① Random Experiment :→

② Out Comes of R.E ⟶ Results

③ Sample Space : ? Set of all possible Out Comes

$S = \{H, T\}$ $S = \{1, 2, 3, 4, 5, 6\}$

$S = \{(1,1)\ (1,2)\ (1,3)\ ---\ (6,6)\}$

Event :- Sub set of S.S A, B $A \cup B$ $A - B$ $A \cap B$ $\bar{A}$

① Equally Likely Outcomes :- ② Exhaustive Out Comes

32.9. PROBABILITY OF AN EVENT

Suppose in a random experiment, there are n exhaustive, equally likely outcome. Let A be an event and there are m outcomes (cases) favourable to the happening of it. The **probability** P(A) of the happening of the event A is defined as :

$$P(A) = \frac{\textbf{Total number of cases favourable to the happening of A}}{\textbf{Total number of exhaustive equally likely cases}} = \frac{m}{n}.$$

It may be observed from this definition, that $0 \le m \le n$.

$S = \{ —— n —— \}$ $A \subset S$ → Event

$A = \{ — m — \}$

$\boxed{P(A) = \frac{m}{n}}$ $0 \le m \le n$

① $0 \le m \le n \Rightarrow \frac{0}{n} \le \frac{m}{n} \le \frac{n}{n}$

$\Rightarrow 0 \le \frac{m}{n} \le 1$

$\Rightarrow \boxed{0 \le P(A) \le 1}$

② $P(A) = 0$ A – Impossible Event — 0%

$P(A) = 1$ A – Sure Event — 100%

$P(A) = 0.23$

23%

③ $\bar{A}$ = Not the Event A or Not A

$n(\bar{A}) = n - m$

$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$

$\boxed{P(\bar{A}) = 1 - P(A)}$ or $\boxed{P(A) + P(\bar{A}) = 1}$

32.10. 'ODDS IN FAVOUR' AND 'ODDS AGAINST' AN EVENT

Let A be an event of a random experiment. The ratio $P(A) : P(\bar{A})$ is called the **odds in favour** of happening of the event A. The ratio $P(\bar{A}) : P(A)$ is called the **odds against** the happening of the event A.

Odds in Favour
$P(A) : P(\bar{A})$

Odds Against
$P(\bar{A}) : P(A)$

Let odds in favour of an event A be $m : n$.

Let $P(A) = p$. $\therefore$ $p : 1 - p = m : n$

$$\Rightarrow \quad \frac{p}{1-p} = \frac{m}{n} \Rightarrow np = m - mp \Rightarrow p = \frac{m}{m+n} \text{ i.e., } P(A) = \frac{m}{m+n}.$$

$\therefore$ **If odds in favour of A are m : n, then** $\mathbf{P(A) = \frac{m}{m+n}}$.

Similarly, if odds against A are $m : n$ then odds in favour of A are $n : m$ and

$$P(A) = \frac{n}{n+m}.$$

Example 2. *Find the probability of the event A if (i) odds in favour of event A are 5 : 7 (ii) odds against A are 3 : 4.*

$\rightarrow P(A) : P(\bar{A}) = 5 : 7$

$P(A) = \frac{5}{5+7} = \frac{5}{12}$

$P(\bar{A}) : P(A) = 3 : 4$

$\rightarrow \frac{4}{3+7} = \frac{4}{7}$

Example 5. *Two unbiased coins are tossed simultaneously. Find the probability of getting :*

(i) one head *(ii) one tail* *(iii) at most one head*
(iv) at least one tail *(v) more than 2 heads* *(vi) less than 3 tails.*

$S = \{HH, TH, HT, TT\}$ $n(S) = 4$

Example 8. *An unbiased die is thrown. Find the probability of getting :*

(i) number greater than 4 *(ii) an even number*
(iii) a number less than 8 *(iv) a multiple of 4.*

$S = \{1, 2, 3, 4, 5, 6\}$ $n(S) = 6$

(1) $P(A) = \frac{2}{6} = \frac{1}{3}$

Example 9. *Two unbiased dice are thrown simultaneously. Find the probability of :*

(i) getting sum 10
(ii) not getting same number on the dice → Doublet — $\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$
(iii) getting a multiple of 3 as the sum
(iv) an even number on the first die and an odd number on the second die.

$S = \{(1,1)\ (1,2)\ (1,3)\ - - -\ (6,6)\}$ $n(S) = 36$

(1) A = Event "getting Sum 10" $= \{(4,6), (5,5)\ (6,4)\}$

$P(A) = \frac{3}{36} = \frac{1}{12}$

(3) Multiple of 3 as a Sum 3, 6, 9, 12

P(Not Same Number) = 1 − P(Same Number) $= 1 - \frac{1}{6} = \frac{5}{6}$

$\to$ $P(\text{Not Same Number}) = 1 - P(\text{Same Number}) = 1 - \frac{1}{6} = \frac{5}{6}$

$\to$ $A = \{(1,2)(2,1)(1,5)(2,4)(3,3)(4,2)(5,1)$
$(3,6)(4,5)(5,4)(6,3)(6,6)\}$

$n(A) = 12$

$P(A) = \frac{12}{36} = \frac{1}{3}$

(IV) $A = \{(2,1)(2,3)(2,5)(4,1)(4,3)(4,5)(6,1)(6,3)(6,5)\}$

$P(A) = \frac{9}{36} = \frac{1}{4}$

(5) *Find the probability that a leap year, selected at random, will contain* 53 Sunday

No. of days in leap year = 366

$366 = 7 \times 52 + \boxed{2}$ $\to$ Extradays

$$\begin{array}{r} 7\overline{)366}(52 \\ 35 \\ \hline 16 \\ 14 \\ \hline 2 \end{array}$$

Possibilities of 2 Extradays

Sun – Mon, Mon – Tue, Tues – Wed, Wed – Thu, Thu – Fri, Frid – Sat, Sat – Sun.

A = Event " 53 Sunday)

$\boxed{P(A) = \frac{2}{7}}$ — $\frac{2}{7}$

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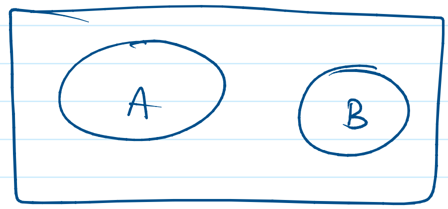
$P(A) = \frac{n(A)}{n(S)}$ $0 \le P(A) \le 1$

$A \subseteq S$ A, B $A \cup B$ A or B $A+B$
$A \cap B$ A and B $A \cdot B$

$P(\bar{A}) = 1 - P(A)$

$P(A) + P(\bar{A}) = 1$



① Mutually Exclusive ↦ $A \cap B = \phi$

$n(A \cap B) = 0$

$\boxed{P(A \cap B) = 0}$

32.13. ADDITION THEOREM (GENERAL)

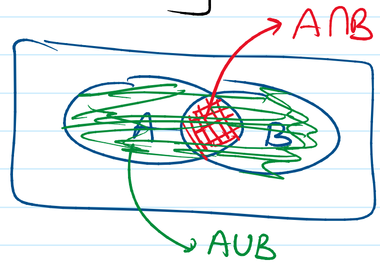
If A and B are two events not necessarily mutually exclusive, associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

① $\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$

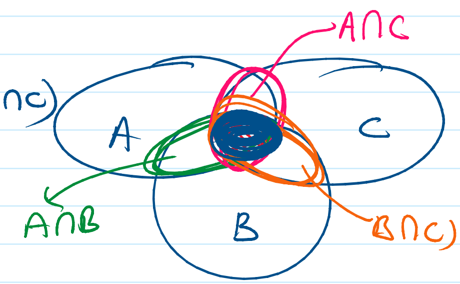
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Note: A&B are m.e event $\therefore P(A \cap B) = 0$

$\boxed{P(A \cup B) = P(A) + P(B)}$ $P(A \text{ or } B) = P(A) + P(B)$

② $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$



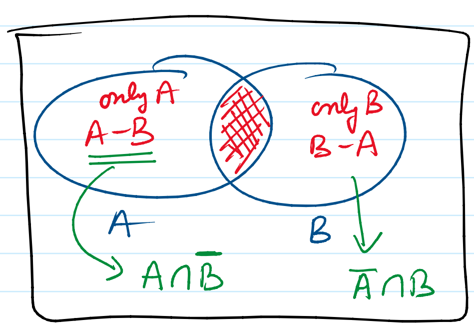
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

③ If A, B, C are m.e pairwise.

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

④ $n(A - B) = n(A) - n(A \cap B)$

$P(A - B) = P(A) - P(A \cap B)$



$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$$P(A\cap\overline{B}) = P(A) - P(A\cap B)$$
$$P(\overline{A}\cap B) = P(B) - P(A\cap B)$$

$\rightarrow A\cap B \qquad \overline{A}\cap B$

Example 4. *If A and B are mutually exclusive events associated with a random experiment such that P(A) = 0.4 and P(B) = 0.5, then find :*

(i) $P(\overline{A})$ (ii) $P(\overline{B}) = 0.5$ (iii) $P(A\cup B)$

(iv) $P(A\cap\overline{B})$ (v) $P(\overline{A}\cap B)$ (vi) $P(\overline{A}\cap\overline{B})$.

$P(A) = 0.4$
$P(B) = 0.5$
$\because$ A&B are m.e
$\therefore P(A\cap B) = 0$

① $P(\overline{A}) = 1 - P(A) = 1 - 0.4 = 0.6$

③ $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.4 + 0.5 - 0 = 0.9$

④ $P(A\cap\overline{B}) = P(A) - P(A\cap B) = 0.4 - 0 = 0.4$

⑤ $P(\overline{A}\cap B) = P(B) - P(A\cap B) = 0.5$

⑥ $P(\overline{A}\cap\overline{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B)$
$= 1 - 0.9$
$= 0.1$

De-Morgan's law
$\overline{A\cup B} = \overline{A}\cap\overline{B}$
$\overline{A\cap B} = \overline{A}\cup\overline{B}$

$$P(A\cup\overline{B}) = P(A) + P(\overline{B}) - P(A\cap\overline{B})$$

Example 1. *If A and B are events associated with a random experiment such that P(A) = 0.3, P(B) = 0.6 and P(A ∩ B) = 0.2, then find :*

(i) $P(\overline{A})$ (ii) $P(\overline{B})$ (iii) $P(A\cup B)$

(iv) $P(A\cap\overline{B})$ (v) $P(\overline{A}\cap B)$ (vi) $P(\overline{A}\cap\overline{B})$.

