Complementary Synthesis for Encoders with Pipeline and Flow Control Mechanism

Abstract—**Complementary synthesis automatically generates an encoder's decoder that recovers the encoder's inputs from its output. This paper proposes the first complementary synthesis algorithm that can handle flow control and pipeline mechanism widely employed in modern encoders. First, it infers the flow control predicate on inputs. Second, it finds out all pipeline stages in the encoder by enforcing the inferred flow control predicate. Third, it infers the flow control predicate for each pipeline stages. Finally, the decoder's Boolean functions that recover each pipeline stage and input are characterized with Craig interpolant. Experimental results indicate that this algorithm can always generate pipelined decoders with flow control mechanism.**

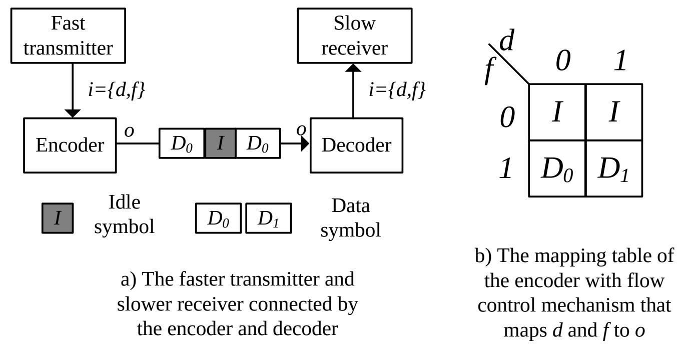


Fig. 1. Encoder with flow control mechanism

I. Introduction

One of the most difficult jobs in designing communication and multimedia chips is to design and verify complex encoder and decoder pairs. The encoder maps its input $\vec{i}$ to its output $\vec{o}$, while the decoder recovers $\vec{i}$ from $\vec{o}$. Complementary synthesis [1]–[6] eases this job by automatically generating a decoder from an encoder, with the assumption that $\vec{i}$ can always be uniquely determined by a bounded sequence of $\vec{o}$.

However, the flow control mechanism [7] in many encoders fails this assumption. As shown in Figure 1a), this mechanism prevents faster transmitter from overwhelming slower receiver by transmitting idle symbols I when the receiver can not keep up with the transmitter. As shown in Figure 1b), the idle symbol I can only uniquely determine a small subset of inputs $\vec{i}$, which is called flow control vector $\vec{f}$. While the normally encoded data symbols D_i can uniquely determine all inputs, including both $\vec{f}$ and data vector $\vec{d}$.

Qin et al. [8] handle such encoders by first finding out all inputs $i \in \vec{f}$ that can by uniquely determined by $\vec{o}$, and then inferring a flow control predicate $valid(\vec{f})$ that can make $\vec{d}$ to be uniquely determined by $\vec{o}$.

At the same time, as shown in Figure 2, many encoders contain pipeline stages $\vec{stg}^j$ to cut their datapath into multiple segments C^j, such that the encoder can run in higher frequency. Just like $\vec{i}$, each pipeline stage $\vec{stg}^j$ can also be partitioned into flow control vector $\vec{f^j}$ and data vector $\vec{d^j}$.

But the decoder generated by Qin et al. [8] doesn't include pipeline stages, which make it much slower than its corresponding encoder. To overcome this problem, this paper proposes a novel algorithm to generate pipelined decoders for flow controlled encoder. It first applies Qin et al. [8]'s algorithm to find out $\vec{f}$ and infers $valid(\vec{f})$. It then finds out all $\vec{d^j}$ and $\vec{f^j}$ in each pipeline stage $\vec{stg}^j$ respectively with and without enforcing $valid(\vec{f})$. It finally characterize the Boolean functions that recover each $\vec{stg}^j$ and $\vec{i}$ with Jiang et al. [9]'s algorithm.

Experimental result indicates that the proposed algorithm can always correctly generate pipelined decoder with flow control mechanism.

The remainder of this paper is organized as follows. Section II introduces the background material; Section III introduces the overall framework of our algorithm. Section IV finds out $\vec{f^j}$ and $\vec{d^j}$ in each pipeline stages $\vec{stg}^j$, while Section V characterize the decoder's Boolean functions that recover each pipeline stage $\vec{stg}^j$ and the input vector $\vec{i}$. Sections VI and VII present the experimental results and related works; Finally, Section VIII sums up the conclusion.

II. Preliminaries

A. Propositional satisfiability

The Boolean value set is denoted as $\mathbb{B} = \{0, 1\}$. A vector of variables is represented as $\vec{v} = (v, \dots)$. $|\vec{v}|$ is the number of variables in $\vec{v}$. If a variable v is a member of $\vec{v}$, then we say $v \in \vec{v}$; otherwise $v \notin \vec{v}$. $v \cup \vec{v}$ is the new vector that contains both v and all members of $\vec{v}$ $\vec{v} - v$ is he new vector that contains all members of $\vec{v}$ except v, $\vec{a} \cup \vec{b}$ is the new vector that contains all members of $\vec{a}$ and $\vec{b}$

The propositional satisfiability problem (SAT) for a Boolean formula F over a variable set V is to find a satisfying

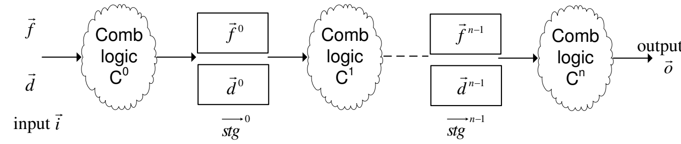


Fig. 2. Encoder with pipeline and flow control mechanism

assignment $A : V \to \mathbb{B}$, so that F can be evaluated to 1. If A exists, then F is satisfiable; otherwise, it is unsatisfiable.

