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# Introduction to Monte Carlo and the Monte Carlo propagation of uncertainties in DN and L to gain and offset

2019

WG4

ahueni

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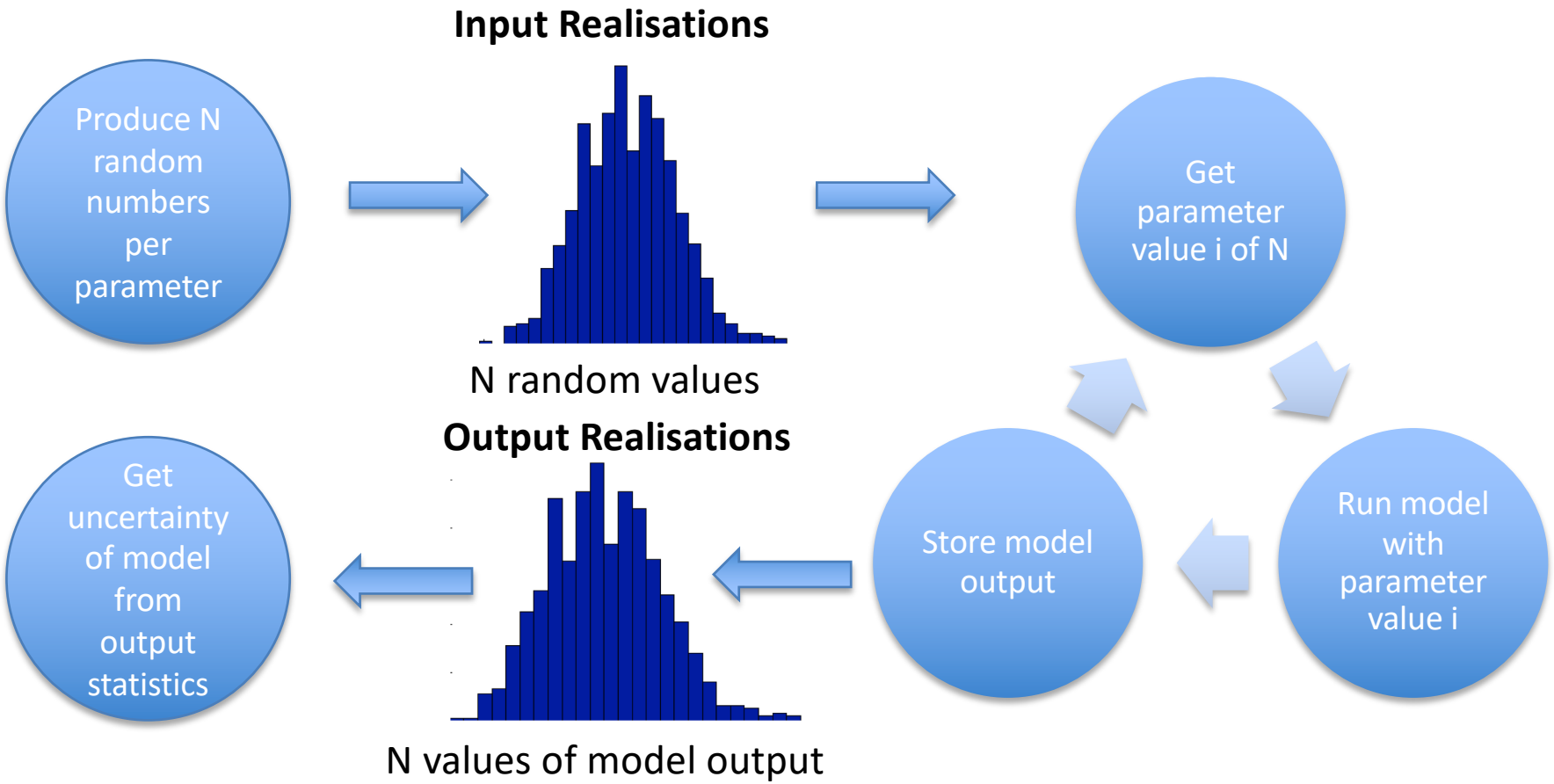
# How Monte Carlo works: Computational Brute-Forcing



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# How to generate Realisations

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Stddev = 1, Rel. Stddev = 2.3810 %

```
% creates realisations, either using a new random generated number
array or
% using the supplied random number array
function [realisations, random_numbers] =
get_realisations_gauss_dist(numSamples, mu, sigma, random_numbers)

    if nargin == 3
        random_numbers = randn(numSamples, length(mu));
    end

    for i=1:numSamples
        realisations(:,i) = mu + sigma.*random_numbers(i,:)';
    end

end
```

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# Testing the Realisation Generation: Is it Gaussian?

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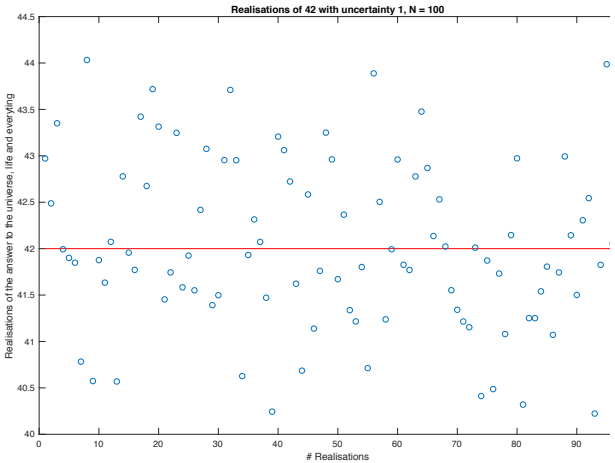


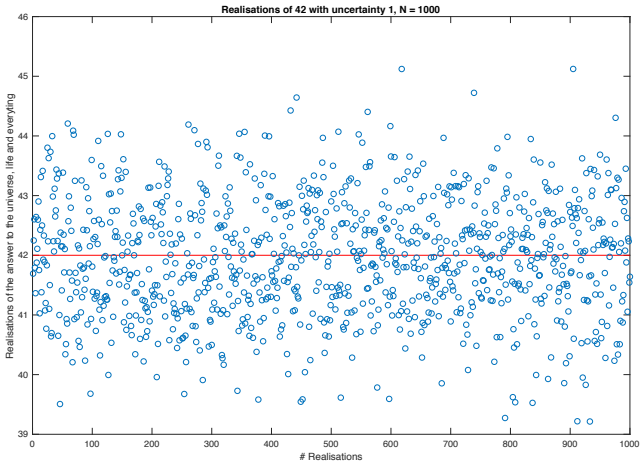


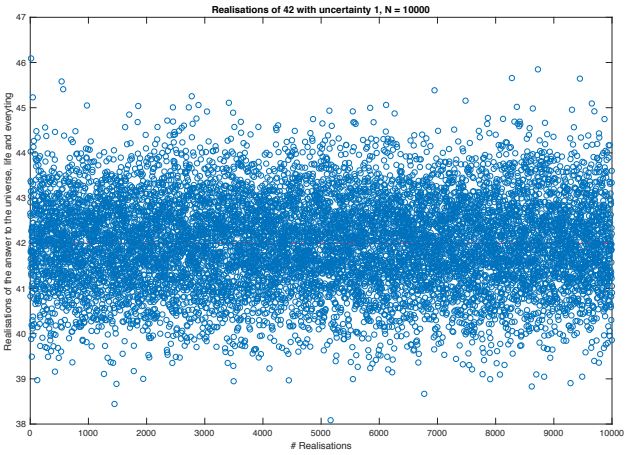


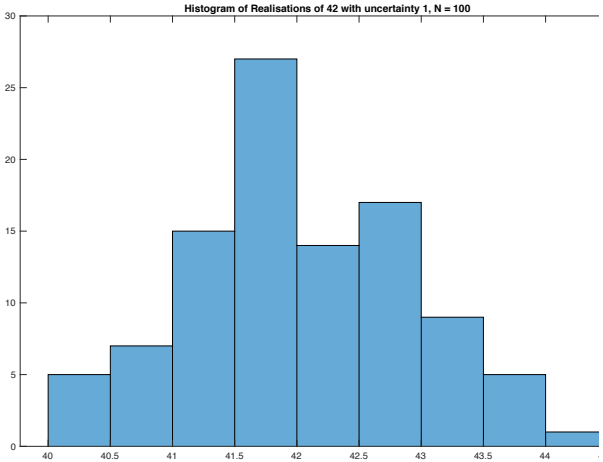


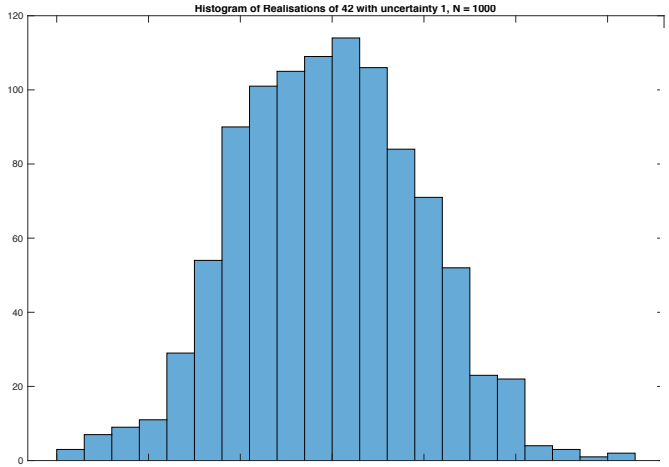


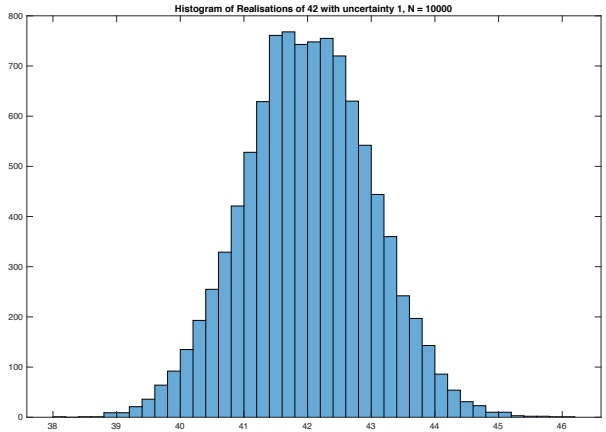

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# Uncertainty Propagation: Creating the Output Realisations

