L16 22 Feb 2021

22 February 2021 11:07

Case No:- 3 :-

$$\frac{1}{f(D)} \cdot x^m = (f(D))^{-1} \cdot x^m$$

$\downarrow$ binomial

. Find the general solution of: $y'' + 25y = 4x^2$

C.F

$D^2 + 25 = 0$

$D^2 = -25$

$D = \pm i5$

$y_c = c_1 \cos 5x + c_2 \sin 5x.$

$$\frac{1}{D^2+25} \cdot 4x^2 = \frac{1}{25\left(1+\frac{D^2}{25}\right)} \cdot 4x^2$$

$$= \frac{4}{25}\left(1+\frac{D^2}{25}\right)^{-1} \cdot x^2$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - - - - - .$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \text{---}$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \text{---}$$

$D(x^2) = 2x$

$D^2(x^2) = 2$

$D^3(x^2) = 0$

$$\frac{4}{25}\left(1+\frac{D^2}{25}\right)^{-1} \cdot x^2 = \frac{4}{25}\left(1 - \frac{D^2}{25} + \frac{D^4}{(25)^2} - \text{----}\right)x^2$$

$$= \frac{4}{25}\left(x^2 - \frac{D^2}{25}(x^2) + \frac{D^4}{(25)^2}(x^2) \text{---}\right)$$

$$= \frac{4}{25}\left(x^2 - \frac{2}{25} + 0 \text{---}\right) = \frac{4}{625}\left(25x^2 - 2\right)$$

Find the general solution of: $y'' - 6y' + 9y = 4x^2 - 1$

C.F

A.E $D^2 - 6D + 9 = 0$

$(D-3)^2 = 0$

$D = 3, 3$

$y_c = (c_1 + c_2 x)e^{3x}$

P.I $\quad \dfrac{1}{D^2-6D+9}\cdot(4x^2-1) = \dfrac{1}{9\left(1+\frac{D^2-6D}{9}\right)}(4x^2-1)$

$D(4x^2-1) = 8x$

$D^2(4x^2-1) = 8$

$D^3(4x^2-1) = 0$

$$= \frac{1}{9}\left(1+\frac{D^2-6D}{9}\right)^{-1}\cdot(4x^2-1)$$

$$= \frac{1}{9}\left[1-\left(\frac{D^2-6D}{9}\right)+\underbrace{\left(\frac{D^2-6D}{9}\right)^2}-\cdots\right](4x^2-1)$$

$(D^2-6D)^2$

$= \cancel{D^4}+36D^2-12\cancel{D^3}$

$$= \frac{1}{9}\left[1-\left(\frac{D^2-6D}{9}\right)+\frac{1}{81}\left(36D^2\right)\right](4x^2-1)$$

$$= \frac{1}{9}\left[(4x^2-1)-\frac{1}{9}\left[8-6(8x)\right]+\frac{36}{81}(8)\right]$$

$$\frac{4}{9}x^2-\frac{1}{9}-\frac{8}{9}+\frac{48}{9}x+\frac{36}{81}\times 8$$

\# $\quad (D^2-5D+6)y = x+e^{mx}, \quad m\neq 2,3$

C.F $\quad$ A.E $\quad D^2-5D+6=0$

$D^2-2D-3D+6=0$

$D(D-2)-3(D-2)=0$

$(D-3)(D-2)=0$

$D=2,3$

$y_c = c_1e^{2x}+c_2e^{3x}$

$[D^2\to -a^2]$

P.I $\quad \dfrac{1}{D^2-5D+6}\cdot(x+e^{mx}) = \dfrac{1}{D^2-5D+6}\cdot x + \dfrac{1}{D^2-5D+6}\cdot e^{mx}$

$D\to m$

$$\Rightarrow \frac{1}{6\left[1+\left(\frac{D^2-5D}{6}\right)\right]}\cdot x + \frac{1}{m^2-5m+6}\cdot e^{mx}$$

$m\neq 2,3$

$$= \frac{1}{6}\left[1+\left(\frac{D^2-5D}{6}\right)\right]^{-1} \cdot x \quad + \frac{1}{m^2-5m+6} \cdot e^{mx}$$

$D(x) = 1$

$D^2(x) = 0$

$$= \frac{1}{6}\left[1-\left(\frac{D^2-5D}{6}\right)\right] \cdot x + \frac{1}{m^2-5m+6} \cdot e^{mx}$$

$$= \frac{1}{6}\left[x + \frac{5}{6}\right] + \frac{1}{m^2-5m+6} \cdot e^{mx}$$

Q34. The general solution of the equation $y'' + 16y = x^2$ is

(a) A cos x + B sin x + $4x^2$

(b) A cos 4x + B sin 4x + $4x^2 - \frac{1}{2}$

(c) A cos 4x + B sin 4x + 4x

(d) None of these ✓

$(D^2+16)y = x^2$

$D^2+16=0$

$D^2 = -16$ $\quad D = \pm 4i$

$y_c = C_1 \cos 4x + C_2 \sin 4x$

$$\frac{1}{D^2+16} \cdot x^2 = \frac{1}{16\left(1+\frac{D^2}{16}\right)} \cdot x^2$$

$D(x^2) = 2x$

$D^2(x^2) = 2$

$D^3(x^2) = 0$

$$\frac{1}{16}\left[\left(1+\frac{D^2}{16}\right)^{-1} \cdot x^2\right] = \frac{1}{16}\left[1 - \frac{D^2}{16}\right] x^2$$

$$\frac{1}{16}\left(x^2 - \frac{1}{\cancel{16}} \cdot \frac{\cancel{2}}{8}\right) = \frac{1}{16}\left(x^2 - \frac{1}{8}\right)$$

\# Case 4 :- $\quad s(x) = e^{ax} \cdot g(x)$

$f(x) = x^m$

$f(x) = \sin$

$$\frac{1}{f(D)} \cdot e^{ax} \cdot g(x) = e^{ax} \cdot \frac{1}{f(D+a)} \cdot g(x)$$

Find the ~~general solution~~ P.I of: $\quad y'' - 4y' + 5y = 24e^{2x} \sin x$

P.I $\quad \frac{1}{D^2-4D+5} \cdot 24 \cdot \underline{e^{2x} \sin x} = 24 \cdot \frac{1}{D^2-4D+5} \cdot e^{2x} \sin x$

$$= 24 \cdot e^{2x} \cdot \frac{1}{(D+2)^2 - 4(D+2) + 5} \cdot \sin x$$

$$D^2 \to -1$$

$$= 24 \cdot e^{2x} \cdot \frac{1}{D^2 + 4 + \cancel{4D} - \cancel{4D} - 8 + 5} \cdot \sin x$$

$$= 24 \cdot e^{2x} \frac{1}{D^2 + 1} \sin x \quad = \quad 24 \cdot e^{2x} \cdot \frac{1}{\cancel{-1+1}\ 0} \cdot \sin x$$

$\downarrow$ Case of failure

$$= 24 \cdot e^{2x} \cdot \frac{1}{2D} \sin x$$

$$D$$

$$= \frac{\overset{12}{\cancel{24}}}{\cancel{2}} \cdot e^{2x} \cdot \frac{1}{D} \sin x$$

$$D^{-1} = \frac{1}{D}$$

$$= 12 \cdot e^{2x} \cdot (-\cos x) = -12 e^{2x} \cos x$$

Find the general solution of: $y'' - y' - 6y = xe^{-2x}$