Given two Boolean formulas ϕ_A and ϕ_B, with $\phi_A \wedge \phi_B$ unsatisfiable, there exists a formula ϕ_I referring only to the common variables of ϕ_A and ϕ_B such that $\phi_A \Rightarrow \phi_I$ and $\phi_I \wedge \phi_B$ is unsatisfiable. We call ϕ_I the **interpolant** [10] of ϕ_A with respect to ϕ_B and use McMillan's algorithm [11] to generate it.

B. Finite state machine

The encoder is modeled by a finite state machine(FSM) $M = (\vec{s}, \vec{i}, \vec{o}, T)$, consisting of a state variable vector $\vec{s}$, an input variable vector $\vec{i}$, an output variable vector $\vec{o}$, and a transition function $T : \vec{s} \times \vec{i} \to \vec{s} \times \vec{o}$ that computes the next state and output variable vector from the current state and input variable vector.

The behavior of FSM M can be reasoned by unrolling transition function. The state variable $s \in \vec{s}$, input variable $i \in \vec{i}$ and output variable $o \in \vec{o}$ at the n-th step are respectively denoted as s_n, i_n and o_n. Furthermore, the state, the input and the output variable vectors at the n-th step are respectively denoted as $\vec{s}_n$, $\vec{i}_n$ and $\vec{o}_n$. A **path** is a state sequence $< \vec{s}_n, \ldots, \vec{s}_m >$ with $\exists \vec{i}_j \vec{o}_j (\vec{s}_{j+1}, \vec{o}_j) \equiv T(\vec{s}_j, \vec{i}_j)$ for all $n \le j < m$. A **loop** is a path $< \vec{s}_n, \ldots, \vec{s}_m >$ with $\vec{s}_n \equiv \vec{s}_m$.

C. The algorithm to find out flow control vector $\vec{f}$

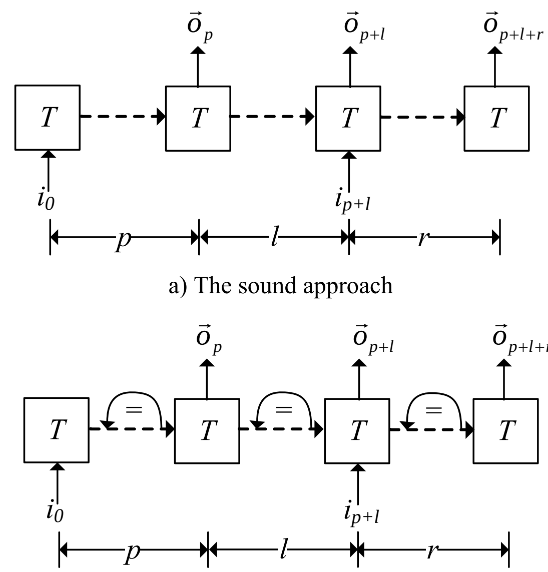
Qin et al. [8] proposed a halting algorithm to find out $\vec{f}$ by iteratively calling a sound and a complete approaches until they converge.

1) The sound approach: As shown in Figure 3a), on the unrolled transition functions, an input variable $i \in \vec{i}$ can be uniquely determined, if there exist three integers p, l and r, such that for any particular valuation of the output sequence $< \vec{o}_p, \ldots, \vec{o}_{p+l+r} >$, i_{p+l} cannot be 0 and 1 at the same time. This is equal to the unsatisfiability of $F_{PC}(p, l, r)$ in Equation (1). Line 1 corresponds to the path in Figure 3a), while Line 2 is a copy of it. Line 3 forces these two paths' output sequences to be the same, while Line 4 forces their i_{p+l} to be different. This approach is sound because when (1) is unsatisfiable, i is definitely a member of $\vec{f}$.

$$F_{PC}(p,l,r) := \left\{ \begin{array}{lc} & \bigwedge_{m=0}^{p+l+r} \{(\vec{s}_{m+1}, \vec{o}_m) \equiv T(\vec{s}_m, \vec{i}_m)\} \\ \wedge & \bigwedge_{m=0}^{p+l+r} \{(\vec{s'}_{m+1}, \vec{o'}_m) \equiv T(\vec{s'}_m, \vec{i'}_m)\} \\ \wedge & \bigwedge_{m=p}^{p+l+r} \vec{o}_m \equiv \vec{o'}_m \\ \wedge & i_{p+l} \equiv 1 \wedge i'_{p+l} \equiv 0 \end{array} \right\} \quad (1)$$

2) The complete approach: For $F_{PC}(p, l, r)$ presented above, there are two possibilities: **(1)**. i_{p+l} can be uniquely determined by $< \vec{o}_p, \ldots, \vec{o}_{p+l+r} >$ for some p, l and r; or **(2)**. i_{p+l} can't be uniquely determined for any p, l and r.

