Mathematical Reasoning

1. Statements (Proposition) :

In reasoning we communicate our ideas or thoughts with the help of sentences in a particular language. "A sentence is called a mathematically acceptable statement or proposition if it is either true or false but not both". A statement is assumed to be either true or false. A true statement is known as a **valid statement** and a false statement is known as an **invalid statement**.

Example # 1 : Which of the following sentences are statements :

(i) Three plus two equals five.

(ii) The sum of two negative number is negative

(iii) Every square is a rectangle.

Solution : Each of these sentences is a true sentence therefore they all are statements.

Example # 2 : Which of the following sentences are statements :

(i) Three plus four equals six.

(ii) All prime numbers are odd.

(iii) Every relation is a function.

Solution : Each of these sentences is a false sentence therefore all of these are statements.

Example # 3 : Which of the following sentences are statements :

(i) The sum of x and y is greater than 0.

(ii) The square of a number is even.

Solution : Here, we are not in a position to determine whether it is true of false unless we know what the numbers are. Therefore these sentences are not a statement.

Example # 4 : Which of the following sentences are statements :

(i) Give me a glass of water.

(ii) Is every set finite ?

(iii) How beautiful ?

(iv) Tomorrow is Monday.

(v) May God bless you !

Solution : None of these sentences is a statement.

Note : (a) Imperative (expresses a request or command), exclaimatory sentences (expresses some strong feeling), Interrogative sentences (asks some question), Optative sentences (blessing & wishes) are not considered as a statement in mathematical language.

(b) Sentences involving variable time such as "today", "tomorrow" or "yesterday" are not statements.

(c) Scientifically established facts are considered true.

(d) Sentences involving word "here", "there" are not statements.

Self Practice Problems :

(1) Which of the following are statements :

(i) open the door (ii) square of an odd number is even

Ans. (1) (i) is not a statement (ii) is a statement

2. Truth table :

Truth table is that which gives truth values (The **truth or falsity of a statement** is called its truth value) of statements. It has a number of rows and columns.

Note that for **n** statements, there are 2^n rows.

(i) Truth table for single statement p:

Number of rows = $2^1 = 2$

p
T
F

(ii) Truth table for two statements p and q :

Number of rows = $2^2 = 4$

p	q
T	T
T	F
F	T
F	F

(iii) Truth table for three statements p, q and r.

Number of rows = $2^3 = 8$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

3. Negation of a statement :

The denial of a statement p is called its negation and is written as ~ p and read as 'not p'. Negation of any statement p is formed by writing "It is not the case that ...."

p	~p
T	F
F	T

Truth table

or "It is false that ...."

or inserting the word "not" in p.

Example # 5 : Write negation of following statements :

(i) "All cats scratch" (ii) $\sqrt{5}$ " is a rational number".

Solution : (i) Some cats do not scratch

OR

There exist a cat which does not scratch

OR

At least one cat does not scratch.

(ii) $\sqrt{5}$ is an irrational number

Self Practice Problems :

(2) Write negation of statement '2 + 2 = 7'

Ans. $2 + 2 \neq 7$

4. **Compounds statements :**

If a statement is combination of two or more statements, then it is said to be a compound statement. Each statement which form a compound statement is known as its sub-statement or component statement.

5. **Basic connectives :**

In the compound statement, two or more statements are connected by words like 'and', 'or', 'if . . . . . then', 'only if', 'if and only if', 'there exists', 'for all' etc. These are called connectives. When we use these compound statements, it is necessary to understand the role of these words.

Basic logical connective	*Symbol*	*Compound statement*
AND	$\wedge$	Conjunction
OR	$\vee$	Disjunction
IF....... THEN	$\rightarrow$	Conditional statement
IF AND ONLY IF	$\leftrightarrow$	Biconditional statement

6. **The word "AND" (Conjuction) :**

Any two statements can be connected by the word "and" to form a compound statement. The compound statement with word "and" is true if all its component statements are true. The compound statement with word "and" is false if any or all of its component statements are false. The compound statement "p and q" is denoted by "p q".

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

7. **The word "OR" (Disjunction) :**

Any two statements can be connected by the word "OR" to form a compound statement. The compound statement with word "or" is true if any or all of its component statements are true. The compound statement with word "or" is false if all its component state ment are false. The compound statement "p or q" is denoted by "p v q."

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

8. **Types of "OR" :**

(i) **Exclusive OR :** If in statement $p \vee q$ i.e. p or q, happening of any one of p, q excludes the happening of the other then it is exclusive OR. Here both p and q cannot occur together. For example in statement "I will go to delhi either by bus or by train", the use of 'or' is exclusive.

(ii) **Inclusive OR :** If in statement p or q, both p and q can also occur together then it is inclusive OR. The statement 'In senior secondary exam, you can take optional subject as physical education or computers' is an example of use of inclusive OR.

Example # 6 : Find the truth value of the statement "2 divides 4 and 3 + 7 = 8"

Solution. 2 divides 4 is true and 3 + 7 = 8 is false. so given statement is false.

Example # 7 : Write component statements of the statement "All living things have two legs and two eyes".

Solution : Component statements are :

All living things have two legs

All living things have two eyes

Example # 8 : Find truth value of compound statement "All natural numbers are even or odd"

Solution : p : all natural numbers are even, q : all natural numbers are odd.

Here compound statement $(p \vee q)$ is exclusive OR

Truth value of p is false and truth value of q is also false.

So truth value of compound statement $(p \vee q)$ is false.

Self Practice Problems :

(3) Find the truth values of $\sim p \vee q$

(4) Find the truth values of the compound statement $(p \vee \sim r) \wedge (q \vee \sim r)$

Ans. (3)

p	q	~p	~p∨q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(4)

p	q	r	~r	p∨~r	q∨~r	(p∨~r)∧(q∨~r)
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	F
F	T	T	F	F	T	F
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	F	F
F	F	F	T	T	T	T

9. Implication :

There are three types of implications which are "if . . . . . then", "only if" and "if and only if".

