

# A Halting Algorithm to Determine the Existence of Decoder

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**Abstract**—Complementary synthesis automatically synthesizes the decoder circuit of an encoder. It determines the existence of the decoder by checking whether the encoder's input can be uniquely determined by its output. However, this algorithm will not halt if the decoder does not exist.

To solve this problem, a novel halting algorithm is proposed in this paper to check the existence of the decoder. For every unfolded instance of the encoder's transition function, this algorithm not only check whether the encoder's input can be uniquely determined by its output, but also check whether there exists a loop-like path that reaches a particular state, from which two more successive loop-like paths can produce an output sequence that corresponds to two different input letters. These two checks will confirm respectively the existence and non-existence of the decoder, and thus ensure the halting character of the new algorithm.

To illustrate its usefulness and efficiency, this algorithm had been run on several complex encoders, including PCI-E and Ethernet. Experimental results indicate that the new algorithm always halts properly by distinguishing correct encoders from the incorrect ones, and it can be more than three times faster than previous work.

**Index Terms**—Halting Algorithm, Complementary Synthesis

## I. INTRODUCTION

One of the most difficult jobs in designing communication and multimedia chips is to design and verify the complex complementary circuit pair  $(E, E^{-1})$ , in which the encoder  $E$  transforms information into a format suitable for transmission and storage, while its complementary circuit(or decoder)  $E^{-1}$  recovers this information. In order to facilitate this job, the complementary synthesis algorithm is proposed in [1], [2] to automatically synthesize the decoder circuit of an encoder, by checking Parameterized Complementary Condition(PC), that is, whether the encoder's input letter can be uniquely determined by its output sequence.

However, this algorithm will not halt if  $E^{-1}$  does not exist. Another algorithm had been proposed in [3] to solve this problem by constructing an onion-ring between  $PC$  and its over-approximation, and checking whether  $E$  is in all these rings. This new algorithm is very slow and complicated. So yet another halting algorithm is proposed in this paper, which is both faster and more straightforward:

For every unfolded instance of  $E$ 's transition function, this algorithm not only check whether the encoder's input can be uniquely determined by its output, but also check whether

there exists a loop-like path that reaches a particular state, from which two more successive loop-like paths can produce an output sequence that corresponds to two different input letters. These two checks will confirm respectively the existence and non-existence of  $E^{-1}$ , and thus ensure the halting character of the new algorithm.

This algorithm had been implemented with the OCaml language, and solved the generated SAT instances with Zchaff SAT solver [4]. The benchmark set includes several complex encoders from industrial projects (e.g., PCI-E and Ethernet), and their slightly modified variants without corresponding decoders. Experimental results indicate that the new algorithm always halts properly by distinguishing correct encoders from incorrect ones, and can be three times faster than the other halting algorithm [3] proposed by us in FMCAD'10. All these experimental results and programs can be downloaded from <http://www.ssympub.org>.

There is one issue that needs to be clarified. There are two classes of complementary circuit pairs, each with its own character and design methodology:

- 1) **Standard datapath-intensive circuits:** Such circuits often work as digital signal processing components, including Fast Fourier Transform (FFT), Discrete Cosine Transform(DCT), and so on. These circuits and their complementary circuits usually have standard and highly optimized implementations from various foundries and IP vendors, such as Xilinx core generator [13] and Synopsys DesignWare library [14]. Therefore this paper's algorithm isn't design for them.
- 2) **Non-standard and control-intensive circuits:** These circuits, such as PCI-E and Ethernet, are often used to handle communication protocols, and typically don't have standard implementations. This paper's algorithm is designed for these circuits.

The remainder of this paper is organized as follows. Section II presents background materials. Section III introduces this paper's algorithm, while Section IV describes how to remove redundant output letters to minimize the circuit area. Section V and VI present the experimental results and related works respectively. Finally, Section VII concludes this paper.

## II. PRELIMINARIES

### A. Propositional Satisfiability

The Boolean value set is denoted as  $B = \{0, 1\}$ . For a Boolean formula  $F$  over a variable set  $V$ , the **Propositional**

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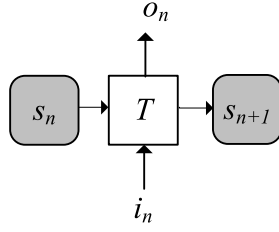


Fig. 1. Mealy finite state machine

**Satisfiability Problem**(abbreviated as **SAT**) is to find a satisfying assignment  $A : V \rightarrow B$ , so that  $F$  can be evaluated to 1. If such a satisfying assignment exists, then  $F$  is **satisfiable**; otherwise, it is **unsatisfiable**.

A computer program that decides the existence of such a satisfying assignment is called a **SAT solver**, such as Zchaff [4], Grasp [5], Berkmin [6], and MiniSAT [7]. A formula to be solved by a SAT solver is also called a **SAT instance**.

### B. Recurrence Diameter

The encoder  $E$  can be modeled by a Mealy finite state machine [9].

**Definition 1: Mealy finite state machine** is a 5-tuple  $M = (S, s_0, I, O, T)$ , consisting of a finite state set  $S$ , an initial state  $s_0 \in S$ , a finite set of input letters  $I$ , a finite set of output letters  $O$ , a transition function  $T : S \times I \rightarrow S \times O$  that computes the next state and output letter from the current state and input letter.

