Unit 2 - M1

Find Eigen Values & Eigen Vectors.

for $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}_{3\times3}$

Step 1: To find eigen values

The Ch eq $= A - \lambda I = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$$

$+24 - 14 + 2\lambda$

$2\lambda + 10$

~~(8-λ) {-6(3-λ)}~~

$$8-\lambda\left[(7-\lambda)(3-\lambda)\right] + 6\left[-6(3-\lambda) - 2(-4)\right]$$

$$+ 2\left\{-6(-4) - 2(7-\lambda)\right]$$

$24 - 14 = 10$

$24 - 8 = 16$

$= 8 \quad -24 + 6\lambda + 8 \quad \boxed{6\lambda - 16}$

$7(3-\lambda) - \lambda(3-\lambda)$

$21 - 7\lambda - 3\lambda + \lambda^2$

$\boxed{\lambda^2 - 10\lambda + 21}$

$$8(\lambda^2-10\lambda+21)-\lambda(\lambda^2-10\lambda+21)+6(6\lambda-16)+2(2\lambda+10)$$

$$8\lambda^2 \underline{-80\lambda}+168-\lambda^3+10\lambda^2\underline{-21\lambda}+\underline{36\lambda}-96+\underline{4\lambda}+20$$

$$-\lambda^3+18\lambda^2-6\lambda+92.$$

$$\lambda^3-18\lambda^2+6\lambda-92=0.$$

$x_1 = 17.95$

$x_2 = 0.02.$

$x_3 = 0.024.$

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}_{3\times 3}$$

trace A = 12.

trace (Adj A) =

gt

M_1 Unit 4

Rolles Theorem

Statement :- Let $f(x)$ be a polynomial defined in $[a,b]$ such that

i) $f(x)$ be a continuous function in $[a,b]$.

$\{a \le x \le b\}$.
$x \in [a,b]$.

(ii) $f'(x)$ be a derivable function in (a,b).

$\to x \in (a,b)$.

$a < x < b$.

iii) $f(a) = f(b)$.

There exists atleast one value, where 'c' $\in (a,b)$ such that $\boxed{f'(c) = 0}$

Note :- ① Every derivable func is cont func. $\downarrow$ converse $\uparrow$ may not be true.

① Verify Rolles theorem for
$f(x) = 2x^3 + x^2 - 4x - 2$ in $[-\sqrt{2}, \sqrt{2}]$

A) * Given polynomial is $f(x) = 2x^3 + x^2 - 4x - 2$ in $[-\sqrt{2}, \sqrt{2}]$.

* By Rolles theorem

i) $f(x) = 2x^3 + x^2 - 4x - 2$ is a continuos in closed $[-\sqrt{2}, \sqrt{2}]$

$f(x) = 2x^3 + x^2 - 4x - 2$

$f(-\sqrt{2}) =$

$2(-\sqrt{2})^3 + (\sqrt{2})^{\cancel{2}} - 4(\sqrt{2}) - 2$

$-2(\sqrt{2})^3 + 2 - 4\sqrt{2} - 2$

finite . cont

$f(\sqrt{2}) =$ finit Result

* Note every polynomial is a continuos function without denominators

$\boxed{\frac{x}{0} = \infty}$

(ii) $f'(x) = 2x^3 + x^2 - 4x - 2$.

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}\left[2x^3 + x^2 - 4x - 2\right]$$

$$= 6x^2 + 2x - 4 - \cancel{0}0$$

$$\boxed{f'(x) = 6x^2 + 2x - 4}$$

it is a derivable function in $(-\sqrt{2}, \sqrt{2})$.

check again $(-\sqrt{2}, \sqrt{2})$

no denominator

∴ It is cont

∴ finite

(iii) $f(a) = f(b)$.

$\therefore a = -\sqrt{2}, b = \sqrt{2}$.

$f(-\sqrt{2}) = f(\sqrt{2})$

$f(-\sqrt{2}) = 2(-\sqrt{2})^3 + (-\sqrt{2})^2 - 4(-\sqrt{2}) - 2 =$

$\cancel{f(\sqrt{2})} = -2(\sqrt{2})^2 \cdot (\sqrt{2}) + 4(\sqrt{2}) - \cancel{2}$.

$f(\sqrt{2}) = 2(\sqrt{2})^2 \cdot (\sqrt{2}) + 2 - 4\sqrt{2} - \cancel{2}$.

$-4(\sqrt{2}) + 4(\sqrt{2}) = 4\sqrt{2} - 4\sqrt{2}$

$0 = 0$.

$\{\because f(a) = f(b)\}$

All 3 conditions are satisfied.

** $\exists$ atleast one point $c \in (-\sqrt{2}, \sqrt{2})$ s.t. $f'(c) = 0$.

$(\because f'(x) = 6x^2 + 2x - 4$. — eq.

To find c value

put c in eq and equate it to 0

$f'(c) = 6c^2 + 2c - 4 = 0$.

$\frac{2}{3}, -1$

$\boxed{\frac{2}{3} \in (-\sqrt{2}, \sqrt{2}), \text{ \& } -1 \in (-\sqrt{2}, \sqrt{2})}$ $\rightarrow$ (-1)

Q). Verify Rolles Theorem for Tan x in $[0, \pi]$.
$0^\circ - 180^\circ$.

A) * Given $f(x) = \tan x$ defined in $[0, \pi]$.

* By Rolles Theorem,

i). $f(x) = \tan x$ is continuos / discontinuos in $[0, \pi]$.

$\{0, \pi)$

$\tan 0 = 0$

$\tan 30 = \frac{1}{\sqrt{3}}$

$\tan 45 = 1$

$\tan 60 = \sqrt{3}$

$\boxed{\tan 90 = \infty}$

Therefore the function is a discontinuos function.

Discontinuos at $x = 90 = \frac{\pi}{2}$

$\therefore f(x) = \tan x$ is not continuos form.

(ii). $f(x) = \tan x$ is discontinuos at $x = \frac{\pi}{2} \in [0, \pi]$

$\{f(\pi/2) = \tan(\frac{\pi}{2}) = \infty\}$.

$\therefore$ Hence $f(x) = \tan x$, the rolles Theorem is not satisfied for the given function.

Q) Verify Rolles theorem of

$f(x) = x^3$ in $[1, 3]$.

1, 27.

i) $f(x)$ is a polynomial so therefore it is a continuos function without any denominator

$\therefore$ $f(x) = x^3$ is a polynomial in $[1, 3]$.

$\therefore$ $f(x) = x^3$ is $\boxed{\text{continuous}}$ in $[1, 3]$.

(ii) $f'(x) = x^3$

$= 3x^2$.

It $f'(x)$ is also $\boxed{\text{derivable}}$ in $(1, 3)$.

(iii) $f(a) = f(b)$

$f(1) = f(3)$

$f(1) = f(x^3) = 1^3$.

$f(3) = x^3 = 3^3 = 27$.

$1^3 \neq 3^3$. ✗.

3rd condition is not satisfied

$\therefore$ Rolles Theorem is not satisfied

Lagrange's Theorem.

$\boxed{f(x)} = x^3 - x^2 - 5x + 3$ in $[0,4]$.

given polynomial function $f(x) = x^3 - x^2 - 5x + 3$.
in $[0,4]$, w.k.t. By lagrange's theorem,

(i). Every polynomial fun is cont fn.
$f(x) = x^3 - x^2 - 5x + 3$ is cont.

(ii). $f'(x) = 3x^2 - 2x - 5$ is derivable
on $(0,4)$

2 condition are satisfied.

c value that $\exists$ atleast one part.

$c \in (0,4)$ s.t. $\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$ $\left\{\begin{matrix} a=0 \\ b=4 \end{matrix}\right)$

$= \frac{f(4) - f(0)}{4 - 0}$ $= \frac{\boxed{f(4)} - f(0)}{4}$

$=$

$f(4)$ is of the form $f(x)$.

$= 3c^2 - 2c - 5$
$= 64$

$\frac{4^3 - 4^2 - 5(4) + 3}{4}$

$$3c^2 - 2c - 5 = \frac{64 - 16 - 20 + 3}{\cancel{3}\,4}$$

$$3c^2 - 2c - 5 = \frac{31}{4}$$

$$12c^2 - 8c - 20 - 31 = 0$$

$$12c^2 - \cancel{0} - 8c - 51$$

$$= \cancel{\frac{\sqrt{33}}{6}} \quad \cancel{- \frac{\sqrt{33}}{6}}$$

$$\frac{2 + \sqrt{157}}{6}, \quad \frac{2 - \sqrt{157}}{6},$$

-1.75, 1.75. $\quad 2.421$, -1.754.

$12c^2 - 8c - 51$ $\quad \in (0, 4)$

$f(x) = \log_e x$ in $[1, e]$.

$\log 1 = 0$.
$\log 0 = \infty$.
$\log \infty = (\frac{1}{0})$.
$\log(1) - \log(0)$
$1 - \infty = \infty$.

The given fun $f(x) = \log_e x$ in $[1, e]$.

(i) $f(x) = \log x$ is continuos on $[1, e]$
$(0, \infty \in [1, e])$

(ii) $f'(x) = \frac{1}{x}$. $(1, e)$.
$x = 1 \Rightarrow 1$.
$\frac{1}{0} = \infty$. $x = 5 \Rightarrow \frac{1}{1.5}$ + Fn.
not there in given int. $x = 0, \frac{1}{0} = \infty$ (1e)

$f'(x)$ is derivable on $(1, e)$

(iii). to find c then there exit at least 1 point. $\boxed{\log c}$

$\frac{1}{c} = \frac{f(b) - f(a)}{b - a}$.

$= \frac{f(e) - f(1)}{e - 1}$.

$\frac{\log e - \log 1}{e - 1}$

$\frac{1}{c} = \frac{1 - 0}{e - 1} \Rightarrow \frac{1}{c} = \frac{1}{e - 1}$

The lagrange's thm is verify $\boxed{c = e - 1} \in (1, e)$.

(7). If $a < b$, Prove that $\frac{b-a}{(1+b^2)} < \tan^{-1}(b) - \tan^{-1}(a) < \frac{b-a}{(1+a^2)}$ using LMVT.

400
500
$\times 120$

120
120
240

500

i). $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$.

(ii) $\frac{5\pi + 4}{20} < \tan^{-1}(2) < \frac{\pi + 2}{4}$

Sol Now the interval is $[a, b]$.

Le $f(x) = \tan^{-1} x$ in $[a, b]$.

i) $f(x) = \tan^{-1} x$ is cont fn in $[a, b]$.

(ii) $f'(x) = \frac{1}{1+x^2}$ is derivable in (a, b).

iii). There exist atleast 1 point $c \in (a, b)$.

$a \leq c < b$

s.t $\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$.

$a < c\ b$

x^2 undu in $f'(x)$ so.

$\boxed{S.O.B.S}$

$a^2 < c^2 < b^2$

$\boxed{A.1.O.B.S}$

$1 + a^2 < 1 + c^2 < 1 + b^2$.

R.O.B.S.

$$\boxed{\frac{1}{1+a^2} > \frac{1}{1+c^2} > \frac{1}{1+b^2}} \quad -(2)$$

ly eqn (1).

