# The Trickling Up of Excess Savings

Adrien Auclert* Matthew Rognlie† Ludwig Straub‡

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**Abstract**

We provide a simple framework connecting the distribution of excess savings across households to the dynamics of aggregate demand. Deficit-financed fiscal transfers generate excess savings. The poorest households with the highest MPCs spend down their excess savings the fastest, increasing other households' incomes and their excess savings. This leads to a long-lasting increase in aggregate demand until, ultimately, excess savings have "trickled up" to the richest savers with the lowest MPCs, raising wealth inequality.

In the wake of the Covid pandemic, households accumulated a very large stock of "excess savings", which they have only recently begun to deplete. Figure 1(a) shows that the U.S. personal savings rate first rose very rapidly in 2020, more than doubling relative to its long-term average, then started falling below that average in late 2021. Figure 1(b) shows an estimate of the resulting stock of excess savings by the Federal Reserve Board (Aditya Aladangady, David Cho, Laura Feiveson and Eugenio Pinto 2022). This stock has only modestly fallen from its peak. In mid-2022, it still stood at $1.7trn, or 6.7% of GDP.

Because excess savings and their distribution across the population intuitively matter for aggregate demand, economists have paid a considerable amount of attention to estimating both. In this paper, we provide a tractable Heterogeneous Agent New Keynesian model that explicitly maps the distribution of excess savings to the path of output, and that explains the process by which their effect dissipates. We use this framework to estimate the likely contribution of excess savings to aggregate spending in the coming years under various assumptions about the marginal propensities to consume (MPCs) of agents holding the savings and scenarios for monetary policy.

Our framework recognizes that one person's spending is another person's income. As we show, taking this fact into account implies that excess savings from debt-financed transfers have

*Stanford University and NBER. Email: aauclert@stanford.edu.

†Northwestern University and NBER. Email: matthew.rognlie@northwestern.edu.

‡Harvard University and NBER. Email: ludwigstraub@fas.harvard.edu.

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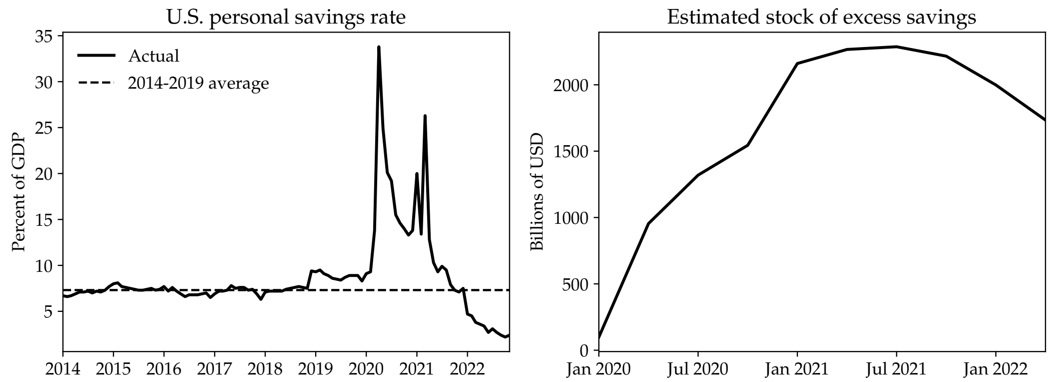


Source: The personal savings rate is from the Bureau of Economic Analysis (FRED code: PSAVERT). The estimated stock of excess savings is from Aladangady et al. (2022).

Figure 1: U.S. personal savings rate and excess savings

much longer-lasting effects than a naive calculation would suggest. In a closed economy, unless the government pays down the debt used to finance the transfers, excess savings do not go away as households spend them down. Instead, the effect of excess savings on aggregate demand slowly dissipates as they "trickle up" the wealth distribution to agents with lower MPCs. Tight monetary policy speeds up this process, but this effect is likely to be quantitatively modest.

# 1 Model

We consider a continuous time model with $N$ types of households, $i = 1, \dots N$. Agents with higher $i$ have lower instantaneous marginal propensities to consume $m_i$, with agent $N$ having an MPC of zero, $m_1 > m_2 > \cdots > m_N = 0$. Motivated by the empirical evidence on the negative correlation between MPCs and wealth, we think of agents with higher $i$ as being initially richer, with agent $N$ being the richest. While this is a useful interpretation, it is not strictly necessary: what is important is the distribution of $m_i$ across types $i$.

At $t = 0$, the government distributes a transfer $a_{i0}$ to households, issuing debt $B = \sum_{i=1}^{N} a_{i0}$ to finance the transfer, and maintaining a constant debt level thereafter. We first consider an "easy monetary policy" scenario in which the central bank responds by holding constant the real interest rate at its steady state level of zero, $r = 0$. This implies, in particular, that the additional debt requires no change in taxes.

Each type's behavior is described by a utility function over consumption and assets. Agents understand the central bank's announcements of future real interest rates $r_t$ (here $r = 0$), but they assume that future aggregate income $Y_t$ remains permanently at its steady state level.[1] Agent type $i$ earns a fixed proportion $\theta_i \in (0, 1)$ of total income $Y_t$.

