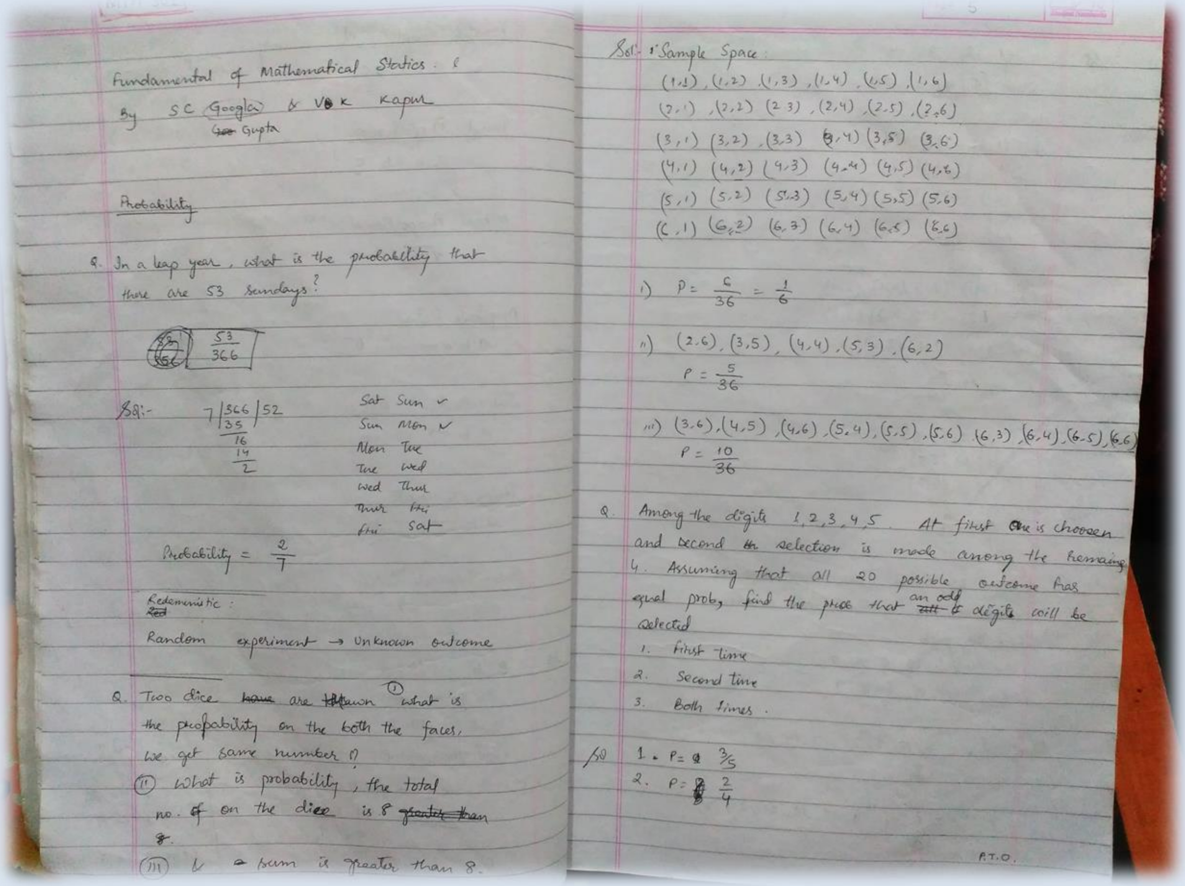


Fundamental of Mathematical Statics : 8

By SC Googla & V.K Kapur
~~Goo~~ Gupta

Probability

Q. In a leap year, what is the probability that there are 53 Sundays?

$\boxed{\frac{53}{366}}$

Sol:- $7 \overline{)366} 52$
35
16
14
2

Sat Sun ✓
Sun Mon ✓
Mon Tue
Tue Wed
Wed Thur
Thur Fri
Fri Sat

Probability $= \frac{2}{7}$

Redeministic :

Random experiment → Unknown outcome

Q. Two dice ~~have~~ are thrown (i) what is the probability on the both the faces, we get same number?
(ii) what is probability, the total no. of on the dice is 8 ~~greater than 8~~.
(iii) & sum is greater than 8.

Sol:- Sample Space

(1,1), (1,2) (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2) (2,3), (2,4), (2,5), (2,6)
(3,1) (3,2), (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

i) $P = \frac{6}{36} = \frac{1}{6}$

ii) (2,6), (3,5), (4,4), (5,3), (6,2)
$P = \frac{5}{36}$

iii) (3,6), (4,5), (4,6), (5,4), (5,5), (5,6) (6,3), (6,4), (6,5), (6,6)
$P = \frac{10}{36}$

Q. Among the digits 1, 2, 3, 4, 5. At first one is choosen and second the selection is made among the remaining 4. Assuming that all 20 possible outcome has equal prob, find the prob that an odd digits will be selected
1. First time
2. Second time
3. Both times.

Sol 1. $P = 3/5$
2. $P = \frac{2}{4}$

P.T.O.

Sol

1 2 3 4 5

$\boxed{5\,|\,4} = 20$

i) (1,2) (1,3) (1,4) (1,5)
(3,1) (3,2) (3,4) (3,5)
(5,1) (5,2) (5,3) (5,4)

$P = \frac{12}{20}$

ii) (2,1), (3,1), (4,1), (5,1)
(1,3) (2,3) (4,3) (5,3)
(1,5) (2,5) (3,5) (4,5)

$P = \frac{12}{20}$

iii) (1,3) (1,5)
(3,1) (3,5)
(5,1) (5,3)

$P = \frac{6}{20}$

Q. From 25 tickets mark with first 25 numbers. One is drawn at random find the prob

a. Multiple of 5 or 7
b. Multiple of 3 or 7

Sol a. 5, 10, 15, 20, 25, 7, 14, 21

$P = \frac{8}{25}$

b. 3, 6, 9, 12, 14, 15, 18, 21, 24, 7,

$P = \frac{10}{25}$

Q Four cards are drawn at random from a pack of 52 cards. Find the prob. (a) They are king, queen, jack, ace
(b) Two are king, two are queen
(c) Two are black, two are red
(d) Two are heart, two are diamond.

Sol a) $P = \frac{\overset{K}{4} \times \overset{Q}{4} \times \overset{J}{4} \times \overset{A}{4}}{{}^{52}C_4}$

$P = \frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_4}$

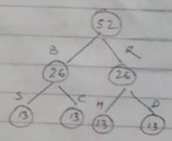
${}^nC_r = \frac{n!}{r!\,(n-r)!}$

↓

To find no. of ways

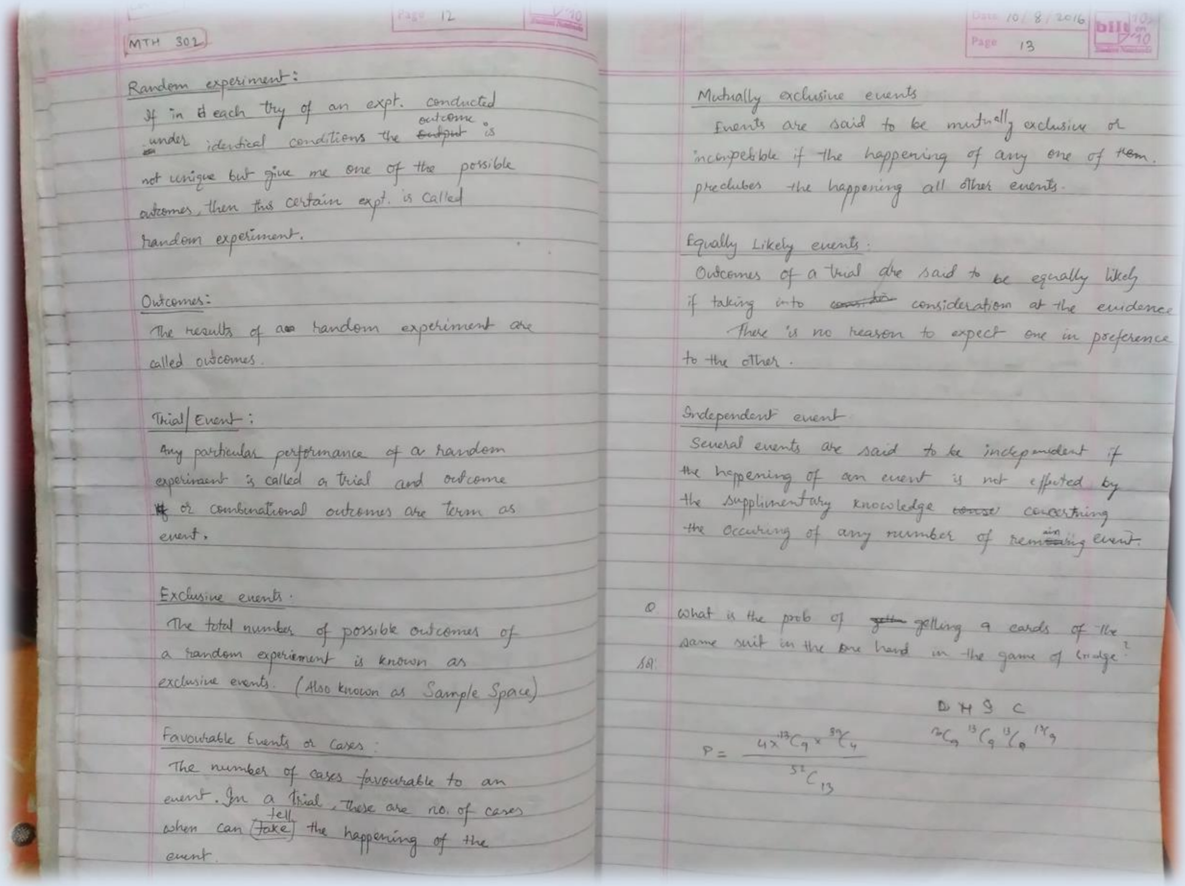
And → ∩ → ×
or → ∪ → +

b) $P = \frac{{}^4C_2 \times {}^4C_2}{{}^{52}C_4}$



c) $P = \frac{{}^{26}C_2 \times {}^{26}C_2}{{}^{52}C_4}$

d) $P = \frac{{}^{13}C_2 \times {}^{13}C_2}{{}^{52}C_4}$



MTH 302

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Random experiment:

If in each try of an expt. conducted under identical conditions the ~~output~~ outcome is not unique but give me one of the possible outcomes, then this certain expt. is called random experiment.

