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## UNIT-3

①

Noam chomsky gave a mathematical model of Grammar which is effective for writing Computer language.

The four type of Grammars according to Noam chomsky are :

Grammars Type	Grammar Accepted	Language Accepted	Automaton
Type 0	unrestricted grammar	Recursively Enumerable Language.	<del>Enumerable</del> Turing machine
Type 1	Context Sensitive Grammars	Context sensitive Language.	Linear Bounded Automaton
Type 2	Context free Grammar	Context free language	Push down Automata
Type 3	Regular Grammar	Regular language	finite state Automaton

(2)

## Grammars :-

A Grammar ' $G$ ' can be formally described using 4 tuples as  $G = (V, T, S, P)$  where,

$V$  = Set of Variables or non-Terminal symbols  
Terminal

$T$  = Set of Terminal symbols

$S$  = Start symbol

$P$  = Production rules for Terminals and non-terminals

A production rules has the form  $\alpha \rightarrow \beta$  (union)  
where  $\alpha$  and  $\beta$  are strings on  $V \cup T$  and  
at least one symbol of ' $\alpha$ ' belongs to ' $V$ '

Example :-  ~~$G = ((S, A, B), (a, b), S,$~~   
 $(S \rightarrow AB, A \rightarrow a, B \rightarrow b))$

$V = \{S, A, B\}$  non terminals       $P$  = Production rules

$T = \{a, b\}$  terminal

$S = S$  start symbol

$S \rightarrow AB$  } This is  
 $A \rightarrow a$  } The  
 $B \rightarrow b$  } Production  
rules

(3)

$$\text{Eg: } S \rightarrow A B$$

$\rightarrow A$  (instead of A we can put a)

$\rightarrow aB$  (instead of B we can put b)

$\rightarrow \underline{ab}$

Now need to  $\Rightarrow$  discuss Regular Grammar.

Regular Grammar:-

Regular Grammar can be divided into two types.

Right Linear Grammar :-

A Grammar is said to be Right linear if all productions are of the form

$$A \rightarrow XB$$

$$A \rightarrow X$$

where  $A, B \in V$  and  $X \in T$  (Terminal)

(non  
Terminals)

left Linear Grammar (4)

A grammar is said to be left linear if all productions are of the form

$$A \rightarrow BX$$
$$A \rightarrow X$$

where  $A, B \in V$  and  $X \in T$ .

Ex :-  $S \rightarrow abS | b$

$S$  which is a non terminal symbol and right of  $S$  i.e.  $ab$  and  $b$  are the terminal symbols so above is the right linear grammar.

$$S \rightarrow Sbb | b$$

$S$  which is a non terminal symbol and  $b$  is a non terminal symbol. so this is a left. linear grammar.

4/12/21 Unit-3 Ambiguity in CFG (1)

A grammar  $G$  is ambiguous if there exists some string  $w \in L(G)$  for which there are two or more distinct derivation trees, or there are two or more distinct leftmost derivations.

Example: Consider CFG  $S \rightarrow S+S/S * S/a/b$  and string  $w = a * a + b$ , and derivations as follows.

Solution:-

first leftmost derivations for  $w = a * a + b$

$$S \Rightarrow S * S \quad (\text{why } S \rightarrow S * S)$$

$$\Rightarrow a * S \quad (\text{why } S \rightarrow a)$$

$$\Rightarrow a * S + S \quad (\text{why } S \rightarrow S + S)$$

$$\Rightarrow a * a + S \quad (\text{why } S \rightarrow a)$$

$$\Rightarrow a * a + b \quad (\text{why } S \rightarrow b)$$

Second leftmost derivation for  $w = a * a + b$

$\Rightarrow S \rightarrow S + S$  (Using  $S \rightarrow S + S$ )

(2)

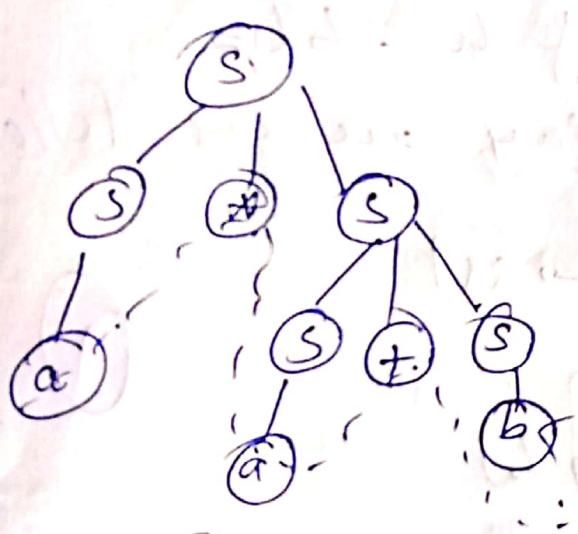
$\Rightarrow S \rightarrow S * S + S$  (Using  $S \rightarrow S * S$ )

$\Rightarrow a * S + S$  (Using  $S \rightarrow a$ )

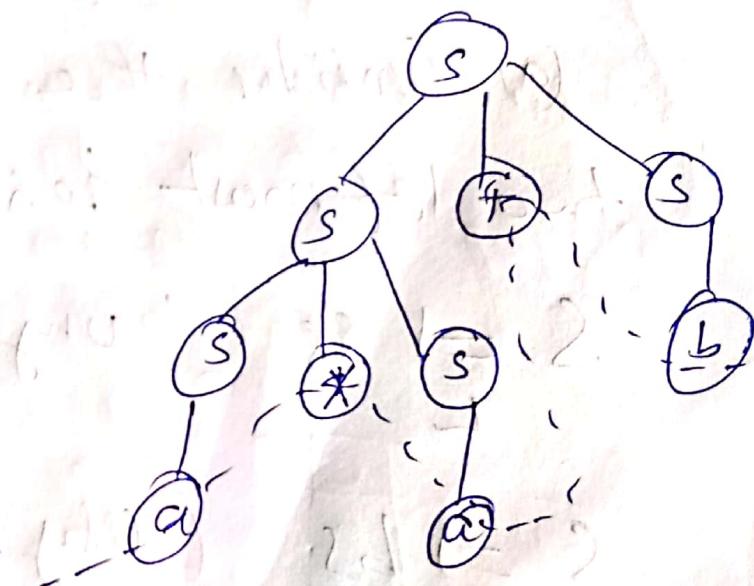
$\Rightarrow a * a + S$  (Using  $S \rightarrow a$ )

$\Rightarrow a * a + b$  (Using  $S \rightarrow b$ )

two distinct parse trees are shown figure (a) and figure (b)



(a)



(b)

(a) Parse tree for  $a * a + b$       (b) : parse tree for  $a * a + b$

Since there are two distinct leftmost derivations (two parse tree) for string  $w$ . Hence  $w$  is ambiguous. There is ambiguity in grammar G.

