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Practice Summary: Solving the External Candidates Exam Schedule in Norway

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Abstract. We developed a mixed-integer linear programming model to plan exam sessions for external candidates in the Vestfold region, Norway. With our model, the administration planned the last session of 2018, the two sessions of 2019, and the first session of 2020. The plans produced are of high quality and saved three weeks of person effort per session.

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1. The External Candidates Exam Schedule

The Norwegian Ministry of Education has delegated the administrative responsibility for upper secondary education to the county authorities. Norway is divided into 11 counties, and each county is therefore in charge of (among other things) arranging the periodic exams for students in both the public and private upper secondary schools as well as external candidates. There are two exam sessions each year, one in the spring and one in the fall. In Vestfold, there are around 3,000 oral exams arranged for external candidates each year, approximately one-third in the second session and two-thirds in the first session. The administration of these exams is one of the main tasks of one employee for a couple of months every year. The task involves roughly the following. First, the candidates are divided into groups of five or six candidates based on which subject they are taking and on which other exams each candidate has as well as where the candidate lives. Second, each group must have available examiners with the right qualifications and experience. Third, each group needs a suitable room for preparation for the exam and a room for the actual examination, preferably in a school close to where the candidates live. Finally, each group needs a date that is within the exam period and fits other criteria specified later. In addition, dates and room assignments will have to maximize some collective benefits (see appendix). We call this problem *External Candidates Examination-Timetabling Problem* (EC-ETP). In 2018, the Department of Education in Vestfold County decided to test the benefits of automatizing this process, involving the University of Sannio and Naples in Italy and SINTEF Digital in Norway. Sponsors of the project in Vestfold County were the Director of Examinations Anne Fischer Bjelland and the Advisor Helle Bergan.

Examination timetabling models are in general very related, all sharing a core problem as defined in the survey by Qu et al. (2009), which consists of assigning a set of exams into a limited number of ordered time slots (time periods) and rooms of certain capacity in each time slot. However, models then differ for special constraints deriving from the specific real-life case at hand. For instance, in Woumans et al. (2016), the core problem is tackled by column generation, where the specific emphasis is on student satisfaction, namely, the spread between the exam sessions of each individual student. The core ETP is instead directly addressed in Al-Hawari et al. (2020) by using a heuristic decomposition.

Our specific practical case also includes the classical ETP as the core problem, but with some original requirements, as we are also asked to assign to each student group two examiners with the right qualifications.

We want also to point out that, despite a significant number of scientific publications on the ETP, to our knowledge, there are only two papers (Müller 2016 and Wang et al. 2010) reporting on the implementation of methods and software actually used in solving real-life ETP cases.

2. Modeling the EC-ETP

To attack the problem, we developed a mixed-integer linear programming (MILP) model (see Nemhauser and Wolsey 1988). Other recent integer programming approaches to ETP can be found in Woumans et al. (2016) and Arbaoui et al. (2019).

In this section, we briefly summarize the variables and main quantities of our model. The main constraints are instead described in the appendix. The EC-ETP requires arranging the final exam sessions for a group of external candidates. All exams must be carried out within a fixed time period, the *exams window*. Each exam session is for a specific subject and takes place in a suitable room of an available school on an available date/time. Moreover, each session must have an official *examiner*, typically (but not necessary) a professor from the school where the session takes place, and an external supervising professor called *sensor*. The number of candidates in each session cannot exceed a prescribed threshold (around six units). So, the EC-ETP consists of assigning a time, a room, a subject, two professors, and a group of candidates for that subject to exam sessions according to different rules and availability. For the entire set of exam sessions, there is a preferred due date, typically a few days before the end of the exams window. The primary objective is to minimize exam sessions that are scheduled after the preferred due date, whereas the secondary objective is to maximize the preferences of the students about the location of their exam sessions. The secondary objective is weighted in the objective function by using a priority parameter E whose effect on the quality of the solutions will be discussed in Section 3. Now we introduce some notation to model the variables and major constraints of our problem.

Each session takes place in a room from the set R of available rooms, each located in a school l from the set L of involved schools. The exam horizon $D = \{1, \ldots, |D|\}$ is divided in days, and the preferred due date is $\boldsymbol{d^*} \leq |D|$. Because each session takes place in a day and in a room, we introduce the set $T \subseteq D \times R$ of available combinations of rooms and days. Each element of T is called a ***space-time slot***. For each space-time slot $\boldsymbol{t}$, we let $\boldsymbol{d(t)} = 0$ if the time slot t comes before the due date $\boldsymbol{d^*}$; otherwise, we let $\boldsymbol{d(t)}$ be equal to the number of days from the due date of the day of the space-time slot $\boldsymbol{t}$. Our problem can then be regarded as an assignment of subjects, students, and professor to space-time slots.