10. Conditional connective 'IF . . . . THEN' :

If p and q are any two statements then the compound statement in the form "If p then q" is called a conditional statement. The statement "If p then q" is denoted by $p \rightarrow q$ or $p \Rightarrow q$ (to be read as p implies q). In the implication $p \rightarrow q$, p is called the antecedent (or the hypothesis) and q the consequent (or the conclusion)

If p then q reveals the following facts :

(i) p is a sufficient condition for q

(ii) q is a necessary condition for p

(iii) 'If p then q' has same meaning as that of 'p only if q'

(iv) $p \rightarrow q$ has same meaning as that of $\sim q \rightarrow \sim p$

p	q	p→q	q→p
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Truth table

Note : The conditional statement $p \rightarrow q$ is defined to be true except in case p is true and q is false.

Examples :

(i) If $x = 4$, then $x^2 = 16$

(ii) If ABCD is a parallelogram, then AB = CD

(iii) If Mumbai is in England, then $2 + 2 = 5$

(iv) If Shikha works hard, then it will rain today.

We also need to be aware that in the English language, there are other ways for expressing the conditional statement $P \rightarrow Q$ other than "If P, then Q." Following are some common ways to express the conditional statement $P \rightarrow Q$ in the English language :

- If P, then Q.
- P implies Q.
- P only if Q.
- Q if P.
- Whenever P is true, Q is true.
- Q is true whenever P is true.
- Q is necessary for P. (This means that if P is true, then Q is necessarily true.)
- P is sufficient for Q. (This means that if you want Q to be true, it is sufficient to show that P is true.)

Converse of a conditional statement : If p and q are two statements, then converse of statement $p \Rightarrow q$ is $q \Rightarrow p$

Example # 9 : Write converse of statement "if two lines are parallel then they do not intersect in same plane"

Solution Converse of statement $p \Rightarrow q$ is $q \Rightarrow p$ so converse of given statement is "if two lines do not intersect in same plane then they are parallel".

11. Biconditional connective "IF and only IF" :

If p and q are any two statements then the compound statement in the form of "p if and only if q" is called a biconditional statement and is written in symbolic form as $p \leftrightarrow q$ or $p \Leftrightarrow q$.

Statement $p \leftrightarrow q$ reveals the following facts :

(i) p if and only if q

(ii) q if and only if p

(iii) p is necessary and sufficient condition for q

(iv) q is necessary and sufficient condition for p

p	q	$p \leftrightarrow q$	$q \leftrightarrow p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

Truth table

Note : The biconditional statement $p \leftrightarrow q$ is defined to be true only when p and q have same truth value.

For Example : The following statements are biconditional statements

(i) A number is divisible by 3 if and only if the sum of the digits forming the number is divisible by 3.

(ii) One is less than seven if and only if two is less than eight.

(iii) A triangle is equilateral if and only if it is equiangular.

Example # 10 : Let p and q stand for the statement 'Bhopal is in M.P.' and '3 + 4 = 7' respectively. Describe the conditional statement $\sim p \rightarrow \sim q$.

Solution : $\sim p \rightarrow \sim q$: If Bhopal is not in M.P. then $3 + 4 \neq 7$

Example # 11 : Find the truth values of $(p \leftrightarrow \sim q) \leftrightarrow (q \rightarrow p)$.

Solution :

p	q	~q	p↔~q	q→p	(p↔~q)↔(q→p)
T	T	F	F	T	F
T	F	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F

Self Practice Problems :

(5) If statements p and q are respectively : 4 + 5 = 9 and 2 + 3 = 5, then write the conditional statement $p \rightarrow q$.

(6) Find the truth values of the compound statement $(p \wedge q) \rightarrow \sim p$

Ans. (5) If 4 + 5 = 9, then 2 + 3 = 5

(6)

p	q	(p∧q)→~p
T	T	F
T	F	T
F	T	T
F	F	T

12. Tautology and fallacy :

(i) **Tautology :** This is a statement which is true for all truth values of its components.It is denoted by t. Consider truth table of $p \vee \sim p$

p	~p	p∨~p
T	F	T
F	T	T

we observe that last column is always true. Hence $p \vee \sim p$ is a tautology.

(ii) **Fallacy :** This is statement which is false for all truth values of its components. It is denoted by f or c. Consider truth table of $p \wedge \sim p$

p	~p	p∧~p
T	F	F
F	T	F

We observe that last column is always false. Hence $p \wedge \sim p$ is a fallacy

13. Logically equivalent statements :

If truth values of statements p and q are same then they are logically equivalent and written as $p \equiv q$.

14. Algebra of statements :

If p, q, r are any three statements and t is a tautology, c is a contradiction then

(i) Commutative Law :

(a) $p \vee q = q \vee p$ (b) $p \wedge q = q \wedge p$

p	q	(p∧q)	(q∧p)	(p∨q)	(q∨p)
T	T	T	T	T	T
T	F	F	F	T	T
F	T	F	F	T	T
F	F	F	F	F	F

(ii) Associative Law :

(a) $p \vee (q \vee r) = (p \vee q) \vee r$ (b) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

p	q	r	$(p \wedge q)$	$(q \wedge r)$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

(iii) Distributive Law :

(a) $p \wedge (q \;\; r) = (p \;\; q) \;\; (p \;\; r)$ (b) $p \vee (q \;\; r) = (p \;\; q) \;\; (p \;\; r)$

(c) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge (p \wedge r)$ (d) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

p	q	r	$(q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

(iv) Identity Law :

(a) $p \vee t = t$ (b) $p \wedge t = p$ (c) $p \vee c = p$ (d) $p \wedge c = c$

p	t	c	$(p \wedge t)$	$(p \vee t)$	$(p \wedge c)$	$(p \vee c)$
T	T	F	T	T	F	T
F	T	F	F	T	F	F

(v) Complement Law :

(a) $p \vee (\sim p) = t$ (b) $p \wedge (\sim p) = c$ (c) $\sim t = c$

(d) $\sim c = t$ (e) $\sim(\sim p) = p$

p	$\sim p$	$(p \wedge \sim p)$	$(p \vee \sim p)$
T	F	F	T
F	T	F	T

(vi) Idempotent Law :

(a) $p \vee p = p$ (b) $p \wedge p = p$

p	$(p \wedge p)$	$(p \vee p)$
T	T	T
F	F	F

(vii) De Morgan's law :

(a) $\sim (p \vee q) = (\sim p) \wedge (\sim q)$ (b) $\sim (p \wedge q) = (\sim p) \vee (\sim q)$

p	q	$\sim p$	$\sim q$	$(p \wedge q)$	$\sim (p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(viii) Involution laws (or Double negation laws) :