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

$$\boxed{\frac{1}{1+c^2} = \frac{\tan^{-1}(b) - \tan^{-1}a}{b-a}} \quad (3)$$

put eq (3) in 2.

$$\frac{1}{1+a^2} > \frac{\tan^{-1}(b) - \tan^{-1}a}{b-a} > \frac{1}{1+b^2}.$$

$\boxed{\text{M. } (b-a) \text{ O.B.S.}}$

$$\frac{b-a}{1+a^2} > \tan^{-1}(b) - \tan^{-1}a > \frac{b-a}{1+b^2}.$$

$$\therefore \frac{b-a}{1+b^2} < \tan^{-1}(b) - \tan^{-1}a < \frac{b-a}{1+a^2}.$$

i)

$$\boxed{\frac{b-a}{1+b^2} < \tan^{-1}(b) - \tan^{-1}(a) < \frac{b-a}{1+a^2}.}$$

choose $b = \frac{4}{3}$ & $a = \tan\left(\frac{\pi}{4}\right) = 1$.

$$\frac{\frac{4}{3}-1}{1+\frac{16}{9}} < \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}(1) < \frac{\frac{4}{3}-1}{1+1}.$$

$$\frac{\frac{4-3}{1}}{\frac{9+16}{9}} < \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\tan\frac{\pi}{4}\right) < \frac{\frac{4-3}{3}}{2}.$$

$$\frac{1}{\cancel{3}_1} \times \frac{\cancel{9}^3}{25} < \tan^{-1}\left(\frac{4}{3}\right) - \frac{\pi}{4} < \frac{1}{3} \times \frac{1}{2}.$$

$$\frac{3}{25} + \frac{\pi}{4} < \tan^{-1}\left(\frac{4}{3}\right) - \frac{\pi}{4} + \frac{\pi}{4} < \frac{1}{6} + \frac{\pi}{4}.$$

$$\boxed{\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}.}$$

(ii) $\frac{5\pi+4}{20} < \tan^{-1}(2) < \frac{\pi+2}{4}$. whose a & b

$b = 2$.

$a = \tan\left(\frac{\pi}{4}\right) = 1$

$$\frac{2-1}{1+4} < \tan^{-1}(2) - \tan^{-1}(1) < \frac{2-1}{1+1}.$$

$$= \frac{1}{5} < \tan^{-1}(2) - \tan^{-1}\left(\tan\frac{\pi}{4}\right) < \frac{1}{2}.$$

$$\frac{1}{5} < \tan^{-1}(2) - \frac{\pi}{4} < \frac{1}{2} =$$

A.~~tte~~ $\frac{\pi}{4}$. O. B. S

$$\boxed{\frac{1}{5} + \frac{\pi}{4} < \tan^{-1}(2) < \frac{\pi}{4} + \frac{1}{2}}$$

Cauchy's nth root test

method :- Cauchy's n root test

Statement :- If ΣU_n is a infinite series.

$\ell = \lim_{n \to \infty} (U_n)^{1/n}$ then

i) ΣU_n is cgt if $\ell < 1$.

2) ΣU_n is dvt if $\ell > 1$.

3) Test fails if $\ell = 1$.

Comp test

basic def cans

de Alembers

Ratio

↓

Roabes.

P-series. ÷ $p>1$ cgt } Σ $\frac{\Sigma v_n}{u_n}$.
$p<1$ dgt.

Ratio test. $l>1$ cgt.
$l<1$ dgt.
$l=1$ fail

3rd unit.
$l>1$ cgt.
4th unit Cauchy
$l<1$ cgt.

Rabees. $l>1$ cgt.
$l<1$ dgt.

(1). Test for convergence of series

$$\Sigma \frac{1}{(1+\frac{1}{n})^{n^2}}$$

for nth root, the given series has total power 'n'

we is

$$U_n = \frac{1}{(1+\frac{1}{n})^{n^2}}.$$

$$\boxed{l = \lim_{n\to\infty} (u_n)^{1/n}.}$$

$$\lim_{n\to\infty} \left[\frac{1}{(1+\frac{1}{n})^{n^2}}\right]^{1/n}$$

$$\lim_{n\to\infty} \frac{1}{(1+\frac{1}{n})^{\cancel{n^2} \times \frac{1}{\cancel{n}}}}$$

$$\lim_{n\to\infty} \frac{1}{(1+\frac{1}{n})^{n}}$$

$$= \frac{1}{\lim_{n\to\infty} (1+\frac{1}{n})^{n}} \qquad \left\{\because \lim_{n\to\infty} (1+\frac{1}{n})^{n} = e\right\}$$

$$\frac{1}{e} = \frac{1}{2.718}$$

$$\frac{1}{2.718} < 1 \qquad \text{cgt}$$

hence ΣU_n is cgt

By cauchy nth root test

(2) Test for convergence of

$$\sum \left[\left(\frac{n+1}{n}\right)^{n+1} - \frac{n+1}{n}\right]^{-n}$$

we use Cauchy nth root test which is of the form $\boxed{\sum U_n}$.

$$U_n = \left[\left(\frac{n+1}{n}\right)^{n+1} - \frac{n+1}{n}\right]^{-n}.$$

W.K.T

$$\lim_{n\to\infty} (U_n)^{1/n}.$$

$$= \lim_{n\to\infty} \left[\left[\left(\frac{n+1}{n}\right)^{n+1} - \frac{n+1}{n}\right]^{-\cancel{n}}\right]^{1/\cancel{n}}$$

$$\lim_{n\to\infty} \left[\left(\frac{n+1}{n}\right)^{n+1} - \frac{n+1}{n}\right]$$

$$\lim_{n\to\infty} \left[\frac{(n+1)^{n+1}}{n^{n+1}} - \frac{n+1}{n}\right]^{-1}.$$

$$\frac{1}{\lim_{n\to\infty} \left[\frac{(n+1)^{n+1}}{n^{n+1}} - \frac{n+1}{n}\right]^{+1}}.$$

$$\frac{1}{\underset{n\to\infty}{lt}\left[\frac{(n+1)^{n+1}}{n^{n+1}}-\frac{n+1}{n}\right]}$$

$$= \frac{1}{\underset{n\to\infty}{lt}\left[\frac{\left[n\left(1+\frac{1}{n}\right)\right]^{n+1}}{\cancel{\left[n\left\{1+\frac{1}{n}\right\}\right]^{n}}\ \cancel{n^{n+1}}}-\frac{\cancel{n}\left(1+\frac{1}{n}\right)}{\cancel{n}}\right]}$$

$$\frac{1}{\underset{n\to\infty}{lt}\left(1+\frac{1}{n}\right)^{n+1}-\left(1+\frac{1}{n}\right)}$$

$$\cancel{\frac{1}{e^{n+1}}}$$

$$\frac{1}{\underset{n\to\infty}{lt}\underbrace{\left(1+\frac{1}{n}\right)^{n}}_{e}\cdot\left(1+\frac{1}{n}\right)^{1}-\underset{n\to\infty}{lt}\left(1+\frac{1}{n}\right)}$$

$$= \frac{1}{\underset{n\to\infty}{lt}\left(1+\frac{1}{n}\right)^{n}\cdot\underset{n\to\infty}{lt}\left(1+\frac{1}{n}\right)^{1}-\left(1+\underbrace{\frac{1}{\infty}}\right)}$$

$$-1$$

$$\frac{1}{e\cdot\left(1+\frac{1}{\infty}\right)-1}\quad\frac{1}{e(1+0)-1}\quad\frac{1}{e-1}\quad=\frac{1}{2.781-1}$$

$\Sigma U_n =$ cgt

$$\frac{1}{1.78} < 1$$

cgt

(2) $\Sigma\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$.

Cauchy nth root test.

$\Sigma (U_n)^{1/n}$.

$U_n = \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$.

$(U_n)^{1/n} = \left[\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}\right]^{\frac{1}{n}}$.

$-n^{\frac{3}{2}\times\frac{1}{n}} \cdot \frac{3}{2n}$.

$-n^{\frac{3}{2n}}$.

$= \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2n}}}$.

$\lim_{n\to\infty}\left(\cancel{\quad} 1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2n}}}$.

$\frac{1}{\lim_{n\to\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2n}}}} \Rightarrow \frac{1}{\lim_{n\to\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{n^{\frac{3}{2}\times\frac{1}{n}}}}$.

$\cancel{\frac{1}{(\sqrt{e})^{-\infty}}}$ $\cancel{\frac{1}{(2.718)^{\infty}}}$ cgt.

e

$= \frac{1}{\lim_{n\to\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{n^{\frac{1}{2}}}} \Rightarrow \lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{\sqrt{n}}\right)^{\sqrt{n}}}$.

$\frac{1}{e} = \frac{1}{2.7} < 1 =$ cgt

(4) $\sum \left[\frac{(n+1)^{n+1}}{n^{n+1}}\right]^{n} \cdot x^{n}$.

Cauchy or nth root test.

$U_n = \left[\frac{n+1}{n^{n+1}}\right]^{n} \cdot x^{n}$.

$U_n^{\frac{1}{n}} = \left[\left[\frac{n+1}{n^{n+1}}\right]^{n} \cdot x^{n}\right]^{1/n}$

$\cancel{= \left[\frac{n+1}{n^{n+1}}\right]^{n} \cdot x}$

$\frac{n+1}{n^{n}+1^{(n)}} \cdot x$.

$\frac{n+1}{n^{n}+n} \cdot x$.

$\cancel{\underset{n\to\infty}{lt} = \left[\frac{n\left(1+\frac{1}{n}\right)}{n^{n+1}}\right]^{n} \cdot x}$

$\underset{n\to\infty}{lt} \frac{(n+1)^{n+1}}{n^{n} \cdot n^{1}} \cdot x$.

$\underset{n\to\infty}{lt} \frac{\cancel{n^{n+1}}\left[1+\frac{1}{n}\right]^{n+1}}{\cancel{n^{n+1}}} \cdot x$. $e^{n+1} \cdot x$.

$$\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^n\cdot\left[1+\frac{1}{n}\right]^1.$$

$$\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^n\cdot\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^1\cdot x.$$

$$x\cdot\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^n\cdot\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^1.$$

$$x\cdot e\cdot 1.$$

$$l = xe.$$

by nth root test.

$ex < 1$ cgt.

$ex > 1$ dgt.

$ex = 1$ fail.

$$x = \frac{1}{e}.$$

At $x = \frac{1}{e}$.

$$U_n = \left[\frac{(n+1)^{n+1}}{n^{n+1}}\cdot x\right]^n$$

sub $x = \frac{1}{e}$.

$$\left[\frac{(n+1)^{n+1}}{n^{n+1}} \cdot \frac{1}{e}\right]^n.$$

By basic eqn.

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty}\left[\frac{(n+1)^{n+1}}{n^{n+1}} \cdot \frac{1}{e}\right]^n$$

$$\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^{n+1}\right]^n \cdot \lim_{n\to\infty}\left(\frac{1}{e}\right)^n.$$

$$\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n+1}\right]^n \lim_{n\to\infty}\left(\frac{1}{2.718}\right)^n.$$

∞.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1}.$$

$\infty \neq 0$.

By basic def

$\Sigma U_n = 0 = \text{cgt}$.

$\Sigma U_n \neq 0$. dgt.

$\therefore$ it is dvgt.

Find the eigen values & eigen vectors for

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

Step 1 To find Eigen Values

The char eq is

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}.$$

$\Rightarrow 8-\lambda\ (\cancel{-6}(3\cancel{-\lambda}))\ \cancel{+6}(\cancel{-6}(1$

$$8-\lambda\left[(7-\lambda)(3-\lambda) + \overset{-16}{4(-4)}\right] + 6\left[-6(3-\lambda) + 2(-4)\right]$$

$$+2\left[(-6)(-4) - 2(7-\lambda)\right].$$

$\lambda^2 - 4\lambda + 5$ $\qquad\qquad$ $6\lambda - 28$

$$8-\lambda\left\{21-\lambda-3\lambda+\lambda^2-16\right] + 6\left[-18+6\lambda-8\right].$$

$2\lambda + 14$

$$+2\left[24 - 14 + 2\lambda\right].$$

28×6

$24 - 14 = 10$

$$\Rightarrow 8\lambda^2 - 32\lambda + 40 - \lambda^3 + 4\lambda^2 - 5\lambda + 36\lambda - 168$$

$$+4\lambda + 28$$

0, 3, 15

$$\Rightarrow -\lambda^3 + 12\lambda^2 - 3\lambda - 100$$

(1)

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}_{3\times 3}$$

Det A = 0.

ch eq = $\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$.