We linearize this model around the steady state where each agent type owns a certain stock of assets (with higher-type agents plausibly holding more wealth). This delivers the following

[1]In the words of Emmanuel Farhi and Iván Werning (2019), agents have level-$k$ thinking with $k = 1$. This makes the model particularly tractable. We later consider the case with rational expectations.

equations:

$$c_{it} = m_i a_{it}; \qquad \dot{a}_{it} = \theta_i Y_t - c_{it}; \qquad Y_t = \sum_{i=1}^{N} c_{it} \tag{1}$$

where $Y_t$ is aggregate demand and income, $c_{it}$ is type $i$'s consumption, $a_{it}$ his asset holdings (all relative to their steady state level), and $m_i \in [0, \infty)$ his instantaneous MPC out of liquid assets. The $\theta$'s, which satisfy $\sum_{i=1}^{N} \theta_i = 1$, are the income shares across the types. The equations in (1) give a tractable version of the intertemporal Keynesian cross (Adrien Auclert, Matthew Rognlie and Ludwig Straub 2018).

An unconventional feature of our model is that it assumes the presence of agents with zero MPC. One can interpret these type-$N$ agents as standard permanent-income agents, in the limit where their discount rate goes to zero, but alternative interpretations are possible. First, they could stand in for the rest of the world. Second, they could represent the government receiving a fraction of aggregate income via taxation, and using it to pay down the debt. Finally, they could represent zero MPC financial accounts, such as retained earnings saved by firms or pension funds.

One natural objection to the model in (1) is that it assumes that monetary policy maintains an easy stance of $r = 0$ in the face of high demand. To address this, we extend our model by assuming that monetary policy tightens as it sees higher demand, reacting with a rule $r_t = \phi Y_t$.[2] Since higher demand will naturally be associated with higher inflation, an alternative interpretation of this rule is that monetary policy tightens in reaction to the inflation generated by excess savings. Online appendix A derives the equations characterizing the model in this case.

Another objection to the model in (1) is that it relies on imperfect foresight by agents. Online appendix A also derives the equations characterizing the model when agents have rational expectations about interest rates $r_t$ as well as income $Y_t$.

**Partial equilibrium analysis.** A naive partial equilibrium approach to calculating the effect of excess savings on spending would be to ignore the endogeneity of output, instead assuming that $Y_t$ remains at its normalized steady state level of 0 forever. Solving out for (1) in this case, we find that aggregate demand is given by:

$$C_t = \sum_{i=1}^{N-1} m_i e^{-m_i t} a_{i0} \tag{2}$$

Equation (2) delivers a simple way to map a distribution of MPCs and excess savings by type into an effect on aggregate spending: take type $i$'s initial stock of savings, and apply to it an exponential distribution for spending with mean $1/m_i$.[3] A simple back-of-the envelope calculation using

[2] To neutralize the income effects of changing interest rates, in this extension, we assume that all agents types start with a steady state level of wealth of 0. We think of this as proxying for the presence of long duration assets, which hedge agents against interest rate risk.

[3] This functional form characterizes the intertemporal marginal propensities to consume of agents with assets in the utility; once multiple types of such agents are mixed together, the model's aggregate dynamics are similar to those of alternative heterogeneous-agent models. See Auclert, Rognlie and Straub (2018).

this equation suggests that, for the United States, the remaining excess savings might only affect aggregate demand for a few quarters (see table 1).

This approach, however, fails to recognize that one agent's spending is another agent's income. Ignoring this fact has important consequences: if agents simply spent down their excess savings without raising anyone else's income, then no one would be purchasing the assets they sold in the process. But this is inconsistent with the government keeping its debt constant. As we show next, recognizing this fact implies a much greater persistence of excess savings and output than equation (2) suggests.

## 2 The trickling-up effect

We now explicitly solve the dynamical system in (1). We begin with a simple observation about the steady state of this system.

**Proposition 1** (Long-run trickling up)**.** *In the long run, type $N$ owns all the debt:* $\lim_{t\to\infty} a_{Nt} = B$.

This result follows immediately from the fact that type $N$ has $m_N = 0$, so that its asset dynamics are given by $\dot{a}_{Nt} = \theta_N \left( \sum_{i=1}^{N-1} m_i a_{it} \right)$. Hence, as long as other agents have excess savings, they spend them down, increasing the income and therefore the savings of the richest type. Since the government keeps its debt position constant, $\sum_{i=1}^{N} a_{it} = B$ at all times, in the long run all types have zero assets, except for type $N$, which owns all of $B$. At this point, excess savings have "trickled up" to agents with the highest $i$. Given our interpretation of type $N$ as being initially the richest agent, we see that any initial transfer, no matter how targeted it is to the poor, eventually ends up raising wealth inequality.

**Proposition 2** (Trickling-up dynamics)**.** *Assume that $m_i a_{i0}/\theta_i$ decreases in $i$. Then the distribution of assets across types $i$ at any later date $t'$ first-order stochastically dominates the distribution at any earlier date $t < t'$: $\sum_{i=1}^{n} a_{it'} < \sum_{i=1}^{n} a_{it}$ for all $n < N$.*

This result, proved in online appendix B, shows the exact sense in which excess savings trickle up: no matter where we look in the distribution of excess savings, as time passes, the wealth held by all lower types is falling, and the wealth held by all higher types is rising. The only necessary condition is that excess savings initially cause a larger percentage increase in spending among poorer agents, which is easily satisfied since they have higher MPCs.

**Proposition 3** (Slow dissipation)**.** *In the long-run, $Y_t \sim e^{-\lambda t}$: aggregate demand and excess savings dissipate at rate $\lambda$, where $\lambda < m_{N-1}$. Hence, excess savings have a strictly longer-lasting effect on demand than the naive partial equilibrium calculation in (2) would suggest.*

In the partial equilibrium calculation from equation (2), spending eventually becomes dominated by type $N-1$ agents, decaying at rate $m_{N-1}$. Proposition 3 shows that general equilibrium spending dissipates strictly more slowly than this. Intuitively, this is because the spending from any type sustains income from any other type as the wealth of all agents goes to zero.