Ex:- Show that the following grammar  
are ambiguous.

(3)

$$(a) S \rightarrow ss \mid ab$$

$$(b) S \rightarrow A \mid B \mid s,$$

$$A \rightarrow aAB \mid ab,$$

$$B \rightarrow abB \mid e$$

Solution:

(a) Consider the string  $w = bbb$ .

two leftmost derivations are as follows

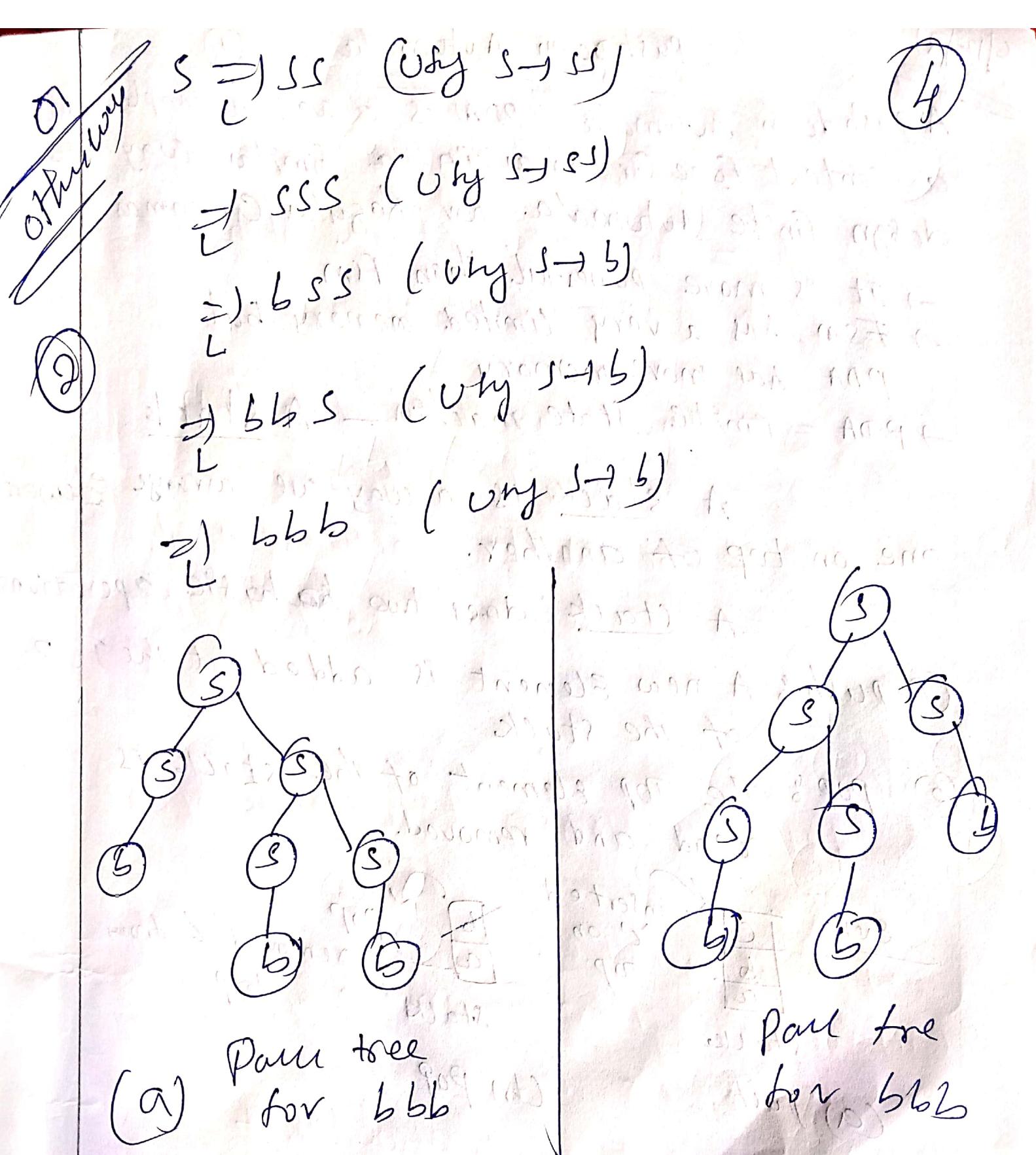
$$\begin{array}{l} S \xrightarrow{\quad} ss \quad (\text{by } S \rightarrow ss) \\ \downarrow \end{array}$$

$$\begin{array}{l} S \xrightarrow{\quad} bs \quad (\text{by } S \rightarrow b) \\ \downarrow \end{array}$$

$$\begin{array}{l} \xrightarrow{\quad} bss \quad (\text{by } S \rightarrow ss) \\ \downarrow \end{array}$$

$$\begin{array}{l} \xrightarrow{\quad} bbs \quad (\text{by } S \rightarrow b) \\ \downarrow \end{array}$$

$$\begin{array}{l} \xrightarrow{\quad} bbb \quad (\text{by } S \rightarrow b) \\ \downarrow \end{array}$$



so, given two grammar are ambiguous

② Context Free Language

①

Derivation Tree

or

Parse Tree

A Derivation Tree or Parse Tree is an ordered rooted tree that graphically represents the semantic information of strings derived from a context free grammar.