$\lambda^3 - s_1\lambda^2 + s_2\lambda - |A| = 0$.

$S_1 = 8+7+3 = 18$

$S_2 = [21-16]\,[24-4]\,[56-36]$

$= [5+20+20] = 45$.

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$.

15, 3, 0

<u>Step 2</u> Finding Eigen Vector.

<u>case i</u>. corresponding to $\boxed{\lambda = 0}$.

$(A - \lambda I)x = 0$.

$$\begin{pmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{pmatrix} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

1st row x 1st colm.

$8x - 6y + 2z = 0$ — ①

$-6x + 7y - 4z = 0$ — ②

$2x - 4y + 3z = 0$ — ③

abca

Select ① & ② & write co-effs

$$\begin{array}{cccc} z & x & y & \\ 8 & -6 & 2 & 8 \\ -6 & 7 & -4 & -6 \end{array}$$

$$\frac{x}{24-14} = \frac{y}{-12+32} = \frac{z}{56-36}$$

$$\frac{x}{10} = \frac{y}{20} = \frac{z}{20} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 20 \end{bmatrix}$$

56
-36
20

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{2} = k$$

$x = k, y = 2k, z = 2k$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ 2k \\ 2k \end{bmatrix}$$

$$\boxed{X_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = k \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}$$

corres to $\lambda = 0$

$X_2 = 3$.

$A - \lambda I = 0$.

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$5x - 6y + 2z = 0 \quad -(1)$

$-6x + 4y - 4z = 0 \quad -(2)$

$2x - 4y + 0 = 0 \quad -(3)$

$$\begin{array}{cccc} & z & x & y \\ 5 & -6 & 2 & 5 \\ -6 & 4 & -4 & -6 \end{array}$$

$$\frac{x}{24-8} = \frac{y}{-12+20} = \frac{z}{20-36}$$

$$\frac{x}{16} = \frac{y}{8} = \frac{z}{-16}$$

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{-2} = K$$

$x = 2K,\ y = K,\ z = -2K$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2K \\ K \\ -2K \end{bmatrix}.$$

$$X_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

$$\boxed{X_2 = K \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}}$$

(iii) root 15.

$$A - \lambda I = 0$$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & 4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-7x - 6y + 2z = 0$$
$$-6x - 8y + 4z = 0$$
$$2x - 4y - 11z = 0$$

$$\begin{array}{cccc} z & x & y & \\ -7 & -6 & 2 & -7 \\ -6 & -8 & 4 & -6 \end{array}$$

$$\frac{x}{-24+16} = \frac{y}{-12+28} = \frac{z}{-49-36}$$

$$\frac{x}{-8} = \frac{y}{16} = \frac{z}{-85} = k$$

$$x = -10k, \quad y = 16k, \quad z = -85k$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = k \begin{bmatrix} -8 \\ 16 \\ -85 \end{bmatrix} = \begin{bmatrix} 2k \\ -2k \\ k \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

cor to $\lambda = 0 = X_1 = \left[\frac{2}{2}\right.$

$$\begin{array}{c} X_1 \\ X_2 \\ X_3 \end{array} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}.$$

$Y \neq 0$ = non zero vector

Diagonalize the matrix

Diagonalize the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$.

Step 1 find eigen values.

$|A| = 8$.

$$\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & -1 & 3-\lambda \end{bmatrix}.$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0.$$

$S_1 = 7.$,

$S_2 = [9-1] + [3-0] + [3-0]$

$S_2 = 8+3+3 = 14.$

$$\lambda^3 - 7\lambda^2 + 14\lambda - 8 = 0.$$

4, 2, 1.

1, 2, 4.

Case step 2 Finding the eigen vectors

Case 1. $\lambda = 1,$

$$\begin{bmatrix} 1-1 & 0 & 0 \\ 0 & 3-1 & -1 \\ 0 & -1 & 3-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$0 = 0.$

$2y - z = 0 - (1).$

$-y + 2z = 0 - (2).$

we dont have x variable so put $x = k$

	z	x	y	
0	2		-1	0.
0	-1		2	0

$$\frac{x}{5} = \frac{y}{\cancel{+0}} = \frac{z}{0} = k.$$

$x = 5k, \; y = 0k, \; z = 0k.$ $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}.$

$2y - z = 0 - (1).$ $\Rightarrow$ $2y = z.$

$-y + 2z = 0 - (2)$

$\cancel{-y + 4y} = 0.$

$x/0/0$

$y = 2z.$ (if $y = 0, 2z = 0$

$-y + 2z = 0.$

$\cancel{-}y = \cancel{-}2z.$

Case (ii) $\lambda = 2$.

$$\begin{bmatrix} 1-2 & 0 & 0 \\ 0 & 3-2 & -1 \\ 0 & -1 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$-1x = 0 - (1) \Rightarrow x = 0.$

$1y - 1z = 0 - (2) \Rightarrow y = z.$

$-1y + 1z = 0 - (3) \Rightarrow z = y.$

If $y = 1$, $z = 1$.

$$X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Case 3 $\lambda = 4$.

$$\begin{bmatrix} 1-4 & 0 & 0 \\ 0 & 3-4 & -1 \\ 0 & -1 & 3-4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x = 0 - (1) \Rightarrow x = 0.$

$-y - z = 0 - (2) \quad -y = z \Rightarrow y = -z.$

$-y - z = 0 - (3) \quad -z = \cancel{y} + y$

~~If $y = 1$, $z = 1$.~~

put $z = 1$.

$-y = z.$

$-z = y.$

$$\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} &-y - 1 = 0 \\ &-y = 1 \\ &y = -1. \end{aligned}}$$

$R = \Big[$

Step 3 :- Find P matrix.

$P \rightarrow$ Normalized Matrix.

$\rightarrow \quad x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Normalized value of $x_1 = \sqrt{1^2+0+0} = 1$.

$\|x_1\| = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$\rightarrow x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

Normalized value of $x_2 = \sqrt{0+1^2+1^2} = \sqrt{2} =$

$\|x_2\| = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$.

$\rightarrow$ ~~Also~~ $x_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

Normalized value of $x_3 = \sqrt{0+1^2+1^2} = \sqrt{2}$.

$\|x_3\| = \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$.

$$P = [\,\|x_1\| \cdot \|x_2\| \;\; \|x_3\|\,].$$

$$P = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -1/\sqrt{2} \\ 0 & \frac{1}{\sqrt{2}} & 1/\sqrt{2} \end{bmatrix} \end{array}$$

<u>Step 4</u> To find P^{-1}.

$\therefore P^{-1} = P^{T}.$ $\{\because A$ is symetri

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^{T}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{-1}{\sqrt{2}} & \sqrt{2} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = D.$$ using orthogonalization

$$\begin{bmatrix} 4 & 4 & 4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1(1)(1) & 0(0)(0) \\ -1(-1)(1) & 1(2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = D.$$

$$\begin{bmatrix} 1(1)(1). & 0(0)\ (0). \\ 0 & \frac{1}{\sqrt{2}}(3)(\frac{1}{\sqrt{2}}). \\ 0. & \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}(1)(-\frac{1}{\sqrt{2}}).$$

$-1.$

Normal form

Reduce the given matrix into Normal form

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}_{4\times 4}$$

where Normal = Identity

$[\diagdown]$ dia = 1, non-dig = 0

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \quad R_2 \leftrightarrow R_1$$

Step 1

$R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$0 - 3(1)$
$0 - 3$

or

$2 - 3(1)$
$2 - 3$

$C_3 \to C_3 - C_1, \quad C_4 - C_4 - C_1.$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

<u>Step 2</u>

$R_3 \to R_3 - R_2.$

$R_4 \to R_4 - R_2.$

$-3-(-3)$

$-3+3 \quad 0.$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-3+3(1)$

$-3+3$

$=0.$

$\cancel{C_2 \leftrightarrow}\ C_3 \to C_3 + 3C_2.$

$C_4 \to C_4 + C_2$

$-1+1$

$=0.$

$\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank $=2$.

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}_{3\times 4}$$

(4)

Step 1

$R_2 \to R_2$
$R_3 \to R_3 + 8R_1$

$-8+8$

$-1+8$

$-3+8(3)$

$-3+24$

$4+8(6)$

$4+48$

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ 0 & 7 & 21 & 52 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$-8+8=0$

$-1+1=0$

$-3+3=0$

$4+6$

$C_2 \to 8C_2 - C_1$

$C_3 \to 8C_3 - 3C_1$

$C_4 \to 6C_1 - 8C_4$

$8(2)-3(0)$

$16-3$

$6(0)-8(10)$

$0-80$

$6(8)-8(6)$

$6(0)-8(2)$

$0-16$

-16

$8-8$

$$\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 24 & 24 & -16 \\ 0 & 0 & 0 & -180 \end{bmatrix}$$

$8(3)-0$

$6(0)-8(10)$ 24

$0-18$

$8(3)-3(8)$

$8(2)-3(0)$

$24-0$

$R_1 \to R_1/8$

$R_2 \to R_2/3$

$R_3 \to R_3/18$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

<u>colm</u>

$C_2 \rightarrow C_2 - \frac{1}{8} C_1$ ✓

$C_3 \rightarrow C_3 - \frac{1}{8} C_2$

$C_4 \rightarrow C_4 - \frac{6}{8} C_1$

$$\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$3 - \frac{1}{8}$

$\frac{24-1}{8}$

$\frac{23}{8}$

$C_3 \rightarrow C_3 - \frac{2}{3} C_2$

$C_4 \rightarrow C_4 - \frac{2}{3} C_2$

$2 - \frac{2}{3}\left(\frac{2}{1}\right)$

$$\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$2 - \frac{4}{3}$

$\frac{6-4}{3} = \frac{2}{3}$

$$\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 10 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1/8$

$R_2 \rightarrow R_2/3$

$R_3 \rightarrow R_3/10$

M1 partial diff

properties of partial differention.

① If $z=K$, $\frac{\partial z}{\partial x}=0$, $\frac{\partial z}{\partial y}=0$.

② If $z=f(y) = \frac{\partial z}{\partial x}=0$

③ If $z = \sin(x+y)$, $\frac{\partial z}{\partial x} = \cos(x+y) \cdot \frac{\partial}{\partial x}(x+y)$

$$= \cos(x+y).$$

$$\frac{\partial z}{\partial y} = \cos(x+y).$$

④ $z=e^{x+y}$, $\frac{\partial z}{\partial x} = e^{x+y}$, $\frac{\partial z}{\partial y} = e^{x+y}$.

⑤ $z = \log(x+y)$, $\frac{\partial z}{\partial x} = \frac{1}{x+y}$, $\frac{\partial z}{\partial y} = \frac{1}{x+y}$.

