

Class 3

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Contents

- Conditional Expectation $E[X | Y]$
 - ★ $E[X | Y = y]$: Constant
 - ★ $E[X | Y]$: Function of Y (Random Variable)
 - Law of iterated expectation $E[E[X|Y]] = E[X]$
- Stochastic Simulation
 - ★ Inverse Transform Method
 - Sample from $Uniform(0, 1)$ and then compare with the PMF of the Random Variable and map to the value in the associated Range.
 - For continuous $X = F^{-1}(U)$ where F is CDF of X .
- Convergence of Random Variables
 - ★ Pointwise or sure convergence
 - $\{X_n, n \geq 0\}$ converges to X **pointwise** or **surely** if

$$\forall \omega \in \Omega, \quad \lim_{n \rightarrow \infty} X_n(\omega) = X(\omega)$$

- ★ Almost sure convergence

$$P(\omega \in \Omega : \lim_{n \rightarrow \infty} X_n(\omega) = X(\omega)) = 1$$

- ★ Strong Law of Large Numbers

$$S_n := \sum_{i=1}^n X_i \text{ then } \frac{S_n}{n} \rightarrow \mu \text{ a.s.}$$

- Interchanging limits and expectation
 - ★ Suppose $X_n \rightarrow X$ then when $\lim_{n \rightarrow \infty} E[X_n]$ equal to $E[\lim_{n \rightarrow \infty} X_n] = E[X]$
 - Counterexample where this doesn't work
 - $U \sim U(0, 1)$ and $X_n = n1_{\{U < 1/n\}}$
 - ★ for when we can swap the expressions above; or limit and expectation:
 - Monotone Convergence Theorem
 - Dominated Convergence Theorem