Example 9. *Two unbiased dice are thrown. If A and B be the events "sum of numbers appearing on the two dice is even" and "at least one die shows the number 2" respectively. Find the probabilities of the following events :*

(i) both A and B occur (ii) at least one of the events A and B occurs

(iii) none of the events A and B occurs (iv) A but not B occurs. $P(A-B) = P(A) - P(A\cap B)$

$S = \{(1,1)\ (1,2)\ ---\ (6,6)\}$ $\boxed{n(S) = 36}$

A = " Sum of Number is Even " B = " Atleast one die show 2 "

$A = \{(1,1)\ (1,3)\ (2,2)\ (3,1)\ (1,5)\ (2,4)\ (3,3)\ (4,2)\ (5,1)\ (2,6)\ (3,5)\ (4,4)\ (5,3)$
$(6,2),\ (4,6)\ (5,5)\ (6,4)\ (6,6)\}$

$n(A) = 18$

$P(A) = \frac{18}{36} = \frac{1}{2}$

$B = \{(1,2)\ (2,1)\ (2,2)\ (2,3)$
$(2,4)\ (2,5)\ (2,6)$
$(3,2)\ (4,2)\ (5,2)\ (6,2)\}$

$\boxed{P(B) = \frac{11}{36}}$

① A and B $\qquad A\cap B$

$P(B) = \frac{11}{36}$

① A and B $\quad A \cap B$

$\boxed{P(A \cap B) = \frac{5}{36}}$

② $A \cup B \qquad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

③ $\overline{A} \cap \overline{B} \qquad P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = 1 - ? =$

$A \cup B \qquad$ A or B $\qquad A + B$

$A \cap B \qquad$ A and B $\qquad A \cdot B$

18-1-22

Tuesday, January 18, 2022 2:43 PM

Ex. 52 cards of playing cards are equally divided among four players. What is the probability that :
(*i*) any player may have four kings ?
(*ii*) a specified player may have four kings ?

Sol

How Many Players = 4   No. of Kings = 4

No. of Cards for each player = 13   No. of other Cards = 48

(i) ① ② ③ ④

$\begin{bmatrix} {}^4C_4 = 1 \\ {}^{48}C_9 \end{bmatrix} \longrightarrow$ " $\longrightarrow$ Total No. of Cases = ${}^{52}C_{13}$

$P\left(\text{Player } 1\right) = \dfrac{1 \times {}^{48}C_9}{{}^{52}C_{13}}$

Req. Prob = $4\left[\dfrac{1 \times {}^{48}C_9}{{}^{52}C_{13}}\right] = \dfrac{4 \times {}^{48}C_9}{{}^{52}C_{13}}$

(ii) P(Sepcified Player) = $\dfrac{{}^{48}C_9}{{}^{52}C_{13}}$

Ex. Find the probability that in a random arrangement of the letters of the word UNIVERSITY, the two I's do not come together.

UNIVERSITY

Req. Prob = P(Two I's do not Come Together) = 1 − P(I, Comes together)

I Comes 2 Time

Total No. of Possible Cases = $\dfrac{\lfloor 10}{\lfloor 2} = \dfrac{40320 \times 9 \times 10}{2} = 40320 \times 45$

(I I) U N V E R S T Y
1 2 3 4 5 6 7 8 9

No. of ways of Arng. = $\lfloor 9$ ${}^9P_9 = \lfloor 9$

No " " " Two I's = 1

F.P.C   No. of fav. Cases = $\lfloor 9 \times 1 = \lfloor 9$

P(Two I's Together) = $\dfrac{\lfloor 9}{\lfloor 10/2} = \dfrac{2 \cdot \cancel{\lfloor 9}}{10 \cdot \cancel{\lfloor 9}} = \dfrac{1}{5}$

Req. Prop = $1 - \dfrac{1}{5} = \dfrac{4}{5}$

$$\frac{\lfloor 11}{\lfloor 2 \cdot \lfloor 2 \cdot \lfloor 2} = ?$$

A occure 2 time
T ,, 2 time
M ,, 2 time

Ex A box contains 100 bulbs, 20 of which are defective. 10 bulbs are drawn at random. Find the probability that :

(i) 4 bulbs are defective
(ii) all bulbs are defective
(iii) more than 8 are defective
(iv) none is defective.

No. of Bulbs = 100 No. of Defective = 20
,, ,, Non-Defective = 80

Since 10 bulbs are sected

$n(S)$ = Total No. of Possible Cases = $^{100}C_{10}$

(1) $P(4 \text{ Def. Bulb}) = \dfrac{^{20}C_4 \times {}^{80}C_6}{^{100}C_{10}}$

(2) $P(\text{all Def.}) = \dfrac{^{20}C_{10}}{^{100}C_{10}}$

(3) $P(\quad) = P(\text{Either 9 or 10 Def})$

$= P(9 \text{ Def}) + P(10 \text{ Def})$

$= \dfrac{^{20}C_9 \times {}^{80}C_1 + {}^{20}C_{10}}{^{100}C_{10}}$

(4) $P(\text{None Def}) = \dfrac{^{80}C_{10}}{^{100}C_{10}}$

Ex The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination ?

$\cap$

A = Event " Pass in English" B = Event " Pass in Hindi"

$P(A \cap B) = 0.5$ $\quad P(\bar{A} \cap \bar{B}) = 0.1$ $\quad P(A) = 0.75$ $\quad P(B) = ?$

$\rightarrow P(\overline{A \cup B}) = 0.1$

$\Rightarrow 1 - P(A \cup B) = 0.1$

$\Rightarrow P(A \cup B) = 1 - 0.1 = 0.9$

By Addition theorem $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.9 = 0.75 + P(B) - 0.5$

$\Rightarrow 1.4 - 0.75 = P(B)$

$\Rightarrow \boxed{P(B) = 0.65}$

$$\Rightarrow 1.4 - 0.75 = P(B)$$

$$\Rightarrow \boxed{P(B) = 0.65}$$

Ex

(*i*) Two dice are tossed together. Find the probability of getting a doublet or a total of 6.

Union

(*ii*) A pair of dice is rolled. Find the probability of getting a doublet or sum of numbers to be at least 10.

A = Doublet = $\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$ $n(A) = 6$ $P(A) = \frac{6}{36}$

B = Sum 6 = $\{(1,5)\ (2,4)\ (3,3)\ (4,2)\ (5,1)\}$ $n(B) = 5$ $P(B) = \frac{5}{36}$

$A \cap B = \{(3,3)\}$ $P(A \cap B) = \frac{1}{36}$

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{10}{36} = \frac{5}{18}$$

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Thursday, January 20, 2022 10:37 AM

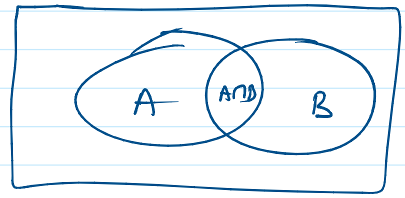
Conditional Probability :→

Let A and B be any two events associated with a random experiment. The probability of occurrence of event A when the event B has already occurred is called the **conditional probability of A** when B is given and is denoted as P(A/B). The conditional probability P(A/B) is meaningful only when P(B) ≠ 0, *i.e.*, when B is not an impossible event.

$P(A/B)$ = Cond. Prob. of Event A when event B has already occured.

$P(B) \neq 0$

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$



$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

\# $\boxed{P(A \cap B) = P(A) \cdot P(B/A)}$ or $\boxed{P(A \cap B) = P(B) \cdot P(A/B)}$

$P(A \text{ and } B) = P(A) \cdot P(B/A)$

$\boxed{P(A \cdot B) = P(A) \cdot P(B/A)}$ $\quad$ $\downarrow$ $\quad$ $\boxed{P(A \cdot B) = P(B) \cdot P(A/B)}$

Multiplication Theorem.

Independent Event :→

... of special importance.

Two events associated with a random experiment are said to be **independent events** if the occurrence or non-occurrence of one event does not affect the probability of the occurrence of the other event. For example, the events A and B are independent events when P(A/B) = P(A) and P(B/A) = P(B).

$\boxed{P(A/B) = P(A)}$ $\quad$ $\boxed{P(B/A) = P(B)}$

\# A and B are Independent Event iff $\boxed{P(A \cap B) = P(A) \cdot P(B)}$

$P(A \cap B) = P(A) \cdot P(B/A)$ $\quad$ ∵ A, B are Indep

$\boxed{P(A \cap B) = P(A) \cdot P(B)}$ $\quad$ ∴ $P(B/A) = P(B)$

\# If $P(A \cap B) \neq P(A) \cdot P(B)$; A, B are not Independent (Dependent)

\# If A, B are Indep. then $P(A \cap B) = P(A) \cdot P(B)$ here $P(A) \neq 0$, $P(B) \neq 0$

$\Rightarrow P(A \cap B) \neq 0$

$\Rightarrow A \cap B \neq \phi \Rightarrow$ A, B are not m.e Events

∴ Indep. Events Cannot be m.e Events.

If A, B, C are three Events.

$$P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B)$$

If A, B, C are Independent $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

Theorem II. Let A and B be events associated with a random experiment. If A and B are independent, then show that the events (*i*) $\bar{A}$, B (*ii*) A, $\bar{B}$ (*iii*) $\bar{A}$, $\bar{B}$ are also independent.

(ii) Find p so that A and B are independent.

3. (*i*) If P(not B) = 0.65, P(A ∪ B) = 0.85 and A, B are independent events, find P(A).

(*ii*) If P(not A) = 0.4, P(A ∪ B) = 0.75 and A, B are given to be independent events, find the value of P(B).

(1) $P(\bar{B}) = 0.65$ $\quad P(A \cup B) = 0.85$ $\quad P(A) = ?$

$P(B) = 1 - P(\bar{B})$

$= 1 - 0.65$

$= 0.35$

Addition theorem

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) \cdot P(B)$

$0.85 = P(A) + 0.35 - P(A)(0.35)$

6.6. INDEPENDENT EXPERIMENTS

Two random experiments are said to be **independent if**, for every pair of events A and B where A is associated with the **first** and B with the **second experiment**, the probability of simultaneous occurrence of the events A and B, when the two experiments are performed, is equal to the product of the probabilities P(A) and P(B) calculated separately on the basis of two experiments. The simultaneous occurrence of events A and B is denoted by A ∩ B or

A $\quad$ B

I $\quad$ II

$P(A \cap B) = P(A) \cdot P(B)$

P(A and B)

Example 1. *A and B appeared for an interview for two posts. Probability of A's rejection is 2/5 and that of B's selection is 4/7. Find the probability that only one of them is selected.*

E = "Selection of A" $\qquad F$ = "Selection of B"

$A\cancel{B}$ or $\cancel{A}B$

$P(\bar{E}) = \frac{2}{5}$ $\qquad P(F) = \frac{4}{7}$

$P(E) = 1 - \frac{2}{5} = \frac{3}{5}$ $\qquad P(\bar{F}) = 1 - \frac{4}{7} = \frac{3}{7}$

Req. Prob $= P(\text{only one is Selected}) = P(E\bar{F} \text{ or } \bar{E}F)$

$= P(E\bar{F}) + P(\bar{E}F)$

$= P(E)P(\bar{F}) + P(\bar{E})P(F) = \frac{3}{5} \cdot \frac{3}{7} + \frac{2}{5} \cdot \frac{4}{7} = \frac{9+8}{35} = \frac{17}{35}$

Example 11. *A can hit a target 3 times in 5 shots, B 2 times in 5 shots, C 3 times in 4 shots. Each fire a volley, what is the probability that 2 shots hit the target ?*

Example 11. A can hit a target 3 times in 5 shots, B 2 times in 5 shots, C 3 times in 4 shots. Each fire a volley, what is the probability that 2 shots hit the target ?