$\sim(\sim p) \equiv p$

p	~q	~(~p)
T	F	T
F	T	F

(ix) Contrapositive Laws : $p \to q \equiv \sim q \to \sim p$

p	q	~p	~q	p→q	~q→~p
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

(x) Truth table of biconditional statement :

p	q	p↔q	q↔p	p→q	q→p	p↔q ≡ (p→q)∧(q→p)
T	T	T	T	T	T	T
T	F	F	F	F	T	F
F	T	F	F	T	F	F
F	F	T	T	T	T	T

15. <u>Negation of compound statements</u> :

If p and q are two statements then

(i) Negation of conjunction : $\sim(p \wedge q) \equiv \sim p \vee \sim q$

p	q	~p	~q	(p∧q)	~(p∧q)	(~p∨~q)
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(ii) Negation of disjunction : $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	~p	~q	(p∨q)	~(p∨q)	(~p∧~q)
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

(iii) Negation of conditional : $\sim(p \to q) \equiv p \wedge \sim q$

p	q	~q	(p→q)	~(p→q)	(p∧~q)
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

(iv) Negation of biconditional : $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ or $p \leftrightarrow \sim q$

p	q	~p	~q	(p→q)	~(p→q)	(p∧~q)	(p↔q)	~(p↔q)	p↔~q	q∧~p	(p∧~q)∨(q∧~p)
T	T	F	F	T	F	F	T	F	F	F	F
T	F	F	T	F	T	T	F	T	T	F	T
F	T	T	F	T	F	F	F	T	T	T	T
F	F	T	T	T	F	F	T	F	F	F	F

we know that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\therefore$ $\sim(p \leftrightarrow q) \equiv \sim[(p \to q) \wedge (q \to p)]$

$\equiv \sim(p \to q) \vee \sim(q \to p) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

SUMMARY

(i) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

(ii) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

(iii) $\sim(p \Rightarrow q) \equiv \sim(\sim p \vee q) \equiv p \wedge (\sim q)$

(iv) $\sim(p \Leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ or $p \Leftrightarrow \sim q$ or $\sim p \Leftrightarrow q$

Note : $p \to q \equiv \sim p \vee q$

$p \to q \equiv (\sim p \vee q) \wedge (p \vee \sim q)$

Example # 12 : Write the negation of the following compound statements :

(i) All the students completed their homework and the teacher is present.

(ii) Square of an integer is positive or negative.

(iii) If my car is not in workshop, then I can go college.

(iv) ABC is an equilateral triangle if and only if it is equiangular

Solution : (i) The component statements of the given statement are :

p : all the students completed their homework.

q : The teacher is present.

The given statement is p and q, so its negation is ~p or ~q = Some of the students did not complete their home work or the teacher is not present.

(ii) The component statement of the given statements are :

p : Square of an integer is positive.

q : Square of an integer is negative.

The given statement is p or q, so its negation is ~p and ~q = Their exists an integer whose square is neither positive nor negative.

(iii) Consider the following statements :

p : My car is not in workshop

q : I can go to college

The given statement in symbolic form is $p \rightarrow q$

Now, $\sim(p \rightarrow q)\ \ p \wedge (\sim q)$

$\Rightarrow$ $\sim(p \rightarrow q)$: My car is not in workshop and I cannot go to college.

Hence the negation of the given statements is "My car is not in workshop and i can not go to college".

(iv) Consider the following statements :

p : ABC is an equilateral triangle.

q : It is equiangular

Clearly, the given statement is symbolic form is $p \leftrightarrow q$.

Now, $\sim(p \leftrightarrow q) \cong (p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow$ $\sim(p \rightarrow q)$: Either ABC is an equilateral triangle and it is not equiangular or ABC is not an equilateral triangle and it is equiangular.

Example # 13 : Show that $p \rightarrow (p \vee q)$ is a tautology

Solution :

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Example # 14 : By using laws of algebra of statements show that $(p \vee q) \wedge \sim p \equiv \sim p \wedge q$.

Solution. $(p \vee q) \wedge \sim p \equiv (\sim p) \wedge (p \wedge q)$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q)$

$\equiv f \vee (\sim p \wedge q)$

$\equiv \sim p \wedge q$

Example # 15 : Find the negation of statement $p \wedge \sim q$

Solution : Negation of $(p \wedge \sim q) = \sim(p \wedge \sim q)$ $\quad = \sim p \vee \sim\sim q = \sim p \vee q$

Self Practice Problems :

(7) Show that $[(\sim p \wedge q) \wedge (q \wedge r)] \wedge \sim q$ is a fallacy.

(8) By using laws of algebra of statements show that $p \wedge (p \vee q) \equiv p \vee (p \wedge q)$.

(9) By using laws of algebra of statements show that $p \vee (p \wedge q) \equiv p$.

(10) Find the negation of statement of $(p \rightarrow q) \rightarrow (q \rightarrow p)$.

Ans. (10) $(\sim p \vee q) \wedge (q \wedge \sim p)$

16. Contrapositive of a conditional statement :

If p and q are two statements then

Let $p \Rightarrow q$ Then
(Contrapositive of $p \Rightarrow q$)is$(\sim q \Rightarrow \sim p)$

Note : A statement and its contrapositive convey the same meaning.

Example # 16 : Write the contrapositive of the following statement : "If Mohan is poet, then he is poor"

Solution : Consider the following statements :

p : Mohan is a poet

q : Mohan is poor

Clearly, the given statement in symbolic form is $p \rightarrow q$.Therefore, its contrapositive is given by $\sim q \rightarrow \sim p$.

Now, $\sim p$: Mohan is not a poet.

$\sim q$: Mohan is not poor.

$\therefore \sim q \rightarrow \sim p$: If Mohan is not poor, then he is not a poet.

Hence the contrapositive of the given statement is "If Mohan is not poor, then he is not a poet".

17. Quantifiers :

There exists (atleast one), For all (for every)

Quantifiers are phrases like "There exists" ($\exists$) and " For all" ($\forall$). In general, a mathematical statement that says "for every" can be interpreted as saying that all the members of the given set S satisfies that property. While "There exists" can be interpreted as saying that "atleast one" member of the given set S satisfies that property.

Example :

P : For every real number x, $x^2 \geq 0$.

Q : There exists a natural no, which is even and prime.

