

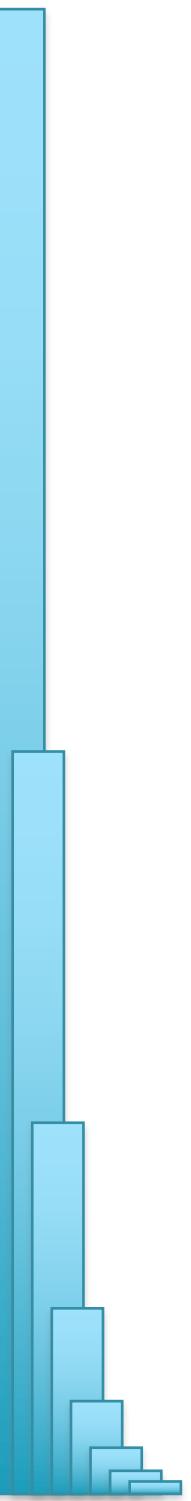
CS 600.226: Data Structures

Michael Schatz

Sept 19 2018
Lecture 9. Stacks



Agenda

- 
- 1. Review HW2***
 - 2. Recap on Sorting***
 - 3. Stacks***

Assignment 2: Due Friday Sept 21 @ 10pm

<https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md>

Assignment 2: Arrays of Doom!

Out on: September 14, 2018

Due by: September 21, 2018 before 10:00 pm

Collaboration: None

Grading:

Functionality 65%

ADT Solution 20%

Solution Design and README 5%

Style 10%

Overview

The second assignment is mostly about arrays, notably our own array specifications and implementations, not just the built-in Java arrays. Of course we also once again snuck a small ADT problem in there...

Note: The grading criteria now include **10% for programming style**.

Make sure you use [Checkstyle](#) with the correct configuration file from [Github](#)!

Assignment 2: Due Friday Sept 21 @ 10pm

<https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md>

Problem 1: Revenge of Unique (30%)

You wrote a small Java program called Unique for Assignment 1. The program accepted any number of command line arguments (each of which was supposed to be an integer) and printed each unique integer it received back out once, eliminating duplicates in the process.

For this problem, you will implement a new version of Unique called ***UniqueRevenge*** with two major changes:

- First, you are no longer allowed to use Java arrays (nor any other advanced data structure), but you can use our Array interface and our SimpleArray implementation from lecture (also available on github)
- Second, you're going to modify the program to read the integers from standard input instead of processing the command line.

Assignment 2: Due Friday Sept 21 @ 10pm

<https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md>

Problem 2: Flexible Arrays (20%)

Develop an algebraic specification for the abstract data type `FlexibleArray` which works like the existing `Array` ADT for the most part **except** that both its **lower** and its **upper** index bound are set when the array is created. The lower as well as upper bound can be **any** integer, provided the lower bound is **less than or equal** the upper bound.

Write up the specification for `FlexibleArray` in the format we used in lecture and **comment** on the design decisions you had to make. Also, tell us what kind of array **you** prefer and why.

Hints

- A `FlexibleArray` for which the lower bound equals the upper bound has exactly one slot.
- Your `FlexibleArray` is **not** the `Array` ADT we did in lecture; it doesn't have to support the exact same set of operations.

Assignment 2: Due Friday Sept 21 @ 10pm

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Problem 3: Sparse Arrays (35%)

A **sparse** array is an array in which **relatively few** positions have values that differ from the initial value set when the array was created. For sparse arrays, it is wasteful to store the value of **all** positions explicitly since **most of them never change** and take the default value of the array. Instead, we want to store positions that **have actually been changed**.

For this problem, write a class `SparseArray` that implements the `Array` interface we developed in lecture (the same interface you used for Problem 1 above). **Do not modify the Array interface in any way!** Instead of using a plain Java array like we did for `SimpleArray`, your `SparseArray` should use a **linked list** of `Node` objects to store values, similar to the `ListArray` from lecture (and available in [github](#)). However, your nodes no longer store just the **data** at a certain position, they also store **the position itself!**

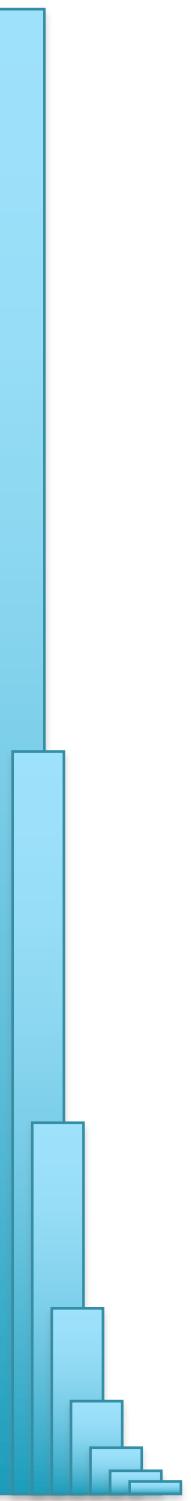
Introduction to Checkstyle

<http://checkstyle.sourceforge.net/>

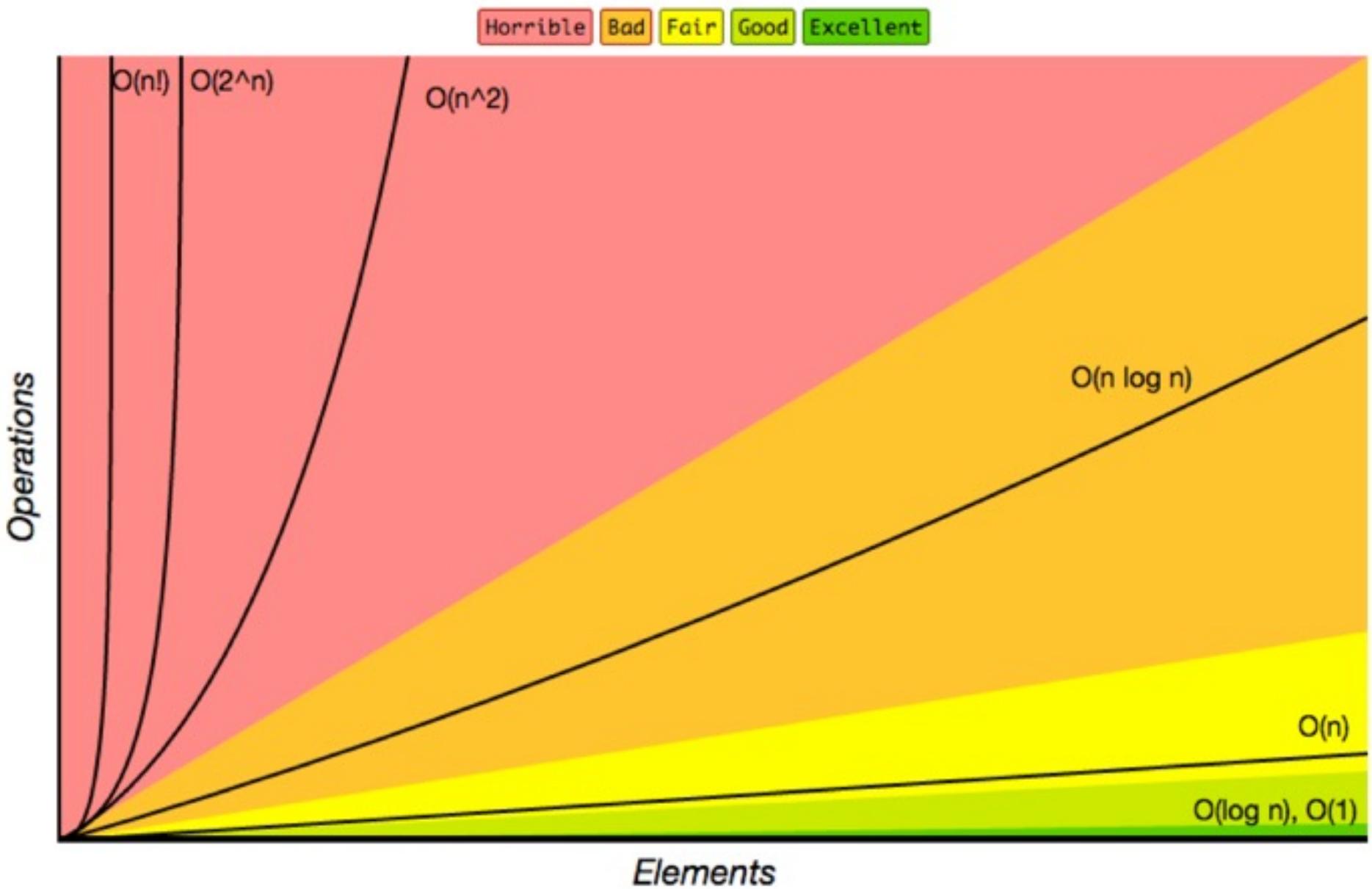
```
2. bash
mschatz@schatzmac:23:11:48:~/Dropbox/Documents/Teaching/2016/JHU/DataStructures/Lectures/02.Practicals $ java -jar checkstyle-6.15-all.jar -c cs226_checks.xml HelloWorld.java
Starting audit...
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:1: Missing a Javadoc comment. [JavadocType]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:1:1: Utility classes should not have a public or default constructor. [HideUtilityClassConstructor]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:2:1: '{' at column 1 should be on the previous line. [LeftCurly]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:3: 'method def modifier' have incorrect indentation level 2, expected level should be 4. [Indentation]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:3:3: Missing a Javadoc comment. [JavadocMethod]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:3:33: 'String' is followed by whitespace. [NoWhitespaceAfter]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:4: 'method def curly' have incorrect indentation level 2, expected level should be 4. [Indentation]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:4:3: '{' at column 3 should be on the previous line. [LeftCurly]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:5: 'method call' child have incorrect indentation level 4, expected level should be 8. [Indentation]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:5: 'method def' child have incorrect indentation level 4, expected level should be 8. [Indentation]
[ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:6: 'method def curly' have incorrect indentation level 2, expected level should be 4. [Indentation]
Audit done.
Checkstyle ends with 11 errors.
mschatz@schatzmac:23:11:52:~/Dropbox/Documents/Teaching/2016/JHU/DataStructures/Lectures/02.Practicals $
```

```
$ java -jar datastructures2018/resources/checkstyle-8.12-all.jar \
-c datastructures2018/resources/cs226_checks.xml HelloWorld.java
```

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Growth of functions



Trying every permutation

```
public static void main(String [] args) {  
    if (args.length == 0) {  
        System.out.println("Permute num");  
        return;  
    }  
  
    int len = Integer.parseInt(args[0]);  
    int [] keys = new int[len];  
    for (int i = 0; i < len; i++) { keys[i] = i+1; }  
    permute(keys, 0, len-1);  
    System.err.println("There are " + numtries  
                      + " permutations of " + len + " items.");  
}  
}
```

```
$ for i in `seq 1 20`;  
    do echo $i; java Permute $i > $i.log ; done  
1  
There are 1 permutations of 1 items.  
2  
There are 2 permutations of 2 items.  
3  
There are 6 permutations of 3 items.  
4  
There are 24 permutations of 4 items.  
5  
There are 120 permutations of 5 items.
```

Why Sort?

Sorting is very powerful to organize large amounts of data
Becomes a core routine in many data structures

When data are sorted you can do **binary search!**

- I'm thinking of a number between 1 and 1,000,000
- **How many hi/lo guesses will it take to figure it out?**

$$\lg(1,000,000) = 20$$

How many hi/lo guesses to find my special number?

26 05 38 28 93 81 71 15 96 33 99 13 58 96 09

Same Data, Sorted Order

05 09 13 15 26 28 33 38 58 71 81 93 96 96 99

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26 05 38 28 93 81 71 15 96 33 99 13 58 96 09

Same Data, Sorted Order

05	09	13	15	26	28	33	38	58	71	81	93	96	96	99
----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

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----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

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Same Data, Sorted Order

05	09	13	15	26	28	33	38	58	71	81	93	96	96	99
----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

Selection Sort

Quickly sort these numbers into ascending order:

14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

Find the minimum

14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

O(N)

Flip it into the right place

6, 29, 14, 31, 39, 64, 78, 50, 13, 63, 61, 19

O(1)

Find the next smallest

6, 29, 14, 31, 39, 64, 78, 50, 13, 63, 61, 19

O(N-1)

Flip it into the right place

6, 13, 14, 31, 39, 64, 78, 50, 29, 63, 61, 19

O(1)

Find the next smallest

6, 13, 14, 31, 39, 64, 78, 50, 29, 63, 61, 19

O(N-2)

Flip it into the right place

6, 13, 14, 31, 39, 64, 78, 50, 29, 63, 61, 19

O(1)

...

Selection Sort Analysis

```
public static void selectionSort(int[] a) {  
    for (int i = 0; i < a.length - 1; i++) {  
        int min = i;  
        for (int j = i + 1; j < a.length; j++) {  
            if (a[j] < a[min]) {  
                min = j;  
            }  
        }  
        int t = a[i]; a[i] = a[min]; a[min] = t;  
    }  
}
```

Analysis

- Outer loop: $i = 0$ to n
- Inner loop: $j = i$ to n
- Total Running time: $\text{Outer} * \text{Inner} = O(n^2)$

$$T = n + (n - 1) + (n - 2) + \dots + 3 + 2 + 1 = \sum_{i=1}^n i = \frac{n(n + 1)}{2} = O(n^2)$$

Selection Sort Analysis

```
public static void selectionSort(int[] a) {  
    for (int i = 0; i < a.length - 1; i++) {  
        int min = i;  
        for (int j = i + 1; j < a.length; j++) {  
            if (a[j] < a[min]) {  
                min = j;  
            }  
        }  
        int t = a[i]; a[i] = a[min]; a[min] = t;  
    }  
}
```

Formally you would prove the recurrence using induction: Discrete Math class!

Requires almost no extra space: In-place algorithm

Analysis

- Outer loop: $i = 0$ to n
- Inner loop: $j = i$ to n
- Total Running time: Outer * Inner = $O(n^2)$

“Feels slow” since inner loop only seeks out one number (the next biggest)

$$T = n + (n - 1) + (n - 2) + \dots + 3 + 2 + 1 = \sum_{i=1}^n i = \frac{n(n + 1)}{2} = O(n^2)$$

Bubble sort

Sort these values by bubbling up the next largest value

14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

[14, 29], 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

[14, 29], 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

14, [29, 6], 31, 39, 64, 78, 50, 13, 63, 61, 19

14, [6, 29], 31, 39, 64, 78, 50, 13, 63, 61, 19

14, 6, [29, 31], 39, 64, 78, 50, 13, 63, 61, 19

14, 6, [29, 31], 39, 64, 78, 50, 13, 63, 61, 19

14, 6, 29, [31, 39], 64, 78, 50, 13, 63, 61, 19

14, 6, 29, [31, 39], 64, 78, 50, 13, 63, 61, 19

14, 6, 29, 31, [39, 64], 78, 50, 13, 63, 61, 19

14, 6, 29, 31, [39, 64], 78, 50, 13, 63, 61, 19

14, 6, 29, 31, 39, [64, 78], 50, 13, 63, 61, 19

14, 6, 29, 31, 39, [64, 78], 50, 13, 63, 61, 19

14, 6, 29, 31, 39, 64, [78, 50], 13, 63, 61, 19

14, 6, 29, 31, 39, 64, [50, 78], 13, 63, 61, 19

14, 6, 29, 31, 39, 64, 50, [78, 13], 63, 61, 19

14, 6, 29, 31, 39, 64, 50, [13, 78], 63, 61, 19

14, 6, 29, 31, 39, 64, 50, 13, [78, 63], 61, 19

14, 6, 29, 31, 39, 64, 50, 13, [63, 78], 61, 19

14, 6, 29, 31, 39, 64, 50, 13, 63, [78, 61], 19

14, 6, 29, 31, 39, 64, 50, 13, 63, [61, 78], 19

14, 6, 29, 31, 39, 64, 50, 13, 63, 61, [78, 19]

14, 6, 29, 31, 39, 64, 50, 13, 63, 61, [19, 78]

14, 6, 29, 31, 39, 64, 50, 13, 63, 61, 19, 78

On the first pass, sweep list
to bubble up the largest element
(also move smaller items down)

Bubble sort

Sort these values by bubbling up the next largest value

14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

[14, 6], 29, 31, 39, 64, 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, 13, [64, 63], 61, 19, 78

[6, 14], 29, 31, 39, 64, 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, 13, [63, 64], 61, 19, 78

6, [14, 29], 31, 39, 64, 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, 13, 63, [64, 61], 19, 78

6, [14, 29], 31, 39, 64, 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, 13, 63, [61, 64], 19, 78

6, 14, [29, 31], 39, 64, 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, 13, 63, 61, [64, 19], 78

6, 14, [29, 31], 39, 64, 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, 13, 63, 61, [19, 64], 78

6, 14, 29, [31, 39], 64, 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, 13, 63, 61, 19, 64, 78

6, 14, 29, [31, 39], 64, 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, 13, 63, 61, 19, 64, 78

6, 14, 29, 31, [39, 64], 50, 13, 63, 61, 19, 78

On the second pass, sweep list
to bubble up the second largest element
(also move smaller items down)

6, 14, 29, 31, [39, 64], 50, 13, 63, 61, 19, 78

6, 14, 29, 31, 39, [64, 50], 13, 63, 61, 19, 78

6, 14, 29, 31, 39, [50, 64], 13, 63, 61, 19, 78

6, 14, 29, 31, 39, 50, [64, 13], 63, 61, 19, 78

6, 14, 29, 31, 39, 50, [13, 64], 63, 61, 19, 78

Bubble sort

Sort these values by bubbling up the next largest value

14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

[6, 14], 29, 31, 39, 50, 13, 63, 61, 19, 64, 78
6, [14, 29], 31, 39, 50, 13, 63, 61, 19, 64, 78
6, 14, [29, 31], 39, 50, 13, 63, 61, 19, 64, 78
6, 14, 29, [31, 39], 50, 13, 63, 61, 19, 64, 78
6, 14, 29, 31, [39, 50], 13, 63, 61, 19, 64, 78
6, 14, 29, 31, 39, [50, 13], 63, 61, 19, 64, 78
6, 14, 29, 31, 39, [13, 50], 63, 61, 19, 64, 78
6, 14, 29, 31, 39, 13, [50, 63], 61, 19, 64, 78
6, 14, 29, 31, 39, 13, 50, [63, 61], 19, 64, 78
6, 14, 29, 31, 39, 13, 50, [61, 63], 19, 64, 78
6, 14, 29, 31, 39, 13, 50, 61, [63, 19], 64, 78
6, 14, 29, 31, 39, 13, 50, 61, [19, 63], 64, 78
6, 14, 29, 31, 39, 13, 50, 61, 19, 63, 64, 78

On the third pass, sweep list
to bubble up the third largest element

How many passes will we need to do?