P = Event "A hit the Target)　$P(P) = \frac{3}{5}$　$P(\bar{P}) = \frac{2}{5}$

Q = " B " "　$P(Q) = \frac{2}{5}$　$P(\bar{Q}) = \frac{3}{5}$

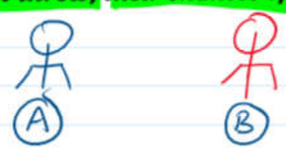
R = " C " "　$P(R) = \frac{3}{4}$　$P(\bar{R}) = \frac{1}{4}$

$$P(PQ\bar{R} \text{ or } \bar{P}QR \text{ or } P\bar{Q}R) = P(\quad) + P(\quad) + P(\quad)$$

$$= P(\,)\cdot P(\,)\cdot P(\,) + \_\_\_\_ + \_\_\_\_$$

$$= \frac{9}{20}$$

Example 17. A and B take turn in throwing two dice. The first to throw sum 9 being awarded. Show that if A has the first throw, their chances of winning are in the ratio 9 : 8.



$\boxed{n(S) = 36}$

$E = \{(3,6)(4,5)(5,4)(6,3)\}$

$n(E) = 4$

$P(E) = \frac{4}{36} = \frac{1}{9}$

$P(A_i) = \frac{1}{9}$　　$P(B_i) = \frac{1}{9}$

$P(\bar{A_i}) = \frac{8}{9}$　　$P(\bar{B_i}) = \frac{8}{9}$

i — Turn

$$P(\text{Winning of } A) = P[A_1 \text{ or } \bar{A_1}\bar{B_1}A_2 \text{ or } \bar{A_1}\bar{B_1}\bar{A_2}\bar{B_2}A_3 \text{ or } \text{------}]$$

$$= P(A_1) + P(\bar{A_1}\bar{B_1}A_2) + P(\qquad) \text{------}$$

$$= P(A_1) + P(\bar{A_1})P(\bar{B_1})P(A_2) + P(\,) \text{---} + \text{---}$$

$$= \frac{1}{9} + \frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9} + \frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9} + \text{------}$$

$$= \frac{1}{9}\left[1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \text{----}\right]$$

$\left\{ \begin{array}{l} \text{Infinite G.P} \\ S_\infty = \frac{a}{1-r} \end{array} \right.$

$a = 1 \quad r = \left(\frac{8}{9}\right)^2$

$$= \frac{1}{9}\left[\frac{1}{1-\left(\frac{8}{9}\right)^2}\right] = \frac{\cancel{1}}{\cancel{9}}\left[\frac{81}{81-64}\right] = \frac{9}{17}$$

$$P(\text{Winning of } B) = 1 - \frac{9}{17} = \frac{8}{17}$$

$$P(A \text{ win}) : P(B \text{ win}) = \frac{9}{17} : \frac{8}{17} = 9 : 8$$

$$P\left(\bar{A}_1 B_1 \text{ or } \bar{A}_1 \bar{B}_1 \bar{A}_2 B_2 \text{ or } \bar{A}_1 \bar{B}_1 \bar{A}_2 \bar{B}_2 \bar{A}_3 B_2 \text{ or } \cdots\right)$$

25-1-22

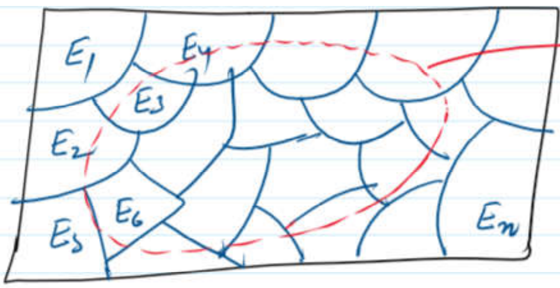
Tuesday, January 25, 2022 4:02 PM

Total Probability Rule

6.7. THEOREM REGARDING TOTAL PROBABILITY

Let $E_1, E_2, \ldots\ldots, E_n$ be n mutually exclusive and exhaustive events, with non-zero probabilities, of a random experiment. If A be any arbitrary event of the sample space of the above random experiment with $P(A) > 0$, then

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + \ldots\ldots + P(E_n)\,P(A/E_n).$$



'A' Arbitrary Event

$$P(A) = P(E_1A \text{ or } E_2A \text{ or } E_3A \text{ or } E_4A \text{ or } \cdots E_nA)$$

$$P(A) = P(E_1A) + P(E_2A) + P(E_3A) + \cdots + P(E_n A)$$

$$\boxed{P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2) + \cdots + P(E_n)P(A/E_n)}$$

Total probability Rule

$30-1 = 29$

A bag contains 12 white and 18 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and second is black?

(a) $\frac{18}{145}$ (b) $\frac{18}{29}$ (c) $\frac{36}{135}$ (d) $\frac{36}{145}$

$W = 12$, $B = 18$ Total 30

$$P(WB) = P(W)\cdot P(B/W) = \frac{\cancel{12}^{2}}{\cancel{30}_{5}} \times \frac{18}{29} = \frac{36}{145}$$

Out of 10 persons working on a project, 4 are graduates. If 3 are selected, what is the probability that there is at least one graduate among them?

(a) 5/6 (b) 1/6 (c) 2/3 (d) 1/3

Total = 10

G = 4, N.G = 6

G	N.G
3	0
2	1
1	2
0	3

No. Grad

All N.G

$1 - P(\text{Non Graduate})$

$$1 - \frac{{}^6C_3}{{}^{10}C_3} = 1 - \frac{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}}{\cancel{10}\cdot\cancel{9}\cdot\cancel{8}} = 1 - \frac{1}{6} = \frac{5}{6}$$

A box contains 3 blue marbles, 4 red, 6 green marbles and 2 yellow marbles. If four marbles are picked at random, what is the probability that none is blue?

(a) 32/91 (b) 30/91 (c) 33/91 (d) 28/91

$$\frac{{}^{12}C_4}{{}^{15}C_4} = \frac{\cancel{12}\cdot 11\cdot\cancel{10}\cdot\cancel{9}}{\cancel{15}\cdot\cancel{14}\cdot 13\cdot\cancel{12}} = \frac{33}{91}$$

Example 2. *One bag contains 5 white and 4 black balls. Another bag contains 7 white and 9 black balls. A ball is transferred from the first bag to the second bag and then a ball is drawn from the second. Find the probability that the ball is white.*

One Ball

W

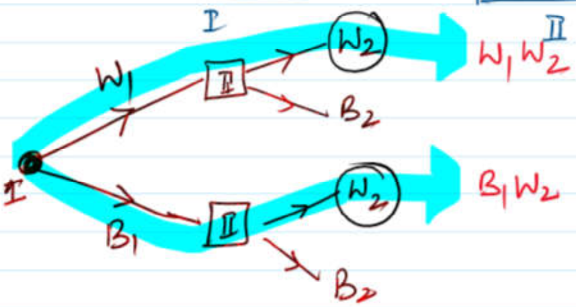
the second. Find the probability that the ball is white.

$$\boxed{\begin{matrix} W-5 \\ B-4 \end{matrix}}_{\text{I}} \xrightarrow{\text{One Ball}} \boxed{\begin{matrix} W-7 \\ B-9 \end{matrix}}_{\text{II}} \longrightarrow ?\ (\text{white Ball})$$

W: $7+1=8$

$B=9+1=10$

$w=7$



$$P(W_2) = P(W_1W_2 \text{ or } B_1W_2)$$

$$= P(W_1W_2) + P(B_1W_2)$$

$$= P(W_1)P(W_2/W_1) + P(B_1)P(W_2/B_1)$$

$$= \frac{5}{9}\times\frac{8}{17} + \frac{4}{9}\times\frac{7}{17} = \frac{4}{9}$$

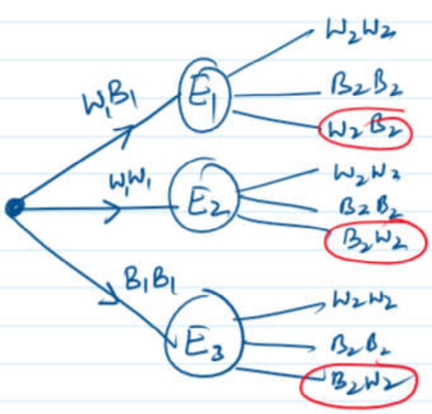
Example 4. *There are two bags. The first bag contains 5 white and 3 black balls and the second bag contains 3 white and 5 black balls. Two balls are drawn at random from the first bag and are put into the second bag, without nothing their colours. Then two balls are drawn from the second bag. Find the probability that the balls drawn are white and black.*

$$\boxed{\begin{matrix} W-5 \\ B=3 \end{matrix}}_{\text{I}} \xrightarrow{\text{Two Balls}} \boxed{\begin{matrix} W-3 \\ B-5 \end{matrix}}_{\text{II}} \longrightarrow \text{Two Balls } (WB)\ ?$$

Only three possibilities

$$\underbrace{WB \quad WW \quad BB}$$

$E_1 \quad E_2 \quad E_3$



$E_1\ (\underset{A}{W_2B_2})$ or $E_2\ (\underset{A}{B_2W_2})$ or $E_3\ (\underset{A}{B_2W_2})$

E_1A or E_2A or E_3A

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$$

E_1 — WB

$$P(E_1) = \frac{5\times 3}{8C_2}$$

$$\boxed{\begin{matrix} W-5 \\ B-3 \end{matrix}} \qquad \boxed{\begin{matrix} W-3+1=4 \\ B-5+1=6 \end{matrix}}$$

$$A \longrightarrow \frac{{}^4C_1\times{}^6C_1}{{}^{10}C_2}$$

$$P(E_1)\,P(A/E_1) = \frac{5\times 3}{28}\times\frac{4\times 6}{45}$$

E_2 — WW

$$P(E_2) = \frac{5C_2}{8C_2}$$

$$\boxed{\begin{matrix} W=3+2=5 \\ B=5=5 \end{matrix}}$$

$$P(A) = \frac{5C_1\times 5C_1}{10C_2}$$

27-1-22

Thursday, January 27, 2022 12:04 PM

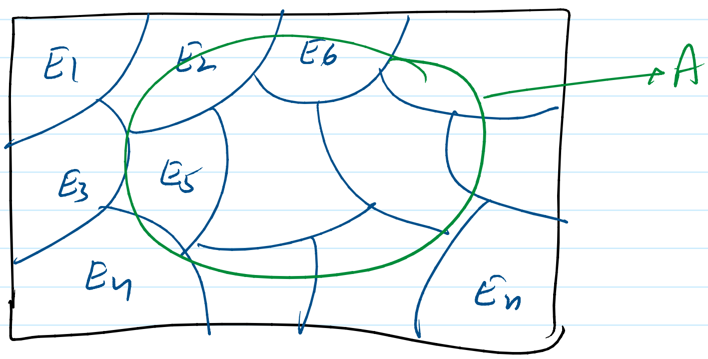
37.2. BAYES' THEOREM

Inverse Probability Rule

Let $E_1, E_2, \ldots\ldots, E_n$ be n mutually exclusive and exhaustive events, with non-zero probabilities, of a random experiment. If A be any arbitrary event of the sample space of the above experiment which occurs with E_1 or E_2 or …… or E_n and $P(A) \neq 0$, then