Example for the Grammar  $G = \{V, T, P, S\}$   
where  $S \rightarrow SB$ ,  $A \rightarrow AA | E$ ,  
 $B \rightarrow A AA$

Root vertex: must be labelled by the start symbol

vertex: labelled by non-terminal symbol

leaves:- labelled by terminal symbols  
or  $E$  (Epsilon)

## Context free Grammars:

A grammar  $G = (V, T, P, S)$  is said to be a CFG if the productions of  $G$  are of the form:

$$A \rightarrow \alpha, \text{ where } \alpha \in (V \cup T)^*$$

The right hand side of a CFG is not restricted and it may be null or a combination of variables and terminals.

The possible length of right hand sentential form ranges from 0 to  $\infty$  ie,  $0 \leq |L| < \infty$

As we know that a CFG has no context ~~regarding~~ neither left nor right. This is why, it is known as Context-free.

Ex:- Consider grammar  $G = (V, T, P, S)$  ②  
having productions  $S \rightarrow aSa \mid bSb \mid \epsilon$ .  
check the productions and find the language generated.

solution:-

Let  $P_1: S \rightarrow aSa$  ( $RHS$  is terminal variable terminal)

$P_2: S \rightarrow bSb$  ( $RHS$  is terminal variable terminal)

$P_3: S \rightarrow \epsilon$  ( $RHS$  is null string)

Since, all productions are of the form  $A \rightarrow \alpha$ , where  $\alpha \in (V \cup T)^*$ . hence

$G$  is a CFG.

given grammar

Language Generated :-  $S \rightarrow aSa \mid bSb$

③

$\Rightarrow a^m a^n$  or  $b^m b^n$

$\Rightarrow a^n a^m$  or  $b^m b^n$  (using n-step derivation)

$\Rightarrow a^n b^m a^m b^n$  or  $b^m a^m a^n b^n$  (using m-step derivation)

$\Rightarrow a^n b^m b^m a^n$  or  $b^m a^m a^m b^n$  (using S  $\rightarrow$  E)

$\therefore \text{So, } L(G) = \{ww^R : w \in (a+b)^*\}$

## ① Derivations from a Grammar

The set of all strings that can be derived from a Grammar is said to be the Language generated from that Grammar.

Ex:- ①

Consider the grammar  $G_1 = \{S, A\}, \{a, b\}, S,$   
 $\{ S \rightarrow aAb,$   
 $aA \rightarrow aaAb,$   
 $A \rightarrow \epsilon \}$ )

$S \rightarrow aAb$  (by  $S \rightarrow aAb$ )

$\rightarrow a_aAb b$  (by  $aA \rightarrow aaAb$ )

$\rightarrow a_aaAb b b$  (by  $aA \rightarrow aaAb$ )

$\rightarrow a_aa_b b b$  (by  $A \rightarrow \epsilon$ )

Ex ② Grammar  $G_2 = \{ (S, A, B), \{a, b\}, S, \{ \begin{matrix} S \rightarrow AB, \\ A \rightarrow a, \\ B \rightarrow b \end{matrix} \}$

$$S \rightarrow AB$$

$$\rightarrow ab$$

Language generated by  $G_2 \Rightarrow L(G_2) = \{ab\}$

Ex ③

$G_3 = \{ (S, A, B), \{a, b\}, S, \{ \begin{matrix} S \rightarrow AB, \\ A \rightarrow aA \\ B \rightarrow bB \end{matrix} \} \}$

①  $S \rightarrow AB$  [  $A \rightarrow a$   $B \rightarrow b$  ]  
 $\rightarrow ab$

④  $S \rightarrow AB$   
 $\rightarrow abB$   
 $\rightarrow abb$

depend on choice  
product by we take  
we can generate  
string.

③  $S \rightarrow A^*$   
 $\rightarrow aAb$   
 $\rightarrow aab$

$L(G_3) \in \{ab, a^2b^2, a^2b, ab^2, \dots\}^*$

$L(G_3) = \{a^m b^n\} \mid m > 0 \text{ and } n > 0\}$

This is the generalized way to generate  
the grammar  $G$ .

A  
C  
S  
C

# ① Language To Context free (Conversion)

Problem:- Show that the language  $L = \{a^m b^n \mid m \neq n\}$  is Context free

Solution:-

If it is possible to construct a CFG to generate this language then we say that the language is context free.

Let us construct the CFG for the language defined.

Assume that  $m = n$

i.e., m number of a's should be followed by m number of b's.

So, the CFG for this can be

$$S \rightarrow a S b / \epsilon \quad \text{--- } ①$$

But,  $L = \{a^m b^n \mid m \neq n\}$  means, a's should be followed by b's and number of a's should not be equal to number of b's.

② i.e.,  $m \neq n$ .

Let us see the different cases when  $m > n$  and when  $m < n$ .

Case - I :-

$m > n$

This case occurs if the number of a's more compared to number of b's.

The extra a's can be generated using the production

$$A \rightarrow aA/a$$

and the extra a's generated from this production should be appended towards left of the string generated from the production shown by production I.

This can be achieved by introducing one more production.

$$S_1 \rightarrow AS$$

③ So, even though from S we get a number of a's followed by a number of b's since it is preceded by a variable A from which we could generate extra a's, number of a's, ~~number of~~ followed by number of b's are different.

Case 2<sup>o</sup>

$m < n$  :-

This case occurs if the number of b's are more compared to number of a's. The extra b's can be generated using the production

$$B \rightarrow bB/b$$

and the extra b's generated from this production should be appended towards right of the string generated from the production shown in production (1).

this can be achieved by introducing one more production. (4)

$$S_1 \rightarrow SB$$

the context free grammar  $G = (V, T, P, S)$  where

$$V = \{S_1, S, A, B\}, T = \{a, b\}$$

$$P = \{$$

$$S_1 \rightarrow AS | SB$$

$$S \rightarrow asb | \epsilon$$

$$A \rightarrow aA | a$$

$$B \rightarrow bB | b$$

If  $S_1$  is the start symbol.

generates the language ~~as many~~

$$L = \{a^m b^n \mid m \neq n\}.$$

Since a CFG exists for the language.

so, the language is context free.  
Hence proved.

Problem:- Draw a CFG to generate a language consisting of equal number of a's and b's.

Solution:-

Note that initial production can be of the form

$$S \rightarrow aB/bA$$

If the first symbol is 'a', the second symbol should be a non-terminal from which we can obtain either 'b' or one more 'a' followed by two B's denoted by 'aBB' or a 'b' followed by S denoted by 'bS'.

