Image denoising based on steepest descent OMP and K-SVD

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***Abstract*—Noise suppression is one of the key problems in image processing. In recent years, sparse representation theory is applied in image denoising successfully. The primary idea is to denoise an image via over-complete dictionary trained by K-SVD algorithm based on OMP (Orthogonal Matching Pursuit) algorithm. This method receives good performance on the quality of image denoising but slow computation speed because of high computational complexity. In order to speed up the computation while keeping image quality. This paper discusses a denoising method via the adaptive over-complete dictionary trained from noisy image using improved K-SVD algorithm and the steepest descent OMP algorithm. In this work, we replace OMP with the steepest descent OMP. Simulation results show this method leads to a better balance between denoising quality and the computation speed, and can improve performance than other methods. The PSNR values are used to measure the denoising quality, and it has been proven the PSNR values can be increased by our method meanwhile the running time can also be reduced to some extent.**

***Index Terms*—image denoising, K-SVD, OMP, steepest descent OMP.**

I. INTRODUCTION

Images are usually inevitably polluted by noise during the transmission and the reception of images. The reliability and the quality of an image can be reduced by the noise. Image denoising has been widely concerned through decades studies. The effectiveness of image denoising contains two aspects: First is to remove noise as much as possible; Second is to preserve details and edge information of the image. In general, an image denoising model can be expressed as:

$$y=x+n \qquad (1)$$

In the above expression, x is an original image, n is supposed to a Gaussian white noise of zero mean. i.e $n \sim N(0,\sigma^2)$. The image denoising can be viewed as a process to recover denoised image x from the polluted image y. Various methods which can be mainly classified into spatial domain method and transform domain method have been adopted in the field of image denoising over the past few decades .The spatial domain technique usually processes the gray scale values of the image directly and determines the center pixel value according to the values of the surrounding pixels, such as median filter[1]. A median filter is a kind of non-linear signal processing technology that resists noise based on an order statistics theory. The algorithm is simple but is not applicable in the image with many points , lines and steeples. The transform domain technique is applied to process the coefficients of the image after the transform such as wavelet transform. Wavelet transform has been successfully applied in the image denoising for its fine time-frequence property over the past decades[2]-[3]. The main idea in wavelet denoising is to set a threshold to filter the noise coefficients from the mixing coefficients to reach the goal of denoising. Wavelet transform method considers the noise is often in high frequency, and image information is often in low frequency. However, sometimes the image edges and other details are in the high frequency. The noise is sometimes in low frequency. Using wavelet transform can get better performance but also leads to the lost of details during denoising[4].

In recent years, a new method called sparse representation attracted significant attention of many scholars in the field of image denoising. The main idea is to decompose image sparsely by OMP to receive sparse cofficients, then update the atoms of the dictionary via K-SVD. As we all know, K-SVD algorithm is a dictionary update algorithm over learning and has achieved good results in image compression[5],denoising[6], and super-solution reconstruction[7]. Using the trained dictionary to represent an image sparsely can reach the goal of denoising. It shows obviously that the combination of K-SVD and OMP can reach the better performance in image denoising. But its high computational complexity also can not be ignored. It also can affect the quality of image denoising.

The algorithm of reconstruction in compressed sensing has a significant effect on the image denoising: the convex optimization algorithm and greedy algorithm. As we all know, the steepest descent method is a common algorithm applied in

ICSPCC2015
978-1-4799-8920-1/15/$31.00 ©2015 IEEE

the convex optimization theory. Generally speaking, the convex optimization algorithm has more accurate reconstruction value but slower speed. OMP is a normal greedy algorithm and can faster convergency speed but also lead the loss of accuracy. In order to reach a balance between the image denoising and the denoising speed. This paper endeavors to improve the denoising algorithm by replacing OMP with the steepest descent OMP—a new algorithm with the merit of the steepest descent algorithm in convex optimization algorithm and the OMP. And then use the improved K-SVD. As a result of the lack of adaptiveness of the dictionary trained by the traditional K-SVD algorithm. We discusses the improved K-SVD algorithm by training dictionary on corrupted image patches then receive a good dictionary adapt noisy image itself. Numerical results show the denoising quality based on this method can be improved and gain the higher PSNR values. The running time of our method can be reduced to some extent.