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A/E_j)}, \quad 1 \le i \le n.$$

A is any Event from S.S



$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3) + \cdots + P(E_n)P(A|E_n)$$

$$\boxed{P(E_i/A) = \frac{P(E_i) \cdot P(A|E_i)}{P(A)}}$$

$$\boxed{P(E_3/A) = \frac{P(E_3)\,P(A|E_3)}{P(A)}}$$

Example 2. *Assume that a factory has two machines. Past records shows that machine I produces 20% of the items of output and machine II produces 80% of the items. Further, 6% of the items produced by machine I were defective and only 1% produced by machine II were defective. If a defective item is drawn at random. What is the probability that it was produced by (i) machine I (ii) machine II ?*

Example 7. *In a bolt factory, machines A, B and C manufacture 60%, 25% and 15% respectively. Of the total of their output 1%, 2% and 1% are defective bolts. A bolt is drawn at random from the total production and found to be defective. From which machine, the defective bolt is expected to have been manufactured ?*

15%

machine A, B and C

Example 7. *In a bolt factory, machines A, B and C manufacture 60%, 25% and* 15% *respectively. Of the total of their output 1%, 2% and 1% are defective bolts. A bolt is drawn at random from the total production and found to be defective. From which machine, the defective bolt is expected to have been manufactured ?*

bolt produced by machine A, B and C

A — E_1 — 60%   B — E_2 — 25%   C — E_3 — 15%

$60+25+15 = 100\%$

$P(E_1) = \frac{60}{100}$ $P(E_2) = 25/100$ $P(E_3) = 15/100$

D = "Bolt is Defective"

$P(D/E_1) = \frac{1}{100}$ $P(D/E_2) = \frac{2}{100}$ $P(D/E_3) = \frac{1}{100}$

$$P(D) = P(E_1)P(D/E_1) + P(E_2)P(D/E_2) + P(E_3)P(D/E_3)$$

$$= \frac{60}{100}\cdot\frac{1}{100} + \frac{25}{100}\cdot\frac{2}{100} + \frac{15}{100}\cdot\frac{1}{100} = \frac{125}{10000}$$

$$P(E_1|D) = \frac{P(E_1)P(D/E_1)}{P(D)} = \frac{60/100 \cdot \frac{1}{100}}{125/10000} = \frac{60}{125} = \frac{12}{25}$$

$$P(E_2/D) = \frac{\frac{25}{100}\times\frac{2}{100}}{125/1000} = \frac{50}{125} = \frac{10}{25}$$

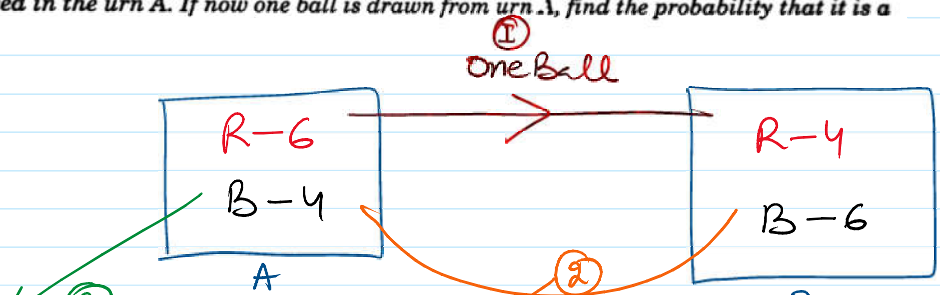
$$P(E_3/D) = \frac{\frac{15}{100}\cdot\frac{1}{100}}{125/10000} = \frac{15}{125} = \frac{3}{25}$$

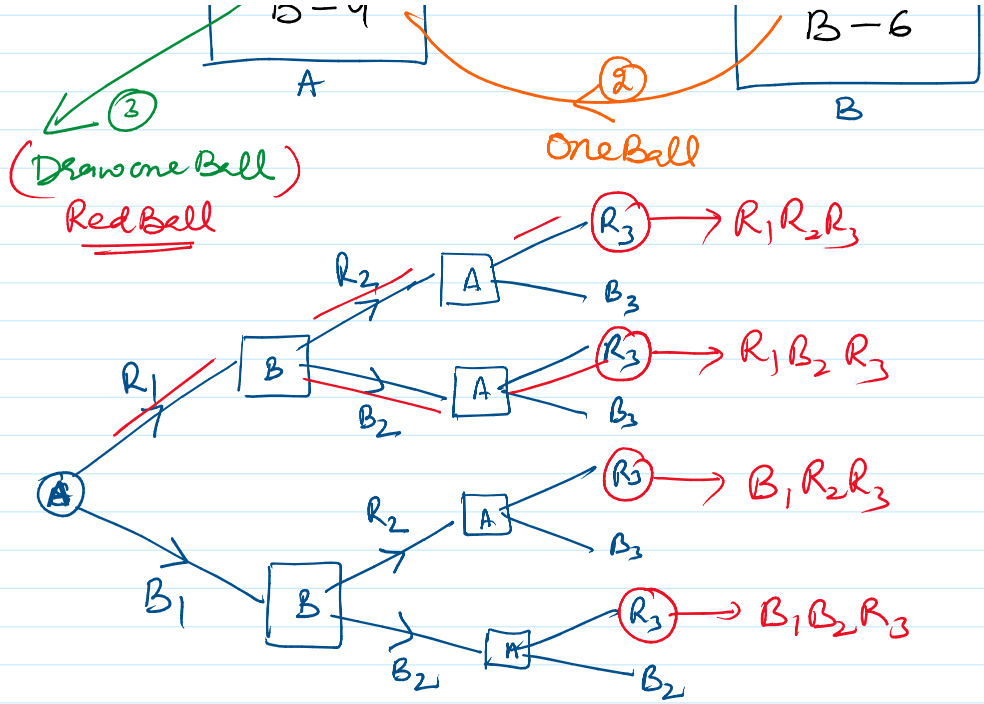
$P(\text{Mach. A}|D) = \frac{12}{25}$ $P(\text{Mach. B}|D) = \frac{10}{25}$

$P(\text{Mach. C}|D) = \frac{3}{25}$

$\therefore$ there are more chances that the Defective Bolt comes from Machine A

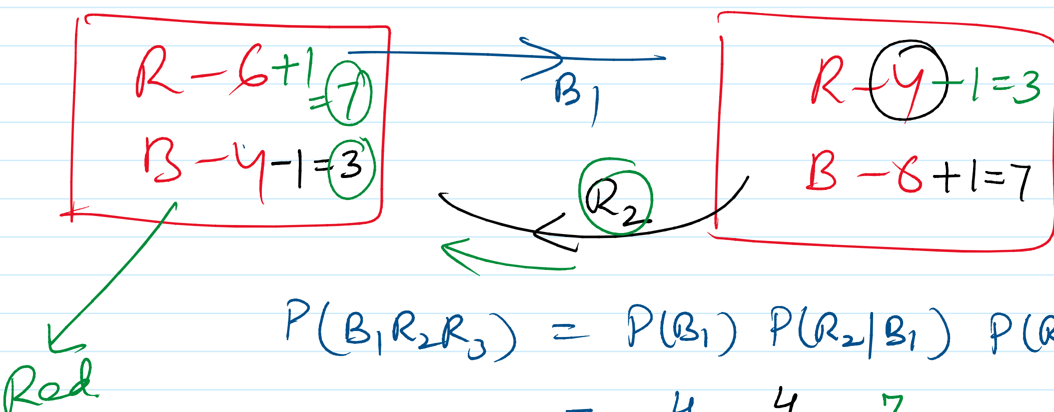
Example 9. *Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B, then one ball is drawn from the urn B and placed in the urn A. If now one ball is drawn from urn A, find the probability that it is a red ball.*





$$P(\text{Red Ball}) = P(R_3) = P(R_1R_2R_3) + P(R_1B_2R_3) + P(B_1R_2R_3) + P(B_1B_2R_3)$$

$$= P(R_1)P(R_2|R_1)\,P(R_3|R_1R_2) + - - - - -$$



$$P(B_1R_2R_3) = P(B_1)\;P(R_2|B_1)\;P(R_3|B_1R_1)$$

$$= \frac{4}{10}\cdot\frac{4}{11}\cdot\frac{7}{10}$$

29-1-22

Saturday, January 29, 2022 1:59 PM

Random Variable :→

Random Variable is a Variable whose Value is a numerical Outcomes of the Random Experiment.

Real Number X which is Connected with the Out Come of R.E.

Let S be the Sample Space of R.E, A Real Valued function X defined on the Sample Space S of the R.E is Called R.V

$$X: S \rightarrow R \text{ (Real Number) } (-\infty, \infty)$$

Ex Tossing a Coin $S = \{H, T\}$

A: " Head "

'X' the R.V. Assoc. to the event " Head"

$X = 1, 0$

$$X = \begin{cases} 1 & : \text{Head} \\ 0 & : \text{Tail} \end{cases}$$

$$X = \underset{\text{No Head}}{0}, \underset{\text{Head}}{1}, \underset{\text{Two Head}}{2}$$ Two Coin are tossed.

Types Two types ① Descrete Random Variable

② Continuous " "

Def 1: If a Sample space Contains a finite number of possibilities or an unending sequence with a many elements as they are whose number. It is Called a discrete Sample Space.

Def 2:- If a Sample Space Contains an Infinite number of possibilities equal to number of points on a line Segment, it is called Continuous Sample Space.