We let $\boldsymbol{C}$ be the set of external candidates and $\boldsymbol{S}$ be the possible subjects. Each candidate $\boldsymbol{c} \in \boldsymbol{C}$ will take part to a certain number of exam sessions, each in a specific subject $\boldsymbol{s}$ from a set of subjects $\boldsymbol{S}$. A pair $\boldsymbol{(c, s)} \in \boldsymbol{C} \times \boldsymbol{S}$ is called *exam*, and we denote by $\boldsymbol{E} \subseteq \boldsymbol{C} \times \boldsymbol{S}$ the set of exams to be planned. The set of professors is $\boldsymbol{P} = \boldsymbol{P_S} \cup \boldsymbol{P_E}$, where $\boldsymbol{P_S}$ is the subset of sensors (i.e., external supervising professors), whereas $\boldsymbol{P_E}$ is the subset of professors that can act as examiners. The sets $\boldsymbol{P_E}$ and $\boldsymbol{P_S}$ overlap in general, so a professor can act as examiner and as sensor but in different exam sessions. To model the EC-ETP as an MILP model, we introduce three types of binary variables, each type corresponding to a specific class of assignments:

- x_{et}, which is one if exam $\boldsymbol{e} \in \boldsymbol{E}$ is carried out in space-time slot $\boldsymbol{t} \in \boldsymbol{T}$, and zero otherwise
- y_{st}, which is one if subject $\boldsymbol{s} \in \boldsymbol{S}$ is assigned to space-time slot $\boldsymbol{t} \in \boldsymbol{T}$, and zero otherwise
- z_{pt}, which is one if professor $\boldsymbol{p} \in \boldsymbol{P}$ is assigned to space-time slot $\boldsymbol{t} \in \boldsymbol{T}$, and zero otherwise

The interested reader can find the main constraints and the objective function of the model in the appendix.

3. Implementation and Computational Results

Vestfold County used our model to plan four examination sessions, namely, the second session of 2018, the first and second session of 2019, and the first session of 2020. Details are given in Table 1.

The project was deployed applying agile prototyping methodology, which involves continuous prototyping and full interaction between the Norwegian Department of Education and our team.

The mathematical model was implemented in JuMP, an algebraic modeling language embedded in the Julia programming language (Dunning et al. 2017). JuMP accepts in input closed-form algebraic expressions and returns mathematical programming formulations in any standard format (e.g., lp-format). Using JuMP allowed us to easily handle the sequence of requirement

Table 1. Instances Features

	Second 2018	First 2019	Second 2019	First 2020
Candidates (set $\|C\|$)	534	538	715	763
Exams ($\|E\|$)	777	1,024	777	1,218
Rooms ($\|R\|$)	20	20	24	28
Professors ($\|P\| = \|PE \cup PS\|$)	130	125	132	135
Start date	2018-11-12	2019-04-23	2019-11-11	2020-04-14
End date	2018-12-13	2019-06-06	2019-12-12	2020-06-18
Working days ($\|D\|$)	20	24	20	36
Due date	2018-11-29	2019-05-15	2019-12-02	2020-05-18

Table 2. Solution of Real-Life Instances, Adopted by the Vestfold School Administration

Periods	Exams	Sessions	Late sessions	Unsatisfied preferences	Times (seconds)
Second 2018	777	143	8	157	2,385
First 2019	1,024	192	2	205	1,078
Second 2019	777	155	14	215	234
First 2020	1,218	228	0	305	661

Note. Stopping condition: *integrality gap* $\leqslant$20%.

adjustments and accelerated the releasing process. JuMP uses a generic solver-independent interface, making it easy to switch between solvers, if necessary. To develop, test, and run our models, we used Gurobi (Gurobi Optimization 2019). Encrypted instances were provided by the exam administrators and solved on our PCs, and encrypted results were returned to the school. This scheme relieves the school from purchasing an expensive solver license, at least in this phase.

All computations were carried out on a workstation with an Intel Core i7-8700 CPU, 3.20 GHz processor, and 16 Gb RAM. We set the Gurobi parameters ***mipfocus*** = 1 to force the MIP solver to find feasible solutions quickly. On the other hand, the quality of the feasible solutions is also affected by the quality of the lp-relaxation, so we set ***cuts*** = 3 to tighten the formulation. All the other Gurobi parameters are kept at their default settings.