A squared log transformation of the answer to the universe, life and everything …

```
% simple model to draw from the generated realisations

output = zeros(size(realisations)); % allocate output vector

% loop over all realisations and calculate the model output
for i=1:length(realisations)

    parameter = realisations(i);

    output(i) = (log(parameter))^2;

end
```

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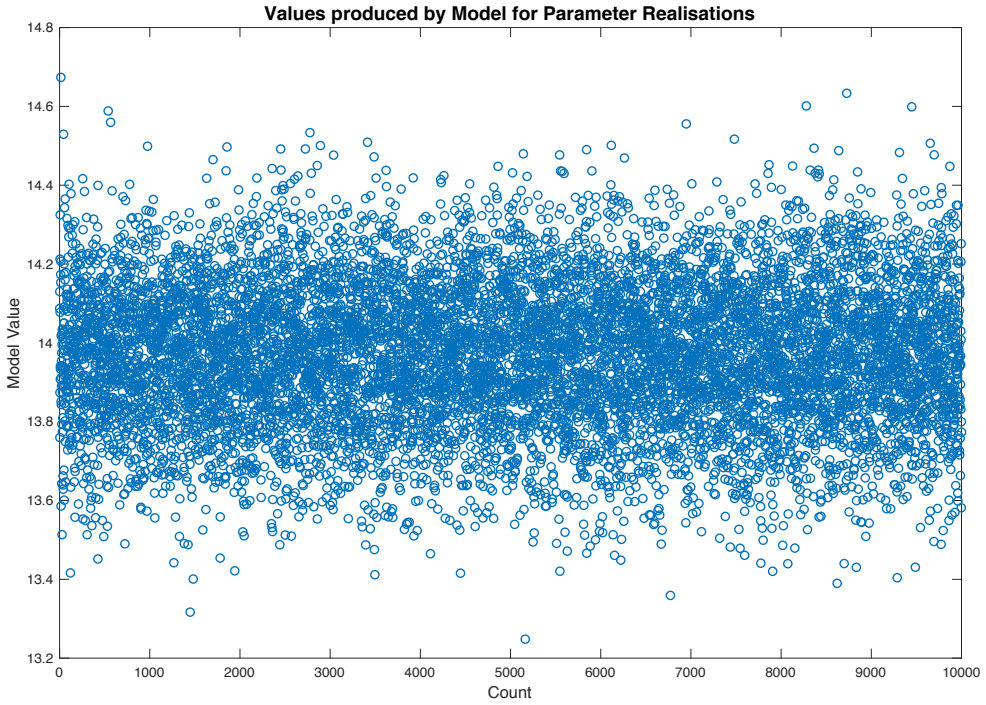
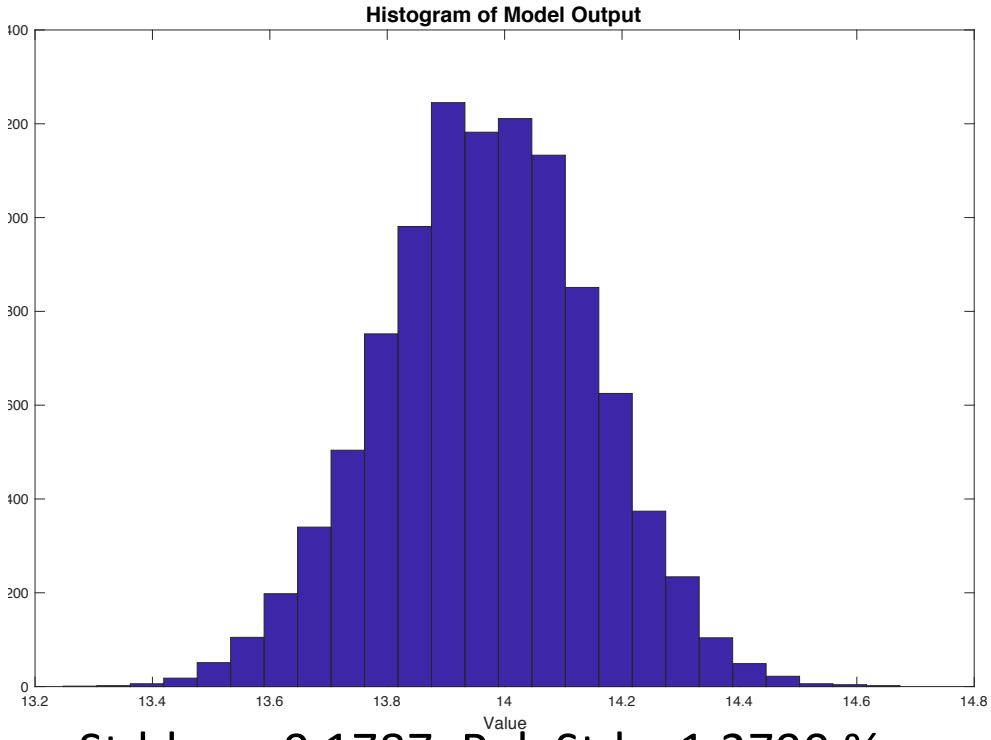
# Uncertainty Propagation: Creating the Output Realisations

A squared log transformation of the answer to the universe, life and everything …




Mean = 13.9694




Stddev = 0.1787, Rel. Std = 1.2790 %

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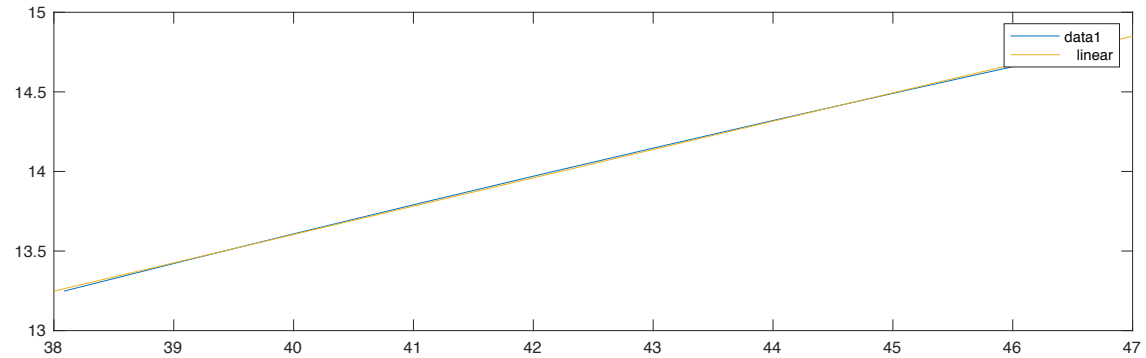
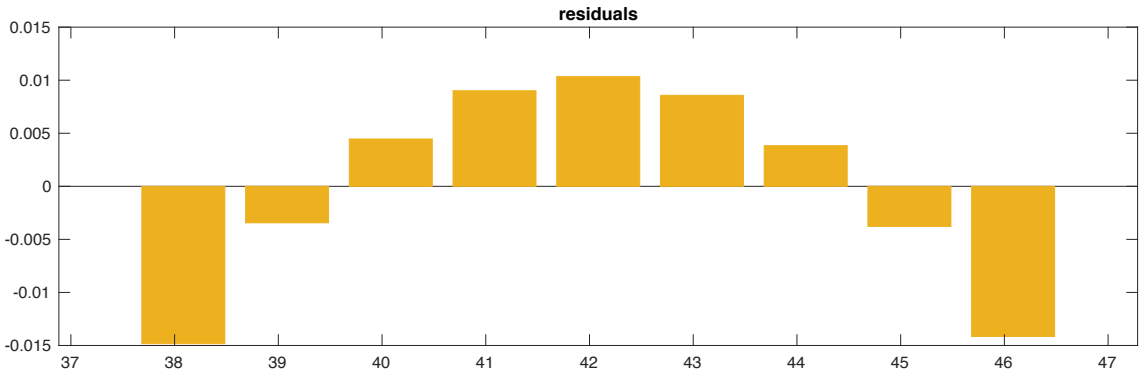
# Analytical Deduction of Uncertainty




```
c = (2*log(mu)/mu);

u2_y = c^2 * sigma^2;
u_y = u2_y ^ (0.5)


u_y = 0.1780
```

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# Analytical Deduction of Uncertainty: Comparison with Monte Carlo

| N | MC u(y) | Delta to Analytical |
|---|---|---|
| 100 | 0.1540 | -0.0240 |
| 1000 | 0.1686 | -0.0094 |
| 10000 | 0.1763 | -0.0017 |
| 100000 | 0.1785 | 5.5467e-04 |

```
Analytical u(y) = 0.1780
```