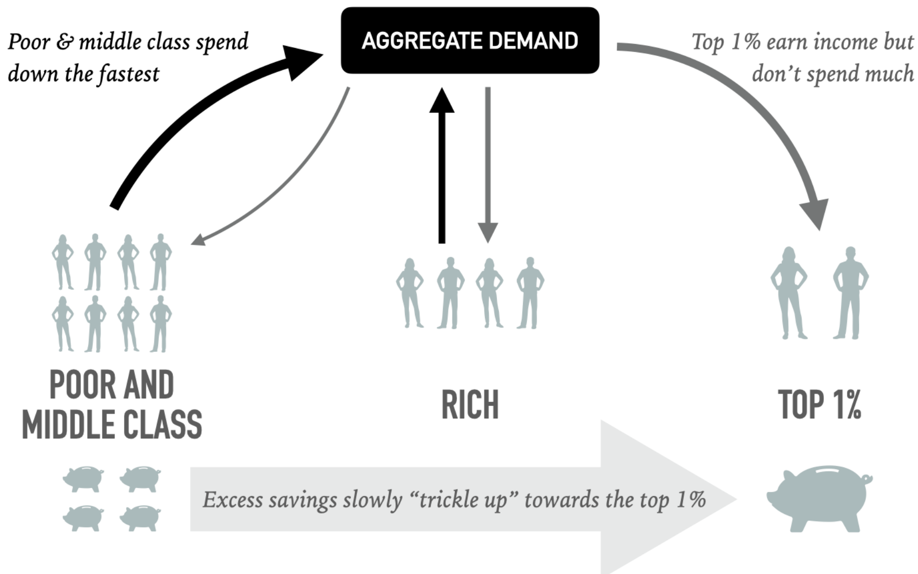


Figure 2: The trickling up effect.

Figure 2 illustrates the adjustment process characterized by Propositions 1–3. Dark arrows flow from agent types to aggregate demand $Y_t$ via their spending ($m_i$), with lower types spending down their assets faster. Gray arrows flow from aggregate demand to the income of these agents, and are more equally distributed across the population ($\theta_i$), with type-$N$ agents receiving a significant share. Running this system forward, we see that excess savings slowly trickle up the wealth distribution, until type $N$ agents own all of the assets.

**Three-type example** We now specialize the model to a case with $N = 3$ types. This case is simple to analyze graphically, and provides additional analytical insights into the trickling up of excess savings. We think of type 1 as representing the poor and the middle class; type 2 as representing the rich; and type 3 as representing the super-rich. Manipulating the equations in (1), we see that the dynamics of excess savings for the first two types satisfy:

$$\begin{pmatrix} \dot{a}_1 \\ \dot{a}_2 \end{pmatrix} = \begin{pmatrix} -m_1(1-\theta_1) & \theta_1 m_2 \\ \theta_2 m_1 & -m_2(1-\theta_2) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Once we have solved for $(a_{1t}, a_{2t})$, it is easy to back out $a_{3t} = B - (a_{1t} + a_{2t})$.

Figure 3 visualizes this dynamical system using a phase diagram for $(a_1, a_2)$. The locus for $\dot{a}_1 = 0$ is given by $a_2 = \frac{\theta_2+\theta_3}{1-(\theta_2+\theta_3)} \frac{m_1}{m_2} a_1$; to the right of this locus, the assets of type 1 agents decline. The locus for $\dot{a}_2 = 0$ is flatter, at $a_2 = \frac{\theta_2}{1-\theta_2} \frac{m_1}{m_2} a_1$; to the right of this locus, type 2 assets increase. The dynamics of the wealth distribution are then given by the arrows on the graph, splitting the positive quadrant into three regions: two regions close to the axes in which agents' assets move in opposite directions, and a middle cone in which both agents' assets decline together. In the

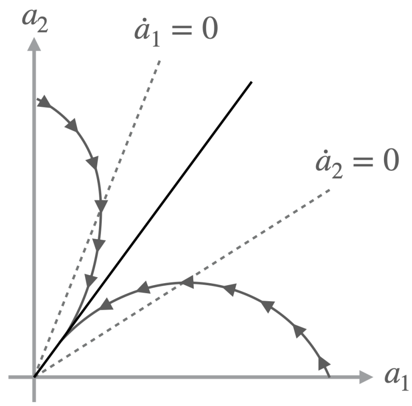


Figure 3: Phase diagram for $N = 3$ case

scenario where initial $a_2$ is low relative to $a_1$, type 2 agents initially increase their assets as the spending by type 1 agents initially boosts their incomes and savings, before reaching a second phase in which both types' assets decline as the super-rich accumulate. We formalize this situation in the following proposition:

**Proposition 4.** *Assume that type-1 agents initially own a sufficiently large share of assets, $\theta_2 m_1 a_{10} > (1 - \theta_2) m_2 a_{20}$. Then, type 2 agents first accumulate assets before spending them down.*

The hump-shaped response of savings of type 2 agents is a simple manifestation of the trickling up effect from proposition 2.

## 3 Application to the U.S.

We use our model to quantify the likely impact of the stock of excess savings estimated by Aladangady et al. (2022) on aggregate demand and its likely duration. We follow the three type classification outlined in section 2. We set the time units so that $t = 1$ corresponds to a quarter. The parameters of the model are $\theta_i$, $m_i$, and $a_{i0}$ for each $i$.