For the 1st case, by iteratively increasing p, l and r, $F_{PC}(p, l, r)$ will eventually become unsatisfiable. But for the 2nd case, this method will never terminate. So, to obtain a halting algorithm, we need the approach shown in Figure 3b) to check the 2nd case, which is similar to Figure 3a) but with three additional constraints used to detect loops on the three state sequences $< \vec{s}_0, \ldots, \vec{s}_p >$, $< \vec{s}_{p+1}, \ldots, \vec{s}_{p+l} >$ and $< \vec{s}_{p+l+1}, \ldots, \vec{s}_{p+l+r} >$. It is formally defined in Equation (2) with the last three lines corresponding to the three new constraints. It is a complete approach because if it is satisfiable, then by unrolling these three loops, we can prove the 2nd case and be sure that $i \notin \vec{f}$.



a) The sound approach

b) The complete approach with loops

Fig. 3. The sound and complete approximative approaches

$$F_{LN}(p,l,r) := \left\{ \begin{array}{lc} & F_{PC}(p,l,r) \\ \wedge & \bigvee_{x=0}^{p-1} \bigvee_{y=x+1}^{p} \{\vec{s}_x \equiv \vec{s}_y \wedge \vec{s'}_x \equiv \vec{s'}_y\} \\ \wedge & \bigvee_{x=p+1}^{p+l-1} \bigvee_{y=x+1}^{p+l} \{\vec{s}_x \equiv \vec{s}_y \wedge \vec{s'}_x \equiv \vec{s'}_y\} \\ \wedge & \bigvee_{x=p+l+1}^{p+l+r-1} \bigvee_{y=x+1}^{p+l+r} \{\vec{s}_x \equiv \vec{s}_y \wedge \vec{s'}_x \equiv \vec{s'}_y\} \end{array} \right\} \quad (2)$$

3) Identifying flow control vector $\vec{f}$ with Algorithm 1: At Line 6, the input i that can be uniquely determined will be moved to vector $\vec{f}$. If $F_{LN}(p, l, r)$ is satisfiable at Line 7, the input i that can NOT be uniquely determined will be moved to vector $\vec{d}$. Please refer to [8] for its termination and correctness proof.

D. Inferring $valid(\vec{f})$ that enables $\vec{d}$ to be uniquely determined

This is also proposed by Qin et al. [8]. It first introduces Algorithm 2 to characterize a function that makes a Boolean formula satisfiable. And then as shown in Figure 4, Algorithm 2 is used to characterize $\neg FSAT_{PC}(p, l, r)$, a monotonically growing under-approximation of $valid(\vec{f})$,