Outcomes:

The results of ~~an~~ random experiment are called outcomes.

Trial/Event:

Any particular performance of a random experiment is called a trial and outcome ~~#~~ or combinational outcomes are term as event.

Exclusive events:

The total number of possible outcomes of a random experiment is known as exclusive events. (Also known as Sample Space)

Favourable Events or Cases:

The number of cases favourable to an event. In a trial, these are no. of cases when can ~~take~~ tell the happening of the event.

Date 10/8/2016

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Mutually exclusive events

Events are said to be mutually exclusive or incompetible if the happening of any one of them precludes the happening all other events.

Equally Likely events:

Outcomes of a trial are said to be equally likely if taking into ~~consider~~ consideration at the evidence. There is no reason to expect one in preference to the other.

Independent event

Several events are said to be independent if the happening of an event is not effected by the supplimentary knowledge ~~course~~ concerning the occuring of any number of remaining event.

Q. What is the prob of ~~gett~~ getting 9 cards of the same suit in the one hand in the game of bridge?

Sol:

D H S C

${}^{13}C_9 \; {}^{13}C_9 \; {}^{13}C_0 \; {}^{13}C_9$

$$P = \frac{4 \times {}^{13}C_9 \times {}^{39}C_4}{{}^{52}C_{13}}$$

Page 14.

Q. A man has 4 spade card from an ordinary 52 cards. If he has given 3 more cards. find the prob. that atleast an additional card is spade.

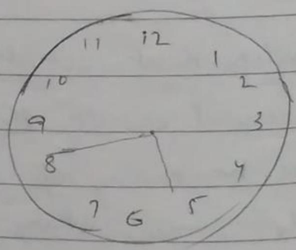
Sol:
$$P(\text{atleast}) = 1 - P(\text{None})$$
$$= 1 - \frac{{}^{39}C_3}{{}^{48}C_3}$$

Q. Angle ~~an~~ between minute hand and hour hand ~~be~~ when 5:40.

Sol: Speed:

$$M_h = \frac{360°}{60°} = 6°/min$$

$$H_h = \frac{360°}{12 \times 60} = \frac{1}{2}°/min$$



$$H = 150 + 20 = 170°$$
$$M = 240°$$
$$240° - 170° = 70°$$

General Formula

$$|30x - 5.5y|$$

x = hour

y = min

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Q. A committee of 4 persons is to be appointed from the 3 officers from production dept, 4 officers from the purchase dept, 2 officers from sells dept, & 1 charter accountant. Find the prob. of forming the comittee in the following manner.

i. There must be one from each category.

ii. ~~It will be~~ should have atleast 1 person purchase dept.

iii. The charter accountant must be in the comittee.

Sol: i. $P = \dfrac{{}^3C_1 \times {}^4C_1 \times {}^2C_1 \times {}^1C_1}{{}^{10}C_4}$

$= \dfrac{3 \times 4 \times 2 \times 1}{\frac{10!}{4!\,6!}}$

$= \dfrac{\cancel{3} \times 4 \times \cancel{2} \times 1 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}{7 \times \cancel{8} \times \cancel{9} \times \cancel{10}5}$

$= \dfrac{4}{35}$

ii. $\dfrac{{}^4C_1 \cdot {}^6C_3}{{}^{10}C_4} + \dfrac{{}^4C_2 \cdot {}^6C_2}{{}^{10}C_4} + \dfrac{{}^4C_3 \cdot {}^6C_1}{{}^{10}C_4} \times \dfrac{{}^4C_4 \cdot {}^6C_0}{{}^{10}C_4}$

or $P(\text{at least one from Purchase dept}) = 1 - P(\text{none from Pur dept})$

$= 1 - \dfrac{{}^6C_4}{{}^{10}C_4}$

iii. $P = \dfrac{{}^1C_1 \times {}^9C_3}{{}^{10}C_4}$

Q. In a random arrangement of the letter of the words COMMERCE. find the prob that all vowels come together.

Sol. COMMERCE - 8

$$P = \frac{\frac{6!}{2!\,2!} \times \frac{3!}{2!}}{\frac{8!}{2!\,2!\,2!}}$$

$$= \frac{6! \times 3 \times 2}{6! \times 7 \times 8}$$

$$= \frac{3}{28}$$

$$= 0.10$$

Q. JOSHIVARUN = 10

OIAU = 4

Total outcome = 10!

~~Fav~~ Favourable outcome = 7!

$$P = \frac{7! \times 4!}{10!} = \frac{7! \times 4 \times 3 \times 2 \times 1}{7! \times 8 \times 9 \times 10}$$

$$= \frac{1}{30}$$

Q. 25 books are placed at random in a shelf find the probability that a particular pair of books shall be i) always together ii) never together

Sol. (i) $\frac{24!}{25!} \times 2!$ (ii) $1 - \frac{2}{25} = \frac{23}{25}$

Q. 8 persons are seated on 8 chairs at a random around a round table. find the probability that two specified persons are sitting next to each other.

Sol. $\frac{7!}{8!} \times 2!$

In circular we don't know where to start so, prob = $(n-1)!$

coz. we fix 1 item that don't count

Total = 7!

favourable = $6! \times 2!$

Probability = $\frac{6! \times 2!}{7!}$

$= \frac{2}{7}$

16/8/2016.

Probability Functions.

A function $P(A)$ is a probability function defined on field (B) of events is defined with the following properties

i) if $A \in B$ then $P(A) \geq 0$

ii) $P(S) = 1$

iii) if A_i are any finite or infinite set of events which are disjoint, $P(\cup A_i) = \Sigma P(A_i)$

contd. on P- 37

Contd from P-21

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Q. A, B, C are three arbitrary events. Find the expressions for the events below.

1. Only A occurs $\rightarrow$ $A \cap \bar{B} \cap \bar{C}$
2. Both and B but not occurs $\rightarrow$ $B \cap C \cap \bar{A}$
3. All three occurs $\rightarrow$ $A \cap B \cap C$
4. Atleast one $\rightarrow (A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$

   or $\rightarrow 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$
5. Atleast two occurs $\rightarrow 1 - \{ (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$
6. Only 1 occurs
7. None occurs $\rightarrow$ $\bar{A} \cap \bar{B} \cap \bar{C}$ or $\overline{A \cup B \cup C}$

if $P(A) = P$, then $P(\bar{A}) = 1 - P$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

Q. A, B and C are three mutually exclusive and exhaustive event associated with a random expt find the prob. of A given that, $P(B) = \frac{3}{2} P(A)$ and $P(C) = \frac{1}{2} P(B)$

Sol: exhaustive event means $\Rightarrow P + B + C = S$

$P(A) + P(B) + P(C) = 1$

$\Rightarrow P(A) + \frac{3}{2} P(A) + \frac{1}{2} \times \frac{3}{2} P(A) = 1$

$\Rightarrow P(A) \left(1 + \frac{3}{2} + \frac{3}{4}\right) = 1$

$\Rightarrow \frac{4 + 6 + 3}{4} P(A) = 1$

$\Rightarrow P(A) = 1 \times \frac{4}{13} = \frac{4}{13}$

P.T.O.

Q. 5 salesmen A, B, C, D, E of a company are considered for a team members trade dedication to represent their company in an International Trade. Make sample space &

1. A is selected
2. A is not selected
3. Either A or B not both.

Sol. Sample Space = $\boxed{5 \times 4 \times 3 = 60}$

$^5C_3$

$\frac{5!}{3! \times 2!}$

10

$^5C_3 = 10$

ABC
ABD
ABE
ACD
ACE
BCD
BCE
CDE
ADE
BDE

i) $P = \frac{6}{10}$

ii) $P = \frac{4}{10}$

iii) $P = \frac{6}{10}$

Q. If two dice are thrown what is the prob. that the sum is neither 7 nor 11.

Sol. Sum is 7 = (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

Sum is 11 = (5,6), (6,5)

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$P = 1 - \frac{8}{36}$

$\Rightarrow P = \frac{28}{36}$

$\Rightarrow P = \frac{7}{9}$

Q. One integer is chosen from first 200 digits. What is the probability that the integer is divisible by 6 or 8.

Sol.

$P(A) = \frac{33}{200}$ (divisible by 6)

$P(B) = \frac{25}{200}$ (divisible by 8)

$P(A \cap B) = \frac{8}{200}$ (divisible by 6 & 8) taking LCM - 6 & 8

$P(A \cup B) = \frac{33}{200} + \frac{25}{200} - \frac{8}{200}$

$= \frac{50}{200} = \frac{1}{4}$

198

$6 \overline{)200}$ 33
18
20
18
2

$8 \overline{)200}$ 25
16
40
40

$2 \overline{)6, 8}$
3, 4
= 24

$24 \overline{)200}$ 8
192
8

Contd. on P-44

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Q. An MBA student applys for a job into two form X and Y. The prob. of getting selected in the form X is 0.7 and being rejected in form Y is 0.5. The prob. of atleast one of his application being rejected is 0.6. what is the probability that he will be selected in one of the form?