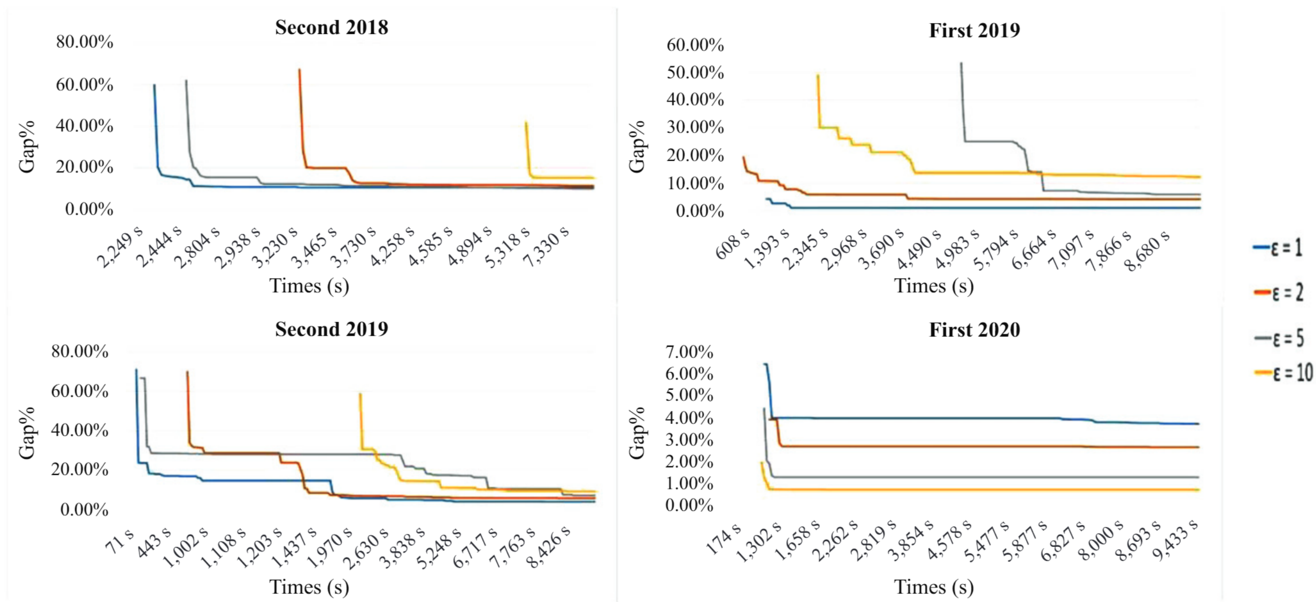
Note that planning exam timetables inevitably involves a trial-and-error process, where the person responsible for examination planning produces a sequence of tentative schedules that are iteratively amended until a satisfactory plan is released. This is because several new facts may happen during the process and some key inputs, such as room or professor availability, may change over time. In order to accomplish this trial-and-error process, any planning tool must support interactive sessions and a suitable compromise between computing time and quality of the solution returned must be achieved. The Norwegian Department of Education indeed wished to obtain a (reasonably good) solution within 20 minutes of computation time. In our tests, we find out that, for all our instances, a good compromise between quality of the solution and computing time could be achieved by letting $\boldsymbol{E} = 1$ in the objective function and by stopping the computation when the integrality gap fell below 20%. All instances were indeed solved within the 20-minute threshold; the quality of the solutions found, described in Table 2, was satisfactory (as also confirmed by the more extensive experiments reported in Table 3). In particular, for each instance, Table 2 describes the solutions obtained and approved by the Norwegian Department of Education.

To better assess the quality of the solutions found for the real-life instances and investigate the trade-off between number of late sessions and number of unsatisfied student preferences, we ran some experiments with different values of E and longer running times. Table 3 reports computational results on the Vestfold instances by letting $\boldsymbol{E} \in \{1, 2, 5, 10\}$ and by using, as the stopping criterion, a time limit to 10,000 seconds. Recall that our primary objective is to minimize the number of sessions scheduled after the preferred deadline $\boldsymbol{d^*}$, whereas the secondary objective is to minimize the number of unsatisfied student preferences. The graphs reported in Figure 1 show that a good compromise is obtained for $\boldsymbol{E} = 1$. In fact, by using $\boldsymbol{E} = 1$, we always obtain the best solution in terms

