L13 15 Feb 2021

15 February 2021 11:12

Solution of Initial Value Problems (IVP) and Boundary Value Problems (BVP).

(1) $y'' - y = 0,\ y(0) = 0, y'(0) = 2.$ At $x=0$ $y=0$ and at $x=0$ $y'(x)=2$

Problem: Solve the Boundary value problem: $y'' - 4y' + 3y = 0$ such that $y(0) = 1, y(1) = 0.$

(1) $y''-y=0$ $y(0)=0$ $y'(0)=2$

$(D^2-1)y=0$

A.E $D^2-1=0$

$D^2=1$

$D=\pm 1$

$y = c_1e^{-x} + c_2e^{x}$

General Sol

$y' = -c_1e^{-x} + c_2e^{x}$

$y(0)=0$

$0 = c_1e^{-0} + c_2e^{0} \Rightarrow 0 = c_1+c_2$ — (1)

$y'(0)=2$

$2 = -c_1e^{-0} + c_2e^{0} \Rightarrow 2 = -c_1+c_2$ — (2)

$2 = 2c_2$

$c_2=1$

$c_1=-1$

Sol $y = -e^{-x} + e^{x}$ Ans

Problem: Solve the Boundary value problem: $y'' - 4y' + 3y = 0$ such that $y(0) = 1, y(1) = 0.$

$y''-4y'+3y=0$

$(D^2-4D+3)y=0$

A.E $D^2-4D+3=0$

$D^2-3D-D+3=0$

$D(D-3)-(D-3)=0$

$(D-1)(D-3)=0$

$D=1, D=3$

$y = c_1e^{x} + c_2e^{3x}$

$y(0)=1 \Rightarrow 1 = c_1e^{0} + c_2e^{0} \Rightarrow 1 = c_1+c_2$ — (1)

$y(1)=0 \Rightarrow 0 = c_1e + c_2e^{3} \Rightarrow 0 = ec_1 + e^3c_2$ — (2)

(1) $\times e$ $\quad e = ec_1 + ec_2$

$-e = (e^3-e)c_2$

$c_2 = \frac{1}{1-e^2}$

$\Rightarrow c_2 = \frac{-e}{e(e^2-1)}$

$1-e$ | 2 $e(e^2-1)$

$1-c_2=c_1 \Rightarrow c_1 = 1-\frac{1}{1-e^2} = \frac{1-e^2-1}{1-e^2}$

$c_1 = \frac{e^2}{e^2-1}$

$y = \frac{e^2}{e^2-1}\cdot e^x + \frac{1}{1-e^2}\cdot e^{3x}$ Ans

Solution of Higher order Homogeneous LDE with Constant coefficients

Find the general solution of the following differential equations:

Problem 1. $y''' - 9y' = 0$

$(D^3-9D)y=0$

A.E $D^3-9D=0$

$D(D^2-9)=0$

$D=0 \quad D^2=9$

$D=\pm 3$

$D=0,-3,3$

Ans $y = c_1e^{0x} + c_2e^{-3x} + c_3e^{3x}$ Ans

Problem 2. $3y''' - 2y'' - 3y' + 2y = 0$

$(3D^3-2D^2-3D+2)y=0$ 2

A.E $3D^3-2D^2-3D+2=0$

$D=1$ ✓

1	3	-2	-3	2
		3	1	-2
-1	3	1	-2	0
		-3	2	
	3	-2	0	

$(3+1)-(3+1)$

$3x^2+x-2=0$

$3x-2=0$

$x = 2/3$

$1, -1, 2/3$

Sol $y = c_1 e^{-x} + c_2 e^{x} + c_3 e^{2/3x}$ 2

Problem 3. $y''' - 2y'' + y' = 0$

$(D^3 - 2D^2 + D)y = 0$

A.E $D^3 - 2D^2 + D = 0$

$D(D^2 - 2D + 1) = 0$

$D(D-1)^2 = 0$

$D = 0, 1, 1$

Sol $y = c_1 e^{0x} + (c_2 + c_3 x) e^{x}$ 2

If $D = 2, 2, 2$

Sol $y = (c_1 + c_2 x + c_3 x^2) e^{2x}$

Problem 4. $27y''' - 27y'' + 9y' - y = 0$

A.E $(27D^3 - 27D^2 + 9D - 1) = 0$

$(3D-1)^3 = 0$

$D = \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

$y = (c_1 + c_2 x + c_3 x^2) e^{1/3x}$

$4y^{IV} + 101y'' + 25y = 0$

A.E $(4D^4 + 101D^2 + 25) = 0$

$4D^4 + 100D^2 + D^2 + 25 = 0$

$4D^2(D^2 + 25) + (D^2 + 25) = 0$

$(4D^2 + 1)(D^2 + 25) = 0$

$4D^2 + 1 = 0$ | $D^2 = -25$

$D^2 = -\frac{1}{4}$ | $D = \pm i5$

$y = e^{0x}(c_1 \cos 5x + c_2 \sin 5x) + e^{0x}\left(c_3 \cos \frac{1}{2}x + c_4 \sin \frac{1}{2}x\right)$

$D = -\frac{1}{4}$

$D = \pm i\frac{1}{2}$

$D = \pm i5$

Ans: $D = \alpha \pm i\beta, \alpha \pm i\beta$

$$y = e^{\alpha x}\left((C_1 + C_2 x)\cos\beta x + (C_3 + C_4 x)\sin\beta x\right)$$

$$y^{IV} + 50y'' + 625y = 0 \quad \Rightarrow \quad D = \pm i5, \pm i5$$