Sol. $P(X) = 0.7 \quad P(\bar{Y}) = 0.5$

$\cancel{P(\overline{X \cup Y}) = 0.6}$

$P(\bar{X}) = 0.3 \quad P(Y) = 0.5$

$P(X \cup Y) = 1 - P(\overline{X \cup Y})$

$= 1 - P(\bar{X} \cap \bar{Y})$

$= 1 - (P(\bar{X}) + P(\bar{Y}) - P(\bar{X} \cup \bar{Y}))$

$= 1 - (0.3 + 0.5 - 0.6)$

Q. A card is drawn from a pack of 52 cards ~~fo~~. Find the prob. of getting a king or a red card or a heart card?

Sol. $P(K \cup H \cup R) = P(K) + P(H) + P(R) - P(K \cap H) - P(H \cap R) - P(K \cap R) + P(K \cap H \cap R)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$

$= \frac{44 - 16}{52} = \frac{\cancel{28}\ 14\ 7}{\cancel{52}\ 26\ 13}$

$= \frac{7}{13}$

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Conditional Probability :

1. $P(A|B) = \frac{P(A \cap B)}{P(B)}$ prob. of A w.r.t. B (dependent)

2. $P(A \cap B) = P(A|B)\, P(B)$ - Multiplication Theorem

3. $P(A \cap B) = P(A) \cdot P(B)$ A and B are independent

or $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(A_1 \cap A_2 \cap A_3 \cap \cdots A_n) = P(A_1) \cdot P(A_2) \cdot P(A_3) \cdots P(A_n)$ {All are independent

Q. One shoot is fired from each of the 3 guns. E_1, E_2, E_3 denotes the events that the target is hit by the 1st, 2nd & 3rd gun respectively. If $P(E_1) = 0.5$, $P(E_2) = 0.6$, $P(E_3) = 0.8$ and E_1, E_2 and E_3 are independent. Find the prob. (a) exactly one hit is the register (b) atleast two hits are register

Sol. $P(E_1) = 0.5 \quad P(\bar{E_1}) = 0.5$

$P(E_2) = 0.6 \quad P(\bar{E_2}) = 0.4$

$P(E_3) = 0.8 \quad P(\bar{E_3}) = 0.2$

a) $(E_1 \cap \bar{E_2} \cap \bar{E_3}) \cup (\bar{E_1} \cap E_2 \cap \bar{E_3}) \cup (\bar{E_1} \cap \bar{E_2} \cap E_3)$

$\Rightarrow P(E_1 \cap \bar{E_2} \cap \bar{E_3}) + P(\bar{E_1} \cap E_2 \cap \bar{E_3}) + P(\bar{E_1} \cap \bar{E_2} \cap E_3)$

$= P(E_1) P(\bar{E_2}) P(\bar{E_3}) + P(\bar{E_1}) P(E_2) P(\bar{E_3}) + P(\bar{E_1}) P(\bar{E_2}) P(E_3)$

$= (0.5 \times 0.4 \times 0.2) + (0.5 \times 0.6 \times 0.2) + (0.5 \times 0.4 \times 0.8)$

$= 0.040 + .060 + 0.160$

$= 0.26$

(b) $(E_1 \cap E_2 \cap \bar{E}_3) \cup (E_1 \cap \bar{E}_2 \cap E_3) \cup (\bar{E}_1 \cap E_2 \cap E_3)$

$\cup (E_1 \cap E_2 \cap E_3)$

$= P(E_1)P(E_2)P(\bar{E}_3) + P(E_1)P(\bar{E}_2)P(E_3) + P(\bar{E}_1)P(E_2)P(E_3)$

$+ P(E_1)P(E_2)P(E_3)$

$= (0.5 \times 0.6 \times 0.2) + (0.5 \times 0.4 \times 0.8) + (0.5 \times 0.6 \times 0.8) +$

$(0.5 \times 0.6 \times 0.8)$

$= 0.060 + 0.160 + 0.240 + 0.240$

$= 0.700$

$= 0.7$

Day on 17 July 1776

$$F = K + \left[\frac{13 \times M - 1}{5}\right] + D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C$$

K = date

M = month & starts from March

D = last 2 digit of year

C = first 2 digit of year

eg. $F = 17 + \left(\frac{13 \times 5 - 1}{5}\right) + 76 + \left(\frac{76}{4}\right) + \left(\frac{17}{4}\right) - 2 \times C$

$= 17 + 12 + 76 + 19 + 4 - 34$

$= \frac{94}{7}$ // divide by 7

$= 3$ } Remainder

Sun	0
Mon	1
Tue	2
Wed	3
Thur	4
Fri	5
Sat	6

MTH 302

Q. The odds against the manager X settling ~~a~~ the wage dispute with the workers are 8:6 and odds in the favour of the manager Y settling the same dispute is 14:16.

a. What is the chance ~~with~~ that neither settle the dispute?

b. What is the prob. that the dispute will be settle?

Sol: odds A : B (A → favour, B → unfavour)

$X = 8:6$ & $Y = 14:16$

$P(X) = \frac{6}{14}$ $\quad P(\bar{X}) = \frac{8}{14}$

$P(Y) = \frac{14}{30}$ $\quad P(\bar{Y}) = \frac{16}{30}$

a) $P(\bar{X} \cap \bar{Y}) = P(\bar{X}) \cdot P(\bar{Y})$

$= \frac{8}{14} \times \frac{16}{30} = \frac{32}{105}$

b) $P(X \cup Y) = 1 - P(\overline{X \cup Y})$

$= 1 - P(\bar{X} \cap \bar{Y})$

$= 1 - \frac{32}{105} = \frac{73}{105}$

Q. The odds that the person X speaks the truth are 3:2 and the odds that the person Y speaks the truth are 5:3. In what percentages of cases are they likely to contradict each other on an identical point.

Sol: X 3:2

Y 5:3

P.T.O.

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$P(X) = \frac{3}{5}$ $\quad P(\bar{X}) = \frac{2}{5}$

$P(Y) = \frac{5}{8}$ $\quad P(\bar{Y}) = \frac{3}{8}$

$P(X \cap \bar{Y}) \cup P(\bar{X} \cap Y)$

$= P(X) \cdot P(\bar{Y}) + P(\bar{X}) \cdot P(Y) = \frac{3}{5} \times \frac{3}{8} + \frac{2}{5} \times \frac{5}{8}$

$= \frac{9}{40} + \frac{10}{40} = \frac{19}{40}$

Q. An urn contains 4 tickets no 1, 2, 3, 4 and another contains 6 tickets no 2, 4, 6, 7, 8, 9. In one of the two urn is choosen at random and the ticket is drawn at ~~second~~ random.

Find the prob. that the ticket draw bears the no.

(i) 2 or 4. (ii) 3 (iii) 1 or 9

Sol: (i) $\frac{1}{2} \times \left(\frac{1}{4} + \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{6} + \frac{1}{6}\right)$ $\quad 2/4$

$= \frac{1}{2} \times \frac{2}{4} + \frac{1}{2} \times \frac{2}{6} = \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12}$

(ii) $\frac{1}{2} \times \frac{1}{4} + 0 = \frac{1}{8}$ $\quad 4(8, 12 / 2, 3)$

(iii) $\frac{1}{2} \times \left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{6}\right) = \frac{1}{8} + \frac{1}{12} = \frac{3+2}{24} = \frac{5}{24}$

Q. A box contains 6 red, 4 white, 5 black balls. A person draws four balls from the box at random. Find the prob. that among the balls drawn that is atleast one ball of each colour. (0.5)

Sol: 2R, 1W, 1B + 1R, 2W, 1B + 1R, 1W, 2B

$= \boxed{\left(\frac{2}{6} \times \frac{1}{4} \times \frac{1}{5}\right) + \left(\frac{1}{6} \times \frac{2}{4} \times \frac{1}{5}\right) + \left(\frac{1}{6} \times \frac{1}{4} \times \frac{2}{5}\right) \\ \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = 3 \times \frac{1}{60} = \frac{1}{20}}$

P.T.O.

Next ~~the~~

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$$= \frac{{}^6C_2 \cdot {}^4C_1 \cdot {}^5C_1}{{}^{15}C_4} + \frac{{}^6C_1 \cdot {}^4C_2 \cdot {}^5C_1}{{}^{15}C_4} + \frac{{}^6C_1 \cdot {}^4C_1 \cdot {}^5C_2}{{}^{15}C_4}$$

$$= \frac{15 \times 4 \times 5}{\quad}$$

$$\frac{4!\,5 \times x^3}{4!\,2}$$

23/8/2016

Q. Three group of children containing ~~de~~ respectively 1st gr (3 girls & 1 boy), 2nd gr (2 boy 2 girl) and 3rd gr (1 girl 3 boys.) One child is selected at random from each group so that the chance that they ~~are~~ selected ~~by~~ (1 girl and 2 boys) is 13/32.

Sol: 1st group = 3 G and 1 B $= \frac{3}{4} + \frac{1}{4}$

2nd group = 2 G and 2 B $= \frac{2}{4} + \frac{2}{4}$

3rd group = 1 G and 3 B $= \frac{1}{4} + \frac{3}{4}$

GBB + BGB + BBG

$\frac{16}{\frac{4}{64}}$

$$\left(\frac{3}{4} \times \frac{2}{4} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{2}{4} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{2}{4} \times \frac{1}{4}\right)$$

$$= \frac{18}{64} + \frac{6}{64} + \frac{2}{64}$$

$$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32}$$

$$= \frac{13}{32}$$

Contd. from P – 52

MTH 302

Date 23/8/2016

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Q. If it is given 8:5 against the wife who is 40 years old living till she is 70 and 4:3 against her husband now at the age of 50 till his 80.

Soln. $P(W) = \frac{5}{13}$ $\quad$ 1. find the prob. both will be alive.

2. None will be alive

$P(H) = \frac{3}{7}$ $\quad$ 3. Only wife will be alive

4. Only husband will be alive.

1. $\cancel{P(W) \cap P(H)}$ $P(W \cap H) = P(W).P(H) = \frac{5}{13}.\frac{3}{7} = \frac{15}{91}$

2. $P(\overline{W \cup H}) = P(\bar{W} \cap \bar{H}) = P(\bar{W}).P(\bar{H}) = \frac{8}{13}.\frac{4}{7} = \frac{32}{91}$

3. $P(W \cap \bar{H}) = P(W).P(\bar{H}) = \frac{5}{13}.\frac{4}{7} = \frac{20}{91}$

4. $P(\bar{W} \cap H) = \frac{8}{13} \times \frac{3}{7} = \frac{24}{91}$

5. Atleast one is alive $= 1 - P(\bar{W} \cap \bar{H}) = \frac{59}{91}$

* Sum of infinite GP $= 1 + r + r^2 + r^3 + \ldots = \frac{a}{1-r}$

Q. A and B alternative cut a pack of cards and pack is suffled after each cut. If A starts and game is continued untill one cuts a diamond, what are the respective changes of A & B first cutting a diamond.