① If $u = e^{x} \cdot \sin x \cdot \sin y$, find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$

$\rightarrow u = \overset{u}{e^{x}} \cdot \overset{v}{\sin x} \cdot \sin y$

y k term treat as const

$\boxed{\text{diff. } u \text{ partially w.r.t } x}$

$$\frac{\partial u}{\partial x} = \sin y \left[e^{x} \cdot \cos x + \sin x \cdot e^{x} \right].$$

$$= e^{x} \cdot \cos x \cdot \sin y + \sin x \cdot \sin y \cdot e^{x}$$

$\boxed{\text{diff } u \text{ partially w.r.t } y}$

x k term treat as const

$$\frac{\partial u}{\partial y} = e^{x} \cdot \sin x \left[\cos y \right]$$

② If $z(x+y) = x-y$, find $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2$

$$\rightarrow z = \left(\frac{x-y}{x+y} \right) \frac{\rightarrow u}{\rightarrow v}$$

$$\frac{u}{v} = $$

$$\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

diff z partially w.r.t x.

$$\frac{\partial z}{\partial x} = \frac{\overset{u}{(x+y)} \frac{\partial}{\partial x} \overset{v}{(x-y)} - \overset{u}{(x-y)} \frac{\partial}{\partial x} \overset{v}{(x+y)}}{\underset{v}{(x+y)^2}}$$

$\frac{\partial}{\partial x}(x-y)$

$x = 1$

$y = 0$

$$\frac{(x+y)(1) - (x-y)(1)}{}$$

$$\frac{\cancel{x}+y-\cancel{x}+y}{(x+y)^2}$$

diff z partially w.r.t y.

$$\frac{\partial z}{\partial y}=\frac{(x+y)\frac{\partial}{\partial y}(x-y)-(x-y)\frac{\partial}{\partial y}(x+y)}{(x+y)^2}$$

$$=\frac{(x+y)(-1)-(x-1)(1)}{(x+y)^2}=\frac{-x-y-x+y}{(x+y)^2}=\frac{-2x}{(x+y)^2}$$

$$\therefore\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=\left[\frac{2y}{(x+y)^2}-\frac{-2x}{(x+y)^2}\right]^2$$

$$=\left[\frac{2y+2x}{(x+y)^2}\right]^2=\left[\frac{2(x+y)}{(x+y)^2}\right]^2$$

$$\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=\frac{4}{(x+y)^2}$$

(2) If $z(x+y)=(x^2+y^2)$ prove that

$$\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$$

$$\therefore z = \frac{(x^2+y^2)}{(x+y)}$$ $\frac{y}{x} = 0$

diff z wrt x.

$$\boxed{\frac{\partial z}{\partial x}} = \frac{(x+y)\frac{\partial}{\partial x}(x^2+y^2) - (x^2+y^2)\frac{\partial}{\partial x}(x+y)}{(x+y)^2}$$

$$= \frac{(x+y)(2x) - (x^2+y^2)(1)}{(x+y)^2}$$

$$= \frac{2x^2+2xy - x^2 + y^2}{(x+y)^2} = \frac{x^2 - y^2 + 2xy}{(x+y)^2} = 1$$

$$\boxed{\frac{\partial z}{\partial y}} = \text{diff w.r.t } y$$

$$\frac{(x+y)\frac{\partial}{\partial y}(x^2+y^2) - (x^2+y^2)\frac{\partial}{\partial y}(x+y)}{(x+y)^2}$$

$$\frac{(x+y)(2y) - (x^2+y^2)(1)}{(x+y)^2}$$

$$= \frac{2y^2+2xy - x^2 - y^2}{(x+y)^2} =$$

*.

Gauss Elimination Method:

(1) Solve the equations by Gauss Elimination method.

$2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16$

Matrix Notation.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}.$$

Augmented Matrix $[A:B]$.

$$= \begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix} \quad \text{-(1)}.$$

$$\begin{array}{r} 16 \\ \times 2 \\ \hline 32 \end{array} \quad \begin{array}{r} 18 \\ \times 2 \\ \hline 36 \end{array}$$

Echlon form.

$R_2 \to 2R_2 - 3R_1, \quad R_3 \to 2R_3 - R_1.$

$$\begin{array}{rrrr} 6 & 4 & 6 & 36 \\ -6 & 3 & 3 & 30 \\ \hline 0 & 1 & 3 & 6 \end{array}$$

$$\begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 7 & 22 \end{bmatrix}$$

$$\begin{array}{rrrr} 2 & 8 & 18 & 32 \\ -2 & 1 & 1 & 10 \\ \hline 0 & 7 & 17 & 22 \end{array}$$

$R_3 \to R_3 - 7R_2$.

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix} \quad -(2).$$

$$\begin{array}{rrrr} 0 & 7 & 17 & 22 \\ -0 & 7 & 21 & 42 \\ \hline 0 & 0 & -4 & -20 \end{array}$$

$$\begin{array}{r} 42 \\ -22 \\ \hline 20 \end{array}$$

eqn 2

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 20 \end{bmatrix}$$

$2x + y + z = 10 \quad -(3)$

$\cancel{0}\, y + 3z = 6 \quad -(4)$

$-4z = 20 = (5) \Rightarrow z = 4 - (5)$

put $\boxed{z=4}$ in eq(4).

$-y + \cancel{4}12 = 6$.

$-y = 6 - 12$

$-y = -6$.

$\boxed{y=6}$.

put $z=4$ & $y=6$ in eq(1).

$2x + 6 + 4 = 10$.

$2x + 10 = 10$.

$2x = 0$.

$\boxed{x=0}$.

$\therefore z = 5$

~~y = 9~~

$x = \cancel{8}\, 7$.

$y = -11$.

(2) $3x+y+2z=3$.
$2x-3y-z=-3$.
$x+2y+z=4$.

$AX=B$.

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}.$$

Aug.

$$\begin{bmatrix} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{bmatrix}.$$

$13 \times 5 = 65$ $\quad 99 - 45 = 4$ $\quad 35 - 11 = 24$ $\quad 11 \times 5 = 55$

Ech form..

$R_2 \to 3R_2 - 2R_1$, $R_3 \to 3R_3 - R_1$.

$$\begin{bmatrix} 3 & 1 & 2 & 3 \\ 0 & 11 & 7 & 15 \\ 0 & 5 & 1 & 9 \end{bmatrix}$$

$$\begin{array}{rrrr} 6 & -9 & -3 & -9 \\ -\;6 & 2 & 4 & 6 \\ \hline 0 & 11 & 7 & 15 \end{array}$$

$R_3 \to 11R_3 - 5R_2$.

$$\begin{bmatrix} 3 & 1 & 2 & 3 \\ 0 & 11 & 7 & 15 \\ 0 & 0 & -24 & 24 \end{bmatrix}$$

$$\begin{array}{rrrr} 3 & 6 & 3 & 12 \\ (-)\;3 & 1 & 2 & 3 \\ \hline 0 & 5 & 1 & 9 \end{array}$$

$$\begin{array}{rrrr} 0 & 55 & 11 & 99 \\ -\;0 & 55 & 35 & 75 \end{array}$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 11 & 7 \\ 0 & 0 & -24 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 15 \\ 24 \end{bmatrix}.$$

$3x + y + 2z = 3$ — (3)

$11y + 7z = 15$ — (4)

$-24z = 24$ — (5)

$$z = \frac{\cancel{24}\,1}{\cancel{24}} = \boxed{z = -1}$$

sub in eq (4)

$11y + 7(-1) = 15$

$11y - 7 = 15$

$11y = 15 + 7$

$11y = 22$

$\boxed{y = 2}$

sub z & y in eq (1). 3.

$3x + y + 2 = 3.$

$3x + \cancel{2} + \cancel{2} = 3$

$3x + 4 = 3.$

$3x = 3 - 4.$

$3x = -1$

$x = -1$

$\cancel{3x + 2 + 1 = 3}.$

$\cancel{3x + 1 = 3}$

$\cancel{3x = 2}.$

$\boxed{x = \frac{2}{3}}$

$\cancel{3x + 1 = 3}.$

$\cancel{3x = 3 + 1}.$

$\cancel{3x = 2}$

$\cancel{x = \frac{2}{3}}.$

$\boxed{x = 1}$

Imp

$$x+2y-3z=9$$
$$2x-y+z=0$$
$$4x-y+z=4$$

$$3x+y+2z=3$$
$$2x-3y-z=-3$$
$$x+2y+z=4$$

$$x+2y+3z=1$$
$$2x+3y+8z=2$$
$$x+y+z=2$$

(1) Find Inverse by Gauss Jordan

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}_{3\times 3}$$

$A = IA$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Change LHS matrix into Normal form

$R_2 \to R_2 - R_1$, $R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + R_2$, $R_1 \to R_1 - \frac{1}{2}R$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + R_2$

$$\begin{array}{rrr} 0 & -2 & 2 \\ 0 & 2 & -6 \\ \hline 0 & 0 & -4 \end{array}$$

$$\begin{array}{rrrr} & 2 & 0 & 1 \\ + & -1 & 1 & 0 \\ \hline & 1 & 1 & 1 \end{array}$$

$$\begin{array}{rrrr} & 1 & 3 & -3 \\ (-) & 1 & 1 & 3 \\ \hline & 0 & 2 & -6 \end{array}$$

$$\begin{array}{rrrr} & -2 & -4 & -4 \\ + & 2 & 2 & 6 \\ \hline & 0 & -2 & 2 \end{array}$$

$$\begin{array}{rrrr} & 0 & 1 & 0 \\ - & 1 & 0 & 0 \\ \hline & -1 & 1 & 0 \end{array}$$

$$\begin{array}{rrr} 0 & 0 & 1 \\ 2 & 0 & 1 \\ \hline 2 & 0 & 0 \end{array}$$

① Find Inverse by Gauss Jordan

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}_{3\times 3}$$

$A = IA$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Change LHS matrix into Normal form

$R_2 \to R_2 - R_1$, $R_3 \to R_3 + 2R_1$.

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + R_2$, $R_1 \to R_1 - \frac{1}{2}R$.

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + R_2$.

$$\begin{array}{ccc} 0 & -2 & 2 \\ 0 & 2 & -6 \\ \hline 0 & 0 & -4 \end{array}$$

$$\begin{array}{cccc} & 2 & 0 & 1 \\ + & -1 & 1 & 0 \\ \hline & 1 & 1 & 1 \end{array}$$

$$\begin{array}{cccc} & 1 & 3 & -3 \\ (-) & 1 & 1 & 3 \\ \hline & 0 & 2 & -6 \end{array}$$

$$\begin{array}{cccc} & -2 & -4 & -4 \\ + & 2 & 2 & 6 \\ \hline & 0 & -2 & 2 \end{array}$$

$$\begin{array}{cccc} & 0 & 1 & 0 \\ - & 1 & 0 & 0 \\ \hline & -1 & 1 & 0 \end{array}$$

$$\begin{array}{ccc} 0 & 0 & 1 \\ 2 & 0 & 1 \\ \hline 2 & 0 & 0 \end{array}$$

$R_3 \to R_3 \left(-\frac{1}{4}\right)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -6 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1 & 1 & 0 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$\boxed{R_1 \to R_1 - 6R_3 \ , \ R_2 \to R_2 + 6R_3}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -13/2 & 3/2 \\ -5/2 & -1/2 & -3/2 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

Multiply $\frac{1}{2}$ to R_2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -13/2 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix} A$$

This is the inverse of the given Matrix By using Gauss Jordan method

Reduce the Q.F $3x^2+5y^2+3z^2-\underline{2y}z\,\bar{2}xy+2z$ into canonical form into orthogonal

$$A = \begin{array}{c|ccc} & x & y & z \\ \hline x & 3 & -2/2 & 2/2 \\ y & -2/2 & 5 & -2/2 \\ z & 2/2 & -2/2 & 3 \end{array}$$

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

The char eqn is $|A-\lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 5-\lambda & 1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda + |A| = 0$$

$$\begin{array}{r} 14 \\ 14 \\ 8 \\ \hline 36 \end{array}$$

$S_1 = 11$

$S_2 = |15-1| + |9-1| + |15-1|$

$\cancel{44}$

$S_2 = |14+8+14| = 36$

$$\lambda^3 - 11\lambda^2 + 36\lambda \mp \overset{36}{\cancel{44}} = 0$$

$\lambda = 6, 3, 2.$

Step 3. find eigen vectors

$$A - \lambda I = 0$$

$\lambda = 2$.