As shown in Figure 1, as well as in the remainder of this paper, the state is represented as a gray round corner box, and the transition function  $T$  is represented by a white rectangle. The state, the input letter and the output letter at the  $n$ -th cycle are respectively denoted as  $s_n, i_n$  and  $o_n$ . The sequence of state, input letter and output letter from the  $n$ -th to the  $m$ -th cycle are respectively denoted as  $s_n^m, i_n^m$  and  $o_n^m$ .

**A loop is a state sequence  $s_n^m$  with  $s_n \equiv s_m$ . A loop-like path is a state sequence  $s_n^m$  with  $s_i \equiv s_j$ , where  $n \leq i < j \leq m$ .**

Kroening et al. [8] defined the **state variables recurrence diameter** with respect to  $M$ , denoted by  $rrd(M)$ , as the longest state sequence without loop in  $M$  starting from an initial state.

(1)

In this paper, a similar concept: the **uninitialized state variables recurrence diameter** with respect to  $M$ ,  $uirrd(M)$ , is defined as the longest state sequence without loop in  $M$ .

(2)

The only difference between these two definitions is that  $uirrd$  does not consider the initial state.

These definitions are only used in proving the theorems below. This paper's algorithm does not need to compute these diameters.

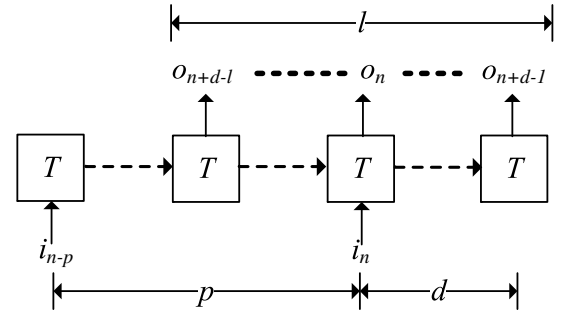


Fig. 2. The parameterized complementary condition

### C. The Original Algorithm to Determine the Existence of the Decoder

The complementary synthesis algorithm [1] includes two steps: determining the existence of the decoder and characterizing its Boolean function. Only the first step is introduced here.

According to our previous research [1], and our engineering experience in designing communication chips for the TH-1A supercomputers [11], many communication protocols are loseless, that is, every input letter can be recovered from its output sequence.

More formally, a sufficient condition for the existence of  $E^{-1}$  is, there exist three parameter values  $p, d$  and  $l$ , so that  $i_n$  of  $E$  can be uniquely determined by the output sequence  $o_{n+d-l}^{n+d-1}$ . As shown in Figure 2,  $d$  is the relative delay between  $o_{n+d-l}^{n+d-1}$  and the input letter  $i_n$ , while  $l$  is the length of  $o_{n+d-l}^{n+d-1}$ , and  $p$  is the length of the prefix state transition sequence used to rule out some unreachable states. Thus, as shown in Figure 2, the parameterized complementary condition(PC) [1] can be defined formally as:

**Definition 2: Parameterized Complementary Condition (PC) :** For encoder  $E$ ,  $E \models PC(p, d, l)$  holds if  $i_n$  can be uniquely determined by  $o_{n+d-l}^{n+d-1}$  on the state transition sequence  $s_{n-p}^{n+d-1}$ . This equals the unsatisfiability of  $F_{PC}(p, d, l)$  in Equation (3).  $E \models PC$  is further defined as  $\exists p, d, l : E \models PC(p, d, l)$ .

(3)

The 2nd and 3rd lines of Equation (3) correspond respectively to two unfolded instances of  $E$ 's transition function. The only difference between them is that a prime is appended to every variable in the 3rd line. The 4th line forces the output sequences of these two unfolded instances to be the same, while the 5th line forces their input letters to be different.

This non-halting algorithm [1] just iterate through all valuations of  $d, l$  and  $p$ , from small to large, until one valuation of  $d, l$  and  $p$  that makes Equation (3) unsatisfiable is found. Then the existence of the decoder is proved. However, if the decoder does not exist, this algorithm will never halt. This problem will be solved in the next section.

**There is one more problem to be explained. According to Figure 2, if  $l > d$ , then, to compute the value of input  $i_n$ , one may need to know the output  $o_m$  where  $m < n$ . It**

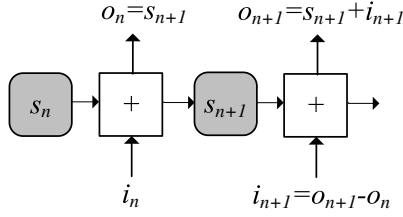


Fig. 3. The circuit that break the causal relation

looks like breaking the causal relation. At the same time, in Table II, XFI and scrambler also have  $l$  larger than  $d$ .

This problem will be explained with the circuit in Figure 3. Assume its transition function  $T$  is defined as :

(4)

Intuitively, this circuit add its input to its current state, and put the result to its output and next state. So, it is obvious that, to recover the input letter  $i_{n+1}$ , the output letter  $o_n$  must be subtracted from  $o_{n+1}$ . In this case,  $l$  is 1, and  $d$  is 0. This explains why  $l$  can be larger than  $d$ .

