BAN402

Project 2

11.10.22 – 19.10.22

NHH

Candidate numbers:

47, 146

Part A

Question 1:

In order to identify a procedure to find the optimal plan of trips we have used a combination of the set partitioning problem and the traveling salesman problem in order to first divide the pieces that needs to be collected into partitions, and then creating an optimal route for each partition.

Sets and indexes:
$f \in F$: Family members
$i \in P$: Set of ChessmonGo stops
$j \in n$: Cluster j out of all alternatives n
K_j: Set of all links in the network connecting to city j
$B(S)$: Set of all links between the stops in the subset S
L_i: Set of all links connecting to stop i, $\quad \forall i \in n$

Parameters:
a_{ij}: Binary. 1 if piece i is included in alternative j, 0 otherwise
c_k: Distance of link k, $\quad \forall k \in K$

Decision variables:
x_j: 1 if alternative j is used. 0 otherwise
y_k: Binary. 1 if link k is included in the TSP trip. 0 otherwise, $\quad \forall k \in K$

Objective function:

$$Min\ z = \sum_{j=1}^{n} x_j \sum_{k \in K} c_k y_k \quad \forall j \in n$$

Constraints:
Each stop *i* must only be included exactly once for all alternatives *j*.
(1)

$$\sum_{j=1}^{n} a_{ij}\, x_j = 1 \quad \forall i \in P$$

The family as a whole must only have to links to each stop *j* as to not travel to it twice.
(2)

$$\sum_{k \in K_j} y_k = 2 \quad \forall j \in N$$

The trips must be continuous
(3)

$$\sum_{k \in B(S)} y_k \leq |S| - 1 \quad \forall S \subset N: |S| \geq 2$$

The number of trips within all clusters must be equal to the amount of family members.
(4)

$$\sum_{j \in N} x_j = |F|$$

Each family members trip to cluster *j* must collect between 25 and 50 Chessmon Go pieces
(5)

$$25 \leq x_j \sum_{i \in P} a_{ij} \leq 50 \quad \forall j \in n$$

Binary constraints on the decision variables
(6)

$$x_j \in \{0,1\} \quad \forall j \in n$$
$$y_k \in \{0,1\} \quad \forall k \in K$$

In summary, this model will create an optimal route for the family as a whole to collect ass the pieces with a minimized travel distance.

Question 2: Will Jacob Ingebrigtzen travel a shorter distance than the Karlsen Family?

As the true distances between the various stops is unknown to us, it might seem impossible to infer whether or not Jakob will travel a shorter or longer route than the family. However, we can gain insight into the problem by using logic derived from the differences between our combined model for the family and a standard TSP which Jakob will be using (as he is one person, we do not need to divide the route into clusters, therefore his route can be defined as a standard TSP problem).

We can approach the problem from two directions. The first alternative is by using the constraints of the model. The only limiting constraint on the family model in comparison to a normal TSP problem is constraint (6) which states that each family member must go to at least 25 stops. This might cause a family member to go to a stop which is inefficient to their route in order to make it to at least 25 stops, which might make the length of the family route longer than Jakob's which does not have this limitation. Jakob's trip must therefore be equal or shorter than the families when it comes to this aspect of the model.

Another approach to the problem is by using logic outside of the model. In order to fulfil the criteria of the model, each person has to start and stop their trip at the *Timez Square Hotel*, which can be defined as the *start* and *end* of the trip. As Jakob is only one person, he does not need to travel back to the hotel for each cluster he finishes. Therefore, for each family member after the first one Jakob "saves" a *start* and *end* trip. On the other hand, he does need to travel from cluster *j* to cluster *j+1*. So, if the distance between two clusters is on average smaller than the average *start* and *end* trip then Jakob will travel less distance than the family.

In summary, both of these approaches points to Jakob travelling a shorter route than the family, so we can assume that given the information we have at hand that Jakob's route is shorter.

Part B

Scenario 1: Family relations

In this scenario we must add a constraint to the model in order for it to fulfil the requirement that an examiner and a candidate should not be related to each other. By stating that the sum of two of the existing binary variables should be less or equal to one, we define a relationship where only one of them can be equal to 1 at the same time for all values of t.

