L6 01 Feb 2021

01 February 2021 11:04

① $M\,dx + N\,dy = 0$ $\qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Exact

Sol $\displaystyle\int_{y\ \text{const}} M\,dx + \int (\text{Terms of } N \text{ free from } x)\,dy = c$

② I.F

① Homog. I.F: $\frac{1}{Mx+Ny}$, $Mx+Ny \neq 0$

② $f(xy)\,y\,dx + g(xy)\,x\,dy = 0$ $\quad$ I.F $\frac{1}{Mx-Ny}$, $Mx-Ny \neq 0$

③ $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ $\quad$ I.F $= e^{\int f(x)\,dx}$

④ $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = g(y)$, $\quad$ I.F $= e^{-\int g(y)\,dy}$

Equations of first order and higher degree

?

$\frac{dy}{dx}$ $\qquad \left(\frac{dy}{dx}\right)^2$ $\qquad \cancel{\frac{d^2y}{dx^2}}$ $\qquad \cancel{\frac{d^2y}{dx^2}}$, $\cancel{\frac{d^3y}{dx^3}}$

or $\left(\frac{dy}{dx}\right)^3$

$\left(\frac{dy}{dx}\right)^{2} + 2\frac{dy}{dx} + y = x \longrightarrow$ Non Linear ?

① y and $\frac{dy}{dx}$ are not multiplied

② powers of y and $\frac{dy}{dx}$ is equal to 1.

$\frac{dy}{dx} + 3xy = e^{x}$

Equations of First order and higher degree:

$\frac{dy}{dx} = p$ $\qquad \left(\frac{dy}{dx}\right)^2 = p^2$, $\quad p^3 = \left(\frac{dy}{dx}\right)^3 \ldots$ $\qquad \cancel{p^2}\ \cancel{\frac{d^2y}{dx^2}}$

Equations solvable for p :-

$$y\left(\frac{dy}{dx}\right)^2 + (x-y)\left(\frac{dy}{dx}\right) - x = 0$$

$\frac{dy}{dx} = p$

$yp^2 + (x-y)p - x = 0$

$yp^2 + xp - yp - x = 0$

$p(yp+x) - (yp+x) = 0$

$(yp+x)(p-1) = 0$

$yp + x = 0$

$\Rightarrow p = -\frac{x}{y}$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

$\Rightarrow y\,dy = -x\,dx$

$\Rightarrow \frac{y^2}{2} = -\frac{x^2}{2} + C_1$

$\frac{y^2}{2} + \frac{x^2}{2} - C_1 = 0$

$\boxed{y^2 + x^2 - 2C_1 = 0}$

$p = 1$

$\frac{dy}{dx} = 1$

$\Rightarrow dy = dx$

$\Rightarrow y = x + C_2$

$\Rightarrow \boxed{(y - x - C_2) = 0}$

General sol $(y^2 + x^2 - 2C_1)(y - x - C_2) = 0$

$$xy\left(\frac{dy}{dx}\right)^2 - (x^2+y^2)\left(\frac{dy}{dx}\right) + xy = 0$$

$\frac{dy}{dx} = p$

$xyp^2 - (x^2+y^2)p + xy = 0$

$xyp^2 - x^2p - y^2p + xy = 0$

$xp(yp - x) - y(py - x) = 0$

$(xp - y)(py - x) = 0$

$xp - y = 0$ $\qquad$ $py - x = 0$

$xp - y = 0$

$p = \frac{y}{x}$

$\frac{dy}{dx} = \frac{y}{x} \Rightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$\log y = \log x + C_1$

$\log \frac{y}{x} = C_1$

$\boxed{y/x = C_1}$

$\boxed{y - xC_1 = 0}$

$\Rightarrow \frac{y}{x} = e^{C_1} = C_3$

$y - xC_3 = 0$

$py - x = 0$

$p = \frac{x}{y}$

$\frac{dy}{dx} = \frac{x}{y} \Rightarrow$

$\int y\,dy = \int x\,dx$

General sol

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c_2$$

$$\boxed{(y^2 - x^2 - 2c_2) = 0}$$

$2c_2 \to c_4$

General sol

$$(y^2 - x^2 - 2c_2)(y - xc_1) = 0$$

$$\boldsymbol{p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0}$$

$$P(P^2 + 2xP - y^2P - 2xy^2) = 0$$

$$P[P(P+2x) - y^2(P+2x)] = 0$$

$$P[P+2x][P-y^2] = 0$$

$P = 0$

$\frac{dy}{dx} = 0$

$\boxed{y = c_1}$

$P + 2x = 0$

$\frac{dy}{dx} = -2x$

$\int dy = \int -2x\,dx$

$y = -\frac{2x^2}{2} + c_2$

$(y + x^2 - c_2) = 0$

$P - y^2 = 0$

$$\frac{dy}{dx} = y^2 \Rightarrow \int \frac{1}{y^2} dy = \int dx$$

$$\Rightarrow -\frac{1}{y} = x + c_3 \Rightarrow \left(-\frac{1}{y} - x - c_3\right) = 0$$

General

$$(y - c_1)(y + x^2 - c_2)\left(-\frac{1}{y} - x - c_3\right) = 0$$

$$(y - c_1)(y + x^2 - c_2)\left(\frac{1}{y} + x + c_3\right) = 0$$

$$\boldsymbol{p(p + y) = x(x + y)}$$

$$P^2 + Py - x^2 - xy = 0$$

$$(P^2 - x^2) + (Py - xy) = 0$$

$P + (x + y) = 0$

$\frac{dy}{dx} = -(x + y)$

$\frac{dy}{dx} + Py = Q$

$(p^2 - x^2) + (py - xy) = 0$

$(p-x)(p+x) + y(p-x) = 0$

$(p-x)(p+x+y) = 0$

$p = x$

$\frac{dy}{dx} = x$

$\int dy = \int x\,dx$

$y = \frac{x^2}{2} + c_1 \Rightarrow (2y - x^2 - 2c_1) = 0$

$\frac{dy}{dx} = -(x+y)$

$x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - 1 = -t^2$

$\frac{dt}{dx} = 1 - t^2$

$\int \frac{dt}{1-t^2} = \int dx$

$\frac{1}{1-t^2} = \frac{A}{1+t} + \frac{B}{1-t}$

$\frac{dy}{dx} + Py = Q$

$\Rightarrow \frac{dy}{dx} + y = -x$

$I.F. = e^{\int P dx}$

$= e^{\int dx} = e^x$

$y \cdot e^x = \int -x \cdot e^x dx$

Home work ?