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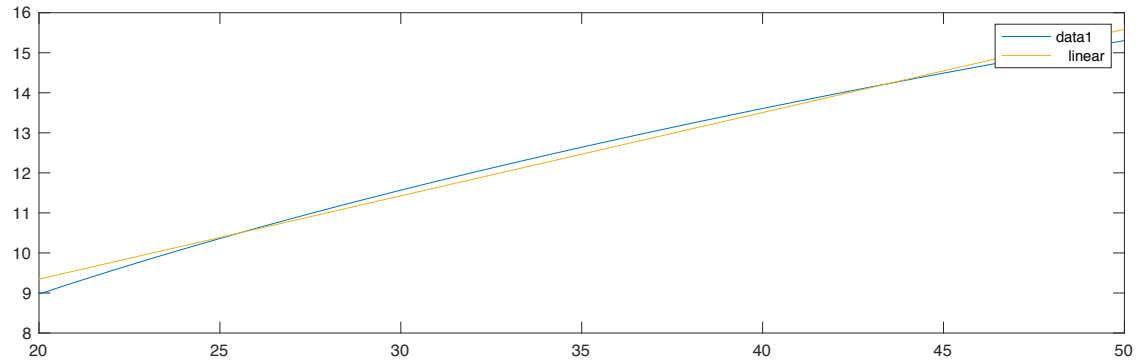
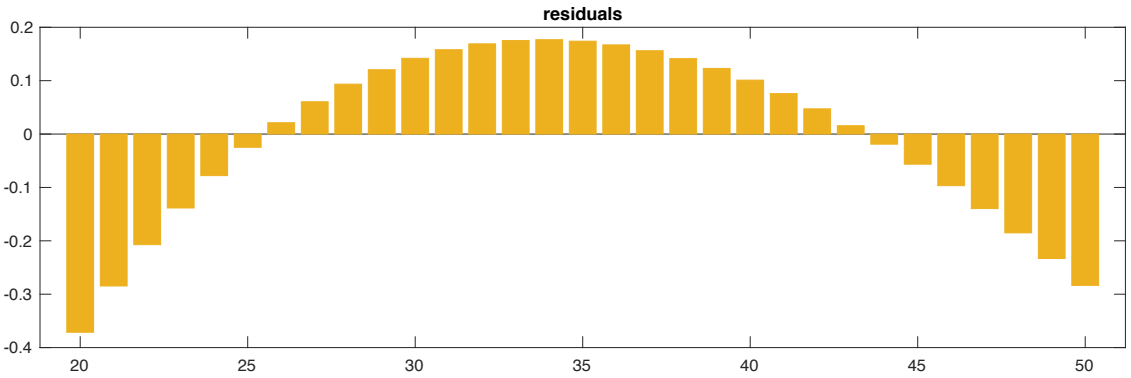
# Limits of Analytical Deduction of Uncertainty




Partial derivatives assume linearity over the range of values.

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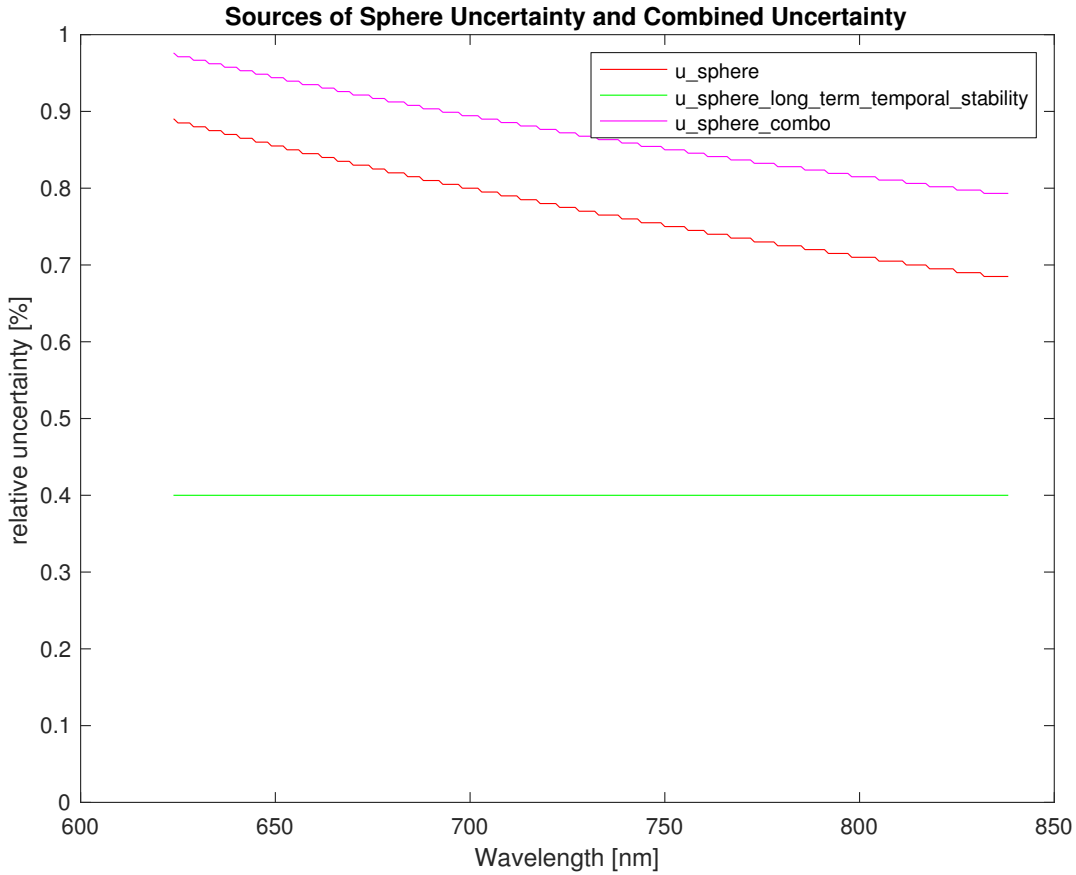
# Combined Uncertainties of Sphere




Added in quadrature as error factors are multiplicative

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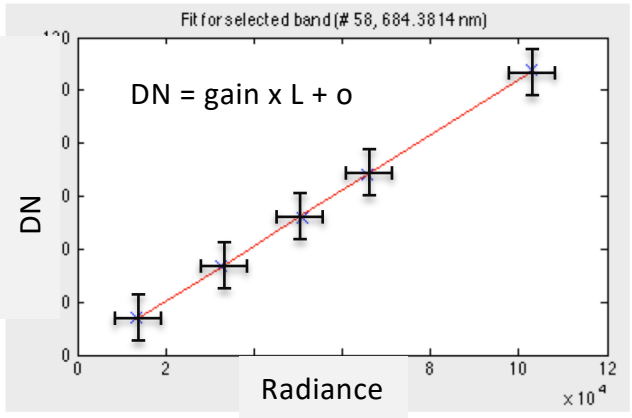
# MC applied to straight line calibration

u(DN)