We interpret types as follows: type 1 is the bottom 80% of the U.S. wealth distribution, type 2 is the next 19%, and type 3 is the top 1%. In the 2019 Survey of Consumer Finances, the bottom 80% of the U.S. wealth distribution earns 47% of income, the next 19% earns 38%, and the top 1% earns 15%. We assume that marginal income is distributed like average income; this implies our $\theta_i$'s. Next, we assume a realistically high quarterly MPCs for the middle class and the rich, $mpc_1 = 0.4$ and $mpc_2 = 0.2$. We then convert these numbers to instantaneous MPCs using the formula $1 - e^{-m_i} = mpc_i$. Finally, we assume that the excess savings have only started to trickle up the wealth distribution, with the middle class owning 60% and the rich owning 30% of the stock of excess savings. Finally, we take the total stock to be $B = 6.7\%$ of GDP, as estimated by these authors. While the exact numbers entering our calculations are highly uncertain, table 1 shows that our results are robust to reasonable alternative calibrations.

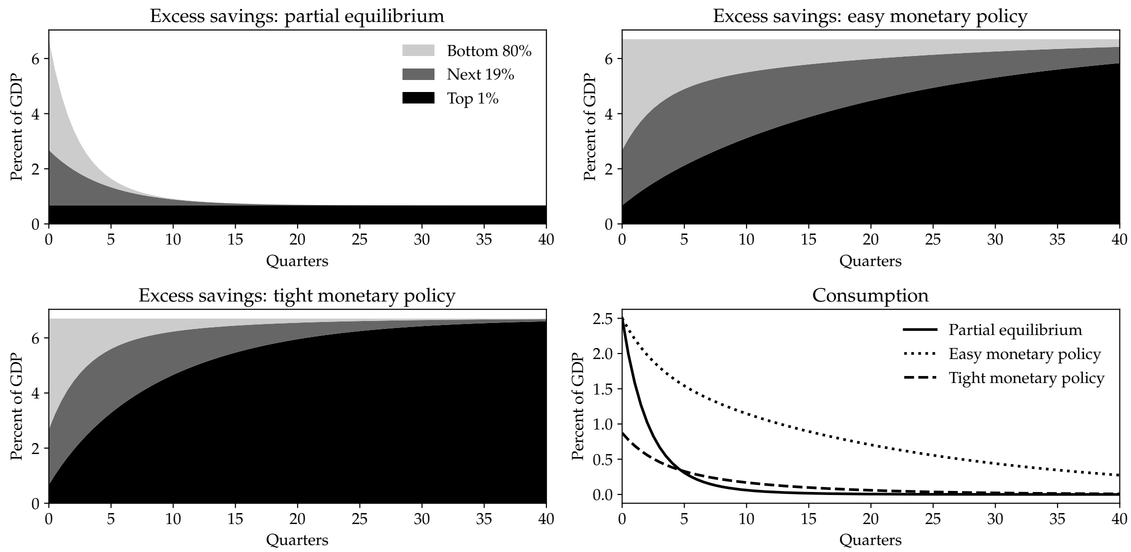


Figure 4: Dynamic evolution of the distribution of excess savings and aggregate demand

| | Duration of output and excess savings | | |
|---|---|---|---|
| Scenario | Output $Y$ | Middle-class $a_1$ | Rich $a_2$ |
| Partial equilibrium | 3 | 2 | 4 |
| Benchmark | 20 | 19 | 22 |
| Lower MPCs ($mpc_1 = 0.3$, $mpc_2 = 0.1$) | 38 | 34 | 43 |
| More excess savings to rich ($a_{10} = a_{20} = 0.45B$) | 21 | 20 | 22 |
| More earnings to rich ($\theta_1 = 0.3$, $\theta_2 = 0.55$) | 23 | 19 | 26 |
| Rational expectations | 8 | 6 | 10 |
| Tight monetary policy ($\phi = 1.5$) | 8 | 7 | 11 |

Note: the time unit is a quarter. Given that $r = 0$, the duration of a variable $X_t$ is defined as $\int tX_t dt / \int X_t dt$. Our benchmark calibration has $mpc_1 = 0.4$, $mpc_2 = 0.2$, with $m_i = -\log(1 - mpc_i)$; income shares $\theta_1 = 0.47, \theta_2 = 0.38$, $\theta_3 = 0.15$; and initial assets $a_{10} = 0.6 \cdot B$, $a_{20} = 0.3 \cdot B$, with $B = 6.7\%$ of GDP. For the monetary response scenario, we assume that agents have an elasticity of intertemporal substitution of 1/2.

Table 1: Duration of output and excess savings by type under alternative scenarios (in quarters)

Figure 4 reports the evolution of the distribution of savings across types in three alternative scenarios. The top left panel shows the outcome of a partial equilibrium analysis: all types except the first quickly run down their excess savings, and after a few years only 10% of the U.S. debt is held by super-rich U.S. residents. The top right panels shows our general equilibrium benchmark instead, in which the debt is continuously held domestically.[4] This visualizes the trickling up phenomenon: the share of wealth held by the rich initially rises (the parametric restriction for a hump shape is satisfied), and the super-rich keep accumulating assets until they hold all of the excess savings. The bottom left panel shows what happens under a tight monetary policy scenario, with $\phi = 1.5$. The qualitative trickling up patterns are unchanged, but the monetary response does speed up the adjustment process. The bottom right panel summarizes the effect of excess savings on aggregate consumption. These effects are long-lasting and significant. In addition to speeding up the adjustment, the monetary response brings down the level (not shown).

