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Lecture-1 (Review)

Differential equation :- equation which contains derivatives of various orders and the variables

Ordinary differential equation :- A differential equation which contains one independent variable = ODE

$y = x^2$

$\frac{dy}{dx} + y = x^2$ one independent variable = x

$\frac{dy}{dx} = y'$, $\frac{d^2y}{dx^2} = y''$, $\frac{d^3y}{dx^3} = y'''$

Orders :- highest order derivative involve in the differential equation.

Order = 1.

Degree :- degree (power) of the highest order derivative

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right) + \frac{dy}{dx} = x^5$$

Order = 3

Degree = 1

Linear :-

Linear: A differential equation is linear, when the dependent variable and its derivatives occur only in first degree and no products of the dependent variable and its derivatives or of various order derivatives occur.

$\frac{dy}{dx}$

$y\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx} + Py = Q$

$y\frac{dy}{dx}$ X

$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 4y = x^5$

Linear

ax' ax Linear

$y\frac{dy}{dx} + 3x = 0$ → Non Linear

$\frac{dy}{dx} + 3xy^2 = x^5$ → Non Linear

$\frac{dy}{dx} + 3xy = x^5$ → Linear

Non Linear :- An equation which is not Linear.

The equation: $yy' + y^2 - x^2 = 0$ is a: $y' = \frac{dy}{dx}$

(A) A linear ODE with order 1 and degree 1

(B) A non-linear ODE with order 1 and degree 1

(C) A linear ODE with order 1 and degree 2

(D) A non-linear ODE with order 2 and degree 2

Solution of ODE

① Variable Separable :- $(x\log x)y' = y$

$$(x\log x)\frac{dy}{dx} = y$$

$$(x\log x)\,dy = y\,dx \Rightarrow \int\frac{dy}{y} = \int\frac{dx}{x\log x}$$

$\log x = t$
$\frac{1}{x}dx = dt$

$$\Rightarrow \log y = \int\frac{\frac{1}{x}dx}{\log x}$$

$$\Rightarrow \log y = \int\frac{dt}{t} \Rightarrow \log y = \log t + c$$

$$\Rightarrow \log y = \log(\log x) + c$$

$$\Rightarrow \log y = \log(\log x) + c$$

$$\Rightarrow \log y - \log(\log x) = c$$

$$\Rightarrow \log\left(\frac{y}{\log x}\right) = c$$

$$\Rightarrow \frac{y}{\log x} = e^c = A$$

$$\Rightarrow \boxed{y = A\log x}$$

(2)

$$y' = e^{x+y} + x^2 e^y \Rightarrow \frac{dy}{dx} = e^x \cdot e^y + x^2 \cdot e^y$$

$$= e^y(e^x + x^2)$$

$$\Rightarrow \frac{dy}{dx} = e^y(e^x + x^2)$$

$$\int \frac{dy}{e^y} = \int (e^x + x^2)\,dx \Rightarrow \int e^{-y}\,dy = \int (e^x + x^2)\,dx$$

$$= -e^{-y} = e^x + \frac{x^3}{3} + C$$

Homogeneous equation

$$\frac{dy}{dx} = f(x,y)$$ → Homogeneous

$$\boxed{f(\lambda x, \lambda y) = \lambda^n f(x,y)}$$

$$y' = \frac{x^3 - y^3 - y^2x}{x^3 + y^3}$$ → 3 → yes Homogeneous

$$y' = \frac{x^2+y^2+xy}{x^2} \qquad \frac{dy}{dx} = \frac{x^2+y^2+xy}{x^2} \quad (*)$$

$y = vx \qquad \frac{dy}{dx} = v \cdot 1 + x \cdot \frac{dv}{dx}$

$\Rightarrow$ (*) $\quad v + x\frac{dv}{dx} = \frac{x^2 + \sqrt{x^2}x^2 + x \cdot vx}{x^2}$

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{x^2(1 + v^2 + v)}{x^2}$

$\Rightarrow \quad v + x\frac{dv}{dx} = 1 + v^2 + v$

$\Rightarrow \quad x\frac{dv}{dx} = 1 + v^2 \Rightarrow \int \frac{dv}{1+v^2} = \int \frac{dx}{x}$

$\Rightarrow \quad \tan^{-1}(v) = \log x + c$

$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \log x + C$ Ans

\# $\frac{dy}{dx} + Py = Q.$

$I.F = e^{\int P dx}$

Sol $\quad y \cdot e^{\int P dx} = \int Q \cdot e^{\int P dx} \cdot dx$

$e^{\int P dx}\left(\frac{dy}{dx} + Py\right) = Q \cdot e^{\int P dx}$

$e^{\int P dx}\frac{dy}{dx} + e^{\int P dx} P \cdot y = Q \cdot e^{\int P dx}$

$\int \frac{d}{dx}\left(e^{\int P dx} \cdot y\right) = \int Q \cdot e^{\int P dx}$

$e^{\int P dx}\frac{dy}{dx} + y \cdot e^{\int P dx} \cdot P$

Exact differential equation