### III. A HALTING ALGORITHM TO DETERMINE THE EXISTENCE OF THE DECODER

#### A. Determining the Non-existence of the Decoder

$PC$  in Definition 2 only defines how to determine the existence of decoder  $E^{-1}$ . But how to determine the non-existence of  $E^{-1}$  is not defined. So the key to a halting algorithm is to find out a sufficient and necessary condition for the non-existence of  $E^{-1}$ . According to Definition 2 and Figure 2,  $E^{-1}$  exists if there is a parameter value tuple  $\langle p, d, l \rangle$ , such that  $E \models PC(p, d, l)$  holds.

So, intuitively,  $E^{-1}$  does not exist if for every parameter value tuple  $\langle p, d, l \rangle$ , another tuple  $\langle p', d', l' \rangle$  with  $p' > p, l' > l$  and  $d' > d$  can always been found, such that  $E \models PC(p', d', l')$  does not hold. This case can be detected by the SAT instance in Figure 4, which is similar to Figure 2, except that three new constraints are inserted to detect loops on state sequences  $s_{n-p}^{n+d-l}, s_{n+d-l}^{n+d}$  and  $s_{n+1}^{n+d}$ .

**If this SAT instance is satisfiable, for any parameter value  $\langle p, d, l \rangle$ , these three loops can be unfolded until  $\langle p', d', l' \rangle$  that is larger than  $\langle p, d, l \rangle$  is found. It is obvious that this unfolded instance is still satisfiable, which means  $E \models PC(p', d', l')$  does not hold. So the decoder does not exist.**

According to line 2 and 3 of Equation (3), there are actually two unfolded instances of transition function  $T$ , so these loops must be detected on both of them, i.e., on the productive machine  $M^2$  defined below:

**Definition 3: Productive machine :** For Mealy machine  $M = (S, s_0, I, O, T)$ , its Productive machine is  $M^2 = (S^2, s_0^2, I^2, O^2, T^2)$ , where

- 1)  $S^2 = S \times S$
- 2)  $s_0^2 = s_0 \times s_0$
- 3)  $I^2 = I \times I$
- 4)  $O^2 = O \times O$

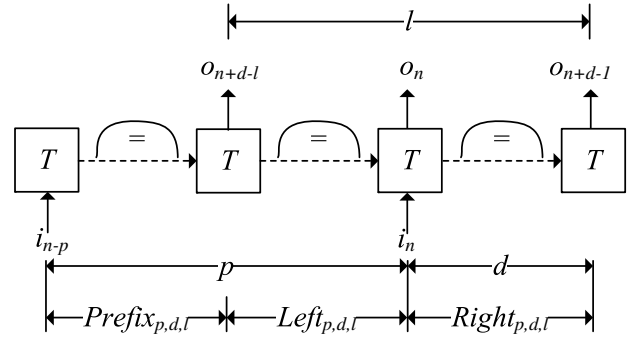


Fig. 4. The loop-like non-complementary condition

- 5)  $T^2$  is defined as  $(\langle s_{m+1}, s'_{m+1} \rangle, \langle o_m, o'_m \rangle) = T^2(\langle s_m, s'_m \rangle, \langle i_m, i'_m \rangle)$  with  $(s_{m+1}, o_m) = T(s_m, i_m)$  and  $(s'_{m+1}, o'_m) = T(s'_m, i'_m)$ .

Thus, the loop-like non-complementary condition is defined below to determine the non-existence of  $E^{-1}$ :

**Definition 4: Loop-like Non-complementary Condition (LN) :** For encoder  $E$  and its Mealy machine  $M = (S, s_0, I, O, T)$ , assume its productive machine is  $M^2 = (S^2, s_0^2, I^2, O^2, T^2)$ , then  $E \models LN(p, d, l)$  holds if  $i_n$  can not be uniquely determined by  $o_{n+d-l}^{n+d-1}$  on the state transition sequence  $s_{n-p}^{n+d-1}$ , and there are loops on  $(s^2)_{n-p}^{n+d-l}, (s^2)_{n+d-l}^{n+d-1}$  and  $(s^2)_{n+1}^{n+d}$ . This equals the satisfiability of  $F_{LN}(p, d, l)$  in Equation (5).  $E \models LN$  is further defined as  $\exists p, d, l : E \models LN(p, d, l)$ .

(5)

By comparing Equation (3) and (5), it is obvious that their only difference is the last three newly inserted lines in (5), which will be used to detect loops on the following three state sequences:

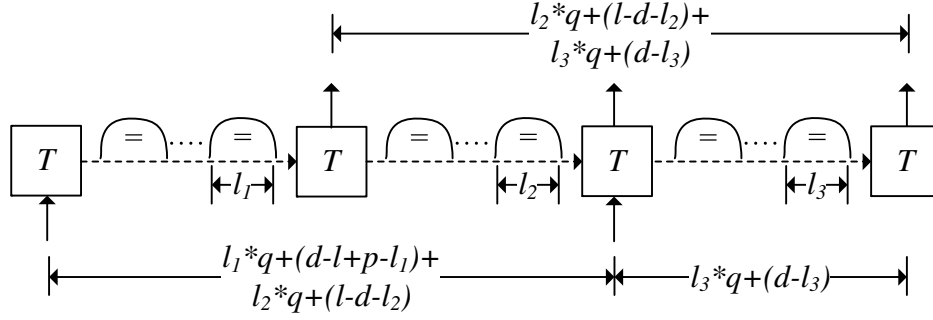
(6)

The correctness of this approach will be proved in the next subsection.