Table 1 summarizes our results by displaying the duration of output and excess savings for the middle class and the rich under each of our scenarios. The partial equilibrium scenario summarizes the conventional wisdom according to which the effect of excess savings will dissipate in a few quarters. By contrast, our benchmark scenario suggests that these effects will stick around for roughly 5 years. These numbers are larger if MPCs are lower, and are robust to plausible alternative calibrations. Rational expectations about the future boom make the response much larger on impact due to current spending out of anticipated income, which turns out to speed up the trickling up process. Tight monetary policy, on the other hand, also speeds up trickling up, but it does so by mitigating the effects of excess savings on demand. In either case, however, the duration of excess savings and output remains more than twice as long as the conventional wisdom suggests.

## References


**Aggarwal, Rishabh, Adrien Auclert, Matthew Rognlie, and Ludwig Straub.** 2022. "Excess Savings and Twin Deficits: The Transmission of Fiscal Stimulus in Open Economies." *NBER Macroeconomics Annual 2022*, Forthcoming.

**Aladangady, Aditya, David Cho, Laura Feiveson, and Eugenio Pinto.** 2022. "Excess Savings during the COVID-19 Pandemic." *FEDS Notes*, Washington: Board of Governors of the Federal Reserve System.

**Auclert, Adrien, Matthew Rognlie, and Ludwig Straub.** 2018. "The Intertemporal Keynesian Cross." National Bureau of Economic Research, Working Paper 25020.

**Farhi, Emmanuel, and Iván Werning.** 2019. "Monetary Policy, Bounded Rationality, and Incomplete Markets." *American Economic Review*, 109(11): 3887–3928.


[4] Rishabh Aggarwal, Adrien Auclert, Matthew Rognlie and Ludwig Straub (2022) consider an intermediate case where the U.S. is a partially open economy. With home bias in spending, the outcome is similar to our closed-economy simulations, except that we can interpret the top 1% as the foreigners.

# The Trickling Up of Excess Savings

Adrien Auclert, Matthew Rognlie, and Ludwig Straub

Online Appendix

## A Deriving the main model and its extensions

### A.1 Characterizing household with bonds-in-utility preferences.

The objective of each household $i$ is to choose consumption to maximize discounted flow utility

$$\int_0^\infty e^{-\rho t} \left(u(c_{it}) + v_i(a_{it})\right) dt \tag{3}$$

where $u$ is flow utility from consumption and $v_i$ is type-specific utility from assets, subject to the flow budget constraint

$$\dot{a}_{it} = r_t a_{it} + \theta_i Y_t - c_{it} \tag{4}$$

where $\theta_i$ is the type's share of aggregate income.

This problem delivers the standard intertemporal Euler equation

$$\frac{\dot{c}_{it}}{c_{it}} = -\frac{u'(c_{it})}{u''(c_{it})c_{it}} \left(\frac{v_i'(a_{it})}{u'(c_{it})} + r_t - \rho\right) \tag{5}$$

Now, multiplying both sides by $c_{it}$ and taking a first-order approximation of (5) around the steady state, we have

$$d\dot{c}_{it} = \sigma_i^{-1} c_i \left(\frac{v_i''(a_i)}{u'(c_i)} da_{it} + \sigma_i(\rho - r)\frac{dc_{it}}{c_i} + dr_t\right) \tag{6}$$

where we define $\sigma_i \equiv -\frac{u''(c_i)c_i}{u'(c_i)}$. Linearizing (4) gives

$$d\dot{a}_{it} = r da_{it} + a_i dr_t - dc_{it} + \theta_i dY_t \tag{7}$$

**Characterizing policy function around the steady state: relating to flow MPCs.** First, we want to characterize the consumption policy function for this agent around the steady state in the absence of shocks to future $r_t$ or $Y_t$. Suppose that it is given locally by $dc_{it} = m_i da_{it}$. Plugging this into (6) gives

$$m_i d\dot{a}_{it} = \sigma_i^{-1} c_i \left(\frac{v_i''(a_i)}{u'(c_i)} da_{it} + \sigma_i(\rho - r)\frac{m_i da_{it}}{c_i}\right)$$

and then plugging in $d\dot{a}_{it} = (r - m_i) da_{it}$ from (7) and dividing the above by $da_{it}$ gives the relation

$$m_i(r - m_i) = \sigma_i^{-1} c_i \frac{v_i''(a_i)}{u'(c_i)} + (\rho - r) m_i \tag{8}$$

Under the assumption of $r = 0$ in the steady state, simplifies to just

$$m_i^2 + \rho m_i + \sigma_i^{-1} c_i \frac{v_i''(a_i)}{u'(c_i)} = 0 \tag{9}$$

(9) gives a quadratic equation that we can solve for the flow MPC $m_i$ in terms of primitives around the steady state.[5] Additionally, if we plug (8) into (5), and again enforce $r = 0$ in steady state, we obtain a linearized Euler equation where the curvature of the bond-in-utility function shows up entirely through $m_i$:

$$d\dot{c}_{it} = -(m_i^2 + \rho m_i)da_{it} + \rho dc_{it} + \sigma_i^{-1} c_i dr_t \tag{10}$$