Note that from all these symbols definitely we obtain equal number of a's and b's.

The productions corresponding to these can be of the form

$$B \rightarrow b/aBB/bS$$

on similar lines we can write  
A-productions as

(2)

$$A \rightarrow a/bAA/a$$

from which we obtain a 'b' followed by  
either

1.  $a'$  or

2. a 'b' followed by AA's denoted by

$bAA$  OR

3. symbol 'a' followed by S denoted by as

The context free grammar  $G = (V, T, P, S)$

where  $V = \{S, A, B\}$ ,  $T = \{a, b\}$

$$P = \{S \rightarrow aB/bA$$

$$A \rightarrow aS/bAA/a$$

$$B \rightarrow bS/aBB/b$$

If S is the start symbol

generates the language consisting  
of equal number of a's and b's.

Hence proved

problem :- obtain a CFG to generate integers.

solution :-

The sign of a number can be '+' or '-' or  $\epsilon$ .

The production for this can be written as

$$S \rightarrow + | - | \epsilon$$

A number can be formed from any of the digits 0, 1, 2, ..., 9. The production to obtain these digits can be written as

$$D \rightarrow 0 | 1 | 2 | 3 | \dots | 9$$

A number  $N$  can be recursively defined as follows.

1. A number  $N$  is a digit  $D$  (i.e.,  $N \rightarrow D$ )
2. The number  $N$  followed by digit  $D$  is also a number (i.e.,  $N \rightarrow ND$ )

The production for the recursive definition can be written as

$$N \rightarrow D$$

$$N \rightarrow ND$$

An integer number  $I$  can be a number  $N$  or the sign  $S$  of a number followed by number  $N$ .

The production for this can be written as  $I \rightarrow N | SN$

so, the grammar  $G$  to obtain integer number can be written as  $G = \{V, T, P\}$  where  $V = \{D, S, N, I\}$

$$T = \{+, -, 0, 1, 2, 3, -9\}$$

$$P = \{I \rightarrow N | SN\}$$

$$N \rightarrow D | ND$$

$$S \rightarrow + | - | E$$

$$D \rightarrow 0 | 1 | 2 | 3 | -9$$

$I$   $\Rightarrow S = I$  which is start symbol.

Hence Proved

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UNIT-3  
Continue

LEFTmost AND RIGHTmost DERIVATIONS

left most derivation:-

If  $G = (V, T, P, S)$  is a CFG and  $w \in L(G)$  then a derivation  $S \xrightarrow{L} w$  is called leftmost derivation if and only if all steps involved in derivation have leftmost variable replacement only.

Right most derivation:-

If  $G = (V, T, P, S)$  is a CFG and  $w \in L(G)$  then a derivation  $S \xrightarrow{R} w$  is called rightmost derivation if and only if all steps involved in derivation have rightmost variable replacement only.

Examples:-

Example 1: Consider the grammar

Examp  $S \rightarrow S+S/S*S/a/b$ . find left most and right most derivations for string w.

$$\text{ie, } w = a * a + b.$$

Solution:-

leftmost derivation for  $w = a * a + b$

$$S \xrightarrow{L} S * S \quad (\text{using } S \xrightarrow{\alpha} S * S)$$

$\Rightarrow a * s$  (the first left hand symbol is a  
so using  $s \rightarrow a$ )

$\Rightarrow a * s + s$  (using  $s \rightarrow s + s$ , in order to get  $a+b$ )

$\Rightarrow a \# a + s$  (second symbol from the left is  $a$ , so using  $S \rightarrow a$ )

$\stackrel{L}{\Rightarrow} a * a + b$  (the last symbol from the left is  $b$ , using  $S \rightarrow b$ )

Right most derivation:-

for  $w = a * a + b$

$\begin{array}{l} S \Rightarrow S * S \\ R \end{array}$  (using  $S \rightarrow S * S$ )

$\begin{array}{l} \Rightarrow S * S * S \\ R \end{array}$  (Since, in the above sentential form second symbol from the right is \* so we can not use  $S \rightarrow a / b$ .  
Therefore, we use  $S \rightarrow S * S$ )

$\begin{array}{l} \Rightarrow S * S + b \\ R \end{array}$  (using  $S \rightarrow b$ )

$\begin{array}{l} \Rightarrow S * a + b \\ R \end{array}$  (using  $S \rightarrow a$ )

$\begin{array}{l} \Rightarrow a * a + b \\ R \end{array}$  (using  $S \rightarrow a$ )

problem 2:-

Consider a CFG  $S \rightarrow bA|aB$ ,  $A \rightarrow aa|aaa|a$ ,  $B \rightarrow bs|aBB|b$ . Find leftmost and rightmost derivations for  $w = aaaabbbaabbba$ .

solution:-

leftmost derivation for  $w = aaaabbbaabbba$

$\begin{array}{l} S \xrightarrow[L]{} aB \\ \quad \text{(using } S \rightarrow aB \text{ to generate first symbol of } w) \end{array}$

$\Rightarrow \begin{array}{l} a \xrightarrow[L]{} aB \\ \quad \text{(since, second symbol is } a, \\ \quad \text{so we use } B \rightarrow aBB) \end{array}$

$\Rightarrow \begin{array}{l} aa \xrightarrow[L]{} aB \\ \quad \text{(since, third symbol is } a, \\ \quad \text{so we use } B \rightarrow aBB) \end{array}$

$\Rightarrow \begin{array}{l} aab \xrightarrow[L]{} B \\ \quad \text{(since, fourth symbol is } b, \text{ so we use } B \rightarrow b) \end{array}$

$\Rightarrow \begin{array}{l} aabb \xrightarrow[L]{} B \\ \quad \text{(since, fifth symbol is } b, \text{ so we use } B \rightarrow b) \end{array}$