Before proceeding to next subsection, we need to define how to unfold these loops in above three state sequences. Assume that the length of loops in  $Prefix_{p,d,l}$ ,  $Left_{p,d,l}$  and  $Right_{p,d,l}$  are  $l_1$ ,  $l_2$  and  $l_3$  respectively. We further assume that we unfold these loops for  $q$  times. Then, the SAT instance generated from this unfolding is shown in Figure 5, It is obvious that, the unfolded SAT instance corresponds to  $F_{LN}(p'', d'', l'', R)$ , where:

(7)

It is obvious that for every particular  $\langle p, d, l \rangle$  and  $\langle p', d', l' \rangle$ , there always exists a  $q$ , such that  $Prefix_{p'', d'', l''}$ ,  $Left_{p'', d'', l''}$  and  $Right_{p'', d'', l''}$  resulted from this unfolding are **not shorter** than  $Prefix_{p', d', l'}$ ,  $Left_{p', d', l'}$  and  $Right_{p', d', l'}$  respectively.

Fig. 5. The loop-like non-complementary condition unfolded for  $q$  times

### B. Proving Correctness

Before proving correctness of this approach, some lemmas are needed.

**Lemma 1 ():** For  $F_{LN}(p'', d'', l'')$  in Figure 5, we have  $F_{LN}(p, d, l) \rightarrow F_{LN}(p'', d'', l'')$

*Proof:* According to Equation (5) and Figure 5, this is obvious. ■

**Lemma 2 ():** For two tuples  $\langle p, d, l \rangle$  and  $\langle p', d', l' \rangle$ , if  $Prefix_{p', d', l'}, Left_{p', d', l'}$  and  $Right_{p', d', l'}$  are **not shorter** than  $Prefix_{p, d, l}, Left_{p, d, l}$  and  $Right_{p, d, l}$  respectively, then  $E \models PC(p, d, l) \rightarrow E \models PC(p', d', l')$ .

*Proof:* It is obvious that  $F_{PC}(p, d, l)$  is a sub-formula of  $F_{PC}(p', d', l')$ , so the unsatisfiability of the former one implies the unsatisfiability of the latter one. Thus,  $E \models PC(p, d, l) \rightarrow E \models PC(p', d', l')$  holds. ■

The following two theorems will prove that  $E \models LN \leftrightarrow \neg\{E \models PC\}$ .

**Theorem 1 ():**  $E \models LN \rightarrow \neg\{E \models PC\}$

*Proof:* We can prove it by contradiction. Assume that  $E \models LN$  and  $E \models PC$  both hold. This means there exist  $\langle p, d, l \rangle$  and  $\langle p', d', l' \rangle$ , such that  $E \models PC(p, d, l)$  and  $E \models LN(p', d', l')$ .

On one hand,  $E \models LN(p', d', l')$  implies that there are loops in the state sequences  $Prefix_{p', d', l'}, Left_{p', d', l'}$  and  $Right_{p', d', l'}$ . By unfolding these loops, we can get another tuple  $\langle p'', d'', l'' \rangle$ , so that :

- 1)  $Prefix_{p'', d'', l''), Left_{p'', d'', l'}$  and  $Right_{p'', d'', l'}$  are not shorter than  $Prefix_{p, d, l}, Left_{p, d, l}$  and  $Right_{p, d, l}$  respectively
- 2) According to Lemma 1,  $F_{LN}(p'', d'', l'')$  is satisfiable.

$F_{PC}(p'', d'', l'')$  is a sub-formula of  $F_{LN}(p'', d'', l'')$ , so  $F_{PC}(p'', d'', l'')$  is also satisfiable, which means that  $E \models PC(p'', d'', l'')$  does not hold.

On the other hand, according to Lemma 2,  $E \models PC(p'', d'', l'')$  holds.

This contradiction concludes the proof. ■

**Theorem 2 ():**  $E \models LN \leftarrow \neg\{E \models PC\}$

*Proof:* Let's prove it by contradiction. Assume that  $E \models LN$  and  $E \models PC$  both do not hold. Then for every  $\langle p, d, l \rangle$  and  $\langle p', d', l' \rangle$ ,  $F_{PC}(p, d, l)$  is satisfiable, while  $F_{LN}(p', d', l')$  is unsatisfiable.

Thus, assume the  $uirrd(M^2)$  is the uninitialized state variables recurrence diameter of  $E$ 's productive machine. Let's

define  $\langle p, d, l \rangle$  as:

(8)

With this definition, it is obvious that  $Prefix_{p, d, l}, Left_{p, d, l}$  and  $Right_{p, d, l}$  are both longer than  $uirrd(M^2)$ . This means that there are loops in all these three paths, which will make  $F_{LN}(p, d, l)$  satisfiable. This contradicts with the fact that  $F_{LN}(p', d', l')$  is unsatisfiable for every  $\langle p', d', l' \rangle$ .