Negation of statement that says "for every" can be interpreted as saying that atleast one member of given set S does not satisfies that property.

(using " There exists")

Example :

~P : There exists a real number x, such that $x^2 < 0$.

Negation of statement that says "There exists" can be interpreted as saying that every member of given set S does not satisfies that property. (Using "For every")

Example :

~Q : Every natural number is not even or not prime.

Self Practice Problems :

(11) Negate the following sentences using quantifiers.

(i) There exists $x \in Z$, such that $x^2 + x = 0$ (ii) For all $x \in R$, $\sqrt{x}$ is a real number.

Ans. (i) For all $x \in Z$, $x^2 + x \neq 0$.

(ii) There exists $x \in R$ such that $\sqrt{x}$ is not a real number.

Exercise-1

✍ Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Statements, Truth table, compound statement, Tautology , Fallacy, Algebra of statements

A-1.✍ Which of the following is a logical statement ?
(1) Open the door.
(2) What an intelligent student !
(3) Are you going to Delhi ?
(4) All prime numbers are odd numbers.

A-2. Which of the following is not a logical statement ?
(1) Two plus two equals four.
(2) The sum of two positive numbers is positive.
(3) Tomorrow is Friday.
(4) Every equilateral triangle is an isosceles triangle.

A-3. Consider the statement p : "New Delhi is a city". Which of the following is not negation of p?
(1) New Delhi is not a city.
(2) It is false that New Delhi is a city.
(3) It is not the case that New Delhi is not a city.
(4) It is not the case that New Delhi is a city

A-4. The negation of the statement $\sqrt{2}$ " is not a complex number" is
(1) $\sqrt{2}$ is a rational number.
(2) $\sqrt{2}$ is an irrational number.
(3) $\sqrt{2}$ is a real number.
(4) $\sqrt{2}$ is a complex number.

A-5. Which of the following is not a component statement of the statement '100 is divisible by 5, 10 and 11'?
(1) 100 is divisible by 5
(2) 100 is divisible by 10
(3) 100 is not divisible by 11
(4)100 is divisible by 11

A-6.✍ Which of the following statements is using an "inclusive Or" ?
(1) A number is either rational or irrational.
(2) All integers are positive or negative.
(3) The office is closed if it is a holiday or a Sunday.
(4) Sum of two integers is odd or even.

A-7. For the compound statement
"All prime numbers are either even or odd". Which of the following is true?
(1) Both component statements are false
(2) Exactly one of the component statements is true
(3) At least one of the component statements is true
(4) Both the component statements are true

A-8. Statement p : "Kota is in Rajasthan"
Statement q : "Bhopal is capital of Madhya Pradesh"
then $p \Rightarrow q$ is written as
(1) If Kota is in Rajasthan then Bhopal is not capital of Madhya Pradesh.
(2) Kota is in Rajasthan and Bhopal is capital of Madhya Pradesh.
(3) Kota is in Rajasthan or Bhopal is capital of Madhya Pradesh.
(4) If Kota is in Rajasthan then Bhopal is capital of Madhya Pradesh.

A-9. Statement p : "Ashok is honest"
Statement q : "Ashok is hardworker"
then statement "Ashok is honest if and only if he is hardworker" can be written mathematically as
(1) $p \wedge q$
(2) $p \vee q$
(3) $q \Rightarrow p$
(4) $p \Leftrightarrow q$

A-10. Which one statement gives the same meaning of statement "The Banana trees will bloom if it stays warm for a month."
(1) It stays warm for a month and the banana trees will bloom.
(2) If it stays warm for a month, then the Banana trees will bloom.
(3) It stays warm for a month or the banana trees will bloom.
(4) It stays warm for a month or the banana trees will not bloom.

A-11. The statement "x is an even number implies that x is divisible by 4" means the same as
(1) x is divisible by 4 is necessary condition for x to be an even number.
(2) x is an even number is a necessary condition for x to divisible by 4.
(3) x is divisible by 4 is a sufficient condition for x to be an even number.
(4) x is divisible by 4 implies that x is not always an even number.

A-12. If p and q are any two statements then $p \Rightarrow q$ is not equivalent to
(1) p is sufficient for q (2) q is necessary for p (3) p only if q (4) q only if p

A-13. Consider statement "If you drive over 100 km/hr, then you will get a fine". Now choose the correct option related with this statement

$p \to q.$

(1) 'Getting fine' is necessary condition.
(2) 'Driving over 100 km/hr' is necessary condition.
(3) 'Getting fine' is sufficient condition.
(4) If you donot drive over 100 km/hr then you will not get a fine

$p \to q.$

A-14. If p is true and q is false, then which of the following statement is not true?
(1) $p \vee q$ (2) $p \Rightarrow q$ (3) $p \wedge (\sim q)$ (4) $q \Rightarrow p$

A-15. Converse of statement $p \Rightarrow q$ is
(1) $p \wedge q$ (2) $p \vee q$ (3) $q \Rightarrow p$ (4) $p \Rightarrow \sim q$

A-16. If p, q, r and s are true propositions, then the truth values of
(i) $(p \wedge q) \to s$ (ii) $(q \wedge r) \to \sim s$ (iii) $(p \wedge \sim q) \wedge (q \to s)$ are respectively
(1) T, T and F (2) F, T and F (3) T, F and T (4) T, F and F

$\sim p \vee q$
$p \wedge \sim q$

A-17. Converse of statement "If Ram works hard then he is rich" is
(1) If Ram does not work hard then he is rich. (2) Ram works hard and he is rich.
(3) Ram works hard or he is rich. (4) If Ram is rich then he works hard.