When candidate c1 and professor p1 is related, an exam in time-slot *t* cannot contain both of these persons simultaneously:

$$\sum_{e \in E_C(c1)} x_{et} + z_{p1\,t} \leq 1 \qquad \forall\, t \in T$$

where $E_C(c1) \subseteq E$ is the set of exams containing the student c1.

Scenario 2: Bad luck sequences

In order to modify the model in a way which allows us to fulfil the constraints the task has given us; we need to add two new decision variables and add a few new constraints.

Decision variable

$b_{c,d}$: Binary, 1 if student *c* has an exam on day *d* and day *d+2*, 0 otherwise.
k_c: Binary, 1 if student c has had a bad luck sequence, 0 otherwise

Constraints

Limits the number of bad-luck sequences (exam on *d* and *d+2*) for a candidate to maximum 2 times. We define $b_{c,d}$ as a binary variable that is equal to 1 when a bad-luck sequence occurs for a given candidate, where M is a sufficiently large number.

(1)

$$\sum_{t \in T(d) \cup T(d+2)} x_{e,t} \geq 2 \cdot b_{c,d} \quad \forall c \in C$$

(2)

$$\sum_{t \in T(d) \cup T(d+2)} x_{e,t} \leq (M \cdot b_{c,d}) + 1 \quad \forall c \in C$$

When $b_{c,d}$ is defined, we can use it in order to allow for a maximum of 2 bad-luck sequences for any given candidate.

(3)

$$\sum_{d \in D} b_{c,d} \leq 2 \quad \forall c \in C$$

At least 60% of the candidates must have no bad-luck sequences.
We create a set of constraints in order to determine the value of k_c, where M is a sufficiently large number.

(4)

$$\sum_{d \in D} b_{c,d} \geq k_c \quad \forall c \in C$$

(5)

$$\sum_{d \in D} b_{c,d} \leq k_c \cdot M \quad \forall c \in C$$

The total amount of candidates with a bad-luck sequence must not exceed 60% of the number of candidates in the set.

(6)

$$\sum_{c \in C} k_c \leq |C| \cdot 0.6$$

This series of additional decision variables and constraints fulfils the additional requirements set by the task description for this scenario.

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Part C

Task 1

Sets and indexes

$i \in I$: Set of crude oils
$b \in B$: Set of components
$p \in P$: Set of final products.
$j \in J$: Set of CDUs.
$d \in D$: Set of depots.
$k \in K$: Set of markets.
$m \in M$: Set of running modes.
$t \in T$: Time periods {0, 1, …, 12}.

Parameters

$c_{i,t}$: Cost of purchasing one unit of crude *i* in time period *t.*
$R_{i,b,j,m}$: Amount of component *b* obtained from refining one unit of crude *i* in CDU *j* at mode *m.*
$N_{b,p}$: Amount of component *b* needed in recipe for one unit of product *p.*
S_p: Sales price of product *p.*
$f_{i,j,m}$: Cost of refining one unit of crude *i*, in CDU *j* at running mode *m.*
$a_{j,m}$: Capacity of CDU *j* at running mode *m.*
r_p: Cost of producing one unit of final product *p* at mixer facility.
CT^1: Cost of transporting one unit of *b* from refining to blending department.
CT_d^2: Cost of transporting one unit of any product *p* to depot d.
$CT_{d,k}^3$: Cost of transporting one unit of any product *p* from depot *d* to market *k.*
CS^{invi}: Cost of storing one unit of any crude *i* at the refinery.
CS^{invb}: Cost of storing one unit of any component *b* at the tanks.
CS_d^{invp}: Cost of storing one unit of any product at depot *d.*
$\delta_{p,k,t}$: Demand for product *p* in market *k* in period *t.*
$o_{j,m}$: Fixed cost of operating CDU *j* at running mode *m* per period.
g: Cost of changing running mode at a CDU.
SLQ: Price of *lowqc*.
$RMI_{j,m}$: Initial running mode of CDU *j*.
$IN_{p,d}^{ZERO}$: Initial inventory of product *p* at depot *d*.
$IN_{p,d}^{FINAL}$: Final inventory of product *p* at depot *d*.