$O(n)$

How much work does each pass take?

$O(n)$

What is the total amount of work?

n passes, each requiring $O(n) \Rightarrow O(n^2)$

**Note, you might get lucky and finish
much sooner than this**

**“Feels faster”: multiple swaps on
the inner loop, but...**

**“Feels slow” because inner loop
sweeps entire list to move one
number into sorted position**

Insertion Sort

Quickly sort these numbers into ascending order:

14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19	To be sorted
14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19	
6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19	
6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19	
6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19	
6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19	
6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19	
6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19	
6, 14, 29, 31, 39, 50, 64, 78, 13, 63, 61, 19	
6, 13, 14, 29, 31, 39, 50, 64, 78, 63, 61, 19	
6, 13, 14, 29, 31, 39, 50, 63, 64, 78, 61, 19	
6, 13, 14, 29, 31, 39, 50, 61, 63, 64, 78, 19	
6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78	

Sorted elements

Base Case: Declare the first element as a correctly sorted array

Repeat: Iteratively add the next unsorted element to the partially sorted array at the correct position

Slide the unsorted element into the correct position:

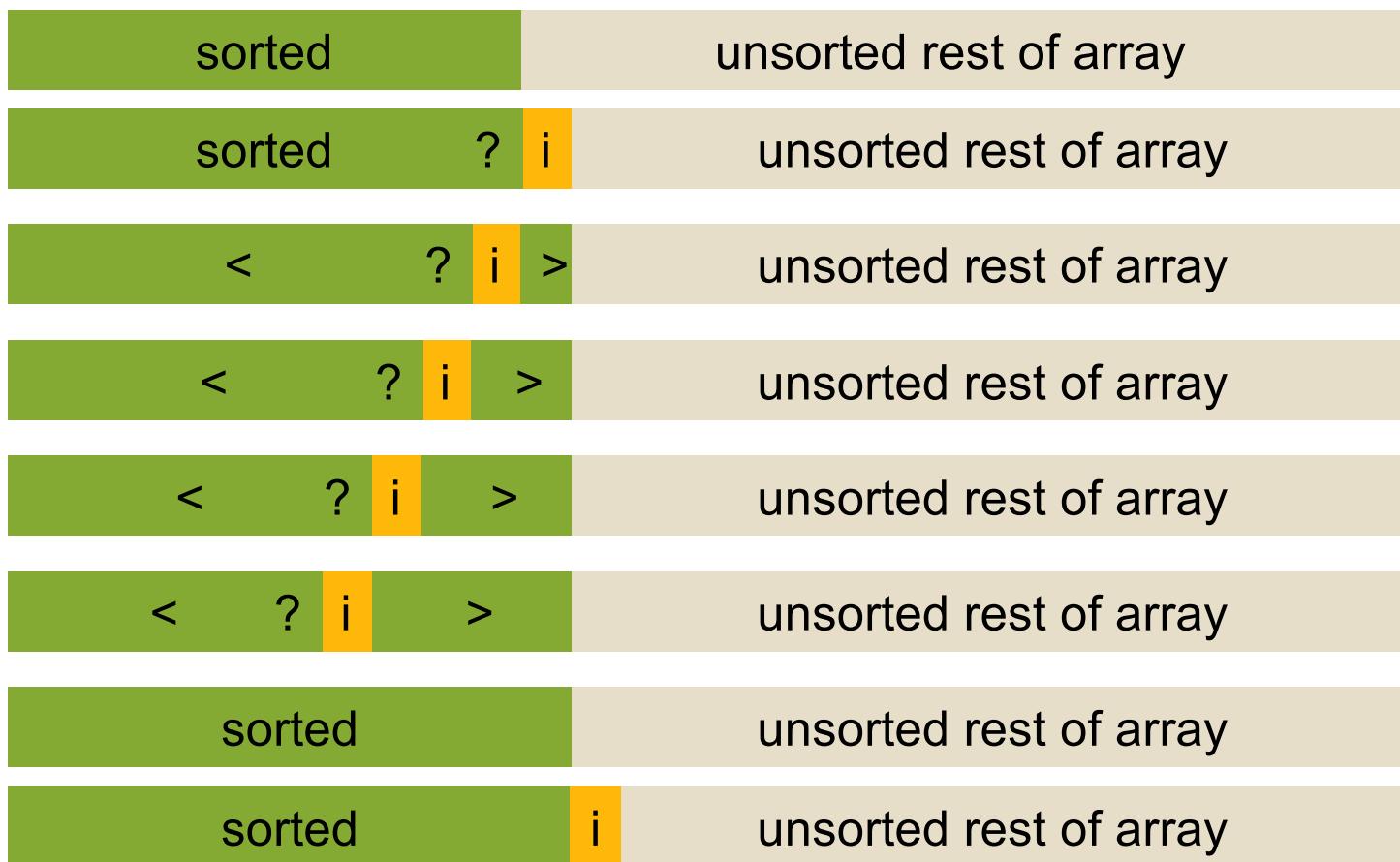
14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19
14, 6, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19
6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19

“Feels fast” because you always have a partially sorted list, but some insertions will be expensive

Insertion Sort

Quickly sort these numbers into ascending order:

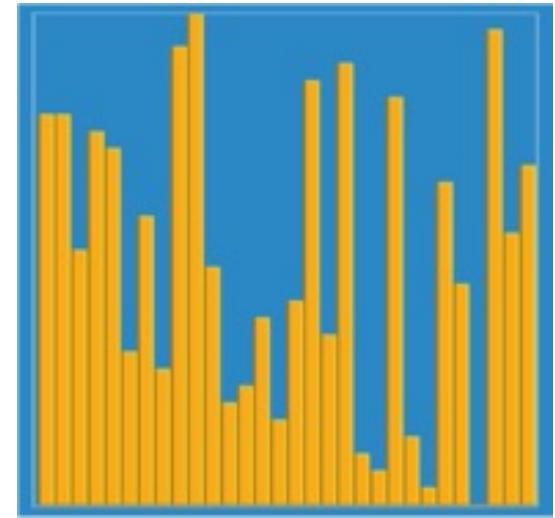
14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19



Outer loop: n elements to move into correct position
Inner loop: $O(n)$ work to move element into correct position

Total Work: $O(n^2)$

Quadratic Sorting Algorithms



Selection Sort

Move next smallest
into position

Bubble Sort

Swap up bigger
values over smaller

Insertion Sort

Slide next value into
correct position

*Asymptotically all three have the same performance
... but can perform significantly different on some datasets*

Sorting Race!

Problem 2: All Sorts of Sorts (50%)

Your second task for this assignment is to explore some of the basic sorting algorithms and their analysis. All of these algorithms are quadratic in terms of their asymptotic performance, but they nevertheless differ in their actual performance. We'll focus on the following three algorithms:

- Bubble Sort (with the "stop early if no swaps" extension)
- Selection Sort
- Insertion Sort

The archive for this assignment contains a basic framework for evaluating sorting algorithms. You'll need a working `StataitableArray` class from Problem 1, and you'll need to understand the following interface as well (again compressed; be sure to to use and read the full interface available on [Piazza](#)):

```
public interface SortingAlgorithm<T extends Comparable<T>> {  
    void sort(Array<T> array);  
    String name();  
}
```

Let's look at the simple stuff first: An object is considered an algorithm suitable for sorting in this framework if (a) we can ask it to sort a given `Array` and (b) we can ask it for its name (e.g. "Insertion Sort").

The more complicated stuff is at the top: The use of `extends` inside the angle brackets means that any type `T` we want to sort must implement the interface `Comparable` as well. It obviously can't just be any old type, it must be a type for which the expression "`a` is less than `b`" actually makes sense. Using `Comparable` in this form is Java's way of saying that we can order the objects; you should definitely read up on the details [here](#).

As an example for all this, we have provided an implementation of `SelectionSort` on [Piazza](#) already. (Actually, there are also two other algorithms, `NullSort` and `GnomeSort`, just so you start out with a few to run right away.)

You need to write classes implementing `BubbleSort` and `InsertionSort` for this problem. Just like our example algorithms, your classes have to implement the `SortingAlgorithm` interface.

All of this should be fairly straightforward once you get used to the framework. Speaking of the framework, the way you actually "run" the various algorithms is by using the `PolySort.java` program we've provided as well. You should be able to compile and run it without yet having written any sorting code yourself. Here's how:

```
$ java PolySort 4000 <random.data  
Algorithm      Sorted?  Size     Reads    Writes   Seconds  
Null Sort      false    4,000     0        0       0.000007  
Gnome Sort     true     4,000    32,195,307  8,045,828  0.243852  
Selection Sort  true     4,000    24,009,991  7,992    0.252085
```

This will read the first 4000 strings from the file `random.data` and sort them using all available algorithms. As you can see, the program checks if the algorithm actually worked (`Sorted?`) and reports how many operations of the underlying `StataitableArray` were used in order to perform the sort (`Reads`, `Writes`). Finally, the program also prints out long it took to sort the array (`Seconds`) but that number will vary widely across machines so you can really only use it for relative comparisons on the machine actually running the experiment.

However, the main point of all this is not the coding work. Instead, the main point is to evaluate and compare the sorting algorithms on different sets of data. We've provided three sets of useful test data on [Piazza](#) and you can use the command line argument to vary how much of it is used (thereby changing the size of the problem). You should try to quantify how the various algorithms differ and explain why they differ as well (i.e. what about a given algorithm makes it better or worse than another one for a given data set). In your `README` file you should describe the series of experiments you ran, what data you collected, and what your conclusions about the performance of these algorithms are. Some ideas for what to address:

- Does the actual running time correspond to the asymptotic complexity as you would expect?
- What explains the practical differences between these algorithms?
- Does it matter what kind of data (random, already sorted in ascending order, sorted in descending order) you are sorting?

Just to be clear: Yes, we'll need the code, and it should be up to the usual standards. But the "report" you put in your `README` is just as important as the code!

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    void sort(Array<T> array);  
    String name();  
}
```

Do you think quadratic sort is the best you can do?

Let's look at the simple stuff first: An object is considered an algorithm suitable for sorting in this framework if (a) we can ask it to

array and (b) we can ask it for its name (e.g. "Insertion Sort").

The more complicated stuff is at the top: The use of `extends` inside the angle brackets means that any type `T` we want to sort in `b` actually makes sense. Using `Comparable` in this form is Java's way of saying that we can order the objects; you should do

the interface `Comparable` as well. It obviously can't just be any old type, it must be a type for which the expression "`a` is less than `b`" makes sense. See the interface `Comparable` for details.

;-)

As an example for all this, we have provided an implementation of `SelectionSort` on [Piazza](#) already. (Actually, there are also two other algorithms, `NullSort` and `GnomeSort`, just so you start out with a few to run right away.)

You need to write classes implementing `BubbleSort` and `InsertionSort` for this problem. Just like our example algorithms, your classes have to implement the `SortingAlgorithm` interface.

All of this should be fairly straightforward once you get used to the framework. Speaking of the framework, the way you actually "run" the various algorithms is by using the `PolySort.java` program we've provided as well. You should be able to compile and run it without yet having written any sorting code yourself. Here's how:

```
$ java PolySort 4000 <randon.data  
Algorithm      Sorted?  Size     Reads    Writes   Seconds  
Null Sort      false    4,000     0        0       0.000007  
Gnome Sort     true     4,000    32,195,307  8,045,828  0.243852  
Selection Sort  true     4,000    24,009,991  7,992    0.252085
```

This will read the first 4000 strings from the file `randon.data` and sort them using all available algorithms. As you can see, the program checks if the algorithm actually worked (`Sorted?`) and reports how many operations of the underlying `StataitableArray` were used in order to perform the sort (`Reads`, `Writes`). Finally, the program also prints out long it took to sort the array (`Seconds`) but that number will vary widely across machines so you can really only use it for relative comparisons on the machine actually running the experiment.

However, the main point of all this is not the coding work. Instead, the main point is to evaluate and compare the sorting algorithms on different sets of data. We've provided three sets of useful test data on [Piazza](#) and you can use the command line argument to vary how much of it is used (thereby changing the size of the problem). You should try to quantify how the various algorithms differ and explain why they differ as well (i.e. what about a given algorithm makes it better or worse than another one for a given data set). In your `README` file you should describe the series of experiments you ran, what data you collected, and what your conclusions about the performance of these algorithms are. Some ideas for what to address:

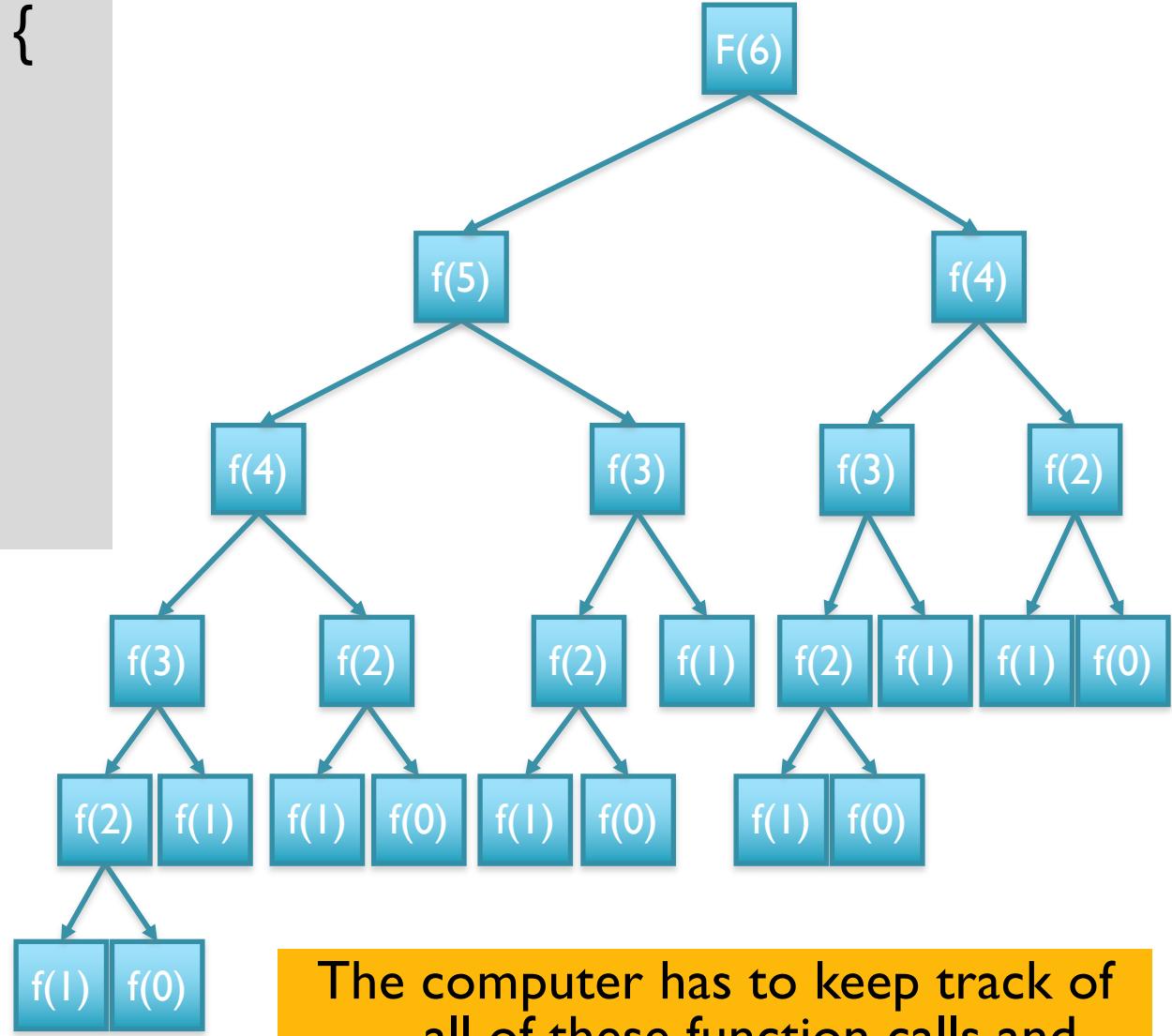
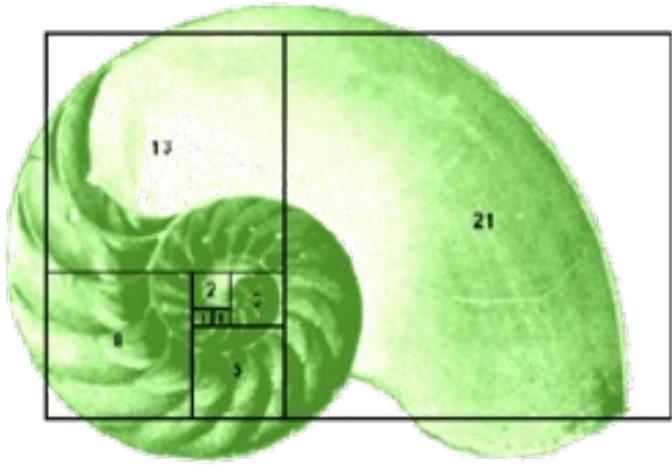
- Does the actual running time correspond to the asymptotic complexity as you would expect?
- What explains the practical differences between these algorithms?
- Does it matter what kind of data (random, already sorted in ascending order, sorted in descending order) you are sorting?

Just to be clear: Yes, we'll need the code, and it should be up to the usual standards. But the "report" you put in your `README` is just as important as the code!

Part 3: Stacks

Fibonacci Sequence

