L9 06 Feb 2021

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Linear Differential Equations (LDE)

A linear ordinary differential equation of order n, is written as

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots\cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)\cdot y = \boxed{s(x)}$$

where y is dependent variable and x is independent variable and $a_0(x) \neq 0$

or

$$a_0 y^n + a_1 y^{n-1} + \cdots\cdots + a_{n-1}y' + a_n y = s(x)$$

$y' = \frac{dy}{dx}$

$y'' = \frac{d^2y}{dx^2}$

$y^n = \frac{d^ny}{dx^n}$

Example

$$y'' + 2y' + 3y = e^{2x}$$

LD non Homogen eqn with order 2.

If $s(x) = 0$ then it is called as Homogenous equation

If $s(x) \neq 0$ then it is non Homogenous equation.

$\frac{dy}{dx} = \boxed{f(x,y)}$

Homo.

$I.F = \frac{1}{1+x+My}$

If $a_i(x)$, $i = 0, 1, 2, 3, \ldots$ are constant then

the equations are linear diff. eqn with constant coefficient.

#

$$x^2 y'' + xy' + (x^2 - 4) = 0$$

Linear Homogenous with variable coeff of order 2.

#

The equation: $y'' + 4y' + xy = x^2 e^x$ is:

(A) 1st order Homogeneous LDE with variable coefficients.

(B) 2nd order Homogeneous LDE with constant coefficients.

(C) 2nd order Non-homogeneous LDE with variable coefficients.

(D) 2nd order Non-homogeneous LDE with constant coefficients

1 4 x

Order = 2 {highest order derivative}

Linear :- powers of y, y', y''

Homo / Non Homo. :- $s(x) \neq 0$ so Non Homo.

Constant / variable coeff :- 1, 4, x

(D) 2^{nd} order Non-homogeneous LDE with constant coefficients

Constant/variable Coff :→ 1, 4, x variable coff.

Thm :→ If the functions $a_0(x), a_1(x), \ldots\, a_n(x)$ and $s(x)$ are continuous over I and $a_0(x) \neq 0$ on I then there exists a unique solution to the problem (*) with initial condition $y(x_0)=c_1,\ y'(x_0)=c_2 \ldots\ y^{n-1}(x_0)=c_n$ where $x_0 \in I$

If the above conditions are satisfied then the given eqn is normal on I

Find the intervals on which the following differential equations are normal.

Problem 1. $(1-x^2)y'' - 2xy' + 3y = 0$

$a_0 = 1-x^2$	$a_1 = -2x$	$a_2 = 3$	$s(x)=0$
$(-\infty,\infty)$	$(-\infty,\infty)$	$(-\infty,\infty)$	

$\downarrow$

$(-\infty,\infty)$

① $a_0(x), a_1(x) \ldots a_n(x)$ and $s(x)$ are cont. on I
② $a_0(x) \neq 0$

$a_0(x)=0$
$1-x^2=0$
$x^2=1$
$x=\pm 1$

normal

$(-\infty,-1), (-1,1), (1,\infty)$

② $x^2y'' + xy' + (n^2-x^2)y = 0$; n is real.

$a_0 = x^2$	$a_1 = x$	$a_2 = n^2-x^2$
$(-\infty,\infty)$	$(-\infty,\infty)$	$(-\infty,\infty)$

$\downarrow$

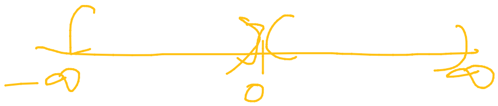
$(-\infty,\infty)$

① $a_0(x), a_1(x), \ldots a_n(x)$ and $s(x)$ are continuous on I
② $a_0(x) \neq 0$ on I

$a_0(x) = x^2 = 0$
$\Rightarrow \boxed{x=0}$

✓ $(-\infty,\infty)$ ✓ $\quad a_0(x) = x = 0$

$\Rightarrow \boxed{x=0}$ ✓



$(-\infty,0)$, $\underline{(0,\infty)}$

\# ③ $y'' + 3y' + \sqrt{x}\, y = \sin x$

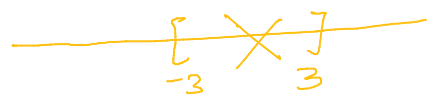
$a_0 = 1$	$a_1 = 3$	$a_2 = \sqrt{x}$	$g(x) = \sin x$
$(-\infty,\infty)$	$(-\infty,\infty)$	$[0,\infty)$ ✓	$(-\infty,\infty)$

Nbhd on $I = [0,\infty)$ ✓

$a_0 = 1 \neq 0$

$$y'' + 9y' + y = \log(x^2 - 9)$$

$a_0 = 1$	$a_1 = 9$	$a_2 = 1$	$g(x) = \log(x^2 - 9)$
$(-\infty,\infty)$	$(-\infty,\infty)$	$(-\infty,\infty)$	$x^2 - 9 > 0$
			$\lvert x \rvert > 3$
			$(-\infty,-3) \cup (3,\infty)$



Nbhd $(-\infty,-3)$, $(3,\infty)$