Decision Variables

$u_{i,t}$: Amount of crude oil *i* purchased on day *t.*
$z_{i,j,t}$: Amount of crude oil *i* distilled in CDU *j* on day *t.*
$c_{b,t}$: Amount of component *b* produced on day *t.*
$y_{b,t}$: Amount of component *b* sent to the blending department on day *t* (for blending in *t+1*).
$w_{p,t}$: Amount of product *p* produced at the blending department on day *t.*
$x_{p,d,t}$: Amount of product *p* sent from the blending department to depot *d* on day *t* (available at depot in *t+1*).
$v_{p,d,k,t}$: Amount of product *p* sent from depot *d* to market *k* on day *t* (to satisfy demand in *t+1*).
$IO_{i,t}$: Inventory of crude oil *i* at the refining department at the end of day *t.*
$IC_{b,t}$: Inventory of component *b* at the refining department at the end of day *t.*
$IP_{p,d,t}$: Inventory of product *p* at depot *d* at the end of day *t.*
$LQ_{b,t}$: Amount of component *lowqc* produced on day *t.*
$RM_{j,m,t}$: Binary. 1 if running mode *m* is active on CDU *j* on day *t*. 0 otherwise.

Objective Function

$$\begin{aligned}\max Profit = &\sum_{p\in P}\sum_{d\in D}\sum_{k\in K}\sum_{t\in T:t>0\cup t\leq|T|-2} S_p\, v_{p,d,k,t} - \sum_{i\in I}\sum_{t\in T:t>0} c_{i,t}\, u_{i,t} + \sum_{t\in T} LQ_{lowqc,t}\, SLQ \\ &- \sum_{i\in I}\sum_{t\in T:t>0} c_{i,t}\, u_{i,t} - \sum_{j\in J}\sum_{m\in M}\sum_{t\in T:t>0} o_{j,m}\, RM_{j,m,t} \\ &- \sum_{j\in J}\sum_{m\in M}\sum_{t\in T:t>0} g\left(RM_{j,m,t} - RM_{j,m,t-1}\right)^2 \\ &- \sum_{i\in I}\sum_{j\in J}\sum_{m\in M}\sum_{t\in T:t>0} z_{i,j,t}\, RM_{j,m,t}\, f_{i,j,m} - \sum_{p\in P}\sum_{t\in T:t>0} r_p\, w_{p,t} \\ &- \sum_{i\in I}\sum_{t\in T:t>0} CS^{invi}\, IO_{i,t} - \sum_{b\in B}\sum_{t\in T:t>0} CS^{invb}\, IC_{b,t} - \sum_{p\in P}\sum_{d\in D}\sum_{t\in T:t>0} CS_d^{invp}\, IP_{p,d,t} \\ &- \sum_{b\in B}\sum_{t\in T:t>0} CT^1\, y_{b,t} - \sum_{p\in P}\sum_{d\in D}\sum_{t\in T:t>0} CT_d^2\, x_{p,d,t} \\ &- \sum_{p\in P}\sum_{d\in D}\sum_{k\in K}\sum_{t\in T:t>0} CT_{d,k}^3\, v_{p,d,k,t}\end{aligned}$$

Constraints

Change in inventory of crude oil *i* at refinery in time *t*.

(1)

$$IO_{i,t} = IO_{i,t-1} + u_{i,t} - \sum_{j\in J}\sum_{m\in M} z_{i,j,m,t} \quad \forall i\in I, t\in T: t>0$$

CDU *j* can only be set to one mode *m* at the same time *t*.

(2)

$$\sum_{m\in M} RM_{j,m,t} = 1 \quad \forall j\in J, t\in T: t>0$$

The amount of crude oil *i* distilled in CDU *j* in mode *m* on day *t* cannot exceed the production capacity of a given CDU in mode *m*.

(3)

$$\sum_{i\in I} z_{i,j,m,t} \leq a_{j,m}\, RM_{j,m,t} \quad \forall j\in J, m\in M, t\in T: t>0$$

Amount of component *b* produced on day *t* must be equal to the amount of crude oil *i* distilled in CDU *j* on day *t* times amount of component *b* obtained from distilling one unit of crude oil *i* in CDU *j* at running mode *m*.

(4)

$$c_{b,t} = \sum_{i\in I}\sum_{j\in J}\sum_{m\in M} z_{i,j,m,t}\, R_{i,j,b,m} \quad \forall b\in B, t\in T: t>0$$

Amount of component *b* sent to blending on day *t* cannot exceed the inventory of component *b* on day *t-1* plus production of component *b* on day *t*.