$\sim q \vee p.$
$\sim p \vee q.$

A-18. If p, q, r are simple propositions, then the truth value of $(\sim p \vee q) \wedge \sim r \Rightarrow p$ is
(1) true if truth values of p, q, r are T, F, T respectively
(2) false if truth values of p, q, r are T, F, T respectively
(3) true if truth values of p, q, r are T, F, F respectively
(4) true if truth values of p, q, r are T, T, T respectively

$(\sim p \vee \sim q) \wedge (\sim q \vee p)$

$\sim q \vee p$

A-19. If p and q are simple propositions, then $p \Leftrightarrow \sim q$ is true when
(1) p is true and q is true (2) p is false and q is true
(3) both p and q are false (4) p and q both are not true

$p \vee \sim q.$

A-20. Consider the following statements :
p : f is a continuous function
q : f is an odd function
r : f is an even function
The proposition " f is a continuous function only if it is either even or odd" is represented is
(1) $p \to (q \vee r)$ (2) $(q \vee r) \to p$ (3) $p \wedge q \to r$ (4) $p \to (q \to r)$

$(q \vee r) \to p.$

A-21. Which of the following is logically equivalent to $\sim(p \leftrightarrow q)$

(1) $(\sim p) \leftrightarrow q$ (2) $(\sim p) \leftrightarrow (\sim q)$ (3) $p \rightarrow (\sim q)$ (4) $p \rightarrow q$

A-22. Let q : you have to start early to get success

p : success comes with luck

The statement " If success does not come with luck then you have to start early to get success" isrepresented by

(1) $(p \rightarrow q) \wedge (q \rightarrow p)$ (2) $(\sim p \rightarrow q) \wedge (\sim q \rightarrow p)$

(3) $(p \vee q) \wedge (\sim(p \wedge q))$ (4) $\sim p \leftrightarrow \sim q$

A-23. The statement

$[p \wedge (p \rightarrow q)] \rightarrow q$, is :

(1) a fallacy (2) a tautology

(3) neither a fallacy nor a tautology (4) not a compound statement

A-24. The proposition $(p \rightarrow \sim p) \wedge (\sim p \rightarrow p)$ is

(1) a tautology (2) a contradiction (3) equivalent to $p \rightarrow p$ (4) equivalent to $\sim p \rightarrow \sim p$

A-25. Which of the following statements is a fallacy ?

(1) $(p \rightarrow q) \leftrightarrow (q \vee \sim p)$ (2) $(\sim(\sim p \wedge q) \wedge (p \vee q)) \leftrightarrow p$

(3) $(\sim p \leftrightarrow q) \leftrightarrow (p \leftrightarrow \sim q)$ (4) $\sim(\sim p \leftrightarrow \sim q) \leftrightarrow \sim(\sim p \leftrightarrow q)$

A-26. If p is any logical statement, then :

(1) $p \wedge p = p$ (2) $p \vee (\sim p) = p$

(3) $p \wedge (\sim p)$ is a tautology (4) $p \vee (\sim p)$ is a fallacy

Section (B) : Negation of compound statements, Contrapositive of conditional statements, Quantifiers

B-1. The negation of the statement "Ramesh is cruel or he is strict" is

(1) Ramesh is neither cruel nor strict. (2) Ramesh is cruel or he is not strict.

(3) Ramesh is not cruel or he is strict. (4) Ramesh is not cruel and he is strict.

B-2. The negation of the statement "The sand heats up quickly in the sun and does not cool down fast at night" is

(1) The sand does not heat up quickly in the sun and it does not cool down fast at night.

(2) Either the sand does not heat up quickly in the sun or it cools down fast at night.

(3) The sand heats up quickly in the sun and it cools down fast at night.

(4) The sand heats up quickly in the sun or it cools down fast at night.

B-3. The negation of the statement "If a quadrilateral is a square then it is a rhombus".

(1) If a quadrilateral is not a square then it is a rhombus.

(2) If a quadrilateral is a square then it is not a rhombus.

(3) A quadrilateral is a square and it is not a rhombus.

(4) A quadrilateral is not a square and it is a rhombus.

B-4. "If India beats Australia, then India qualifies for the world cup" Negation of the above is:

(1) If India doesn't beat Australia, then India does not qualify for the world cup.

(2) India beats Australia and India does not qualify for the world cup.

(3) Neither India beasts Australia, not India qualifies for the world cup.

(4) India does not beat Australia and India qualifies for the world cup.

B-5. The negation of the statement "Two lines are parallel if and only if they have the same slope" is
(1) Two lines are not parallel and they have the same slope.
(2) Two lines are parallel and they do not have the same slope.
(3) Two lines are not parallel and they do not have the same slope.
(4) Either two lines are parallel and they have different slopes or two lines are not parallel and they have the same slope.

B-6. Negation of the following is
" Demonetisation is a successful step, if and only if Modi Ji is the prime minister"
(1) Demonetisation is a not successful step, if Modi Ji is not the prime minister.
(2) Demonetisation is a successful step and Modi Ji is the prime minister and demonetisation is not a successful step.
(3) Demonetisation is not a successful step and Modi Ji is not the prime minister or demonetisation is a successful step and modi ji is the prime minister.
(4) Demonetisation is a successful step if and only if modi ji is not the prime minister.

B-7. Negation of the statement $p \rightarrow (q \wedge r)$ is
(1) $\sim p \rightarrow \sim(q \wedge r)$ (2) $\sim p \vee (q \wedge r)$ (3) $(q \wedge r) \rightarrow p$ (4) $p \wedge (\sim q \vee \sim r)$

B-8. Consider the following statements :
S_1: Negation of $(\sim p \rightarrow q)$ is $[\sim(p \vee q)] \wedge [p \vee (\sim p)]$.
S_2: Negation of $(p \leftrightarrow q)$ is $(p \wedge \sim q) \vee (\sim p \wedge q)$.
S_3: Negation of $(p \vee q)$ is $\sim p \wedge \sim q$.
S_4: $p \leftrightarrow q$ is equivalent to $(\sim p \vee q) \wedge (p \vee \sim q)$.
State, in order, whether S_1, S_2, S_3, S_4 are true or false
(1) TTTT (2)TFTF (3)FFTT (4)FTFT

B-9. The negation of $A \rightarrow (A \vee \sim B)$ is
(1) a fallacy (2) a tautology
(3) equivalent to $(A \vee B) \rightarrow A$ (4) equivalent to $A \rightarrow (A \wedge \sim B)$

B-10. Let p : you got a seat in IIT
q : you got selected in BITS
The statement " you got a seat in both IIT and BITS " is represented by
(1) $p \rightarrow \sim q$ (2) $\sim p \vee \sim q$ (3) $\sim(p \rightarrow \sim q)$ (4) $\sim(\sim p \wedge \sim q)$

B-11. Negation of the statement $(p \wedge r) \rightarrow (r \vee q)$ is
(1) $(p \wedge r) \wedge (r \vee q)$ (2) $(\sim p \vee \sim r) \wedge (r \vee q)$ (3) a tautology (4) a fallacy

B-12. Which one statement gives the same meaning of statement
"If you watch television, then your mind is free and if your mind is free then you watch television"
(1) You watch television if and only if your mind is free.
(2) You watch television and your mind is free.
(3) You watch television or your mind is free.
(4) None of these

B-13. The contrapositive of statement "Something is cold implies that it has low temperature" is
(1) If something does not have low temperature, then it is not cold.
(2) If something does not have low temprerature then it is cold.
(3) Something is not cold implies that it has low temperature.
(4) If something have low temperature, then it is not cold.

