Obstacle Map

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Square

This is a convex shape, so this can be plotted using the half planes concept.

Half Plane

A line can divide the plane into two halfs, we can differentiate the region in which the point lies i.e left, right or on the line. The equation of the line can be written in the form $f(x, y) = ax + by + c = 0$, the region in which the point lies depends on whether the value of $f(x, y) > 0, f(x, y) < 0 or f(x, y) = 0$.

For the square we divide it into 4 lines, and arrange the points in a counterclockwise direction. So for the given square the points will be $[(100, 82.5), (100, 37.5), (50, 37.5), (50, 82.5)]$. We write the line equation for these points using the slope intercept form of the line equation, so the line equations will be

$$y = mx + c$$

where $m = \frac{y2-y1}{x2-x1}$ and $c = y1 - (m * x1)$
For first line, slope $m1 = \frac{82.5-37.5}{100-100}$ for this case line equation will be $x = 100$ as it is parallel to y-axis for which the slope is Infinity. Similarly for the other points

$$m2 = \frac{37.5 - 37.5}{50 - 100} = 0, \quad c = 37.5 - (0 * 100)$$

Therefore the line equations in the $f(x, y)$ form will be

$$f_1 = x - 100 \quad f_2 = -y + 37.5 \quad f_3 = -x + 50 \quad f_4 = y - 82.5$$

The image in fig 1 shows the line dividing the plane into two halfs.

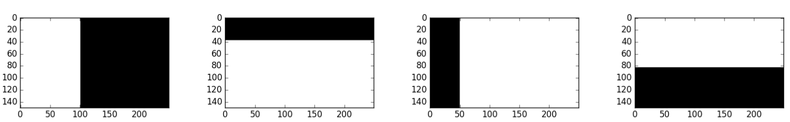

Figure 1: The plots showing the half planes for the square

Now we need to find the intersection of the white region shown in the images in fig. 1. After the intersection we get as shown in fig. 2

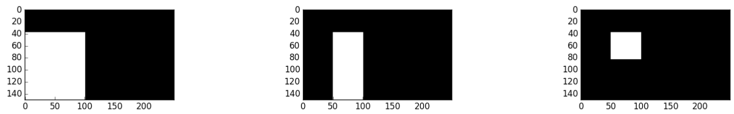

Figure 2: The plots showing the intersection of half planes for the square

Hexagon

This is a non-convex shape, for this we divide the shape into two convex shapes. Then use the same concept of half planes to form the two convex shapes and perform a union on them to get the final non-convex shape.
The hexagon given is divided into two convex shapes as quadrilaterals. The points for the quadrilaterals will be as follows

$$Quad_1 = [(173, 15), (193, 52), (170, 90), (163, 52)]$$

$$Quad_2 = [(163, 52), (125, 56), (150, 15), (173, 15)]$$

By using the slope-intercept form of the line as above in half planes we get the line equations as follows,

$$f_{11} = -1.85x + y + 305.05 \quad f_{12} = -1.652x - y + 370.869$$

$$f_{13} = 5.428x - y - 832.857 \quad f_{14} = 3.7x + y - 655.1$$

$$f_{21} = -0.105x - y + 69.158 \quad f_{22} = 1.64x + y - 261$$

$$f_{23} = y - 15 \quad f_{24} = -3.7x - y + 655.1$$

Using these we get the half planes as shown in the fig. 3 and 4

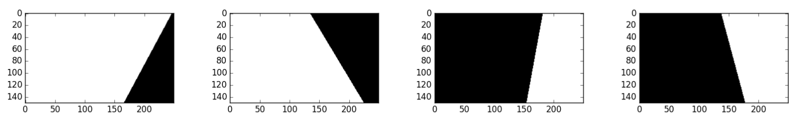

Figure 3: The plots showing the half planes for the first quadrilateral points

After performing the intersection of the planes shown, we get the plots for quadrilateral 1 and 2 as fig. 5 and 6 respectively.

Then we do the union of both these shapes to get the final hexagon.