$$A - 2I = 0$$

$$\begin{bmatrix} 3-2 & -1 & 1 \\ -1 & 5-2 & -1 \\ 1 & -1 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} = x - y + z &= 0 \quad -① \\ -x + 3y - z &= 0 \quad -② \end{aligned}\right\}$$

$$x - y + z = 0 \quad -③$$

$x = -2K$.
$y = 0K$.
$z = 4K$.

$$X_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K \begin{bmatrix} \cancel{-2}\ -1 \\ 0 \\ \cancel{4}\ 1 \end{bmatrix}$$

$$K = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{array}{cccc} z & x & y & \\ 1 & -1 & 1 & 1 \\ -1 & 3 & -1 & -1 \end{array}$$

$$\frac{x}{1-3} = \frac{y}{-1+1} = \frac{z}{3-1} = \frac{x}{-2} = \frac{y}{0} = \frac{z}{2} = K$$

$\boxed{\lambda = 3}$

$A - 3I = 0$

$$\begin{bmatrix} 3-3 & -1 & 1 \\ -1 & 5-3 & -1 \\ 1 & -1 & 3-3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$-y + z = 0 \quad -(1)$ $\quad \boxed{z = y}$ $\quad \therefore \boxed{x = y = z}$

$-x + 2y - z = 0 \quad -(2)$

$x - y = 0 \quad -(3) \Rightarrow \boxed{y = x} \quad -(4)$

$$\begin{array}{cccc} & z & y & x \\ 0 & -1 & 1 & 0 \\ -1 & 2 & -1 & -1 \end{array}$$

$-(-1 \times 0)$

$-(0)$

$$\frac{x}{-1-0} = \frac{y}{1-2} = \frac{z}{0-1}$$

$$= \frac{x}{-1} = \frac{y}{-1} = \frac{z}{-1} = k$$

$x = -k, \ y = -k, \ z = -k.$

$$X_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = k \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}.$$

Step

eigen value

Normalised form $= \sqrt{(-1)^2+(-1)^2+(-1)^2} = \sqrt{3}$

Normalised eigen value vector $= X2 = \begin{bmatrix} -1/\sqrt{3} \\ -1\sqrt{3} \\ -1\sqrt{3} \end{bmatrix}$.

$\boxed{\lambda=6}$

$A - 6I = 0$.

$$\begin{bmatrix} 3-6 & -1 & 1 \\ -1 & 5-6 & -1 \\ 1 & -1 & 3-6 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix}.$$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x - y + z = 0$ — (1).

$-x - y - z = 0$ — (2).

$x - y - z = 0$ — (3)

$$\begin{array}{cccc} z & x & & y \\ -3 & -1 & 1 & -3 \\ -1 & -1 & -1 & -1 \end{array}$$

$$\frac{x}{1+1} = \frac{y}{-1+3} = \frac{z}{-3-1}$$

$\frac{x}{2} = \frac{y}{2} = \frac{z}{4} = K$.

$x = 2K$

$y = 2K$.

$z = 4K$.

$X3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} = K\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

N Va $= \sqrt{1^2+1^2+2^2} = \sqrt{6}$

N Ve $\|x_3\|$

$\begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$.

4.

$P = [\, ||x_1|| \quad ||x_2|| \quad ||x_3|| \,]$.

$$= \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & -1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}.$$

$\therefore$ A is symmetric & P is Orthogonal

then $P^T = P^{-1}$

$P^{-1} = P^T$.

$$P^{-1} = \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \end{bmatrix}.$$

Dont calculate.

5. $P^{-1}AP = P^TAP$.

$$\begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & -1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

The lambda will be the principal values.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$ which is formed by 2, 3 & 6.

6) $\boxed{y^TDy}$ $\quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \Rightarrow y^TDy = \underbrace{[y_1 \; y_2 \; y_3] \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$$[2y_1 \; 3y_2 \; 6y_3] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$CF = 2y_1^2 + 3y_2^2 + 6y_3^2$

Matrix:

A system of mn no's arranged in a rectangular formation along m rows and n coloums and bounded by the brackets is called as matrix written as m×n matrix and is denoted by a single capital letter.

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

is a matrix of order $m \times n$.
It has m rows and n columns.
It can also be noted by $A = [a_{ij}]$.

Rows and Column Matrices:
they are called as rows or vector & column vector respectively.

Square matrix: A matrix having n rows and n columns is called a square matrix of order n.

The diagonal of this matrix containing the elements 1, 3, 5 is called the leading or principal diagonal.

Trace :- The sum of the diagonal elements of a square matrix A is called the trace of A

Singular :- A square matrix is said to be singular if its determinant is zero otherwise non singular.

Diagonal matrix : A square matrix all of whose elements, except those in the leading diagonal is zero is called diagonal matrix.

Scalar matrix :- A diagonal matrix whose all the leading diagonal elements are equal is called scalar matrix $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

Unit matrix :- A diagonal matrix of order n which has unity for all its diagonal elements is called a unit matrix or an identity matrix of order n & is denoted by I_n $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Null Matrix :- If all the elements of a matrix are zero, it is denoted by '0' $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Symmetric Matrix :- A square matrix $A = (a_{ij})$ is said to be symmetric when $a_{ij} = a_{ji}$ for all i & j.

Eg:- $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$. & $\boxed{A = A^T}$

Skew - Symmetric :- If $a_{ij} = -a_{ji}$ for all i and j so that all the leading diagonal elements are 0, then the matrix is SSM.

Eg :- $A = \begin{pmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{pmatrix}$ $\boxed{A = -A^T}$

Triangular matrix :- A square matrix all of whose elements below the leading diagonal are zero is called upper triangular matrix.

Eg :- $A = \begin{pmatrix} 2 & 3 & 5 \\ 0 & 4 & 2 \\ 0 & 0 & -1 \end{pmatrix}$.

A square matrix all of whose elements above the leading diagonal are zeros is called lower triangular matrix.

eg:- $\begin{pmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ -1 & 2 & 1 \end{pmatrix}$

Orthogonal matrix:- A square matrix A is said to be orthogonal if $\boxed{AA^T = A^TA = I}$.

i.e $\boxed{A^T = A^{-1}}$

Theorem :- (5M).

1) Every square matrix can be expressed as the sum of a symmetric matrix & skew symmetric matrices in one and & only way (uniquely).

Show that any square matrix can be expressed as $A = B + C$ where B is symmetric & C is skew symmetric.

Proof :- Let A be any square matrix.

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$P + Q$ where $P =$ $Q =$

$$P^T = \left\{\frac{1}{2}(A + A^T)\right\}^T = \frac{1}{2}(A + A^T)^T = \frac{1}{2}(A^T + (A^T)^T)$$

$\frac{1}{2}(A^T+A) = \frac{1}{2}(A+A^T) = P$

$\therefore$ P is symmetric.

$Q = \frac{1}{2}(A-A^T)$

$Q^T = \left[\frac{1}{2}(A-A^T)\right]^T$

$\Rightarrow \frac{1}{2}\left(A^T-(A^T)^T\right)$

$\therefore \frac{1}{2}\left(A^T-A\right) = -\frac{1}{2}(A-A^T)$

$\Rightarrow Q^T = -Q$

$\therefore$ Q is skew symmetric

$\therefore A = P+Q$ where P is symmetric & Q is skew symmetric.

To prove the uniqueness:-

If possible, let $A = R+S$ be another such representation of A where R is sym & S is skew

$A = R+S$

R is symmetric $\Rightarrow R^T = R$.

S is skew symetric $\Rightarrow S^T = -S$.

$A^T = (R+S)^T = R^T+S^T = R-S$.

$P = \frac{1}{2}(A+A^T) = \frac{1}{2}\{R+\cancel{S}+R-\cancel{S}\} = \frac{1}{\cancel{2}}\times\cancel{2}R = R$

$Q = \frac{1}{2}(A-A^T) = \frac{1}{2}\{R+S-R+S\} = \frac{1}{\cancel{2}}\cancel{2}S = S$

$\therefore$ $P = R$ & $Q = S$.

$\therefore$ The representation is unique.

Rank :- Echelon form :- Reduce the matrix to echolon form.

Q) Find the rank of matrix by reducing it to echlon form

PD

pivot.

$$A = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{pmatrix}$$

$1-2(3)$
$1-6$

make these 0 below PD

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 4R_1$ $\Rightarrow$
$R_4 \to R_4 - 4R_1$

$$\begin{pmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -15 & -21 \end{pmatrix}$$

$R_3 \to R_3 - R_2$ $\Rightarrow$
$R_4 \to R_4 - 3R_2$

$$\begin{pmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

non zero rows.

$\Rightarrow$ is echlon

There are 2 non zero rows.

$\rho(A) = 2$

Rank of Matrix

A matrix is said to be r when.

1) . . . .

Reduction to Normal form:

Every $m \times n$ matrix of rank r be reduced to form $(I_r \; 0)$ or $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ by a chain of elementary row or coloumn operations where I_r is the unit matrix of order r. The above form is called normal form of canonical form.

Linear System of eqn's.

$$\left.\begin{aligned} a_{11}\, x + a_{1} y + a_{13} z &= b_1 \\ a_{21}\, x + a_{22} y + a_{23} z &= b_2 \\ a_{31}\, x + a_{32} y + a_{33} z &= b_3 \end{aligned}\right\} - ①$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}_{3\times 3} \quad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3\times 1} + X \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{3\times 1}$$

$$AX = B$$

Rouche's theorem.

$f(A) = f(A, B)$.

Procedure to test consistancy

$f(A)$ & $\rho(A, B)$.

1) If $\rho(A) \neq \rho(A, B)$ = Inconsistant = no sol.
2) If $\rho(A) = \rho(A, B) = n$, = consistant = unique
3). If $\rho(A) = \rho(A, B) < n$, = consistant = ∞ solution

Normal form

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

$R_2 \to R_2 + 2R_1$ $R_1 \to R_2$

$R_3 \to 2R_3 -$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$.

$R_4 \to R_4 - 6R_1$

$$\Rightarrow \begin{pmatrix} 1 & -1 & 2 & -4 \\ 0 & & & \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

Gauss Jordan Method (Only row operations)

$$A = AI$$

$$AI = \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right).$$

$$R_2 \to R_2 - R_1, \quad R_3 \to R_3 + 2R_1.$$

$$A^{-1} =$$

Gauss Elimination Method (Only row operation)

$$AX = B.$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots \\ & & \end{pmatrix} \qquad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

$$\text{AUG MAT } (A, B) = \left(\begin{array}{ccc} a_{11} & a_{12} \cdots & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31}. & a_{32} & b_3 \end{array}\right)$$

$\rho(A,B)$ $\rho(A)$ compare $\neq$ no sol. (inconsistent)

$$\rho(A,B) = \rho(A) = \begin{cases} = n, \text{ unique} \\ < n, \text{ infinite} \end{cases}$$

Test the consistancy & solve

$5x + 3y + 7z = 4$

$3x + 26y + 2z = 9.$

$7x + 2y + 10z = 5.$

$AX = B.$

$$\begin{pmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 5 \end{pmatrix}.$$

a_{11}

$$(A, B) = \begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}.$$

$45 - 3(4)$
$-12 \quad 12$
33

$\therefore R_2 \to 5R_2 - 3R_1.$
$R_3 \to 5R_3 - 7R_1.$ $\Rightarrow \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{pmatrix}.$

$R_2 \to \dfrac{R_2}{11} \sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & -11 & 1 & -3 \end{pmatrix}.$

$R_3 \to R_3 + R_2. \sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Check Rank.