In general, we will consider the $\rho \to 0$ limit, as well as a case where $u$ is CRRA, at which point (10) simplifies further to just

$$d\dot{c}_{it} = -m_i^2 da_{it} + \sigma^{-1} c_i dr_t \tag{11}$$

Similarly, with the assumption of $r = 0$, (7) simplifies to

$$d\dot{a}_{it} = a_i dr_t - dc_{it} + \theta_i dY_t \tag{12}$$

**Characterizing household policy functions.** Define $dc_{it}^P \equiv dc_{it} - m_i da_{it}$ to be the first-order change in a household's consumption policy function, which equals the first-order change in consumption relative to steady state, minus the effect of assets. Note that then plugging into (7) and assuming $r = 0$ gives

$$\begin{aligned} d\dot{c}_{it}^P &= d\dot{c}_{it} - m_i d\dot{a}_{it} \\ &= d\dot{c}_{it} - m_i a_i dr_t + m_i dc_{it} - m_i \theta_i dY_t \\ &= d\dot{c}_{it} - m_i a_i dr_t + m_i dc_{it}^P + m_i^2 da_{it} - m_i \theta_i dY_t \end{aligned} \tag{13}$$

Now, substituting (10) for $d\dot{c}_{it}$ into this, and writing $dc_{it} = dc_{it}^P + m_i da_{it}$, we get

$$\begin{aligned} d\dot{c}_{it}^P &= -(m_i^2 + \rho m_i)da_{it} + \rho dc_{it}^P + \rho m_i da_{it} + \sigma_i^{-1} c_i dr_t - m_i a_i dr_t + m_i dc_{it}^P + m_i^2 da_{it} - m_i \theta_i dY_t \\ &= (\rho + m_i) dc_{it}^P + \sigma_i^{-1} c_i dr_t - m_i a_i dr_t - m_i \theta_i dY_t \end{aligned} \tag{14}$$

where we see that the $(m_i^2 + \rho m_i)da_{it}$ cancel. (14) implies that

$$dc_{it}^P = \int_0^\infty e^{-(\rho + m_i)s} (-\sigma_i^{-1} c_i dr_{t+s} + m_i a_i dr_{t+s} + m_i \theta_i dY_{t+s}) ds \tag{15}$$

i.e. that the change in consumption policy is the discounted forward-looking average of substitution effects of interest rates $-\sigma_i^{-1} c_i dr_{t+s}$, income effects of interest rates $m_i a_i dr_{t+s}$, and changes in aggregate income $m_i \theta_i dY_{t+s}$. The discount factor is $\rho + m_i$.

[5] See Auclert, Rognlie and Straub (2018) for the equivalent quadratic equation in discrete time.

## A.2 Description of benchmark general equilibrium environment

We suppose that each type $i = 1, \ldots, N$ supplies $n_{it}$ hours of effective labor to the market in steady state, leading to aggregate labor supply of $N_t = \sum_i n_{it}$. Each unit of labor produces one unit of goods, $Y_t = N_t$, and the goods market is competitive, so that the real wage is always 1. We assume that nominal wages are sticky, and that for any level of aggregate labor demand $N_t$ that deviates from the steady state, the rationing rule increases the effective labor of each type proportionately: $n_{it} = \frac{N_t}{N} n_i$. We define $\theta_i = n_i / N$ to be the share of effective labor supplied by type $i$; differences in $\theta_i$ across groups can reflect differences in population or differences in productivity. Then labor income of each group is $n_{it} = \theta_i Y_t$.

Since nominal wage inflation will not matter for real outcomes under our assumptions, we leave the Phillips curve for wages (and the underlying disutility function from labor) unspecified. See Auclert, Rognlie and Straub (2018) for more details.

We assume that we are in the neighborhood of the steady state, that $r_t$ is held constant by monetary policy, and that agents assume future aggregate income will be at its steady state level. Then because all forward-looking inputs to their problems are fixed, agents follow the their steady-state consumption policy function $dc_{it} = m_i da_{it}$, and the budget constraint will be given by (12) with $dr_t = 0$, i.e. $d\dot{a}_{it} = -dc_{it} + \theta_i dY_t$. Further, goods market clearing implies that $dY_t = \sum_i dc_{it}$.

These three equations describe our benchmark model; for economy of notation, in the paper, we replace $dc_{it}$ with $c_{it}$, $da_{it}$ with $a_{it}$, and $dY_t$ with $Y_t$, with all variables implicitly denoting first-order deviations from steady state.

Note that, in the absence of a reaction of monetary policy ($r_t = 0$ for all $t$), we have that the cumulative output response is

$$\frac{\int_0^\infty Y_t dt}{B} = \left(1 - \frac{a_{N0}}{B}\right) \frac{1}{\theta_N} = \frac{\text{Share of initial transfer not given to super-rich}}{\text{Income share of super-rich}} \tag{16}$$

This follows from the fact that $\dot{a}_{Nt} = \theta_N Y_t$, so applying Proposition 1, $\theta_N \int_0^\infty Y_t dt = \int_0^\infty \dot{a}_{Nt} dt = B - a_{N0}$. Equation (16) expresses the cumulative multiplier from the deficit-financed transfer as a simple ratio of two sufficient statistics, the share not initially given to the super-rich to their income share.