- $\Rightarrow aaabbbaB$  (since first symbol is a, so we use  $B \rightarrow aBB$ )  
 $\Rightarrow aaabbbaB$  (since seventh symbol is b, so we use  $B \rightarrow b$ )  
 $\Rightarrow aaabbbaS$  (since eighth symbol is b, so we use  $B \rightarrow bS$ )  
 $\Rightarrow aaa\cancel{b}babbbA$  (since ninth symbol is b, so we use  $S \rightarrow bA$ )  
 $\Rightarrow aaa\cancel{b}babbbA$  (since tenth symbol is a, so using  $A \rightarrow a$ )  
 $\therefore S \Rightarrow aaa\cancel{b}babbbA$

Rightmost derivations for  $w = aaabbba$

- $S \xrightarrow{R} aB$  (using  $S \rightarrow aB$  to generate first symbol of w)  
 $\xrightarrow{R} aaBB$  (we need a as the rightmost symbol and second symbol from the left side, so we use  $B \rightarrow aBB$ )

$\Rightarrow aABbs$  (we need ~~a~~ A as the right most symbol and thus is obtained from A only, we use  $B \rightarrow bs$ )

$\Rightarrow aaBbba$  (using  $S \rightarrow bA$ )

$\Rightarrow aaBbba$  (using  $A \rightarrow a$ )

$\Rightarrow aaaBBbba$  (we need b as the fourth symbol from the right)

$\Rightarrow aaaBbbba$  (using  $B \rightarrow b$ )

$\Rightarrow aaa bSbbbba$  (using  $B \rightarrow bs$ )

$\Rightarrow aaa.bba.bbbba$  (using  $S \rightarrow bA$ )

$\Rightarrow aaaabbabbba$  (using  $A \rightarrow a$ )

R

Right most derivation  $S \xrightarrow[R]{} aaaabbabbba$

S

S

problem 1°-

①

Let  $G = (V, T, P, S)$  where  $V = \{S, C\}$ ,  $T = \{a, b\}$

$$P = \{ S \rightarrow aCa \\ C \rightarrow aCa \mid b \}$$

3 S is the starting symbol.

what is the language generated by this grammar?

solution:-

Consider the derivation

$S \Rightarrow aCa \Rightarrow aba$  (by applying 1<sup>st</sup> & 3<sup>rd</sup> production)  
so, the string  $aba \in L(G)$

Consider the derivation

$S \Rightarrow aCa$  by applying  $S \rightarrow aCa$

$\Rightarrow aacaa$  by applying  $C \rightarrow aCa$

$\Rightarrow aaacaaa$  by applying  $C \rightarrow aCa$

$\Rightarrow \cdots$

②

$\Rightarrow \cdots$

$\Rightarrow a^n c a^n$  by applying  $C \rightarrow a c a$  (n times)

$\Rightarrow a^n b a^n$  by applying  $C \rightarrow b$

So, the Language  $L$  accepted by the grammar  $G$  is  $L(G) = \{a^n b a^n \mid n \geq 1\}$

i.e, the language  $L$  derived from the grammar  $G$  is "The string consisting of  $n$  number of  $a$ 's followed by a ' $b$ ' followed by  $n$  number of  $a$ 's.

problem 2<sup>o</sup>  
what is the language generated by  
the grammar  $S \rightarrow OA | \epsilon$   
 $A \rightarrow IS$

solution: The null string  $\epsilon$  can be obtained by applying the productions  $S \rightarrow \epsilon$  and the derivations is shown below

$S \Rightarrow \epsilon$  (By applying  $S \rightarrow \epsilon$ )

Consider the derivation

$S \Rightarrow OA$  (By applying  $S \rightarrow OA$ )

$\Rightarrow OIS$  (By applying  $A \rightarrow IS$ )

$\Rightarrow OIOA$  (By applying  $S \rightarrow OA$ )

$\Rightarrow OIOIS$  (By applying  $A \rightarrow IS$ )

$\Rightarrow OIOI$  (By applying  $S \rightarrow \epsilon$ )

② so, alternatively applying the productions  
 $S \rightarrow OA$  and  $A \rightarrow IS$  and finally  
applying the production  $S \rightarrow E$ , we  
get string consisting of only 01's.  
so, both null string i.e.  $\epsilon$   
and string consisting 01's can be  
generated from this grammar.  
so, The language generated by this  
grammar is

$$L = \{w \mid w \in \{01\}^*\} \text{ or } L = \{(01)^n \mid n \geq 0\}$$

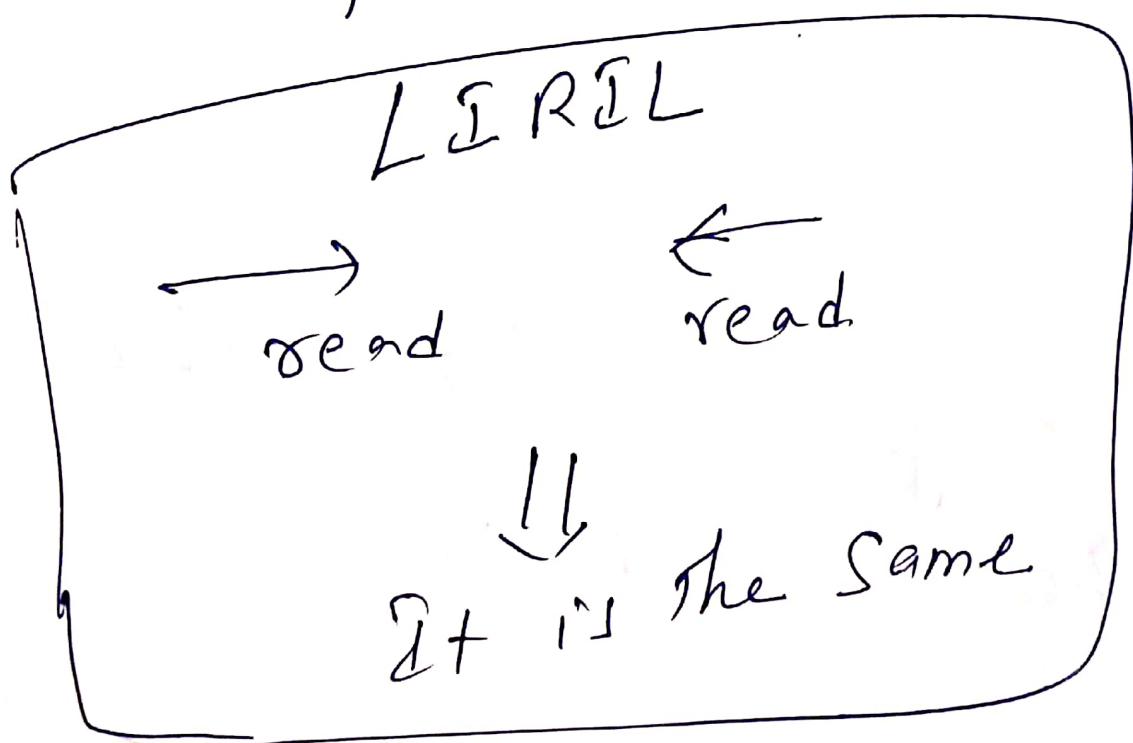
problem 3:-

①  
Construct CFG. for the language L which has all the strings which are all palindromes over  $T = \{a, b\}$