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A: A is wining the game

1. A_1
2. $\bar{A}_1 \bar{B}_1 A_2$
3. $\bar{A}_1 \bar{B}_1 \bar{A}_2 \bar{B}_2 A_3$
4. $\bar{A}_1 \bar{B}_1 \bar{A}_2 \bar{B}_2 \bar{A}_3 \bar{B}_3 A_4$

$\frac{13}{52} = \frac{1}{4}$

$\frac{3}{4}$

$$P(A) = \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^4 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^6 \cdot \frac{1}{4} + \cdots$$

$$= \frac{a}{1-r} \qquad a = \frac{1}{4},\ r = \left(\frac{3}{4}\right)^2$$

$$= \frac{1/4}{1-\left(\frac{3}{4}\right)^2}$$

$$= \frac{1/4}{1-9/16} = \frac{1/4}{7/16}$$

$$= \frac{16}{28} \frac{4}{7} = \frac{4}{7}$$

$$P(B) = 1 - P(A)$$

$$= 1 - \frac{4}{7}$$

$$= 3/4$$

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Random variable

Random variable 'x' is a function between outcomes and ~~[illegible]~~ numbers.

X : outcome → numbers.

Types :-

~~Speed random~~ ① Discrete random variable

② Continuous random variable. – (weight of students)

Discrete distribution function

$F(x_i) = P(X \leq x_i)$

$= P(X \leq 0) = P(X=0)$

$= P(X \leq 1) = P(X=0)+P(X=1)$

x	$P(x)$	$F(x_i)$
0	$1/8$	
1	$3/8$	
2	$3/8$	
3	$1/8$	

Q. A random variable X as a following probability f^n

i.e.

X :	0	1	2	3	4	5	6	7
$P(X)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

A. Find K,

B. Evaluate $P(X<6)$, $P(X \geq 6)$, $P(0<X<5)$

C. $P(X \leq a) > \frac{1}{2}$, then find a.

Sol. A. $0+K+2K+2K+3K+K^2+2K^2+7K^2+K = 1$

$\Rightarrow 9K + 10K^2 = 1$

$\Rightarrow 10K^2 + 9K - 1 = 0$

$= \frac{-9 \pm \sqrt{81-40}}{20} = \frac{41}{20}$

$= K = \frac{1}{10}$

P.T.O

$P(X<6) = P(X=0)+P(X=1)+P(X=2)$
$+P(X=3)+P(X=4)+P(X=5)$

$P(X \leq a) > \frac{1}{2}$

min value of $a = 4$

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$$P(x) = \begin{cases} \frac{x}{15} & ; x = 1,2,3,4,5 \\ 0 & , \text{otherwise} \end{cases}$$

(i) $P(X=1 \text{ or } 2)$ (ii) $P\left(\frac{1}{2} < x < \frac{5}{2} \,\middle|\, X=1\right)$

$P(X=1)+P(X=2)$

$\frac{1}{15}+\frac{2}{15} = \frac{3}{15} = \frac{1}{5}$

$$\frac{P\left(\left(\frac{1}{2}<x<\frac{5}{2}\right)\cap(X=1)\right)}{P(X=1)}$$

$$= \frac{P(X=1)}{P(X=1)}$$

Q.

Q. Two dice are drawn. Let x be random variable which counts the total no. of ~~coins~~ sum on the up~~turned~~ faces. Construct a table giving the non zero values of probability mass f^n and also find the distribution function of x.

Sol

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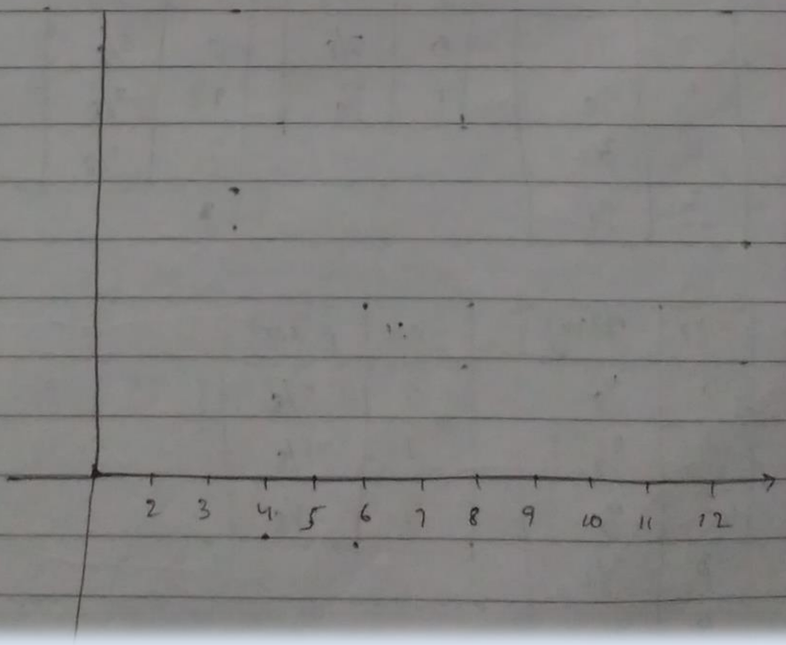
Sol:

X		PMF $P(x)$	$F(x)$	
2	(1,1)	1/36	1/36	F(2)
3	(1,2)(2,1)	2/36	3/36	F(3)
4	(1,3)(3,1)(2,2)	3/36	6/36	F(4)
5	(1,4)(4,1)(2,3)(3,2)	4/36	10/36	F(5)
6	(1,5)(5,1)(2,4)(4,2)(3,3)	5/36	15/36	F(6)
7	(1,6)(6,1)(2,5)(5,2)(3,4)(4,3)	6/36	21/36	F(7)
8	(2,6)(6,2)(3,5)(5,3)(4,4)	5/36	26/36	F(8)
9	(3,6)(6,3)(4,5)(5,4)	4/36	30/36	F(9)
10	(4,6)(6,4)(5,5)	3/36	33/36	F(10)
11	(5,6)(6,5)	2/36	35/36	F(11)
12	(6,6)	1/36	36/36 = 1	F(12)



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Q. An expt consists 3 independent tosses of a fair coin. X = no. of head, Y = no. of head runs, Z = length of head runs. Find the prob. funcn of X, Y, Z

Sol

	X	Y	Z	X+Y	X·Y
HHH	3	1	3	4	3
HHT	2	1	2	3	2
HTH	2	0	0	2	0
HTT	1	0	0	1	0
THH	2	1	2	3	2
THT	1	0	0	1	0
TTH	1	0	0	1	0
TTT	0	0	0	0	0

X	P(X)
0	1/8
1	3/8
2	3/8
3	1/8

Y	P(Y)
0	5/8
1	3/8

Z	P(Z)
0	5/8
2	2/8
3	1/8

X+Y	P(X+Y)
0	1/8
1	3/8
2	1/8
3	2/8
4	1/8

X·Y	P(X·Y)
0	5/8
2	2/8
3	1/8

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D.R.V.	C.R.V.
① $P(x) \geq 0$	① $f(x) \geq 0$
② $0 \leq P(x)$ or PMF ≤ 1	② $0 \leq f(x) \leq 1$
③ $\sum_{i=1} P(x_i) = 1$	③ $\int f(x)\,dx = 1$
Discrete Random Variable	Continuous Random variable

Q. A continuous random variable X has a PDF (prob. density funcn) $f(x) = 3x^2$, $0 \leq x \leq 1$. Find a and b such that

(i) $P(X \leq a) = P(X > a)$

(ii) $P(X > b) = 0.05$

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Sol (i) $P(X \leq a) = P(X > a)$

$P(X \leq a) = \frac{1}{2}$

$\int_0^a 3x^2\,dx = \frac{1}{2}$

$\Rightarrow \left[\frac{3x^3}{3}\right]_0^a = \frac{1}{2}$

$\Rightarrow a^3 - 0 = \frac{1}{2} \Rightarrow a = \left(\frac{1}{2}\right)^{1/3}$

i) $\int_0^a 3x^2\,dx = \int_a^1 3x^2\,dx$

$\Rightarrow \left[\frac{3x^3}{3}\right]_0^a = \left[\frac{3x^3}{3}\right]_a^1$

$\Rightarrow a^3 - 0 = 1 - a^3$

$\Rightarrow 2a^3 = 1$

$\Rightarrow a = \left(\frac{1}{2}\right)^{1/3}$

ii) $\int_b^1 3x^2\,dx = 0.05$

$\Rightarrow \left[\frac{3x^3}{3}\right]_b^1 = 0.05$

$\Rightarrow 1^3 - b^3 = 0.05$

$\Rightarrow b^3 = 0.95$

$\Rightarrow b = (0.95)^{1/3}$

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$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax+3a & 2 \le x \le 3 \\ 0 & \text{other} \end{cases} \quad \text{pdf}$$

(i) Find a

(ii) $P(X \le 1.5)$

Sol. (i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{3} (-ax+3a)\,dx + \int_{3}^{\infty} 0\,dx$

$\Rightarrow 0 + \left[\frac{ax^2}{2}\right]_0^1 + \left[ax\right]_1^2 + \left[-\frac{ax^2}{2} + 3ax\right]_2^3 + 0 = 1$

$\cancel{\Rightarrow \frac{ax^2}{2} + ax + -\frac{ax^2}{2} + 3ax = 1}$

$\cancel{\Rightarrow 4ax = 1}$

$\cancel{\Rightarrow a = \frac{1}{4x}}$

$\Rightarrow \frac{a}{2} + 2a - a - \frac{9a}{2} + 9a + 2a - 6a = 1$

$\Rightarrow \frac{-8a}{2} + 8a = 1$

$\Rightarrow \frac{8}{2}a = 1$

$\Rightarrow 4a = 1 \quad \Rightarrow a = \frac{1}{4}$

$\cancel{9a} + \cancel{4a} - 2a - \cancel{9a} + 18a + \cancel{4a}$

$-12a = 2$

$4a = 2$

$a = \frac{2}{4} = \frac{1}{2}$

(ii) $\therefore P(X \le 1.5)$

$= \int_{-\infty}^{1.5} f(x)\,.dx$

P.T.O.