Circle

This can be modelled using the semi algebraic concept, which says if a point (x, y) lies inside the conic section with $f(x, y)$ as the equation then $f(x, y) < 0$ and point outside we get

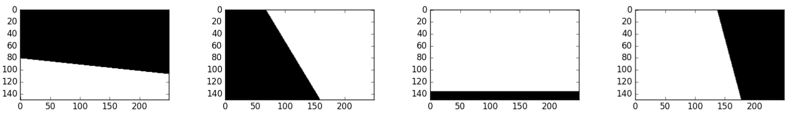

Figure 4: The plots showing the half planes for the second quadrilateral points

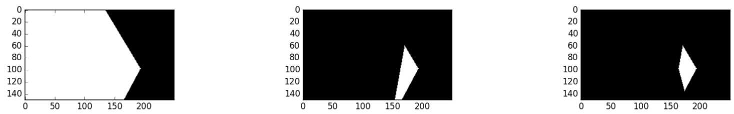

Figure 5: The plots showing the intersection of half planes for the first quadrilateral points

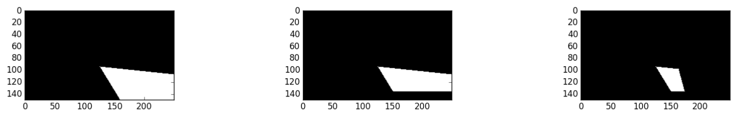

Figure 6: The plots showing the intersection of half planes for the second quadrilateral points

$f(x, y) > 0$. The general equation of circle is $(x-x_c)^2+(y-y_c)^2-radius^2 = 0$ where (x_c, y_c) is the centre of the circle.

For the given circle the centre is $(140, 120)$ and $radius = 15$, so the equation of the circle is

$$f_{cir} = (x-140)^2 + (y-120)^2 - 15^2 = 0$$

. Using this the points in the circle i.e the points for which $f_{cir}(x, y) \leq 0$ is the obstacle points. They are plotted in black color.

Ellipse

The semi algebraic concept can be made use of in this case also. The equation of the ellipse is given for horizontal major axis as

$$f_{hor} = \frac{(x-x_c)^2}{majorRadius^2} + \frac{(y-y_c)^2}{minorRadius^2} - 1$$

and for vertical major axis,

$$f_{ver} = \frac{(x-x_c)^2}{minorRadius^2} + \frac{(y-y_c)^2}{majorRadius^2} - 1$$

Here we use the horizontal major axis with centre at $(190, 130)$ and $majorRadius = 15 \quad minorRadius = 6$. So the equation of the ellipse will be

$$f_{ellip} = \frac{(x-190)^2}{15^2} + \frac{(y-130)^2}{6^2} - 1$$

. The points inside the ellipse can be obtained by using the condition $f_{ellip}(x, y) \leq 0$.

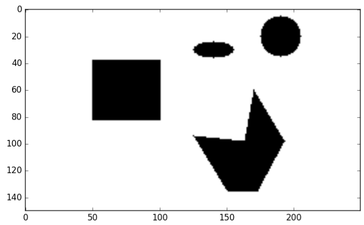

Figure 7: The map produced without expanding obstacles

Enlarging the Obstacles

The obstacles are enlarged by placing the robot centre on every edge point and finiding the union of all the points which are in the intersection between robot and free space. All these points will be the part of the obstacle now, this region is marked in grey in the map just to differentiate. This procedure is known as Minikowski sum.

Also the border of the map is enlarged accordingly, so that when the robot is considered as a point robot we make sure that it doesnot hit the border of the map. The final map can be seen in the fig. 8

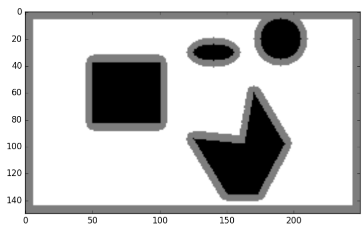

Figure 8: The final map produced

Code

To run the code with custom robot disc size or changing the clearance and also whether or not to save the intermediate step images, the input can be given by using argument parser in python.

To run the code use

$$python \quad Map.py$$

To change the values

$$python \quad Map.py \quad --DiaDisc \quad 20 \quad --clearance \quad 2 \quad --vis \quad True$$

where robot size is changed to 20mm diameter with clearance 2 and this saves the intermediate plots.
Also the location or the polygon shapes can be changed by tweaking the values in the $main()$ function in the code.