This contradiction concludes the proof. ■

Theorems 1 and 2 illustrate that, a halting algorithm can be implemented by enumerating all combinations of  $\langle p, d, l \rangle$  from small to large, and checking  $E \models PC(p, d, l)$  and  $E \models LN(p, d, l)$  in every iteration. This process will eventually terminate with one and only one answer between  $E \models PC$  and  $E \models LN$ . The implementation of this algorithm will be presented in the next subsection.

### C. Algorithm Implementation

#### Algorithm 1 check\_PCLN

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```

1: for bound = 1  $\rightarrow$   $\infty$  do
2:   for p = 0  $\rightarrow$  bound - 1 do
3:     d = bound - 1 - p
4:     for l = d  $\rightarrow$  bound do
5:       if  $F_{PC}(p, d, l)$  is unsatisfiable then
6:         print " $E^{-1}$  exists with  $\langle p, d, l \rangle$ "
7:         halt;
8:       else if  $F_{LN}(p, d, l)$  is satisfiable then
9:         print " $E^{-1}$  does not exist"
10:        halt;
11:      end if
12:    end for
13:  end for
14: end for

```

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In line 1 of Algorithm 1, *bound* is the total length of the unfolded transition function. Placing the enumeration of *bound* on the out most loop can avoid generating the SAT instance of unfolded transition function repeatedly for  $F_{PC}$  and  $F_{LN}$ . And then, in line 2,3 and 4, the valuation of  $p, d$  and  $l$  are enumerated respectively. According to Theorems 1 and 2, Algorithm 1 will eventually terminate at line 6 or 9.

**Algorithm 2** *RemoveRedundancy*( $p, d, l$ )

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```

1: for  $p' = p \rightarrow 0$  do
2:   if  $F_{PC}(p' - 1, d, l)$  is satisfiable then
3:     break
4:   end if
5: end for
6: for  $d' = d \rightarrow 0$  do
7:   if  $F_{PC}(p', d' - 1, l)$  is satisfiable then
8:     break
9:   end if
10: end for
11: for  $l' = l - (d - d') \rightarrow 0$  do
12:   if  $F_{PC}(p', d', l' - 1)$  is satisfiable then
13:     break
14:   end if
15: end for
16: print "final result is  $\langle p', d', l' \rangle$ "

```

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## IV. REMOVING REDUNDANT OUTPUT LETTERS

Although Algorithm 1 is sufficient to determine the existence of  $E^{-1}$ , the parameter values found by line 6 of Algorithm 1 contain some redundancy, which will cause unnecessary large overheads on the circuit area and run time of characterizing.

For example, as shown in Figure 6, assume that  $l$  is the smallest parameter value that leads to  $E \models PC(p, d, l)$ , and  $l < d$ , which means that  $i_n$  is uniquely determined by some output letters  $o_k$  with  $k > n$ . Further assume that line 6 of Algorithm 1 prove  $E \models PC(p, d, l')$ . It is obvious that  $l' > d$ , which makes  $i_n$  depend on some redundant  $o_k$  with  $k \leq n$ . So  $o_{n+d-l'-1}^{n+d-l-1}$  is the sequence of redundant output letters, which should be removed to prevent them from being instantiated as latches in circuit  $E^{-1}$ . At the same time, also according to Figure 6, larger  $p$  and  $d$  lead to larger SAT instances for the characterization algorithm.

So, Algorithm 2 is used to minimize  $\langle p, d, l \rangle$  before passing it to the characterization algorithm:

## V. EXPERIMENTAL RESULTS

This algorithm had been implemented with the OCaml language, and solved the generated SAT instances with Zchaff SAT solver [4]. All experiments are run on a PC with a

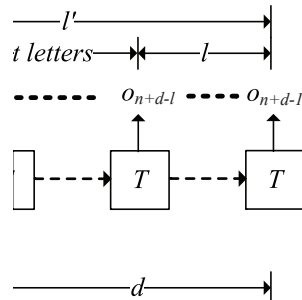


Fig. 6. Redundant Output Letters

TABLE I  
INFORMATION OF BENCHMARKS

	XGXS	XFI	scrambler	PCI-E	T2 ethernet
Line number of verilog source code	214	466	24	1139	1073
#regs	15	135	58	22	48
Data path width	8	64	66	10	10

TABLE II  
EXPERIMENTAL RESULTS ON PROPERLY DESIGNED ENCODERS

		XGXS	XFI	scrambler	PCI-E	T2 ethernet
[3]	time chk PC(sec)	1.06	70.52	5.74	2.40	66.37
	$d, p, l$	1,1,1	0,3,2	0,2,2	2,1,1	4,1,1
This paper	time chk PC(sec)	0.39	21.31	3.27	0.61	65.34
	$d, p, l$	1,2,1	0,3,2	0,2,2	2,2,1	4,2,1

2.4GHz Intel Core 2 Q6600 processor, 8GB memory and CentOS 5.2 linux. All experimental results and programs can be downloaded from <http://www.ssypub.org>.

## A. Benchmarks

Table I shows information of the following benchmarks.

- 1) A XGXS encoder compliant to clause 48 of IEEE-802.3ae 2002 standard [10].
- 2) A XFI encoder compliant to clause 49 of the same IEEE standard.
- 3) A 66-bit scrambler used to ensure that a data sequence has sufficiently many 0-1 transitions, so that it can run through a high-speed noisy serial transmission channel.
- 4) A PCI-E physical coding module [12].
- 5) The Ethernet module of Sun's OpenSparc T2 processor.