B-14. If $x = 5$ and $y = -2$ then $x - 2y = 9$. The contrapositive of this statement is
(1) If $x - 2y = 9$ then $x = 5$ and $y = -2$ (2) If $x - 2y \neq 9$ then $x \neq 5$ and $y \neq -2$
(3) If $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$ (4) If $x - 2y \neq 9$ then either $x \neq 5$ or $y = -2$

B-15. The contrapositive of the following statement,
" If the side of a square doubles, then its area increases four times", is
(1) If the area of a square does not increase four times, then its side is not doubled.
(2) If the area of a square increases four times, then its side is not doubled.
(3) If the area of a square increases four times, then its side is doubled.
(4) If the side of a square is not doubled, then its area does not increase four times.

B-16. The contrapositive of the statement "If it is raining, then I will not come", is :
(1) If I will come, then it is raining (2) If I will not come, then it is not raining
(3) If I will not come, then it is raining (4) If I will come, then it is not raining

B-17. Consider the following statements :
p : I have the raincoat
q : I cannot walk in the rain
The proposition " If I walk in the rain then I do not have the raincoat " is represented by
(1) $p \to \sim q$ (2) $q \to \sim p$ (3) $p \to q$ (4) $\sim q \to p$

B-18. The contrapositive of the statement "p implies q" is
(1) ~p implies ~ q (2) q implies p (3) ~q implies ~p (4) p only if q

B-19 The contrapositive of $(p \wedge q) \Rightarrow r$ is
(1) $\sim r \Rightarrow (p \vee q)$ (2) $r \Rightarrow (p \vee q)$ (3) $\sim r \Rightarrow (\sim p \vee \sim q)$ (4) $p \Rightarrow (q \vee r)$

B-20. The contrapositive of $p \to (\sim q \to \sim r)$ is
(1) $(\sim q \wedge r) \to \sim p$ (2) $(q \wedge \sim r) \to \sim p$ (3) $p \to (\sim r \vee q)$ (4) $p \wedge (q \vee r)$

B-21. Negation of the following statement "Every natural number is greater than 0" is
(1) Every natural number is less than 0.
(2) Every natural number is less than or equal to 0.
(3) There exists a natural number which is not greater than 0.
(4) Atleast one natural numbers is greater than 0.

B-22.✍ Consider the statement p : "Everyone in Germany speaks German" which of the following is not negation of p
(1) Not everyone in Germany speaks German.
(2) No one in Germany speaks German.
(3) There are persons in Germany who do not speak German.
(4) There is atleast one person in Germany who does not speak German.

Exercise-2

✍ **Marked Questions may have for Revision Questions.**

PART - I : OBJECTIVE QUESTIONS

1. Which of the following is not a logical statement
(1) There are 12 month in a year.
(2) The sun is a star.
(3) Product of two irrational numbers is irrational.
(4) Sum of two rational numbers is irrational.

2. If p : Mumbai is in Japan and q : Delhi is in South Africa then
(1) $p \to q$ is true (2) $p \to q$ is false (3) $p \to \sim q$ is false (4) $\sim q \to \sim p$ is false

3.✍ The converse of the statement "If it is a national holiday, then kids go to picnic with their parents" , is
(1) If it is a national holiday, then kids go to picnic with their parents.
(2) kids go to picnic with their parents and it is a national holiday.
(3) If kids go to picnic with their parents, then it is a national holiday.
(4) kids go to picnic with their parents or it is a national holiday.

4. If p, q, r are three statements then converse of $p \Rightarrow (q \sim r)$ is

(1) $\sim (r \vee \sim q) \rightarrow p$ (2) $(r \vee \sim q) \rightarrow p$ (3) $(r \vee q) \rightarrow p$ (4) $\sim (r \wedge \sim q) \rightarrow p$

5. Consider statement "If you are born in India then you are a citizen of India". Which of the following is logical equivalent to the given statement ?

(1) If you are not born in India then you are not a citizen of India.

(2) If you are a citizen of India then you are not born in India.

(3) You are born in India only if you are a citizen of India.

(4) Taking birth in India is not sufficient condition to be a citizen of India.

6. Consider statement "If there are clouds in the sky then it will rain". Which of the following give same meaning?

(1) Having clouds in the sky is sufficient to have rain.

(2) It is not necessary to have rain if there are clouds in the sky.

(3) If it is raining then there are clouds in the sky.

(4) There are clouds in the sky implies it will not rain

7. Statements "If the traders do not reduce the price then the government will take action against them" is equivalent to

(1) It is not true that the trader do not reduce the prices and government does not take action against them.

(2) It is true that the trader do not reduce the prices and government does not take action against them.

(3) It is not true that the trader do not reduce the prices and government take action against them.

(4) It is not true that the trader do not reduce the prices or government take action against them.

8. Statement $p \wedge (\sim p \vee q)$ is equivalent to

(1) $p \vee q$ (2) $p \wedge q$ (3) $\sim p \wedge q$ (4) $\sim p \vee q$

9. Statement $(p \wedge q) \vee \sim p$ is equivalent to

(1) $p \vee q$ (2) $\sim p \wedge q$ (3) $\sim p \vee q$ (4) $\sim p \vee \sim q$

10. Statement $((\sim q) \wedge p) \vee (p \vee \sim p)$ is a

(1) Tautology (2) Fallacy (3) $\sim p \wedge \sim q$ (4) $p \vee q$

11. Statement $[(p \leftrightarrow q) \wedge ((q \rightarrow r) \wedge r)] \rightarrow r$ is a

(1) Fallacy (2) Tautology (3) $\sim p \wedge \sim q$ (4) $p \vee q$

12. The proposition $\sim(p \vee \sim q) \vee \sim(p \vee q)$ is logically equivalent to :

(1) p (2) q (3) $\sim p$ (4) $\sim q$

13. Let p, q, r denote arbitarary statements then the logically equivalent of the statement $p \Rightarrow (q \vee r)$ is

(1) $(p \Rightarrow q) \vee (p \Rightarrow r)$ (2) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$

(3) $(p \vee q) \Rightarrow r$ (4) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$

14. Negation of the compound proposition $p \vee (\sim p \vee q)$ is

(1) Tautology (2) Fallacy (3) $\sim p \wedge q$ (4) $\sim p \vee q$

15. Negation of statement "If ΔABC is right angled at B, then $AB^2 + BC^2 = AC^2$" is

(1) ΔABC is right angled at B and $AB^2 + BC^2 \neq AC^2$.

