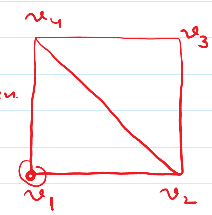
Thm: The number of odd degree vertices in any graph are always even.

Soln: Let G be a graph with n vertices and e edges.

$\therefore$ By Hand shaking theorem.

$$\sum_{i=1}^{n} deg(v_i) = 2e \longrightarrow (1)$$



$2+4+6$ = even.

$$\underset{v_i \text{ is odd degree vertex}}{\sum deg(v_i)} + \underbrace{\underset{v_j \text{ is even degree vertex}}{\sum deg(v_j)}}_{\text{Even no}} = \underbrace{\boxed{2e}}_{\text{Even no.}}$$

$V = \{v_1, v_2, v_3, v_4\}$

$A = \{v_2, v_4\} \leftarrow$

$B = \{v_1, v_3\} \leftarrow$

$$\underset{v_i \text{ is odd degree vertex}}{\sum deg(v_i)} = \text{Even no} - \text{Even no}$$

$$\underset{v_i \in odd}{\sum deg(v_i)} = \text{Even no.}$$

$3+5+7 = 15$

$3+5+7+9 = 24$

This is possible only when no of odd degree vertices are even.

—— X ——

Can we have a graph, degree of whose vertices are given as.

$\underbrace{1, 3, 5}_{odd}, 2, 4, 6$

This is not possible.

Theorem: ~~Deg~~ Maximum degree of any vertex in a simple graph with n vertices can be only n-1.

Soln As there are n vertices

Let $v_1, v_2, \ldots, v_n$ be n vertices.

As the graph is simple, so self-loops and parallel edges are not allowed in the graph.

$\therefore$ Any vertex v_j

then this vertex can be joined with the remaining (n-1) vertices.

$\therefore$ Max degree of any vertex in a simple graph can be n-1.

—— X ——

Can we have a simple graph degree of whose vertices are given as 1, 3, 5, 1, 2, 8

Can we have a simple graph degree of whose vertices are given as 1, 3, 5, 1, 2, 8

Sol

Solⁿ As we have 6 vertices and the maximum degree of any vertex can be only 5.

But here we have one vertex whose degree is 8.

$\therefore$ We can not have simple graph whose vertices have degree 1, 3, 5, 1, 2, 8

——×——

Ques: The max no of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

Solⁿ Here we have n vertices in the graph.

$\therefore$ By hand shaking theorem.

$$\sum_{i=1}^{n} \deg(v_i) = 2e$$

$$\deg(v_1) + \deg(v_2) + \cdots + \deg(v_n) = 2e$$

$$\text{Max}\deg(v_1) + \text{Max}\deg(v_2) + \cdots + \text{Max}\deg(v_n) = 2e$$

$$\underbrace{(n-1) + (n-1) + \cdots + (n-1)}_{} = 2e$$

$$n(n-1) = 2e$$

$$e = n(n-1)/2 = \frac{n(n-1)}{2} \text{ Ans.}$$

The no of edges in a complete graph with n vertices is $\frac{n(n-1)}{2}$

Solⁿ Here we have n vertices.

$$\sum_{i=1}^{n} \deg(v_i) = 2e$$

$$\deg(v_1) + \deg(v_2) + \cdots + \deg(v_n) = 2e$$

$$(n-1) + (n-1) + \cdots + (n-1) = 2e$$

$$n(n-1) = 2e \implies e = \frac{n(n-1)}{2} \checkmark$$

V.V.V Imp: ~~Ma~~ No. of Edges in K_{10} graph are.

(a) 50 (b) 56 (c) 60 (d) 55 (e) 45 . ✓

No of edges in $K_n = \frac{n(n-1)}{2}$

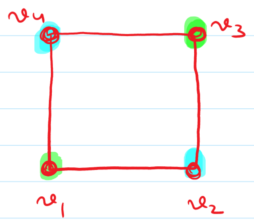
$= \frac{\cancel{10}^{5} \times 9}{\cancel{2}} = 45.$

No of Edges in K_{100} graph are ~~...~~ . 4950

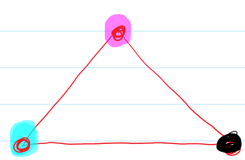
— X —

Chromatic number of a Graph: Let G be a graph with n vertices. The Minimum no. of colors required to paint the vertices of graph so that no two adjacent vertices receive the same color is called the chromatic number of the graph. It is denoted by $\chi(G)$.

~~S~~ky

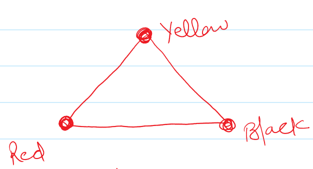


$\chi(G) = 2$



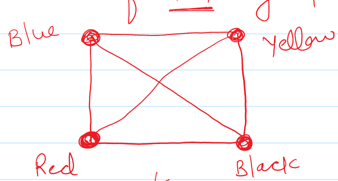
$\chi(G) = 3$

Q1) Find the chromatic no of K_n graph.