## B. Determining the existence of the decoder for Properly Designed Encoders

The 2nd and 4th rows of Table II compare the run time of checking  $E \models PC$  between [3] and this paper. It is obvious that this paper's approach is much faster than that of [3].

The 3rd and 5th rows compare the discovered parameter values, and some minor differences are found on parameter value  $p$ . This is caused by the different orders in checking various parameter combinations.

## C. Comparing Decoder Area

Table III compares the circuit area of hand-written decoders, and decoders built by this paper's algorithm. These decoders are synthesized with LSI10K technology library from Synopsys DesignCompiler.

**From Table III, it is obvious that: Except for the most complex XFI, synthesis results of this paper's algorithm are more compact than hand-written decoders. However, this comparison is unfair when hand-written decoders include error-detection and reporting functions, or other additions not required by the main functionality.**

On the other hand, for the XFI case, circuit area of this paper's algorithm is about 2 times larger than that of hand-written decoders. This means the circuit area must be improved in the future work.

#### D. Comparing Decoder Timing

Table IV compares the critical path latencies of hand-written decoders, and decoders built by this paper's algorithm. Their synthesis settings are same as Subsection V-C. According to Table IV, for all those circuits, the critical path latencies of the decoders built by this paper's algorithm are better than that of those hand-written decoders.

#### E. Determining the non-existence of the decoder for Improperly Designed Encoders

To further show the usefulness of this paper's algorithm, some improperly designed encoders without corresponding decoders are needed. These improperly designed encoders are obtained by modifying each benchmark's output statements, so that they can explicitly output the same letter for two different input letters. In this way, input letter  $i_n$  can never be uniquely determined by  $E$ 's output sequence.

The 2nd row of Table V shows the run time of [3] on checking these improperly designed encoders, while the 3rd row shows the run time of this paper's algorithm. These results indicate that this paper's algorithm always terminate correctly, and is much faster than [3].

## VI. RELATED WORKS

### A. Complementary Synthesis

The concept of complementary synthesis was first proposed by us [1] in ICCAD 2009. Its major shortcomings are that it is not halting, and its run-time overhead of building complementary circuit is too large.

The halting problem are handled by building a set of over-approximations that are similar to onion-rings [3]. The second shortcoming are addressed by simplifying the SAT instance with unsatisfiable core extraction [2].

### B. Program Inverse

According to Gulwani [17], program inverse is the problem that derive a program  $P^{-1}$  that negate the computation of a given program  $P$ . So the definition of program inverse is very similar to complementary synthesis.

TABLE III  
COMPARING DECODER AREA

	XGXS	XFI	scrambler	PCI-E	T2 ethernet
hand-written decoders	921	6002	1629	852	1446
decoders built by this paper's algorithm	700	12754	1455	455	552

TABLE IV  
COMPARING CRITICAL PATH LATENCIES IN NANOSECOND

	XGXS	XFI	scrambler	PCI-E	T2 ethernet
hand-written decoders	12.33	46.65	6.54	19.03	23.36
decoders built by this paper's algorithm	11.96	28.13	6.54	9.09	12.69

Initial work on deriving program inverse used proof-based approaches [18], but it can only handle very small programs and very simple syntax structures.

Glück et.al [19] inverses the first-order functional programs by eliminating nondeterminism with LR-based parsing methods. The requirement that the program to be inverse should be expressed in functional language makes it impossible to apply it to complementary synthesis.

Srivastava et.al [20] assume that an inverse program is typically related to the original program, so the space of possible inverses can be inferred by automatically mining the original program for expressions, predicates, and control flow. This algorithm inductively rules out invalid paths that can't fulfill the requirement of inversion, to narrow down the space of candidate programs until only the valid ones remain. So it can only guarantee the existence of a solution, but not the correctness of this solution if its assumptions do not hold.

### C. The Completeness of Bounded Model Checking

Bounded model checking(BMC) [15] is a model checking technology that considers only limited length path. Many researchers try to find out complete approaches for BMC.

One line of research [8], [15] tries to find out a bound  $b$ , which can guarantee the correctness of a specification, if the specification is correct on all paths shorter than  $b$ .

The other line of research [16] tries to find out a pattern for induction, such that the correctness of a specification within any bound  $b$  implies the correctness on bound  $b + 1$ .

This paper achieves completeness without following these two approaches. Instead, it defines two complement uniqueness conditions,  $LP$  and  $LL$ , and find out proper algorithms to check them.

### D. Protocol Converter Synthesis

The protocol converter synthesis is the problem that automatically generates a translator between two different communication protocols.

Avnit et.al [21], [22] first defines a general model for describing the different protocols. Then it provides an algorithm

TABLE V  
COMPARING RUN TIME OF IMPROPERLY DESIGNED ENCODERS

	XGXS	XFI	scrambler	PCI-E	T2 ethernet
[3](sec)	0.98	35.08	2.54	1.36	17.39
This paper's algorithm(sec)	0.33	14.44	2.43	0.56	10.14

to decide whether there are some functionality of a protocol that can't be translated into another. Finally, it synthesizes the translator by computing a greatest fixed point for the update function of buffer's control states. Avnit et.al [23] improved the algorithm mentioned above with a more efficient design space exploration algorithm.