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$$= \int_{-\alpha}^{0} 0.\,dx + \int_{0}^{1} \frac{1}{2}x\,dx + \int_{1}^{1.5} \frac{1}{2}\,dx$$

$$= \left[\frac{1}{2}\frac{x^2}{2}\right]_0^1 + \left[\frac{1}{2}x\right]_1^{1.5}$$

$$= \frac{1}{4} + \frac{1.5}{2} - \frac{\cancel{1.5}\,1}{2}$$

$$=$$

$$= \frac{1}{2}$$

Q. Suppose that the light in hour of a certain unit of radio tube is a continuous random variable with pdf.

$$f(x) = \begin{cases} \frac{100}{x^2}, & \text{when } x \geq 100 \\ 0, & \text{otherwise} \end{cases}$$

a. What is the prob. all three tubes in a given radio set ~~a~~ will have to be replaced during the first 150 hour of the operation.

b. What is the prob. that non of the of the original tubes will have to replaced during the week ~~a~~ 150 hours.

c. What is the prob that the tube will last less than 200 hrs, if it is known that the tube is still functioning after 150 hr.

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d) What is the max no. of tubes that will be inserted ~~a~~ into a set so that there is a prob. of 0.5 that after 150 hr of service all of them are in functioning?

Sol: a)

$$f(x) = \begin{cases} \frac{100}{x^2} & x \geq 100 \\ 0 & \text{otherwise} \end{cases}$$

$$P(X < 150) = \int_{100}^{150} \frac{100}{x^2}\,dx = \left[\frac{100\,x^{-1}}{-1}\right]_{100}^{150}$$

$$= \left[-\frac{100}{x}\right]_{100}^{150} = -\frac{100}{150} + \frac{100}{100}$$

$$= \frac{1}{3}$$

$$= \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} = \frac{1}{27}$$

b) $P(X > 150) = \int_{150} \frac{100}{x^2}\,dx$

b) $P(X > 150) = 1 - \frac{1}{3} = \frac{2}{3}$

$$= \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \frac{8}{27}$$

c) $P(X < 200 \mid X > 150)$

$$= \frac{P((X<200) \cap (X>150))}{P(X>150)}$$

$$= \frac{P(150 < X < 200)}{P(X > 150)}$$

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$$= \frac{\int_{150}^{200} \frac{100}{x^2}\,dx}{2/3} = \frac{\left[\frac{100\, x^{-1}}{-1}\right]_{150}^{200}}{2/3}$$

$$= \frac{-\frac{100}{200} + \frac{100}{150}\ \frac{2}{3}}{2/3}$$

$$= \frac{1/6}{2/3} = \frac{3}{12} = \frac{1}{4}$$

d) $n\ P(X > 150) = 0.5$

$$\left(\frac{2}{3}\right)^n = 0.5$$

$$\Rightarrow \left(\frac{2}{3}\right)^n = \frac{5}{10} = \frac{1}{2}$$

$$\frac{3-4}{6}$$

$$n \log\left(\frac{2}{3}\right) = \log\left(\frac{1}{2}\right)$$

$$\Rightarrow n = \log\left(\frac{1}{2}\right) \div \log\left(\frac{2}{3}\right)$$

$$= \log\left(\frac{1}{2} - \frac{2}{3}\right) = \log\left(\frac{-1}{6}\right) \quad 1.7$$

$$= 1.7$$

$$F_x(x) = P(X \le x) = \int_{-\alpha}^{x} f(x) \cdot dx$$

$$P(X \le 2) = \int_{-\alpha}^{1} f(x) \cdot dx + \int_{1}^{x} f(x) \cdot dx$$

Q. The diameter (say X) of an electric cable is assumed to be continuous random variable with p.d.f $= 6x(1-x) \quad 0 \le x \le 1$

1. Check this is a pdf function.
2. Obtain an expression for cdf
3. Compute the probability $P(X \le \frac{1}{2} \mid \frac{1}{2} \le X \le \frac{2}{3})$
4. Find K such that $P(X < K) = P(X > K)$

Sol:- (1) $\int_{-\alpha}^{\alpha} f(x) = 1$

$$\int_0^1 6x(1-x)\,dx = \int_0^1 (6x - 6x^2)\,dx$$

$$= \left[\frac{6x^2}{2} - \frac{6x^3}{3}\right]_0^1 = (3 - 2 - 0 + 0)$$

$$= 1$$

(2) $F(0) = P(X \le 0) = \int_{-\alpha}^{x} 0 . dx = 0$

$$F(1) = P(X \le 1) = 0 + \int_0^x 6x(1-x)\,dx$$

$$= 0 + \int_0^x (6x - 6x^2)\,dx$$

$$= 3x^2 - 2x^3$$

$$P(X > 1) = 0 + \int_0^1 6x(1-x)\,dx + \int_1^{\alpha} 0 \, dx$$

$$= \int_0^1 6x(1-x)\,dx$$

$$= 1$$

(3) $P(X \le \frac{1}{2} \mid \frac{1}{2} \le X \le \frac{2}{3}) \cdot P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{P((X \le \frac{1}{2}) \cap (\frac{1}{2} \le X \le \frac{2}{3}))}{P(\frac{1}{2} \le X \le \frac{2}{3})} = \frac{P(X = \frac{1}{2})}{\int_{1/2}^{2/3} 6x(1-x) \cdot dx}$$

$$= \frac{6 \times \frac{1}{2}(1 - \frac{1}{2})}{\left[3x^2 - 2x^3\right]_{1/2}^{2/3}}$$

$$= \frac{3(1/2)}{3 \cdot (\frac{2}{3})^2 - 2(\frac{2}{3})^3 - 3(\frac{1}{2})^2 + 2(\frac{1}{2})^3}$$

$$= \frac{3/2}{\frac{4}{3} - \frac{16}{27} - \frac{3}{4} + \frac{2}{8}}$$

$$= \frac{3/2}{\frac{144 - 64 - 81 + 24}{108}}$$

$$= \frac{108 \times 3}{23 \times 2} = \frac{162}{23}$$

(4) $P(X < K) = P(X > K)$

$$\int_0^K 6x(1-x)\,dx = \int_K^1 6x(1-x)\,dx$$

$$\Rightarrow \left[3x^2 - 2x^3\right]_0^K = \left[3x^2 - 2x^3\right]_K^1$$

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$\Rightarrow 3k^2 - 2k^3 = 3 - 2 - 3k^2 + 2k^3$

$\Rightarrow 6k^2 - 4k^3 - 1 = 0$

$\Rightarrow 4k^3 - 6k^2 + 1 = 0$

When $k = \frac{1}{2}$

$\underline{LHS} = 4 \cdot \left(\frac{1}{2}\right)^3 - 6 \cdot \left(\frac{1}{2}\right)^2 + 1$

$= \frac{1}{2} - \frac{3}{2} + 1$

$= -1 + 1 = 0$ $\underline{RHS}$

$\therefore$ Value of $k = \frac{1}{2}$

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Q.
$$f(x) = \begin{cases} kx & 0 \le x \le 1 \\ k & 1 \le x \le 2 \\ -kx+3k & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$= \frac{1}{2}k$

$= k \cdot \frac{1}{2}$

$= -\frac{k}{2} + \frac{3}{2}$

(1) Find k, (2) Determine cdf (3) If x_1, x_2, x_3 are the three independent obs. what is the prob that exactly one of these is larger than 1.5

Sol: (1) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{-\infty}^{0} 0\,dx + \int_0^1 kx\,dx + \int_1^2 k\,dx + \int_2^3 (-kx+3k)\,dx + \int_3^{\infty} 0\,dx = 1$

$\Rightarrow 0 + \left[\frac{kx^2}{2}\right]_0^1 + \left[kx\right]_1^2 + \left[\frac{-kx^2}{2} + 3kx\right]_2^3 + 0 = 1$

$\Rightarrow \frac{k}{2} + k + \left(\frac{-k9}{2} + 9k + 2k - 6k\right) = 1$

$\Rightarrow \frac{k}{2} + k - \frac{9}{2}k + k = 1$

$\Rightarrow \frac{k + 2k - 9k + 2k}{2} = 1$

$\Rightarrow \frac{-4k}{2} = 1 \quad \Rightarrow k = \frac{2}{-4} = \frac{1}{2}$

(ii) $x < 0$

$F(x<0) = 0$

$F(x<1) = 0 + \int_0^x \frac{1}{2}x\,dx = 0 + \left[\frac{1}{2}\frac{x^2}{2}\right]_0^x = \frac{x^2}{4}$

$1 \le x < 2$: $0 + \int_0^1 \frac{1}{2}x\,dx + \int_1^x \frac{1}{2}\,dx = \left[\frac{1}{2}\frac{x^2}{2}\right]_0^1 + \left[\frac{1}{2}x\right]_1^x$

$= \frac{1}{4} + \frac{1}{2}x - \frac{1}{2} = \frac{1}{2}x - \frac{1}{4}$

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$2 \le x < 3 \quad = 0 + \int_0^1 \frac{1}{2}x\,dx + \int_1^2 \frac{1}{2}\,dx + \int_2^x \left(-\frac{x}{2} + \frac{3}{2}\right) dx$

(ii) $P(X > 1.5) = \int_{1.5}^{2} \frac{1}{2}\,dx + \int_{2}^{3} \left(-\frac{1}{2}x + \frac{3}{2}\right) dx$

(Apply on cdf)

$= \left[\frac{1}{2}x\right]_{1.5}^{2} + \left[-\frac{1}{2}\cdot\frac{x^2}{2} + \frac{3}{2}x\right]_{2}^{3}$

$= 1 - \frac{1.5}{2} + \left(-\frac{9}{4} + \frac{9}{2} + 1 - 3\right)$ $\quad \frac{1.5}{2}$

$= 1 - \frac{3}{4} - \frac{9}{4} + \frac{9}{2} - 2$ $\quad \frac{\cancel{15}\,3}{\cancel{20}\,4}$

$= \frac{4 - 3 - 9 + 18 - 8}{4} = \frac{2}{4}$ $\quad \frac{22}{-20}\,2$

$= \frac{1}{2}$

(3) $x_1 . \overline{x_2} . \overline{x_3} + \overline{x_1}\, x_2 \overline{x_3} + \overline{x_1}, \overline{x_2}\, x_3$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3$

$= \frac{3}{8}$

Q. A petrol pump is supplied with petrol on a day. If its ~~dra~~ daily volume of ~~set~~ sells in thousands of litres distributed by

$f(x) = 5(1-x)^4 \qquad 0 \le x \le 1$

What must be the capacity of each tank in order that the prob. that the supplier will be exhausted in a given day shall be 0.01.