Table 3. Computational Results with Time Limit = 10,000 Seconds

Periods	E	Exams	% Gap	Sessions	Late sessions	Unsatisfied preferences
Second 2018	1	777	10.5	143	**8**	**104**
	2	777	11.4	157	**8**	**96**
	5	777	9.93	162	**8**	**83**
	10	777	15.2	164	**10**	**81**
First 2019	1	1,024	1.05	192	**2**	**197**
	2	1,024	4.2	192	**6**	**182**
	5	1,024	5.92	192	**11**	**152**
	10	1,024	12.2	192	**17**	**135**
Second 2019	1	777	4.21	155	**14**	**175**
	2	777	5.99	155	**22**	**142**
	5	777	7.4	159	**31**	**113**
	10	777	9.46	162	**39**	**89**
First 2020	1	1,218	3.74	228	**0**	**301**
	2	1,218	2.68	228	**0**	**301**
	5	1,218	1.31	228	**0**	**301**
	10	1,218	0.74	228	**0**	**301**

Figure 1. (Color online) Gap Trend for $E = 1, 2, 5$, and 10; Time Limit 10,000 Seconds



of our primary objective (number of late sessions). Note that, with an exception made for the first instance (second 2018), a greater value of $\boldsymbol{E}$ results in more late sessions, which means that $\boldsymbol{E} = 1$ is the best choice for $\boldsymbol{E}$ in its range.

Finally, the results on the last instance (first 2020) are independent on the value of $\boldsymbol{E}$ and always optimal. Indeed, because of the COVID-19 pandemic, the Department of Education made available more rooms and days. This allows us to obtain solutions with no late sessions. Finally, the integrality gap is reported in a column of Table 3; its behavior is depicted in Figure 1. As we can see, with an exception made for the first instance, the final integrality gap is almost always below 5%, which is a good outcome from a practical standpoint. Also, after the first feasible solution is found, there is a fast drop in gap, which quickly stabilizes at its final value.

4. Conclusions

The Norwegian Department of Education realized that applying Operations Research methods can lead to dramatic speedups in their processes and expressed interest in extending the scope of the project. One natural future direction is to consider other types of examinations, with different constraints and rules. Another crucial improvement will be the development of an interactive and user-friendly system to allow an operator to iteratively adapt timetables and to factor in small changes in input data. For instance, a professor may get sick or a classroom may become unavailable. In such cases, a new timetable must be provided very quickly, possibly in a few minutes. Reducing the response time from minutes to seconds is a very desirable feature, which requires further research for developing suitable fast heuristics or stronger models.

Appendix. Main Constraints and Objective Function

Observe first that, for different reasons, many assignments of subjects, professors, and exams to space-time slots are not allowed. For instance, students and professors may not be available on certain dates because they are taking or supervising other exams already scheduled or for other reasons. Certain rooms may be equipped for certain subjects or for specific days, etc. From now on, we will assume that the corresponding assignment variables are fixed to zero, so we will not take care explicitly of such forbidden assignments in our constraints. More formally, denoting by $\bar{E}(t) \subset E, \bar{P}(t) \subset P$, and $\bar{S}(t) \subset S$ respectively, the set of exams, professors, and subjects, which cannot be assigned to space-time slot $t \in T$, we have $x_{et} = 0$ for $e \in \bar{E}(t)$, $y_{st} = 0$ for $s \in \bar{S}(t)$, and $z_{pt} = 0$ for $p \in \bar{P}(t)$.

Next, for every exam, $e \in E$ must be carried out exactly once: $\sum_{t \in T} x_{et} = 1$.

The condition that at most one subject $s \in S$ can be assigned to a (session in a) space-time slot $t \in T$ writes as $\sum_{s \in S} y_{st} \leq 1$. Note that assigning a subject to a slot t corresponds, in some sense, to activating t.

An exam e can be assigned to a space-time slot t only if its subject is assigned to t and the number of assigned exams does not exceed the maximum number n^s of candidates allowed for subject $s \in S$ writes as $\sum_{e \in E_S(s)} x_{et} \leq n^s \cdot y_{st}$, where $E_S(s) \subseteq E$ is the subset of exams on subject $s \in S$.

A candidate c cannot undergo two distinct exams in the same day d or in two adjacent days d and $d+1$: $\sum_{e \in E_C(c)} \sum_{t \in T(d) \bigcup T(d+1)} x_{et} \leq 1$, where $E_C(c) \subset E$ is the set of exams of candidate $c \in C$ and $T(d) \subseteq T$ is the set of space-time slots available on day d.