```
public static int fib(int n) {  
    if (n <= 1) {  
        return 1;  
    }  
    return fib(n-1) +  
        fib(n-2);  
}
```



The computer has to keep track of all of these function calls and keep everything in order!

Introducing the call stack

```
100:    int f(int a) {  
101:        int y = 42;  
102:        int z = 13;  
103:        ...  
400:        int x = g(13 + a);  
104:        ...  
497:        return x + y + z;  
498:    }  
  
500:    int g(double a) {  
501:        return h(a*2);  
502:    }  
  
504:    int h(double x) {  
505:        int k = (int) x;  
506:        return k+1;  
507:    }  
  
509:    static public void main() {  
510:        int val = f(15);  
511:        System.out.println(val);  
512:    }
```

h

Return val to g line 501
x=56
k=56
val = 57

g

Return val to f line 400
a=28
val = <h(56)>

f

Return val to main line 510
a = 15
y = 42
z = 13
x = <g(28)>

main

val = <f(15)>

Introducing the call stack

The **call stack** keeps track of the local variables and return location for the current function. This makes it possible for program execution to jump from function to function without loosing track of where the program should resume

A **stack frame** records the information for each function call, with local variables and the address of where to resume processing after this function is complete.

=> **Take a computer architecture course for more info**

Importantly the computer only needs to **add or remove items from the very top of the stack**, making it easy for the computer to keep track of where to go next!

More generally, stacks are a very useful data structure for **Last-In-First-Out (LIFO)** processing

h

Return val to g line 501
x=56
k=56
val = 57

g

Return val to f line 400
a=28
val = <h(56)>

f

Return val to main line 510
a = 15
y = 42
z = 13
x = <g(28)>

main

val = <f(15)>

Stacks

Stacks are very simple but surprisingly useful data structures for storing a collection of information

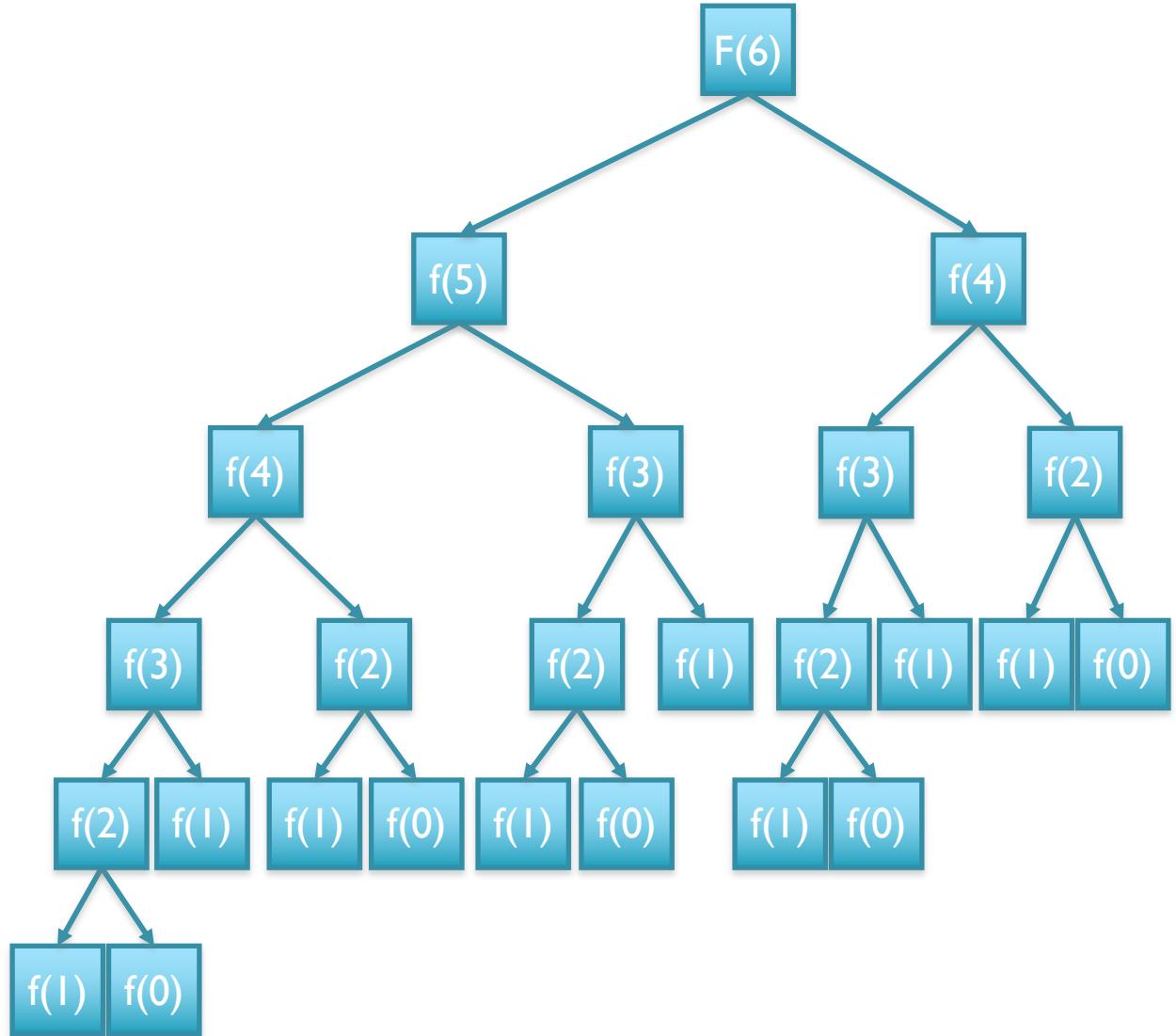
- Any number of items can be stored, but you can only manipulate the top of the stack:
 - **Push:** adds a new element to the top
 - **Pop:** takes off the top element
 - **Top:** Lets you peek at top element's value without removing it from stack

Many Applications

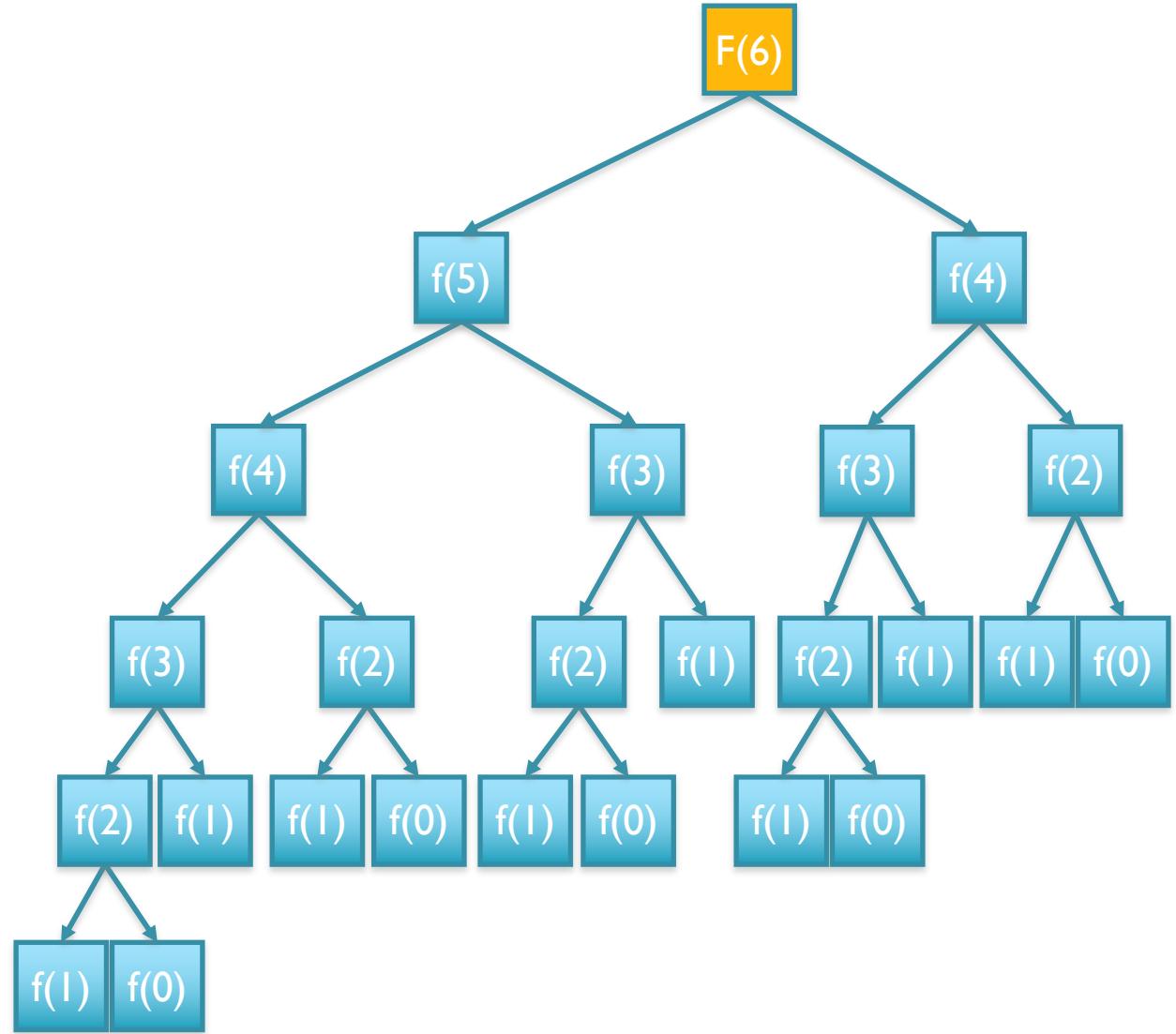
- In hardware call stack
- Memory management systems
- Parsing arithmetic instructions:
 $((x+3) / (x+9)) * (42 * \sin(x))$
- Back-tracing, such as searching within a maze



Fibonacci Sequence

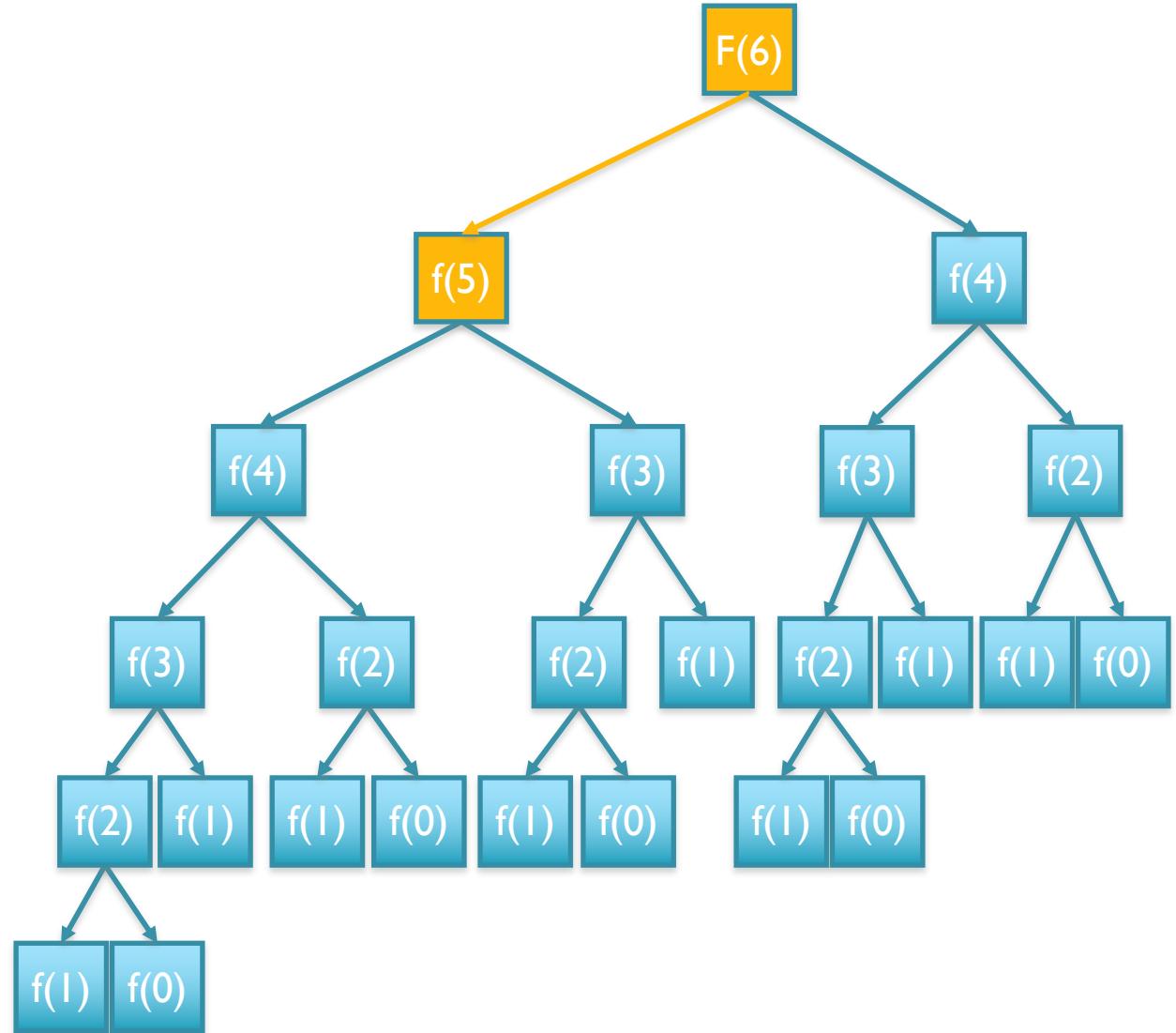


Fibonacci Sequence



$F(6)$
 $a = F(5)$

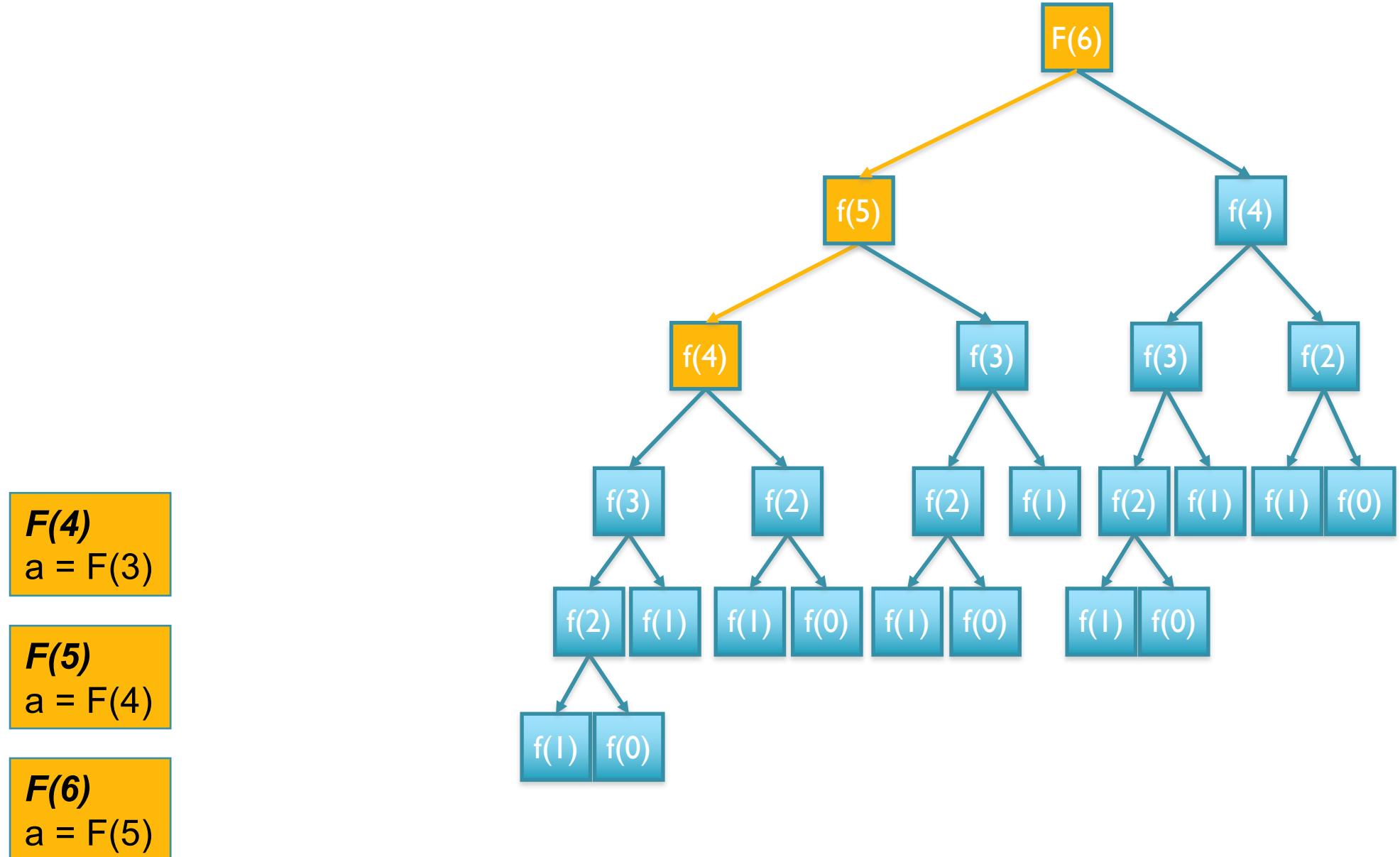
Fibonacci Sequence



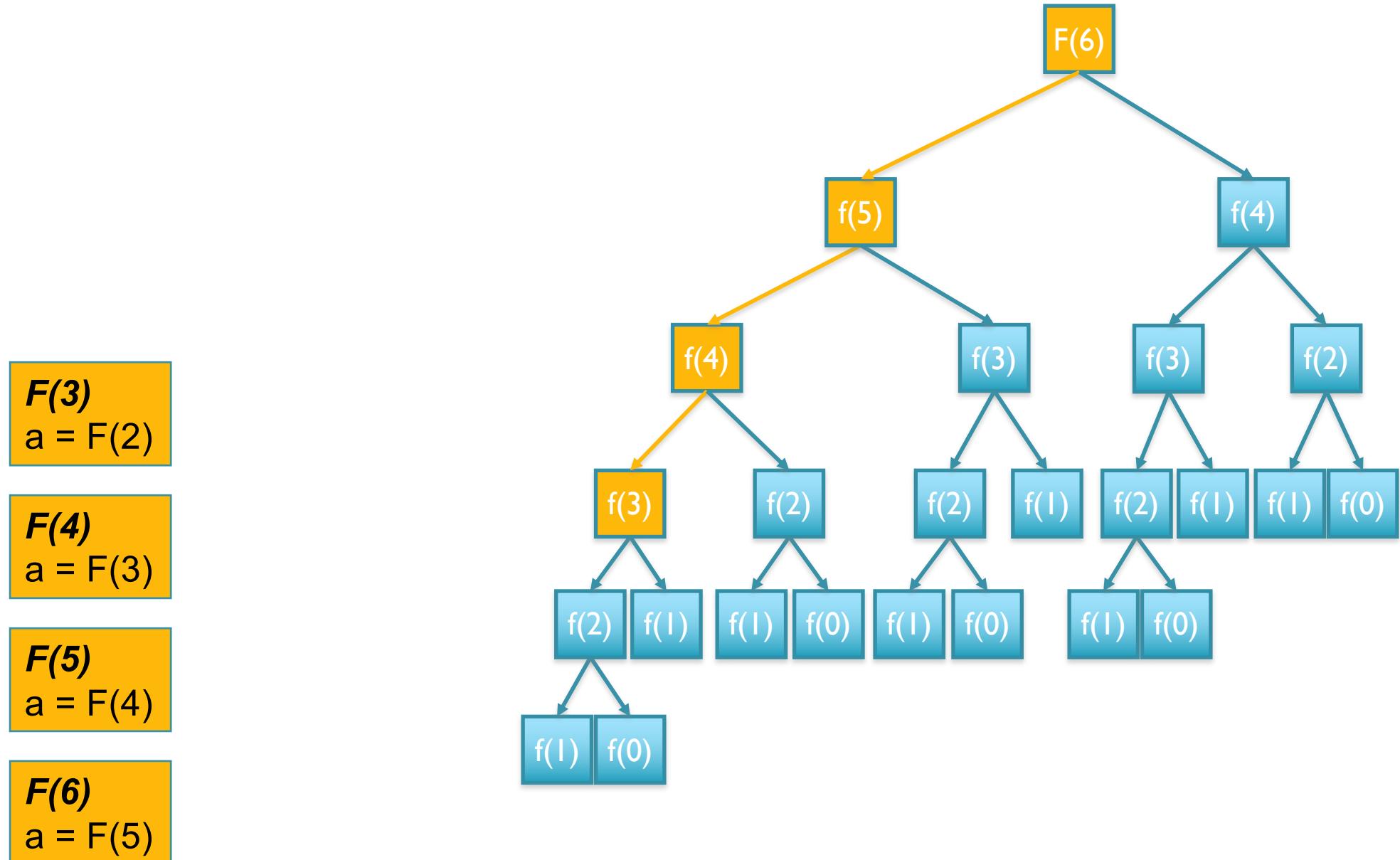
$$F(5) \\ a = F(4)$$

$$F(6) \\ a = F(5)$$

Fibonacci Sequence