solution:-

As we know the strings are palindrome if they posses same alphabets from forward as well as from backward

Ex:- Example string 'LIRIL' is Palindrome because



Since the language  $L$  is over  $T = \{a, b\}$ , we want the production rules to be build a's and b's. As  $E$  can be the Palindrome,  $a$  can be Palindrome even so we can  $b$  can be palindrome. So we can write the production rules as

$$G = (\{S\}, \{a, b\}, P, S)$$

P can be  $S \rightarrow aSa$   
 $S \rightarrow bSb$   
 $S \rightarrow a$   
 $S \rightarrow b$   
 $S \rightarrow E$

The string can be 'abaaba' will be derived as

$s \rightarrow a s a$

(3)

$\rightarrow a b s b a \quad (s \rightarrow b s a)$

$\rightarrow a b a s a b a \quad (s \rightarrow a s a)$

$\rightarrow a b a E a b a \quad (s \rightarrow E)$

$\rightarrow a b a a b a$

which is a palindrome.

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## DERIVATION TREES

Let  $G = (V, T, P, S)$  is a CFG. Each production of  $G$  is represented with a tree satisfying the following conditions:

- ① If  $A \rightarrow \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n$  is a production in  $G$ , then  $A$  becomes the parent of nodes labeled  $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$  and
- ② The collection of children from left to right yields  $\alpha_1 \alpha_2 \alpha_3 \dots \alpha_n$

Example—

Consider a CFG  $S \rightarrow S + S \mid S * S \mid a \mid b$  and construct the derivation trees for all productions.

Solution— for production  $S \rightarrow S + S$ .

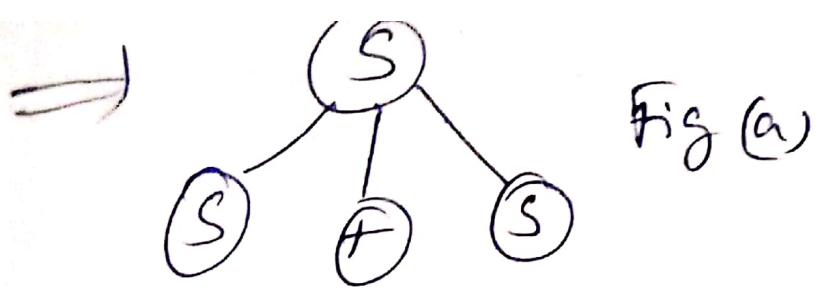


Fig (a)

for production  $S \rightarrow S * S \Rightarrow$

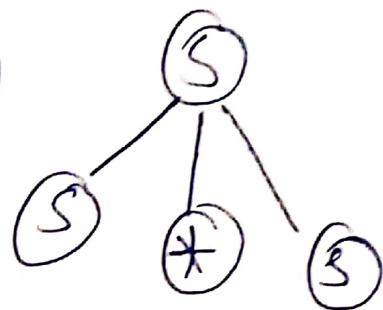


Fig (b)

For production  $S \rightarrow a \Rightarrow$

```

graph TD
    S1((S)) --- @
  
```

fig(c)

for production  $S \rightarrow b \Rightarrow$

```

graph TD
    S1((S)) --- b
  
```

fig(d)

If  $w \in L(G)$  then it is represented by a tree called derivation tree or Parse tree satisfying the following conditions:

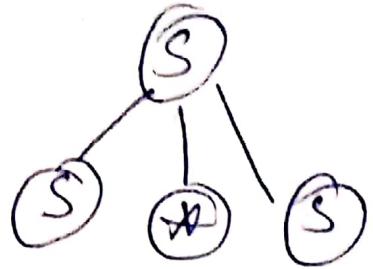
- ① The root has label S (the starting symbol)
  - ② The all internal vertices (or nodes) are labeled with variables.
  - ③ The leaves (or terminal) nodes are labeled with  $\epsilon$  or terminal symbol.
  - ④ If  $A \rightarrow d_1 d_2 d_3 \dots d_n$  is a production in G then A becomes the parent of nodes labeled  $d_1 d_2 d_3 \dots d_n$  and
  - ⑤ ~~The~~ The collection of leaves from left to right yields the string w.
- 

Example:- Consider the grammar  
 $S \rightarrow S + S \mid S * S \mid a/b$ . Construct derivation tree for string  $w = a * b + a$ .

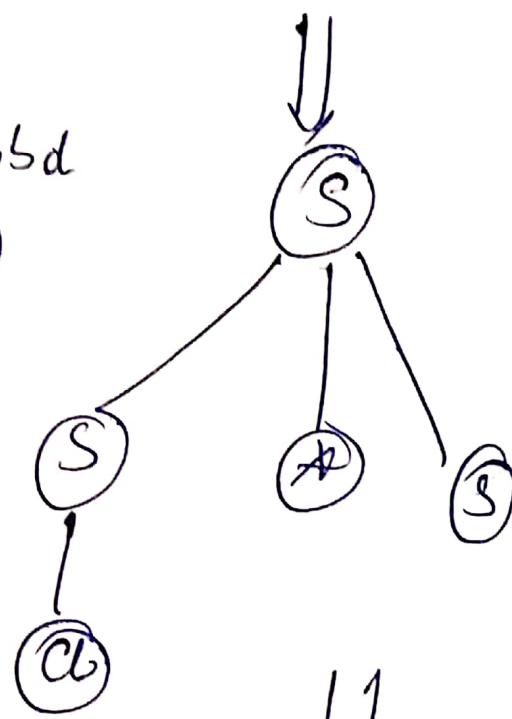
Solution:- The derivation tree or parse tree is shown as

leftmost derivation: for  $w = a * b + a$ .