## A.3 Extensions

**With rational expectations.** When agents have rational expectations and do perceive future $dY_t$, in the limit $\rho \to 0$, then their consumption is simply characterized by (11). We further assume that monetary policy keeps the real interest rate constant, $dr_t = 0$, so this gives

$$d\dot{c}_{it} = -m_i^2 da_i$$

Finally, we assume no steady state assets $a_i = 0$, so that equation (12) is

$$d\dot{a}_{it} = \theta_i dY_t - dc_{it}$$

Redefining $c_{it} \equiv dc_{it}$, $a_{it} \equiv da_{it}$ , $Y_t \equiv dY_t$ for simplicity, the model is now:

$$\dot{c_{it}} = -m_i^2 a_{it}; \qquad \dot{a}_{it} = \theta_i Y_t - c_{it}; \qquad Y_t = \sum_{i=1}^{N} c_{it} \tag{17}$$

**With monetary response.** Now suppose that $r_t$ does vary over time according to some monetary rule $dr_t = \phi dY_t$ that increases the real interest rate to offset a boom in demand. Assume that this path of real interest rates is perfectly anticipated by households, but that households still do not anticipate changes in aggregate income. (For instance, households might see the term structure of borrowing rates directly from financial markets, but not have similar exposure to their own incomes; these are level-1 households in Farhi and Werning 2019)

To avoid large instantaneous income effects (since in reality assets will have longer duration and their returns will be insulated from interest rate changes), and to avoid needing to specify a taxation rule for the government, we assume here that steady-state assets of all types are zero. Also, changes in future incomes do not appear in (14), since the household does not perceive them when choosing policy. Hence, together with our other simplifications, (14) becomes simply $d\dot{c}_{it}^P = m_i dc_{it}^P + \sigma^{-1} c_i dr_t$, and $dc_{it} = m_i da_{it} + dc_{it}^P$. This modification to consumption is the only first-order departure from the benchmark framework.

Note that since the effects of monetary policy are discounted by $m_i$, high $i$ types with lower $m_i$ will have a larger consumption response to interest rates. Therefore, a rise in real interest rates in response to excess savings will cause high $i$ to spend relatively less, leaving them with more wealth and speeding the process of trickling up.

To summarize, the equations are:

$$\begin{aligned} d\dot{c}_{it}^P &= m_i dc_{it}^P + \sigma^{-1} c_i dr_t \\ dc_{it} &= m_i da_{it} + dc_{it}^P \\ d\dot{a}_{it} &= -dc_{it} + \theta_i dY_t \\ dY_t &= \sum_{i=1}^{N} dc_{it} \end{aligned}$$

Plugging in the monetary response $dr_t = \phi dY_t$, assuming further that steady state $c_i = \theta_i$, and switching notation back to levels, we obtain:

$$\begin{aligned}
\dot{c}_{it}^P &= m_i c_{it}^P + \sigma^{-1}\phi\theta_i Y_t \\
c_{it} &= m_i a_{it} + c_{it}^P \\
\dot{a}_{it} &= -c_{it} + \theta_i Y_t \\
Y_t &= \sum_{i=1}^{N} c_{it}
\end{aligned}$$

Note that in particular type $N$ agent is Ricardian, with Euler equation $dc_{Nt} = \sigma^{-1} c_N dr_t$. In condensed form, these equations read:

$$\dot{c}_{it}^P = m_i c_{it}^P + \sigma^{-1}\theta_i\phi Y_t; \qquad \dot{a}_{it} = \theta_i Y_t - m_i a_{it} - c_{it}^P; \qquad Y_t = \sum_{i=1}^{N}\left(m_i a_{it} + c_{it}^P\right) \tag{18}$$

# B Proofs of propositions 2 and 3

## B.1 Proof of proposition 2

We prove the following claim:

**Claim** $(N)$**:** Let $\{\theta_j\}$ be positive and sum to 1, let $m_1 > \ldots > m_N = 0$, let $a_{j0} \geq 0$, and let $a_{jt}$ solve the system of differential equations

$$\dot{a}_{jt} = -m_j a_{jt} + \theta_j \left(\sum_{i=1}^{N} m_i a_{it} + x_t\right)$$

where $x_t \geq 0$ is an exogenous inflow. Assume $m_j a_{j0}/\theta_j$ strictly falls in $j$. Then: For any $J \geq 1$ and $t$

$$\sum_{j=J}^{N} \dot{a}_{jt} \geq \left(\sum_{j=J}^{N} \theta_j\right) x_t \tag{19}$$

and for any $t$

$$\sum_{j=1}^{N} m_j \dot{a}_{jt} \leq x_t \sum_{j=1}^{N} m_j \theta_j \tag{20}$$

Claim $(N)$ is strictly more general than proposition 2. Indeed, setting $x_t = 0$, the claim implies $\sum_{j=J}^{N} \dot{a}_{jt} \geq 0$ for any $J \geq 1$, from which it follows that

$$\sum_{j=J}^{N} a_{jt'} \geq \sum_{j=J}^{N} a_{jt}$$

for any dates $t' > t$. The flip-side is $\sum_{j=1}^{J-1} a_{jt'} \leq \sum_{j=1}^{J-1} a_{jt}$.

We proceed to prove claim $(N)$ by induction over $N$. The induction start with $N = 1$ is trivial.

Next, suppose claim $(N)$ holds. We intend to prove claim $(N+1)$. For that, take $\{\theta_j\}_{j=0}^N$ with $\sum_{j=0}^N \theta_j = 1$, $m_0 > \ldots > m_N = 0$, $a_{j0} \geq 0$ and $a_{jt}$ described by

$$\dot{a}_{jt} = -m_j a_{jt} + \theta_j \left( \sum_{i=0}^{N} m_i a_{it} + x_t \right)$$

Assume $m_j a_{j0} / \theta_j$ decreases monotonically in $j$.