Note:- If a random Variable takes a finite set of Values, it is Called a discrete Random Variable or discrete Variable. On the other hand if it assumes a infinite number of Uncountable Values It is Called a Continuous Variate / Variable

$$X = 0, 1, 2, 3, 4, 5 ---$$

$$X = (0, 2) =$$

Ex) Tossing of Two Coins "No. of Heads"

$S = \{HH, HT, TH, TT\}$

X = Dis. R.V. "No. of Heads"

$X = 0, 1, 2$

$P(x=0) = \frac{1}{4}$

$P(x=1) = \frac{2}{4}$

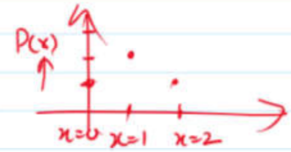
$P(x=2) = \frac{1}{4}$

x	0	1	2
$P(x)$	$1/4$	$2/4$	$1/4$

Probability Distribution

$P(X=x) > 0$

$\sum P(X=x) = \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = 1$



Discrete Probability Distribution :→

The Set of Ordered pair $(x, f(x))$ where $f(x) = P(X=x)$ is a probability function or discrete probability distribution or probability Mass function or probability distribution if

(i) $f(x) \geq 0$ $P(x) \geq 0$

(ii) $\sum f(x) = 1$ $\sum P(X=x) = 1$

(iii) where $f(x) = P(X=x)$ or $f(x) = p(x_i)$

$P(X=x) = f(x)$

$f(0) = P(X=0)$

$f(1) = P(X=1)$

$f(2) = P(X=2)$

$(0, \frac{1}{4})$ $(1, \frac{1}{2})$ $(2, \frac{1}{4})$

26.8 (1) DISCRETE PROBABILITY DISTRIBUTION

Suppose a discrete variate X is the outcome of some experiment. If the probability that X takes the values x_i is p_i, then

$$P(X = x_i) = p_i \text{ or } p(x_i) \text{ for } i = 1, 2, \ldots$$

where (i) $p(x_i) \geq 0$ for all values of i, (ii) $\Sigma p(x_i) = 1$

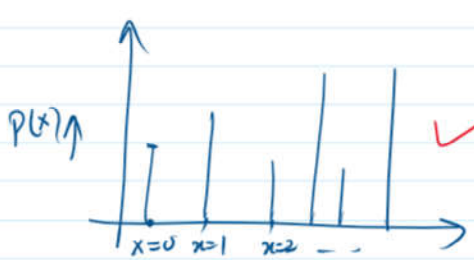
The set of values x_i with their probabilities p_i constitute a **discrete probability distribution** *of the discrete variate X.*

(2) Distribution function. *The distribution function* F (x) *of the discrete variate X is defined by*

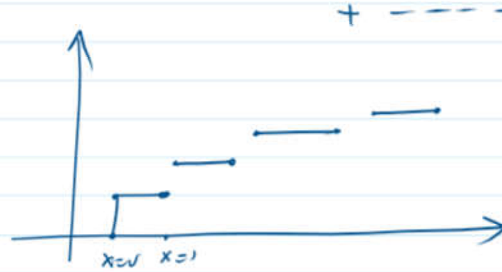
$$F(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$$ where x is any integer. The graph of $F(x)$ will be stair step form (Fig. 26.2). The distribution function is also sometimes called *cumulative distribution function.*

$$F(x) = P(X \leq x) = \sum_{i=1}^{x} P(x_i)$$

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} P(x_i) = P(X=0) + P(X=1) + P(X=2) + \cdots + P(X=x)$$



P.d



d. function

P.d

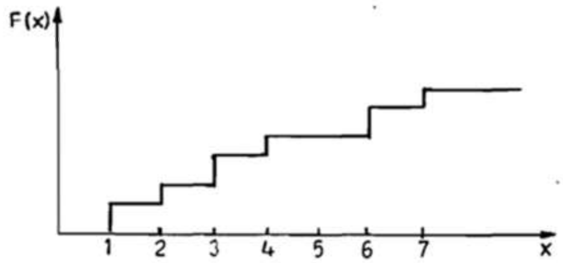
d. function

5·3·2. Discrete Distribution Function. In this case there are a countable number of points $x_1, x_2, x_3, \ldots$ and numbers $p_i \geq 0$, $\sum_1^{\infty} p_i = 1$ such that $F(X) = \sum_{(i : x_i \leq x)} p_i$. For example if x_i is just the integer i, $F(x)$ is a *"step function"* having jump p_i at i, and being constant between each pair of integers.

$X = x_1, x_2 - - - x_n$

$p_i \geq 0 \qquad \sum_{i=1}^{\infty} p_i = 1$

$F(x) = \sum_{x_i \leq x} p_i$



Ex ③ The probability distribution of Variable X is given by

X	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

(i) Find $P(X \leq 4)$ (ii) $P(x \geq 5)$, $P(3 < X \leq 6)$

(ii) What will be the minimum Value of k so that $P(X \leq 2) > 0.3$

$f(0) = k \quad f(1) = 3k \quad f(2) = 5k \qquad f(6) = 13k$

Sol

$\sum p_i = 1 \qquad k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$\Rightarrow 49k = 1 \Rightarrow \boxed{k = \frac{1}{49}}$

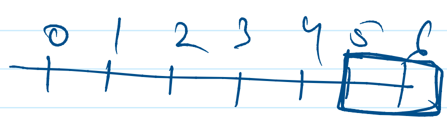
① $P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= k + 3k + 5k + 7k + 9k$

$= 25k = 25 \times \frac{1}{49} = \frac{25}{49}$

② $P(x \geq 5) = 1 - P(x \leq 4)$

$= 1 - \frac{25}{49} = \frac{24}{49}$



$P(x=5) + P(x=6)$

$11k + 13k = 24k = \frac{24}{49}$

③ $P(3 < x \leq 6) = P(x=4) + P(x=5) + P(x=6)$

$= 9k + 11k + 13k$

$= 33k = \frac{33}{49}$

$P(x \leq 2) > 0.3$

$$P(x \le 2) > 0.3$$

$$\Rightarrow P(x=0) + P(x=1) + P(x=2) > 0.3$$

$$k + 3k + 5k > 0.3$$

$$9k > 0.3$$

$$k > \frac{0.3}{9}$$

$$k > \frac{1}{30}$$

Minimum $k = \frac{1}{30}$

L-10

Saturday, February 5, 2022 2:05 PM

Note :→

① X, Y are r.v and $Y \leq X$ then $E(Y) \leq E(X)$

② $|E(X)| \leq E(|X|)$

③ Covariance $Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = E[(X-E(X))(Y-E(Y))]$

$$\boxed{Cov(X,Y) = E(XY) - E(X)E(Y)}$$

④ If X, Y are Independent r.v then $\boxed{Cov(X,Y) = 0}$

↳ $E(XY) = E(X) \cdot E(Y)$

$E(XY) - E(X)E(Y) = 0$

⑤ $Cov(aX, bY) = ab\,Cov(X,Y)$

⑥ $Cov(X+a, Y+b) = Cov(X,Y)$

Jenson's Inequalities :- (i) $E(X^2) \geq (E(X))^2$

(ii) $X > 0$; and $E(X)$, $E\left(\frac{1}{X}\right)$ exist then $E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)}$

(iii) $X > 0$; $E(X^{1/2}) = (E(X))^{1/2}$

(iv) $X > 0$; $E(\log X) \leq \log(E(X))$

(v): $E(f(X) \cdot g(X)) \geq E(f(X)) \cdot E(g(X))$

or $\leq E(f(X)) \cdot E(g(X))$ Accordingly as $f(x), g(x)$ are monotones in the same or opposite direction.

the variance of getting head when two coins are tossed

(a) 1 (b) 0.5 (c) 0.8 (d) 3.5

X: Getting Head

$Var(X) = E(X^2) - (E(X))^2$

$= \sum x^2 f(x) - \left(\sum x f(x)\right)^2$

$= \left(\frac{3}{2}\right) - 1$

$= \frac{1}{2} = 0.5$

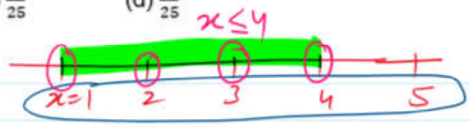
$X = x$	$f(x)$	$x f(x)$	x^2	$x^2 f(x)$
0	$\frac{1}{4}$	0	0	0
1	$2/4 = \frac{1}{2}$	$1/2$	1	$1/2$
2	$1/4$	$1/2$	4	1
		$\sum x f(x) = 1$		$\sum x^2 f(x) = \frac{3}{2}$

$\boxed{Mean = 1}$

If X is a discrete random variable, and its probability mass function is given by $p(x) = \frac{x+2}{25}$; $x = 1,2,3,4,5$ then $P(X \le 4)$ is

(a) $\frac{18}{25}$ (b) $\frac{2}{25}$ (c) $\frac{10}{25}$ (d) $\frac{6}{25}$

$p(x) = \frac{x+2}{25}$ $x = 1, 2, 3, 4, 5$



$P(X \le 4) = 1 - P(X=5)$

$\boxed{P(x=1)+P(x=2)+P(x=3)+P(x=4)} + P(x=5) = 1$

$\sum p(x) = 1$

$= 1 - \left(\frac{5+2}{25}\right) = 1 - \frac{7}{25} = \frac{18}{25}$ $P(x \le 4) + P(x=5) = 1$

Moment Generating functions (m.g.f)

The m.g.f of a r.v X about origin having probability function $f(x)$ is given by

$$M_X(t) = E(e^{tX}) = \begin{cases} \sum_x e^{tx} f(x) & : X \text{ is Discrete} \\ \int_x e^{tx} f(x)\,dx & : X \text{ is Continuous} \end{cases}$$

Where 't' is the parameter and it is assumed that the R.H.S of above result is absolutely Convergent for some +ve number h then $-h < t < h$

$$M_X(t) = E(e^{tX}) = E\left[1 + tX + \frac{(tX)^2}{\lfloor 2} + \frac{(tX)^3}{\lfloor 3} + \cdots + \frac{(tX)^r}{\lfloor r} + \cdots \infty\right]$$

$\left\{ e^x = 1 + \frac{x^1}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots \infty \right\}$

$$= E(1) + E(tX) + E\left(\frac{t^2}{\lfloor 2} X^2\right) + \cdots + E\left(\frac{t^r}{\lfloor r} X^r\right) + \cdots \infty$$

$$= 1 + t\,E(X) + \frac{t^2}{\lfloor 2} E(X^2) + \cdots + \frac{t^r}{\lfloor r} E(X^r) + \cdots \infty$$

$$M_X(t) = 1 + t\mu_1' + \frac{t^2}{\lfloor 2} \mu_2' + \cdots + \frac{t^r}{\lfloor r} \mu_r' + \cdots \infty$$

Where $\mu_r' = E(X^r) = \begin{cases} \sum x^r f(x) & : \text{Discrete} \\ \int x^r f(x)\,dx & : \text{Continuous} \end{cases}$

μ_r' is called the r^{th} moment of r.v X about Origin.

$$\boxed{\mu_r' = E(X^r)}$$

I^{st} Moment of X about Origin $= \mu_1' = E(X) =$ Coeff of t in the Exp. of $M_X(t)$

2^{nd} " " " " $= \mu_2' = E(X^2) =$ Coeff. of $\frac{t^2}{\lfloor 2}$ " " "

3^{rd} " " " " $= \mu_3' = E(X^3) =$ " " $\frac{t^3}{\lfloor 3}$ " " "

$\lfloor 3$

r^{th} " " " " $= \mu_r' = E(x^r) =$ Coeff. of $\dfrac{t^r}{\lfloor r}$ " " "

$\therefore$ $M_x(t)$ generate the moment of x about origin, so It is called moment generating function.