(5)

$$y_{b,t} \leq IC_{b,t-1} + c_{b,t} \quad \forall b\in B\backslash\{lowqc\}, t\in T: t>0$$

The amount of component *b* sent to the blending department in *t-1* must be equal to the amount of component *b* used to produce product *p*.
(6)

$$y_{b,t-1} = \sum_{p \in P} N_{b,p}\, w_{p,t} \quad \forall b \in B \backslash \{lowqc\}, t \in T: t > 0$$

Inventory of component *b* at refining department in period *t* cannot exceed the inventory in the refining department in *t-1* plus the production of component *b* minus the amount of component *b* sent to blending department
(7)

$$IC_{b,t} = IC_{b,t-1} + c_{b,t} - y_{b,t} \quad \forall b \in B \backslash \{lowqc\}, t \in T: t > 0$$

As the blending department do not possess any storage, the amount of product *p* produced must equal the amount of product *p* sent to depot *d* in time *t*.
(8)

$$w_{p,t} = \sum_{d \in D} x_{p,d,t} \quad \forall p \in P, t \in T: t > 0$$

Inventory at depot *d* must be equal to the inventory in depot *d* in *t-1* plus the inflow of product *p* to depot *d* in *t-1* minus the amount of product *p* from depot *d* sold to market *k* on day *t*.
(9)

$$IP_{p,d,t} = IP_{p,d,t-1} + x_{p,d,t-1} - \sum_{k \in K} v_{p,d,k,t} \quad \forall p \in P, d \in D, t \in T: t > 0$$

The amount of product *p* from all depots sold to market *k* in *t-1* must be less or equal to the demand of product *p* in market *k* on day *t*.
(10)

$$\sum_{d \in D} v_{p,d,k,t-1} \leq \delta_{p,k,t} \quad \forall p \in P, k \in K, t \in T: t > 0$$

Initial values for various inventories and transported materials between departments.
(11)

$$\begin{aligned} IO_{i,0} &= 0 && \forall i \in I \\ IC_{b,0} &= 0 && \forall b \in B \\ IP_{p.d,0} &= IN_{p,d}^{ZERO} && \forall p \in P, d \in D \\ y_{b,0} &= 0 && \forall b \in B \\ x_{p,d,0} &= 0 && \forall p \in P, d \in D \end{aligned}$$

Ending values for inventories.
(12)

$$\begin{aligned} IC_{b,12} &\geq 80 \quad \forall b \in B \backslash \{lowqc\} \\ IP_{p,d,12} &= IN_{p,d}^{FINAL} \quad \forall p \in P, d \in D \end{aligned}$$

Non-negativity condition.
(13)

$$u_{i,t}, z_{i,j,m,t}, c_{b,t}, y_{b,t}, w_{p,t}, x_{p,d,t}, v_{p,d,k,t}, IO_{i,t}, IC_{b,t}, IP_{p,d,t}, RM_{j,m,t} \geq 0$$
$$\forall j \in J, b \in B, p \in P, j \in J, d \in D, k \in K, m \in M, t \in T$$

With the model defined we can implement it in AMPL and apply it to the dataset given as part of the task description in order to answer the questions at hand.

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(a) *What is optimal profit?*

By running the model, we find that the optimal profit given the dataset is $5 197 910.

(b) *1. Which running modes are selected for each of the CDUs throughout the different time periods?*

This is the running modes used for **CDU 1**:

t	*HighMode*	*LowMode*	*Shutdown*
0	0	0	1
1	1	0	0
2	1	0	0
3	1	0	0
4	1	0	0
5	1	0	0
6	1	0	0
7	1	0	0
8	1	0	0
9	0	0	1
10	0	0	1
11	0	0	1
12	0	0	1

This is the running modes used for **CDU 2**:

t	*HighMode*	*LowMode*	*Shutdown*
0	0	0	1
1	1	0	0
2	1	0	0
3	1	0	0
4	1	0	0
5	1	0	0
6	1	0	0
7	1	0	0
8	0	1	0
9	0	1	0
10	0	1	0
11	0	1	0
12	0	1	0

2. How much of the capacity of the CDUs is used in each time period?

In order to answer how much of the capacity in the CDUs is used for each time period we will first of all present the production capacity cap given in the dataset and then presenting how much was actually used for each time period.