Fibonacci Sequence



Fibonacci Sequence

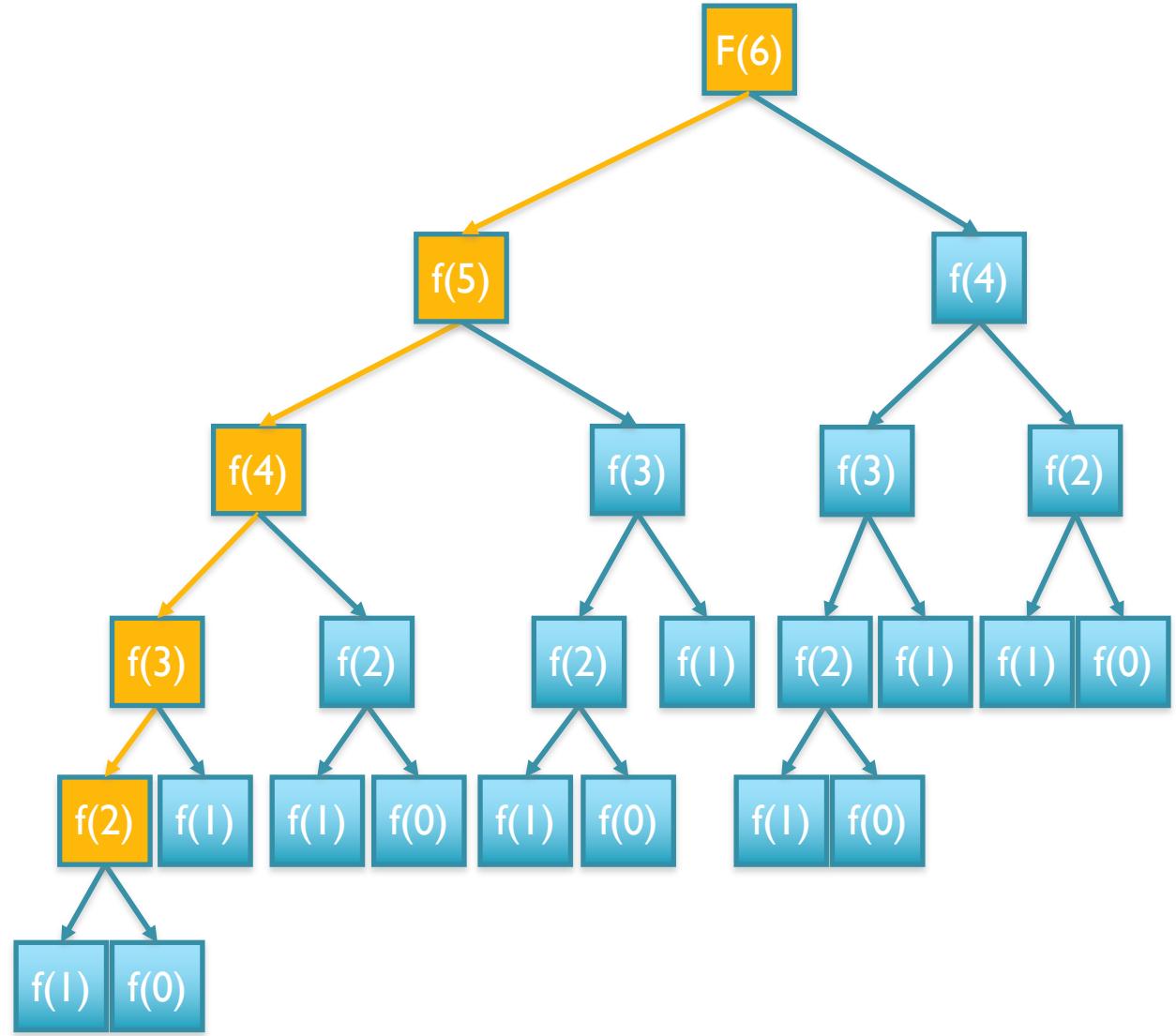
$F(2)$
 $a = F(1)$

$F(3)$
 $a = F(2)$

$F(4)$
 $a = F(3)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Fibonacci Sequence

$F(1)$
return 1

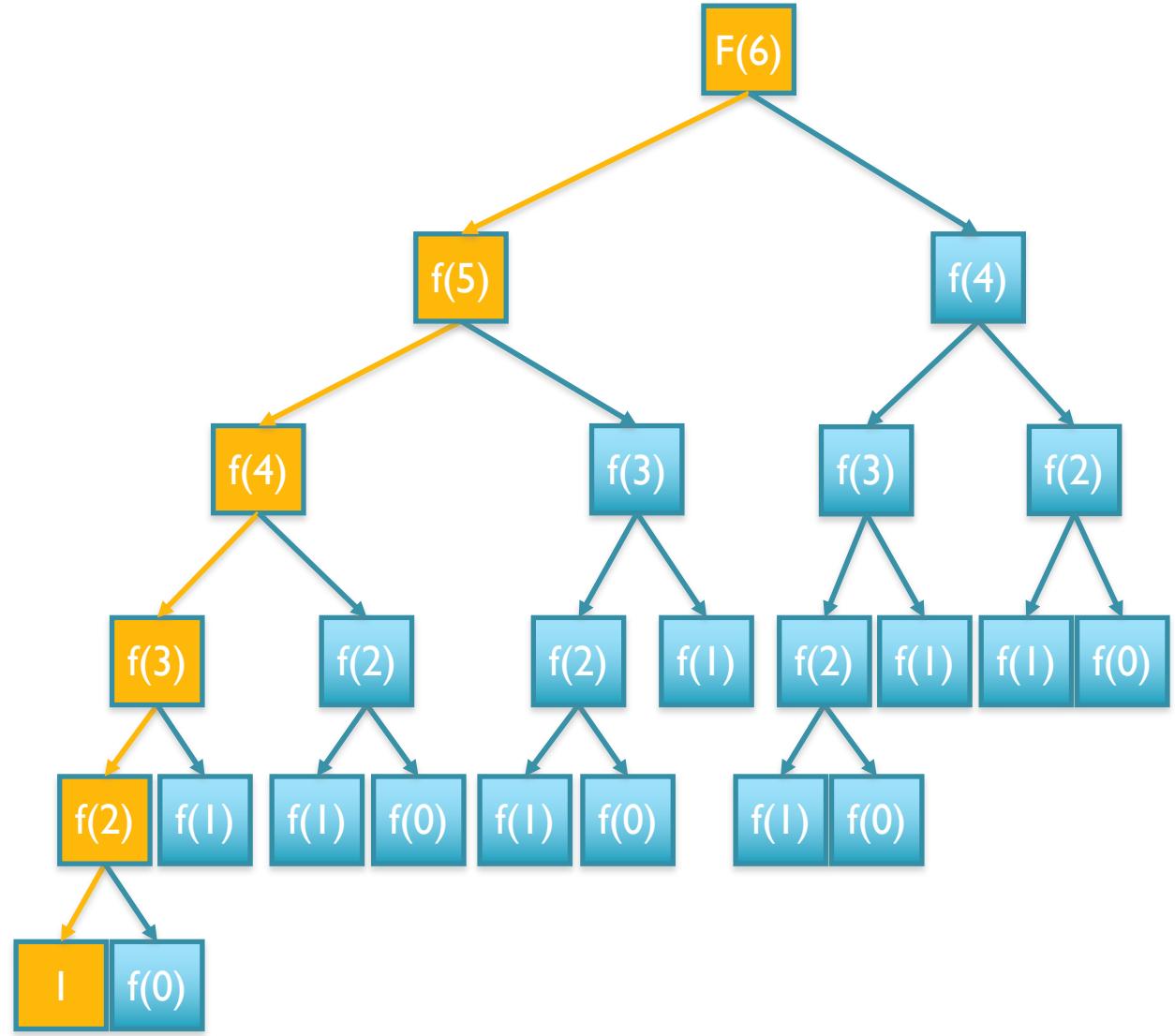
$F(2)$
 $a = F(1)$

$F(3)$
 $a = F(2)$

$F(4)$
 $a = F(3)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Fibonacci Sequence

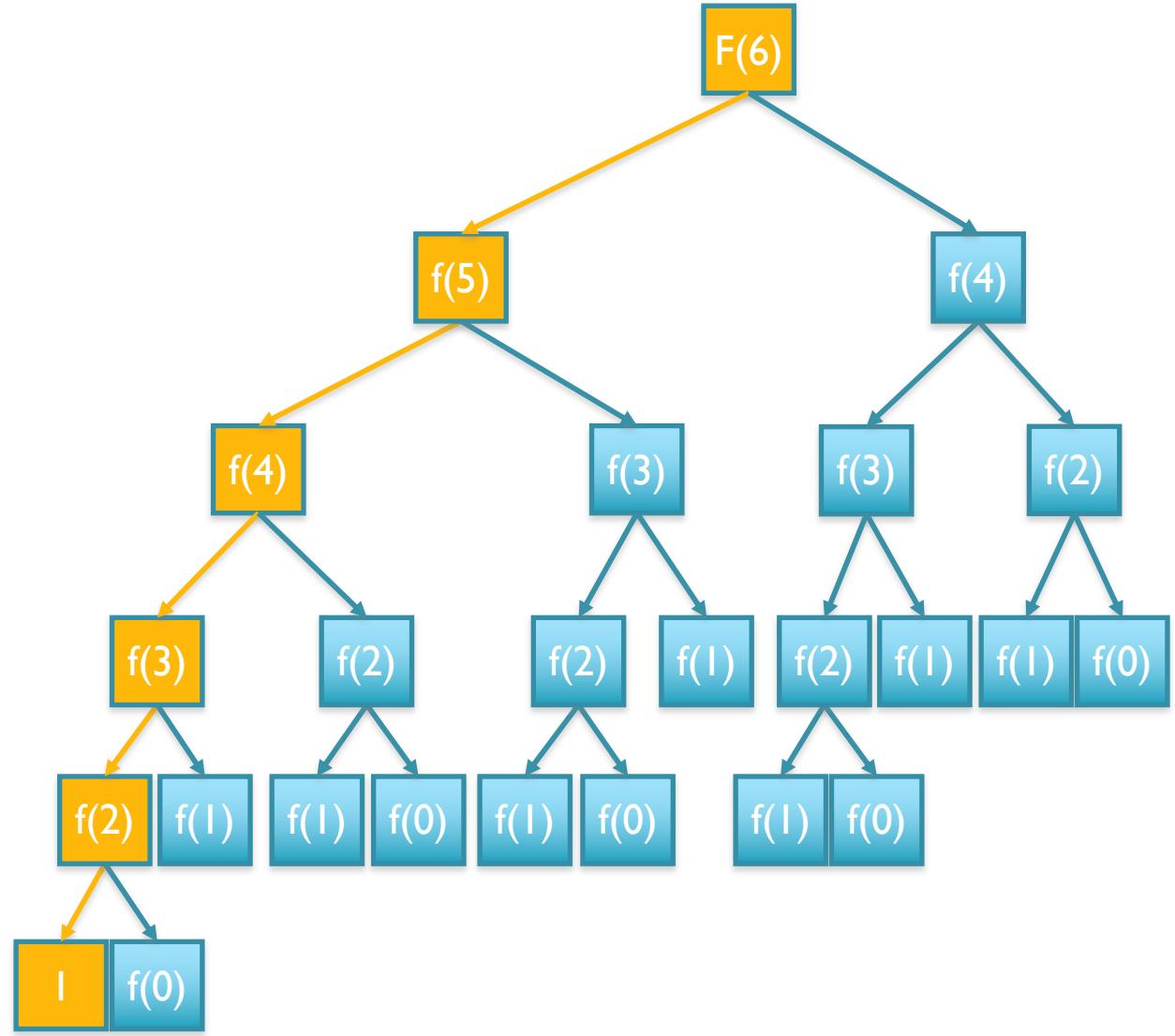
$F(2)$
 $b = F(0)$

$F(3)$
 $a = F(2)$

$F(4)$
 $a = F(3)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Fibonacci Sequence

$F(0)$
return 1

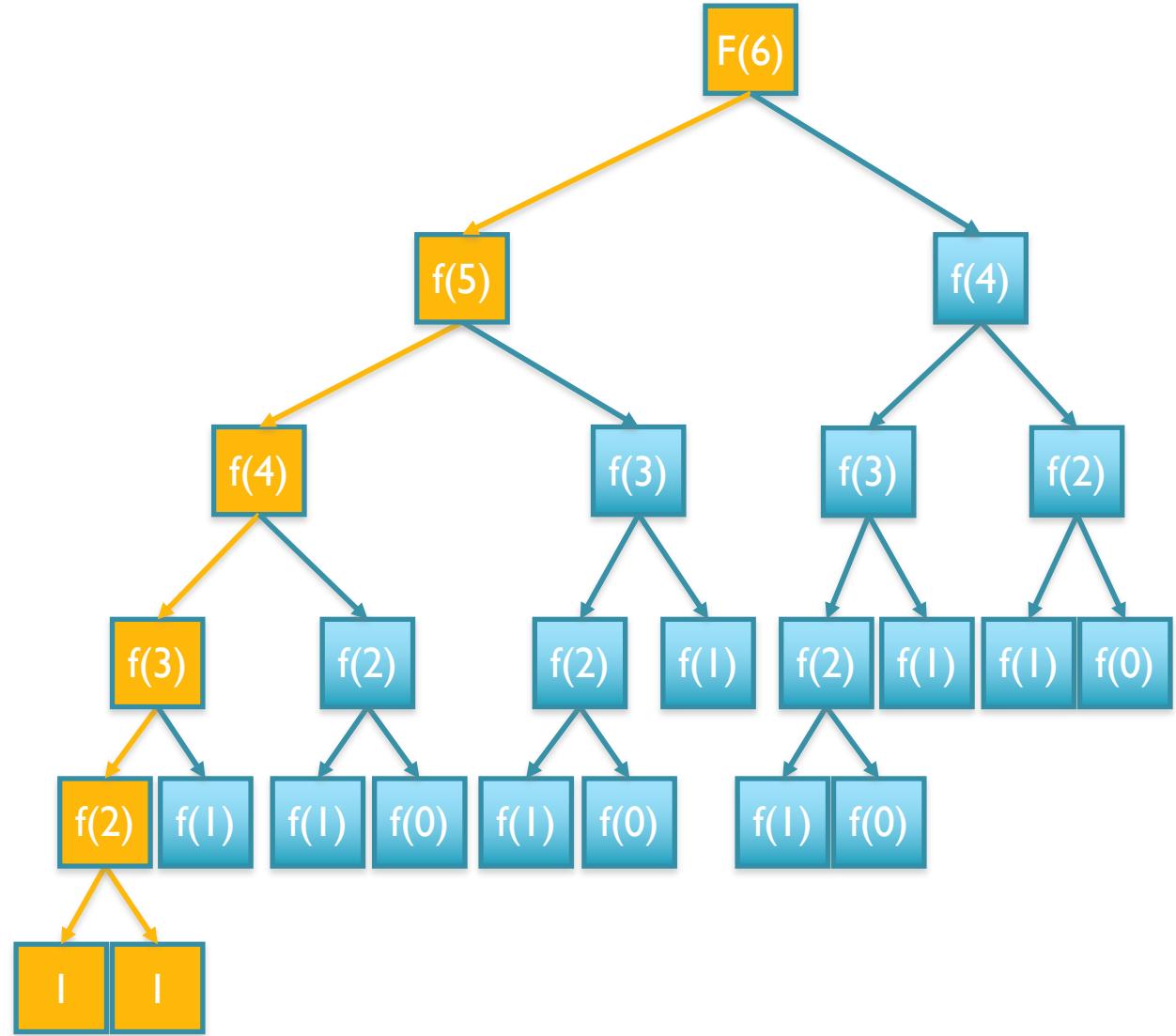
$F(2)$
 $b = F(0)$

$F(3)$
 $a = F(2)$

$F(4)$
 $a = F(3)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Fibonacci Sequence

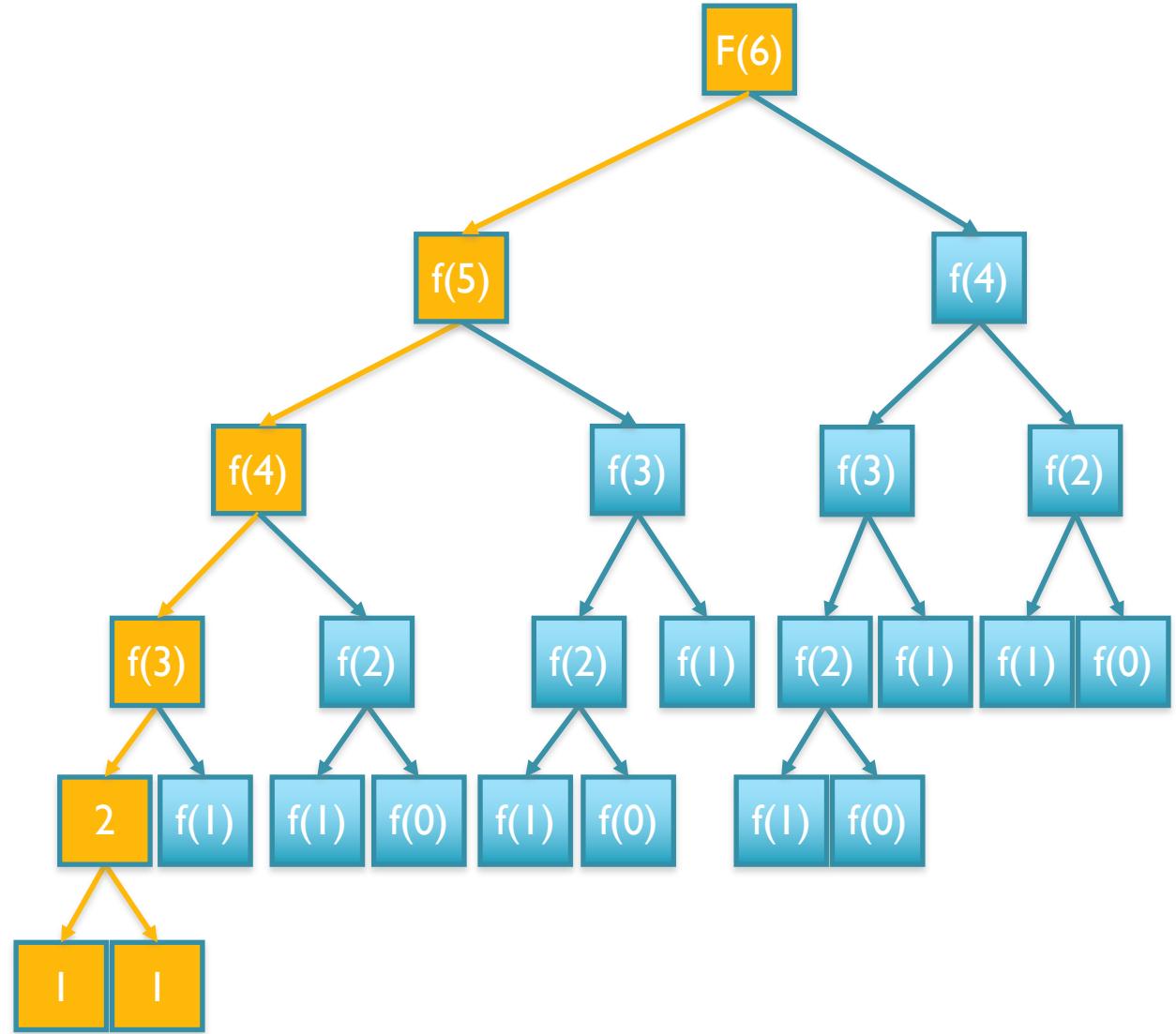
$F(2)$
return 2

$F(3)$
 $a = F(2)$

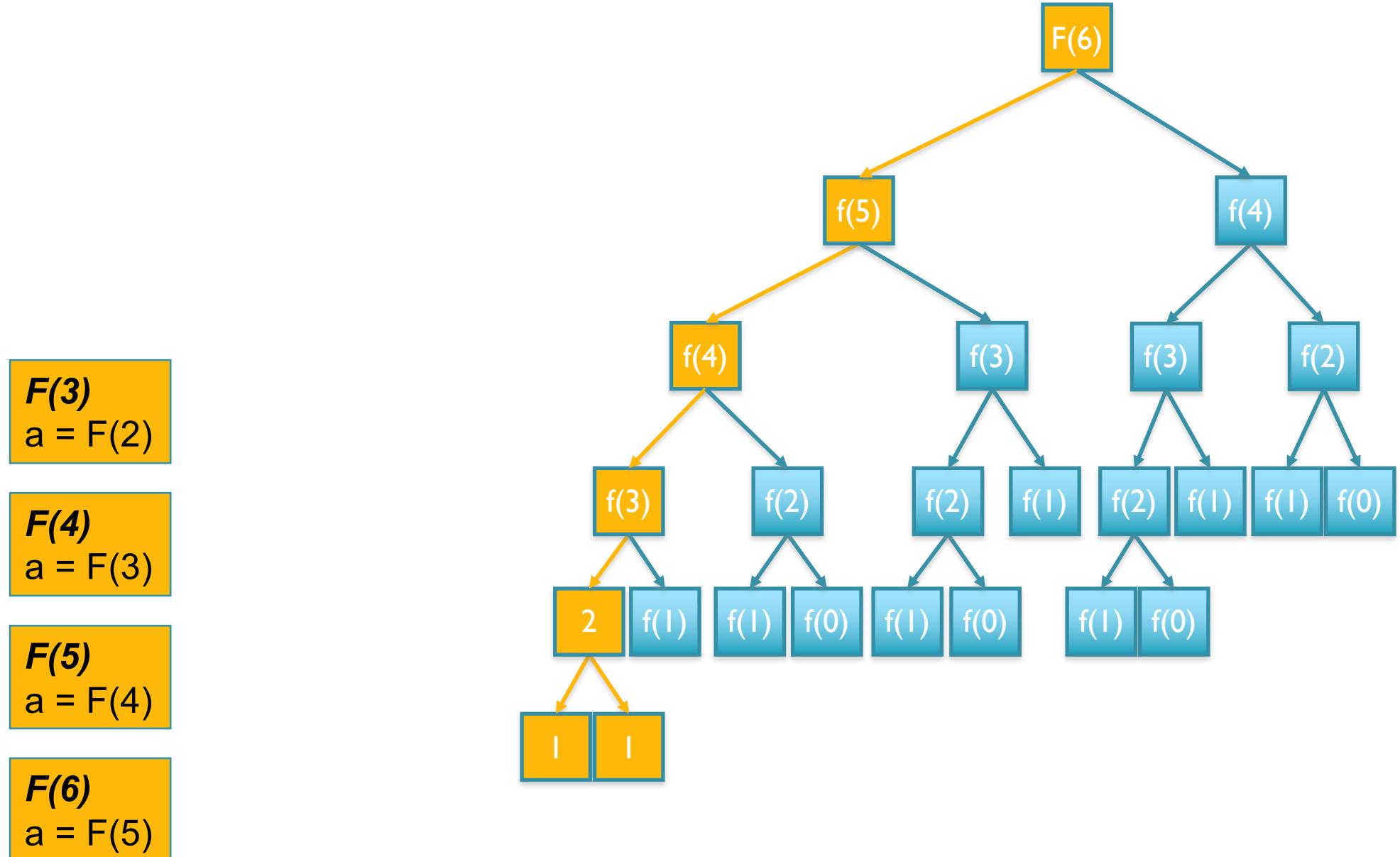
$F(4)$
 $a = F(3)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Fibonacci Sequence



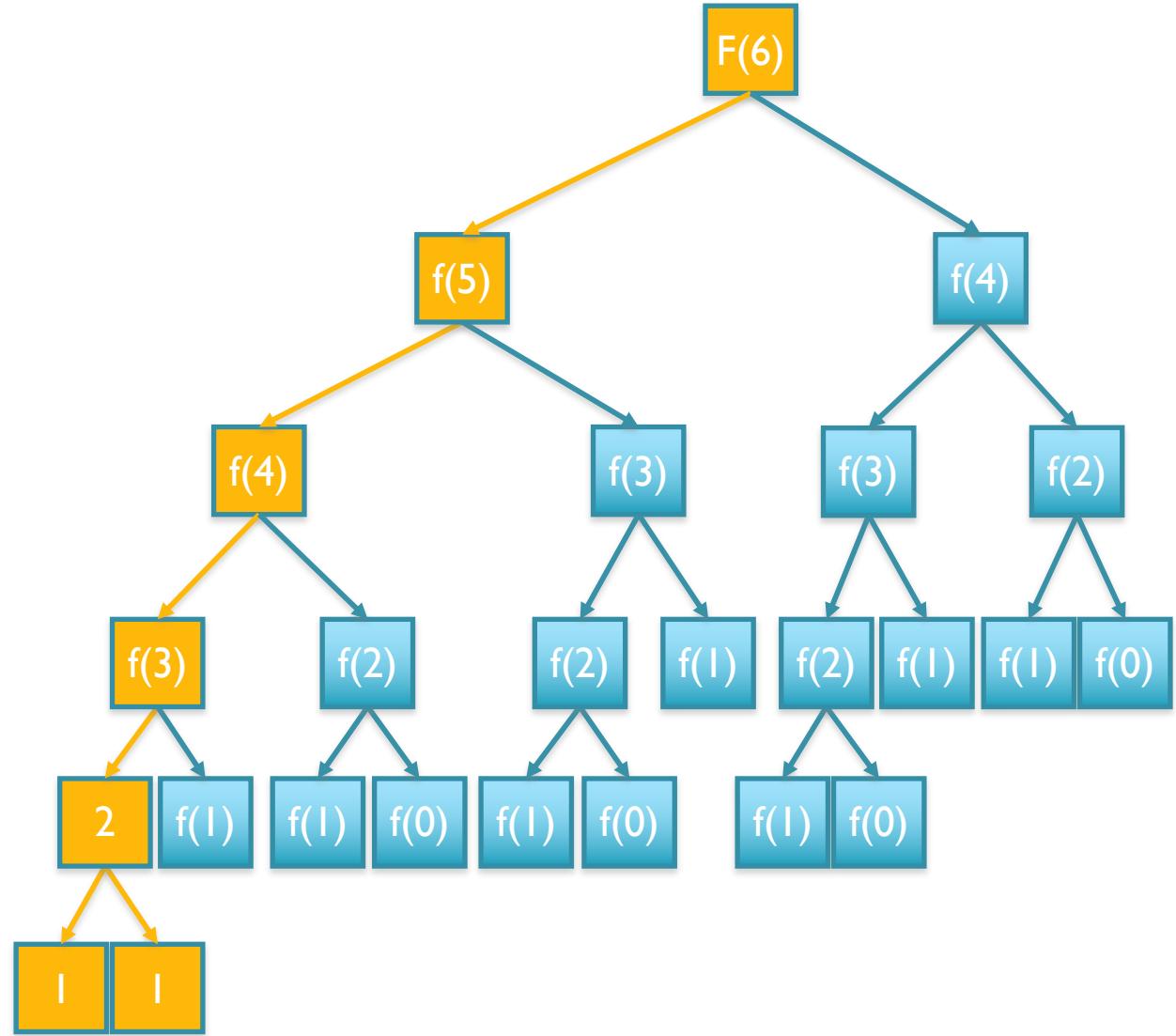
Fibonacci Sequence

$F(3)$
 $b = F(1)$

$F(4)$
 $a = F(3)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Fibonacci Sequence

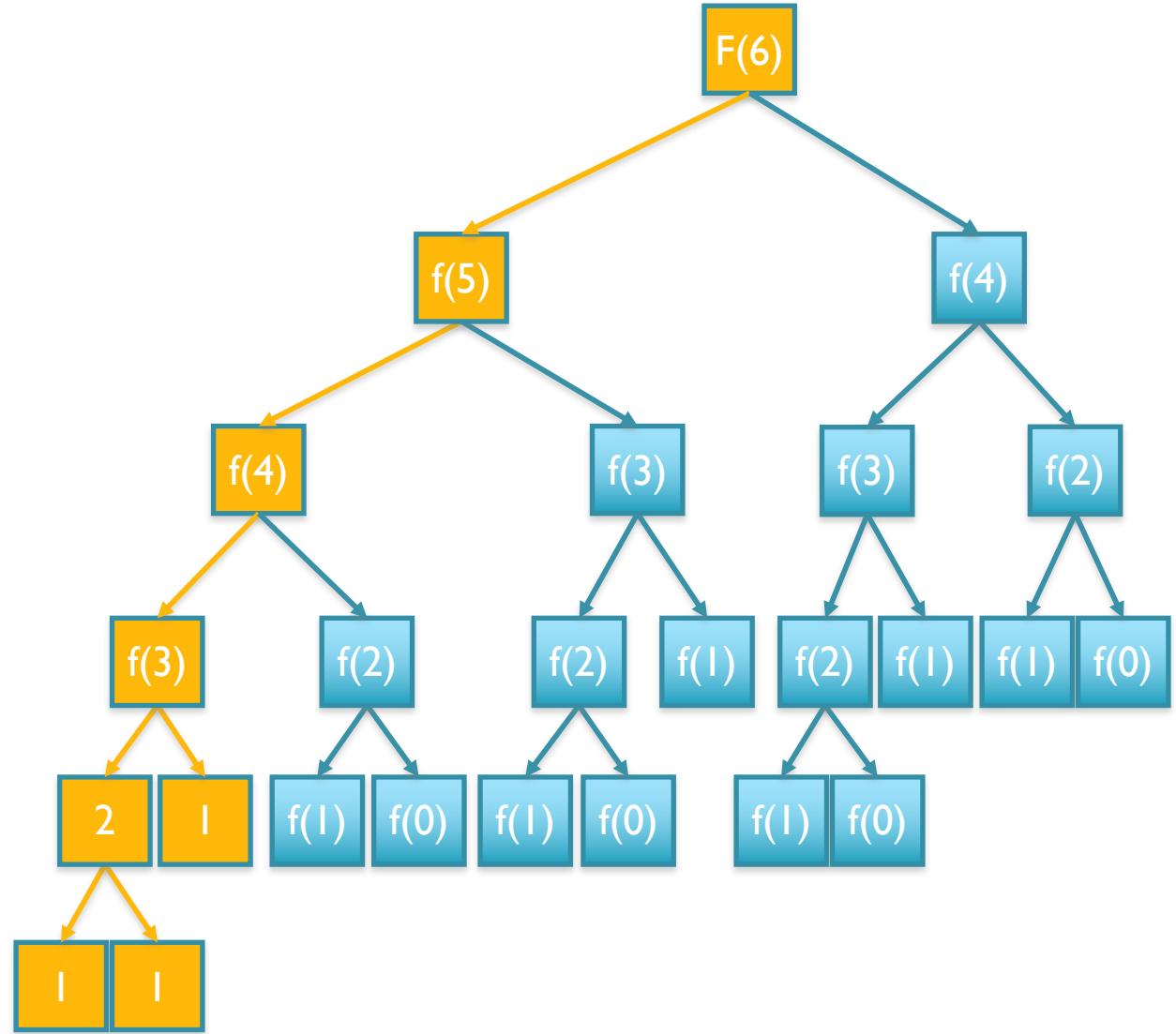
$F(1)$
return 1

$F(3)$
 $b = F(1)$

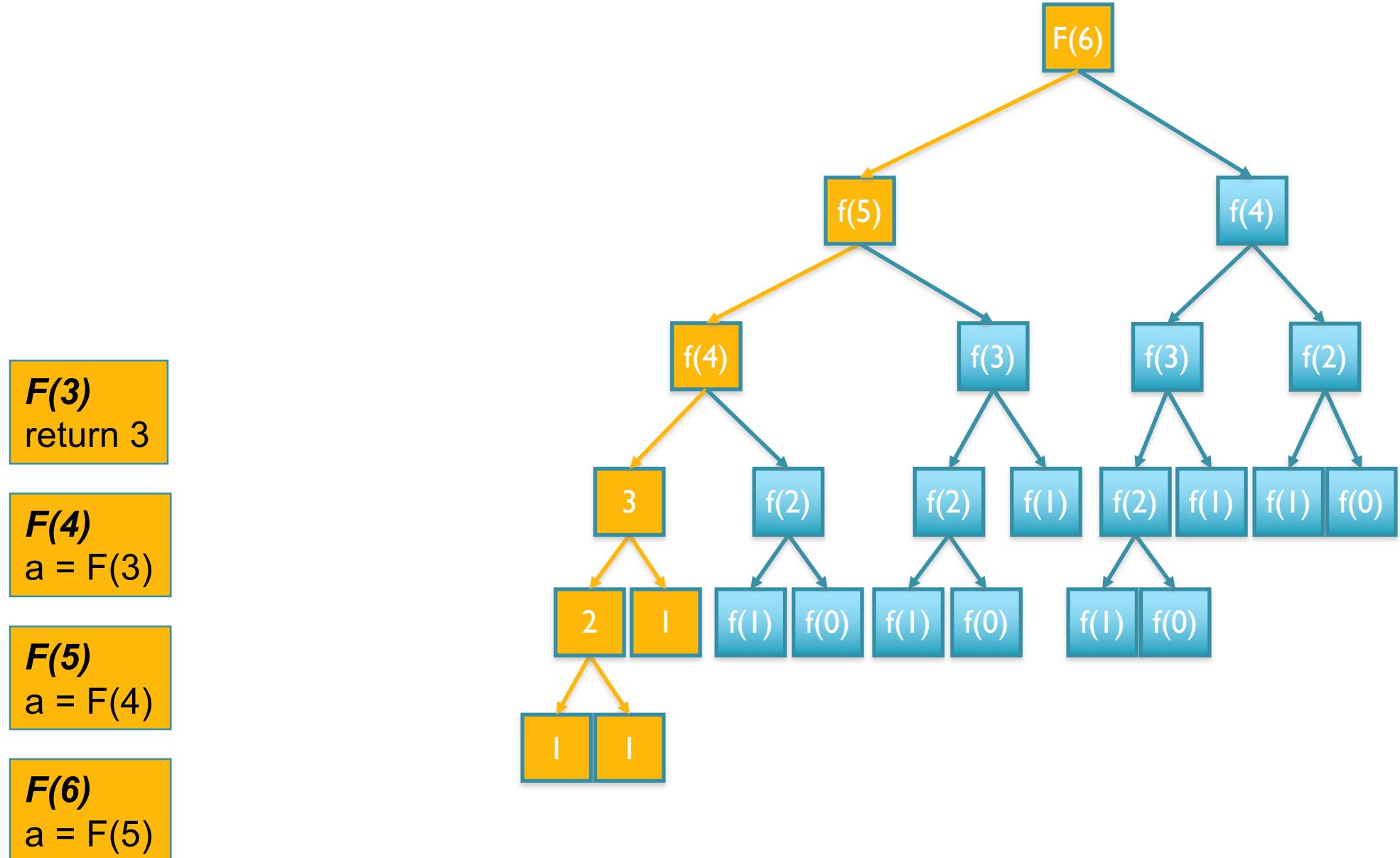
$F(4)$
 $a = F(3)$

$F(5)$
 $a = F(4)$

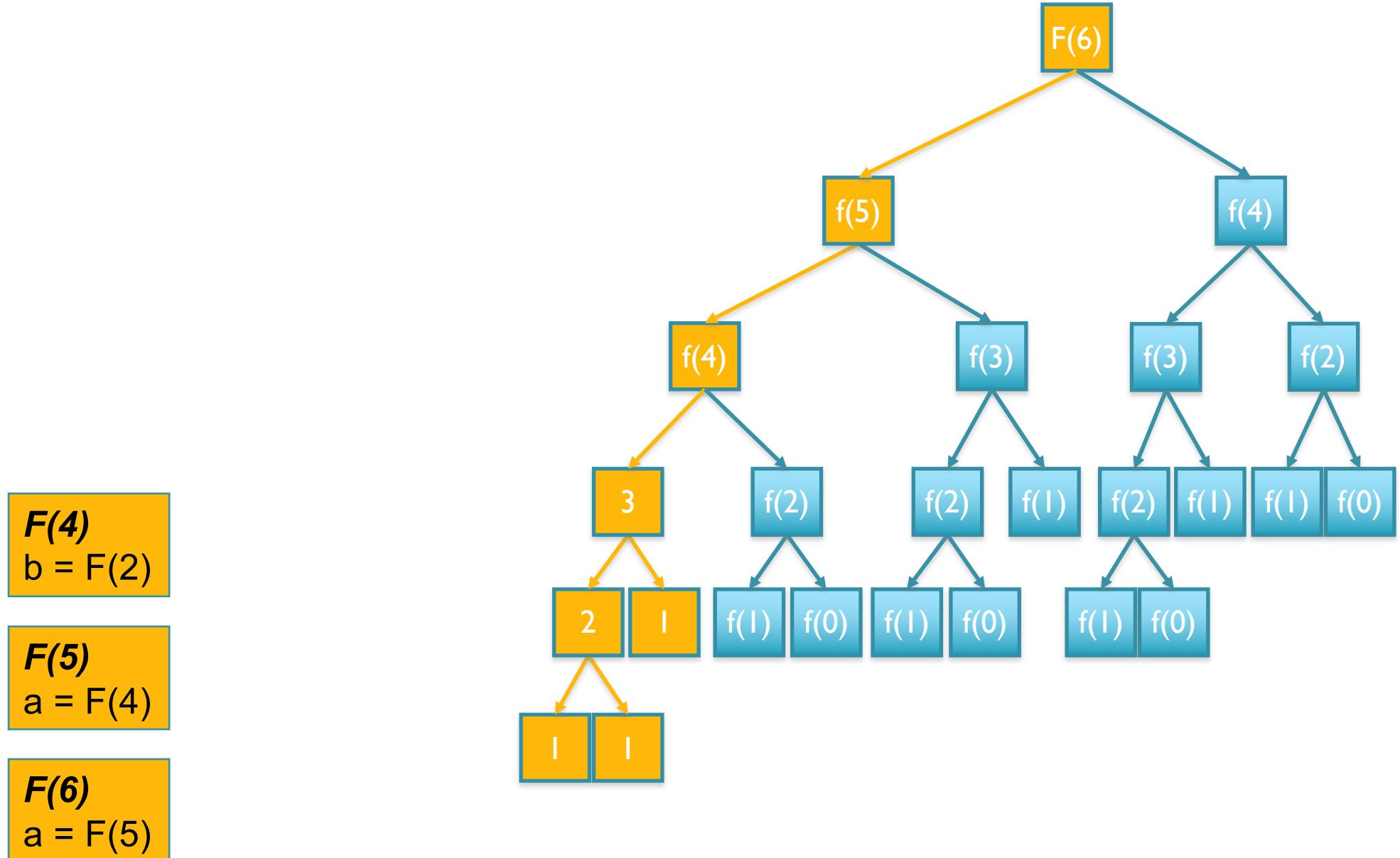
$F(6)$
 $a = F(5)$



Fibonacci Sequence



