L37 12 April 2021

12 April 2021 11:06

① If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ Show that $(\vec{u}\cdot\nabla)\vec{r} = \vec{u}$

Let $\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$

$$(\vec{u}\cdot\nabla) = (u_1\hat{i} + u_2\hat{j} + u_3\hat{k})\cdot\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

$$= \left(u_1\frac{\partial}{\partial x} + u_2\frac{\partial}{\partial y} + u_3\frac{\partial}{\partial z}\right)$$

$$(\vec{u}\cdot\nabla)\vec{r} = \left(u_1\frac{\partial}{\partial x} + u_2\frac{\partial}{\partial y} + u_3\frac{\partial}{\partial z}\right)(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$$

$$= \vec{u}$$

\# It is given that $\nabla f(P) = 3\hat{i} + 4\hat{j}$. Find a unit vector $\hat{b}$ such that $D_{\hat{b}}f(P)$ is maximum.

$$\vec{b} = 3\hat{i} + 4\hat{j}$$

$$\hat{b} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{9+16}} = \frac{3\hat{i} + 4\hat{j}}{5}$$

f $\quad \vec{b}$

$\nabla f \cdot \hat{b}$

$|\nabla f|\,|\hat{b}|\cos\theta$

\# If $\nabla f(P) = 3\hat{i} + 4\hat{j}$. Find a unit vector $\hat{b}$ such that $D_{\hat{b}}f(P)$ is minimum.

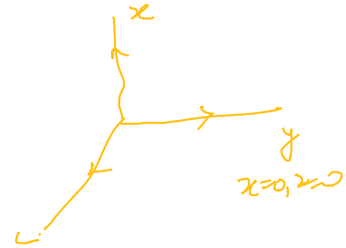
$$\hat{b} = -\frac{(3\hat{i} + 4\hat{j})}{5}$$

\# Find the values of the constant a, b and c such that the maximum value of the directional derivative of

" Find the values of the constant a, b and c such that the maximum value of the directional derivative of $f(x,y,z) = axy^2 + byz + cx^2z^2$ at $(1,-1,1)$ is in the direction ||el to the axis of y and has magnitude 6.

$$\nabla f = \hat{i}(ay^2 + 2cxz^2) + \hat{j}(2axy + bz) + \hat{k}(by + 2cx^2z)$$

$$\nabla f\Big|_{(1,-1,1)} = \hat{i}(a+2c) + \hat{j}(-2a+b) + \hat{k}(-b+2c)$$



$a + 2c = 0$ $\qquad$ $-b + 2c = 0$

$\boxed{a = -2c}$ $\qquad$ $\boxed{b = 2c}$

$$|\nabla f| = 6 \Rightarrow \sqrt{(a+2c)^2 + (-2a+b)^2 + (-b+2c)^2} = 6$$

$$= \sqrt{0 + (-2a+b)^2 + 0} = 6$$

$-2a + b = 6$

$-2(-2c) + 2c = 6$

$4c + 2c = 6$

$c = 1$

$a = -2$
$b = 2$
$c = 1$

Q52. The directional derivative of $\emptyset(x,y,z) = x^2yz + 4xz^2$ at the point (1,0,-1) in the direction of PQ where P=(1,2,-1) and Q=(-1,2,3) is D

(a) $\frac{12}{\sqrt{5}}$ (b) $\frac{-28}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{-25}{\sqrt{5}}$

$$\vec{PQ} = (-1-1)\hat{i} + (2-2)\hat{j} + (3+1)\hat{k}$$

$$= -2\hat{i} + 0\hat{j} + 4\hat{k}$$

$$\vec{b} = \vec{PQ} = -2\hat{i} + 4\hat{k}$$

$$\nabla f = \hat{i}(2xyz + 4z^2) + \hat{j}(x^2z + 0) + \hat{k}(x^2y + 8xz)$$

$$\nabla f_{(1,0,-1)} = \hat{i}(4) + \hat{j}(-1) + \hat{k}(-8) = 4\hat{i} - \hat{j} - 8\hat{k}$$

$$\nabla f \cdot \hat{b} = (4\hat{i} - \hat{j} - 8\hat{k}) \cdot \frac{(-2\hat{i} + 4\hat{k})}{\sqrt{4+16}} = \frac{-8 - 32}{\sqrt{20}}$$

$$= \frac{-40}{\sqrt{20}} = \frac{-20}{\sqrt{5}}$$

#

If $f(x,y) = x^2 - xy - y + y^2$. Find all points where the directional derivative in the direction $\vec{b} = \frac{\hat{i} + \sqrt{3}\hat{j}}{2}$ is 0.

$\sqrt{1+3} = 2$

$$\nabla f = \hat{i}(2x - y) + \hat{j}(-x - 1 + 2y)$$

$$\nabla f \cdot \hat{b} = \left[\hat{i}(2x-y) + \hat{j}(-x-1+2y)\right] \cdot \left[\frac{\hat{i} + \sqrt{3}\hat{j}}{2}\right] = 0$$

$$\frac{1}{2}(2x - y) + \frac{\sqrt{3}(-x - 1 + 2y)}{2} = 0$$

$$(2x - y) + \sqrt{3}(-x - 1 + 2y) = 0$$

$$(2 - \sqrt{3})x + (2\sqrt{3} - 1)y = \sqrt{3}$$