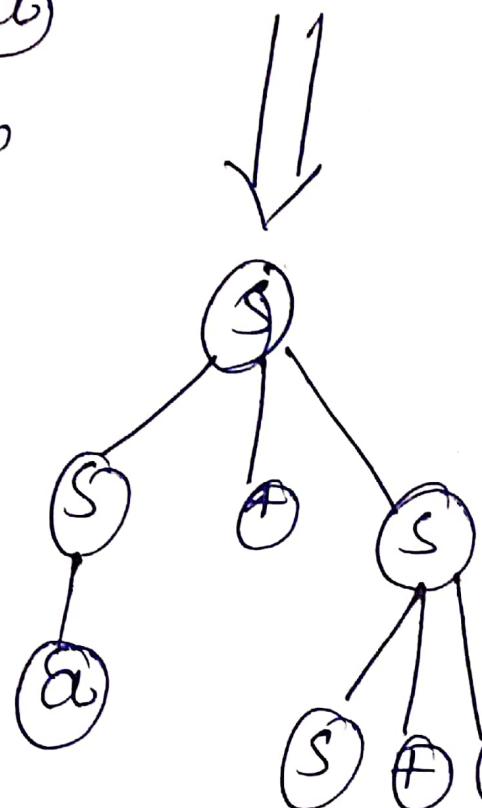
$S \Rightarrow S * S$  (using  $S \rightarrow S * S$ )  $\Rightarrow$



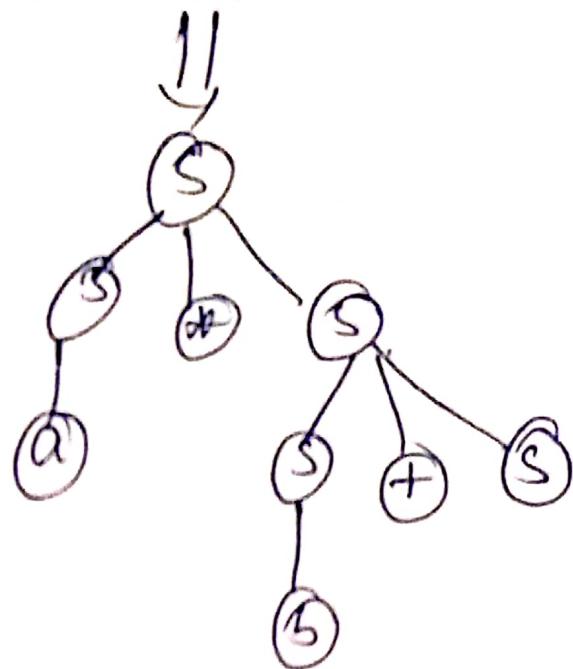
$\Rightarrow a * S$  (the first left hand symbol is a, so using  $S \rightarrow a$ )



$\Rightarrow a * S + S$  (using  $S \rightarrow S + S$ , in order to get  $b + a$ )



$a * b + a$  (second symbol from left is  $b$ ,  
so try  $S \rightarrow b$ ).



$a * b + a$  (the last symbol from the left  
is  $a$ , so try  $S \rightarrow a$ )

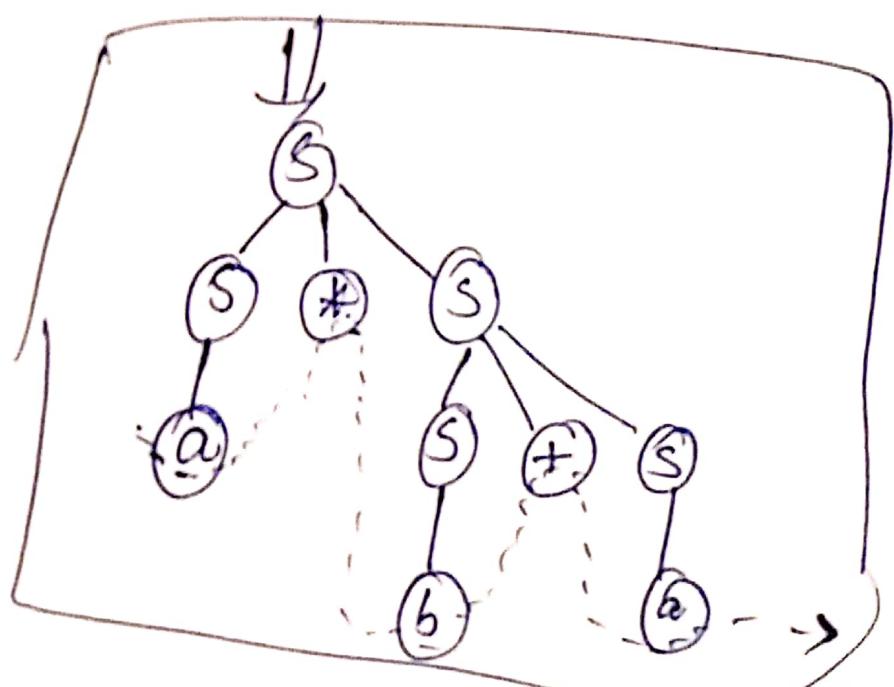


figure 8 Parse tree for  
 $a * b + a$ .

Example 2 :-

Consider a grammar G having production  
 $S \rightarrow aAS/a, A \rightarrow \text{~~s~~} bA \mid ss/ba.$

Show that  $S \Rightarrow aa\text{ }bb\text{ }aa$  and construct  
a derivation tree whose yield is ~~aa~~  
aa bb aa.

Solution :-

$$S \Rightarrow aAS$$

$$\Rightarrow ASbAS$$

$$\Rightarrow aAbAS$$

$$\Rightarrow aabbAs$$

$$\Rightarrow aa bb aa.$$

Hence  $S \Rightarrow aa bb aa$   
Parse tree is shown  
in figure.