The production cap on units of crude each CDU can process in the various modes is listed below: (They are equal for all t in T)

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	HighMode	*LowMode*	*Shutdown*
CDU1	950	1050	0
CDU2	900	1000	0

CDU 1:

	Crude A			*Crude B*			
t	HighMode	LowMode	Shutdown	HighMode	LowMode	Shutdown	Sum
0	0	0	0	0	0	0	0
1	0	0	0	897,813	0	0	897,813
2	0	0	0	950	0	0	950
3	0	0	0	950	0	0	950
4	0	0	0	918,313	0	0	918,313
5	0	0	0	950	0	0	950
6	0	0	0	592,604	0	0	592,604
7	0	0	0	950	0	0	950
8	0	0	0	924,5	0	0	924,5
9	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0

As CDU 1 only operates in either HighMode or Shutdown in this scenario, we can see that most of the days the amount of crude oil processed is close to the cap of the CDU, and it exclusively produces Crude B, as it doesn't produce any Crude A during this period.

CDU 2:

	Crude A			*Crude B*			
t	HighMode	LowMode	Shutdown	HighMode	LowMode	Shutdown	Sum
0	0	0	0	0	0	0	0
1	754,113	0	0	145,887	0	0	900
2	647,238	0	0	252,762	0	0	900
3	601,524	0	0	298,476	0	0	900
4	657,637	0	0	242,363	0	0	900
5	628,238	0	0	271,762	0	0	900
6	770,292	0	0	129,708	0	0	900
7	0	0	0	900	0	0	900
8	0	1000	0	0	0	0	1000
9	0	1000	0	0	0	0	1000
10	0	286,667	0	0	0	0	286,667
11	0	0	0	0	0	0	0
12	0	266,667	0	0	0	0	266,667

From this table we can infer that for most of the days the CDU 2 is running at full capacity distilling both crude oil A and crude oil B, except in time period 10 through 12 where production is significantly lower than capacity.

(c) *How much inventory of crude oils, components and final products is left at the end of the planning horizon? If there is any difference among them, briefly discuss why this occurs.*

Inventory of crude oils in refining department at the end of day 12:

t	*Crude A*	*Crude B*
12	0	0

The inventory for crude oils is empty at the end of day 12. The reasoning behind why this might be optimal is that crude oil in of itself does not have any value in the model other than being used to produce compounds. If crude oil was being held up in storage it would accumulate storage costs and would not be used to produce the compounds which in turn are turned into products which generate income. It is also possible to regulate how much crude oil is bought so an optimal model would not buy crude which would just end up in storage within the timeframe of the model.

Inventory of components in the refining department at the end of day 12:

t	*distilA*	*distilB*	*lowqc*	*naphtha1*	*naphtha2*
12	1595,24	1060,41	0	80	80

This balance of inventory might seem odd at first glance. It might be hard to believe that it would be optimal to accumulate a very large amount of some components whilst being at the lower limit for some other components. This can be explained by the production process of the various components. By looking at which components is accumulated in storage, and which is at their lower bound one can assume that naphtha1 and 2 is the more profitable components as to what they can produce. Therefore, distilA and B will become similar to lowqc as they are treated as biproducts of naphtha in the model, and the relatively small cost of storage is made up for the increased profits from producing more naphtha 1 and 2. In summary, the demand for products made from distilA and B is simply not large enough to warrant production of the products that use distilA and B as input, and the components end up in storage as biproducts of naphtha1 and naphtha2.

Inventory of final products at the two depots at the end of day 12:

	Depot 1				*Depot 2*			
t	distilF	premium	regular	super	distilF	premium	regular	super
12	20	25	50	15	30	25	50	25

The inventory at the depots is at their lower limit defined in the model. This indicates that the model has not produced any superficial products, and all products not used to fulfil the end requirements of a minimum inventory at the depots have been sold to the various markets. This is in line with expectations, as an optimal model would not use funds to produce products it does not intent to sell.

Task 2

In this task we will consider three different scenarios in which the cost of purchasing crude oil is changed to various degrees. We will implement these changes to the model presented in task 1 and use it to analyse how the total profit is affected by the different price changes.

