Integrated Planning Model in a Supply Chain of Oil Products

A company buys several types of crude oils daily at its main refinery location. The cost of purchasing one unit of crude oil j on day t is $C^{CRU}_{j,t}$. The crude oil is converted to components in the distillation department. A crude oil which goes through a distillation process provides different components in given proportions. We will refer by $R_{j,b}$ to the amount of component b obtained from processing one unit of crude oil j. There is a maximum processing capacity of Cap units of crude oil per day. The cost of processing one unit of crude oil j is C^{DIS}_j. The crude oil is ready to be used for distillation within the same day of purchase or, alternatively, it can be stored at the distillation department.

The components generated can either be stored in tanks located at the same distillation department or sent to the blending department of the company. Assume that the components sent from the distillation department on day t will be used at the blending department on day $t+1$, due to some lead time of transportation. The cost of transporting one unit of any component from the refining to the blending department is C^{TRA1}. It is not possible to store components at the blending department, so the amount of components sent from the refining department on one day must be the exact amount required for blending the following day.

When the components arrive at the blending department, they are mixed according to recipes for generating final products. The recipe for producing one unit of product p needs $N_{b,p}$ units of component b. The cost of producing one unit of product p is C^{PRO}_p.

From the blending department, the products are sent to depots (there is no storage at the blending department). The cost of transporting one unit of any product from the blending department to depot d is C^{TRA2}_d. Assume that what is produced during one day arrives to the depots at the beginning of the following day.

Once the products arrive at the depots, they are ready to be shipped to the markets. The cost of shipping one unit of any product from depot d to market k is $C^{TRA3}_{d,k}$. Alternatively, the products may be stored at the depots.

There is a maximum demand limit for product p from market k on day t, which we will refer by $\delta_{p,k,t}$. Assume that what is shipped from the depots one day arrives to the markets the following day. Also, assume that it is possible to fulfil demand of a same market partly from different depots.

The price of one unit of product p in all markets is S_p.

The cost of storing one unit of any type of crude oil at the refining department is C^{INVI} per day. The cost of storing one unit of any type of component at the refining department is C^{INVB} per day. The cost of storing one unit of any type of product at depot d is C^{INVP}_d per day. (Note all these inventory costs are incurred per any unit stored at the end of each day.)

Formulate a linear programming model for the multi-period planning problem of the company, including decisions on procurement, production, transportation, storage and sales. The objective is to maximize the total contribution over a given planning horizon. Assign value zero to any initial inventory or initial flow variable that you may require in your formulation, except for the initial inventory of product p at depot d which is equal to a given quantity $I^{ZERO}_{p,d}$.

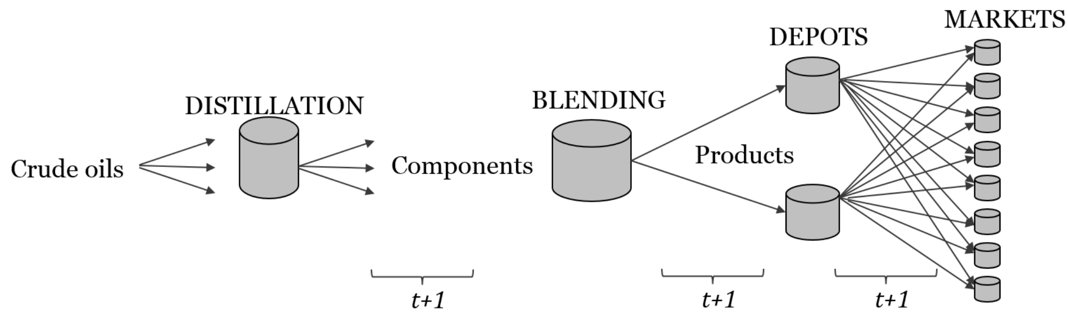

Figure 1: Illustration of the supply chain.

Indexes and sets

$j \in J$: set of crude oils.
$b \in B$: set of components.
$p \in P$: set of final products.
$d \in D$: set of depots.
$k \in K$: set of markets.
$t \in T$: set of days (including the day "0" preceding the planning horizon).

Parameters

$R_{j,b}$: amount of component b obtained from one unit of crude oil j.
$N_{b,p}$: amount of component b necessary in the recipe for generating one unit of product p.
$C_{j,t}^{CRU}$: purchase price of one barrel of crude oil j on day t.
C_{j}^{DIS}: cost of processing one unit of crude oil j.
C_{p}^{PRO}: cost of producing one unit of product p.
C^{TRA1}: cost of transporting one unit of component from the distillation to the blending department.
C_{d}^{TRA2}: cost of transporting one unit of product from the blending department to depot d.
$C_{d,k}^{TRA3}$: cost of transporting one unit of product from depot d to market k.
S_p: sale price of one unit of product p.
$\delta_{p,k,t}$: demand for product p in market k on day t.
Cap: maximum processing capacity of crude oil per day.
C^{INVI}: cost of storing one unit of crude oil at the distillation department.
C^{INVB}: cost of storing one unit of component at the distillation department.
C_{d}^{INVP}: cost of storing one unit of product at depot d.

