

Class 2

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Contents

- Random Experiments and Sample Space (Ω)
- Probability Measure (\mathbb{P}) and it's axioms
- Sigma-Algebra (\mathcal{F})
 - ★ Null set
 - ★ Element and it's complement
 - ★ Closure under countable union of disjoint element
- Borel σ -algebra $\mathcal{B}(\mathbb{R})$
 - ★ When $\Omega = \mathbb{R}$
- Conditional Probability
 - ★ Chain rule

$$P(A_1 \cap A_2 \dots A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \dots P(A_n|A_1 \cap A_2 \cap \dots A_{n-1})$$

- Independent Events
- Conditional Independence $P(AB|C) = P(A|C)P(B|C)$
- Mutually exclusive and Independence
 - ★ If one has zero-probability then related.
- Random Variable
 - ★ Map from one probability space to another
 - ★ Most cases the resultant probability space has sample space \mathbb{R} .

$$\therefore (\Omega, \mathcal{F}, \mathbb{P}(.)) \xrightarrow{X} (\mathbb{R}, \mathcal{B}(\mathbb{R}), \mathbb{P}_X(.))$$

- $X^{-1}(B)$ is called the preimage or the inverse image of B .
- ★ Discrete vs Continuous R.V.
- ★ nth moment $E[X^n]$
- ★ Law of unconscious statistician $E[g(X)] = \sum g(x)p_X(x)$.
- ★ Joint Random Variables