Fibonacci Sequence



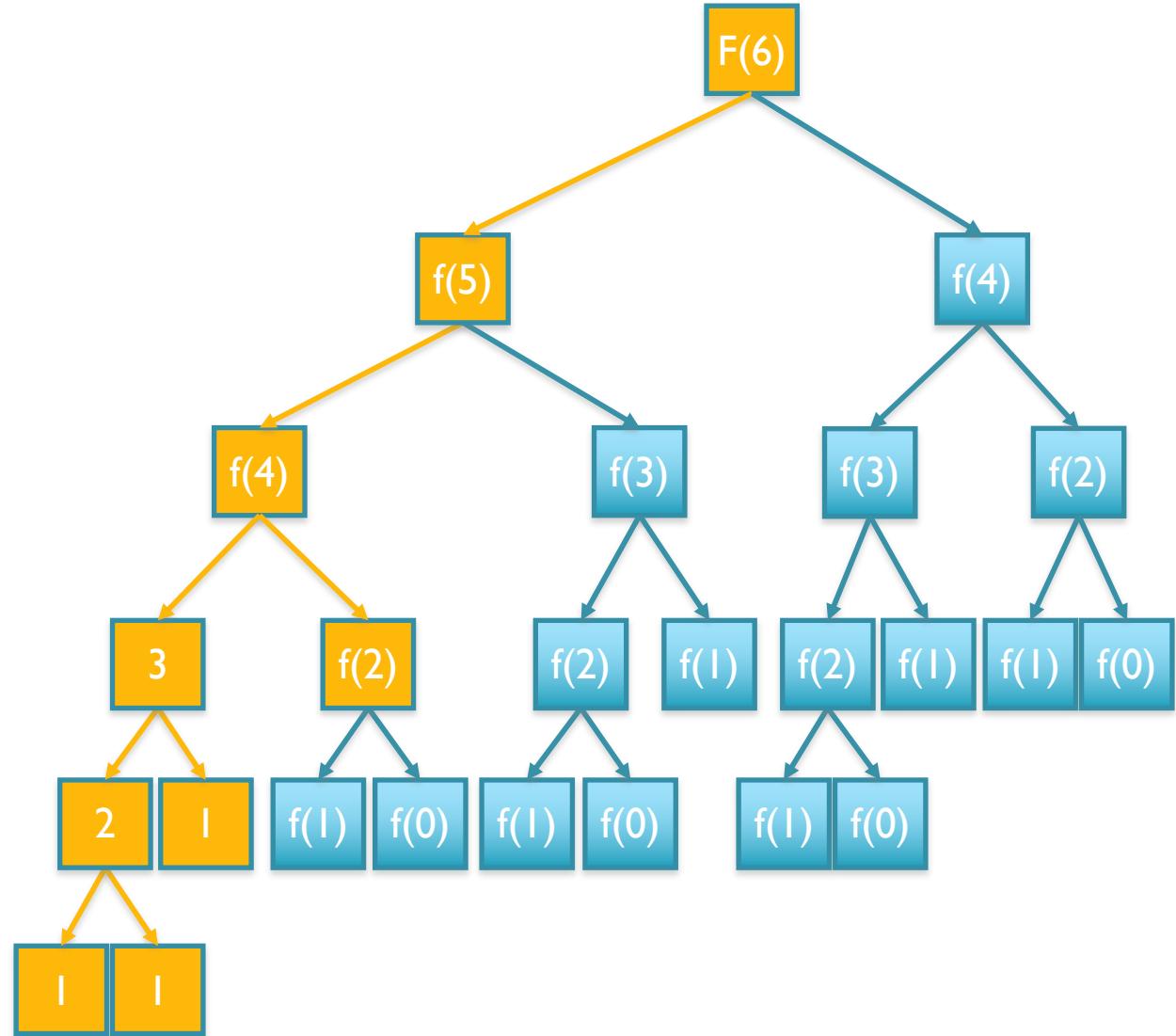
Fibonacci Sequence

$F(2)$
 $A = F(1)$

$F(4)$
 $b = F(2)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Fibonacci Sequence

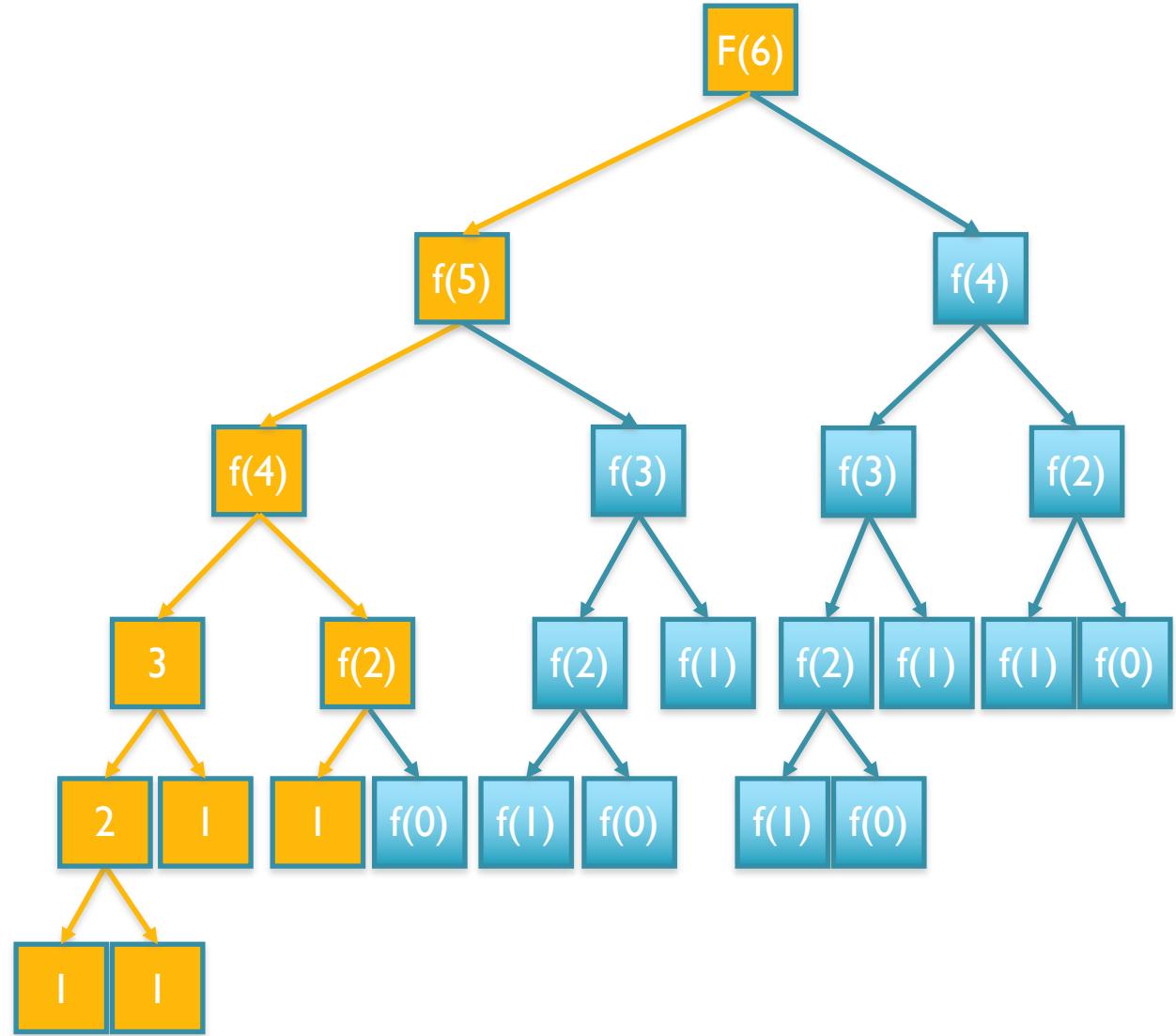
$F(1)$
return 1

$F(2)$
 $A = F(1)$

$F(4)$
 $b = F(2)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



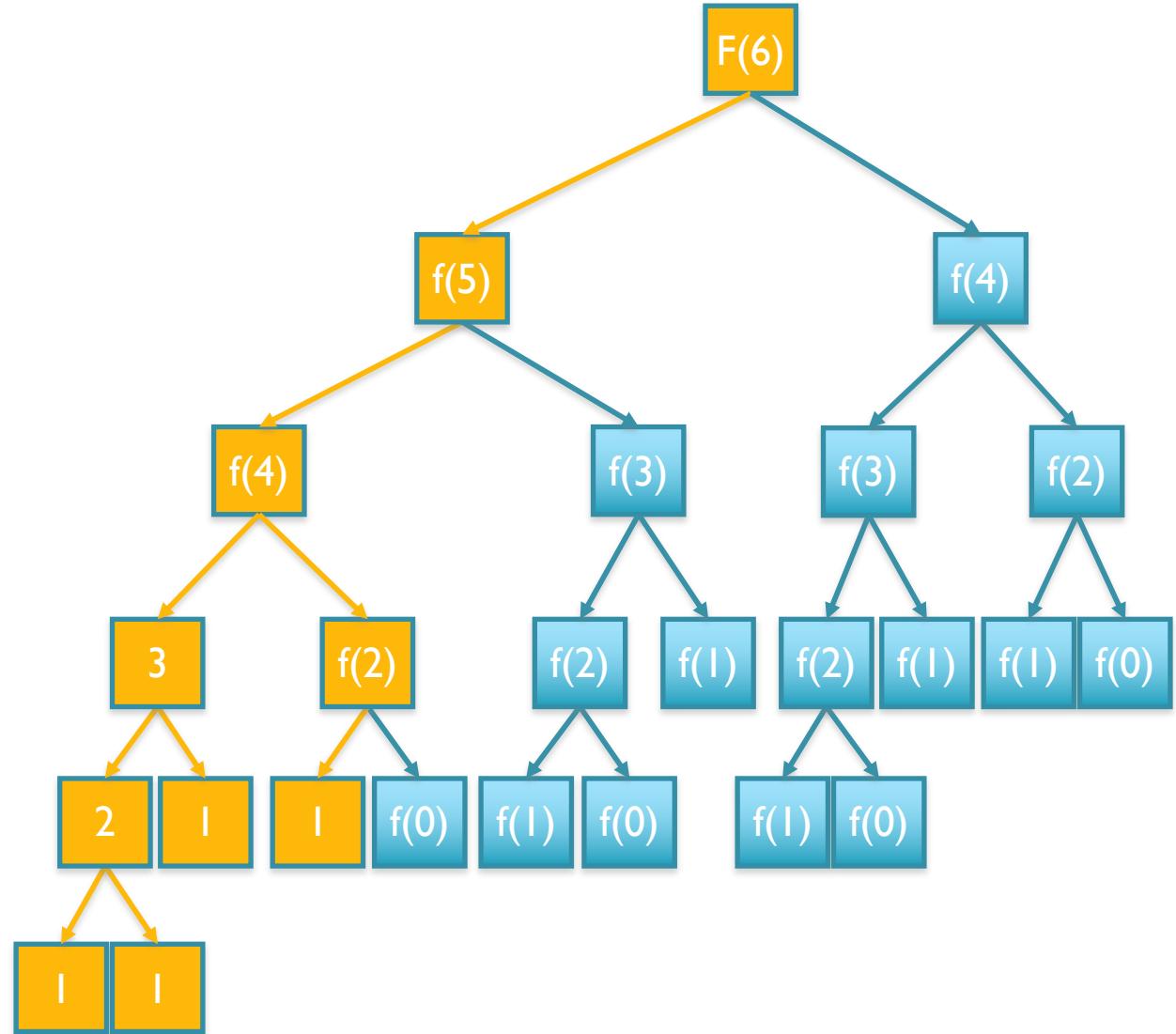
Fibonacci Sequence

$F(2)$
 $b = F(0)$

$F(4)$
 $b = F(2)$

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Fibonacci Sequence

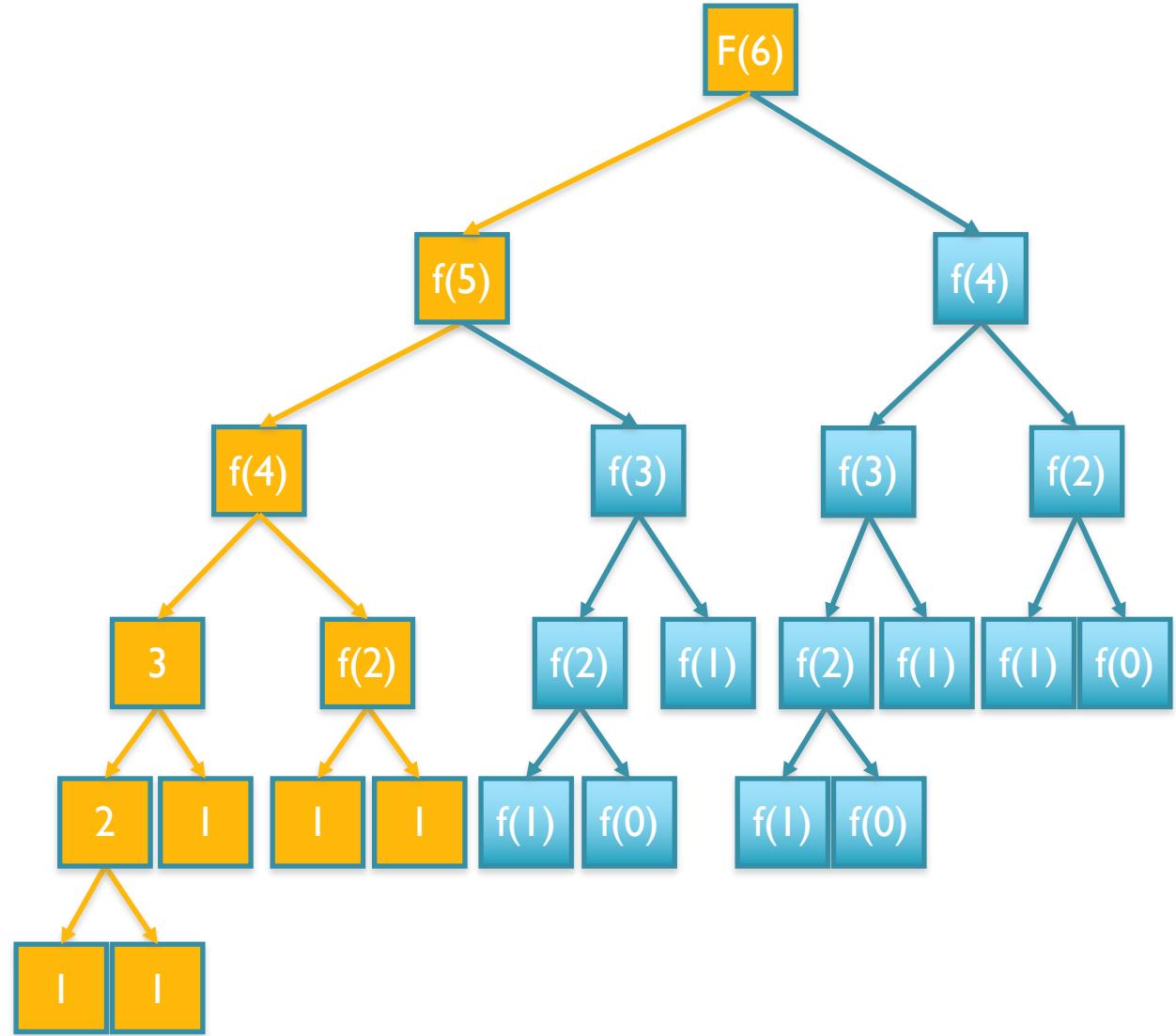
$F(0)$
return 1

$F(2)$
 $b = F(0)$

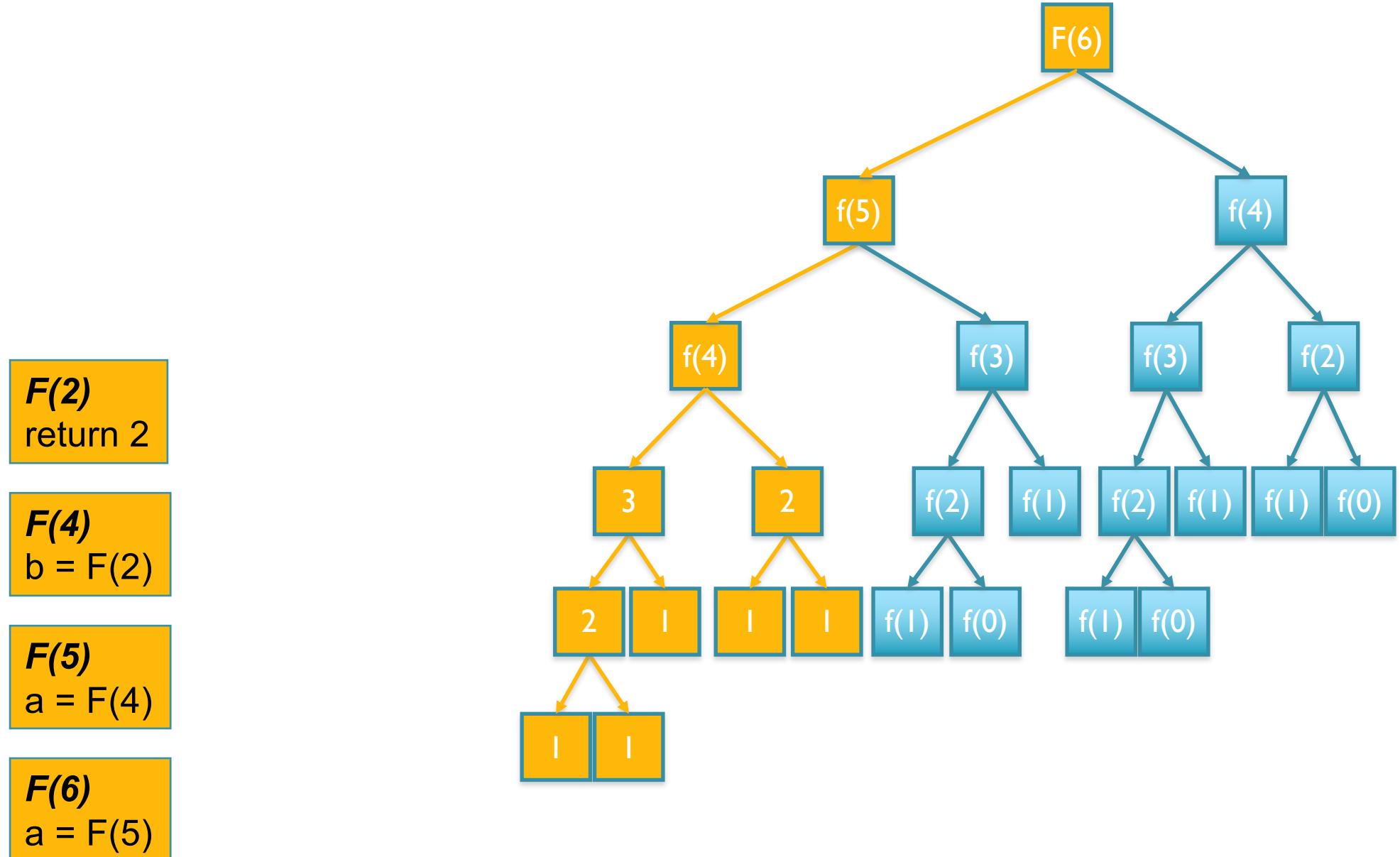
$F(4)$
 $b = F(2)$

$F(5)$
 $a = F(4)$

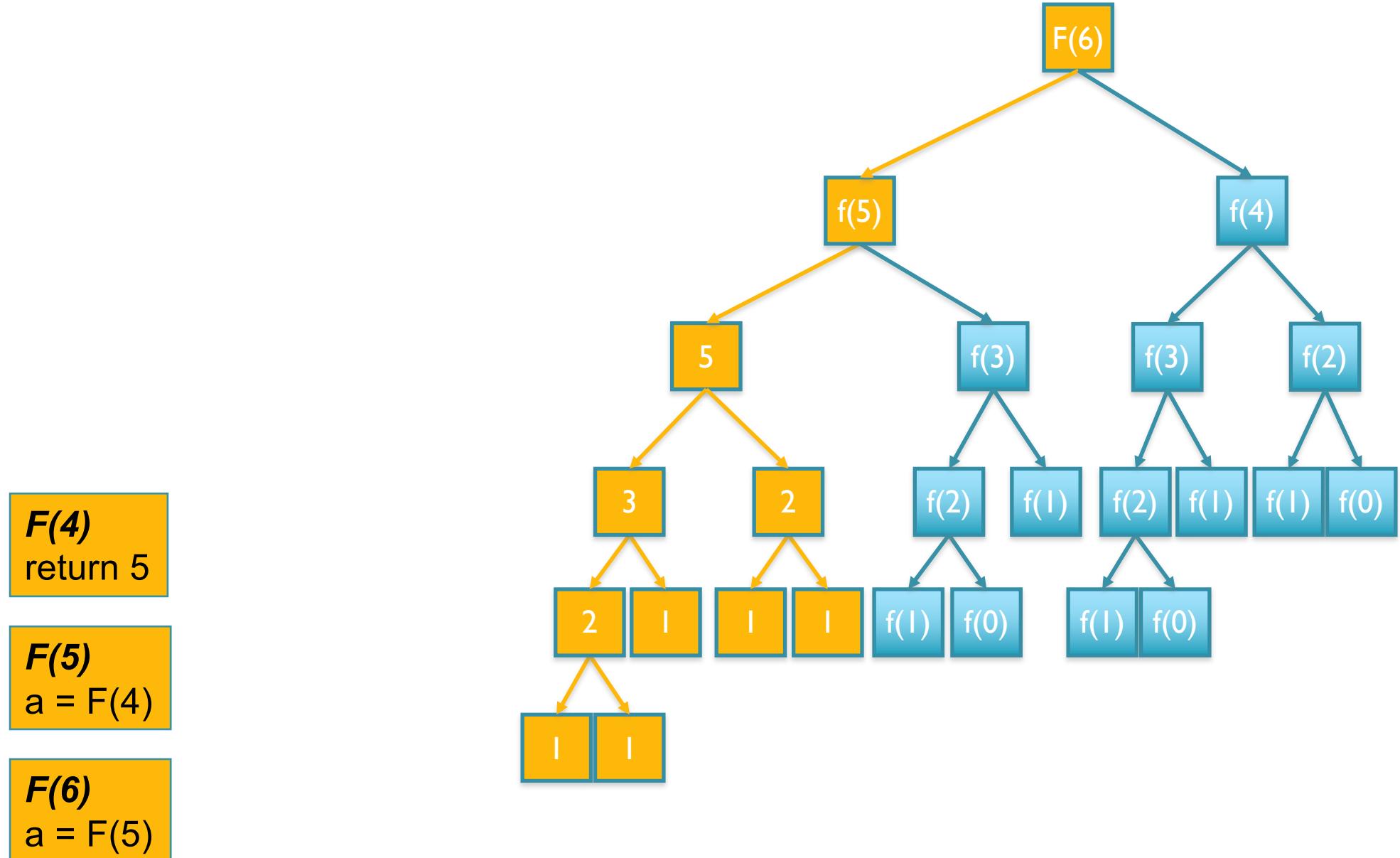
$F(6)$
 $a = F(5)$



Fibonacci Sequence



Fibonacci Sequence



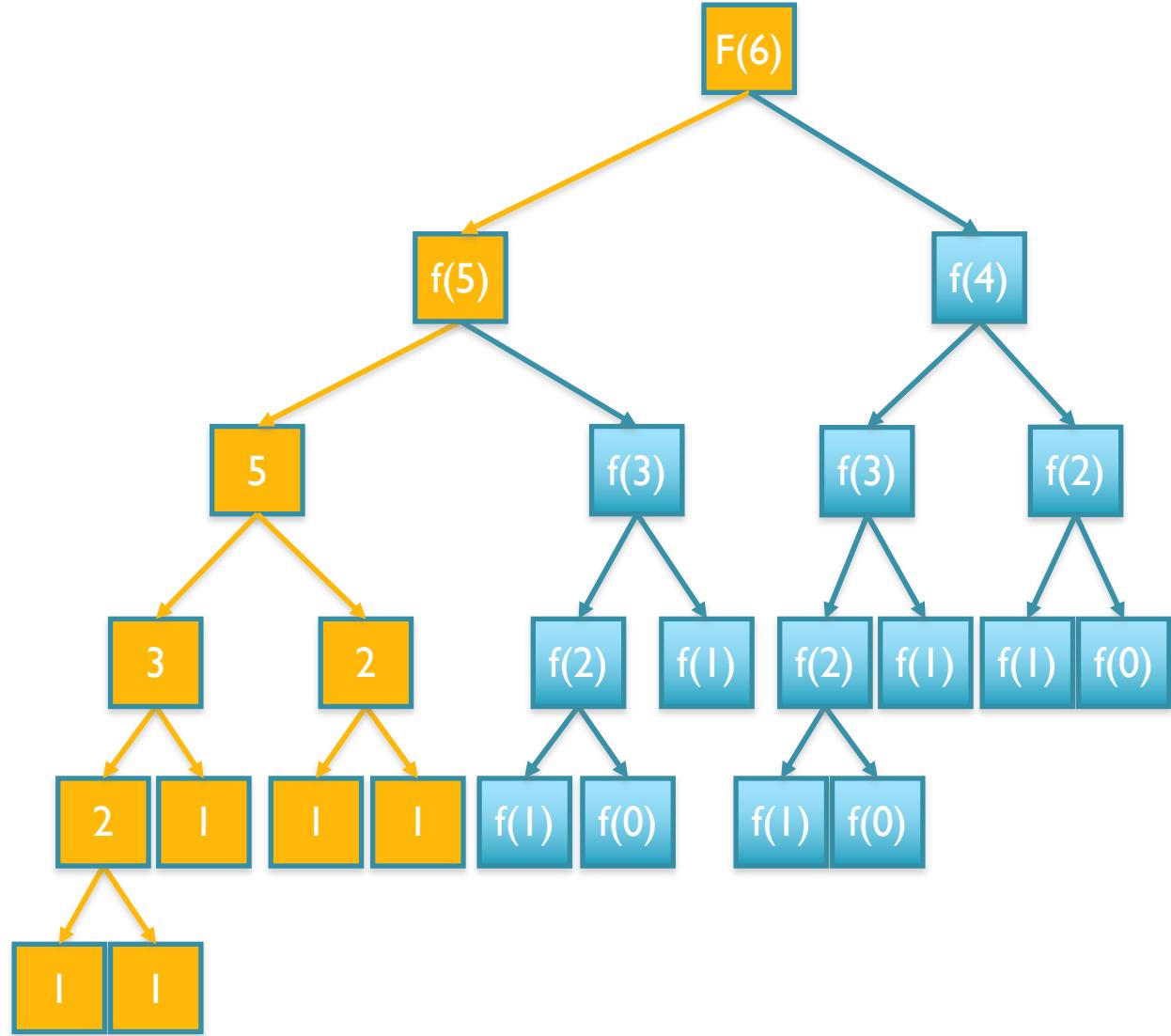
Fibonacci Sequence

Equivalent to running through a maze and always keeping your right hand on the wall

Notice we only look at the top of the stack to keep track of where we have been and where we should go next!

$F(5)$
 $a = F(4)$

$F(6)$
 $a = F(5)$



Stack ADT

adt Stack

uses Any, Boolean

defines Stack<T: Any>

operations

new:

push:

pop:

top:

empty:

preconditions

axioms

Stack ADT

adt Stack

uses Any, Boolean

defines Stack<T: Any>

operations

new: $\text{---} \rightarrow \text{Stack}\langle T \rangle$

