L31 02 March 2021

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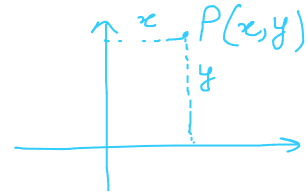
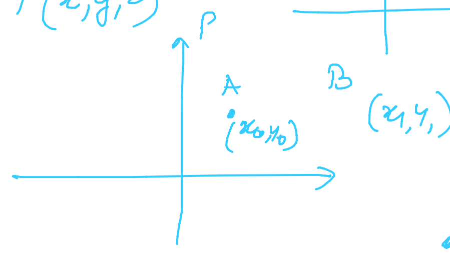
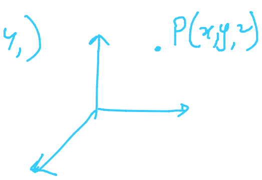
Unit-5 Vector Calculus-I

Scalar function $f(x,y,z)$ value is real and depends only on the point $P(x,y,z)$ in space.







$$f(P)=f(x,y,z)=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$$

$\sqrt{(x_1-x_0)^2+(y_1-y_0)^2}$

is a scalar function

$$f(x,y,z)=x^2+y^2-z$$

vector function A function $\vec{v}=v_1\hat{i}+v_2\hat{j}+v_3\hat{k}$ defined at each point $P \in D$ is called a vector function.

$\vec{v}$ v

$P(x,y,z)$

$v_1=x^2y$ $v_2=y-z$

$v_3=xyz$

Level Surfaces :→

Let $f(x,y,z)$ be a single valued continuous scalar function defined at every point $P \in D$. Then

$$f(x,y,z)=c, \quad (\text{Constant})$$

defines the equation of a surface and is called as level surface of the function.

Level surface of the function.

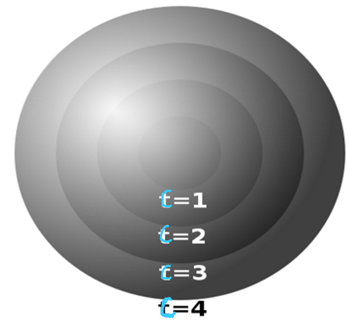
Let us Consider

(1) $f(x,y,z) = x^2+y^2+z^2$

$f(x,y,z) = c$

$x^2+y^2+z^2 = c$

$$\boxed{x^2+y^2+z^2 = (\sqrt{c})^2}$$



(2) $f(x,y,z) = x+y+z$

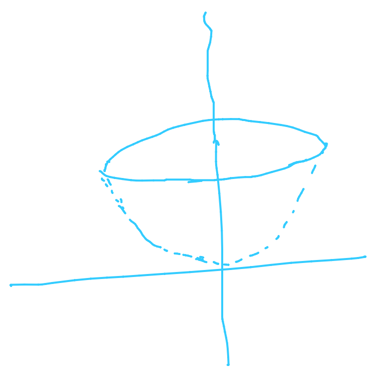
Level surface $=$ $f(x,y,z) = c$

$\Rightarrow$ $\boxed{x+y+z=c}$

$$f = x^2 + y^2 - z$$

$f(x,y,z) = c$

$x^2+y^2-z = c$



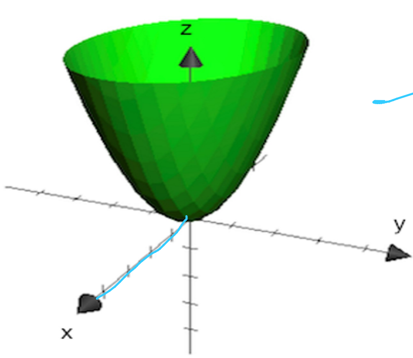
$x^2+y^2 = z$

$\downarrow$

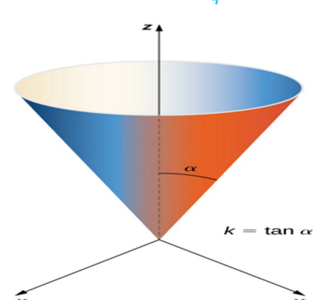
Paraboloid

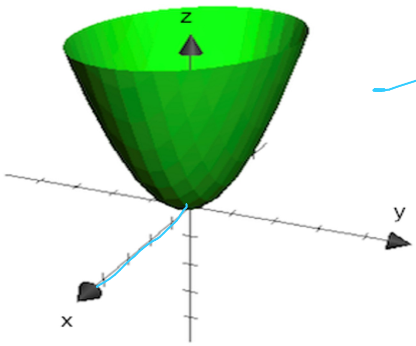
$x^2+y^2 = z^2$ Cone

$\downarrow$



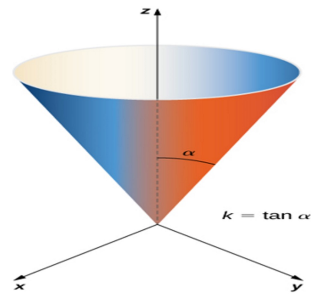
$\rightarrow$ Paraboloid



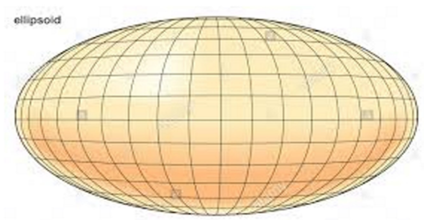


$\longrightarrow$ Paraboloid

$\Rightarrow$ Cone



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



$$f(x,y,z)= z-\sqrt{x^2+y^2}$$

$$f(x,y,z)=c$$

$$z-\sqrt{x^2+y^2}=c$$

$$(z-c)^2=x^2+y^2 \Rightarrow \boxed{x^2+y^2=(z-c)^2}$$

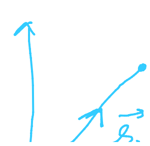
Parametric Representation of Curves

$y^2=4ax$

$x=at^2$, $y=2at$

Line $\quad \dfrac{x-x_0}{l}=\dfrac{y-y_0}{m}=\dfrac{z-z_0}{n}=t$

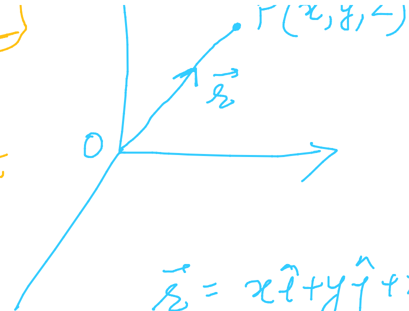
$$\boxed{\vec{r}=(x_0+lt)\,\hat{i}+(y_0+mt)\,\hat{j}+(z_0+nt)\,\hat{k}}$$



$P(x,y,z)$

$\vec{r}$

$$\boxed{\vec{r} = (x_0 + lt)\hat{i} + (y_0 + mt)\hat{j} + (z_0 + nt)\hat{k}}$$



$$x = x_0 + lt \qquad y = y_0 + mt \qquad z = z_0 + nt$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\boxed{\vec{r} = \vec{r}_0 + \vec{b}t}$$

$$\vec{r}_0 = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$$

$$\vec{b} =$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$