II. PROPOSED METHOD: IMPROVED ALGORITHM IN IMAGE DENOISING

A. Steepest descent Orthogonal Matching Pursuit(SD-OMP)

The goal of decomposing an image sparsely is to find a matrix α satisfying :

$$\min \|\alpha\|_0 \quad x = D\alpha \tag{2}$$

At present, sparse decomposition method can be classified in Basis Pursuit (BP), Matching Pursuit(MP) and Orthogonal Matching Pursuit(OMP).Among them OMP is the best matching pursuit algorithm to express functions with the least and applicable bases. Eq.2 can be converted to the unconstrained optimization problems：

$$\min_{\alpha} \lambda \| x - D\alpha\|_2^2 + \|\alpha\|_1 \tag{3}$$

In Eq.3, λ is a regularization parameter.

As we all know, OMP is a common method to decompose signal sparsely, and the process that approximate image patch x using OMP can be described as follows:

- Step 1:Initialization: residual r^0 =x (x is the image patch), α_x is the sparse coefficient of x, D_x is the dictionary of x, D_x□D, ε is the control error ,index set I=∅;
- Step 2:Update the index set I=I∪{i}

  i) Select the atom i and the corresponding sparse coefficient α_x which can make the value of the objective function minimum from the complementary set of I.

  $$\hat{i} = \arg\min_i \{\min_{\alpha_x} \| x - D_{I\cup\{i\}}\alpha\|_2^2\} \tag{4}$$

  ii) Update I, I=I∪{i}; (5)

  iii)Compute sparse coefficients $\alpha_x = (D_x^T D_x)^{-1} D_x^T x$ by solving least square method;

  iv) Update residual;

  r=x-$D_x \alpha_x$.
- Step 3: Stop iteration if $\| r \|_2 < \varepsilon$. Otherwise return to Step 2.

Considering the convex optimization algorithm has a higher reconstruction quality but a slower speed, and OMP has a faster speed but lower quality. In this work, we combine the steepest descent method with OMP algorithm as an improved algorithm—steepest descent OMP.

If Eq.3 is viewed as an unconstrained optimization problem of a multi-function f(x). Its optimization expression can be expressed as Eq.6 [8].

$$\alpha^{k+1} = \alpha^k + t^k p^k \tag{6}$$

Usually considers the direction of the fastest descent at a point is the negative gradient direction of this point. Then there is adaptive descent direction can be defined as

$$p^k = -\nabla f(\alpha^k) \tag{7}$$

The optimal iterative step each time t^k can be obtained from Eq.8

$$f(x^k + t^k p^k) = \min f(x^k + t p^k) \tag{8}$$

Notice that solving Eq.3 based on the steepest descent method is to make $\| x - D\alpha\|_2^2$ minimum. Then the problem can be equivalently written as the optimization problem of the quadratic function[9]:

$$\min \frac{1}{2}\alpha^T G \alpha - b^T \alpha \tag{9}$$

$$G = D^T D, b = D^T x \tag{10}$$

Differentiate to Eq.9, then

$$\nabla f(x^k) = G\alpha - b = -D^T r \tag{11}$$

$$p^k = -\nabla f(x^k) = D^T r \tag{12}$$

$$t^k = -\frac{(-p^k)^T p^k}{(p^k)^T G p^k} \tag{13}$$

With the merit of the steepest descent algorithm in convex optimization algorithm and the OMP. The steepest descent OMP can be summarized as follows[10].

The purpose of the OMP decomposition method is to solve Eq.14.

$$\min_{\alpha} \| x - D\alpha\|_2^2 (\|\alpha\|_0 \le L) \tag{14}$$

- Step 1. Initialization: residual r^0 =x (x is the image patch), α_x is the sparse coefficient of x, D_x is the dictionary of x, $D_x \in D$, ε is the control error ,index set I=∅;
- Step 2. Update the index set I=I∪{i}

  i)Select the atom i and the corresponding sparse coefficient α_x which can make the value of the objective function minimum from the complementary set of I;

  $$\hat{i} = \arg\min_i \{\min_{\alpha_x} \| x - D_{I\cup\{i\}}\alpha\|_2^2\} \tag{15}$$

  ii) Update I, I=I∪{i};

  iii) Update the sparse coefficients of matrix α_x;

$$\alpha_x^{k+1} = \alpha_x^k + t^k p^k \quad (16)$$

iv) Update residual;

$$r_{k+1} = r_k - D_x t^k p^k \quad (17)$$

Step 3. Stop iteration if $\| r_{k+1} \|_2 < \varepsilon$. Otherwise return to Step 2.