Algorithm 1: Identifying the flow control vector $\vec{f}$

```
Input: The input variable vector i⃗.
Output: f⃗ ⊂ i⃗, and the maximal p, l and r reached in
        this searching.
f⃗ := {}; d⃗ := {}; p := 0 ; l := 0 ; r := 0 ;
while i⃗ ≠ {} do
    assume i ∈ i⃗;
    p++;  l++;  r++;
    if F_PC(p,l,r) is unsatisfiable for i then
        f⃗ := i ∪ f⃗ ;  i⃗ := i⃗ − i;
    else if F_LN(p,l,r) is satisfiable for i then
        d⃗ := i ∪ d⃗ ;  i⃗ := i⃗ − i
return (f⃗, p, l, r)
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Algorithm 2: $CharacterizingFormulaSAT(R, \vec{a}, \vec{b}, t)$

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Input: The Boolean formula R(a⃗, b⃗, t).
Output: FSAT_R(a⃗) that makes R(a⃗, b⃗, 1) satisfiable.
1 FSAT_R(a⃗) := 0 ;
2 while R(a⃗, b⃗, 1) ∧ ¬FSAT_R(a⃗) is satisfiable do
3     assume A : a⃗ ∪ b⃗ ∪ {t} → {0,1} is the satisfying
      assignment ;
4     φ_A(a⃗) := R(a⃗, A(b⃗), 1) ;
5     φ_B(a⃗) := R(a⃗, A(b⃗), 0) ;
6     assume ITP(a⃗) is the Craig interpolant of φ_A with
      respect to φ_B ;
7     FSAT_R(a⃗) := ITP(a⃗) ∨ FSAT_R(a⃗) ;
8 return FSAT_R(a⃗)
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and $\neg FSAT_{LN}(p,l,r)$, a monotonically shrinking over-approximation of $valid(\vec{f})$. And finally we show that these two approximations will eventually converge to $valid(\vec{f})$.

1) Characterizing a function that makes a Boolean formula satisfiable: For a particular Boolean relation $R(\vec{a}, \vec{b}, t)$, with $R(\vec{a}, \vec{b}, 0) \wedge R(\vec{a}, \vec{b}, 1)$ unsatisfiable. Algorithm 2 characterize a Boolean function $FSAT_R(\vec{a})$ that covers and only covers all the valuations of $\vec{a}$ that can make $R(\vec{a}, \vec{b}, 1)$ satisfiable. Line 2 finds out new valuation of $\vec{a}$ that can make $R(\vec{a}, \vec{b}, 1)$ satisfiable, but hasn't been covered by $FSAT_R(\vec{a})$. Lines 4, 5 and 6 enlarge this valuation to an interpolant $ITP(\vec{a})$ with McMillan's algorithm [11]. Line 7 adds $ITP(\vec{a})$ to $FSAT_R(\vec{a})$.

2) Computing monotonically growing under-approximation of $valid(\vec{f})$: By replacing i in Equation (1) with $\vec{d}$ inferred in Algorithm 1, we have:

$$F^d_{PC}(p,l,r) :=$$

$$\left\{ \begin{array}{ll} & \bigwedge_{m=0}^{p+l+r} \{(\vec{s}_{m+1}, \vec{o}_m) \equiv T(\vec{s}_m, \vec{i}_m)\} \\ \wedge & \bigwedge_{m=0}^{p+l+r} \{(\vec{s'}_{m+1}, \vec{o'}_m) \equiv T(\vec{s'}_m, \vec{i'}_m)\} \\ \wedge & \bigwedge_{m=p}^{p+l+r} \vec{o}_m \equiv \vec{o'}_m \\ \wedge & \vec{d}_{p+l} \neq \vec{d'}_{p+l} \end{array} \right\} \quad (3)$$

If $F^d_{PC}(p,l,r)$ is satisfiable, then $\vec{d}_{p+l}$ can't be uniquely determined by $< \vec{o}_p, \ldots, \vec{o}_{p+l+r} >$. We define $T_{PC}(p,l,r)$ by collecting the 3rd line of (3):

$$T_{PC}(p,l,r) := \left\{ \bigwedge_{m=p}^{p+l+r} \vec{o}_m \equiv \vec{o'}_m \right\} \quad (4)$$

By substituting $T_{PC}(p,l,r)$ back into $F^d_{PC}(p,l,r)$, we have a new formula:

$$F'^d_{PC}(p,l,r,t) :=$$

$$\left\{ \begin{array}{ll} & \bigwedge_{m=0}^{p+l+r} \{(\vec{s}_{m+1}, \vec{o}_m) \equiv T(\vec{s}_m, \vec{i}_m)\} \\ \wedge & \bigwedge_{m=0}^{p+l+r} \{(\vec{s'}_{m+1}, \vec{o'}_m) \equiv T(\vec{s'}_m, \vec{i'}_m)\} \\ \wedge & t \equiv T_{PC}(p,l,r) \\ \wedge & \vec{d}_{p+l} \neq \vec{d'}_{p+l} \end{array} \right\} \quad (5)$$

Obviously $F^d_{PC}(p,l,r)$ and $F'^d_{PC}(p,l,r,1)$ are equivalent. We further define:

$$\vec{a} := \vec{f}_{p+l} \quad (6)$$

$$\vec{b} := \vec{d}_{p+l} \cup \vec{d'}_{p+l} \cup \vec{s}_0 \cup \vec{s'}_0 \cup \bigcup_{0 \le x \le p+l+r, x \neq (p+l)} (\vec{i}_x \cup \vec{i'}_x) \quad (7)$$

Thus, $\vec{a} \cup \vec{b}$ is the vector that contains all the input variable vectors $< \vec{i}_0, \ldots, \vec{i}_{p+l+r} >$ and $< \vec{i'}_0, \ldots, \vec{i'}_{p+l+r} >$ at all steps for the two sequences of unrolled transition function. It also contains the two initial states $\vec{s}_0$ and $\vec{s'}_0$. So $\vec{a}$ and $\vec{b}$ can uniquely determine the value of t in $F'^d_{PC}(p,l,r,t)$, which means $R(\vec{a}, \vec{b}, 1) \wedge R(\vec{a}, \vec{b}, 0)$ is unsatisfiable. Thus, for a particular combination of p, l and r, the Boolean function over $\vec{f}_{p+l}$ that makes $F'^d_{PC}(p,l,r,1)$ satisfiable can be computed by calling Algorithm 2 with $F'^d_{PC}(p,l,r,t)$, $\vec{a}$ and $\vec{b}$ defined above:

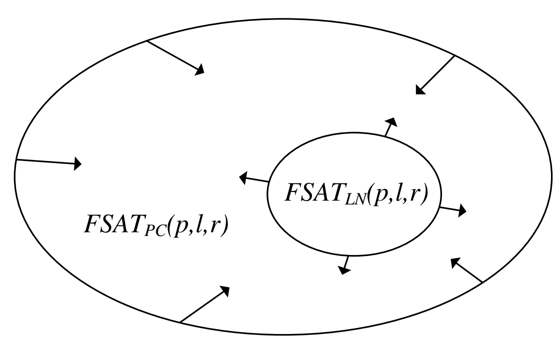

Fig. 4. The monotonicity of $FSAT_{PC}(p,l,r)$ and $FSAT_{LN}(p,l,r)$

Algorithm 3: Inferring $valid(\vec{f}_{p+l})$

p:= 0; l:= 0; r:= 0 ;
