L47 01 May 2021

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Q62. Evaluate surface integral of v=yzi+zxj+xyk over the surface of sphere $x^2 + y^2 + z^2 = a^2$

a. 1 b. 0 c. 3a d. $3a^2$

$$\iint_S \text{Curl}\,\vec{v}\cdot\hat{n}\,dA$$

$$= \iint_{S_1} + \iint_{S_2}$$

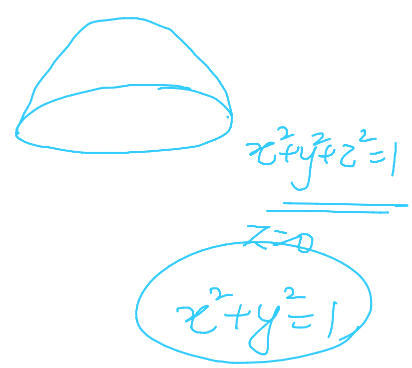
$$= \oint_{C_1} \cancel{v\cdot dr} - \oint_{C_1} \cancel{v\cdot dr} = 0$$

If S is any closed surface then $\iint_S \text{Curl}\,\vec{F}\cdot\hat{n}\,dA = 0$

Q63. Evaluate line integral of v=(2x-y)i-yz²j-y²zk over upper half of $x^2 + y^2 + z^2 = 1$ bounded by its projectior xy-plane, given curl v=k.

a. a² b. πa^2 c. π. d. 1

$$\oint_C v\cdot dr = \iint_S (\text{Curl}\,v)\cdot\hat{n}\;dA$$



$$= \iint (\hat{k})\cdot\cancel{\hat{n}}\;\frac{dx\,dy}{\cancel{\hat{n}\cdot\hat{k}}} = \iint_R dx\,dy$$

$= \text{Area of } x^2+y^2=1$

$= \pi(1)^2 = \pi$

Problem 1: Evaluate $\oint_C \vec{V}.\vec{dr}$ using Stokes' theorem where $\vec{V} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and S is the surface bounded by sphere $x^2 + y^2 + z^2 = 16$.

$$\oint v\cdot d\bar{r} = \iint (\text{Curl}\,\bar{v}\cdot\hat{n})\,dA$$

$$\text{Curl}\,\bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial & \partial & \partial \end{vmatrix}$$

$$\oint_C v\cdot d\bar{s} = \iint_S (\text{Curl}\,\bar{v}\cdot\hat{n})\, dA$$

$$= 0$$

$$= \underline{\underline{0}}$$

$$\text{Curl}\,\bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-0)$$

$$= 0$$

1. If the projection of the surface $z = f(x, y)$ is taken on xy −plane.

Area element, $dA = \sqrt{1 + {f_x}^2 + {f_y}^2}$

Surface Area, $A = \iint_R dA = \iint_R \sqrt{1 + {f_x}^2 + {f_y}^2}\,dxdy$

$z = f(x,y)$

$S:\ z - f(x,y) = 0$

$\text{grad}\, S = \vec{n} = -f_x\hat{i} - f_y\hat{j} + \hat{k}$

$|\vec{n}| = \sqrt{f_x^2 + f_y^2 + 1}$

2. If the projection of the surface $x = g(y, z)$ is taken on yz −plane.

Area element, $dA = \sqrt{1 + {g_y}^2 + {g_z}^2}$

Surface Area, $A = \iint_R dA = \iint_R \sqrt{1 + {g_y}^2 + {g_z}^2}\,dydz$

3. If the projection of the surface $y = h(z, x)$ is taken on zx −plane.

Area element, $dA = \sqrt{1 + {h_z}^2 + {h_x}^2}$

Surface Area, $A = \iint_R dA = \iint_R \sqrt{1 + {h_z}^2 + {h_x}^2}\,dzdx$

Find the surface area of $z^2 = x^2 + y^2, 0 \le z \le 4$

Cone

$\pi r l$

$l = \sqrt{h^2 + r^2} = \sqrt{(4)^2 + (4)^2}$

$x^2 + y^2 = (4)^2$

$z = 4$

$\pi r l$

$l = \sqrt{r^2+z^2} = \sqrt{(4)^2+(4)^2}$

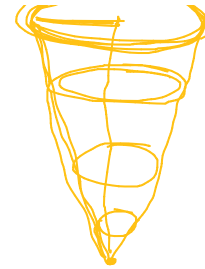
$= \sqrt{16+16} = \sqrt{32}$

Surface $= \pi r l$

$= \pi(4)\sqrt{32}$

$= 16\pi\sqrt{2}$

$z=4$



(d) None of these

Q61. If $\vec{v} = (2xyz + 3x^2)\hat{\imath} + (1 + x^2z)\hat{\jmath} + (x^2y)k$ and C is the boundary of the triangle with vertices (1,0,0),(0,2,0) and (0,0,3) then the integral $\oint_c \vec{v}.\overline{dr}$ is equal to

(a) −1 (b) 1 (c) 6 (d) 0

Stoke's Thm

$$\boxed{\oint_C \vec{v}\cdot d\vec{r} = \iint_S (\text{curl}\,\vec{v})\cdot\hat{n}\,dA} = 0$$

$$\text{curl}\,\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz+3x^2 & 1+x^2z & x^2y \end{vmatrix}$$

$$= \hat{i}\left(\cancel{x^2}-\cancel{x^2}\right) - \hat{j}\left(\cancel{2xy}-\cancel{2xy}\right) + \hat{k}\left(\cancel{2xz}-\cancel{2xz}\right) = 0$$