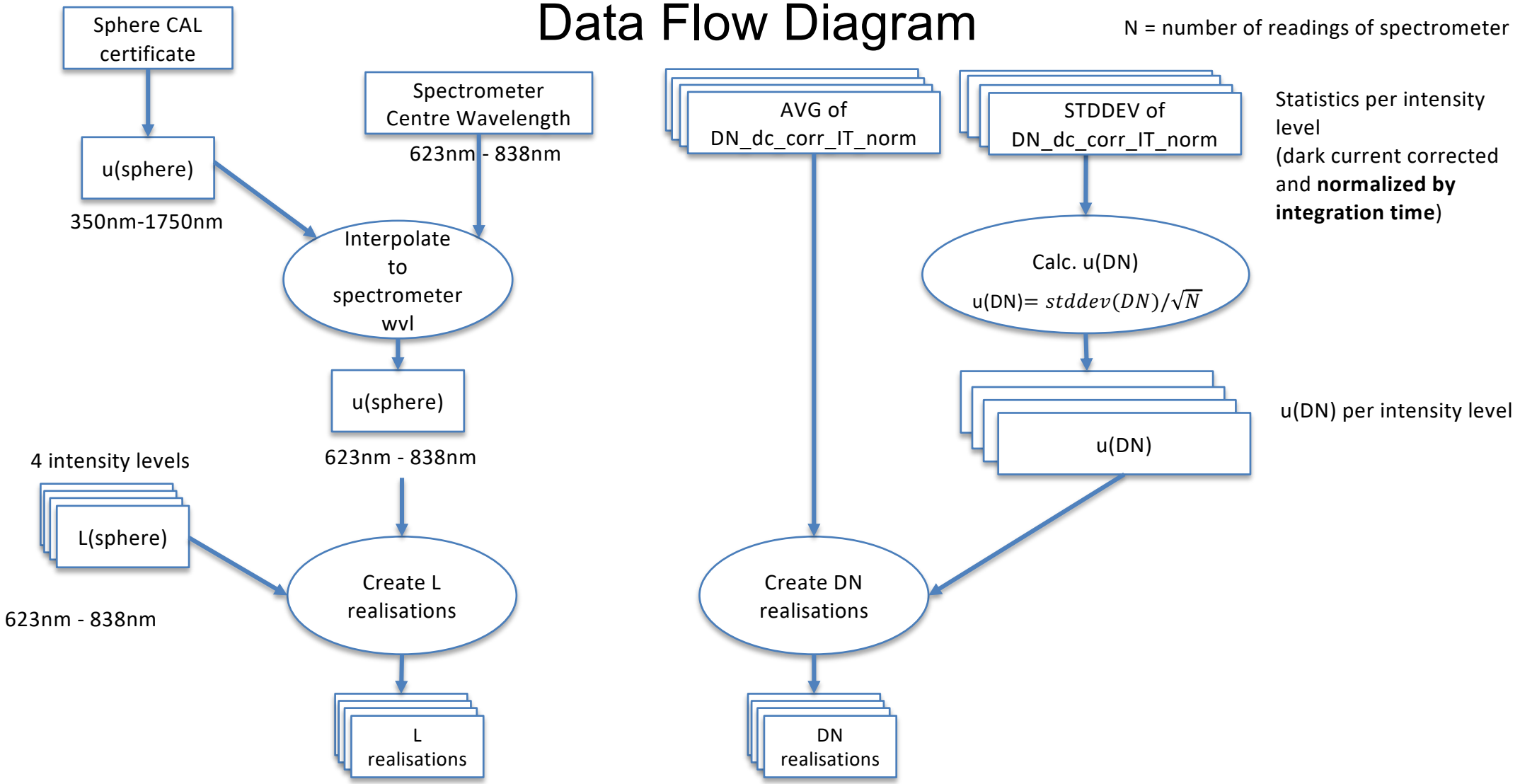



u(L)

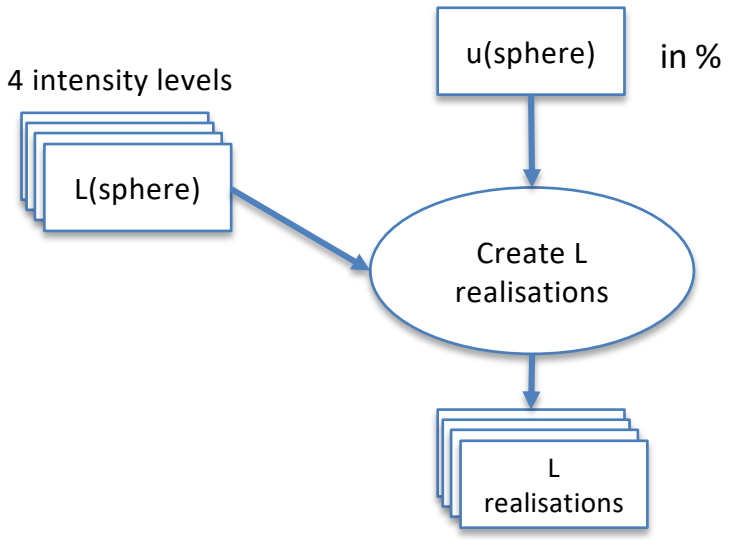
DN = gain x L + o

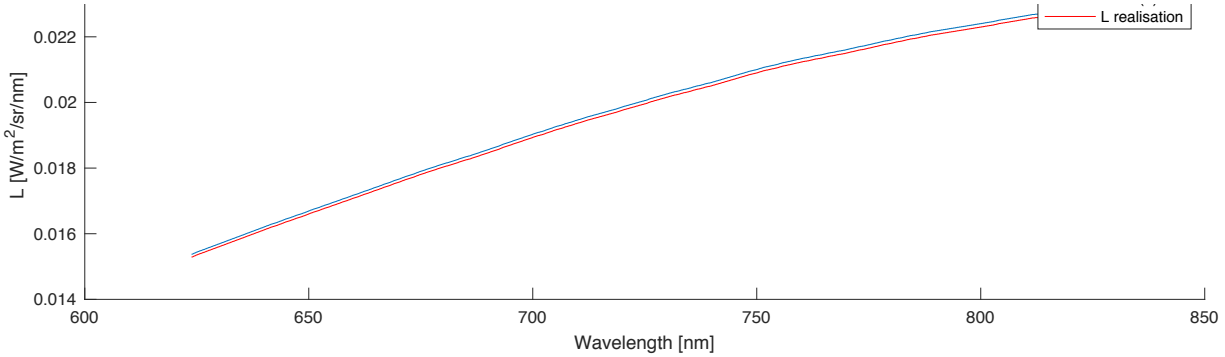
L = (DN – o) / gain

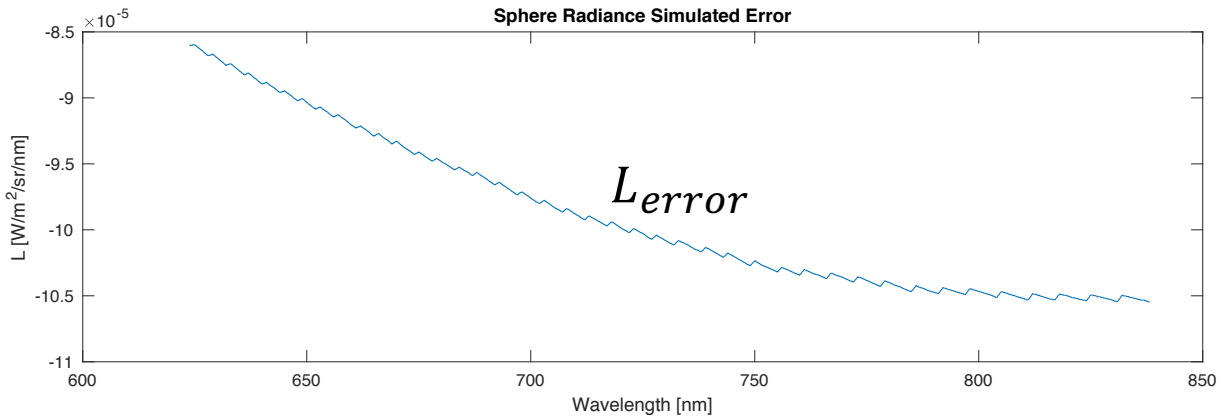
# Data Flow Diagram



# Data Flow Diagram: Create L Realisations




$$L_{realisation} = L_{sphere} + L_{error}$$

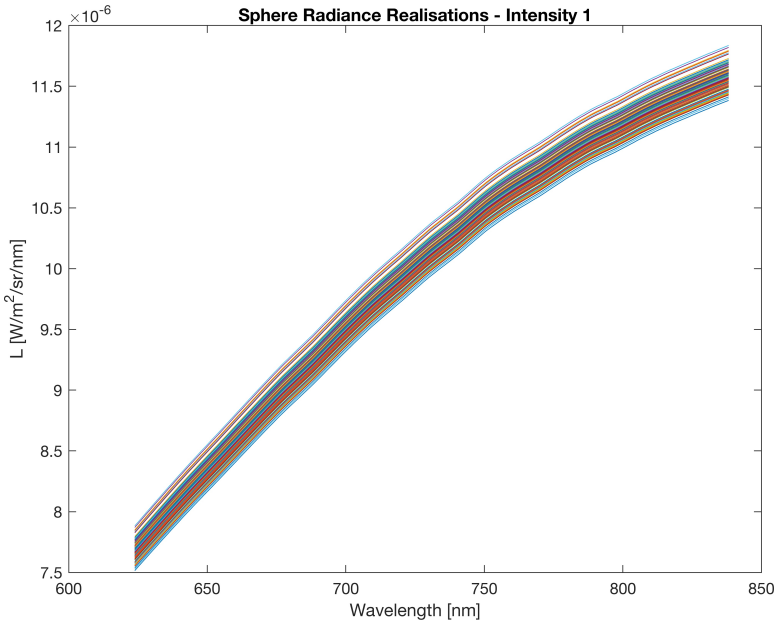
$$u\_abs_{sphere} = \frac{u(sphere) \cdot L_{sphere}}{100}$$

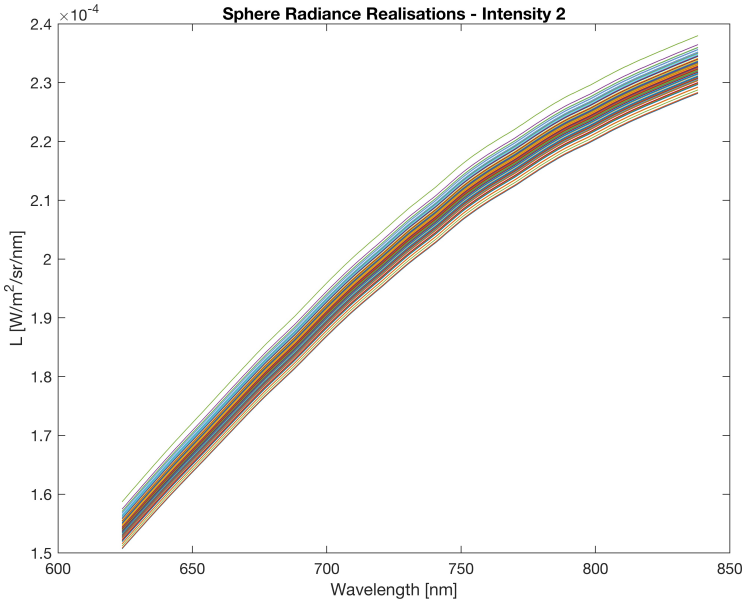
$$L_{error}(\lambda) = u\_abs_{sphere}(\lambda) \cdot rand$$

