L8 05 Feb 2021

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Clairaut Equation

$y = -px + f(p)$

An equation of the form $\boxed{y = px \pm f(p)}$ is called the Clairaut equation where $p = \frac{dy}{dx}$.

Non Singular solutions

$y = px + f(p)$ solvable for y $y = f(x, p)$

$$\frac{dy}{dx} = p.1 + x.\frac{dp}{dx} + f'(p).\frac{dp}{dx}$$

$$\cancel{p} = \cancel{p} + x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

$$0 = x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

$$0 = \frac{dp}{dx}\left(x + f'(p)\right)$$

$\frac{dp}{dx} = 0$

$x + f'(p) = 0$ X Ignore

Singular solution

$f(p) = p^3$

$3p^2$

$x + 3p^2 = 0$

No Constant

$3p^2 = -x$

$p = \sqrt{\frac{-x}{3}}$

$\frac{dy}{dx} = \sqrt{\frac{-x}{3}}$

$p = \text{Constant}$

$\boxed{p = c}$ — (2)

Given => $y = px + f(p)$ — (1)

Sol

$$\boxed{y = cx + f(c)}$$

$$\boldsymbol{xp^2 - yp + a = 0} \Rightarrow$$

$yp = xp^2 + a$

$y = xp + \frac{a}{p}$ → Clairaut Equation

$\boxed{y = cx + \frac{a}{c}}$ → solution

$$p = \log(px - y) \Rightarrow$$

$$e^p = px - y$$

$$y = px - e^p \rightarrow \text{Clairaut Equation}$$

$$\boxed{y = cx - e^c} \rightarrow \text{Solution}$$

$$y = px + \sqrt{a^2p^2 + b^2} \Rightarrow y = px + f(p)$$

$$y = cx + \sqrt{a^2c^2 + b^2}$$

$$\sin px \cos y = \cos px \sin y + p$$

$$\sin px \cos y - \cos px \sin y = p$$

$$\sin(px - y) = p$$

$$px - y = \sin^{-1} p$$

$$y = px - \sin^{-1} p \rightarrow \text{Clairaut form}$$

$$\boxed{y = cx - \sin^{-1} c} \rightarrow \text{Sol}$$

$$y + 2\left(\frac{dy}{dx}\right)^2 = (x+1)\left(\frac{dy}{dx}\right)$$

$$y + 2p^2 = xp + p$$

$$y = xp + (p - 2p^2) \longrightarrow \text{Clairaut Equation}$$

Sol $y = cx + (c - 2c^2)$

$$(px - y)(py + x) = a^2p$$

$$(px + y)^2 = py^2$$

$$p^2xy - py^2 + px^2 - xy = a^2p$$

$$p^2xy - py^2 + px^2 - xy = a^2p$$

$x^2 = t_1 \qquad y^2 = t_2$

$2x\,dx = dt_1 \qquad 2y\,dy = dt_2$

$$p = \frac{dy}{dx} = \frac{dt_2}{\cancel{2}y}\,\frac{\cancel{2}x}{dt_1} = \frac{x}{y}\left(\frac{dt_2}{dt_1}\right) P = \frac{xP}{y}$$

$$\left(\frac{xP}{y}.x - y\right)\left(\frac{x.P}{\cancel{y}}.\cancel{y} + x\right) = a^2.\frac{x.P}{y}$$

$$\left(\frac{x^2P - y^2}{\cancel{y}}\right)(P+1).\cancel{x} = a^2.\cancel{x}.\frac{P}{\cancel{y}}$$

$$(t_1P - t_2) = \frac{a^2P}{P+1}$$

$$t_2 = Pt_1 - \frac{a^2P}{P+1} \rightarrow \text{Clairaut form}$$

$y = px + f(p)$

$t_2 = pt_1 + f(p)$

$$\downarrow$$

$$t_2 = t_1c - \frac{a^2c}{c+1}$$

$$\boxed{y^2 = x^2c - \frac{a^2c}{c+1}}$$

Q6. Which of the following is Clairaut's differential equations?

(a) y = px + f(p) **(b)** y = p^2x + f(p) **(c)** y = p^2 + f(x) **(d)** None of these

$y = px + f(p)$

Q10. The general solution of differential equation $xp^2 - yp + a = 0$ *where* $p = \frac{dy}{dx}$ is given by

(a) $y = cx + \frac{a}{c}$ **(b)** $y = cx - e^c$ **(c)** $(y - cx)^2 = a^2c^2 + b^2$ **(d)** $y = cx - \sin^{-1} c$

$xp^2 - yp + a = 0$

$xp^2 + a = yp$

$xp + \frac{a}{p} = y$

$y = cx + \frac{a}{c}$

(a) Family of straight lines

Q12. If the differential equation $16x^2 + 2p^2y - p^3x = 0$ while solving for y takes the form $y = f(x, p)$, then $f(x, p) =$

(a) $\frac{px}{2} - \frac{8x^2}{p^2}$ (b) $\frac{2x}{p} + \frac{8x^2}{p}$ (c) $\frac{x}{2} + \frac{8x^2}{p^2}$ (d) $\frac{8x}{p^2} - \frac{px}{2}$

$16x^2 + 2p^2y - p^3x = 0$

$y = f(x, p)$

$2p^2y = p^3x - 16x^2$

$y = \frac{p^3x - 16x^2}{2p^2} = \frac{xp}{2} - \frac{8x^2}{p^2}$

$y = xp + \frac{p}{p-1}$

Q10. The general solution of differential equation $(y - xp)(p - 1) = p$ where $p = \frac{dy}{dx}$ is given by

(a) $y = cx + \frac{c}{c-1}$ (b) $y = cx - \frac{c}{c-1}$ (c) $y = c + \frac{cx}{c-1}$ (d) none of these

Q11. The general solution of differential equation $p = \log(px - y)$ where $p = \frac{dy}{dx}$ is given by

(a) $y = cx - e^c$ (b) $y = 2c^2 + cx$ (c) $y = cx + \frac{2}{c}$ (d) none of these

Q12. Which of the following is Clairaut's differential equation ?

(a) $p^2 - 3p + 2 = 0$ (b) $y = px + p - p^3$ (c) $y = px^2 + \frac{p}{p-1}$ (d) none of thes