Note: (1) $Var(x) = E(x^2) - (E(x))^2 = \mu_2' - (\mu_1')^2$

$= \left(2^{nd} \text{ Moment of } x\right) - \left(I^{st} \text{ Moment of } x\right)^2$

(2) m.g.f of X about $x = a$

$$\boxed{M_{x=a}(t) = E\left(e^{t(x-a)}\right)}$$

$$= 1 + t\mu_1' + \frac{t^2}{\lfloor 2}\mu_2' + \cdots + \frac{t^r}{\lfloor r}\mu_r' + \cdots \infty$$

$\mu_r' = E[(x-a)^r] = r^{th}$ Moment of X about the point $X = a$

Epectation

Thursday, February 3, 2022 12:19 PM

$$\sigma^2 = E|(x-\mu)^2| = \sum (x-\mu)^2 f(x)$$

μ = Mean → constant

$$= \sum (x^2+\mu^2-2\mu x) f(x)$$
$$= \sum (x^2 f(x) + \mu^2 f(x) - 2\mu x f(x))$$
$$= \sum x^2 f(x) + \sum \mu^2 f(x) - \sum 2\mu x f(x)$$
$$= \sum x^2 f(x) + \mu^2 \sum f(x) - 2\mu \sum x f(x)$$
$$= \sum x^2 f(x) + \mu^2 (1) - 2\mu\,\mu$$
$$= \sum x^2 f(x) + \mu^2 - 2\mu^2$$
$$= \sum x^2 f(x) - \mu^2 = \sum f(x) x^2 - \left(\sum f(x).x\right)^2$$

$\sum p(x) = 1$

$\mu = \sum x f(x)$

$\sum \mu = ?$ $\boxed{\sum x f(x)}$ $\sum($ → σ^2

Note:

① $E(x) = \sum x f(x)$ or $\int_{-\infty}^{\infty} x f(x)\,dx$

$E(x^2) = \begin{cases} \sum x^2 f(x) d \\ \int x^2 f(x)\,dx \end{cases}$

② $E(ax) = a\,E(x)$

③ $E(ax+b) = a\,E(x) + b$

④ $E(g(x) \pm h(x)) = E(g(x)) \pm E[h(x)]$

⑤ $\boxed{\sigma^2(x) = \sum (x-\mu)^2 f(x) \text{ or } \int (x-\mu)^2 f(x)}$

⑥ $\sigma^2(ax) = a^2 \sigma^2(x) = a^2\sigma^2$

⑦ $\sigma^2(ax+b) = a^2\sigma^2(x) = a^2\sigma^2$

⑧ $\sigma^2(x+y) = \sigma^2(x) + \sigma^2(y)$ where x & y are Independent R. Variable

⑨ $\sigma^2(x-y) = \sigma^2(x) + \sigma^2(y)$

⑩ $\sigma^2(ax+by) = a^2\sigma^2(x) + b^2\sigma^2(y)$

$\sum$ — Discrete.

$\int$ — Continuous

② $E(ax) = aE(x)$

③ $E(ax+b) = aE(x) + b$

⑥ $\sigma^2(ax) = a^2 \cdot \sigma^2(x)$

$\sigma^2(\text{constant}) = 0$

$$\sigma^2(ax+b) = \sigma^2(ax) + \sigma^2(b)$$
$$= a^2\sigma^2(x) + 0$$

③ $E(ax+b) = \sum (ax+b) f(x) = \sum (ax f(x) + b f(x))$

$$= \sum ax f(x) + \sum b f(x)$$

$$E(ax+b) = \sum (ax+b) f(x) = \sum (axf(x) + bf(x))$$

$$= \sum axf(x) + \sum b f(x)$$

$$= a \sum x f(x) + b \sum f(x) \qquad \boxed{\sum f(x) = 1}$$

$$= a E(x) + b \cdot 1 \qquad \sum p_i(x) = 1$$

$$= a \underline{\underline{E(\cancel{x}) + b}}$$

$$\sigma^2(X-Y) = \sigma^2(X+(-Y)) = \sigma^2(X) + \sigma^2(-1 \cdot Y)$$

$$= \sigma^2(X) + (-1)^2 \sigma^2(Y)$$

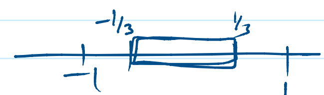
$$= \sigma^2(X) + \sigma^2(Y)$$

$$V(ax+b) = a^2 \,\mathrm{Var}(x)$$

$$V(4x+5)$$

$$= (4)^2 \mathrm{Var}(x) = 16 \,\underline{\underline{\mathrm{Var}}}(x)$$

$$P(a \le x \le b) = \int_a^b f(x)\,dx$$



$$= \int_{-1/3}^{1/3} x^2\,dx = 2 \int_0^{1/3} x^2\,dx$$

$$\left[\int_{-\infty}^{\infty} x f(x)\,dx \right]$$

$\uparrow$

$$\int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^1 x f(x)\,dx$$

$$\int_0^1 (x^5 - x^7)\,dx = \frac{x^6}{6} - \frac{x^8}{8} \Big|_0^1 \qquad = \int_0^1 x(x^4(1-x^2))\,dx$$

$$\frac{1}{6} - \frac{1}{8} = \frac{4-3}{24} = \frac{1}{24}$$

$$M_x(t) = 1 + t\mu_1' + \frac{t^2}{\lfloor 2}\mu_2' + \frac{t^3}{\lfloor 3}\mu_3' + \cdots + \frac{t^r}{\lfloor r}\mu_r' + \cdots \infty$$

r^{th} Moment of $X = \mu_r' = E(X^r) =$ Coeff. of $\frac{t^r}{\lfloor r}$ in $M_x(t)$.

Limitations ① A r.v X may have no moment although its m.g.f Exist.

Ex $f(x) = \begin{cases} \frac{1}{x(x+1)} & ; x = 1, 2, 3 \cdots \\ 0 & ; \text{Otherwise} \end{cases}$

$$E(x) = \sum_{x=1}^{\infty} x f(x) = \sum_{x=1}^{\infty} x\left(\frac{1}{x(x+1)}\right) = \sum_{x=1}^{\infty} \frac{1}{(x+1)}$$ It is divergent series

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \cdots$$

$\therefore \mu_1' = E(x)$ does not Exist.

$$M_x(t) = E(e^{tx}) = \sum_{x=1}^{\infty} e^{tx} f(x) = \sum_{x=1}^{\infty} \frac{e^{tx}}{x(x+1)} = \sum_{x=1}^{\infty} \frac{(z)^x}{x(x+1)} \; ; z = e^t$$

$$= \frac{z}{1\cdot 2} + \frac{z^2}{2\cdot 3} + \frac{z^3}{3\cdot 4} + \cdots \infty \quad \left\{ \frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1} \right.$$

$$= z\left(\frac{1}{1} - \frac{1}{2}\right) + z^2\left(\frac{1}{2} - \frac{1}{3}\right) + z^3\left(\frac{1}{3} - \frac{1}{4}\right) + \cdots \infty$$

$$= \left(z + \frac{z^2}{2} + \frac{z^3}{3} + \cdots \infty\right) - \left(\frac{z}{2} + \frac{z^2}{3} + \frac{z^3}{4} + \cdots \infty\right)$$

$$= \left(\underline{z + \frac{z^2}{2} + \frac{z^3}{3} + \cdots \infty}\right) - \frac{1}{z}\left(\cancel{z} + \underline{\frac{z^2}{2} + \frac{z^3}{3} + \cdots \infty} - \cancel{z}\right)$$

$=$

$$\left[\begin{aligned} &\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots \infty \qquad |x| < 1 \\ &\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} \cdots \infty = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} \cdots \infty\right) \end{aligned} \right]$$

$$= -\log(1-z) - \frac{1}{z}\left[-\log(1-z) - z\right] \qquad \underline{|z| < 1} \qquad \left[\begin{aligned} &|e^t| < 1 \\ &\Rightarrow t < 0 \end{aligned} \right.$$

$$= -\log(1-z) + \frac{1}{z}\log(1-z) + 1$$

$$= 1 + \log(1-z)\left[\frac{1}{z} - 1\right] = 1 + (e^{-t} - 1)\log(1 - e^t) \; ; t < 0$$

$$M_x(t) = 1 + (e^{-t} - 1)\log(1 - e^t) \qquad t < 0$$

② Vice Versa

Note:→ In some cases m.g.f does not Exist even moment of r.v X Exist

Characteristic function

Characteristic function

$$\phi_X(t) = E(e^{itx}) = \begin{cases} \sum e^{itx} f(x) : \text{Disct} \\ \int_x e^{itx} f(x)\,dx : \text{Cont.} \end{cases}$$

Example 6·37. *Let the random variable X assume the value 'r' with the probability law :*

$$P(X=r) = q^{r-1}p\,;\; r = 1, 2, 3, \ldots$$

Find the m.g.f. of X and hence its mean and variance.

Sol

$$f(r) = P(x=r) = q^{r-1} \cdot p$$

$$M_X(t) = E(e^{tx}) = \sum_{r=1}^{\infty} e^{tr} \cdot f(r) = \sum_{r=1}^{\infty} e^{tr} q^{r-1} \cdot p$$

$$= \sum p\, q^{-1} \cdot q^r e^{tr} = \frac{p}{q} \sum_{r=1}^{\infty} (e^t \cdot q)^r$$

$$= \frac{p}{q}\left[(e^t q) + (e^t q)^2 + (e^t q)^3 + \cdots \infty\right]$$

$$= \frac{p}{q}\left[e^t q + e^{2t} q^2 + e^{3t} q^3 + \cdots \infty\right]$$

$$= \frac{p}{\cancel{q}} e^t \cdot \cancel{q}\left[1 + (e^t q) + (e^t q)^2 + \cdots \infty\right]$$

$$= p e^t \left[\begin{matrix}\text{Infinite G.P} \\ S_\infty = \frac{a}{1-r}\end{matrix}\right] = p e^t \left[\frac{1}{1 - e^t q}\right] = \frac{p e^t}{1 - e^t q}$$

which is the required Moment generating function.