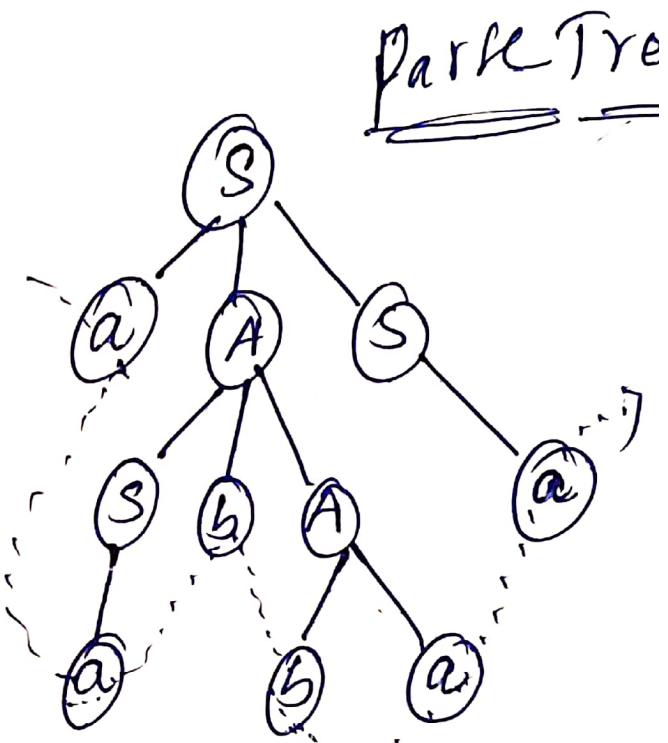


Fig: Parse tree  
yielding  
aa bb aa.

Example 3 :- Consider the grammar G whose productions are

$$S \rightarrow 0B/1A, A \rightarrow 0/0S/1AA, B \rightarrow 1/1S/0BB.$$

Find (a) left most derivation (b) Right most derivation for string 00110101 and construct derivation tree also.

Solution :-

Left most derivation:-

$$S \Rightarrow 0B \Rightarrow 00BB$$

$$\Rightarrow 001B \Rightarrow 0011S$$

$$\Rightarrow 00110B \Rightarrow 001101S$$

$$\Rightarrow 0011010B \Rightarrow 00110101$$

Right most derivation

$$S \Rightarrow 0B \Rightarrow 00BB$$

$\Rightarrow 00B1 \Rightarrow 001S1$

$\Rightarrow 0011A1 \Rightarrow 00110S1$

$\Rightarrow 001101A1 \Rightarrow 00110101$

## ② Derivation tree.

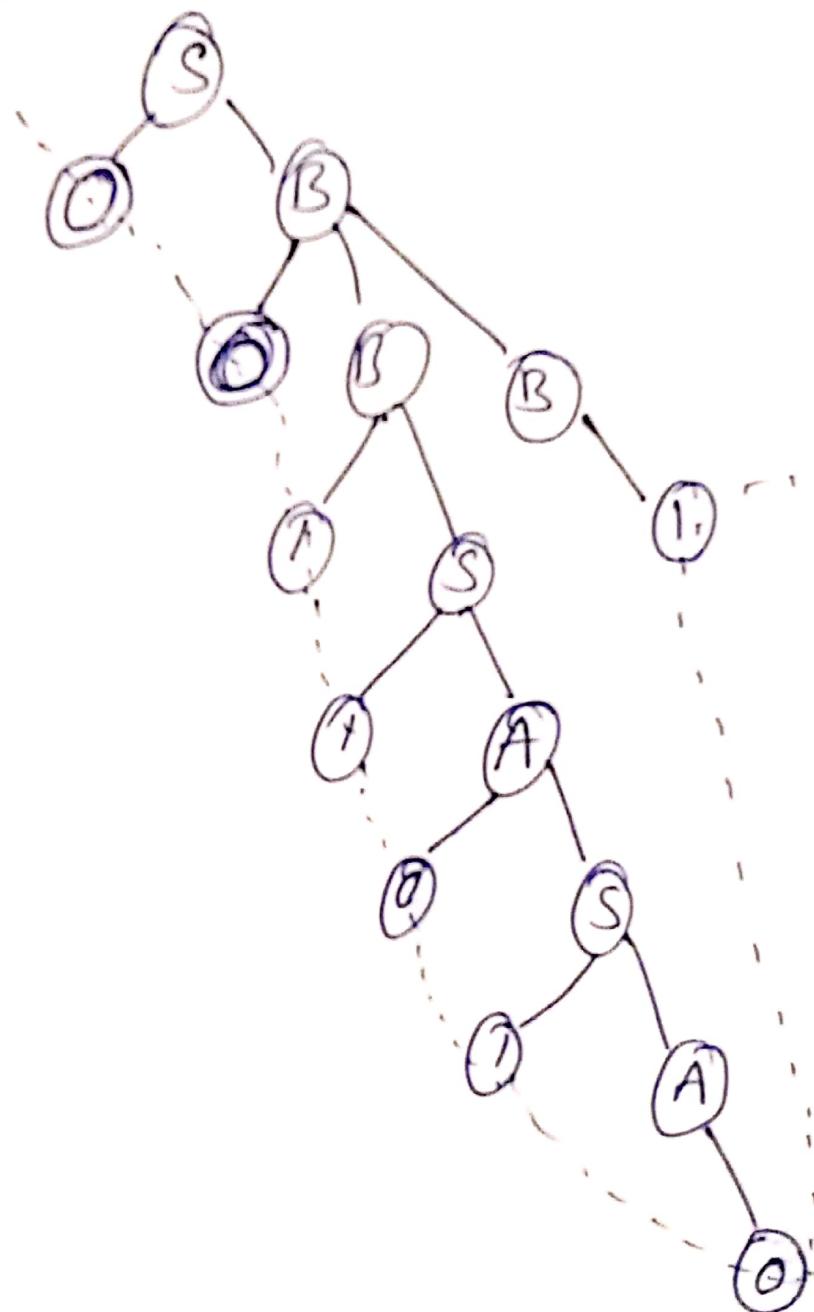


Fig.  $\Rightarrow 00110101$   
in the derivation tree