B. Image denoising via adaptive over-complete dictionary

K-SVD algorithm can be regarded as an algorithm based on the K-means clustering[11]. K-SVD algorithm is a dictionary update algorithm over adaptive learning and trains a good dictionary that can adapt image itself. Traditional K-SVD algorithm is to train a suitable dictionary based on a “clean image”. This method considers multi-features of different images while ignoring the characteristics of the noise. This paper discusses a way of training dictionary based on “ noisy image patches” and can get a good dictionary fit noisy image itself to overcome above drawback. The improved K-SVD algorithm improves efficiency of dictionary training by removing redundancy of the dictionary. Here models the atom which used least and can reduce the complexity of the algorithm. This algorithm gets higher PSNR values and also can be simplified. The process can be summarized as follows[6]:

First ,split a noisy image of size $\sqrt{N} \times \sqrt{N}$ into image patches of size $\sqrt{n} \times \sqrt{n}$ （$N >> n$）.Second, initialize the dictionary as DCT redundant dictionary with redundancy is 4. Train the dictionary of noisy image patch y over K-SVD algorithm. At last, using optimal dictionary to approximate the image patch x. This can viewed as a optimal question[9]:

$$\left\{\hat{\alpha}_{ij}, \hat{X}\right\} = \underset{\alpha_{ij}, X}{\arg\min} (\lambda \| X - Y \|_2^2 + \sum_{ij} \mu_{ij} \| \alpha_{ij} \|_0 + \sum_{ij} \| D\alpha_{ij} - R_{ij} X \|_2^2) \quad (18)$$

Where λ is Lagrange multiplication parameter. The first term of this expression is to measure the proximity between X and Y. Y is measured image and X is denoised image. The second term is the sparsity constraint. And in third term, $R_{ij}X$ represents ij image patch, $D\alpha_{ij}$ represents approximate image after reconstruction. The purpose of the algorithm is to make error smaller. If we don’t suppose that the dictionary is fixed. Then the model can be converted to:

$$\left\{\hat{\alpha}_{ij}, \hat{D}, \hat{X}\right\} = \underset{D, \alpha_{ij}, X}{\arg\min} (\lambda \| X - Y \|_2^2 + \sum_{ij} \mu_{ij} \| \alpha_{ij} \|_0 + \sum_{ij} \| D\alpha_{ij} - R_{ij} X \|_2^2) \quad (19)$$

Solving step can be summarized as follows[6]:

- Step1.Initialization: X=Y, D=overcomplete DCT dictionary.
- Step 2.Repeat J times.
- Step 3.Sparse coding stage: Fix D, then use steepest descent OMP to compute the representation vector α_{ij} for each patch R_{ij} X by approximating the solution of

$$\forall i, j, \hat{\alpha}_{ij} = \underset{\alpha_{ij}}{\arg\min} \| \alpha_{ij} \|_0, (\| R_{ij}\hat{x} - D\alpha_{ij} \|_2^2 \le n(C\sigma)^2)$$

(20)

- Step 4.Update the dictionary: Fix sparse matrix, update each atom $d_x (x \in 1, 2......k)$ in D.

i)Find the set of patches of sparse coefficients are non-zero;

$$h_x = \left\{[i, j] \mid \hat{\alpha}_{ij}(x) \neq 0\right\} \quad (21)$$

ii)For each index(i,j) $\in h_x$, compute its representation error;

$$e_{ij}^x = R_{ij} X_{ij} - \sum_{m \neq x} d_x \alpha_{ij}(m) \quad (22)$$

Set E_x as the matrix whose columns are$\{ e_{ij}^x \}$

Apply SVD decomposition to E_x .Then update the atom $\hat{d}_x$ and its corresponding sparse coefficients $\alpha_{ij}(x)$, after J times iterations. If $\| \hat{\alpha}_{ij}(x) \|_0 < T$ (T is the lower limit of the numbers of the sparse cofficients).i.e the atom $\hat{d}_x$ is less used. Then $\hat{d}_x = normalize(R_{ij}X)$. There are updated dictionary $\hat{D}$ and sparse matrix $\hat{\alpha}$, considering the overlapping of image blocks. Averging the overlapping part of denoised image blocks .At last, the denoised image can be gotten from Eq.23[9]