while $\neg FSAT_{LN}(p,l,r) \wedge FSAT_{PC}(p,l,r)$ *is satisfiable* **do**
  p ++ ; l ++ ; r ++ ;
return $\neg FSAT_{LN}(p,l,r)$

Algorithm 4: Minimizing r

1 **for** $r' := r \to 0$ **do**
2   **if** $r' \equiv 0$ *or* $F_{PC}(p,l,r'-1) \wedge valid(\vec{f}_{p+l})$ *is satisfiable for some* $i \in \vec{i}$ **then**
3   break
4 return r'

$$FSAT_{PC}(p,l,r) := \\ CharacterizingFormulaSAT(F'^{d}_{PC}(p,l,r,t), \vec{a}, \vec{b}, t) \quad (8)$$

As shown in Figure 4, $\neg FSAT_{PC}(p,l,r)$ is an under-approximation of $valid(\vec{f})$ monotonically growing with respect to p, l and r.

3) Computing monotonically shrinking over-approximation of $valid(\vec{f})$: Similarly, we can define :

$$T_{LN}(p,l,r) := \left\{ \begin{array}{cc} & \bigwedge_{m=p}^{p+l+r} \vec{o}_m \equiv \vec{o'}_m \\ \wedge & \bigvee_{x=0}^{p-1} \bigvee_{y=x+1}^{p} \{\vec{s}_x \equiv \vec{s}_y \wedge \vec{s'}_x \equiv \vec{s'}_y\} \\ \wedge & \bigvee_{x=p+1}^{p+l-1} \bigvee_{y=x+1}^{p+l} \{\vec{s}_x \equiv \vec{s}_y \wedge \vec{s'}_x \equiv \vec{s'}_y\} \\ \wedge & \bigvee_{x=p+l+1}^{p+l+r-1} \bigvee_{y=x+1}^{p+l+r} \{\vec{s}_x \equiv \vec{s}_y \wedge \vec{s'}_x \equiv \vec{s'}_y\} \end{array} \right\} \quad (9)$$

$$F'^{d}_{LN}(p,l,r,t) := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+l+r} \{(\vec{s}_{m+1}, \vec{o}_m) \equiv T(\vec{s}_m, \vec{i}_m)\} \\ \wedge & \bigwedge_{m=0}^{p+l+r} \{(\vec{s'}_{m+1}, \vec{o'}_m) \equiv T(\vec{s'}_m, \vec{i'}_m)\} \\ \wedge & t \equiv T_{LN}(p,l,r) \\ \wedge & \vec{d}_{p+l} \neq \vec{d'}_{p+l} \end{array} \right\} \quad (10)$$

$$FSAT_{LN}(p,l,r) := CharacterizingFormulaSAT(F'^{d}_{LN}(p,l,r,t), \vec{a}, \vec{b}, t) \quad (11)$$

As shown in Figure 4, $\neg FSAT_{LN}(p,l,r)$ is an over-approximation of $valid(\vec{f})$ monotonically shrinking with respect to p, l and r.

4) Inferring $valid(\vec{f})$ with Algorithm 3: It just iteratively increases the value of p, l and r, until $FSAT_{PC}(p,l,r)$ and $FSAT_{LN}(p,l,r)$ converge. Please refer to [8] for the proofs of its termination and correctness.

III. Algorithm Framework

A. A general model for the encoder

As shown in Figure 5, we assume that the encoder has n pipeline stages $\vec{stg}^j$, where $0 \le j \le n-1$. And each pipeline stage $\vec{stg}^j$ can be further partitioned into flow control vector $\vec{f^j}$ and data vector $\vec{d^j}$. The input vector $\vec{i}$, as in [8], can also be partitioned into flow control vector $\vec{f}$ and data vector $\vec{d}$. If we take the combinational logic block C^j as a function, then this encoder can be represented by the following equations.

$$\begin{array}{rcll} \vec{stg}^0 & := & C^0(\vec{i}) & \\ \vec{stg}^j & := & C^j(\vec{stg}^{j-1}) & 1 \le j \le n-1 \\ \vec{o} & := & C^n(\vec{stg}^{n-1}) & \end{array} \quad (12)$$

In the remainder of this paper, superscript always means the pipeline stage, while the subscript, as mentioned in Subsection II-B, always means the step index in the unrolled transition function. For example, $\vec{stg}^j$ is the j-th pipeline stage. While $\vec{stg}^j_i$ is the value of this j-th pipeline stage at the i-th step in the unrolled state transition sequence.

B. Algorithm framework

With the encoder model shown in Figure 5, our overall algorithm framework is:

1) Calling Algorithm 1 to partition $\vec{i}$ into $\vec{f}$ and $\vec{d}$.
2) Calling Algorithm 3 to infer $valid(\vec{f})$ that enables $\vec{d}$ to be uniquely determined with parameters p, l and r.
3) In Section IV, finding out $\vec{f^j}$ and $\vec{d^j}$ in each pipeline stage $\vec{stg}^j$.
4) In Section V, characterizing the decoder's Boolean functions that recover each pipeline stages $\vec{stg}^j$ and input vector $\vec{i}$.

IV. Inferring the encoder's pipeline structure

A. Minimizing r and l

As Algorithm 3 increases p, l and r simultaneously, there may be some redundancy in the value of l and r. So we need to first minimize r in Algorithm 4.

In Line 2, we enforce the inferred flow control predicate $valid(\vec{f})$ by conjugating it with $F_{PC}(p,l,r'-1)$. When it is satisfiable, then r' is the last one that makes $F_{PC}(p,l,r') \wedge valid(\vec{f}_{p+l})$ unsatisfiable, we return it directly. On the other hand, when $r' \equiv 0$, $F_{PC}(p,l,0)$ must have been tested in last iteration, and the result must be unsatisfiable. In this case we return 0.

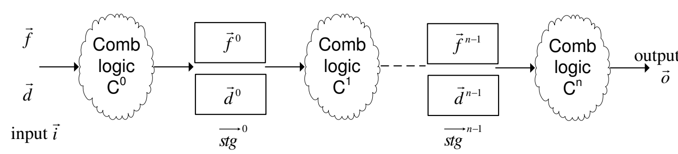

Fig. 5. A general structure of the encoder with pipeline stages and flow control mechanism

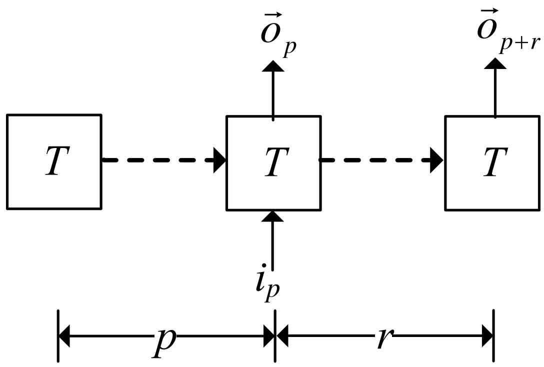