As the model we have presented in Task 1 is built as a general model, we deem it appropriate to modify the data file instead of the model itself in order to simulate these new scenarios. We will therefore not modify the model formulation as specified in the task description, only the data used as input to the model. We will also not present all the values of shutdown for each of the scenarios, but will rather comment on the differences between the scenario and the original model, as the specific run modes can be found in the .run files attached to this document.

Scenario 1: Increase in price of crude oil of 10%

In this scenario, the new optimal profit is $5 123 860 which is less than our original model. This is expected as a price increase will increase our costs and lower our margins. Although the decrease is relatively small, as the percentage change of the profit are $1 - \frac{5\ 123\ 860}{5\ 197\ 910} = 0{,}0142 \approx 1{,}4\%$.

When it comes to the mode of the CDUs (specifically the Shutdown-mode) it doesn't change with the price change that is presented, as the mode in each CDU for each t is equal to the original model in this scenario.

Scenario 2: Increase in price of crude oil of 30%

This increase in price changes the optimal profit of the model to $4 976 450 which again is less than our original model, although the profit decrease of $1 - \frac{4\ 976\ 450}{5\ 197\ 910} = 0{,}0426 \approx 4\%$ is small compared to the increase in price of the crude oil which might suggest that the cost of the crude oil is not the main driver of the costs in the model.

This price change does change the modes of the CDUs in a specific time period T9. In CDU1 the mode changes from "Shutdown" in the original model to "HighMode" in this altered scenario. One theory as to why it seems like the model increases production with lower margins is that whilst the price increases are relative, the objective function measures in absolute terms. This might cause the cheaper Crude B to have a relatively lower opportunity cost compared to the original model, as the more expensive Crude A has become relatively more expensive in absolute terms and might make it relatively more profitable to distill Crude B than before the price increase. This can explain the sudden increase in production with a quite large price increase.

Scenario 3: Increase in price of crude oil of 50%

This further increase in price changes the optimal profit of the model to $4 831 080. In comparison to the original profit the decrease of profits equates to $1 - \frac{4\ 831\ 080}{5\ 197\ 910} = 0{,}0705 \approx 7\%$. Although the decrease in profits is significant, it is relatively small compared to the change in crude oil prices, as we have seen in the previous two scenarios.

When it comes to the modes of the CDUs (specifically the "Shutdown" mode) there is no changes between scenario 2 and scenario 3.

Part D

Task 1

Sets and indexes:
$d \in D$: Set of days $\{1, \dots, 92\}$
$h \in H$: Set of hours $\{0, \dots, 23\}$
$d \in W \subseteq D$: Set of workdays within the set of days.

Parameters:
r: Charging rate
c: Consumption rate
b_{dh}: Battery charge level at time day d and hour h
p_{dh}: Cost of one kWh at day *d* and hour *h*.

Decision variables:
x_{dh}: Amount charged at day *d* and hour *h*
y_{dh}: Binary variable, 1 if consuming at day *d* and hour *h*, 0 if not
b_{dh}: Battery charge at hour h in day *d*

Objective function:

$$Min \sum_{d \in D} \sum_{h \in H} x_{dh} p_{dh}$$

Constraints:

In order to keep continuity in the battery charge level we have defined the battery charge level in day *d* and hour *h* to be equal to the battery charge level in *h-1* plus the amount charged at day *d* and hour *h*, less the consumption of charge level whilst driving on day *d* and hour *h*.

(1)

$$b_{d,h} = b_{d,h-1} + x_{d,h} - c y_{d,h} \quad \forall d \in D, h \in H : h > 0$$
$$b_{d,0} = b_{d-1,23} + x_{d,0} - c y_{d,0} \quad \forall d \in D : d > 1$$

The battery charge level in the first time period of the timeframe and the last period of the timeframe must be equal to 51.2

(2)

$$b_{1,0} = 51.2$$
$$b_{92,23} = 51.2$$

The amount charged on day *d* and hour *h* is limited to a minimum and maximum value.

(3)

$$0 \leq x_{dh} \leq 7.5 \quad \forall d \in D, h \in H$$

The minimum battery charge level for all day's *d* and hours *h* is defined with a lower limit.

(4)

$$b_{dh} \geq 12.8 \quad \forall d \in D, h \in H$$

The maximum battery capacity for all day's *d* and hours *h* is defined with an upper limit.

