L18 24 Feb 2021

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$(D^2-2D+1)y = x^2 e^{3x}$

C.F A.E $D^2-2D+1=0$

$(D-1)^2=0$

$D=1,1$

$y_c = (C_1+C_2x)e^x$

P.I

$$y_p = \frac{1}{D^2-2D+1}\cdot x^2e^{3x} = e^{3x}\cdot\frac{1}{(D+3)^2-2(D+3)+1}\cdot x^2$$

$$= e^{3x}\cdot\frac{1}{D^2+9+6D-2D-6+1}\cdot x^2$$

$$= e^{3x}\cdot\frac{1}{D^2+4D+4}\cdot x^2$$

$$= e^{3x}\cdot\frac{1}{4\left[1+\left(\frac{D^2+4D}{4}\right)\right]}\cdot x^2$$

$$= \frac{e^{3x}}{4}\left[1+\frac{D^2+4D}{4}\right]^{-1}x^2$$

$$= \frac{e^{3x}}{4}\left[1-\left(\frac{D^2+4D}{4}\right)+\left(\frac{D^2+4D}{4}\right)^2\right]x^2$$

$$y_p = \frac{e^{3x}}{4}\left[x^2-\frac{1}{4}\left(2+4(2x)\right)+\frac{1}{\cancel{16}}\left[\cancel{16}(2)\right]\right]$$

$D(x^2)=2x$

$D^2(x^2)=2$

$D^3(x^2)=0$

$(D^2+4D)^2$

$=\cancel{D^4}+16D^2+8\cancel{D^3}$

Case No:-5 $X(x) = x.V$ V is a function of x.

$$\frac{1}{f(D)}\cdot x.V = x.\frac{1}{f(D)}.V + \frac{d}{dD}\left(\frac{1}{f(D)}\right).V$$

$\sin ax$

$D^2 \to -a^2$

$(D^2-1)y = x\sin x$

A.E $D^2-1=0$ | P.I $\frac{1}{D^2-1}\cdot x.\sin x$

A.E $D^2-1=0$

$D^2=1$

$D=\pm 1$

$y_c = c_1 e^{-x} + c_2 e^{x}$

P.I $\frac{1}{D^2-1}\cdot x\sin x$

$= x\cdot\frac{1}{D^2-1}\cdot\sin x + \frac{d}{dD}\left(\frac{1}{D^2-1}\right)\cdot\sin x$

$= x\cdot\frac{1}{-1-1}\sin x + \left(\frac{-1}{(D^2-1)^2}\cdot 2D\right)\sin x$

$= \frac{x}{-2}\sin x - \frac{2D}{(D^2-1)^2}\sin x$

$= \frac{-x}{2}\sin x - \frac{2D}{(-1-1)^2}\sin x$

$= -\frac{x}{2}\sin x - \frac{2}{4}D(\sin x)$

$y_p = -\frac{x}{2}\sin x - \frac{1}{2}\cos x$

Euler formula

$$\boxed{e^{ix} = \cos x + i\sin x}$$

$(D^2-1)y = x\sin x$

P.I $\frac{1}{D^2-1}\cdot x\sin x$ = Imaginary part of $\frac{1}{D^2-1}\cdot x\cdot e^{ix}$

$= e^{ix}\frac{1}{(D+i)^2-1}\cdot x$

$= e^{ix}\frac{1}{D^2-1+2Di-1}\cdot x$

$= e^{ix}\cdot\frac{1}{D^2+2Di-2}\cdot x$

$= e^{ix}\cdot\frac{1}{-2\left[1-\left(\frac{D^2+2Di}{2}\right)\right]}\cdot x$

$D(x)=1$

$D^2(x)=0$

$\left[\quad\right]^{-1}$

$$-2\left\{1-\left(\frac{D^2+2Di}{2}\right)\right\}$$

$$= \frac{e^{ix}}{-2}\left[1-\left(\frac{D^2+2Di}{2}\right)\right]^{-1}\cdot x = -\frac{e^{ix}}{2}\left[1+\left(\frac{\cancel{D^2}+\cancel{2}Di}{\cancel{2}}\right)\right]x$$

$$= -\frac{e^{ix}}{2}\left[1+Di\right]x$$

$$= \frac{e^{ix}}{-2}\left[x+i\right]$$

$$= -\frac{1}{2}(\cos x + i\sin x)(x+i)$$

Imaginary part $\rightarrow$ $= -\frac{1}{2}\cos x - \frac{x}{2}\sin x$

$(D^2+1)y = x\cdot e^x\cos 2x \rightarrow = x\cdot e^x\cdot e^{i2x}$

$$= x\cdot e^{(1+2i)x}$$

Real part

$$\frac{1}{D^2+1}\cdot xe^x\cos 2x = e^x\cdot\frac{1}{(D+1)^2+1}\cdot x\cos 2x$$

$$= e^x\cdot\frac{1}{D^2+D+2}\cdot x\cos 2x$$

$$= e^x\left[x\cdot\frac{1}{D^2+D+2}\cos 2x + \left(-\frac{2D+1}{(D^2+D+2)^2}\right)\cdot\cos 2x\right]$$

$$= e^x\cdot\left[x\cdot\frac{1}{-4+D+2}\cos 2x + \text{---}\ \text{---}\right.$$?

Real part

$$\frac{1}{D^2+1}x\cdot e^x\cdot e^{i2x} = \frac{1}{D^2+1}\cdot x\cdot e^{(1+2i)x}$$

$$= e^{(1+2i)x}\cdot\frac{1}{(D+1+2i)^2+1}\cdot x$$

$$= e^{x(1+2i)}\left[1+(D+1+2i)^2\right]^{-1}\cdot x$$

?

$= e^{x(1+x)} \left[1 + (D+1+x) \right] \cdot a$ $= ?$

Important

$$\boxed{\frac{1}{D-a} Q = e^{ax} \int Q \cdot e^{-ax} \, dx}$$

$$\frac{1}{D^2-4D+3} e^x = \frac{1}{1-4+3} = \frac{1}{4-4} \rightarrow \text{Case of failure}$$

$$= x \cdot \frac{1}{2D-4} \cdot e^x = x \cdot \frac{1}{2-4} e^x = \frac{x \cdot e^x}{-2}$$

$$\downarrow$$

$$\frac{1}{(D-3)(D-1)} e^x$$