(2) ΔABC is right angled at B then $AB^2 + BC^2 \neq AC^2$.

(3) ΔABC is right angled at B or $AB^2 + BC^2 \neq AC^2$.

(4) ΔABC is right angled at B and $AB^2 + BC^2 = AC^2$.

16. p : you want to succeed
q : you will find a way then the negation of $\sim(p \vee q)$ is
(1) If you want to succeed then you can't find way
(2) If you don't want to succeed then you will find a way
(3) you wan't to succeed and you find a way
(4) you wan't to succeed and you don't find a way

17. Negation of $(\sim p \to q)$ is
(1) $\sim p \vee \sim q$
(2) $\sim (p \vee q) \vee (p \vee (\sim p))$
(3) $\sim (p \vee q) \wedge (p \vee (\sim p))$
(4) $(\sim p \vee q) \wedge (p \vee \sim q)$

18. Contrapositive of statement "If you watch television, then your mind is free" is
(1) If your mind is free then you are not watching television
(2) If your mind is not free then you are not watching television
(3) If your mind is not free then you are watching television
(4) If your mind is free then you are watching television

19. Consider the following two statements :
P : If 7 is an odd number, then 7 is divisible by 2.
Q : If 7 is a prime number, then 7 is an odd number.
If V_1 is the truth value of contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals :
(1) (F, T) (2) (T, F) (3) (F, F) (4) (T, T)

20. The contrapositive of the statments " If I am not feeling well , then I will go to the doctor" is
(1) If I will go to the doctor then I am not feeling well.
(2) If I will not go to the doctor then I am feeling well.
(3) If I am feeling well then I will not go to the doctor.
(4) If I will go to the doctor then I am feeling well.

21. Let p : Team India plays well ; q : Virat Kohli is the captain, then the contrapositive of the implication $p \to q$ in the verbal form is-
(1) If team India does not play well then Virat Kohli is not the captain.
(2) If Team India plays well then Virat Kohli is not the captain
(3) If Virat Kohli is not the captain, then team India plays well.
(4) If Virat Kohli is not the captain, then team India does not play well.

22. The negation of the statement "There exists a number which is equal to its square" is
(1) There exists a number which is not equal to its square.
(2) There exists no number which is not equal to its square.
(3) There does not exists a number which is equal to its square.
(4) The square of a number is greater than the number.

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.
(1) Both the statements are true.
(2) Statement-I is true, but Statement-II is false.
(3) Statement-I is false, but Statement-II is true.
(4) Both the statements are false.

A-1. **Statement - 1 :** $\sim (A \Leftrightarrow \sim B)$ is equivalent to $A \Leftrightarrow B$.
Statement - 2 : $A \vee (\sim(A \wedge \sim B))$ a tautology.

A-2. Let p and q be any two propositions.
Statement 1 : $(p \to q) \leftrightarrow q \vee \sim p$ is a tautology.
Statement 2 : $\sim(\sim p \wedge q) \wedge (p \vee q) \leftrightarrow p$ is a fallacy.

A-3. **Statement-1 :** Consider the statements

p : Sachin Tendulakar is a good cricketer.

q : Mukesh Ambani is a rich person in india.

Then the negation of statement $p \vee q$, is 'Sachin Tendulakar is not a good cricketer and Mukesh Ambani is not a rich person in india".

Statement - 2 : For any two statements p and q

$$\sim(p \vee q) = \sim p \vee \sim q$$

$$\sim(p \vee q) = \sim p \vee \sim q$$

A-4. **Statement -1 :** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

Statement -2 : $\sim(p \leftrightarrow \sim q)$ is a tautology

A-5. **Statement-1 :** The type of "OR" used in the statement "You may have a voter card or a PAN card for your identity proof" is inclusive OR.

Statement-2 : Inclusive OR is said to be used in a statement if its component statements both may happen together.

Section (B) : MATCH THE COLUMN

B-1.

	Column - I		Column - II
(A)	$\sim(\sim p \wedge q)$ is equivalent to	(p)	$p \vee (p \wedge q)$
(B)	$p \wedge (p \vee q)$ is equivalent to	(q)	t
(C)	$(p \wedge q) \vee [\sim p \vee (p \wedge \sim q)]$ is equivalent to	(r)	$p \vee \sim q$
(D)	$(p \wedge q) \rightarrow p$ is equivalent to	(s)	$(\sim p \wedge q) \vee t$

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. Which type of sentences are not logical statements.

(1) Imperative sentence (Expresses a request or command)

(2) Exclaimatory sentence (Expresses some strong feeling)

(3) Interrogative sentence (Asks some question)

(4) Optative sentence (Blessing & wishes)

C-2. Which of the following statements is using an "exclusive Or" ?

(1) A polygon is concave or convex.

(2) To apply for a driving license, you should have a ration card or a passport.

(3) The office is closed if it is a holiday or a Sunday.

(4) I will take leave and stay in home or I will go to office.

C-3. In which of the following compound statements, the connective "or" is exclusive?

(1) If x is a real number then x is either rational or irrational.

(2) If x is an integer, then either $x \geq 0$ or $x \leq 0$.

(3) If x is any real number, then either $x \geq 0$ or $x \leq 0$.

(4) Lines are said to be parallel If they are non intersecting or coincident.