(5)

$$b_{d,h} \leq 64 \quad \forall d \in D, h \in H$$

For all workdays *d* the consumption of battery charge level at hour {7,16} changes the binary value of whether the car is consuming charge to 1.
(6)

$$y_{d,7} = 1 \quad \forall d \in W \subseteq D$$
$$y_{d,16} = 1 \quad \forall d \in W \subseteq D$$

For all workdays *d* and hours *h* where the car is driving or parked at work the car cannot increase its battery charge level.
(7)

$$x_{dh} = 0 \quad \forall d \in W \subseteq D, h \in \{7, \dots, 16\}$$
$$\frac{x_{dh}}{r} + y_{d,h} \leq 1 \quad \forall d \in D, h \in H$$

Solving the model in gurobi gives an optimal cost of 116.7€. The average price in the optimal solution is 160.8€ per MWh compared to the average price in the period being 288.2€ per MWh.

When maximizing the optimal cost is 273.8€ with an average price in the optimal solution of 373.2€ per MWh.

The difference between the minimizing and maximizing solutions are $273.8€ - 116.7€ = 157.1€$ or in relative terms $273.8/116.7 - 1 = 134.6\%$ more expensive.

Task 2

Changes to the model:

Sets and indexes:
$d \in G \subseteq D$: Sundays in the period

Parameters:
c^2: Discharging rate while driving recreationally

Decision variables:
$s_{d,h}$: Binary variable for the hour at which the recreation period starts
$j_{d,h}$: Binary variable for the hours spent driving for recreational driving.
$i_{d,h}$: Binary variable for the hours spent at a recreational facility where charging is impossible

Constraints:
With the new discharging rate we need to make a change to the battery charge level constraint from task 1. In order to keep continuity in the battery charge level we have defined the battery charge level in day *d* and hour *h* to be equal to the battery charge level in *h-1* plus the amount charged at day *d* and hour *h*, less the consumption of charge level whilst driving to work on day *d* and hour *h* and whilst driving recreationally on day *d* and hour *h*.
(8)

$$b_{d,h} = b_{d,h-1} + x_{d,h} - cy_{d,h} - c^2 j_{d,h} \quad \forall d \in D, h \in H: h > 0$$
$$b_{d,0} = b_{d-1,23} + x_{d,0} - cy_{d,0} - c^2 j_{d,0} \quad \forall d \in D: d > 1$$

For all workdays and Sundays *d* and hours *h* where the car is driving or parked at work or recreationally the car cannot increase its battery charge level.
(9)

$$\frac{x_{dh}}{r} + y_{d,h} + j_{d,h} + i_{d,h} \leq 1 \quad \forall d \in D, h \in H$$

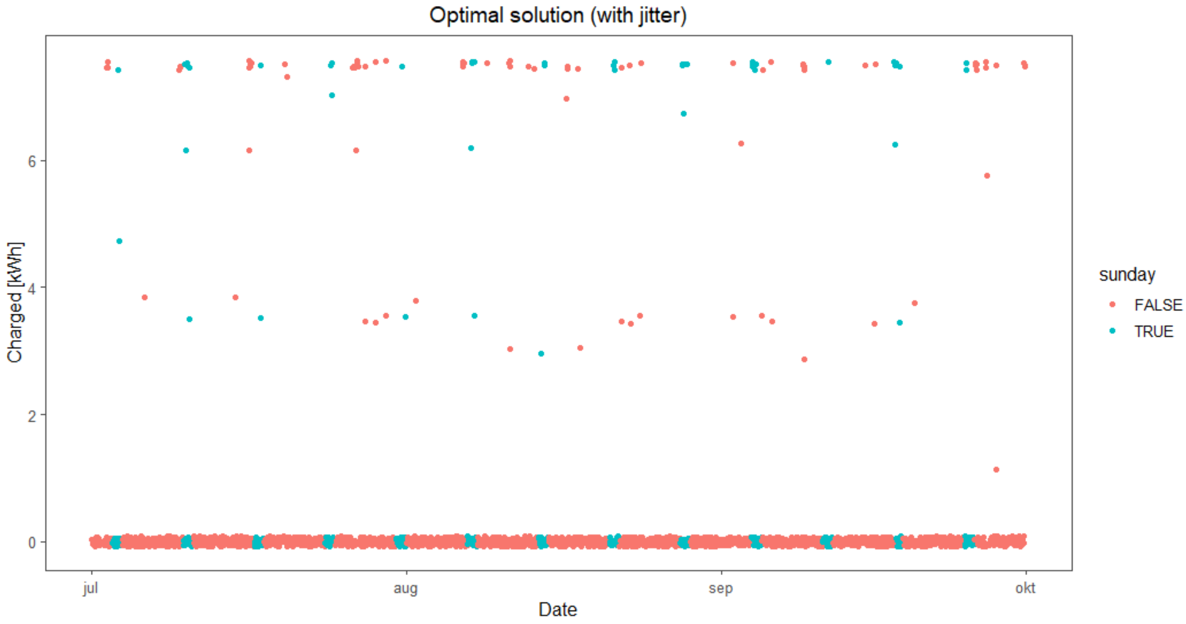
All recreational periods must start on Sunday d and in hours h {10,…,13}.
(10)