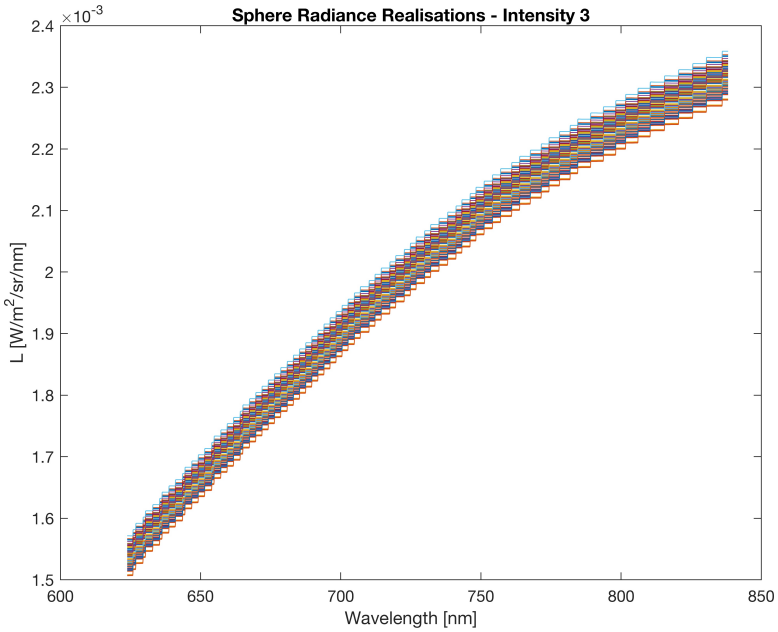
Note: use same random number for all spectral bands to achieve full correlation of uncertainty

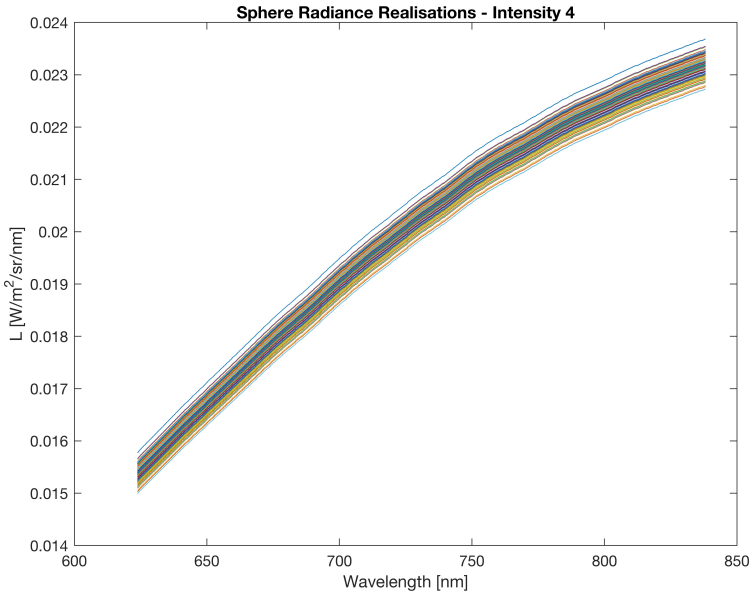
# Creating Sphere Radiance Realisations

Trick: use same random number to draw from PDF. This then assumes full correlation between the spectral bands.

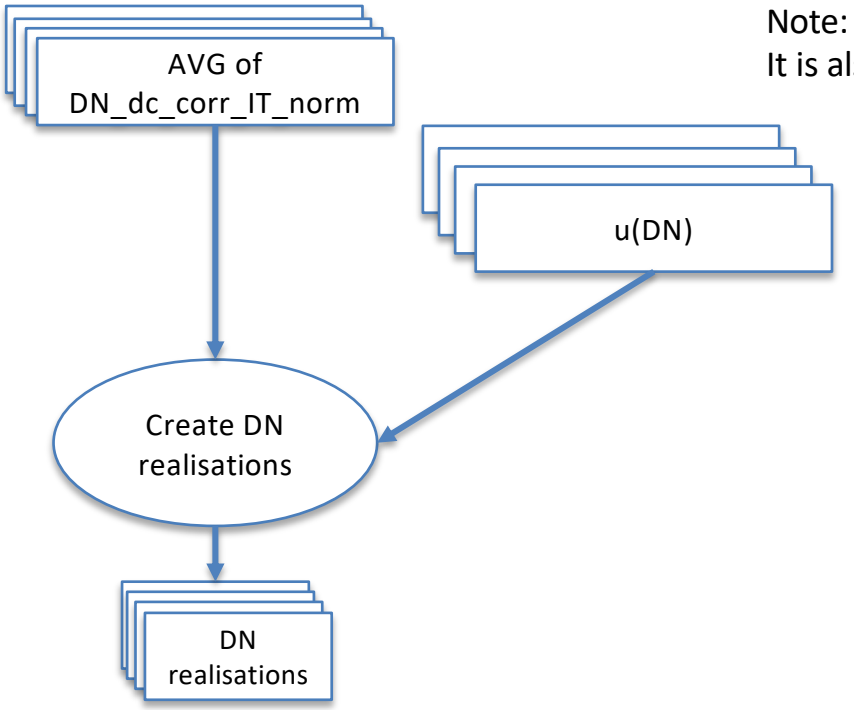


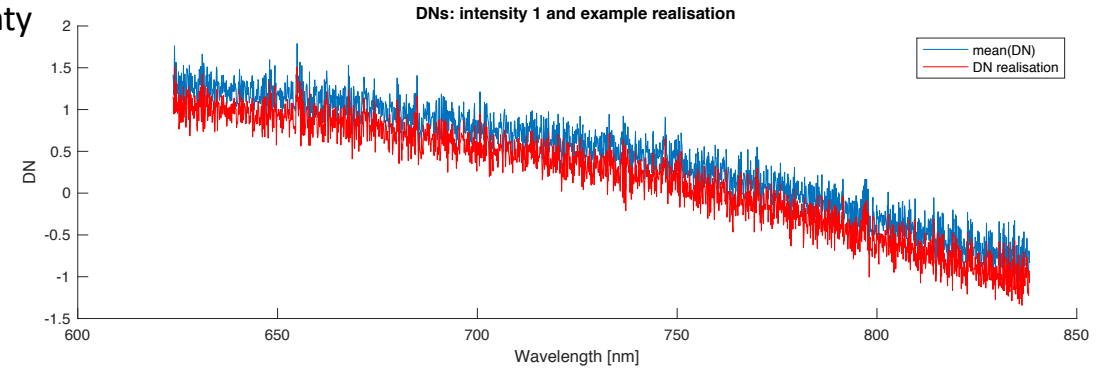



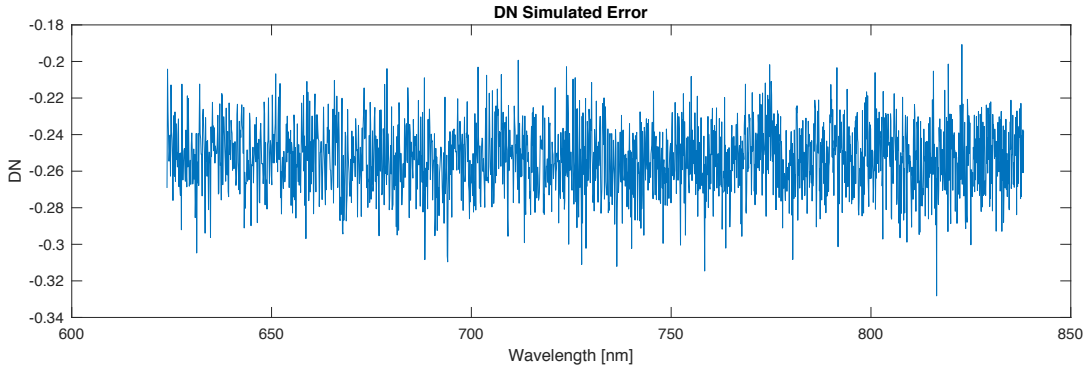
# Data Flow Diagram: Create DN Realisations




Note: the u(DN) are already absolute as they were estimated from the measurements.
It is also important to get the standard deviation of the DNs normalized by integration time.