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Sol: $f(x) = 5(1-x)^4 \qquad 0 \le x \le 1$

$P(X > c) = 0.01$

$\int_c^1 5(1-x)^4\, dx = 0.01$

$\Rightarrow \left[\frac{5(1-x)^5}{-5} \right]_c^1 = 0.01$

$\Rightarrow 0 + (1-c)^5 = 0.01$

$\Rightarrow 1-c = (0.01)^{1/5}$

$\Rightarrow c = 1 - (0.01)^{1/5}$

Mathematical ~~expressions~~ Expectation or expected value of random variable.

$E(X) = \Sigma x \cdot P(x)$ in D.R.V.

$E(X) = \int x \cdot P(x)$ in C.R.V.

	x
HH	2
HT	1
TH	1
TT	0

x	P(x)
0	1/4
1	2/4
2	1/4

$E(X) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{2}{4} + 2 \cdot \frac{1}{4}$

$= 0 + \frac{2}{4} + \frac{2}{4}$

$= \frac{4}{4} = 1$

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Q.1 Find the expection of a number on a die.

Q.2 Two ~~unbi~~ unbiased dice are thrown find the expected values of the sum of the nos of points on them.

1. Sol:

x	$P(x)$
1	$1/6$
2	$1/6$
3	$1/6$
4	$1/6$
5	$1/6$
6	$1/6$

$$E(X) = \frac{1}{6}(1+2+3+4+5+6)$$
$$= \frac{21}{6} = \frac{7}{2} = 3.5$$

2. Sol:

x		$P(x)$
2	(1,1)	$1/36$
3	(1,2) (2,1)	$2/36$
4	(1,3) (3,1), (2,2)	$3/36$
5	(1,4), (4,1), (2,3), (3,2)	$4/36$
6	(1,5), (5,1), (2,4) (4,2), (3,3)	$5/36$
7	(1,6), (6,1), (2,5), (5,2), (4,3), (3,4)	$6/36$
8	(2,6), (6,2) (3,5), (5,3), (4,4)	$5/36$
9	(3,6) (6,3), (4,5), (5,4)	$4/36$
10	(4,6), (6,4), (5,5)	$3/36$
11	(6,5), (5,6)	$2/36$
12	(6,6)	$1/36$

$$P(x) = \frac{1}{36}(2+6+12+20+30+42+40+36+30+22+12)$$

$= \frac{1}{36}(252) = \frac{\cancel{252}\ \cancel{126}\ \cancel{63}\ 21}{\cancel{36}\ \cancel{18}\ \cancel{9}\ 3}$

$= 7$

* 1. $E(X+Y) = E(X) + E(Y)$
2. $E(aX) = aE(X)$
3. $E(X+a) = E(X) + a$
4. $E(X.Y) = E(X).E(Y)$
5. $Var(X) = E(X^2) - (E(X))^2$
6. $Var(X+Y) = Var(X) + Var(Y)$
7. $Var(aX) = a^2 Var(X)$
8. $Var(X+a) = Var(X)$

Q. In 4 tosses of a coin, let X be the no. of heads. Calculate the expected value of Heads.

$\longrightarrow$ Total – 16

H H H H
H H H T
H H T H
H H T T

x :	-3	6	9
$P(X=x)$:	$1/6$	$1/2$	$1/3$

Q. Find $E(X)$, $E(X^2)$, $E(2X+1)^2$, $Var(X)$

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Sol (i) $E(x) = \sum x \cdot p(x)$

$= -3 \cdot \frac{1}{6} + 6 \cdot \frac{1}{2} + 9 \cdot \frac{1}{3}$

$= -\frac{1}{2} + 3 + \frac{3}{}$

$= \frac{-1}{2} + 6 = \frac{-1+12}{2} = \frac{11}{2}$

(ii) $E(X^2) = \sum x^2 p(x)$

$= 9 \cdot \frac{1}{6} + 36 \cdot \frac{1}{2} + 81 \cdot \frac{1}{3}$

$= \frac{3}{2} + 18 + 27$

$= \frac{93}{2}$

$E(c) = c$

$Var(c) = 0$

(iii) $E(2x+1)^2 = 25 \cdot \frac{1}{6} + \cancel{169 \cdot \frac{1}{2}}$

$E(4x^2 + 1 + 4x) = 4(E(x^2))$

$Var(x) = E(x^2) - (E(x))^2$

$= \frac{93}{2} - \left(\frac{11}{2}\right)^2$

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Q. In a cassino there is a disk with a freely revolving needle. The disk is divided into 20 equal parts by thin line and the parts are marked with nos. 0 – 19. The cassino treats 5 or any multiple of 5 as a lucky number and 0 as a special number. Any person to play with disk has to pay 10 rs. When the needle stops at lucky no. he pays back 2ice the sum and for the special lucky no. he will get 5 times of the sum charge.

Q. Is the game fair? what is the expectation of the player?

Sol:

	Gain	P(x)
5, 10, 15	10	$\frac{3}{20}$
0	40	$\frac{1}{20}$
1 —— 19	−10	$\frac{16}{20}$

$$E(x) = \sum x \cdot P(x)$$

$$= 10 \times \frac{3}{20} + 40 \times \frac{1}{20} - 10 \times \frac{16}{20}$$

$$= \frac{30 + 40 - 160}{20}$$

$$= \frac{-90}{20} = -\frac{9}{2}$$

If game is fair, expectation = 0.

Q. A coin is tossed untill head appears. what is the expection of the number of tosses required?

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Sol

x	outcome	
1	H	$\left(\frac{1}{2}\right)$
2	TH	$\left(\frac{1}{2}\right)^2$
3	TTH	$\left(\frac{1}{2}\right)^3$
4	TTTH	$\left(\frac{1}{2}\right)^4$

$$E(x) = \Sigma x . P(x)$$

$$S = 1\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{2}\right)^2 + 3\left(\tfrac{1}{2}\right)^3 + \cdots \quad \text{— (i)}$$

$$\tfrac{1}{2}S = 1\left(\tfrac{1}{2}\right)^2 + 2\left(\tfrac{1}{2}\right)^3 + 3\left(\tfrac{1}{2}\right)^4 + \cdots \quad \text{— (2)}$$

(i) — (ii)

$$S - \tfrac{1}{2}S = \left(\tfrac{1}{2}\right)^1 + \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^3 + \cdots$$

$$\Rightarrow \tfrac{1}{2}S = \frac{1/2}{1-\frac{1}{2}} = 1$$

$$\Rightarrow S = 2$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$\left.\begin{array}{l} E(x) \\ E(x^2) \\ E(x^3) \\ E(x^4) \end{array}\right\} \text{moments}$$

Moment Generating ~~Moment~~ functions

denoted by

$$M_x(t) = E(e^{xt}) = \begin{cases} \sum_{n=0}^{\infty} e^{xt} . P(X=x) & \text{(discrete)} \\ \text{or} \\ \int_{-\infty}^{\infty} e^{xt} . f(x) & \text{(continuous)} \end{cases} = \Sigma \frac{t^r}{r!} . \mu_r$$

$$e^{xt} = 1 + xt + \frac{(xt)^2}{2!} + \frac{(xt)^3}{3!} + \cdots$$

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$$M_x(t) = E(e^{xt}) = E\left(1 + xt + \frac{(xt)^2}{2!} + \frac{(xt)^3}{3!} + \frac{(xt)^4}{4!} + \cdots\right)$$

$$= E(1) + E(xt) + E\left(\frac{(xt)^2}{2!}\right) + E\left(\frac{(xt)^3}{3!}\right) \cdots$$

$$= 1 + t(E(x)) + \frac{t^2}{2!} \cdot E(x^2) + \frac{t^3}{3!} \cdot E(x^3) + \cdots$$

$$\frac{d}{dt} M_x(t) = 0 + E(x) + \frac{2t}{2!} E(x^2) + \frac{3t^2}{3!} E(x^3) \cdots$$

$$\left(\frac{d}{dt} M_x(t)\right)_{t=0} = E(x)$$

$$\boxed{E(x^r) = \frac{d^r}{dt^r}\left(M_x(t)\right)_{t=0}}$$

1. $M_{cx}(t) = M_x(ct) \Rightarrow E(e^{cxt}) = E(e^{xct})$
2. $M_{x_1+x_2}(t) = M_{x_1}(t) \cdot M_{x_2}(t)$

$$\Rightarrow E(e^{(x_1+x_2)t}) = E(e^{x_1 t}) \cdot E(e^{x_2 t})$$

$\cancel{M_x(t)}$

$$= \sum \frac{t^r}{t!} \cdot u_r$$

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Q. Let the random variable X assume the value r with the prob. mass f^n $P(x=r)=q^{r-1}.p$; $r=1,2,3\ldots$ find the MGF of X and hence its mean and variance.