Fig. 6. Recovering input with reduced output sequence

Now, we have a minimized r from Algorithm 4, which can make $\vec{i}_{p+l}$ to be uniquely determined by $< \vec{o}_p, \ldots, \vec{o}_{p+l+r} >$. We further require that :

1) As shown in Figure 6, l can be reduced to 0, which means $\vec{i}_p$ can be uniquely determined by $< \vec{o}_p, \ldots, \vec{o}_{p+r} >$, that is, the set of future outputs.
2) The above mentioned output sequence $< \vec{o}_p, \ldots, \vec{o}_{p+r} >$ can be further reduced to $\vec{o}_{p+r}$. This means $\vec{o}_{p+r}$ is the only output vector needed to recover the input vector $\vec{i}_p$.

Checking these two requirements equals to checking the unsatisfiability of $F'_{PC}(p,r) \wedge valid(\vec{f}_{p+l})$, with $F'_{PC}(p,r)$ defined below:

$$F'_{PC}(p,r) := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+r}\{(\vec{s}_{m+1}, \vec{o}_m) \equiv T(\vec{s}_m, \vec{i}_m)\} \\ \wedge & \bigwedge_{m=0}^{p+r}\{(\vec{s'}_{m+1}, \vec{o'}_m) \equiv T(\vec{s'}_m, \vec{i'}_m)\} \\ \wedge & \vec{o}_{p+r} \equiv \vec{o'}_{p+r} \\ \wedge & i_p \equiv 1 \wedge i'_p \equiv 0 \end{array} \right\} \quad (13)$$

This equation seems much stronger than the general requirement in Equation (1). But we will show in experimental results that they are always fulfilled.

B. Inferring pipeline stages

Now, with the inferred p and r, we need to generalize F'_{PC} in Equation (13) to the following new formula that can determine whether a particular variable v at step j can be uniquely determined by a vector $\vec{w}$ at step k. Now v and $\vec{w}$ can be either input, registers or output variables.

$$F''_{PC}(p,r,v,j,\vec{w},k) := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+r}\{(\vec{s}_{m+1}, \vec{o}_m) \equiv T(\vec{s}_m, \vec{i}_m)\} \\ \wedge & \bigwedge_{m=0}^{p+r}\{(\vec{s'}_{m+1}, \vec{o'}_m) \equiv T(\vec{s'}_m, \vec{i'}_m)\} \\ \wedge & \vec{w}_k \equiv \vec{w'}_k \\ \wedge & v_j \equiv 1 \wedge v'_j \equiv 0 \end{array} \right\} \quad (14)$$

Obviously, when $F''_{PC}(p,r,v,j,\vec{w},k)$ is unsatisfiable, $\vec{w}_k$ can uniquely determine v_j.

For $0 \leq j \leq n-1$, in the j-th pipeline stage $\vec{stg}^j$, its flow control vector $\vec{f^j}$ is exactly the set of registers $s \in \vec{s}$ that can be uniquely determined at the $j-((n-1)-(p+r))$-th step by $\vec{o}$ at the $p+r$-th step without enforcing $valid(\vec{f_p})$. It can be formally defined as:

$$\vec{f^j} := \left\{ s \in \vec{s} \mid \begin{array}{c} F''_{PC}(p,r,s,j-D,\vec{o},p+r) \\ is\ unsatisfiable \end{array} \right\} \quad (15)$$

with:

$$D := (n-1)-(p+r) \quad (16)$$

While the data vector $\vec{d^j}$ in the j-th pipeline stage $\vec{stg}^j$ is the set of registers $s \in \vec{s}$ that can be uniquely determined at the same $j-((n-1)-(p+r))$-th step by $\vec{o}$ at the $p+r$-th step by enforcing $valid(\vec{f_p})$. It can be formally defined as:

$$\vec{d^j} := \left\{ s \in \vec{s} \mid \begin{array}{c} F''_{PC}(p,r,s,j-D,\vec{o},p+r) \wedge valid(\vec{f_p}) \\ is\ unsatisfiable \end{array} \right\} \quad (17)$$

V. Characterizing the Boolean functions recovering input variables and pipeline registers

A. Characterizing the Boolean functions recovering the last pipeline stage

According to Equation (15), every registers $s \in \vec{f}^{n-1}$ can be uniquely determined by $\vec{o}$ at the $p+r$-th step, that is, $F''_{PC}(p,r,s,p+r,\vec{o},p+r)$ is unsatisfiable and can be partitioned into :

$$\phi_A := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+r}\{(\vec{s}_{m+1}, \vec{o}_m) \equiv T(\vec{s}_m, \vec{i}_m)\} \\ \wedge & s_{p+r} \equiv 1 \end{array} \right\} \quad (18)$$

$$\phi_B := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+r}\{(\vec{s'}_{m+1}, \vec{o'}_m) \equiv T(\vec{s'}_m, \vec{i'}_m)\} \\ \wedge & \vec{o}_{p+r} \equiv \vec{o'}_{p+r} \\ \wedge & s'_{p+r} \equiv 0 \end{array} \right\} \quad (19)$$

As $F''_{PC}(p,r,s,p+r,\vec{o},p+r)$ equals to $\phi_A \wedge \phi_B$, so $\phi_A \wedge \phi_B$ is unsatisfiable. And the common variables of ϕ_A and ϕ_B is $\vec{o}_{p+r}$.

According to [9], a Craig interpolant ϕ_I of ϕ_A with respect to ϕ_B can be constructed, which refer only to $\vec{o}_{p+r}$, and covers all the valuations of $\vec{o}_{p+r}$ that can make $s_{p+r} \equiv 1$. At the same time, $\phi_I \wedge \phi_B$ is unsatisfiable, which means ϕ_I covers nothing that can make $s_{p+r} \equiv 0$.

Thus, ϕ_I can be used as the decoder's Boolean function that recovers $s \in \vec{f}^{n-1}$ from $\vec{o}$.