$$\sum_{h\in\{10,\dots,13\}} s_{d,h} = 1 \quad \forall d \in G \subseteq D$$

The battery charge unit cannot increase on Sundays d and hours $h>4$.
(11)

$$\begin{aligned} j_{d,h} &\geq s_{d,h} && \forall d \in G \subseteq D, h \in H{:}\, h > 4 \\ i_{d,h} &\geq s_{d,h-1} && \forall d \in G \subseteq D, h \in H{:}\, h > 4 \\ i_{d,h} &\geq s_{d,h-2} && \forall d \in G \subseteq D, h \in H{:}\, h > 4 \\ j_{d,h} &\geq s_{d,h-3} && \forall d \in G \subseteq D, h \in H{:}\, h > 4 \end{aligned}$$

The optimal cost is 135.1€. The average price in the optimal solution is 169.4€ per MWh compared to the average price in the period being 288.2€ per MWh.



This graph shows the amount charged in kWh for each hour h.

Task 3

Changes to the model:

Sets and indexes:
$u \in U$: Week numbers in the period
$d \in V_u \subseteq D$: Workdays in week number u

Decision variables:
f_d: Binary variable on whether working from home or not at day d

Constraints:
For all workdays d the consumption of battery charge level at hour {7,16} changes the binary value of whether the car is consuming charge to 1, except when *Optidriver* is working from home on day d, which does not consume battery charge level.
(12)

$$y_{d,7} = 1 - f_d \quad \forall d \in W$$

$$y_{d,16} = 1 - f_d \quad \forall d \in W$$

For all workdays *d* and hours *h* whilst at work or driving the car cannot increase its battery charge level.
(13)

$$x_{dh} = rf_d \quad \forall d \in W \subseteq D, h \in \{7, \dots, 16\}$$

For each week *u* the sum of days *d* working from home must be equal to 1.
(14)

$$\sum_{d \in V_u \subseteq D} f_d = 1 \quad \forall u \in U$$

For the days of the partial week at the start of the period, *Optidriver* cannot work from home.
(15)

$$y_{1,7} = 1$$
$$y_{1,16} = 1$$

The cost in the optimal solution is 100.1€ with an average price per MWh of 158.1€. Optidriver charges his car while working from home on three different days: 26.07.2022, 03.08.2022 and 30.09.2022.

Task 4

In this task we will explore how we can reduce the amount of "ON-OFF"-operations in the model by restricting the possibility of turning off the charger when it has begun a charging operation, whilst still minimizing the cost of charging to *Optidriver*.

We did not have sufficient time to implement a model for this task, but we have postulated an outline of how we want to approach the problem:

Define a set E of all possible subsets combinations of consecutive hours that can be used for charging.

The minimum amount that can be charged is $(80 - 50)/7.5 = 4$ hours and maximum is $(100 - 12.8)/7.5 = 11.627$ which means 11 hours. This means that the subset will be all combinations of consecutive hours of lengths 2 through 11.

Define a binary variable q_e that iterates over the subsets and is equal to 1 if charging should be done within all hours in the subset, and 0 otherwise.

The subsets where $q_e = 1$ cannot overlap with each other.

The subsets can also not overlap with any hours where $y_{d,h}, j_{d,h}, i_{d,h} = 1$