(ii) Mean (iii) Variance

$\mu = \mu_1' = \text{I}^{st}$ Moment about origin

$\text{Var} = \mu_2' - (\mu_1')^2$

Note

$$\mu_r' = r^{th} \text{ Moment about origin} = \left\{\frac{d^r}{dt^r}[M_x(t)]\right\}_{t=0}$$

$$\mu_1' = \text{I}^{st} \text{ Moment } t=0 = \left(\frac{d}{dt}(M_x(t))\right)_{t=0}$$

$$= \frac{d}{dt}(M_x(0))$$

$$M_x(t) = \frac{p e^t}{1 - e^t q} = p e^t (1 - e^t q)^{-1}$$

$$M_x(t) = \frac{pe^t}{1-e^tq} = pe^t(1-e^tq)^{-1}$$

$$= p\left[\qquad \right]\left[\overline{\quad B.T \quad} \right]$$

$$= (\quad) + (\underset{\mu_1'}{\downarrow})t + (\underset{\mu_2'}{\downarrow})\frac{t^2}{\lfloor 2} + \text{———}$$

$$\mu_1' = \frac{d}{dt}\left[\frac{pe^t}{1-qe^t}\right]_{t=0} \text{————} = \frac{1}{p}$$

$$\mu_2'' = \frac{d^2}{dt^2}\left[\frac{pe^t}{1-qe^t}\right]_{t=0} \text{————} = \frac{1+q}{p^2}$$

CA-1 Saturday in your Class time

Unit-1

Example 6·40. *Find the moment generating function of the random variable whose moments are*

$\boxed{\mu_r' = (r+1)!\,2^r}$ Also find the Mean and Variance

Sol r^{th} moment of $X = \mu_r' = \lfloor r+1 \cdot 2^r$

$$\boxed{M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{\lfloor r} \mu_r'} = 1 + \frac{t}{\lfloor 1}\mu_1' + \frac{t^2}{\lfloor 2}\mu_2' + \frac{t^3}{\lfloor 3}\mu_3' + \cdots + \frac{t^r}{\lfloor r}\mu_r' + \cdots \infty$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{\lfloor r} \lfloor r+1 \cdot 2^r = \sum_{r=0}^{\infty} \frac{t^r}{\cancel{\lfloor r}} (r+1)\cancel{\lfloor r} \cdot 2^r = \sum_{r=0}^{\infty} (r+1)\, t^r \cdot 2^r$$

$$M_X(t) = \sum_{r=0}^{\infty} (r+1)(2t)^r = 1(2t)^0 + 2(2t)^1 + 3(2t)^2 + 4(2t)^3 + \cdots \infty$$

$$= 1 + 2Z^1 + 3Z^2 + 4Z^3 + \cdots \infty \quad \text{When } Z = 2t$$

Ⓐ $(1-Z)^{-1}$ Ⓑ $(1+Z)^{-1}$ Ⓒ $(1-Z)^{-2}$ Ⓓ $(1+Z)^{-2}$

$(1+Z)^{-1} = 1 - Z + Z^2 - Z^3 + \cdots \infty$ $(1+Z)^{-2} = 1 - 2Z + 3Z^2 - 4Z^3 + \cdots \infty$

$(1-Z)^{-1} = 1 + Z + Z^2 + Z^3 + \cdots \infty$ $(1-Z)^{-2} = 1 + 2Z + 3Z^2 + 4Z^3 + \cdots \infty$

$\therefore M_X(t) = (1-Z)^{-2} = (1-2t)^{-2}$

$\therefore \boxed{M_X(t) = (1-2t)^{-2}}$

ⓐ Mean

$\mu_r' = \lfloor r+1 \cdot 2^r$

Mean $= \mu = E(X) = \mu_1' = \lfloor 1+1 \cdot 2^1$

$= \lfloor 2 \cdot 2$

$= 2 \cdot 2$

$= 4$

$\therefore \boxed{\text{Mean} = 4}$

ⓑ Variance

Variance $= \mu_2' - (\mu_1')^2 = E(X^2) - (E(X))^2$

$\mu_2' = \lfloor 2+1 \cdot 2^2 = \lfloor 3 \cdot 4 = 6 \cdot 4 = 24$

$\mu_1' = 4$

Variance $= 24 - (4)^2 = 24 - 16 = 8$

$\boxed{\text{Variance} = 8}$

Consider the following probability mass function (p.m.f) of a random variable X.

$$p(x,q) = \begin{cases} q, & if\ x = 0 \\ 1-q, & if\ x = 1 \\ 0, & otherwise \end{cases}$$

If $q = 0.4$, then the Variance of X is

(a) 0.24 (b) 0.6 (c) 0.36 (d) 1

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$= \sum_{x=0}^{1} x^2 f(x) - \left(\sum_{x=0}^{1} x f(x)\right)^2$$

$$= [0 \cdot q + 1 \cdot (1-q)] - [0 + 1 \cdot (1-q)]^2$$

$$= (1-q) - (1-q)^2$$

$$= (0.6) - (0.6)^2$$

$$= 0.6 - 0.36$$

$$= \underline{\underline{0.24}}$$

If two dice are thrown at random, then the expected value of sum of numbers of points on them is:

(a) $\frac{1}{3}$ (b) 2 (c) $\frac{7}{2}$ (d) 7

x:	2	3	4	5	6	7	8	9	10	11	12
$f(x)$:	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\mu = E(x) = \sum x f(x) = \frac{2+6+12+20+30+42+40+54+30+22+12}{36}$$

$$=$$

Binomial Distribution

$$\boxed{P(x=r) = {}^nC_r\, p^r q^{n-r}} = b(x;n,p)$$

$$\boxed{b(x,n,p) = {}^nC_r\, p^x q^{n-x}}$$

Note: $\sum_{x=0}^{n} b(x;n,p) = 1$ $\qquad$ $B(r;n,p) = \sum_{x=0}^{r} b(x;n,p)$

Ex) The probability that a patient recover from a rare blood disease is 0.4. if 15 people are known to have contradict this disease what is the prob. that

(i) at least 10 Survive

(ii) from 3 to 8 Survive

(iii) Exactly 5 Survive.

Mean In the Binomial distribution $b(x; n, p)$

$$\boxed{\text{Mean } \mu = np}$$

$$\boxed{\text{Variance } (\sigma^2) = npq}$$

$$\boxed{\begin{aligned} S.D\,(\sigma) &= \sqrt{\text{Variance}} \\ &= \sqrt{npq} \end{aligned}}$$

Mean $\mu = E(x)$ Expected value

Moment of Binomial Distribution:

$$\mu_1 = E(x) = \sum_{x=0}^{n} x \cdot {}^{n}C_{x}\, p^{x} q^{n-x} = np$$

$$\mu_2 = E(x^2) = n(n-1)p^2 + np$$

$$\mu_3 = E(x^3) = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np$$

$$\mu_4 = E(x^4) = n(n-1)(n-2)(n-3)p^4 + 6n(n-1)(n-2)p^3 + 7n(n-1)p^2 + np$$

- - - -

Central Moment :- $\boxed{\bar{\mu}_2 = npq}$ Variance.

$\bar{\mu}_3 = npq(q-p)$

$\bar{\mu}_4 = npq[1+3(n-2)pq]$

$\beta_1 = \frac{(1-2p)^2}{npq}$ $\beta_2 = 3 + \left(\frac{1-6pq}{npq}\right)$

$\gamma_1 = \sqrt{\beta_1} = \frac{1-2pq}{\sqrt{npr}}$, $\gamma_2 = \frac{1-6pq}{npq}$

Note ① Mean $= np$ Variance $= npq$

$\bar{\mu}_3 = npq(q-p)$.

② Variance $= npq < np =$ mean

$Var(x) <$ Mean.

Note $E\left(\frac{X}{n}-p\right)^2 = \frac{pq}{n}$

$Covar\left(\frac{X}{n}, \frac{n-X}{n}\right) = \frac{-pq}{n}$

$E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = p$

$Var\left(\frac{X}{n}\right) = \frac{1}{n^2}Var(x) = \frac{pq}{n}$

Moment Generating function of B.D

$X = B(n, p)$

$M_X(t) = E(e^{tx}) = \sum e^{tx}\, {}^nC_x\, p^x q^{n-x} = (q+pe^t)^n$

Mode:→ If $k = np + p$ is not an Integer then there is only One mode i.e $\boxed{\text{Mode} = np+p}$

If $k = np+p$ is an Integer, there are Two Modes at $np+p-1$ and $np+p$.

$$\text{Mode} = \begin{cases} (n+1)p & ; (n+p)p \text{ is Zero or Non Integer} \\ (n+1)p \text{ or } (n+1)p-1 & ; (n+1)p \in \{1,2,3 \ldots n\} \\ n & ; (n+1)p = n+1 \end{cases}$$

$$\boxed{\text{Mode} = (n+1)p = np+p.}$$

$$\boxed{\text{Mode} = \text{Mean} + p}$$

Poisson Distribution

$$\boxed{P(r) = \frac{e^{-m} \cdot m^{r}}{\lfloor r}} \qquad r = 0, 1, 2, 3, \ldots$$

where $e = 2.7183$

m = mean of PD = np

If n is very large i.e $n \to \infty$ then BD $\to$ P.D

$b(x; n, p) \longrightarrow p(x; \mu)$

$$\boxed{P(x; \mu) = \frac{e^{-\mu} (\mu)^{x}}{\lfloor x}} \qquad \mu = \text{mean} = np.$$

\# n, No. of Trials is very large i.e $n \to \infty$

\# The Constant probability of success i.e p is very very small i.e $p \to 0$

\# mean $= np = \lambda$ or μ $\quad$ λ is the parameter.

The Distribution function

$$F(x) = P(X \le x) = \sum_{r=0}^{x} p(r) = e^{-\lambda} \sum_{r=0}^{x} \frac{\lambda^{r}}{\lfloor r}$$

\# Poisson Distribution is a limiting Case of Binomial Distribution.

$$p(X = r) = {}^{n}C_{r} p^{r} q^{n-r} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{\lfloor r} \left(\frac{\lambda}{n}\right)^{r} \left(1 - \frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{\lambda^{r}}{\lfloor r} \left[1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right)\cdots\left(1-\frac{r-1}{n}\right)\right]\left(1-\frac{\lambda}{n}\right)^{+n}\left(1-\frac{\lambda}{n}\right)^{-r} \quad \text{—(1)}$$

Now as $n \to \infty$ $\quad \lim_{n\to\infty}\left(1 - \frac{\lambda}{n}\right) = 1 \qquad \lim_{n\to\infty}\left(1 - \frac{a}{n}\right) = 1$ where a is finite No.

As $n \to \infty$ $\quad \lim_{\substack{n\to\infty \\ \lambda - \text{Finite}}} \left[P(X=r)\right] = \frac{\lambda^r}{\lfloor r}(1).e^{-\lambda}$

$$= \frac{e^{-\lambda}\lambda^r}{\lfloor r} = P(X=r) \text{ in Poisson Distribution.}$$

Moments

$$\mu_1' = E(X) = \lambda \qquad \mu_2' = E(X^2) = \lambda^2+\lambda$$

$$\mu_3' = \lambda^3+3\lambda^2+\lambda \qquad \mu_4' = \lambda^4+6\lambda^3+7\lambda^2+\lambda$$

Mode

$$\frac{p(x)}{p(x-1)} = \frac{\lambda}{x}$$

Moment Generating functions:

$$M_X(t) = E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx}p(x) = \sum_{x=0}^{\infty} e^{tx}\frac{e^{-\lambda}\lambda^x}{\lfloor x} = e^{\lambda(e^t-1)}$$

$$\boxed{M_X(t) = e^{\lambda(e^t-1)}}$$

Addition/Reproductive Property If X_1, X_2 are Two Independent Poisson Variate with parameter λ_1, λ_2 then.
X_1+X_2 will also be P.R. Variate with parameter $\lambda_1+\lambda_2$

$$\therefore \quad M_{(X_1+X_2)}(t) = e^{(\lambda_1+\lambda_2)(e^t-1)}$$

Characteristic function:

$$\phi_X(t) = \sum_{x=0}^{\infty} e^{itx}p(x,\lambda) = E(e^{itx})$$

$$\boxed{\phi_x(t) = e^{\lambda(e^{it}-1)}}$$

Cumulants of Poisson Distribution:

$$K_x(t) = \lambda(e^t - 1) = \lambda\left[t + \frac{t^2}{\lfloor 2} + \frac{t^3}{\lfloor 3} + \cdots \infty\right]$$

$$K_r = r^{th}\text{ Coefficient } r^{th}\text{ Cumulant} = \text{Coeff. of } \frac{t^r}{\lfloor r} = \lambda.$$

$\therefore$ All the Cumulants of the Poisson distribution are equal.