**Lemma 1.** *There is always a positive net flow from type 0 to everyone else. In math,*

$$m_0 a_{0t} > \theta_0 \sum_{i=0}^{N} m_i a_{it}$$

*which we can rewrite as*

$$m_0 a_{0t} > \frac{\theta_0}{1-\theta_0} \sum_{i=1}^{N} m_i a_{it}$$

*Proof.* We show this by contradiction. Let $\tau$ be the first time at which the are equal,

$$m_0 a_{0\tau} = \frac{\theta_0}{1-\theta_0} \sum_{i=1}^{N} m_i a_{i\tau} \tag{21}$$

This means, up until date $t = \tau$, we can write the evolution of wealth of the types $j > 0$ as

$$\dot{a}_{jt} = -m_j a_{jt} + \theta_j \left( \frac{1}{1-\theta_0} \sum_{i=1}^{N} m_i a_{it} + x_t + m_0 a_{0t} - \frac{\theta_0}{1-\theta_0} \sum_{i=1}^{N} m_i a_{it} \right)$$

where before date $\tau$, $m_0 a_{0t} \geq \frac{\theta_0}{1-\theta_0} \sum_{i=1}^{N} m_i a_{it}$.

Define $\tilde{\theta}_j \equiv \frac{\theta_j}{1-\theta_0}$ for $j = 1, \ldots, N$ and

$$\tilde{x}_t \equiv (1-\theta_0) \left( x_t + m_0 a_{0t} - \frac{\theta_0}{1-\theta_0} \sum_{i=1}^{N} m_i a_{it} \right)$$

Observe that, at date $t = \tau$, $\tilde{x}_\tau = (1-\theta_0)\, x_\tau$. Then, we can apply the induction hypothesis on types $j = 1, \ldots, N$. This establishes that, at date $t = \tau$,

$$\sum_{j=1}^{N} m_j \dot{a}_{j\tau} \leq \left( \sum_{j=1}^{N} \tilde{\theta}_j m_j \right) \tilde{x}_\tau = \left( \sum_{j=1}^{N} \theta_j m_j \right) x_\tau$$

and so

$$m_0\dot{a}_{0\tau} = -m_0^2 a_{0\tau} + m_0\theta_0\left(\sum_{i=0}^{N} m_i a_{i\tau} + x_\tau\right) = m_0\theta_0 x_\tau$$
$$\geq \frac{m_0\theta_0}{\sum_{j=1}^{N}\theta_j m_j}\sum_{j=1}^{N} m_j\dot{a}_{j\tau} \geq \frac{\theta_0}{1-\theta_0}\sum_{j=1}^{N} m_j\dot{a}_{j\tau}$$

where we used the fact that $m_j$ falls monotonically in $j$. This is a contradiction to $\tau$ being the first time for which (21) holds with equality, given that at date 0,

$$\frac{m_0 a_{00}}{\theta_0} > \sum_{i=0}^{N}\theta_i\frac{m_i a_{i0}}{\theta_i}$$

which falls from $m_j a_{j0}/\theta_j$ strictly falling in $j$. □

**Lemma 2.** *The equations* (19) *and* (20) *hold for the economy with* $N+1$ *types.*

*Proof.* Now that we established the positive flow from type 0 to the other types, it follows directly that

$$\sum_{j=J}^{N}\dot{a}_{jt} \geq \left(\sum_{j=J}^{N}\tilde{\theta}_j\right)\tilde{x}_t = \left(\sum_{j=J}^{N}\theta_j\right)x_t$$

for any $J \geq 1$. Moreover, total wealth grows at rate $x_t$, so

$$\sum_{j=0}^{N}\dot{a}_{jt} = x_t$$

Hence (19) holds. (20) follows from (19), because

$$\sum_{j=1}^{N} m_j\dot{a}_{jt} = m_1\sum_{j=1}^{N}\dot{a}_{jt} - \sum_{k=2}^{N}(m_{k-1}-m_k)\sum_{j=k}^{N}\dot{a}_{jt}$$
$$\leq m_1 x_t\sum_{j=1}^{N}\theta_j - \sum_{k=2}^{N}(m_{k-1}-m_k)\left(\sum_{j=k}^{N}\theta_j\right)x_t$$
$$\leq x_t\sum_{j=1}^{N} m_j\theta_j$$

□

Lemma 2 establishes claim $(N+1)$ and thus concludes our proof by induction.

### B.2 Proof of proposition 3

We can write the law of motion for assets (dropping $t$ subscripts) as

$$\dot{a}_i = -m_i a_i + \theta_i \sum_j m_j a_j \tag{22}$$

or, in stacked form,

$$\dot{a} = (-M + \theta m')a$$

where $M = \text{diag}(m)$. Define $A \equiv -M + \theta m'$. Note that $M^{-1/2}AM^{1/2} = -I + (M^{-1/2}\theta)(M^{1/2}m)'$ should have the same eigenvalues as $A$. Perron-Frobenius implies that $(M^{-1/2}\theta)(M^{1/2}m)'$ has a unique largest (real) eigenvalue with corresponding positive eigenvector, and then the largest eigenvalue of $A$ is this minus 1.

Since we have already shown in proposition 1 that this system is globally stable, the largest eigenvalue of $A$ must be negative. Call this $-\lambda$. We see that

$$-\lambda v_i = -m_i v_i + \theta_i \sum_j m_j v_j$$

$$v_i = \frac{\theta_i}{m_i - \lambda} \sum_j m_j v_j$$

Note that the eigenvector $v$ would not be everywhere positive if $\lambda$ was greater than or equal to any $m_i$. We conclude that $\lambda < m_i$.