The Bandy Case

Bandy is a team winter sport played on ice, similar to ice hockey. In Norway, bandy is played by both male and female players in several age categories. In this problem, we will study the case of the Women's *Eliteserien* in the 2021/22 season. The aim of the problem is to formulate a schedule of matches that could help league organizers to arrange this season tournament.[1]

There are seven teams participating in the league: Drammen Bandy, Høvik, Ready, Solberg, Stabæk, Ullevål IL, and NTNUI. For the ease of writing, we will refer to these teams by the following abbreviations: DRAM, HOVI, READ, SOLB, STAB, ULLE, and NTNU. Each team has its own home venue. Most of these are located in the Eastern area of the country, close to Olso, except for NTNU whose home venue is located in Trondheim. The tournament schedule should comply with a double round robin format, that is, every team must play against every other team one home match and one away match (i.e. every pair of teams meets twice during the season, once at the home venue of each of them).

The season spans from mid November until mid February, as it is shown in the calendar in Figure 1. The matches can be played on Tuesdays or during the weekends (see the yellow dates in the figure). On each Tuesday, every team can play at most one match. On each weekend, every team can play at most one match, except for NTNU, which can play at most two matches. However, if NTNU is scheduled two matches on the same weekend, both matches have to be either at home or away (i.e., it cannot happen that one of the matches is at home and the other match is away). There are good reason for this, as NTNU players are mostly students who prefer to play during weekends (during the week they are busy with lectures and course work), and traveling from Trondheim to Oslo is time-consuming and costly.

In some special dates marked in the calendar of Figure 1, there cannot be any match. These dates correspond to: week 51 in 2021 (Christmas), the weekend of week 52 in 2021 (close to New Year's Eve), and the weekend of week 1 in 2022 and the whole week 2 in 2022 (because the 2022 Women's Bandy World Championship will be played in Sweden during January 9th-16th, and the Norwegian national team wants to perform well there). Thus, there are in total 11 weeks available to schedule the *Eliteserien*, but in two of these 11 weeks the weekends are not available. In addition, the teams have specified some dates in which they cannot have a home match, because their home venues will be used for other purposes. These dates are specified for each team in Figure 1 (for example, NTNU cannot play a home match on Tuesday of week 4 in 2022, and neither SOLB nor STAB can play a home match on the weekend of that week).

To secure a balanced distribution of home matches during the season, the league organizers require that during each month, every team should play at least one home match (a *month* here refers to a calendar month, that is, November, December, January, and February).

In addition to all the considerations above, the league organizers have other wishes, which we study independently in tasks 1 and 2, and simultaneously in task 3 below.

1. To keep activity during midweek, it is good when a week has at least two matches on Tuesdays. For NTNU, on the other hand, it is good to play during the weekends. This motivates us to define the concept of "Super Week". A *Super Week* is a week in which there are at least two matches on Tuesday and NTNU plays two matches during the weekend. The wish of the organizers is to maximize the number of *Super Weeks* during the season. Formulate an integer linear model to schedule the league, using this wish as optimization criterion. Implement and solve the model in AMPL, using the solver ***gurobi***. How many *Super Weeks* are obtained? Outline the optimal schedule that you found, either graphically, as a table, commented in words, or through some other type of output that can be readable for an external person. You may use AMPL or any other means to write this output (even doing it manually is fine). Imagine you are communicating the solution to the league organizers, who are not necessarily familiar with mathematical programming, so they want to observe the final schedule instead of a raw display with the optimal value of your variables.

[1] The problem described here is a modified and simplified version of the actual problem, adapted for pedagogical purposes.

2. For every team, except NTNU, we define now the concept of "Tough Week". A team has a *Tough Week* if it has two matches during the week (that is, it plays on Tuesday and on the weekend of the same week). The wish of the organizers is to minimize the sum of the number of *Tough Weeks* over all teams (except NTNU) during the season. Formulate an integer linear model to schedule the league, using this wish as optimization criterion[2]. Implement and solve the model in AMPL, using the solver ***gurobi***. How many *Tough Weeks* are scheduled in total? How many *Tough Weeks* are scheduled for each team? Outline the optimal schedule that you found (using the same readable format that you used in task 1).

3. Can you find a schedule that fulfils the optimal number of *Super Weeks* that you obtained in task 1 and also the optimal number of *Tough Weeks* that you obtained in task 2? If yes, outline such a schedule. If not, what is the minimum number of *Tough Weeks* that you can schedule while fulfilling the optimal number of *Super Weeks* that you obtained in task 1? And what is the maximum number of *Super Weeks* that you can schedule while fulfilling the optimal number of *Tough Weeks* that you obtained in task 2?

Note: When solving mathematical programming models, it is interesting to observe how the algorithms of the solvers approach the optimal solution during the optimization process and some statistics on the dimension of the problem and the solution time. For this purpose, when using *gurobi*, you can add in your .run file the following lines anywhere before the statement *solve*:
option solver gurobi;
option gurobi_options 'outlev=1';
option show_stats 1;

	Monday		Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Nov 2021	15	Week No 46	16 Venue unavailable this Tuesday: NTNU	17	18	19	20	21 Venue unavailable this Weekend: NTNU, DRAM
	22	Week No 47	23 Venue unavailable this Tuesday: NTNU	24	25	26	27	28 Venue unavailable this Weekend: HOVI
	29	Week No 48	30 Venue unavailable this Tuesday: NTNU	1	2	3	4	5 Venue unavailable this Weekend: NTNU, DRAM
Dec 2021	6	Week No 49	7 Venue unavailable this Tuesday: NTNU	8	9	10	11	12 Venue unavailable this Weekend: READ
	13	Week No 50	14 Venue unavailable this Tuesday: NTNU	15	16	17	18	19 Venue unavailable this Weekend: HOVI
	20	Week No 51	21 **NO MATCHES THIS WEEK**	22	23	24	25	26
	27	Week No 52	28 All venues available	29	30	31	1	2 **NO MATCHES THIS WEEKEND**
Jan 2022	3	Week No 1	4 All venues available	5	6	7	8	9 **NO MATCHES THIS WEEKEND**
	10	Week No 2	11 **NO MATCHES THIS WEEK**	12	13	14	15	16
	17	Week No 3	18 Venue unavailable this Tuesday: NTNU	19	20	21	22	23 Venue unavailable this Weekend: READ
	24	Week No 4	25 Venue unavailable this Tuesday: NTNU	26	27	28	29	30 Venue unavailable this Weekend: SOLB, STAB
	31	Week No 5	1 Venue unavailable this Tuesday: NTNU, STAB	2	3	4	5	6 Venue unavailable this Weekend: STAB, ULLE
Feb 2022	7	Week No 6	8 Venue unavailable this Tuesday: NTNU, ULLE	9	10	11	12	13 All venues available

Figure 1: Calendar features of the league season.

[2]Note that on a same week there might be more than one team with a *Tough Week*, and we are interested in the sum over all teams. Thus, for example, if DRAM plays on Tuesday and on the weekend of week 46 and HOVI also plays on Tuesday and on the weekend of week 46, this contributes with 2 to the objective function.