$\rho(A|B) = \cancel{2}\ 2$, $\rho(A) = 2$

$\rho(A) = \rho(A|B) = 2$

$\therefore$ consistant.

rank = 2

no of unknowns 'n' = 3

$2 < 3$

$\therefore$ Infinite no of sol.

write values as eq^n

$\Rightarrow$ $5x + 3y + 7z = 4$

$11y - z = 3$

let $z = k$

$11y = 3 + k$

$$\boxed{y = \frac{3+k}{11}}$$

$5x + 3y + 7z = 4$

$5x = 4 - 7z - 3y$

$4 - 7k - 3\left(\frac{3+k}{11}\right)$

$= 4 - 7k - 3\left(\frac{3+k}{11}\right)$

$5x = \dfrac{44 - 77k - 9 - 3k}{11}$

$$\boxed{x = \frac{35 - 80k}{55}}$$

If $\boxed{k=0}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

If $\boxed{k=1}$ =

(2)

$x + y + z = 3$

$x + 2y + 2z = 5$

$3x + 4y + 4z = 12$

$$(A\,|\,B) = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 2 & 5 \\ 3 & 4 & 4 & 12 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 3R_1$ $\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{pmatrix}$

$R_3 - R_3 - R_2$ $\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ 1, 1 1 1 1

$\rho(A) = 2\ ,\ \rho(A, B) = 3$

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 3 \end{pmatrix}$$

$\rho(A) = 3,\ \rho(A\,|\,B)\ 3$

unique

$1x + 1y + 1z = 3$

$1y + 1z = 2$

$2z = 3$

$z = \frac{3}{2}$

$1y + \frac{3}{2}$

(3) $2x + ~~3y~~\,3z = 5$

$y + z = 2$

$x + y + z = 3$

$\rho(A|B) \neq \rho(A)$

$= \begin{cases} = n & \text{unique} \\ < n & \text{infinite} \end{cases}$

$$\begin{pmatrix} 2 & 3 & 5 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$$

$R_3 \to R_3 - R_2$

0 0 2 3

$$(A, B) = \begin{pmatrix} 2 & 3 & 5 & 5 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 2 & 3 & 5 & 5 \end{pmatrix}$$

$R_3 \to R_3 - 2R_1 =$ $\begin{pmatrix} 0 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 \end{pmatrix}$

Complex Matrices:- If all the elements of a matrix are real then it is called Real matrix. If atleast one element of a matrix is a complex no $a+ib$. then the matrix is called complex matrix.

Conjugate Matrix:- The matrix obtained by replacing the elements of a complex matrix A by corresponding conjugate complex no's is called conjugate of the matrix A & is denoted by $\bar{A}$.

Eg:- If $a+ib$ is a complex no then $a-ib$ is a conjugate matrix.

w.k.t the conjugate of transpose of A and transpose conjugate of A are equal and is denoted by A^*.

$$A^* = (\overline{A^T}) \text{ or } (\bar{A})^T$$

* A square matrix is said to be hermitian if $A^* = A$.

* A square matrix is said to be Skew Hermitian if $A^* = -A$.

Note: In Hermitian matrix, all diagonal elements are all real and every other element is the conjugate of element in the transpose matrix.

$$\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ \left(\begin{matrix} 5 \\ 2-i \\ 3i \end{matrix}\right. & \begin{matrix} 2+i \\ -3 \\ 1+i \end{matrix} & \left.\begin{matrix} -3i \\ 1-i \\ 0 \end{matrix}\right) \end{array} \text{ Real.}$$

for Skew Hermitian

$$\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ 3i & 1+i & 7 \\ -1+i & 0 & -2-i \\ -7 & 2-i & -i \end{array}$$

Unitary Matrix $\quad A^* A = I = AA^*$

Det is 1.

It must be non singular

Observation.

* Every real symmetric matrix is Hermitian
* A real skew-sym is Skew-Her...
* A orthogonal matrix is unitary.

Sequence & Series (Unit 3)

(I) P-Series.
↳ Comparision Test.

P-series. $p > 1$ = Convergent
$P \leq 1$ = dgt.

ΣU_n & ΣV_n behaves like same,

$\left\{ \begin{array}{l} \text{if } \Sigma V_n \text{ is cgt} \Rightarrow \Sigma U_n \text{ cgt.} \\ \text{if } \Sigma V_n \text{ is dgt} \Rightarrow \Sigma U_n = \text{dgt} \end{array} \right.$

Test for convergence.

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \left(\frac{n+2}{n+3}\right)^n \quad \text{—(1)}$$

Sol: $U_n = \frac{1}{n^3} \left(\frac{n+2}{n+3}\right)^n$

$$\boxed{\text{Now check } U_n = \frac{\text{Highest power of } n.}{\text{Highest power of } n}}$$

$$U_n = \frac{1}{n^3} \cdot \frac{\left[n\left(1+\frac{2}{n}\right)\right]^n}{\left[n\left[1+\frac{3}{n}\right]\right]^n}$$

(5).

$$\frac{1}{n^3} \cdot \frac{\cancel{n^n} \left(1+\frac{2}{n}\right)^n}{\cancel{n^n} \left(1+\frac{3}{n}\right)^n}$$

$\boxed{(ab)^n = a^n . b^n}$

$$= \frac{1}{n^3} \cdot \frac{\left(1+\frac{2}{n}\right)^n}{\left(1+\frac{3}{n}\right)^n}$$

$$\boxed{Vn = \frac{1}{n^3}}$$

$$Un = \frac{1}{n^3} \left(\frac{(n+2)^n}{(n+3)^n}\right).$$

& $Vn = \frac{1}{n^3}$.

$$\frac{Un}{Vn} = \frac{1}{n^3} \frac{(n+2)^n}{(n+3)^n} \times \frac{1}{\left(\frac{1}{n^3}\right)}$$

$$= \frac{1}{\cancel{n^3}} \frac{(n+2)^n}{(n+3)^n} \times \cancel{n^3}.$$

$$\frac{Un}{Vn} = \frac{(n+2)^n}{(n+3)^n}.$$

$$\lim_{n\to\infty} \frac{Un}{Vn} = \lim_{n\to\infty} \frac{(n+2)^n}{(n+3)^n} = \cancel{\frac{\infty}{\infty}}$$

$$\lim_{n\to\infty} = \left[\frac{\cancel{n}\left(1+\frac{2}{n}\right)^n}{\cancel{n}\left(1+\frac{3}{n}\right)^n}\right] = \frac{e^2}{e^3} = \boxed{\frac{1}{e} \neq 0.}$$

③

finit $e = 2.718$.

Now $\Sigma V_n = \Sigma \frac{1}{n^3}$.

which is of the form $\frac{1}{n^3} = \frac{1}{n^p}$. $p = 3$
$p > 1 = cgt$

Now ΣV_n is cgt.
$\therefore \Sigma U_n$ is cgt.
$\therefore \Sigma \lim_{n\to\infty} \frac{U_n}{V_n} \neq 0$ we get By comp test.

② Test for converge of.

$$\Sigma \frac{1}{(4n^2-1)}$$

①.

$$\boxed{U_n = \frac{1}{(4n^2-1)}}$$

②.

$$\boxed{V_n = \frac{1}{n^2}}$$

$$\cancel{\Sigma} U_n = \frac{1}{4 n^2 (1 - \frac{1}{n})}$$

③. $\boxed{\Sigma V_n} = \Sigma \frac{1}{n^2}$.

$\Sigma V_n = cgt$ $\boxed{p=2}$

$\therefore \Sigma U_n = cgt$.

④. To show $\lim_{n\to\infty} \frac{U_n}{V_n} = $ finite

i).

$$\frac{1}{4n^2-1} \times \frac{n^2}{1}$$

$$\frac{1}{4\cancel{n^2}(1-\frac{1}{n})} \times \frac{\cancel{n^2}}{1} \quad p = 2.$$

$2 > 1$ cgt.

$$\frac{1}{(4-\frac{1}{n})}$$

ii).

$$\lim_{n\to\infty} = \frac{1}{(4-\frac{1}{n})} \quad \frac{1}{(4-\frac{1}{\infty^2})} = \frac{1}{4-0}$$

$$= \frac{1}{4} \neq 0.$$

$\therefore$ finite value

Hence.

Test the convergence for

$$\Sigma \left[\sqrt{n+1}-\sqrt{n}\right].$$

$$\boxed{U_n} = \sqrt{n+1}-\sqrt{n}$$

$$= \frac{\sqrt{n+1}-\sqrt{n}}{\cancel{\sqrt{n+1}+\sqrt{n}}} \times \frac{(\sqrt{n+1}+\sqrt{n})}{(\sqrt{n+1}+\sqrt{n})}$$

$$\frac{(\sqrt{n+1})^2-(\sqrt{n})^2}{(\sqrt{n+1}+\sqrt{n})} \Rightarrow \frac{\cancel{n}+1-\cancel{n}}{\sqrt{n+1}+\sqrt{n}}$$

$$\boxed{U_n = \frac{1}{\sqrt{n+1}+\sqrt{n}}} \Rightarrow \frac{1}{\sqrt{n\left(1+\frac{1}{n}\right)}+\sqrt{n}}$$

$$\frac{1}{\sqrt{n}\left(\sqrt{1+\frac{1}{n}}+1\right)}$$

$$\Rightarrow \frac{1}{\sqrt{n}\left(\sqrt{1+\frac{1}{n}}+1\right)}$$

$$\boxed{U_n = \frac{1}{n^{\frac{1}{2}}\left[\sqrt{1+\frac{1}{n}}+1\right]}}$$

$$V_n = \frac{1}{n^{\frac{1}{2}}}$$

$$\Sigma V_n = \frac{1}{n^{\frac{1}{2}}}$$

$P = \frac{1}{2} < 1$ it is dgt

$\therefore \Sigma U_n = $ dgt

⑤

$$\lim_{n\to\infty} \frac{U_n}{V_n} \neq 0.$$

$$\frac{1}{\cancel{n^{1/2}}\left[\sqrt{1+\frac{1}{n}}+1\right]} \times \frac{\cancel{n^{1/2}}}{1} = \frac{1}{\left[\sqrt{1+\frac{1}{n}}+1\right]}.$$

$$= \frac{1}{\left[\sqrt{1+\frac{1}{\infty}}+1\right]} = \frac{1}{\left[\sqrt{1}+1\right.} = \frac{1}{2} \neq 0.$$

∴ Condition satisfied.

Hence ΣU_n is dgt.

ΣV_n is dgt.

$$\frac{U_n}{V_n} \neq 0.$$

comp the x^2 - 1 is com.

Test the convergence for

$$\underbrace{\frac{\sqrt{2}-1}{3^2-1} + \frac{\sqrt{3}-1}{4^2-1}} + \frac{\sqrt{4}-1}{5^2-1} + \cdots \qquad \text{Supp } n=1. \quad \frac{\sqrt{n+1}-1}{(n+2)^2-1}$$

① When series is given, find the nth term.

$$U_n \quad \cancel{\frac{\sqrt{n}+1}{(n+1)^2-1}} \quad \boxed{\frac{\sqrt{n+1}-1}{(n+2)^2-1}} \qquad \Sigma U_n = \Sigma \frac{\sqrt{n+1}-1}{(n+2)^2-1}$$

$$U_n = \frac{\sqrt{n\left(1+\frac{1}{n}\right)}-1}{\left[n\left(1+\frac{2}{n}\right)\right]^2-1} \Rightarrow \frac{\sqrt{n}\cdot\sqrt{1+\frac{2}{n}}-1}{n^2\cdot\left(1+\frac{2}{n}\right)^2-1} \qquad \frac{(n+1)^{1/2}-1}{(n+2)^2-1}.$$

20½.