push: $\text{Stack}\langle T \rangle \times T \text{ ---} \rightarrow \text{Stack}\langle T \rangle$

pop: $\text{Stack}\langle T \rangle \text{ ---} \rightarrow \text{Stack}\langle T \rangle$

top: $\text{Stack}\langle T \rangle \text{ ---} \rightarrow T$

empty: $\text{Stack}\langle T \rangle \text{ ---} \rightarrow \text{Boolean}$

preconditions

pop(s): not empty(s)

top(s): not empty(s)

axioms

empty(new())

not empty(push(s, t))

top(push(s, t)) = t

pop(push(s, t)) = s

Note: $\text{pop}(\text{push}(s, t)) = s$
is comparing the whole stack, but we havent defined what it means for one stack to equal another!

Relax this flaw by considering the axioms as rewrite rules:

$\text{top}(\text{pop}(\text{push}(\text{push}(\text{new}(), 1), 2))) = 1$

Can be rewritten as

$\text{top}(\text{push}(\text{new}(), 1)) = 1$

and finally to

1=1

Stack ADT

adt Stack

uses Any, Boolean

defines Stack<T: Any>

operations

new: $\text{---} \rightarrow \text{Stack}\langle T \rangle$

push:

pop: You might be tempted to try to derive a correct
top: equality operation, or add additional constraints (like
empty: maximum number of elements that the stack can
store) but the ADT gets very messy

precon:

pop()

top(): See *lecture notes for details*

axioms

empty(): Instead, lets focus on implementation instead

not

top(push(s, t)) = t

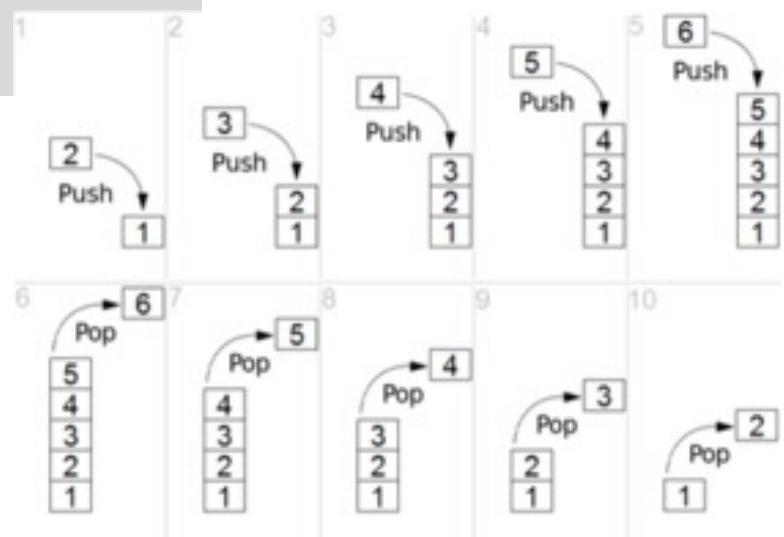
pop(push(s, t)) = s

Stack Interface