K_3

$\chi(K_3) = 3$

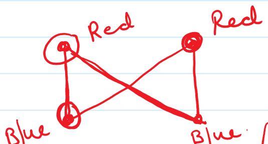


K_4

$\chi(K_4) = 4$

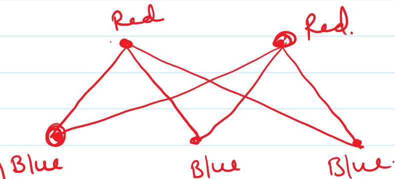
$$\boxed{\chi(K_n) = n}$$

① Find the Chromatic number of Complete Bipartite graph $K_{m,n}$.



$K_{2,2}$

$\boxed{\chi(K_{m,n}) = 2}$

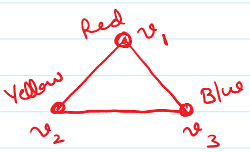


$K_{2,3}$

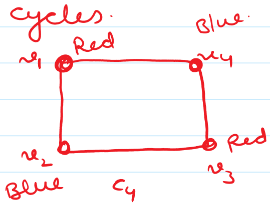
$\chi(K_{2,2}) = 2$ $\quad \chi(K_{100,50}) = 2$ $\quad \chi(K_{2,3}) = 2$

$\chi(C_{101}) = 3$

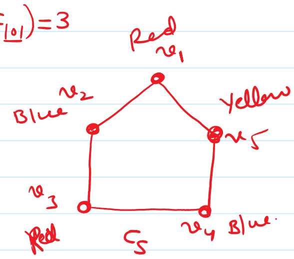
Chromatic number of cycles.



C_3

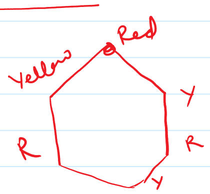


C_4



C_5

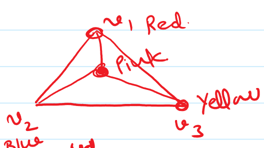
$\chi(C_3) = 3$, $\quad \chi(C_4) = 2$ $\quad \chi(C_5) = 3$



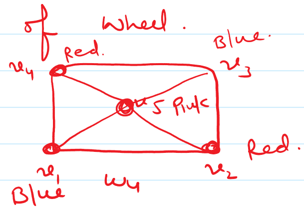
$\chi(C_6) = 2$

$$\boxed{\chi(C_n) = \begin{cases} 3 & \text{if } n \text{ is odd} \\ 2 & \text{if } n \text{ is even} \end{cases}}$$

① Find the Chromatic no of wheel.



W_3

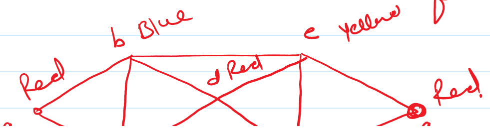


W_4

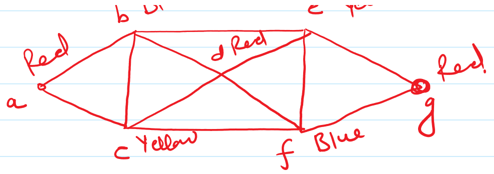
$$\chi(W_n) = \begin{cases} 4 & \text{if } n \text{ is odd} \\ 3 & \text{if } n \text{ is even.} \end{cases}$$

——— × ———

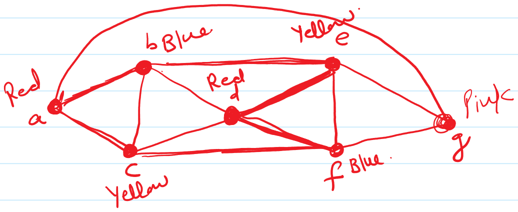
Find the Chromatic no of the following graphs.



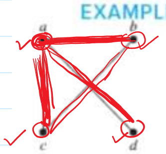
3



3 ?



4 ?



A simple graph.

EXAMPLE Use an adjacency matrix to represent the graph shown in Figure.

Solution: We order the vertices as a, b, c, d. The matrix representing this graph is

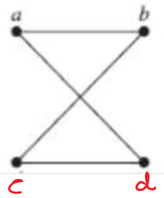
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Soln

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

	a	b	c	d
a	0	1	1	1
b	1	0	1	0
c	1	1	0	0
d	1	0	0	0

EXAMPLE Draw a graph with the adjacency matrix



$$\left\{\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}\right\}$$

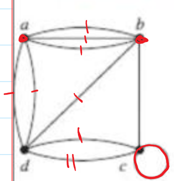
with respect to the ordering of vertices a, b, c, d.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

— X —

	a	b	c	d
a	0	1	0	1
b	1	0	1	0
c	0	1	0	1
d	1	0	1	0

EXAMPLE Use an adjacency matrix to represent the pseudograph shown.



Solution: The adjacency matrix using the ordering of vertices a, b, c, d is

$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}.$$

	a	b	c	d.
a	0	3	0	2
b	3	0	1	1
c	0	1	1	2
d	2	1	2	0

EXAMPLE Represent the graph shown in Figure with an incidence matrix.