Decision variables

$u_{j,t}$: amount of crude oil j purchased on day t.
$z_{j,t}$: amount of crude oil j distilled on day t.
$y_{b,t}$: amount of component b sent to the blending department on day t (for blending in $t+1$).
$w_{p,t}$: amount of product p produced at the blending department on day t.
$x_{p,d,t}$: amount of product p sent from the blending department to depot d on day t (available at depot in $t+1$).
$v_{p,d,k,t}$: amount of product p sent from depot d to market k on day t (to satisfy demand in $t+1$).
$IO_{j,t}$: inventory of crude oil j at the refining department at the end of day t.
$IC_{b,t}$: inventory of component b at the refining department at the end of day t.
$IP_{p,d,t}$: inventory of product p at depot d at the end of day t.

Objective function

$$\max Contribution = \sum_{p \in P} \sum_{d \in D} \sum_{k \in K} \sum_{t \in T: t>0 \text{ and } t \leq |T|-2} S_p v_{p,d,k,t} - \sum_{j \in J} \sum_{t \in T: t>0} C^{CRU}_{j,t} u_{j,t}$$
$$- \sum_{j \in J} \sum_{t \in T: t>0} C^{DIS}_{j} z_{j,t} - \sum_{p \in P} \sum_{t \in T: t>0} C^{PRO}_{p} w_{p,t} - \sum_{j \in J} \sum_{t \in T: t>0} C^{INVI} IO_{j,t}$$
$$- \sum_{b \in B} \sum_{t \in T: t>0} C^{INVB} IC_{b,t} - \sum_{p \in P} \sum_{d \in D} \sum_{t \in T: t>0} C^{INVP}_{d} IP_{p,d,t} - \sum_{b \in B} \sum_{t \in T: t>0} C^{TRA1} y_{b,t}$$
$$- \sum_{p \in P} \sum_{d \in D} \sum_{t \in T: t>0} C^{TRA2}_{d} x_{p,d,t} - \sum_{p \in P} \sum_{d \in D} \sum_{k \in K} \sum_{t \in T: t>0} C^{TRA3}_{d,k} v_{p,d,k,t}$$

Constraints

Balance of crude oils at the refinery: inflow purchase, stored, outflow to distillation.

$$IO_{j,t} = IO_{j,t-1} + u_{j,t} - z_{j,t} \quad \forall j \in J, t \in T : t > 0 \tag{1}$$

Max processing capacity of total crude oil that can be processed per day.

$$\sum_{j \in J} z_{j,t} \leq Cap \quad \forall t \in T : t > 0 \tag{2}$$

Balance of components at the distillation department: inflow from distilled crude oils, stored, outflow to blending.

$$IC_{b,t} = IC_{b,t-1} - y_{b,t} + \sum_{j \in J} R_{j,b} z_{j,t} \quad \forall b \in B, t \in T : t > 0 \tag{3}$$

Quantity of each component sent to blending is according to requirement of recipes for production.

$$y_{b,t-1} = \sum_{p \in P} N_{b,p} w_{p,t} \quad \forall b \in B, t \in T : t > 0 \tag{4}$$

All production from blending is sent to the depots (there is no storage at the blending department).

$$w_{p,t} = \sum_{d \in D} x_{p,d,t} \quad \forall p \in P, t \in T : t > 0 \tag{5}$$

Balance of products at depots: inflow from blending, stored, shipped to markets.

$$IP_{p,d,t} = IP_{p,d,t-1} + x_{p,d,t-1} - \sum_{k \in K} v_{p,d,k,t} \quad \forall p \in P, d \in D, t \in T : t > 0 \tag{6}$$

Quantity of each product shipped to each market cannot be more than the demand for the corresponding product from the corresponding market.

$$\sum_{d \in D} v_{p,d,k,t-1} \quad \leq \quad \delta_{p,k,t} \quad \forall p \in P, k \in K, t \in T : t > 0 \tag{7}$$

Initial condition for all variables that require initial value in this formulation.

$$IO_{j,0} = 0 \quad \forall j \in J \tag{8}$$
$$IC_{b,0} = 0 \quad \forall b \in B \tag{9}$$
$$IP_{p,d,0} = I^{ZERO}_{p,d} \quad \forall p \in P, d \in D \tag{10}$$
$$y_{b,0} = 0 \quad \forall b \in B \tag{11}$$
$$x_{p,d,0} = 0 \quad \forall p \in P, d \in D \tag{12}$$
$$v_{p,d,k,0} = 0 \quad \forall p \in P, d \in D, k \in K \tag{13}$$

Non-negativity condition.

$$u_{j,t}, z_{j,t}, y_{b,t}, w_{p,t}, x_{p,d,t}, v_{p,d,k,t}, IO_{j,t}, IC_{b,t}, IP_{p,d,t} \geq 0 \quad \forall j \in J, t \in T, b \in B, d \in D, k \in K \tag{14}$$