By replacing $F''_{PC}(p,r,s,p+r,\vec{o},p+r)$ with $F''_{PC}(p,r,s,p+r,\vec{o},p+r) \wedge valid(f_p)$, we can similarly characterize the Boolean function that recovers $s \in \vec{d}^{n-1}$.

B. Characterizing the Boolean functions recovering other pipeline stages

According to Figure 5, $\vec{f^j}$ at the $j-D$-step can be uniquely determined by $\vec{stg}^{j+1}$ at the $j-D+1$-th step. So we can partition the unsatisfiable formula $F''_{PC}(p,r,s,j-D,\vec{stg}^{j+1},j-D+1)$ into the following two equations:

$$\phi_A := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+r}\{(\vec{s}_{m+1},\vec{o}_m) \equiv T(\vec{s}_m,\vec{i}_m)\} \\ \wedge & s_{j-D} \equiv 1 \end{array} \right\} \quad (20)$$

$$\phi_B := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+r}\{(\vec{s'}_{m+1},\vec{o'}_m) \equiv T(\vec{s'}_m,\vec{i'}_m)\} \\ \wedge & \vec{stg}^{j+1}_{j-D+1} \equiv \vec{stg'}^{j+1}_{j-D+1} \\ \wedge & s'_{j-D} \equiv 0 \end{array} \right\} \quad (21)$$

Again, a Craig interpolant ϕ_I of ϕ_A with respect to ϕ_B can be constructed, and used as the decoder's Boolean function that recovers $s \in \vec{f^j}$ from $\vec{stg}^{j+1}$.

Similarly, by replacing $F''_{PC}(p,r,s,j-D,\vec{stg}^{j+1},j-D+1)$ with $F''_{PC}(p,r,s,j-D,\vec{stg}^{j+1},j-D+1) \wedge valid(f_p)$, we can characterize the Boolean function that recovers $s \in \vec{d^j}$ from $\vec{stg}^{j+1}$.

C. Characterizing the Boolean functions recovering the encoder's input variables

According to Figure 5, $\vec{f}$ at the p-step can be uniquely determined by $\vec{stg}^0$ at the p-th step. $F''_{PC}(p,r,i,p,\vec{stg}^0,p)$ is unsatisfiable and can be partitioned into :

$$\phi_A := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+r}\{(\vec{s}_{m+1},\vec{o}_m) \equiv T(\vec{s}_m,\vec{i}_m)\} \\ \wedge & i_p \equiv 1 \end{array} \right\} \quad (22)$$

$$\phi_B := \left\{ \begin{array}{cc} & \bigwedge_{m=0}^{p+r}\{(\vec{s'}_{m+1},\vec{o'}_m) \equiv T(\vec{s'}_m,\vec{i'}_m)\} \\ \wedge & \vec{stg}^0_p \equiv \vec{stg'}^0_p \\ \wedge & i'_p \equiv 0 \end{array} \right\} \quad (23)$$

Again, the Craig interpolant ϕ_I of ϕ_A with respect to ϕ_B can be used as the decoder's Boolean function that recovers $i \in \vec{f}$ from $\vec{stg}^0$.

Similarly, by replacing $F''_{PC}(p,r,i,p,\vec{stg}^0,p)$ with $F''_{PC}(p,r,i,p,\vec{stg}^0,p) \wedge valid(\vec{f_p})$, we can characterize the Boolean function that recovers $i \in \vec{d}$ from $\vec{stg}^0$.

VI. Experimental results

We have implemented these algorithms in OCaml language, and solved the generated CNF formulas with MiniSat 1.14 [12]. All experiments have been run on a server with 16 Intel Xeon E5648 processors at 2.67GHz, 192GB memory, and CentOS 5.4 Linux.

TABLE II
INFERRED PIPELINE STAGES OF PCIE

	input	pipeline stage 0	pipeline stage 1
flow control vector	CNTL_TXEnable_P0	InputDataEnable_P0_reg	OutputData_P0_reg[9 OutputElecIdle_P0_r
flow control predicate	CNTL_TXEnable_P0	InputDataEnable_P0_reg	true
data vector	TXDATA[7:0] TXDATAK	InputData_P0_reg[7:0] InputDataK_P0_reg	

A. Comparing timing and area

Table I shows the benchmarks used in this paper. The 2nd and 3rd column show respectively the number of inputs, outputs and registers of each benchmark. The 4th column shows the area of the encoder when mapped to LSI10K library with Design Compiler. In this paper, all area and delay are obtained in the same setting.

The 6th to 8th columns show respectively the run time of [2]'s algorithm to generate the decoder without pipeline, and the delay and area of the generated decoder. While the 9th to 11th columns show respectively the run time of this paper's algorithm to generate the pipelined decoder, and the delay and area of the generated decoder.

Comparing the 7th and the 10th column indicates that the decoders' delay have been significantly improved.

One thing that is a little bit surprise is, the two largest benchmarks scrambler and xfi do not have pipeline stages inside. We study their code and confirm that this is true. Their area are so large because they use much wider datapaths with 64 to 72 bits.

B. Inferred pipeline stages of pcie

For the benchmark pcie, there are two pipeline stages, whose flow control vector and data vector are respectively shown in Table II.

One issue to be noticed that is the data vector at pipeline stage 1 is empty, while all registers in that stages are recognized as flow control vector. We inspect the encoder's source code and find that these registers are directly feed to output. So they can actually be uniquely determined by $\vec{o}$. This doesn't affect the correctness of the generated decoder, because the functionality of flow control vector never depend on the inferred flow control predicate.