Sol.
$$M_x(t) = E(e^{xt}) = \Sigma e^{xt}.P(x)$$

$$M_r(t) = E(e^{rt}) = \sum_{r=1}^{\infty} e^{rt}.q^{r-1}.p$$

$$= p\sum_{r=1}^{\infty} e^{rt}.q^r q^{-1}$$

$$= \frac{p}{q}\sum_{r=1}^{\infty}(e^t.q)^r$$

$$= \frac{p}{q}\left((e^t.q)+(e^t.q)^2+(e^t.q)^3+\cdots\right)$$

$$E(X) = \frac{d}{dt}(Mgf)_{t=0}$$

$$E(X^2) = \frac{d^2}{dt^2}(Mgf)_{t=0}$$

$$M.g.f = M_r(t) = \frac{p}{q}\left(\frac{e^t.q}{1-e^t.q}\right)$$

$$= \frac{p}{q}\left\{\frac{(1-e^t.q)e^t.q - e^t.q(-e^t.q)}{(1-e^t.q)^2}\right\}$$

$$= \frac{p}{q}\,\frac{(1-q)q+q^2}{(1-q)^2}$$

$$= \frac{p}{q}\,\frac{q-\cancel{q^2}+\cancel{q^2}}{(1-q)^2} = \frac{p}{(1-q)^2}$$

$$\frac{p}{q}\left(\frac{e^t.q - e^{2t}.q^2+e^{2t}.q^2}{(1-e^t.q)^2}\right)$$

$$= \frac{p}{q}\left\{\frac{e^t.q}{(1-e^t.q)^2}\right\}$$

P.T.O.

$E(x^2) = \frac{d}{dt}\frac{d}{dt}$ M.g.f.

$$\frac{d}{dt}\frac{e^t.q}{(1-e^t.q)^2} = \frac{(1-e^t.q)^2 \cdot e^t.q - e^t.q \; 2(1-e^t.q)(-e^t.q)}{(1-e^t.q)^4}$$

$$= \frac{p}{q}\,\frac{(1-q)^2 \cdot q + q2(1-q).q}{(1-q)^4}$$

$$= \frac{p}{q}\,\frac{(1-q)^2 \cdot q + 2q^2(1-q)}{(1-q)^4}$$

$$= \frac{p}{q}\,\frac{q+q^3-2q^2+2q^2-2q^3}{(1-q)^4}$$

$$= \frac{p}{q}\,\frac{(q-q^3)}{(1-q)^4}$$

$$= \frac{p(1-q^2)}{(1-q)^4}$$

$$\text{Var} = E(x^2) - (E(x))^2$$

$$= \frac{p(1-q^2)}{(1-q)^4} - \frac{p^2}{(1-q)^4}$$

$$= \frac{p(1-q^2) - p^2}{(1-q)^4}$$

Q. The prob. density function of a random variable

$P(x) = \frac{1}{2\theta} e^{-\frac{|x|+\theta}{\theta}} \qquad -\alpha < x < \alpha$

find m.g.f., mean, variance.

P.T.O.

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Sol: $p(x) = \frac{1}{2\theta} e^{-\frac{|x-\theta|}{\theta}}$

Mgf $= M_x(t) = E(e^{xt})$

$$= \int_{-\alpha}^{\alpha} e^{xt}\cdot\frac{1}{2\theta}\, e^{-\frac{|x-\theta|}{\theta}}\, dx$$

$$= \int_{-\alpha}^{\theta} e^{xt}\cdot\frac{1}{2\theta}\cdot e^{\frac{(x-\theta)}{\theta}}\, dx + \int_{\theta}^{\alpha} e^{xt}\cdot\frac{1}{2\theta}\, e^{-\frac{(x-\theta)}{\theta}}\, dx$$

$$= \frac{1}{2\theta}\int_{-\alpha}^{\theta} e^{x(t+\frac{1}{\theta})}\cdot e^{-1}\, dx + \frac{1}{2\theta}\int_{\theta}^{\alpha} e^{-x(t-\frac{1}{\theta})}\cdot e\, dx$$

$$= \frac{e^{-1}}{2\theta}\left[\frac{e^{x(t+\frac{1}{\theta})}}{t+\frac{1}{\theta}}\right]_{-\alpha}^{\theta} + \frac{e}{2\theta}\left[\frac{e^{-x(t+\frac{1}{\theta})}}{t+\frac{1}{\theta}}\right]_{\theta}^{\alpha}$$

$$= \frac{e^{-1}}{2\theta}\left[\frac{e^{(\theta t+1)}}{t+\frac{1}{\theta}} - 0\right] + \frac{e}{2\theta}\left[0 - \frac{e^{\theta t-1}}{t+\frac{1}{\theta}}\right]$$

$$= \frac{e^{\theta t}}{2\theta(t+\frac{1}{\theta})} + \frac{e^{\theta t}}{2\theta(t+\frac{1}{\theta})}$$

$$= \frac{2e^{\theta t}}{2\theta(t+\frac{1}{\theta})} = \frac{e^{\theta t}}{\theta(t+\frac{1}{\theta})}$$

Mgf $= e^{\theta t}(1+\theta t)^{-1}$

$$= \left(1+\theta t+\frac{(\theta t)^2}{2!}+\cdots\right)\left(1-\theta t+\frac{(\theta t)^2}{2!}-\frac{(\theta t)^3}{3!}+\cdots\right)$$

$\therefore \boxed{1+t(2\theta)+t^2}$

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Binomial Distribution

① $P(X=x) = \begin{cases} {}^nC_x \, (p)^x (q)^{n-x} & ; \; x = 1, 2, 3, 4 \ldots \\ 0 & , \; \text{otherwise} \end{cases}$

②. Mean $= np$

③ Variance $= npq$

Q. 10 coins are tossed simultaneously. find the prob of getting atleast 7 heads and exactly 7 heads

Sol:- ① Exactly 7 heads

$$P(X=7) = {}^{10}C_7 \left(\tfrac{1}{2}\right)^7 \left(\tfrac{1}{2}\right)^{10-7}$$

$$= {}^{10}C_7 \left(\tfrac{1}{2}\right)^7 \left(\tfrac{1}{2}\right)^3$$

$$= \frac{8 \times \cancel{9}^{3} \times \cancel{10}^{5}}{\cancel{3} \times \cancel{2}} \left(\tfrac{1}{2}\right)^{10}$$

(ii) Atleast 7 heads

$$P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_7 \left(\tfrac{1}{2}\right)^7 \left(\tfrac{1}{2}\right)^3 + {}^{10}C_8 \left(\tfrac{1}{2}\right)^8 \left(\tfrac{1}{2}\right)^2 + {}^{10}C_9 \left(\tfrac{1}{2}\right)^9 \left(\tfrac{1}{2}\right) + {}^{10}C_{10} \left(\tfrac{1}{2}\right)^{10}$$

$$= \left({}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right) \left(\tfrac{1}{2}\right)^{10}$$

Mean $= E(x) = 10 \times \frac{1}{2} = 5$

Variance $= 10 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{10}{4} = 2.5$

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Q. A and B play a game in which their chance of wining are in ratio 3:2. Find its chance of wining atleast 3 games out of the 5 games played.

Sol:

A : B

3 : 2

$p = \frac{3}{5}$ $q = \frac{2}{5}$

$$P(X) = {}^5C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2 + {}^5C_4\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right) + {}^5C_5\left(\frac{3}{5}\right)^5$$

Q. A coffee consumer claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of the time. It is aggreed that his claim will be accepted if he correctly identify atleast 5 of 6 staffs. Find his chance of having the claim

a. accepted.

b. rejected.

Sol: Probability = 75% = $\frac{75}{100}$ = $\frac{3}{4}$

$p = \frac{3}{4}$ $q = \frac{1}{4}$

a. $P(X \geq 5) = {}^6C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right) + {}^6C_6\left(\frac{3}{4}\right)^6$

b. $P(X < 5) = 1 - P(X \geq 5)$

P.T.O.

Q. A multi choice test consists 8 questions with 3 answers to each of which ~~one~~ 1 is correct. Each student answer each questions by rolling a die and ticking the 1st option. If he get 1 or 2, he select 1st quest. 3 or 4, 2nd and 5 or 6 in 3rd. to get a distinction, the student must score 75%. If there is no -ve marking, what is the prob that student secure the distinction.

Sol. $P(X \geq 6) = {}^8C_6\left(\frac{2}{6}\right)^6\left(\frac{4}{6}\right)^2 + {}^8C_7\left(\frac{2}{6}\right)^7\left(\frac{4}{6}\right)^1 + {}^8C_8\left(\frac{2}{6}\right)^8$

$= {}^8C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^2 + {}^8C_7\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right) + {}^8C_8\left(\frac{1}{3}\right)^8$

$= \left(\frac{1}{3}\right)^8\left({}^8C_6\, 2^2\right.$

Q. An irregular face die is thrown, an expectation that in 10 throws it will give 5 even nos. is twice the expectation that it will give 4 even nos. How many items in 10,000 sets of 10 throws each. What do you expect to give no even nos.

Sol:- $P(X=5) = 2P(X=4)$

${}^{10}C_5\, p^5 q^5 = 2.\, {}^{10}C_4\, p^4 q^6$

$\Rightarrow \frac{10!}{5!\,5!} p^5 q^5 = 2 \frac{10!}{4!\,6!} p^4 q^6$

$\Rightarrow \frac{p}{5} = \frac{q}{6} \Rightarrow \frac{p}{5} = \frac{1-p}{3}$ $\Rightarrow 3p = 5-5p$ $\Rightarrow 8p = 5 \Rightarrow p = \frac{5}{8}$

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$$P(x=0) = {}^{10}C_0 \left(\frac{5}{8}\right)^0 \left(\frac{3}{8}\right)^{10}$$

17-12

$$= \left(\frac{3}{8}\right)^{10}$$

$$= 10{,}000 \times \left(\frac{3}{8}\right)^{10}$$

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Poison Distribution:

$$P(X=x) = \begin{cases} \dfrac{e^{-\lambda} \cdot \lambda^x}{x!} & ; \; x = 0,1,2,3 \\ 0 & ; \; \text{other} \end{cases}$$

Mean = var = $\lambda = np$.

Q. A manufacturer of pen knows that 5% of his product are defective. If he sells pen in box of 100 and gurantee is that not more than 10 pens are defective in one box. What is the approx. prob. that a box will p. ~~mean~~ fails to meet the guranteed quality.