If there is an exam session with subject $s \in S$ in space-time slot $t \in T$, then exactly two professors qualified for the subject must be assigned to t: $\sum_{p \in P(s)} z_{pt} \geq 2y_{st}$, where $P(s) \subseteq P$ is the subset of professors qualified for subject $s \in S$.

Moreover, (at least) one of the two professors must be an available sensor, $\sum_{p \in P(s)} z_{pt} \geq y_{st}$, and (at least) one must be an available professor, $\sum_{p \in PE} z_{pt} \geq y_{st}$.

A professor p cannot be assigned to two sessions in the same day d and to more than k sessions in total, $\sum_{t \in T(d)} z_{pt} \leq 1$ and $\sum_{t \in T} z_{pt} \leq k$.

Two professors assigned to a same space-time slot t cannot belong to the same school $\sum_{p \in P_L(l)} z_{pt} \leq 1$, where $P_L(l) \subseteq P$ is the set of professors belonging to the school l.

The objective function is the weighted sum of two components. The first accounts for the length of the exam period. The second attempts to maximize the number of times candidates are assigned to exam sessions, which take place in their preferred location.

$$\min \sum_{t \in T_>(d*)} d(t) \sum_{s \in S} y_{st} - \epsilon \sum_{e \in E} \sum_{t \in T_l(e)} x_{et}, \tag{A.1}$$

where $T_>(d^*)$ is the set of space-time slots after the preferred final day d^* and $T_l(e)$ is the set of space-time slots in the school preferred by the candidate of exam $e \in E$.

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Verification Letter

Anine Fischer Bjelland, Director of Examinations, Department of Education, Vestfold County, and Helle Bergan, Adviser, Department of Education, Vestfold County, 3103 Tønsberg, Norway, write:

"In July 2018, the county authority of upper secondary education in Vestfold, Norway, started a collaboration with the University of Sannio and the University of Naples, with the purpose of improving the process of organizing oral exams for students and external candidates in the area.

"A group of scientists from Italy (Professor Pasquale Avella, Professor Maurizio Boccia, system analyst Sandra Viglione) and Norway (Professor Carlo Mannino) developed an optimization model and a tool to generate the best possible solution to the problem.

"We are very happy with the outcome of this collaboration. The system has been successfully applied to the plans of oral examinations in 2019. Our office is responsible for three oral exam sessions a year, spring and fall exam sessions for external candidates (approximately 2,000 exams a year) and one session in spring for students (approximately 2,500 exams). The plans have a high level of complexity and several factors must be considered, such as, amongst many others; available and suitable rooms for the subject and available examiners and sensors with the correct qualifications. Furthermore, the candidates usually have several other exam dates which must be avoided during the same period. It is also important for us that each exam period is as short as possible, and that the assignments are spread out between the qualified professors.

"Not only the plans produced by the algorithm are of high quality, but the entire process has been significantly sped up. Indeed, even if the current implementation is not yet integrated in our planning tools, using the solutions produced by the model allowed us to save approximately three weeks of person effort per session, which means approximately nine to ten weeks of person effort saved a year.

"We believe that the approach developed by the group when fully integrated in an interactive and user-friendly software tool, can lead to further cost savings and a new and more effective way to do exam planning in our department of the school authority.

"We even hope the approach can be extended into other areas of exam planning, such as for example distribution of invigilators on written exam days.

"We are impressed and most grateful for the work they have done, and we hope we can continue the collaboration."

Pasquale Avella is full professor of operations research at the Università del Sannio. His research interests are in computational mixed-integer programming, particularly on network design, location, and scheduling problems.

Maurizio Boccia is associate professor of the department of electrical engineering and information technology. His current research interests include computational mixed-integer programming, in particular network location, network design, routing problems, and scheduling problems.

Carlo Mannino is senior scientist in the department of applied mathematics at SINTEF Digital, Oslo, and part-time full professor at the University of Oslo. He received the 2009 European Parking Association award from the European Conference on Operational Research, the Associazione Italiana di Ricerca Operativa Best Application Paper award in 2014, and the INFORMS Best Telecommunication Paper Award 2014. His main interests are in combinatorial optimization and its applications, in particular, in scheduling and timetable problems in transport, health care, and education.

Sandro Viglione is a software engineer and worked as a researcher for the University of Sannio, Benevento, Italy, until 2020. His current research is focused on the computational mixed-integer programming, in particular on scheduling problems.