## VII. CONCLUSIONS

This paper proposes a faster and more direct halting algorithm that checks whether a particular encoder has corresponding decoder. The theoretical analysis shows that this paper's approach always distinguishes correct encoders from their incorrect variants and halts properly. Experimental results shows that this paper's approach is much faster than that of [3].

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## VIII. RESPONSE TO REVIEWER

I will respond to every review one by one. For every review, my response and corresponding modification will be in **bold font**.

### A. Review Number 1

1) This is a journal paper. I think providing some background information would help understand the motivation better. Could you elaborate a bit on why the protocols you consider have to be lossless? (An informal explanation will do.) I am not a specialist in the area of data transmission but as far as I understand loss of information is quite common in data transmission.

**Yes, you are right.**

**First, I add the 1st paragraph in Section I to provide background and motivation.**

**Second, I add the 2nd paragraph in subsectionII-C to explain why the protocols I consider are lossless.**

2) In a previous paper, you mentioned that your approach does not work for data-intensive protocols. It makes sense to repeat this statement here (unless you have reconsidered it).

**Yes, you are right. I add the 5th, 6th and 7th paragraphs in Section I to mention this point.**

3) Why can't one build a decoder manually? If it is possible, could you say a few words about how the quality of decoders built manually compares with that of decoders generated automatically?

**Yes, you are right. The decoder can be built manually. And as mentioned in the 1st paragraph in Section I, this is a tedious and error-prone job. That is why we propose complementary synthesis.**

**I add Subsections V-C and V-D to compare the quality of decoders built manually and generated automatically.**

4) What is a loop-like path? Is it a reconvergent path?

**I add the 4th paragraph in Subsection II-B to explain the concept of loop-like path.**

A more general remark is that the paper lacks informal explanations. Intuitively, a Mealy machine  $M$  is lossless if for every pair of inputs  $i$  and  $i'$   $M$  either a) produces different outputs  $o$  and  $o'$  or it b) switches into different states  $s$  and  $s'$  such that any two sequences starting in  $s$  and  $s'$  and converging to a state  $s^*$  have to produce different outputs in state  $s^*$ .

I would like to see more informal explanations like that.

**According to Definition 2,  $M$  is lossless if its input can be uniquely determined by its output, with some valuation of  $p$ ,  $d$  and  $l$ . This definition does NOT need the concept of loop-like path.**

**On the other hand, the concept of loop-like path is used in Definition 4, which defines how to determine whether  $M$  is loss. I add the 3th paragraph in Subsection III-A to explain this idea. Please also refer to the two paragraph before it.**

5) Here an example of unnecessary complicating things. In expression (1) describing the recurrent diameter, instead of just saying that the state are different pairwise i.e. that  $(s_j \neq s_k)$  for all  $j, k=1, \dots, i, j \neq k$  you give a more complex expression. In this expression, the indexes are chosen in such a way that the check  $(s_j \neq s_k)$  is performed only once. It would make sense if you were describing an algorithm for finding the recurrence diameter. But you are giving a definition.

**Actually, according to the last paragraph of Subsection II-B, we don't need to compute these diameters in our algorithms. We only need them in proving our theorems.**

6) In Figure 2, if  $l > d$ , then, to compute the value of input  $i_n$ , one may need to know the output  $o_m$  where  $m < n$ . It looks like breaking the causal relation. Could you give an informal explanation of why it may be necessary?

**I add the last three paragraphs in Subsection II-C to explain this.**

7) The miter of your definition 3 is not what people usually call a miter. In your miter, the number of input variables is doubled, while in a regular miter the input variables of two circuits are identified. It is worth mentioning this fact. An alternative is to use a different name, like "pseudo-miter" or "generalized miter".

**Yes, you are right. I change it to "productive machine".**

8) The experimental results are not terribly impressive. Your best result is reducing the runtime from 70 sec. to 21 sec. on XFI. If you found an example where you reduced time from 10 hours to 3 hours it would make more sense. Waiting for 70 sec. instead of 21 sec. is tolerable, unless you can argue that in some scenarios your algorithm may be applied many times.

**Yes, you are right. In our recent research, we try to infer assertion for complementary synthesis. In that new algorithm, this paper's algorithm are called hundreds of**

**times, and result in thousands of seconds of run time. Should I mention this in my paper?**

## B. Review Number 2

\*\*\*\*\*

Comments to the Author \_\_\_\_\_

The paper presents incremental work on earlier conference papers, and in fact is written in conference style.

There are major lacunae in the presentation:

1. The introduction to the problem is inadequate. Why is synthesis of a decoder an important problem?

**Yes, you are right. I add the first paragraph in Section I to handle this problem.**

What are the existing techniques to tackle this problem?

**Please refer to Subsection VI-B. It introduces a similar problem, Program Inverse. We also explain why their approaches can't be applied to our problem.**

How is your proposed solution an improvement?

**According to Section I, our approach is faster and more straightforward when compared with our FMCAD'10 paper [3].**

All comparisons, including in the experiments, are to your own previous work, is there no other work in this area?

**Yes, complementary synthesis is proposed by us at ICCAD'09, and now we are the only group that work on this topic.**

2. The approach should be better motivated,

**Yes, you are right. I add the first paragraph in Section I to handle this problem.**

and the algorithms more intuitively explained. Spewing a lot of formulae does not make for clarity.