Sol: $n = 100$

$p = \frac{5}{100} = 0.05$

$\lambda = np = 100 \times 0.05$

$= 5$

$$P(X \geq 10) = 1 - P(X \leq 10)$$

$$= 1 - \big(P(X=10) + P(X=9) + P(X=8) + \cdots + P(X=0)\big)$$

$$= 1 - \left(\frac{e^{-5} \cdot (5)^{10}}{10!} + \frac{e^{-5} \cdot (5)^9}{9!} + \cdots + \frac{e^{-5} (5)^0}{0!}\right)$$

$$= 1 - e^{-5} \sum_{p=0}^{10} \frac{(5)^n}{n!}$$

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Q. A car hire firm has 2 cars which ~~it~~ hires out day by day. The number of demands for a car on each day is distributed as Poison distribution with ~~mean~~ mean = 1.5. Calculate the proportion of the day on which a) neither car is used.

b) proportion of the day some demands are refused

Sol: Mean = 1.5

(a) $P(X=0) = \dfrac{e^{-1.5} \cdot (1.5)^0}{0!} = e^{-1.5} = (2.7)^{-1.5}$

(b) $P(X>2) = 1 - P(X \leq 2)$

$= 1 - \left(P(X=2) + P(X=1) + P(X=0)\right)$

$= 1 - \left(\dfrac{e^{-1.5}(1.5)^2}{2!} + \dfrac{e^{-1.5}(1.5)^1}{1!} + \dfrac{e^{-1.5}(1.5)^0}{0!}\right)$

$= 1 - e^{-1.5} \sum_{i=0}^{2} \dfrac{(1.5)^n}{n!}$

Q. $P(x=2) = 9P(x=4) + 90P(x=6)$

$\dfrac{\cancel{e^{-\lambda}} \cdot \lambda^2}{2!} = 9\dfrac{\cancel{e^{-\lambda}} \cdot \lambda^4}{4!} + 90\dfrac{\cancel{e^{-\lambda}}\lambda^6}{6!}$

$\Rightarrow \dfrac{1}{\cancel{2} \times 1} = \dfrac{\overset{3}{\cancel{9}} \cdot \lambda^2}{4 \times \cancel{3} \times \cancel{2}} + \dfrac{\overset{15}{\cancel{90}}\lambda^4}{\cancel{6} \times 5 \times 4 \times 3 \times \cancel{2}}$

$\Rightarrow 1 = \dfrac{3\lambda^2}{4} + \dfrac{\lambda^4}{4}$

$\Rightarrow 4 = \lambda^4 + 3\lambda^2$

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Q. An insurance company ensures 4000 company against the lost of both eyes in car accident. Based on the previous data, the rates are computed on the assumption that on the avg 10 p in 1,00,000 will have car accident each year that result in this type of ~~enquire~~ injury. What is the prob. that more than 3 of the ensured person get their claim.

Sol: $n = 4000$

$p = \frac{10}{1,00,000}$

$\lambda = np = 4000 \times \frac{10}{1,00,000} = 0.4$

$P(X > 3)$

Q. 6 coins are tossed 6400 times. Using the poison distribution find the approximate probability of getting 6 heads r times.

Sol: $n = 6400$

$p = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

$\lambda = np = 6400 \times \frac{1}{64} = 100$

$$P(X = r) = \frac{e^{-100}(100)^r}{r!}$$

Q. A manufacturer who produces medicine bottles finds that 0.1% of the bottles are defective. The bottles are packed in a box containing 500 bottles each. A drug manufacturer buys 100 boxes from the producer of the bottle ~~ev~~ find how many boxes will contain

a. No defective bottle.

b. Atleast 2 defective bottle.

Sol^n:

$n = 500$

$p = 0.001$

$\lambda = 0.5$

$$P(X=0) = \frac{e^{-0.5}\,(0.5)^0}{0!} = e^{-0.5}$$

$$100 \text{ boxes} = 100 \times e^{-0.5} = 100 \times 0.606$$

$$= 60.6$$

Normal Distribution:

$$P(X=x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\alpha < x < \alpha$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}(z)^2} \quad \text{where } z = \frac{x-\mu}{\sigma}$$

μ = mean

σ = Standard deviation

$\sqrt{Var} = \sigma$

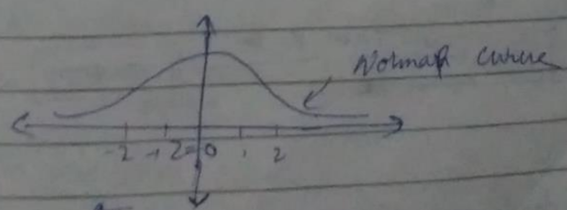

Diagram.

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Properties of Normal Curve

1. Total area under this curve is always 1.
2. In Norman Distribution, mean, median, mode are same
3. Curve is splitted on both sides — 0.5 on left 0.5 on right.

Q. X is normally distributed and the mean of x is 12 and std. deviation is 4. (a) Find the prob. when $P(x \geq 20)$ $P(x \leq 20)$ $P(0 \leq x \leq 12)$

(i) $P(X \geq 20)$

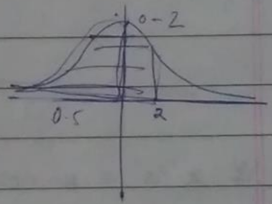
$P\left(\frac{X-\mu}{\sigma} \geq \frac{20-\mu}{\sigma}\right)$

$P\left(z \geq \frac{20-12}{4}\right)$

$= P(z \geq 2)$

$= 0.5 - P(0 \leq z \leq 2)$

$= 0.5 - 0.47725$



(ii) $P(X \leq 20) = 1 - P(X \geq 20)$

or $P\left(\frac{X-\mu}{\sigma} \leq \frac{20-12}{4}\right)$

$= P(z \leq 2)$

$= 0.5 + 0.47725$

(iii) $P(0 \leq x \leq 12)$

$= P\left(\frac{0-12}{4} \leq \frac{x-\mu}{\sigma} \leq \frac{12-12}{4}\right)$

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$= P(-3 \le Z \le 0)$

$= 0.49865$

Q Find x'
when $P(X > x_1') = 0.25$

Sol
$$P\left(\frac{X-\mu}{\sigma} > \frac{x_1' - \mu}{\sigma}\right)$$
$$= P\left(Z > \frac{x_1' - 12}{4}\right) = 0.25$$
$$= 0.5 - P\left(0 \le Z \le \frac{x_1' - 12}{4}\right) = 0.25$$
$$= P\left(0 \le Z \le \frac{x_1' - 12}{4}\right) = 0.25$$
$$\frac{x_1' - 12}{4} = 0.67$$
$$x' = (0.67)4 + 12$$
$$x_1' = 14.6$$

27/9/16

Q. If x is a normal variate with mean $= 30$, $SD = 5$. Find the probability (i) $P(26 \le x \le 40)$ (ii) $P(|x-30| > 5)$

(i)
$$P\left(\frac{26-30}{5} \le \frac{x-\mu}{\sigma} \le \frac{40-30}{5}\right)$$
$$= P\left(-\frac{4}{5} \le Z \le 2\right)$$
$$= P(-0.8 \le Z \le 0) + P(0 \le Z \le 2)$$
$$= 0.28814 + 0.47725$$
$$= 0.76539$$

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(ii) $P(|x-30| > 5)$

$\boxed{|x| \le 3 \Rightarrow -3 \le x \le 3}$ <upto 5 decimal pts>

$= 1 - P(|x-30| \le 5)$

$= 1 - P(-5 \le x-30 \le 5)$

$= 1 - P\left(-1 \le \frac{x-30}{5} \le 5\right)$

$= 1 - P(-1 \le z \le 1) = 1 - 2P(0 \le z \le 1)$

$= 1 - (0.34134) \times 2$

$= 1 - 0.68268 = 0.31732$

Q. The mean yield for 1 acre plot is 662 kilos with SD 32 kilos. Assuming normal distribution. How many 1 acre plot in a batch of 1000 plots what do you expect to have yield, (i) over 700 kilos (ii) below 650 kilos. (iii) what is the lowest yield of best 100 plots.

Sol: $P(X>700) = P\left(\frac{X-\mu}{\sigma} > \frac{700-662}{32}\right)$

$= P(X > 1.19) = 0.5 - P(0 \le Z \le 1.19)$

$= 0.5 - 0.38298$

$= 0.11702$

$\Rightarrow 1000 \times 0.11702 \to 117.02$

$P(X<650) = P\left(\frac{X-\mu}{\sigma} < \frac{650-662}{32}\right) = P\left(Z < \frac{-12}{32}\right)$

$= P(Z < 0.375) = P(Z < 0.38)$

$= 0.5 - P(0 \le Z \le 0.38)$

$= 0.5 - 0.14803$

$= 0.35197$

$= 0.35197 \times 1000 = 351.97$

(iv) $P(z > z_1) = \frac{100}{1000} = 0.1$

$= 0.5 - P(0 \le z \le z_1) = 0.1$

$P(0 \le z \le z_1) = 0.4$

$z_1 = 1.28$

$\frac{x_1 - 662}{32} = 1.28$

$\Rightarrow x_1 - 662 = 1.28 \times 32$

$\Rightarrow x_1 = 662 + 40.96$

$= 702.46$

Q. The local authority in a certain city is cost 10,000 electrical lamp in street of the city. If these lamps have an avg life of 1000 burning hours of SD of 200 hrs. Assuming normality what no. of lamps might be expected to fail in (i) 1st 800 burning hours (ii) between 800 and 1200 burning hours (iii) after what period of burning hours. what would you expect 10% of lamps would fail.

Sol. (i) $P(x < 800) = P\left(\frac{x-\mu}{\sigma} < \frac{800-1000}{200}\right)$

$P(z < -1) = 0.5 - P(0 \le z \le 1)$

~~(ii) P(800 < x < 1200) = P(800 - ...~~

(ii) $P(800 \le x \le 1200) = P\left(\frac{800-1000}{200} \le z \le \frac{1200-1000}{200}\right)$

$= P(-1 \le z \le 1) = 2P(0 \le z \le 1)$

(iii) $P(z > z_1) = 0.1$

$= 0.5 - P(0 \le z \le z_1) = 0.1$

$P(0 \le z \le z_1) = 0.4$

$\Rightarrow z_1 = 1.28$

$\frac{x_1 - 1000}{200} = 1.28$

$\Rightarrow x_1 = 1000 + 1.28 \times 200$