```
public interface Stack<T> {  
    // checks if empty  
    boolean empty();  
  
    // peeks at top value without removing  
    T top() throws EmptyException;  
  
    // removes top element  
    void pop() throws EmptyException;  
  
    // adds new element to top of stack  
    void push(T t);  
}
```

*How would you implement
this interface?*

Why?



```

/**
 * Stack implemented using a linked list.
 *
 * All operations take O(1) time in the worst case; however
 * each push() results in a new object being allocated which
 * may be inappropriate for some applications.
 *
 * @param <T> Element type.
 */
public class ListStack<T> implements Stack<T> {
    private static class Node<T> {
        Node<T> next;
        T data;
    }

    private Node<T> first;

    /**
     * Create an empty stack.
     */
    public ListStack() {
    }

    @Override
    public T top() throws EmptyException {
        try {
            return this.first.data;
        } catch (NullPointerException e) {
            throw new EmptyException();
        }
    }

    @Override
    public void pop() throws EmptyException {
        try {
            this.first = this.first.next;
        } catch (NullPointerException e) {
            throw new EmptyException();
        }
    }

    @Override
    public void push(T t) {
        Node<T> n = new Node<T>();
        n.data = t;
        n.next = this.first;
        this.first = n;
    }

    @Override
    public boolean empty() {
        return this.first == null;
    }

    @Override
    public String toString() {
        String s = "[";
        for (Node<T> n = this.first; n != null; n = n.next) {
            s += n.data.toString();
            if (n.next != null) {
                s += ", ";
            }
        }
        s += "]";
        return s;
    }
}

```

```

/**
 * Stack implemented using a growing array.
 *
 * All operations except push() take O(1) time in the worst
 * case; push() takes O(1) amortized time because the array
 * may need to be resized; however, compared to ListStack,
 * fewer push() operations result in objects being allocated.
 *
 * @param <T> Element type.
 */
public class ArrayStack<T> implements Stack<T> {
    private T[] data;
    private int used;

    /**
     * Create an empty stack.
     */
    public ArrayStack() {
        this.data = (T[]) new Object[1];
    }

    @Override
    public T top() throws EmptyException {
        if (this.empty()) {
            throw new EmptyException();
        }
        return this.data[this.used - 1];
    }

    @Override
    public void pop() throws EmptyException {
        if (this.empty()) {
            throw new EmptyException();
        }
        this.used -= 1;
    }

    private boolean full() {
        return this.data.length == this.used;
    }

    private void grow() {
        T[] bigger = (T[]) new Object[this.data.length * 2];
        for (int i = 0; i < this.used; i++) {
            bigger[i] = this.data[i];
        }
        this.data = bigger;
    }

    @Override
    public void push(T t) {
        if (this.full()) {
            this.grow();
        }
        this.data[this.used] = t;
        this.used += 1;
    }

    @Override
    public boolean empty() {
        return this.used == 0;
    }

    @Override
    public String toString() {
        String s = "[";
        for (int i = this.used - 1; i >= 0; i--) {
            s += this.data[i].toString();
            if (i > 0) {
                s += ", ";
            }
        }
        s += "]";
        return s;
    }
}

```

ListStack vs. ArrayStack

Which is better?

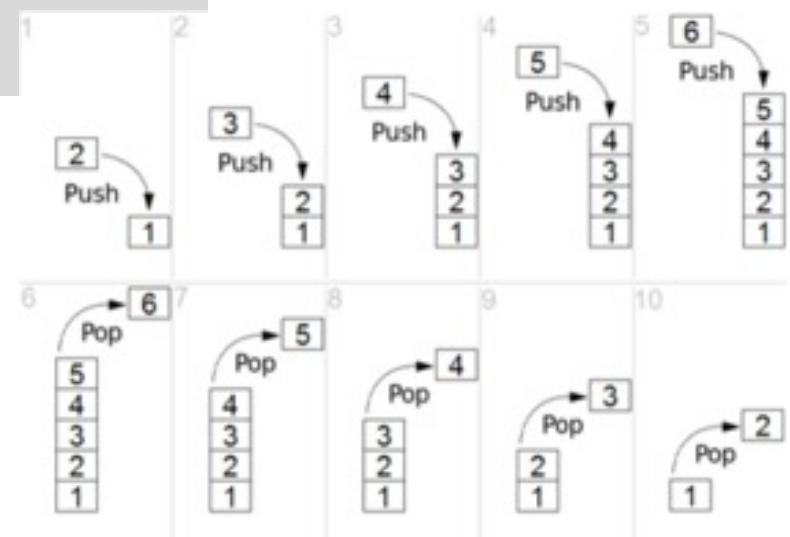
Why?

Stack Interface

```
public interface Stack<T> {  
    // checks if empty  
    boolean empty();  
  
    // peeks at top value without removing  
    T top() throws EmptyException;  
  
    // removes top element  
    void pop() throws EmptyException;  
  
    // adds new element to top of stack  
    void push(T t);  
}
```

*How would you *test*
the implementation?*

Why?



Introducing JUnit

Lecture 2: SimpleCounter.java

```
public class SimpleCounter implements Counter {  
  
    public static void main(String[] args) {  
        Counter c = new SimpleCounter();  
        assert c.value() == 0;  
        System.out.println("Counter is now: " + c.value());  
        c.up();  
        assert c.value() == 1;  
        System.out.println("Counter is now: " + c.value());  
        c.down();  
        assert c.value() == 0;  
        System.out.println("Counter is now: " + c.value());  
        c.down();  
        c.up();  
        c.up();  
        c.up();  
        System.out.println("Counter is now: " + c.value());  
        assert c.value() == 2;  
    }  
}
```

Asserts are very useful for testing, but are very limited especially because first failed assert kills the entire program ☹

Lecture 2: SimpleCounter.java

```
public static void main(String[] args) {
    MyArray a = new MyArray(5, "Mike");

    a.put(3, "Peter");
    a.put(2, 1234);

    for (int i = 0; i < a.length(); i++) {
        System.out.println("a[" + i + "]: " +
                           a.get(i) + " " + a.get(i).getClass());
    }

    try {
        System.out.println("a[" + 57 + "]: " + a.get(57));
    } catch (IndexOutOfBoundsException e) {
        System.out.println("Caught IndexOutOfBoundsException (as expected)");
    }
}
```

Printing is useful while developing, but becomes
unscalable in large programs with lots of methods to test

Introducing JUnit

The screenshot shows a web browser displaying the JUnit 4 Project Documentation. The title bar reads "JUnit - About". The address bar shows the URL "https://junit.org/junit4/". The page header includes the JUnit logo and navigation links for "JUnit 4", "Project Documentation", and "About". A red ribbon banner on the right says "fork me on GitHub". The main content area displays code snippets for JUnit tests:

```
@Test  
public void newArrayListsHaveNoElements() {  
    assertThat(new ArrayList().size(), is(0));  
}  
  
@Test  
public void sizeReturnsNumberOfElements() {  
    List instance = new ArrayList();  
    instance.add(new Object());  
    instance.add(new Object());  
    assertThat(instance.size(), is(2));  
}
```

Below the code, there's a section titled "Annotations" with the instruction "Start by marking your tests with `@Test`". At the bottom of the main content area is a blue button labeled "Let's take a tour »".

The footer contains three columns of links:

- Welcome**
 - Download and install
 - Getting started
 - Release Notes
 - 4.12
 - 4.11
 - 4.10
 - 4.9.1
 - 4.9
 - Maintainer Documentation
 - I want to help!
- Usage and Idioms**
 - Assertions
 - Test Runners
 - Aggregating tests in Suites
 - Test Execution Order
 - Exception Testing
 - Matchers and `assertThat`
 - Ignoring Tests
 - Timeout for Tests
 - Parameterized Tests
 - Assumptions with `Assume`
- Third-party extensions**
 - Custom Runners
 - `net.trajano.commons.common-testing` for `UtilityClassTestUtil` per #646
 - System Rules – A collection of JUnit rules for testing code that uses `java.lang.System`.
 - JUnit Toolbox – Provides runners for parallel testing, a `PollingWait` class to ease asynchronous testing, and a `WildcardPatternSuite` which allow you to `new WildcardPatternSuite("com.mycompany.myapp")`.

Sample Report

The screenshot shows a web browser window with multiple tabs open. The active tab is titled "Surefire Report - Google Search". The main content area displays the "Apache Maven Project" website, specifically the "Surefire Report" section. The page title is "Surefire Report". It includes a "Summary" table and a "Package List" table. Below these are sections for "Test Cases" and "SurefireReportMojoTest", each with its own table. The left sidebar contains navigation links for the Apache Maven Project, including "Downloads", "Checklist", "Source", "Test Source", "Tag List", "JavaDoc", "Test JavaDoc", "FindBugs", "Gherkin", "JIRA Report", "Plugin Documentation", "LTS Status", "ASF PRODUCTS", and "How Apache Works". A decorative bar at the bottom features blue and white geometric shapes.

Surefire Report

Summary

[Summary] [Package List] [Test Cases]

Tests	Errors	Failures	Skipped	Success Rate	Time
11	0	0	0	100%	3.08

Note: Failures are anticipated and checked for with assertions while errors are unanticipated.

Package List

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Package	Tests	Errors	Failures	Skipped	Success Rate	Time
org.apache.maven.plugins.surefire.report	11	0	0	0	100%	3.08

Note: package statistics are not computed recursively, they only sum up all of its testcases numbers.

org.apache.maven.plugins.surefire.report

Class	Tests	Errors	Failures	Skipped	Success Rate	Time
Surefire97Test	1	0	0	0	100%	0.188
SurefireReportMojoTest	10	0	0	0	100%	0.018

Test Cases

[Summary] [Package List] [Test Cases]

Surefire97Test

computedTestCase\$when\$1\$MissingDir\$thenReturnMessage	0.108
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SurefireReportMojoTest

testBaseReport\$closedFromBaseClass	0.297
testBaseReport\$notRebase	0.402
testBaseReport\$closed	0.191
testBaseReport\$report	0.291
testBaseReport\$report\$ReportingImpl	0.201
testBaseReport\$testCase\$ImprintTrace	0.142
testBaseReport\$testCase	0.124
testBaseReport\$testCase\$ImprintTrace	0.124

JUnit According to Peter ☺

So why use a testing framework like JUnit instead of writing the tests like we did so far, using Java's assert instruction and a main method with the test code in it?

- **For one thing, JUnit allows you to modularize your tests better.** It's not uncommon for large software projects to have just as much testing code as actual program code, and so the principles you use to make regular code easier to read (splitting things into methods and classes, etc.) should also apply to test code.
- **Also, JUnit allows you to run all your test cases every time,** it doesn't stop at the first failing test case like assert does. This way you can get feedback about multiple failed tests all at once.
- **Finally, lots of companies expect graduates to have some experience with testing frameworks,** so why not pick it up now? Note that testing is not just for software developers anymore, increasingly people working with software developers but who are themselves not software developers will be asked to contribute to testing a certain application being developed by their company.

So it's a really useful skill to have on your list.

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So why use a testing framework like JUnit instead of writing the tests like we did so far, using Java's assert instruction and a main method with the test code in it?

- **For one thing, JUnit allows you to modularize your tests better.** It's not uncommon for large software projects to have just as much testing code as actual application code.

***From HW3 on out, when we say
“write test cases”***

- **as part of an assignment, we mean**

“write JUnit 4 test cases”

- **as described here.**

With testing frameworks, so why not pick it up now? Note that testing is not just for software developers anymore, increasingly people working with software developers but who are themselves not software developers will be asked to contribute to testing a certain application being developed by their company.

So it's a really useful skill to have on your list.

Next Steps

- I. Work on HW2
2. Check on Piazza for tips & corrections!