(b)

$$\frac{\sqrt{n}\left[\sqrt{1+\frac{1}{n}}-\frac{1}{\sqrt{n}}\right]}{n^2\left[\left(1+\frac{2}{n}\right)^2-\frac{1}{n^2}\right]}$$

$$\frac{1}{n^2 \cdot n^{-\frac{1}{2}}} \cdot \frac{\left[\sqrt{1+\frac{1}{n}}-\frac{1}{\sqrt{n}}\right]}{\left[\left(1+\frac{2}{n}\right)^2-\frac{1}{n^2}\right]}$$

$$= \frac{1}{n^{3/2}} \cdot \frac{\left[\sqrt{1+\frac{1}{n}}-\frac{1}{\sqrt{n}}\right]}{\left[\left(1+\frac{2}{n}\right)^2-\frac{1}{n^2}\right]}$$

$v_n = \frac{1}{n^{3/2}} \Rightarrow \Sigma\, v_n = \frac{1}{n^{3/2}}$

$2)3(1.5$

$p = \cancel{4}\ \frac{3}{2}$ $\quad \boxed{\frac{3}{2} > 1 = cgt^{10}}$

$U_n = cgt$

$V_n = cgt$

$\lim_{n\to\infty} \frac{U_n}{V_n} \neq 0$

$$\frac{\sqrt{n+1}-1}{(n+2)^2-1} \times \frac{n^{3/2}}{1}$$

$\cancel{+10}$

cgt

cgt

(7) Test for convergence of the given series.

$$\frac{1\cdot 2}{3\cdot 4\cdot 5} + \frac{2\cdot 3}{4\cdot 5\cdot 6} + \frac{3\cdot 4}{5\cdot 6\cdot 7} + \cdots \text{ nth term.}$$

$n=1$ $$\frac{n(n+1)}{(n+2)(n+3)(n+4)} = U_n.$$

$$U_n = \frac{\cancel{n}\cdot\cancel{n}\left(1+\frac{1}{n}\right)}{\left[\cancel{n}\left(1+\frac{2}{n}\right)\right]\left[\cancel{n}\left(1+\frac{3}{n}\right)\right]\left[n\left(1+\frac{4}{n}\right)\right]}$$

$$\frac{\left(1+\frac{1}{n}\right)}{\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\left(n\left(1+\frac{4}{n}\right)\right)} \times 1$$

$$\frac{1}{n}\cdot\frac{\left(1+\frac{1}{n}\right)}{\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\left(1+\frac{4}{n}\right)}$$

$$V_n = \frac{1}{n}, \quad \Sigma V_n = \Sigma \frac{1}{n}$$

$P = 1 = 1.$ dgt

$P > 1 = $ cgt.
$P \leq 1 = $ dgt.

$U_n = $ dgt.

$$\lim_{n\to\infty}\frac{U_n}{V_n} = \frac{1+\frac{1}{n}}{\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\left(1+\frac{4}{n}\right)} = \frac{1+0}{1+1+1} = \frac{1}{3} \neq 0.$$

dgt
dgt

(8)

RATIO TEST.

If ΣU_n is series of +ve terms such that.

$\lim_{n\to\infty} \frac{U_n}{U_{n+1}} = l$ then.

① ΣU_n is cgt if $l > 1$. cgt.

② ΣU_n is dgt if $l < 1$.

③ ΣU_n is test failed if $l = 1$.

Test the convergence for.

$$\Sigma \frac{3n-1}{2^n}$$

$$U_n = \frac{3n-1}{2^n}$$

$$U_{n+1} = \frac{3(n+1)-1}{2^{n+1}}$$

$$= \cancel{\frac{3\{n(1+\frac{1}{n})\}-1}{2^{n(1+\frac{1}{n})}}}$$

$$U_{n+1} = \frac{3n+3-1}{2^{n+1}}$$

$$\frac{3n+2}{2^{n+1}}$$

$$\lim_{n\to\infty} \frac{U_n}{U_{n+1}} = \frac{3n-1}{2^n} \times \frac{2^{n+1}}{3n+2}$$

$$= \frac{3n-1}{\cancel{2^n}} \times \frac{\cancel{2^n}\cdot 2^1}{3n+2}$$

$\frac{2}{2}=0$ $\quad 2 \cdot \frac{3n-1}{3n+2}$

$$2\lim_{n\to\infty} \cancel{\frac{3\times\infty-1}{3(\infty)+2}}$$

$$2\lim_{n\to\infty} \frac{\cancel{n}[3-\frac{1}{n}]}{\cancel{n}[3+\frac{2}{n}]}$$

$$\frac{3-0}{3+0}$$

$$= \frac{3}{3} = 1$$

Test fail

$$2 \times \frac{1}{1}$$

$$= 2$$

cgt.

(4)

Find the nature of the series.

$$\frac{3}{4} + \frac{3\cdot 6}{4\cdot 7} + \frac{3\cdot 6\cdot 9}{4\cdot 7\cdot 10} + \cdots \infty.$$

nth term $n+1$. $\cancel{\dfrac{(n+2)(n+2)}{(n+3)(n+6)}}$.

$$U_n \Rightarrow \frac{3\cdot 6\cdot 9 \cdots\cdots (3n)}{4\cdot 7\cdot 10 \cdots\cdots (3n+1)}.$$

$$U_{n+1} \Rightarrow \frac{3\cdot 6\cdot 9 \cdots\cdots [3(n+1)]}{4\cdot 7\cdot 10 \cdots\cdots [3(n+1)+1]}.$$

prev term

$$U_{n+1} \Rightarrow \frac{3\cdot 6\cdot 9 \cdots (3n)}{4\cdot 7\cdot 10 \cdots (3n+1)} \cdot \frac{3n+4}{3n+4}.$$

$$U_{n+1} \Rightarrow \frac{3\cdot 6\cdot 9 \cdots\cdots (3n)(3n+3)}{4\cdot 7\cdot 10 \cdots\cdots (3n+1)(3n+4)}.$$

$\dfrac{U_n}{U_{n+1}} = l = 1$ Ratio test fail

So go for Raabe's test.

(11)

(ii). $\boxed{\dfrac{U_n}{U_{n+1}}}$.

$$l = \lim_{n\to\infty} \frac{\dfrac{3\cdot 6\cdot 9\cdot 12 \cdots (3n)}{4\cdot 7\cdot 10\cdot 13 \cdots (3n+1)}}{\dfrac{3\cdot 6\cdot 9\cdot 12 \cdots (3n)(3n+\cancel{3}4)}{4\cdot 7\cdot 10\cdot 12 \cdots (3n)(3n+3)}}$$

$$= \boxed{\lim_{n\to\infty} \frac{(3n+4)}{(3n+3)}} \to *$$

$$\lim_{n\to\infty} \frac{(\cancel{n}[3+\frac{4}{n}])}{\cancel{n}(3+\frac{3}{n})]}$$

$$\lim_{n\to\infty} \left(\frac{3+0}{3+0}\right) = 1$$

$\boxed{l=1}$

test fail

go for Rabes Test.

RAABEES TEST.

$$l = \lim_{n\to\infty} n\left[\frac{U_n}{U_{n+1}} - 1\right]$$

$$= \lim_{n\to\infty} n\left[\frac{3n+4}{3n+3} - 1\right]$$

$$\lim_{n\to\infty} n\left[\frac{\cancel{3n}+4-\cancel{3n}-3}{3n+3}\right] = \lim_{n\to\infty} n\left[\frac{1}{3n+3}\right]$$

$$\lim_{n\to\infty} \cancel{n}\left[\frac{1}{\cancel{n}(3+\frac{3}{n})}\right].$$

$$\lim_{n\to\infty} \frac{1}{3+0} = \frac{1}{3} =$$

$l = \frac{1}{3} < 0$ dgt

(11)

Test the convergence for

$$U_n = \sum \frac{2^2 \cdot 4^2 \cdot 6^2 \cdots (2n)^2}{3^2 \cdot 4^2 \cdot 5^2 \cdots (2n+2)^2}$$

$$U_{n+1} = \frac{2^2 \cdot 4^2 \cdot 6^2 \cdots (2(n+1)^2)}{3^2 \cdot 4^2 \cdot 5^2 \cdots (2(n+1)+2)^2}$$

$2n+2+2$

$2n+4$

$\boxed{\text{Add prev. term also}}$

$$= \frac{2^2 \cdot 4^2 \cdot 6^2 \cdots (2n+2)^2 \, (2n)^2}{3^2 \cdot 4^2 \cdot 5^2 \cdots (2n+4)^2 \cdot (2n+2)^2}$$

$$\lim_{n\to\infty} \frac{U_n}{U_{n+1}} = \frac{\cancel{(2n)^2}}{\cancel{(2n+2)^2}} \times \frac{(2n+4)^2 \, \cancel{(2n+2)^2}}{(2n+2)^2 \, \cancel{(2n)^2}}$$

$$\lim_{n\to\infty} = \frac{(2n+4)^2}{(2n+2)^2} \quad \text{—*}$$

$$\lim_{n\to\infty} = \frac{\cancel{n}\left(2 + \frac{4}{n}\right)^2}{\cancel{n}\left(2 + \frac{1}{n}\right)^2}$$

$$\lim_{n\to\infty} \frac{\left[2+\frac{4}{n}\right]^2}{\left[2+\frac{2}{n}\right]^2} = \frac{[2+0]^2}{[2+0]^2} = \frac{4}{4} = 1$$

$\boxed{l = 1}$

test fail

(18) (42)

Rabees test.

$$n\left[\frac{U_n}{U_{n+1}}-1\right].$$

$$n\left[\frac{(2n+4)^2}{(2n+2)^2}-1\right] \Rightarrow \lim_{n\to\infty} n\left[\frac{4n^2+16+16n-(4n^2+4+8n)}{(2n+2)^2}\right]$$

U_n

$$n\left[\frac{\cancel{2n}+4-\cancel{2n}-2}{2n+2}\right].$$

U_n

$$\cancel{n\left[\frac{2}{2n+2}\right].}$$

$$\cancel{n\times\frac{1}{n}\left[\frac{2}{4}\right]=\frac{1}{2}. \quad l=\frac{1}{2}.}$$

~~$\therefore \frac{1}{2}<1$ dgt by Rabees test.~~

l

n

$$\lim \frac{n[8n-12]}{n^2\left[2+\frac{2}{n}\right]^2} = \lim_{n\to\infty} \frac{n.n\left[8-\frac{12}{n}\right]}{n\left[2+\frac{2}{n}\right]}$$

$$=\lim_{n\to\infty} \frac{\left[8-\frac{12}{n}\right]}{\left[2+\frac{2}{n}\right]} = \frac{8-\frac{12}{\infty}}{2+\frac{2}{\infty}} = 4.$$

$$\boxed{l=4>1}$$

cgt By Rabe test

(18)

Test the convergence of.

$$\frac{1.\cancel{4}}{3} + \frac{1.4}{3.6} + \frac{1.4.7}{3.6.9} + \dots \infty$$

$\frac{(3n)}{(3n}$

now sub $n=1$ & like

$$U_n = \frac{1.4.7.10 \dots (3n-2)}{3.6.9 \dots (3n)} = 1$$

$$U_{n+1} = \frac{1.4.7.10 \dots (3(n+1)-2)}{3.6.9 \dots (3(n+1))}$$

$$= \dots \frac{(3n-2)}{3n} \frac{3n+1}{3n+3}$$

$$\lim_{n\to\infty} \frac{U_n}{U_{n+1}} = 1 \quad \text{fail}$$

$$n\left[\frac{U_n}{U_{n+1}} - 1\right] = \cancel{n\left(\frac{3n-2}{3n}\ 1-1\right)}$$

$n\{0$

$$\lim_{n\to\infty} n\left[\frac{3n+3}{3n+1} - 1\right]$$

$$\lim_{n\to\infty} n\left[\frac{\cancel{3}n+3-\cancel{3}n-1}{3n+1}\right] = n\left[\frac{2}{3n+1}\right]$$

$\frac{2}{3} < 1$ dgt. $\quad \cancel{n}\left[\frac{2}{3+\frac{1}{n}}\right]$

$\left(\frac{2}{n}\right) = \frac{2}{3}$. dgt.