$$\hat{X} = (\lambda I + \sum_{ij} R_{ij}^T R_{ij})^{-1} (\lambda Y + \sum_{ij} R_{ij}^T \hat{D}\hat{\alpha}_{ij}) \quad (23)$$

III. ALGORITHM FOR IMAGE DENOISING

Denoising procedure using a dictionary trained on patches from the polluted image based on the steepest descent OMP is shown in Fig 1.

In this paper, we discusses the way to denoise image which can reach a balance between image quality and computation speed. The initial step is spliting a noisy image into three parts using improved MCA algorithm. Three parts are structure image X_t, texture image X_n and edge image X_l shown in Fig.2. Furthermore according to the characteristics of structure image with piecewise smooth, using wavelet denoising method process the structure part . For other two parts, adopting over-complete dictionary via improved K-SVD algorithm can reach a good performance but make high computational complexity. In this paper, we replace OMP with the steepest descent OMP to overcome above drawback. At last, add three parts after processed can consist of a denoised image.

Our basic idea is to process three parts aparted from the noisy image individually. According to the different features of three parts, we can adopt corresponding methods to process. Wavelet denoising method can process structure image well, and our method proposed in this paper can denoise other two parts via different dictionaries trained by their features. The algorithm of reconstruction in compressed sensing has a significant effect on the image processing. Steepest descent method, a normal reconstruction algorithm applied in optimization theory and has more accurate reconstruction value but slower computation speed. OMP is also another common reconstruction algorithm and can faster convergency speed but also lead the loss of accuracy. With the merit of the two algorithms, a good balance between the image quality and the running speed can be reached.

IV. RESULTS ANALYSIS

The method in this paper is compared with other denoising methods: Median filter, Wavelet transform, the method via DCT method, the method via over-complete dictionary trained on “clean image”. We set block size as 8×8. Peak-Signal-to-Noise-Ratio (PSNR) values are used to measure the denoising quality.

Figure.2 shows the three parts separated from the noisy image based on the improved MCA algorithm.

Figure.3 shows some completion results produced by our method, median filter method, wavelet threshold method, DCT dictionary method, the method via over-complete dictionary trained on “clean image”. It can be seen obviously that after denoised by median filter , little noise also exists in the image. Wavelet transform can resist more of the noise than median filter, but also lost some details and edges. The DCT dictionary method and the method via dictionary trained on natural image got similar results. Our method retains almost details without any noise and can enhance the visual quality.

Figure.4 shows respectively DCT redundant dictionary, the dictionary trained on natural image of Lena and the dictionary trained on the corrupted image with σ =25.

Table 1 shows the PSNR values of different algorithms under different noise levels. Our method took higher PSNR values than the traditional methods.

Table 2 shows the running time of different methods. It can be seen obviously the running time of our method can be reduced to some extent. Here improved OMP speeds up the computation because of the optimal step at the steepest descent direction. K-SVD proposed in this paper trained the adaptive dictionary while abandoning the atom which is used least. With the merit of steepest descent OMP and improved K-SVD, the new method can simplify the computation complexity.

V. CONCLUSION

In this paper, an image denoising mehod that is a combination of improved K-SVD algorithm via adaptive over-complete dictionary trained on “corrupted image” and steepest descent OMP is carried out. In comparison with other traditional methods , this algorithm not only structure image and texture image but also edge image that plays an important role in image processing were separated from the noisy image. According to the different characteristics of three parts, wavelet denoising method can process structure image well, and the method proposed in this paper process other parts. Considering the convex optimization algorithm has a higher reconstruction quality but a slower speed, and OMP has a faster speed but lower quality. This paper endeavors to replace OMP with steepest descent OMP in the improved algorithm to simplify the computational complexity in the denoising method. Denoise an image via the steepest descent OMP and improved K-SVD can reach a balance between the quality and speed. Numerical results shows our method can get higher PSNR values over other traditional methods. And running time can be reduced to some extent.