Note

Mean $= K_1 = \lambda$.

$\mu_2 = K_2 = \lambda$.

$\mu_3 = K_3 = \lambda$.

$\beta_1 = \frac{1}{\lambda} = \frac{\mu_3^2}{\mu_2^2}$

$\beta_2 = \frac{1}{\lambda} + 3 = \frac{\mu_4^2}{\mu_3^2}$

Recurrence Formula for Probability of Poisson Distribution

$$\boxed{p(x+1) = \frac{\lambda}{(x+1)}\, p(x)}$$

$$p(x) = \frac{e^{-\lambda}\lambda^x}{\lfloor x}$$

Mode: Mode $= [\lambda]$ = Integral part of λ if λ is Non-Integer

Mode $= \lambda, \lambda - 1$ if λ is an Integer.

Ex: if $\lambda = 2.5$ Mode $= [2.5] = 2$

if $\lambda = 4$ Mode $= 4, 3$

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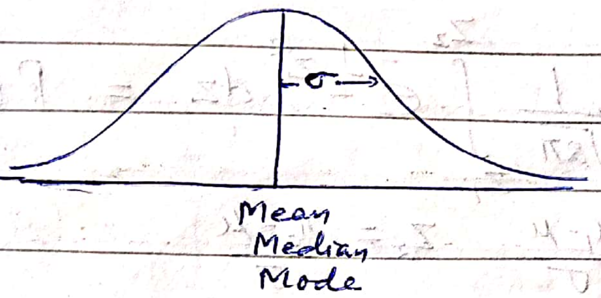
Normal Distribution

$$\boxed{P(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\left(\frac{x-\mu}{2\sigma^2}\right)^2}}$$

X = Value of Continuous Random Variable.
μ = Mean of N. R. Variable $\sigma = S.D$
e = 2.7183
π = 3.1416 $\sqrt{2\pi} = 2.5066$



Normal Curve
Bell-Shaped Curve.

$$= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

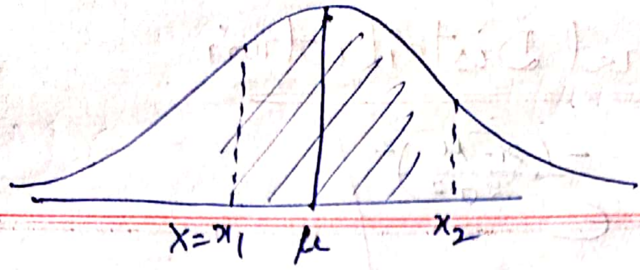
$\boxed{\text{Mean } \mu = E(X)}$, $\boxed{\text{Variance } \sigma^2 = E[(X-\mu)^2]}$

Area Under Normal Curve:

The Area Bounded By Under the Curve $f(x)$ (Coni. Prob. density func) bounded by two Ordinates $x = x_1$ and $x = x_2$ equals to the probability that the Random Variable X assume a value between $x = x_1$ & $x = x_2$

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} n(x;\mu,\sigma)\,dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$



Put $\frac{X-\mu}{\sigma} = Z$ i.e. we transform all the observations of normal Random Variable X into a new set of observations of normal Random Variable Z.

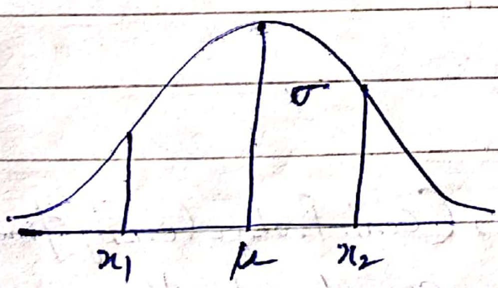
$$\boxed{Z = \frac{X-\mu}{\sigma}}$$

$$P(x_1 < X < x_2) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{1}{2}z^2}\, dz = P(z_1 < Z < z_2)$$

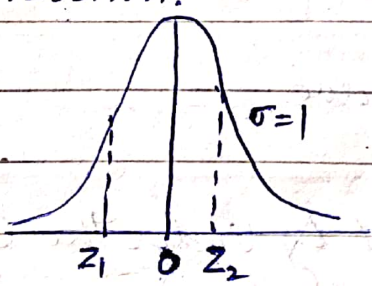
where $z_1 = \frac{x_1-\mu}{\sigma}$ $\quad z_2 = \frac{x_2-\mu}{\sigma}$

① [Z is Normal Random Variable with Mean 0 & Variance 1]

② [The distribution of N.R.Variable with Mean 0 & Varian 1 is Called Standard Normal distribution.]



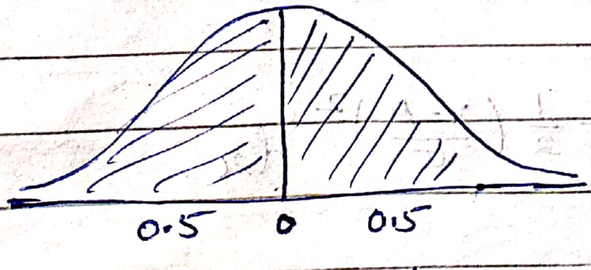
X-Curve



Z-Curve.

Value of Z is ranging from $-3.49 \longleftrightarrow 3.49$

③



Total Area = 1

Notes ① If $X = N(\mu, \sigma^2)$ then $\boxed{Z = \frac{X-\mu}{\sigma}}$ is a Standard normal Variate with $E(Z) = 0$ and $Var(Z) = 1$ then we write $Z = N(0,1)$.

② p.d.f of Standard Normal Variable Z is

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \; ; \; -\infty < Z < \infty$$

and the distribution function.

$$P(Z \le z) = \int_{-\infty}^{z} \phi(u)\,du = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-u^2/2}\,du.$$

③ $\phi(-z) = 1 - \phi(z) \; ; \; z > 0$

④ $P(a \le X \le b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$

$$P(a \le X \le b) = \int_{a}^{b} f(x; \mu, \sigma)\,dx = \int_{Z_1}^{Z_2} \phi(z)\,dz = P(Z_1 \le Z \le Z_2)$$

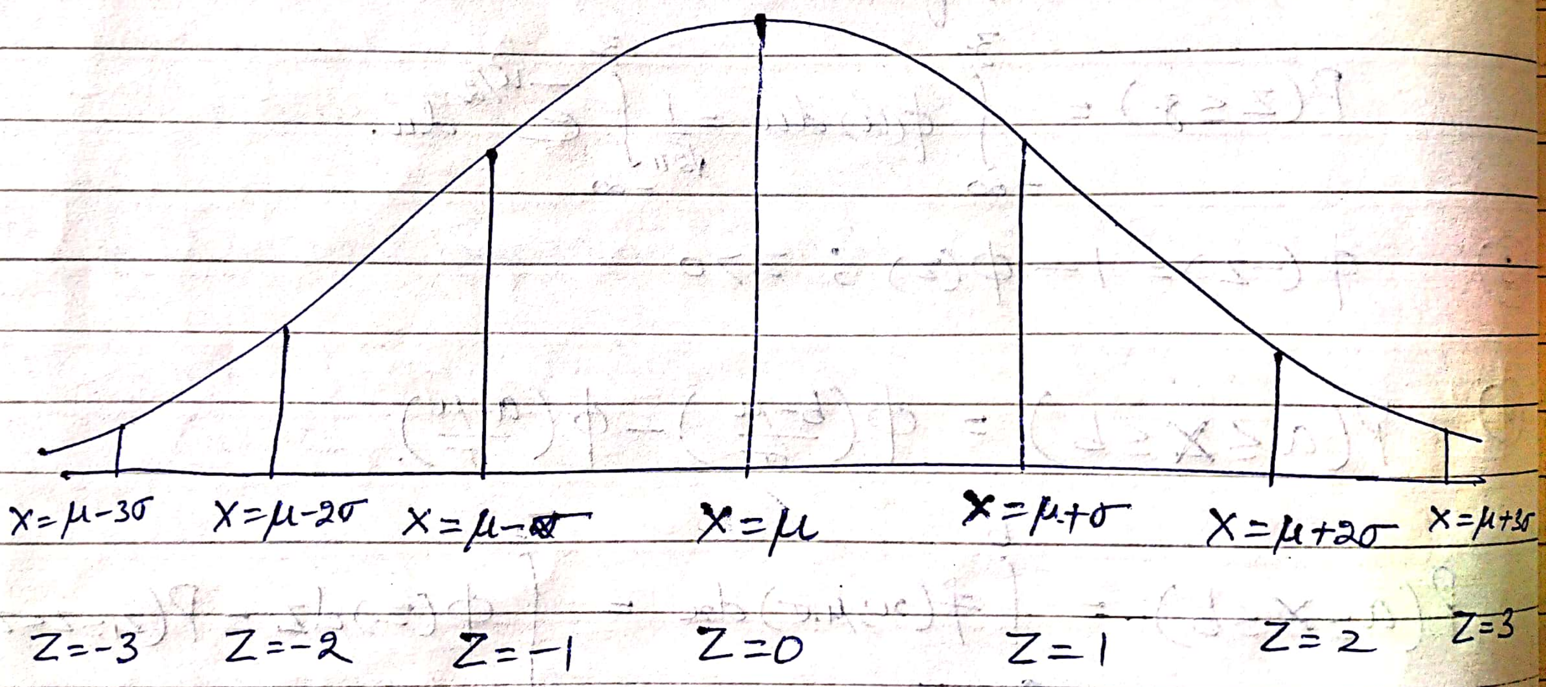
where $Z_1 = \frac{a-\mu}{\sigma}$ $\quad Z_2 = \frac{b-\mu}{\sigma}$.

Moment g.f

$$M_x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\mu+\sigma z)} e^{-\frac{z^2}{2}} dz.$$

$$\boxed{M_x(t) = e^{\left[\mu t + \frac{\sigma^2 t^2}{2}\right]}}$$

$\boxed{E(x) = \mu}$ $\boxed{E(x^2) = \sigma^2 + \mu^2}$



Normal Distribution is a Limiting Case of B.D

① $n \to \infty$

② neither p nor q is very small

$$Z = \frac{X - np}{\sqrt{npq}} = \frac{X-\mu}{\sigma}. \quad X = 0, 1, 2, 3 \ldots$$