Solution: The incidence matrix is

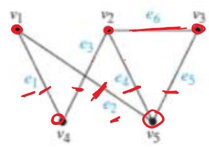


FIGURE **An undirected graph.**

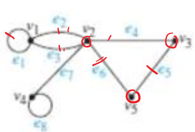
$$
\begin{array}{c}
\\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5
\end{array}
\begin{array}{c}
\begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0
\end{bmatrix}
\end{array}.
$$

(v_1, e_1)

	e_1	e_2	e_3	e_4	e_5	e_6
v_1	1	1	0	0	0	0
v_2	0	0	1	1	0	1
v_3	0	0	0	0	1	1
v_4	1	0	1	0	0	0
v_5	0	1	0	1	1	0

EXAMPLE Represent the pseudograph shown in Figure using an incidence matrix.

Solution: The incidence matrix for this graph is

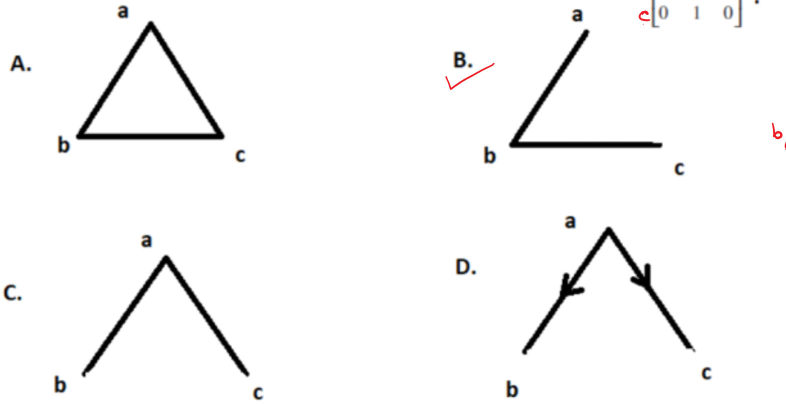


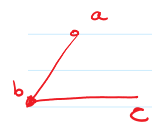
$$
\begin{array}{c}
\\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}
\end{array}.
$$

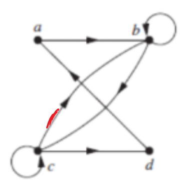
A pseudograph.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
v_1	1	1	1	0	0	0	0	0
v_2	0	1	1	1	0	1	1	0
v_3	0	0	0	1	1	0	0	0
v_4	0	0	0	0	0	0	1	1
v_5	0	0	0	0	1	1	0	0

Quiz 2 : Which of the following graph has the adjacency matrix $\begin{array}{c|ccc} & a & b & c \\ a & 0 & 1 & 0 \\ b & 1 & 0 & 1 \\ c & 0 & 1 & 0 \end{array}$ **?**





Quiz 3 : Which of the following adjacency matrix represents 

?

A. $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

✓ **C.** $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

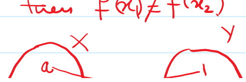
D. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$

	a	b	c	d
a	0	1	0	0
b	0	1	1	0
c	0	1	1	1
d	1	0	0	0

Graph Isomorphism.

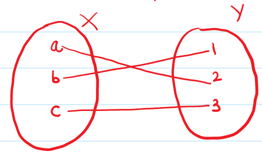
injective

One-one function : Let X and Y be two non-empty sets.
Then a rule $f: X \to Y$ is called a 1-1 function.
if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$



$a \neq b$

if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$



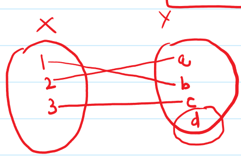
$a \neq b$

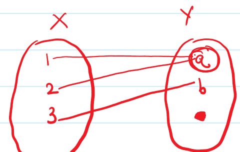
$f(a) \neq f(b)$

$\boxed{\text{if } f(x_1) = f(x_2) \text{ then } x_1 = x_2 \;\; \forall x \in D_f}$

Onto-function:→ Let X and Y be two non-empty sets. then a function $f: X \to Y$ is called an onto-function. ~~iff~~ if for each $y \in Y$ $\exists$ $x \in X$ s.t

$\boxed{f(x) = y}$





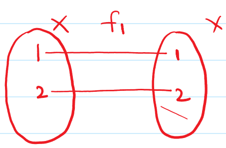
This function is onto function.

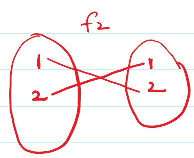
$d \in Y$ has no pre-image in X, ∴ function is not onto.

—×—

Bijective function: A function $f: X \to Y$ is called bijective if it is both 1-1 and onto.

Counting of bijective functions: Let $f: X \to X$ be a bijective map. then no of bijections from $X \to X$ are $= n!$ if X has n elements.

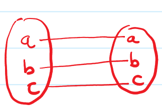




In this case we have 2! bijections = 2!

$= 2 \times 1$

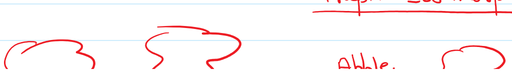
$= 2$



no of bijective functions = 3!

$= 6$

Graph Isomorphism.



Graph Isomorphism.