TABLE I. PSNR OF DIFFERENT METHODS

Denoising method	**σ =25**	**σ =35**	**σ =45**
	PSNR/dB	***PSNR/dB***	***PSNR/dB***
Noisy image	20.1579	17.2554	15.0545
Median filter	23.1123	20.1251	18.3387
Wavelet denoising	23.3205	20.3578	18.7632
DCT dictionary	30.9171	29.2384	27.9701
Traditional OMP+Traditional K-SVD	31.207	29.5883	28.2986
Our method	31.548	30.2568	29.8952

TABLE II. RUNNING TIME OF DIFFERENT METHODS

Image	**DCT**	**Global dictionary**	**K-SVD+OMP**	**Our method**
	Time/s	**Time/s**	**Time/s**	**Time/s**
Lena	80.44	83.72	265.25	230.12
House	22.77	12.78	240.91	201.56
Barbara	114.72	126.41	320.74	287.75

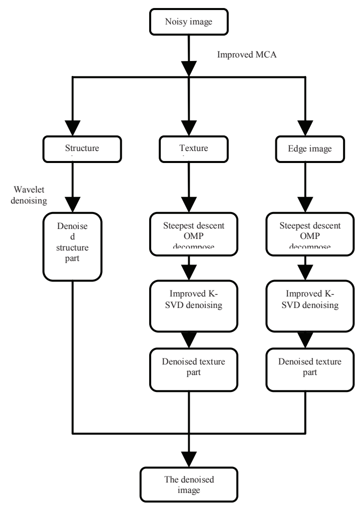
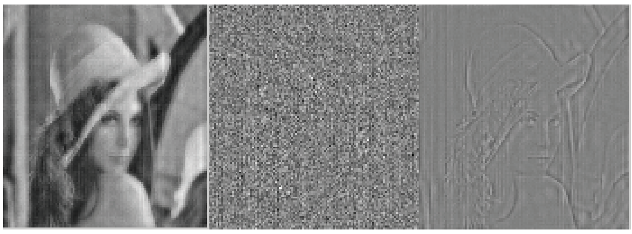


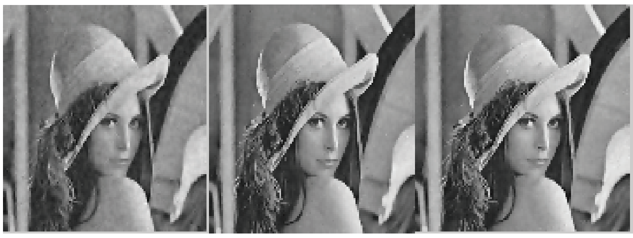
Figure. 1 Denoising procedure via steepest descent OMP and K-SVD.



(a) (b) (c)

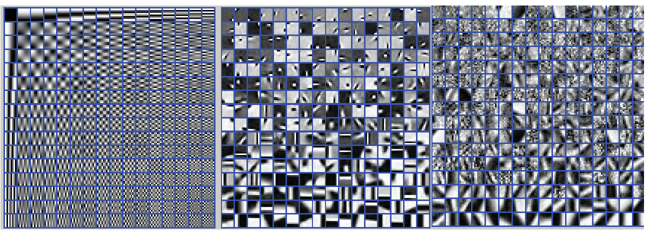
Figure. 2 Three parts separated from the noisy image. (a)the structure part separated from noisy image via improved MCA, (b) the texture part separated from noisy image via improved MCA,(c) the edge part separated from noisy image via improved MCA.

(a) (b) (c)



(d) (e) (f)

Figure.3 Denoised image via different denoising methods.(a)noisy image withσ=25, (b)denoised image by median filter, (c)denoised image by wavelet transform,(d) denoised image via DCT redundant dictionary,(e) denoised image via dictionary trained on "clean image",(f)our method.



(a) (b) (c)

Figure.4 The dictionary based on gray-scale image.(a)DCT redundant dictionary,(b)the dictionary trained on natural image of Lena,(c)the dictionary trained on the corrupted image with σ =25.

ACKNOWLEDGMENT

This work was supported by the Natural Science Foundation of NSFC under the grant No.60872157 and No.61271007.

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