in DN, i.e. absolute uncertainty



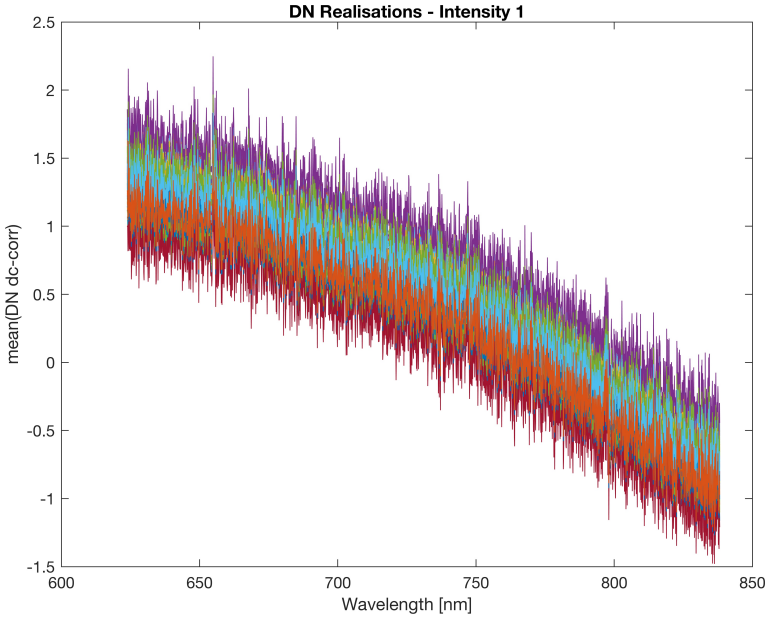

$$DN_{error}\ (\lambda) = u_{DN}(\lambda) \cdot rand(\lambda)$$

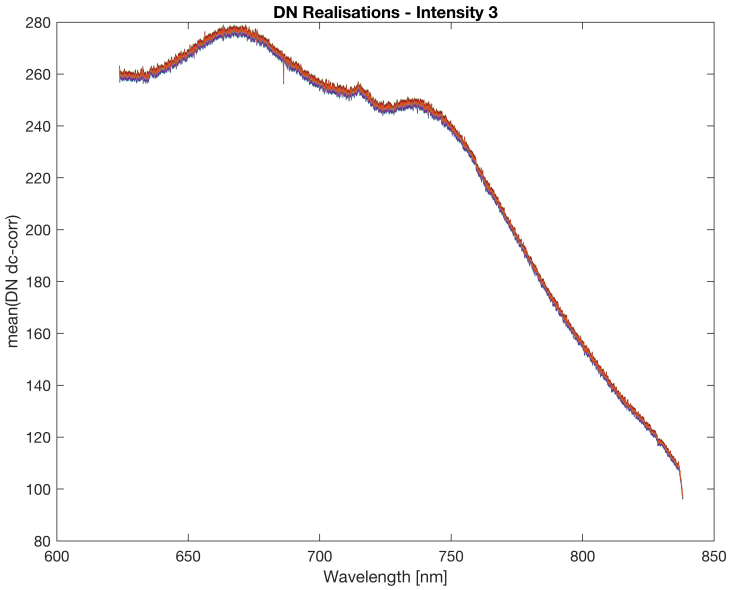
$$DN_{realisation} = DN_{dc\_corr\_IT\_norm} + DN_{error}$$

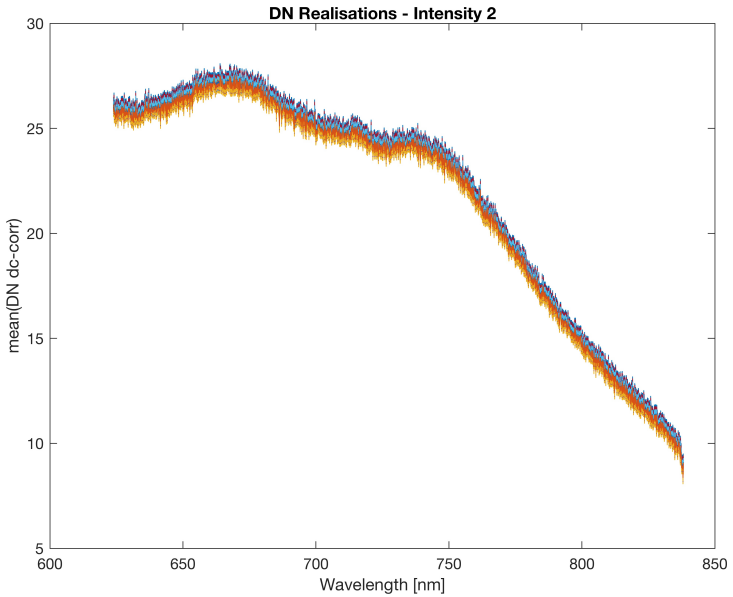
Note: individual random numbers for all spectral bands to achieve uncorrelated noise

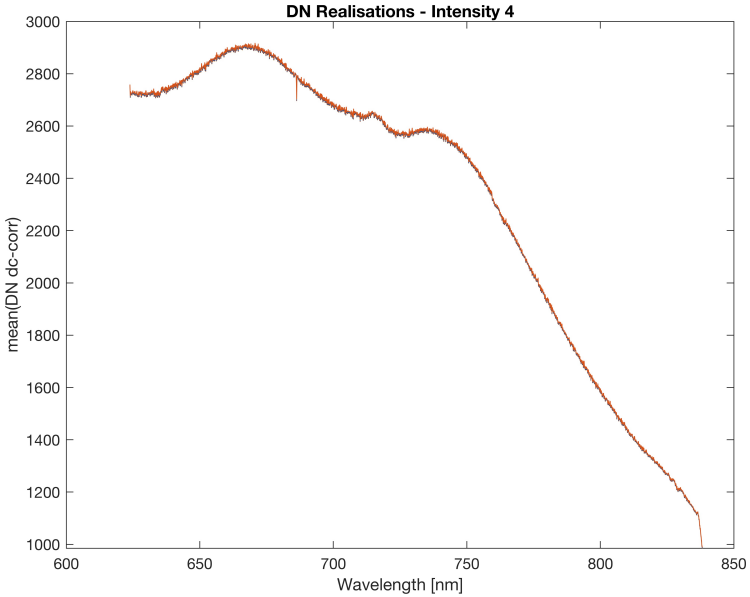
# Creating DN Realisations

Note: noise is random for the DNs

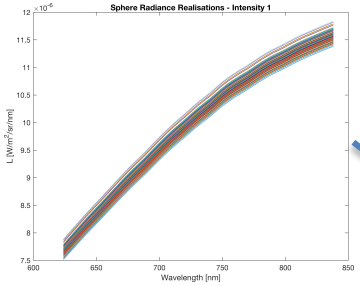


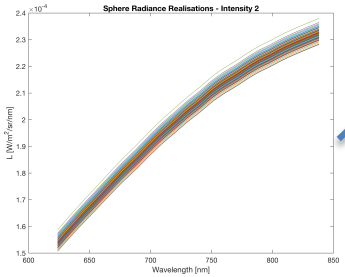



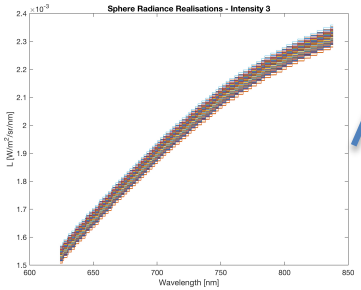



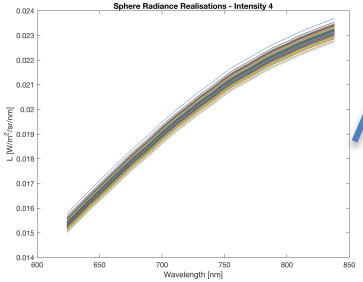



L realisations

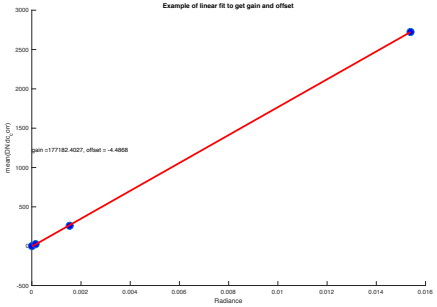


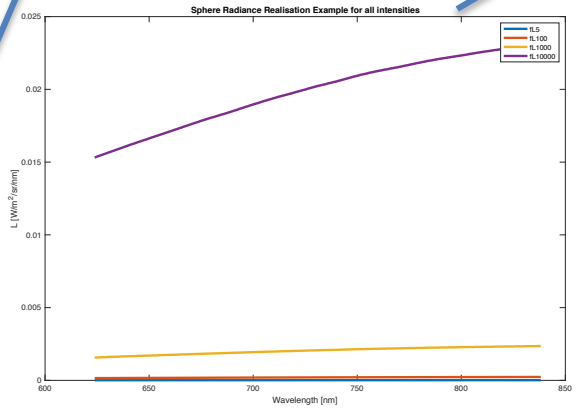

# Dataflow of a combined Monte Carlo Simulation for two PDFs

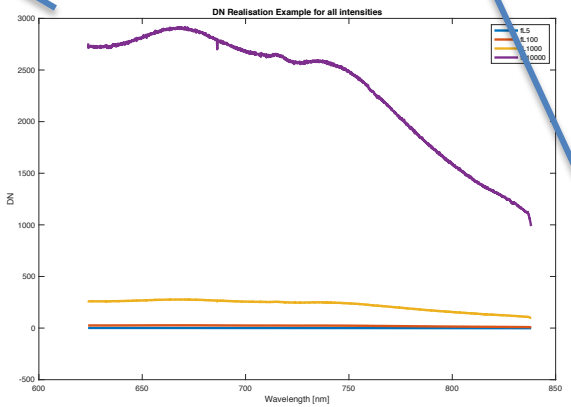



Select a realization from the L realisations

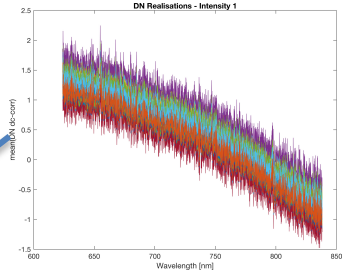
Do first order fitting to get gain and offset for these realisations for all bands

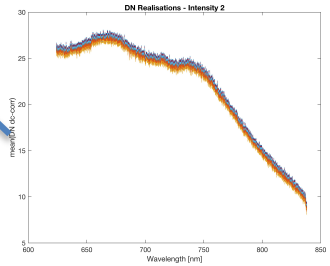
Select a realization from the DN realisations

