L3 27 Jan 2021

27 January 2021 10:25

$$xdy - ydx + a(x^2+y^2)dx = 0$$

$\boxed{x^2\, d(y/x)}$

$\Rightarrow \left(a(x^2+y^2) - y\right)dx + x\,dy = 0$

$M\,dx + N\,dy = 0$

$M = ax^2 + ay^2 - y \qquad \frac{\partial M}{\partial y} = 2ay - 1$

$N = +x \qquad \frac{\partial N}{\partial x} = +1$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ the eqn is not exact.

Integrating factor (I.F)

↳ it is multiplied with the given diff. eqn and given diff. eqn becomes exact.

① Inspection

$(x\,dy - y\,dx)$

I.Fs

(i) $\frac{1}{x^2}$ $\quad d(y/x)$

$\frac{1}{\cancel{y/x}}\left[\frac{x\,dy - y\,dx}{\cancel{x^2}}\right] = \frac{x\,dy - y\,dx}{xy}$

(ii) $\frac{1}{y^2}$ $\quad -d\left(\frac{x}{y}\right)$

iii) $\frac{1}{xy}$ $\quad d\left(\log\left(\frac{y}{x}\right)\right)$ $\qquad \frac{x\,dy - y\,dx}{xy} = d\left(\log\left(\frac{y}{x}\right)\right)$

(iv) $\frac{1}{x^2+y^2}$ $\quad d\left(\tan^{-1}(y/x)\right)$ $\qquad \frac{x\,dy - y\,dx}{x^2+y^2} = d\left(\tan^{-1}(y/x)\right)$

$\frac{\cancel{x^2}}{x^2+y^2/\cancel{x^2}}\left[\frac{x\,dy - y\,dx}{\cancel{x^2}}\right]$

v) $\frac{1}{x^2-y^2}$ $\quad d\left(\frac{1}{2}\log\left(\frac{x+y}{x-y}\right)\right)$

$\frac{1}{2}\left[\log(x+y) - \log(x-y)\right]$

$\frac{1}{2}\left[\frac{1}{x+y}(dx+dy) - \frac{1}{x-y}(dx-dy)\right]$

$$xdy - ydx + a(x^2+y^2)dx = 0$$

$(x\,dy - y\,dx \quad\quad (x^2+y^2)$

$xdy - ydx + a(x^2 + y^2)dx = 0$

$\frac{1}{2}\left[\frac{1}{x+y}(\ldots) - \frac{1}{x-y}(\ldots)\right]$

$$\underbrace{\frac{xdy - ydx}{x^2}} + \underbrace{a\left(\frac{x^2+y^2}{x^2}\right)}dx = 0$$

Not Good to use 1st I.F

$$d\left(\frac{y}{x}\right) +$$

$$\int \frac{xdy - ydx}{x^2+y^2} + \int a\,dx = \int 0$$

$$\int d\left(\tan^{-1}\left(\frac{y}{x}\right)\right) + ax = c$$

$$\boxed{\tan^{-1}(y/x) + ax = c}$$

$$\boldsymbol{ydx - xdy + 3x^2y^2e^{x^3}dx = 0}$$

$I.F = \frac{1}{y^2}$

$$\int \frac{ydx - xdy}{y^2} + \int 3x^2e^{x^3}dx = 0$$

$x^3 = t$

$3x^2dx = dt$

$$\int d\left(\frac{x}{y}\right) + \int e^t dt = c$$

$$\boxed{\frac{x}{y} + e^{x^3} = c}$$

③

$$xdx + ydy + 2(x^2+y^2)dx = 0$$

$$\Rightarrow \int \frac{xdx + ydy}{x^2+y^2} + \int 2\,dx = 0$$

$$\Rightarrow \int \frac{1}{2}d\left(\log(x^2+y^2)\right) + 2x = C$$

$x^2+y^2 = t$

$2xdx + 2ydy = dt$

$(xdx + ydy) = \frac{1}{2}dt$

$$\boxed{\frac{1}{2}\log(x^2+y^2) + 2x = C}$$

$$\boxed{\frac{1}{2}\log(x^2+y^2)+2x=c}$$

② If $Mdx+Ndy=0$ is an Homogeneous diff. eqn then

$I.F = \frac{1}{Mx+Ny}$, $Mx+Ny \neq 0$.

$(x^3+y^3)dx - xy^2dy = 0$

$Mdx+Ndy=0$

$M = x^3+y^3$ $\quad N = -xy^2$

$\frac{\partial M}{\partial y} = 3y^2$ $\quad \frac{\partial N}{\partial x} = -y^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ eqn is not exact

Given eqn is Homogeneous

$\Rightarrow I.F = \frac{1}{Mx+Ny} = \frac{1}{x(x^3+y^3)+(-xy^2)y} = \frac{1}{x^4+\cancel{xy^3}-\cancel{xy^3}}$

$= \frac{1}{x^4}$

$$\boxed{I.F = \frac{1}{x^4}}$$

Given ~~eqn~~ eqn

$\left(\frac{x^3+y^3}{x^4}\right)dx - \left(\frac{xy^2}{x^4}\right)dy = 0 \Rightarrow \left(\frac{1}{x}+\frac{y^3}{x^4}\right)dx - \left(\frac{y^2}{x^3}\right)dy = 0$

Solution $\int_{y\ \text{cost.}} M\,dx + \int (\text{Terms of } N \text{ free from } x)\,dy = c$

$\int_{y\ \text{Constant}} \left(\frac{1}{x}+\frac{y^3}{x^4}\right)dx + 0 = c$

$\log x + y^3\left(-\frac{1}{3x^3}\right) = c$

$$\boxed{\log x - \frac{y^3}{3x^3} = c}$$ Ans

$\int \frac{1}{x^4}dx$

$\int x^{-4}dx$

$\frac{x^{-3}}{-3}$

$(x^2y-2xy^2)dx + (3x^2y-x^3)dy = 0$