L10 08 Feb 2021

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Linear Combination of functions:-

Let $f_1(x), f_2(x), \ldots \ldots f_n(x)$ be n functions Then

$c_1 f_1(x) + c_2 f_2(x) + \ldots \ldots + c_n f_n(x)$ where $c_1, c_2, c_3, \ldots c_n$ are Constants

is called Linear Combination of functions

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$y'' - y = 0$ — (1)

$y = e^{-x}$ and $y = e^{x}$ → Solutions of ~~the~~ eqn.

$y = e^{-x}$

$y' = -e^{-x}$

$y'' = e^{-x}$

put in (1) $\cancel{e^{-x}} - \cancel{e^{-x}} = 0$ so ✓ $y = e^{-x}$ is the solution of (1)

Illy ✓ $y = e^{x}$ is also the solution of (1)

~~Then~~ Linear Combination of function will also be the sol.

means $y = c_1 e^{-x} + c_2 e^{x}$ will also be the solution

$y' = -c_1 e^{-x} + c_2 e^{x}$ ✓

$y'' = c_1 e^{-x} + c_2 e^{x}$

$y'' - y = 0$ ✓

put in (1)

$y'' - y = (\cancel{c_1 e^{-x}} + \cancel{c_2 e^{x}}) - (\cancel{c_1 e^{-x}} + \cancel{c_2 e^{x}})$

$= 0$

$y = e^{x}$ $\quad y = 2e^{x}$

$y = c_1$ $\quad y = 3e^{x}$

$y = c_1 e^{x}$

Linearity principle or Superposition principle

Linear Independence and Linear Dependence :-

Linear Independence and Linear Dependence :-

Let $f_1(x), f_2(x), \ldots f_n(x)$ be n functions.

Then these functions are s.t.b linearly independent on some interval I if

$$\boxed{c_1 f_1(x) + c_2 f_2(x) + \cdots c_n f_n(x) = 0}$$

implies $\Rightarrow$ $c_1 = 0 = c_2 = c_3 = \cdots = c_n$

These functions are s.t.b linearly dependent on I, if $c_1, c_2, c_3 - c_n$ not all zero

e.g. f_1, f_2, f_3

$$c_1 f_1 + c_2 f_2 + c_3 f_3 = 0$$

$c_1 \neq 0$

$$c_1 f_1 = -c_2 f_2 - c_3 f_3$$

$$f_1 = \left(-\frac{c_2}{c_1}\right) f_2 - \left(\frac{c_3}{c_1}\right) f_3$$

f_1 can be written in terms of other functions

f_1 depends upon other two function

$$(2,4) = 2(1,0) + 4(0,1)$$
$$= (2,0) + (0,4)$$
$$= (2,4)$$

$$(x,y) = x(1,0) + y(0,1)$$

$(1,0)$ $(0,1)$

$(1,0)$ $(0,1)$

$(x,y) = x(1,0) + y(0,1)$

$(1,0) = 1\cdot(1,0) + 0\cdot(0,1)$

$(1,0)$ $(0,1)$

$(1,5) = 1(1,0) + 5(0,1)$

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$\boxed{x^2-1, \quad 3x^2, \quad 2-5x^2}$

$c_1(x^2-1) + c_2(3x^2) + c_3(2-5x^2) = 0$

$c_1x^2 - c_1 + 3c_2x^2 + 2c_3 - 5c_3x^2 = 0$

$x^2(c_1 + 3c_2 - 5c_3) + (-c_1 + 2c_3) = 0$

$c_1 + 3c_2 - 5c_3 = 0$ $\quad -c_1 + 2c_3 = 0$

$\boxed{2c_3 = c_1}$

$2c_3 + 3c_2 - 5c_3 = 0$

$3c_2 - 3c_3 = 0$

$\boxed{c_2 = c_3}$

$c_3 = 1$

$c_2 = 1$

$c_1 = 2(1) = 2$

$2, 1, 1$

L.D

Wronskians :→ Let $f_1(x), f_2(x), f_3(x) \ldots f_n(x)$ be n functions.

$$W(f_1, f_2, f_3, \ldots f_n) = \begin{vmatrix} f_1 & f_2 & f_3 & & f_n \\ f_1' & f_2' & f_3' & & f_n' \\ f_1'' & f_2'' & f_3'' & & f_n'' \\ \vdots & & & & \\ f_1^{n-1} & f_2^{n-1} & f_3^{n-1} & & f_n^{n-1} \end{vmatrix}$$

f_1 $\quad f_2$ $\quad f_3$

$$\begin{vmatrix} f_1^{n-1} & f_2^{n-1} & f_3^{n-1} & \cdots & f_n^{n-1} \end{vmatrix}$$

$$\begin{matrix} f_1 & f_2 & f_3 \\ \downarrow & \downarrow & \downarrow \\ x^2-1, & 3x^2, & 2-5x^2 \end{matrix}$$

$$W(f_1, f_2, f_3) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = \begin{vmatrix} x^2-1 & 3x^2 & 2-5x^2 \\ 2x & 6x & -10x \\ 2 & 6 & -10 \end{vmatrix}$$

$$= (x^2-1)\left[-\cancel{60x} + \cancel{60x}\right] - 3x^2\left[-\cancel{20x} + \cancel{20x}\right] + (2-5x^2)\left[\cancel{12x} - \cancel{12x}\right]$$

$$= 0$$

$W(f_1, f_2, f_3) = 0$ then L.D

$W(f_1, f_2, f_3) \neq 0$ then L.I