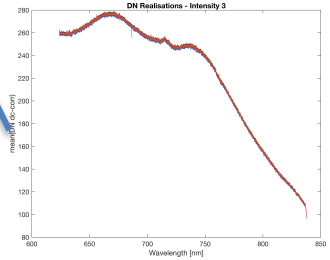


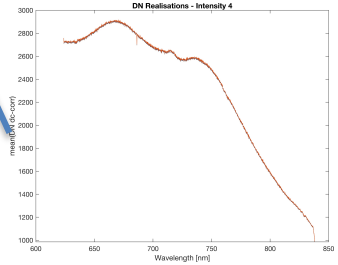




DN realisations



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# Implementation Details: A nested Monte Carlo Run for two PDFs

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```
for n=1:N

    for m = 1:N

        for i=1:size(DN_l_dc,1)

            c = polyfit(L_realisations(:,i,n), DN_realisations(:,i,m), 1);
            gains(cnt,i) = c(1);
            offsets(cnt,i) = c(2);

        end

        cnt = cnt + 1;

    end

end
```

Combine realisations from both L and DN and then do linear fits for all spectral bands

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# How many linear fits will we have to do?

- For N – number of realisations for both DN and L
- N = 100
- For 2047 spectral bands = n_bands

#linear_fits = N * N * n_bands = **20’470’000**

Runtime on a Macbook Pro: 1h 20’

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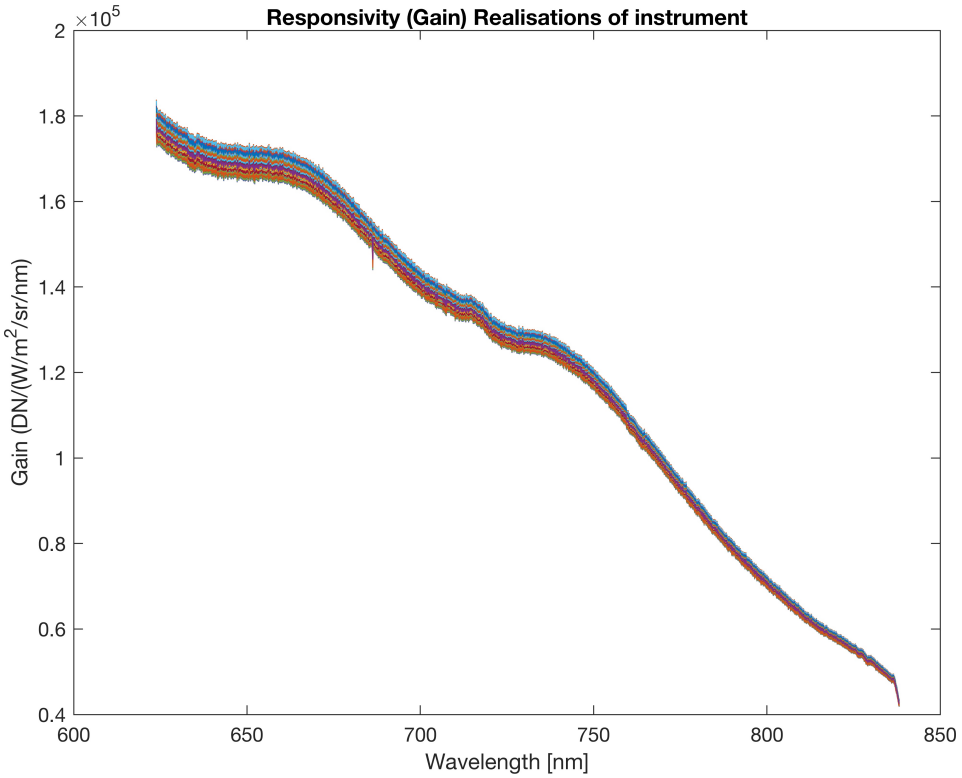
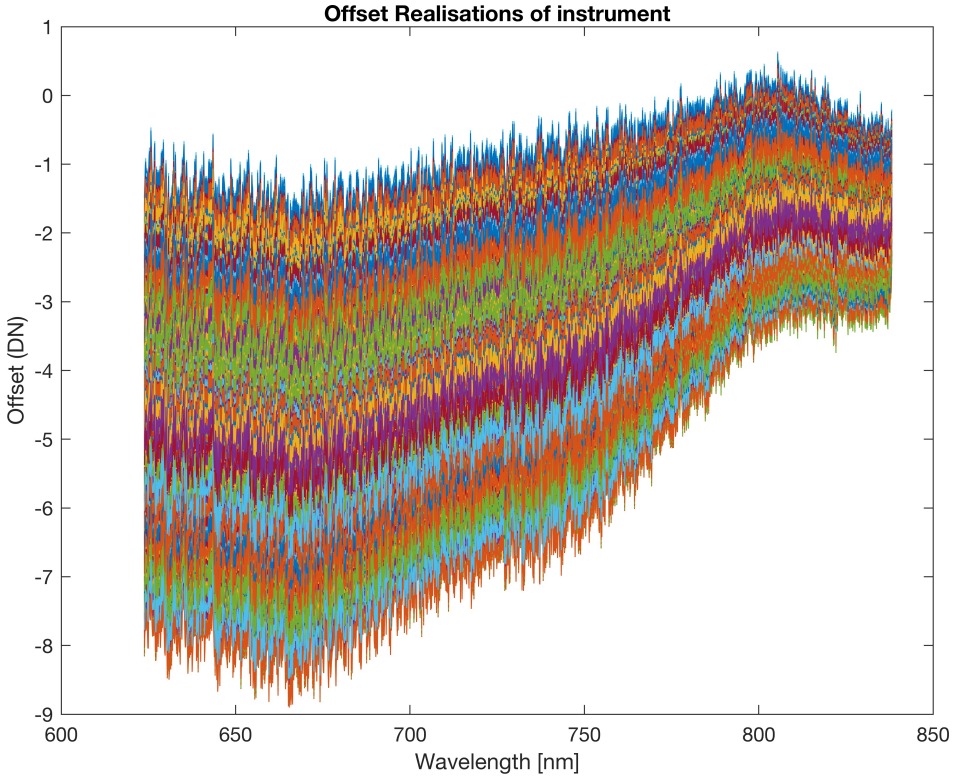
# Realisations of Gains and Offsets



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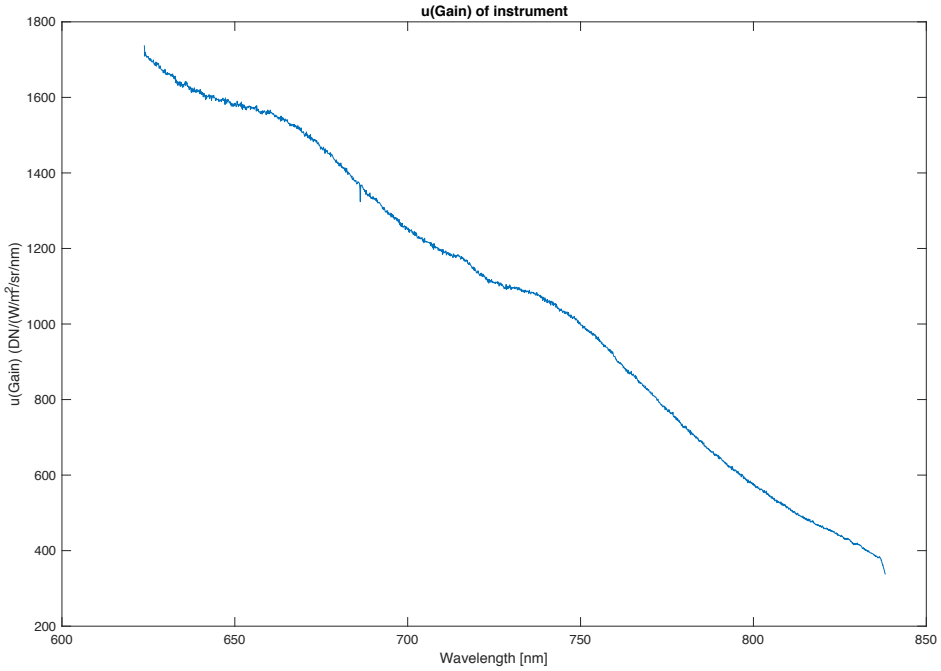
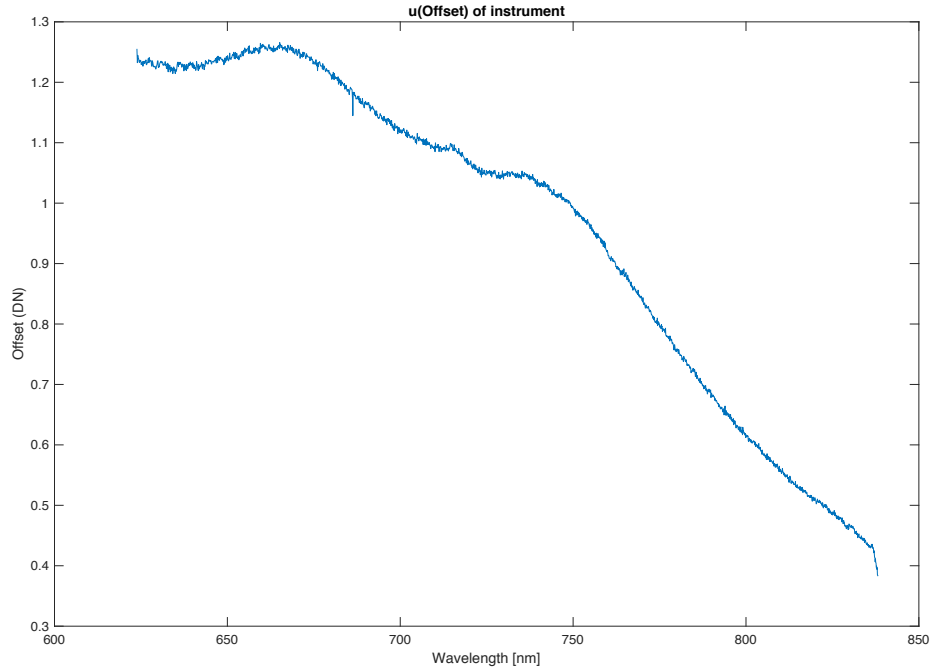
# Uncertainty of Gains and Offsets (Standard deviation of output realisations)



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# Propagating the uncertainty of gain and offset to calibrated radiance

$$u^2(L_{scene}) = \frac{u^2(DN)}{G^2} + \frac{u^2(Offset)}{(-G)^2} + \frac{{L_{scene}}^2 u^2(G)}{(-G)^2} + 2 \cdot u(G, Offset) \frac{L_{scene}}{G^2}$$

