L39 17 April 2021

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Divergence of Curl :- $\text{div}(\text{curl}\,\vec{v}) = \nabla\cdot(\nabla\times\vec{v}) = 0$

$\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$

$$\text{curl}\,\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix} = \hat{i}\left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right) - \hat{j}\left(\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right) + \hat{k}\left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right)$$

$$\nabla\cdot(\nabla\times\vec{v}) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left[\hat{i}\left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right) - \hat{j}\left(\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right) + \hat{k}\left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right)\right]$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right)$$

$$= \cancel{\frac{\partial^2 v_3}{\partial x\partial y}} - \cancel{\frac{\partial^2 v_2}{\partial x\partial z}} - \cancel{\frac{\partial^2 v_3}{\partial y\partial x}} + \cancel{\frac{\partial^2 v_1}{\partial y\partial z}} + \cancel{\frac{\partial^2 v_2}{\partial z\partial x}} - \cancel{\frac{\partial^2 v_1}{\partial z\partial y}}$$

$$= \underline{\underline{0}}$$

Let $\vec{v}$ denote the velocity of a fluid in a medium.

If $\text{div}\,\vec{v} = 0$ then the <u>fluid</u> is s.t.b <u>incompressible</u>

If $\text{div}\,\vec{v} = 0$ then the vector field $\vec{v}$ is s.t.b <u>solenoidal</u>

$\text{Curl}(\vec{v}) = 0$ then $\vec{v}$ is s.t.b an <u>irrotational field</u>

<u>Conservative field</u>

A force field $\vec{F}$ is s.t.b Conservative if it is derivable from a potential function f i.e

$\text{grad}\,f = \vec{F}$

derivable from a potential function f i.e

$$\vec{F} = \text{grad } f.$$

$$\text{grad } f = \vec{F}$$

$$\text{Curl } \vec{F} = \text{curl}(\text{grad } f) = 0$$

If $\vec{F}$ is Conservative
then $\text{curl}(\vec{F}) = 0$
and there exists a Scalar function f. such that

$$\vec{F} = \text{grad } f.$$

$\vec{F} \rightarrow \vec{F} = \text{grad } f.$

$\downarrow$

Conservative

$$\text{Curl } \vec{F} = \text{Curl}(\text{grad } f) = 0$$

Show that the vector field $\vec{v}$ is irrotational and find a scalar function f such that $\vec{v} = \text{grad } f$.

$$\vec{v} = 3x^2y^2z^4\,\hat{i} + 2x^3yz^4\,\hat{j} + 4x^3y^2z^3\,\hat{k}$$

$$\text{Curl } \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^2z^4 & 2x^3yz^4 & 4x^3y^2z^3 \end{vmatrix} = \hat{i}\left(\cancel{8x^3yz^3} - \cancel{8x^3yz^3}\right) - \hat{j}\left(\cancel{12x^2y^2z^3} - \cancel{12x^2y^2z^3}\right)$$

$$+ \hat{k}\left(\cancel{6x^2yz^4} - \cancel{6x^2yz^4}\right) = 0$$

$\text{Curl } \vec{v} = 0 \Rightarrow \vec{v}$ is irrotational

$$\vec{v} = \text{grad } f.$$

$$3x^2y^2z^4\,\hat{i} + 2x^3yz^4\,\hat{j} + 4x^3y^2z^3\,\hat{k} = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$\frac{\partial f}{\partial x} = 3x^2y^2z^4, \quad \frac{\partial f}{\partial y} = 2x^3yz^4, \quad \frac{\partial f}{\partial z} = 4x^3y^2z^3$$

$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$

$\downarrow$ integral

$f(x,y,z) = \cancel{3}\cdot\frac{x^3}{\cancel{3}}\cdot y^2 z^4 + g(y,z)$

$c \to 2, 3, 4$

$g(y,z) = y^2 z$

diff.

$\frac{\partial f}{\partial y} = x^3 2y z^4 + \frac{\partial g}{\partial y}$

$\cancel{2x^3yz^4} = \cancel{x^3 2y z^4} + \frac{\partial g}{\partial y} \Rightarrow \frac{\partial g}{\partial y} = 0$

$\Rightarrow$ g is independent of y

$g = g(z)$

$f(x,y,z) = x^3 y^2 z^4 + g(z)$

$\frac{\partial f}{\partial z} = x^3 y^2 4z^3 + \frac{dg}{dz}$

$\cancel{4x^3y^2z^3} = \cancel{4x^3y^2z^3} + \frac{dg}{dz}$ $\Rightarrow \frac{dg}{dz} = 0$

$\Rightarrow$ g is constant

$\boxed{g = K}$

$$\boxed{f(x,y,z) = x^3 y^2 z^4 + K}$$

Q13. If A and B are two constant vectors then div(A x B) =

(a) 0 (b) 2(A+B) (c) A.B (d) A+B

$$A \times B = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \frac{\partial}{\partial x}(\quad) + \frac{\partial}{\partial y}(\quad)$$

$\downarrow$

$0 + 0 + 0$