L7 03 Feb 2021

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Equations solvable for y :→ $y = f(x,p)$ ✓

(1) $y = x + a\tan^{-1}p$ — (1)

Diff. w.r.t. x

$\frac{dy}{dx} = p$

$$\frac{dy}{dx} = 1 + a\cdot\frac{1}{1+p^2}\cdot\frac{dp}{dx}$$

but $\frac{dy}{dx} = p$

$$p = 1 + \frac{a}{1+p^2}\frac{dp}{dx}$$

$$(p-1) = \frac{a}{1+p^2}\frac{dp}{dx}$$

$$\int\frac{dx}{a} = \int\frac{1}{(p-1)(1+p^2)}dp$$

$$\frac{1}{a}\cdot x = \int\frac{1}{(p-1)(1+p^2)}dp$$

$$\frac{1}{(p-1)(1+p^2)} = \frac{A}{p-1} + \frac{Bp+C}{1+p^2}$$

$$1 = A(1+p^2) + (Bp+C)(p-1)$$

put $p=1$

$$1 = A(1+1) + 0 \Rightarrow \boxed{A = \tfrac{1}{2}}$$

Compare the coeff. of p^2

$$0 = A + B \Rightarrow \boxed{B = -A = -\tfrac{1}{2}}$$

Constant term

$$1 = A - C \Rightarrow \boxed{C = A - 1 = \tfrac{1}{2} - 1 = -\tfrac{1}{2}}$$

$$\frac{x}{a} = \int\left(\frac{\frac{1}{2}}{p-1} + \frac{-\frac{1}{2}p - \frac{1}{2}}{1+p^2}\right)dp$$

$$\frac{x}{a} = \frac{1}{2}\log(p-1) - \frac{1}{2}\left[\int\frac{p}{1+p^2}dp + \int\frac{1}{1+p^2}dp\right]$$

$$\frac{x}{a} = \frac{1}{2}\log(p-1) - \frac{1}{2\times 2}\int\frac{2p}{1+p^2}dp - \frac{1}{2}\int\frac{1}{1+p^2}dp$$

$$\frac{x}{a} = \frac{1}{2}\log(p-1) - \frac{1}{4}\log(1+p^2) - \frac{1}{2}\tan^{-1}p + c$$

$$\boxed{x = \frac{a}{2}\log(p-1) - \frac{a}{4}\log(1+p^2) - \frac{a}{2}\tan^{-1}p + ac} \quad (2)$$

$$y = x + a\tan^{-1}p \quad (1)$$

(1) & (2) Combined form the solution

① and ② Combined form the solution

② $y = xp^2 + p$

$$\frac{dy}{dx} = x\left(2p\frac{dp}{dx}\right) + p^2 + \frac{dp}{dx}$$

$$p = 2xp\frac{dp}{dx} + p^2 + \frac{dp}{dx}$$

$$(p - p^2) = \frac{dp}{dx}(2xp + 1)$$

$$\frac{dx}{dp} = \frac{2xp}{p-p^2} + \frac{1}{p-p^2}$$

$$\frac{dx}{dp} = \frac{2}{1-p}x + \frac{1}{p(1-p)}$$

$$\boxed{\frac{dy}{dx} + Py = Q}$$

$$I.F = e^{-\int\frac{-2}{1-p}dp} = e^{-2\log(1-p)} = e^{\log(1-p)^{-2}} = \frac{1}{(1-p)^2}$$

$$-\int\frac{-2}{1-p}dp = -2\log(1-p)$$

Sol

$$x \cdot I.F = \int (Q \cdot I.F)\,dp$$

$$x \cdot \left(\frac{1}{(1-p)^2}\right) = \int \frac{1}{(p-p^2)(1-p)^2}\,dp$$

$$\frac{x}{(1-p)^2} = \int \frac{1}{p(1-p)^3}\,dp$$

$$\frac{1}{p(1-p)^3} = \frac{A}{p} + \frac{B}{1-p} + \frac{C}{(1-p)^2} + \frac{D}{(1-p)^3}$$

Homework ?

Equations solvable for x

$x = f(y, p)$

Ex (1.1) ②

$p^3y + 2px = y \longrightarrow \boxed{p^3y^2 + 2px = y}$?

$$2px = y - p^3y$$

$$x = \frac{y - p^3y}{2p}$$

$$\frac{dx}{dy} = \frac{(2p)\left[1 - p^3 - y\,3p^2\frac{dp}{dy}\right] - (y - p^3y)\,2\frac{dp}{dy}}{(2p)^2}$$

$$\frac{1}{p} = 2p - 2p^4 - 6yp^3\frac{dp}{dy} - 2y\frac{dp}{dy} + 2p^3y\frac{dp}{dy}$$

$$\frac{1}{\cancel{p}} = \frac{2p - 2p^4 - 6yp^3\frac{dp}{dy} - 2y\frac{dp}{dy} + 2p^3y\frac{dp}{dy}}{4 \cdot p^{\cancel{2}}}$$

$$4p = 2p - 2p^4 + \frac{dp}{dy}\left(-6yp^3 - 2y + 2p^3y\right)$$

$$4p - 2p + 2p^4 = y\frac{dp}{dy}\left(-6p^3 + 2p^3 - 2\right)$$

$$2p + 2p^4 = y\frac{dp}{dy}\left(-4p^3 - 2\right)$$

$$\cancel{2}p\left(p^3 + 1\right) = -\cancel{2}y\,\frac{dp}{dy}\left(2p^3 + 1\right)$$

$$\int \frac{dy}{y} = \int \frac{\left(2p^3 + 1\right)}{p\left(p^3 + 1\right)}\,dp = ?$$