$$u(G, Offset) = u(G) * u(Offset) * corrcoeff(G, Offset)$$

For details see Uncertainty Textbook

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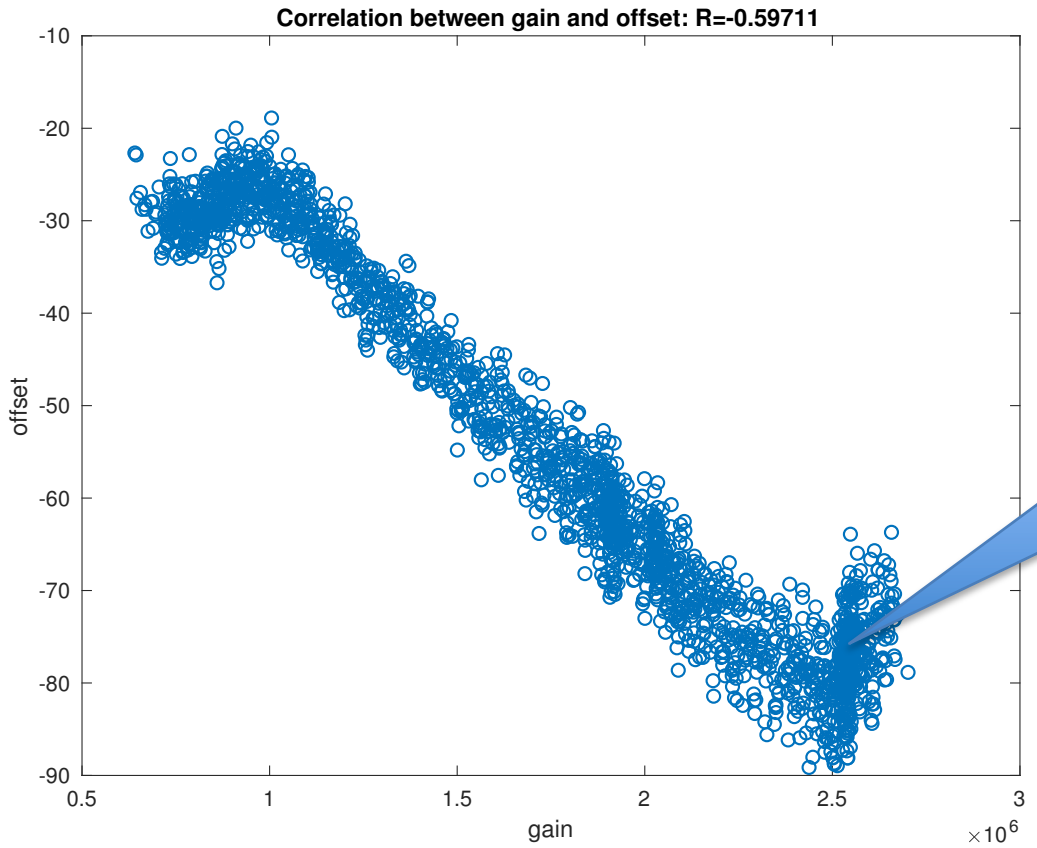
# Correlation of gain and offset




Likely an effect of lowest light setting (sensor non-linearity)

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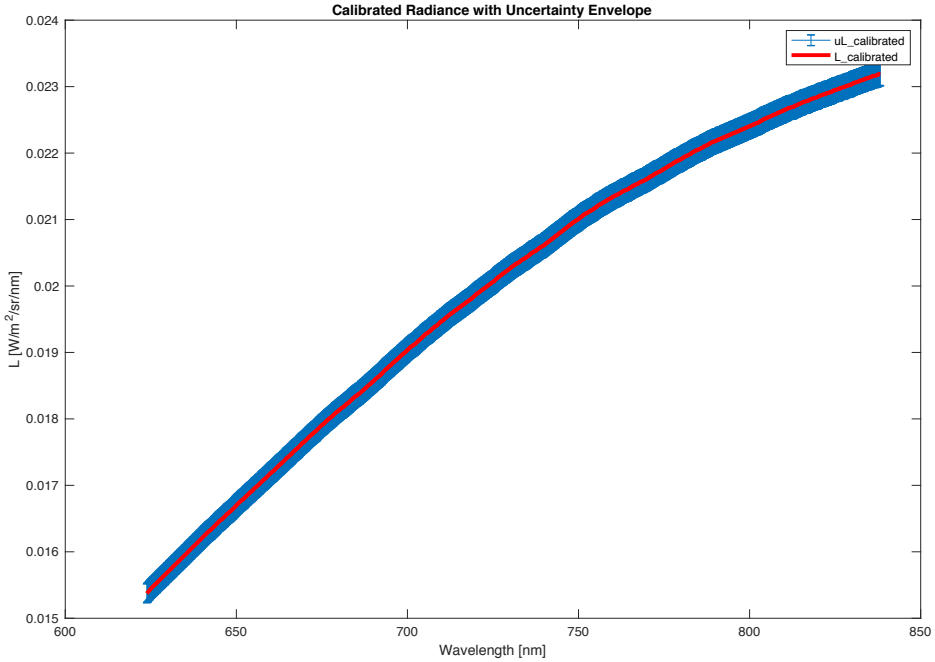
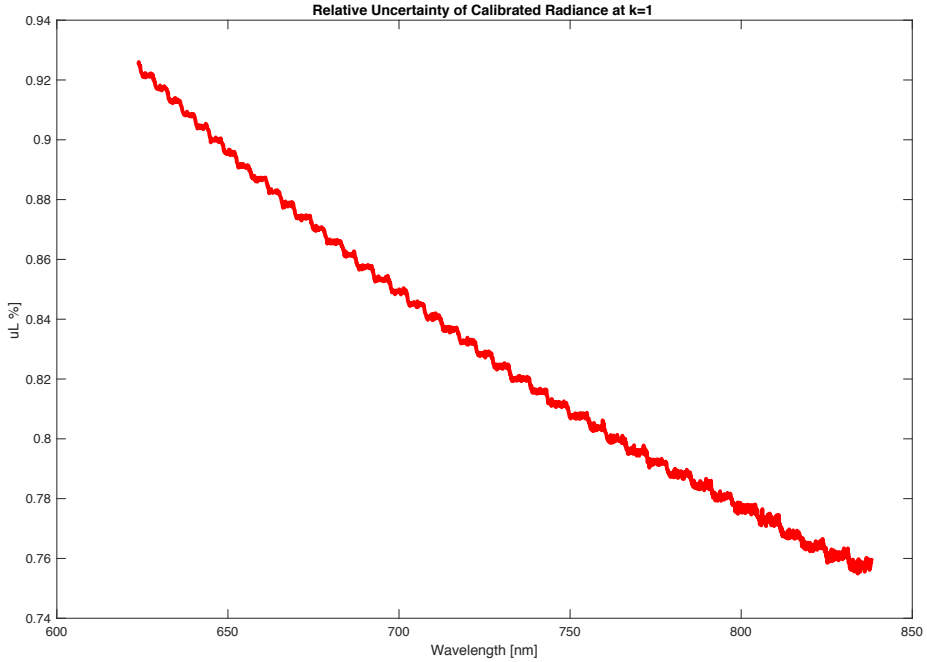
# Computed Uncertainty for a Calibrated Radiance Spectrum



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# Example Code

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| File | Description |
|---|---|
| RAD_CAL_with_Linear_Fit_and_uncertainty_estimation_with_Monte_Carlo.m | Main script |
| print_jpeg.m<br>print_pdf.m | Functions to export figure to JPEG or PDF |
| progressbar.m<br>gui_active.m | Functions for progress bar used to show progress during monte carlo run |
| L_Sphere.mat<br>STD_DN.mat<br>DN_L_CAL.mat<br>uL.mat | Input files for the code:<br>Sphere radiances, standard deviations of DNs, average of DNs the spectrometer recorded during calibration (corrected for dark current), uncertainty of sphere radiances |
| u_rad_coeffs.mat | Output of Monte Carlo run: uncertainties of gain, offset and uncertainty due to gain and offset correlation |
| offset_realisations.mat<br>gain_realisations.mat | Realisation matrices for both gain and offset |
| get_realisations_gauss_dist.m | Function to create realisations |