L18 26 Feb 2021

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$$\boxed{\frac{1}{D-a}(Q) = e^{ax}\int Q.e^{-ax}\,dx}$$

$\int g(x) = e^{ax}$, $\sin ax / \cos ax$, $x^{n}$, $e^{ax} \cdot g(n)$, $x \cdot V$

# $(D^2-3D+2)y = \sin e^{-x}$

A.E. $D^2-3D+2=0$

$D^2-2D-D+2=0$

$D(D-2)-(D-2)=0$

$(D-1)(D-2)=0$

$y_c = c_1e^{x}+c_2e^{2x}$

P.I. $\frac{1}{D^2-3D+2}\cdot \sin e^{-x} = \frac{1}{(D-2)(D-1)}\cdot \sin e^{-x}$

$= \frac{1}{D-2}\left[\frac{1}{D-1}\sin e^{-x}\right]$

$a=1$

$Q=\sin e^{-x}$

$$\frac{1}{D-a}Q = e^{ax}\int Q.e^{-ax}\,dx$$

$= \frac{1}{D-2}\left[e^{x}\int \sin e^{-x}\cdot e^{-x}\,dx\right]$

put $e^{-x}=t$

$-e^{-x}dx=dt$

$= \frac{1}{D-2}\left[e^{x}\int -\sin t\,dt\right]$

$= \frac{1}{D-2}\left[e^{x}\cos t\right] = \frac{1}{D-2}\left[\underbrace{e^{x}\cdot\cos e^{-x}}_{Q}\right]$

$a=2$

$Q=e^{x}\cos e^{-x}$

$= e^{2x}\int e^{x}\cos e^{-x}\cdot e^{-2x}\,dx$

$= e^{2x}\int \cos e^{-x}\cdot e^{-x}\,dx$

$$= e^{2x}\int -\cos t \, dt = e^{2x}\left[-\sin t\right] = -\sin e^{-x} \cdot e^{2x}$$

$$y = \frac{1}{D-a} Q \quad \text{— (1)}$$

$$(D-a)y = Q$$

$$\boxed{\frac{dy}{dx} - ay = Q}$$

Sol

$$I.F = e^{\int -a\,dx} = e^{-ax}$$

$$y \cdot e^{-ax} = \int Q \cdot e^{-ax} \, dx$$

$$(D^2+1)y = \sec x$$

A.E $\quad D^2+1=0 \quad D^2 = -1 \Rightarrow D = \pm i \qquad y_c = C_1\cos x + C_2 \sin x$

P.I $\quad \frac{1}{D^2+1} \cdot \sec x = \frac{1}{(D-i)(D+i)} \cdot \sec x \qquad \frac{1}{(D-i)(D+i)} = \frac{A}{D-i} + \frac{B}{D+i}$

$$= \left[\frac{1}{(D-i)(2i)} + \frac{1}{(-2i)(D+i)}\right] \sec x$$

$$= \frac{1}{2i}\left[\frac{1}{D-i}\sec x - \frac{1}{D+i}\sec x\right]$$

$$\boxed{\frac{1}{D-a} \cdot Q = e^{ax}\int Q \cdot e^{-ax} dx}$$

$$\frac{1}{D-i}\sec x = e^{ix}\int \sec x \cdot e^{-ix} dx$$

$$= e^{ix}\int \frac{1}{\cos x}(\cos x - i \sin x) \, dx$$

$$= e^{ix}\int (1 - i\tan x) \, dx$$

$$\int \sin x \, dx$$

$$= e^{ix}\int (1 - i\tan x)\,dx$$

$$= e^{ix}\left[x + i\log|\cos x|\right]$$

$$\int \frac{\sin x}{\cos x}\,dx$$

$$-\int \frac{1}{t}\,dt$$

$$-\log t$$

$$\frac{1}{D+i}\sec x = e^{-ix}\int \sec x \cdot e^{ix}\,dx$$

$$= e^{-ix}\left[x - i\log|\cos x|\right]$$

$$\frac{1}{2i}\left[x e^{ix} + i e^{ix}\log|\cos x| - x e^{-ix} + i e^{-ix}\log(\cos x)\right]$$

$$\left[x\left(\frac{e^{ix} - e^{-ix}}{2i}\right) + \cancel{i}\log\cos x\left(\frac{e^{ix} + e^{-ix}}{2\cancel{i}}\right)\right]$$

$$= \left[x\sin x + \log(\cos x)\cdot\cos x\right]$$ Ans

① Euler-Cauchy form + Legendre form

② Method of variation of parameters

③ Undetermined Coeff.

④ Simultaneous eqn.

**Q13.** The particular integral of the differential equation $(D^3 - D)y = e^x + e^{-x}$

(a) $\cancel{\frac{e^x+e^{-x}}{2}}$ (b) $x\left(\frac{e^x+e^{-x}}{2}\right)$ (c) $x^2\cancel{\left(\frac{e^x+e^{-x}}{2}\right)}$ (d) $x^2\cancel{\left(\frac{e^x-e^{-x}}{2}\right)}$

(a) $\cancel{\frac{e^x+e^{-x}}{2}}$ (b) $x\left(\frac{e^x+e^{-x}}{2}\right)$ (c) $\cancel{x^2\left(\frac{e^x+e^{-x}}{2}\right)}$ (d) $\cancel{x^2\left(\frac{e^x-e^{-x}}{2}\right)}$

$D^3-D=0$

$D(D^2-1)=0$

$D=0, 1, -1$

$\frac{1}{D^3-D} \cdot e^x+e^{-x}$

$\frac{1}{D^3-D} e^x + \frac{1}{D^3-D} e^{-x}$

$\frac{1}{D^3-D} e^x \quad D=1$

$\frac{1}{1-1} e^x = \frac{1}{0} e^x$

Case of failure

$x \cdot \frac{1}{3D^2-1} e^x + x \cdot \frac{1}{3D^2-1} e^{-x}$

$\frac{x}{2} e^x + \frac{x}{2} e^{-x} = \frac{x(e^x+e^{-x})}{2}$

**Q31.** The general solution of the equation $y'' - 5y' + 9y = \sin 3x$ is

(a) $y = Ae^{-x} + Be^{-4x} + 15\cos 2x$

(b) $y = Ae^{x} + Be^{4x} + 15\sin 2x$

(c) $y = Ae^{-x} + Be^{-x} + 15\sin 2x$

(d) $y = Ae^{x} + Be^{4x} + \frac{1}{15}\cos 2x$

$D^2-5D+9=0$

$D = \frac{5 \pm \sqrt{25-4\times 9}}{2} = \frac{5 \pm \sqrt{25-36}}{2}$

$= \frac{5 \pm \sqrt{11}i}{2}$

$\frac{1}{D^2-5D+9} \cdot \sin 3x$

$\frac{1}{-\cancel{9}-5D+\cancel{9}} \cdot \sin 3x = -\frac{1}{5}\left(-\frac{\cos 3x}{3}\right) = +\frac{1}{15}\cos 3x$