C-4. Which of the following is a fallacy

(1) $\sim(p \Rightarrow q) \Leftrightarrow (\sim p \sim q)$ (2) $(p \wedge \sim q) \wedge (\sim p \vee q)$

(3) $p \wedge \sim p$ (4) $(p \wedge \sim q) \wedge (\sim p \wedge q)$

C-5. Statement p : "An equilateral triangle is equiangular" then negation of statement p is

(1) An equilaterial triangle is not equiangular

(2) It is false that an equilateral triangle is equiangular

(3) It is not the case that an equilateral triangle is not equiangular

(4) There exist at least one equilateral triangle which is not equiangular

Exercise-3

✍ Marked Questions may have for Revision Questions.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number" and r be the statement "x is a irrational number iff y is a transcendental number".
Statement-1 : r is equivalent to either q or p.
Statement-2 : r is equivalent to $\sim(p \rightarrow \sim q)$. **[AIEEE - 2008, (4, –1), 120]**
(1) Statement-1 is False, Statement-2 is True
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(4) Statement-1 is True, Statement-2 is False

2. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to **[AIEEE - 2008, (4, –1), 120]**
(1) $p \rightarrow (p \rightarrow q)$ (2) $p \rightarrow (p \rightarrow q)$ (3) $p \rightarrow (p \vee q)$ (4) $p \rightarrow (p \wedge q)$

3. **Statement -I** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$. **[AIEEE - 2009, (4, –1), 120]**
Statement -II $\sim(p \leftrightarrow \sim q)$ is a tautology.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

4. Let S be a non-empty subset of **R**. Consider the following statement :
P : There is a rational number $x \in S$ such that $x > 0$.
Which of the following statements is the negation of the statement P ? **[AIEEE - 2010, (4, –1), 120]**
(1) There is no rational number $x \in S$ such that $x \le 0$.
(2) Every rational number $x \in S$ satisfies $x \le 0$.
(3) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.
(4) There is a rational number $x \in S$ such that $x \le 0$.

5.✍ Consider the following statements **[AIEEE - 2011(I), (4, –1), 120]**
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest.
The negation of the statement " Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as :
(1) $\sim P \wedge (Q \leftrightarrow \sim R)$ (2) $\sim (Q \leftrightarrow (P \wedge \sim R))$ (3) $\sim Q \leftrightarrow \sim P \wedge R$ (4) $\sim (P \wedge \sim R) \leftrightarrow Q$

6. The only statement among the following that is a tautology is - **[AIEEE-2011(II), (4, –1), 120]**
(1) $A \wedge (A \vee B)$ (2) $A \vee (A \wedge B)$ (3) $[A \wedge (A \rightarrow B)] \rightarrow B$ (4) $B \rightarrow [A \wedge (A \rightarrow B)]$

7.✍ The negation of the statement **[AIEEE - 2012, (4, –1), 120]**
"If I become a teacher, then I will open a school", is :
(1) I will become a teacher and I will not open a school.
(2) Either I will not become a teacher or I will not open a school.
(3) Neither I will become a teacher nor I will open a school
(4)I will not become a teacher or I will open a school.

8. Consider [AIEEE - 2013, (4, –1), 120]

Statement-I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-II : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.

(1) Statement-I is true; Statement-II is true; Statement-II is a correct explanation for Statement-I.

(2) Statement-I is true; Statement-II is true; Statement-II is **not** a correct explanation for Statement-I.

(3) Statement-I is true; Statement-II is false.

(4) Statement-I is false; Statement-II is true.

9. The statement $\sim(p \leftrightarrow \sim q)$ is : **[JEE(Main) 2014, (4, – 1), 120]**

(1) a tautology (2) a fallacy (3) equivalent to $p \leftrightarrow q$ (4) equivalent to $\sim p \leftrightarrow q$

10. The negation of $\sim s \vee (\sim r\, s)$ is equivalent to **[JEE(Main) 2015, (4, – 1), 120]**

(1) $s \wedge \sim r$ (2) $s \wedge (r \wedge \sim s)$ (3) $s \vee (r \vee \sim s)$ (4) $s \wedge r$

11. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $p \wedge q$ (2) $p \vee q$ (3) $p \vee \sim q$ (4) $\sim p \wedge q$

12. The following statement **[JEE(Main) 2017, (4, – 1), 120]**

$(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is :

(1) a tautology (2) equivalent to $\sim p \rightarrow q$

(3) equivalent to $p \rightarrow \sim q$ (4) a fallacy

Answers

EXERCISE # 1

Section (A) :

A-1.	(4)	A-2.	(3)	A-3.	(3)	A-4.	(4)	A-5.	(3)	A-6.	(3)	A-7.	(1)
A-8.	(4)	A-9.	(4)	A-10.	(2)	A-11.	(1)	A-12.	(4)	A-13.	(1)	A-14.	(2)
A-15	(3)	A-16.	(4)	A-17.	(4)	A-18.	(1)	A-19.	(2)	A-20.	(1)	A-21.	(1)
A-22.	(2)	A-23.	(2)	A-24.	(2)	A-25.	(4)	A-26.	(1)				

Section (B) :

B-1.	(1)	B-2.	(2)	B-3.	(3)	B-4.	(2)	B-5.	(4)	B-6.	(4)	B-7.	(4)
B-8.	(1)	B-9.	(1)	B-10.	(3)	B-11.	(4)	B-12.	(1)	B-13.	(1)	B-14.	(3)
B-15.	(1)	B-16.	(4)	B-17.	(3)	B-18.	(3)	B-19	(3)	B-20	(1)	B-21.	(3)
B-22.	(2)												

EXERCISE # 2

PART - I

1.	(3)	2.	(1)	3.	(3)	4.	(1)	5.	(3)	6.	(1)	7.	(1)
8.	(2)	9.	(3)	10.	(1)	11.	(2)	12.	(3)	13.	(1)	14.	(2)
15.	(1)	16.	(2)	17.	(3)	18.	(2)	19.	(1)	20.	(2)	21.	(4)
22.	(3)												

PART - II

Section (A) :

A-1. (1) A-2. (2) A-3. (2) A-4. (2) A-5. (1)

Section (B) :

B-1. (A) → (r), (B) → (p), (C) → (s), (D) → (q)

Section (C) :

C-1. (1,2,3,4) C-2. (1,4) C-3. (1,4) C-4. (2,3,4) C-5. (1,2,4)

EXERCISE # 3

PART - I

1.	(1)	2.	(3)	3.	(3)	4.	(2)	5.	(2)	6.	(3)	7.	(1)
8.	(2)	9.	(3)	10.	(4)	11.	(2)	12.	(1)				