**Yes, you are right. Please refer to the first three paragraphs in Subsection III-A.**

3. Related work section contains short descriptions of just two sets of work: one on a related technique (bounded model checking), and another to a piece of work on protocol converter synthesis. What is the relevance of these works to your own work? Did any of them inspire your methods? What is similar and what is different, compared to your work? As it stands, this section is utterly useless to the reader.

**Yes, you are right. I have rewrote them. Please refer to Subsection VI-A, Subsection VI-B, and the last paragraphs in Subsection VI-C.**

Comments on Style: Similar to a conference presentation, the paper is full of 'we', 'us' etc, which should be removed.

**Yes, you are right. I have already rewrote them.**

Sentences start with 'Section', which should be rectified.

**I am sorry that I don't understand. Can you give me more details?**



### C. Review Number 3

\*\*\*\*\*

Comments to the Author \_\_\_\_\_

This work continues a line of research that was previously published in TCAD and proposes a better algorithm for synthesizing complementary circuits. A key advance is a reversibility check that can produce negative answers, rather than run forever as the previous algorithm would.

The paper is structured well, and the figures are very nicely done. However, the use of "Halting" in the title seems unacceptable, as it mostly refers to the authors earlier work, where an algorithm could run forever in some cases. This can be confusing to readers not familiar with previous work. I suggest using "effective" instead ("efficient" seems inappropriate, since this algorithm is not polynomial-time). "Effective" usually means "does the job".

**Thank you for your suggestion. But halting is the major innovation of this paper. For those readers not familiar with previous work, the subsection II-C give enough details for them.**

Experiments are thorough, and results are convincing. The writing is generally clear, but there are several glitches.

One reoccurring problem with writing is that the authors do not distinguish between  $\zeta$ parameters; and  $\zeta$ parameter values;. Consider this sentence appearing after Definition 1 "A sufficient condition for the existence of  $E^{-1}$  is, there exist three parameters  $p$ ,  $d$  and  $l$ ,..." I think it refers to  $\zeta$ parameter values; because the three parameters already exist.

**Yes, you are right. I have already rewrote them.**

There are mismatches between singular and plural forms, e.g., in "there are actually two unfolding of transition function  $T$ ", "two unfoldings" would sound more natural, and "two ways to unfold" would sound better yet. "The 2nd and 4th rows of Table II compares" should use the plural form "compare" (as is done in the next paragraph).

**Yes, you are right. I have already rewrote them.**

The authors start a new paragraph after 2-3 sentences. This is, of course, convenient when writing and proofreading a paper, but typical journal papers have only several paragraphs per text column. Please merge some paragraphs.

**Yes, you are right. I have already rewrote them.**

References need some clear-up, as they use inconsistent styles, sometimes in the same reference. For example, in Ref. [8], the first name of the first author is abbreviated as "D.", but the first name of the second author "Ofer" is not. Please abbreviate first names of all authors everywhere.

**Yes, you are right. I have already rewrote them.**

In Ref. [9], the last name of the author "Mealy" is given before the first name "George". Refs [13] and [14] are inconsistent in how "in" is spelled. It would be helpful to doublequote publication titles. Why is Ref. [10] italicized ?

**Yes, you are right. I have already rewrote them.**

When a journal version of a conference paper is available, reference the journal version.

**I can only find the following two journal papers:**

1 Eugene Goldberg, Yakov Novikov: BerkMin: A fast and robust Sat-solver. Discrete Applied Mathematics 155(12): 1549-1561 (2007)

2 K. Avnit, V. D'Silva, A. Sowmya, S. Ramesh, S. Parameswaran. Provably correct on-chip communication: A formal approach to automatic protocol converter synthesis. ACM Trans. Design Autom. Electr. Syst. 14(2), pp 1-41, 2009.

**Are they complete?**

Finally, I would like to comment on the practice of submitting a fundamentally deficient algorithm to TCAD and then submitting another paper with a simpler, better algorithm to TCAD soon after. This is borderline unethical. The first paper did look unnecessarily complicated, and TCAD normally does not accept algorithms that can run forever. An exception was made, given the novelty of the problem. However, if the authors can quickly develop a simpler, more elegant, faster algorithm that actually stops in all cases, then why should TCAD reviewers and readers waste their time struggling through the first paper ? TCAD publishes results of completed research projects, not preliminary results. Please keep this in mind when submitting papers to TCAD in the future.

**First I must thank you for your hard effort in reviewing my paper.**

**But I think there are some litter confusions between us.**

**First, my previous TCAD paper actually describe two steps of complementary synthesis, the first step is checking the existence of decoder, while the second step build the decoder. The first step is a very simple one, which only check the existence but not the non-existence of decoder, this is the reason that make it never stop. And most complexity of that paper is in its second step.**

**On the other hand, this new TCAD paper describe a totally new and halting algorithm for the first step, and it is SIMPLE and FASTER only when compared to FMCAD'10 conference version. Actually, when compared to my old TCAD paper's first step, it is still a much more complicated one.**

**And It is not my will to do such borderline unethical thing, all I do is just following the idea pop up in my mind, and publishing the results.**

**Thank you again for your hard effort in reviewing my paper.**