C. Inferred pipeline stages of xgxs

For the benchmark xgxs, there are only 1 pipeline stage, whose flow control vector and data vector are respectively shown in Table III.

D. Inferred pipeline stages of t2ether

For the benchmark t2ether, there are four pipeline stages shown in Table IV. The control flow predicates are fairly

TABLE I
BENCHMARKS AND EXPERIMENTAL RESULTS

Names	The encoders				decoder generated by [2]			decoder generated by this paper		
	# in/out	# reg	area	Description of Encoders	run time	delay (ns)	area	run time	delay (ns)	area
pcie	10/11	23	326	PCIE 2.0 [13]	0.37	7.20	624	8.08	5.89	652
xgxs	10/10	16	453	Ethernet clause 48 [14]	0.21	7.02	540	4.25	5.93	829
t2eth	14/14	49	2252	Ethernet clause 36 [14]	12.7	6.54	434	430.4	6.12	877
scrambler	64/64	58	1034	inserting 01 flipping	no pipeline stages found					
xfi	72/66	72	7772	Ethernet clause 49 [14]						

complex, so we list them below instead of in Table IV. The input control flow predicate f is :

$$\begin{aligned}&(\mathit{tx\_enc\_ctrl\_sel}[2]\ \&\ \mathit{tx\_enc\_ctrl\_sel}[3])|\\&(\mathit{tx\_enc\_ctrl\_sel}[2]\ \&\ !\mathit{tx\_enc\_ctrl\_sel}[3]\ \&\ !\mathit{tx\_enc\_ctrl\_sel}[0]\ \&\ \mathit{tx\_enc\_ctrl\_sel}[1])|\\&(!\mathit{tx\_enc\_ctrl\_sel}[2]\ \&\ \mathit{tx\_enc\_ctrl\_sel}[3])|\\&(!\mathit{tx\_enc\_ctrl\_sel}[2]\ \&\ !\mathit{tx\_enc\_ctrl\_sel}[3]\ \&\ \mathit{tx\_enc\_ctrl\_sel}[0])\end{aligned} \tag{24}$$

The flow control predicate $valid(\vec{f^0})$ for the 0-th pipeline stage is :

$$\begin{aligned}&(\mathit{qout\_reg\_2\_4}\ \&\ \mathit{qout\_reg\_1\_4}\ \&\ !\mathit{qout\_reg\_0\_8})|\\&(!\mathit{qout\_reg\_2\_4}\ \&\ \mathit{qout\_reg\_0\_8})\end{aligned} \tag{25}$$

The flow control predicate $valid(\vec{f^1})$ for the 1-th pipeline stage is :

$$\begin{aligned}&(\mathit{qout\_reg\_2\_5}\ \&\ \mathit{qout\_reg\_1\_5}\ \&\ \mathit{qout\_reg\_0\_10}\ \&\ !\mathit{qout\_reg\_0\_9})|\\&(\mathit{qout\_reg\_2\_5}\ \&\ \mathit{qout\_reg\_1\_5}\ \&\ !\mathit{qout\_reg\_0\_10})|\\&(\mathit{qout\_reg\_2\_5}\ \&\ !\mathit{qout\_reg\_1\_5}\ \&\ !\mathit{qout\_reg\_0\_10})|\\&(!\mathit{qout\_reg\_2\_5}\ \&\ \mathit{qout\_reg\_0\_10}\ \&\ \mathit{qout\_reg\_0\_9})|\\&(!\mathit{qout\_reg\_2\_5}\ \&\ !\mathit{qout\_reg\_0\_10})\end{aligned} \tag{26}$$

The flow control predicates $valid(\vec{f^2})$ and $valid(\vec{f^3})$ for the last two pipeline stages are all $true$.

VII. RELATED PUBLICATIONS

The first complementary synthesis algorithm was proposed by [1]. It checks the decoder's existence by iteratively increasing the bound of unrolled transition function sequence, and generates the decoder's Boolean function by enumerating all satisfying assignments of the decoder's output. Its major shortcomings are that it may not halt and it is too slow in building the decoder.

Shen et al. [2] and Liu et al. [4] tackled the halting problem independently by searching for loops in the state sequence, while the runtime overhead problem was addressed in [3], [4] by Craig interpolant [11].

Shen et al. [3] automatically inferred an assertion for configuration pins, which can lead to the decoder's existence.

Qin et al. [8] proposed the first algorithm can handle encoder with flow control mechanism. But it can not handle pipeline stages.

Tu and Jiang [6] proposed a break-through algorithm that recover the encoder's input by considering its initial and reachable states.

TABLE III
INFERRED PIPELINE STAGES OF XGXS

	input	pipeline stage 0
flow control vector	bad_code	bad_code_reg_reg
flow control predicate	!bad_code	!bad_code_reg_reg
data vector	encode_data_in[7:0] konstant	ip_data_latch_reg[2:0] plus34_latch_reg data_out_latch_reg[5:0] konstant_latch_reg kx_latch_reg minus34b_latch_reg

TABLE IV
INFERRED PIPELINE STAGES OF T2ETHER

	input	pipeline stage 0	pipeline stage 1	pipeline stage 2
flow control vector	tx_enc_ctrl_sel[3:0]	qout_reg_0_8 qout_reg_2_4 qout_reg_1_4	qout_reg_0_9 qout_reg_1_5 qout_reg_2_5 qout_reg_0_10	qout_reg[9:0]_2
data vector	txd[7:0]	qout_reg[7:0]	qout_reg[7:0]_1	

VIII. CONCLUSIONS

This paper proposes the first complementary synthesis algorithm that can handle encoders with pipeline stages and flow control mechanism. Experimental result indicates that the proposed algorithm can always correctly generate pipelined decoder with flow control mechanism.

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