(14)

Alternating Series.

A series in which the terms are alternately positive and negative is called alternative series.

Eg:- $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$

Note:- The series obtained from this by taking each term as a positive is known as absolute series.

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\cdots\cdots$$

Leibnitz Rule:- In Alternating series.

$u_1-u_2+u_3\cdots$ if it

i) Each term is numerically less than its preceding term i.e $|u_{n+1}|\leq|u_n|$ for $n\geq 1$.

(ii) $\lim\limits_{n\to\infty} u_n=0$.

Σu_n positive series. $1+\frac{1}{2}+\frac{1}{3}+\cdots\frac{1}{n}$.

(15)

ΣU_n is alternating series.

$= \Sigma |U_n| \rightarrow$ Absolute series.

Absolute cgt:

If ΣU_n is cgt $\Rightarrow \Sigma |U_n|$ is also cgt.

Hence ΣU_n is called Absolute cgt.

Conditionally cgt:

If ΣU_n is cgt $\Rightarrow \Sigma |U_n|$ is dgt. Hence ΣU_n is called conditionally cgt.

(17)

Unit 2

Jacobian :- det of P.D.

Say $u(x,y,z) = v(x,y,z)$ & $w(x,y,z)$.

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = J\left[\frac{u,v,w}{x,y,z}\right]$$

$$= \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$

2 unit.

$$J\left[\frac{u,v}{x,y}\right]$$

$$= \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$$

$$\begin{bmatrix} \cdot \end{bmatrix}$$

Q) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$.
— (a) — (b) — (c)

$u = \frac{yz}{x}$

P.D with x

$u = yz \frac{\partial}{\partial x}\left(\frac{1}{x}\right)$

$u = yz\left(\frac{-1}{x^2}\right)$

$v = \frac{zx}{y}$

PD with y

$v = zx \frac{\partial}{\partial y}\left(\frac{1}{y}\right)$.

$\cancel{v = zx\left(\frac{-1}{y^2}\right)}$

$\frac{\partial u}{\partial y} =$

$w = \frac{xy}{z}$

PD with z

$w = xy \frac{\partial}{\partial z}\left(\frac{1}{z}\right)$.

$z = \frac{xy}{z}\left(\frac{-1}{z^2}\right)$

(18) ... a into 3 parts

$u = \frac{yz}{x}$ (x)	$u = \frac{yz}{x}$ (y)	$u = \frac{yz}{x}$ (z)
$\frac{\partial u}{\partial x} = yz \frac{\partial}{\partial x}\left(\frac{1}{x}\right)$ $= yz \frac{\partial}{\partial x}\left(\frac{-1}{x^2}\right)$	$\frac{\partial u}{\partial y} = \frac{z}{x}\left[\frac{d}{dy}(y)\right]$ $\frac{\partial u}{\partial y} = \frac{z}{x}\left[1\right]$	$\frac{\partial u}{\partial z} = \frac{y}{x}\frac{\partial}{\partial z}(z)$ $\frac{\partial u}{\partial z} = \frac{y}{x}(1)$
$V = \frac{zx}{y}$	$v = \frac{zx}{y}$	$v = \frac{zx}{y}$
PD with x $\frac{\partial v}{\cancel{\partial y}x} = \frac{z}{y}\frac{\partial}{\partial x}(x)$ $\frac{\partial v}{\cancel{\partial y}x} = \frac{z}{\cancel{y}x}(1)$	PD with y $\frac{\partial v}{\partial y} = zx\left(\frac{\partial}{\partial y}\left(\frac{1}{y}\right)\right)$ $\frac{\partial v}{\partial y} = zx\left(\frac{-1}{y^2}\right)$	PD with z $\frac{\partial v}{\partial z} = \frac{x}{y}\frac{\partial}{\partial z}(z)$ $\frac{\partial v}{\partial z} = \frac{x}{y}(1)$
$w = \frac{xy}{z}$	$w = \frac{xy}{z}$	$w = \frac{xy}{z}$
PD with x $\frac{\partial w}{\partial x} = \frac{y}{z}\frac{\partial}{\partial x}(x)$ $\frac{\partial w}{\partial x} = \frac{y}{z}(1)$	PD with y $\frac{\partial w}{\partial y} = \frac{x}{z}\frac{\partial}{\partial y}(y)$ $\frac{\partial w}{\partial y} = \frac{x}{z}(1)$	PD with z $\frac{\partial w}{\partial z} = xy\frac{\partial}{\partial z}\left(\frac{1}{z}\right)$ $\frac{\partial w}{\partial z} = xy\left(\frac{-1}{z^2}\right)$

$$\begin{bmatrix} -\frac{yz}{x^2} & \frac{z}{x} & \frac{y}{x} \\ \frac{z}{x} & -\frac{zx}{y^2} & \frac{x}{y} \\ \frac{y}{z} & \frac{x}{z} & -\frac{xy}{z^2} \end{bmatrix}_{3\times 3}$$

(1a).

$$\frac{-yz}{x^2}\left[\frac{x^2yz}{y^2z^2}-\frac{x^2}{yz}\right]-\frac{z}{x}\left[\frac{-xyz}{xz^2}-\frac{xy}{yz}\right]$$

$$+\frac{y}{x}\left[\frac{zx}{zx}+\frac{xyz}{y^2z}\right].$$

$$\left(\frac{-zx}{y^2}\right)\left(\frac{-xy}{z^2}\right)$$

$$\frac{1}{y^2z^2}\frac{-x^2}{zy}$$

$-1+1+1+1+1+1$

$=4$

$$J\left[\frac{u,v,w}{x,y,z}\right]=4.$$

(2) $u=x^2-2y$ $v=x+y+z$ $w=x-2y+3z$.

a b c

P.T $J\left(\frac{u,v,w}{x,y,z}\right)$ is $10x+4$.

$[\ \ ]$ (1).

$u=x^2-2y$. $\frac{\partial u}{\partial x}=(2x)-0$ $=2x$	$u=x^2-2y$ $\frac{\partial u}{\partial y}=0-2(1)$ $=-2$.	$u=x^2-2y$. $\frac{\partial u}{\partial z}=0$.
$v=x+y+z$ $\frac{\partial v}{\partial x}=1$	$v=x+y+z$ $\frac{\partial v}{\partial y}=1$	$v=x+y+z$. $\frac{\partial v}{\partial z}=1$
$w=x-2y+3z$ $\frac{\partial w}{\partial x}=1$	$w=x-2y+3z$ $\frac{\partial w}{\partial y}=-2$.	$w=x-2y+3z$. $\frac{\partial w}{\partial z}=3$.

(20)

$$\begin{bmatrix} 2x & -2 & 0 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{bmatrix}.$$

$\cancel{0 + 2[3-1]}$ ~~+e~~

$= \cancel{8-1} = \cancel{5}$

$2x(3+2) + 2[3-1]$

$2x(5) + 2(2).$

$10x + 4.$

Hence proved.

(a) (b)

Find Jacobian for $x = u(1+v)$ & $y = v(1+u)$

$$J = \left(\frac{x, y}{u, v}\right) \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}.$$

$u\,\underline{(1+u)}^{const}.$

$\frac{\partial x}{\partial u} = 1\cdot(1+v).$

$= 1+v.$

$u(1+v).$

$\frac{\partial x}{\partial v}$ $\cancel{(1+v)}$ $u(1+0).$

$= u.$

$y = v(1+u)$

$\frac{\partial y}{\partial u} = v(0+1).$

$= v$

$y = v(1+u).$

$\frac{\partial y}{\partial v} = 1+u.$

$$\begin{bmatrix} 1+v & u \\ v & 1+u \end{bmatrix}.$$

$(1+v)(1+u) - uv$

$1+u+v$//

Cayley Hamilton Theorem

Verify Cayley Hamilton Theorem for the given matrix & also find its Inverse.

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}_{3\times 3}.$$

The char eq. of matrix is.

$|A-\lambda I| = 0.$

$$\begin{vmatrix} 1-\lambda & 0 & 3 \\ 2 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{vmatrix} = 0$$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0.$

$S_1 = 3$

$S_2 = [1-1] + [\overset{1-3}{[illegible]}] + [\overset{1-0}{[illegible]}]$

$= [0] = [0 - 2 + 0]$

$\lambda^3 - 3\lambda^2 + 0\lambda + 9 = 0$

$(1-\lambda)[(1-\lambda)(1-\lambda) - (1)].$

$-0 + 3[-2(1-\lambda)].$

$(1-\lambda)[1-\lambda-\lambda+\lambda^2-1].$

$+3[-2+2\lambda]$

$(1-\lambda)[\lambda^2-2\lambda]$

$+3[-2+2\lambda]$

$= \lambda^2 - 2\lambda - \lambda^3 + 2\lambda^2$

$-6+6\lambda.$

$-\lambda^3 + \cancel{8}\,9\lambda^2 + 4\lambda - 6.$

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}_{3\times3}$$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0.$

$S_1 = 3$

$S_2 = [1-1] + [1-3] + [1-0] = 0 - 2 + 0 = -2.$

$\boxed{\lambda^3 - 3\lambda^2 - 2\lambda + 9 = 0.}$ Req char eq → (1).

By Cayley Hamilton Theorem, w.k.t. every sq matrix satisfies its own char eqn.

put $\lambda = A$ in eq (1).

$A^3 - 3A^2 - 2A + 9I = 0.$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 1 & -3 \\ -2 & -10 & 1 \\ -1 & 1 & -10 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 0.$$

$=$

$$A^3 - 3A^2 - 2A + 9I = 0.$$

$$A^2 = A \cdot A.$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix}.$$

$$A^3 = A^2 \cdot A.$$

$$= \begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} ; \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\therefore A^3 - 3A^2 - 2A + 9I = 0.$$

$$\begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix} - 3 \begin{bmatrix} 4 & -9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$+ 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$\Rightarrow$

$$\begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 4 \\ 0 & -2 & 5 \end{bmatrix} + \begin{bmatrix} -12 & 9 & -18 \\ -9 & -6 & -12 \\ 0 & 6 & -15 \end{bmatrix} + \begin{bmatrix} -2 & 0 & -6 \\ -4 & -2 & 2 \\ -2 & 2 & -2 \end{bmatrix}$$

$$+ \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 4-12-2+9 & -3+9+0+0 & 6-18-6+0 \\ 3-9-4+0 & 2-6-2+9 & 4-12+2+0 \\ 0+0-2+0 & -2+6+2+0 & 5-15-2+9 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \end{bmatrix} = [0] = [0]$$

$A^3 - 3A^2 - A + 9I = 0$

$$\begin{bmatrix} 1-\lambda & 0 & 3 \\ 2 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{bmatrix}$$

$1-\lambda-\lambda+\lambda^2-1-\lambda+\lambda^2+\lambda^2-\lambda^3$

$+\lambda+6\lambda-6$

$-5-10\lambda+3\lambda^2-\lambda^3$

$-\lambda^3+3\lambda^2-10\lambda-5$

$1-\lambda[(1-\lambda)$

$1-\lambda[(1-\lambda)(1-\lambda)-1]+0+3[-2(1-\lambda)]$

$1-\lambda[(1(1-\lambda)-\lambda(1-\lambda)-1]+3[-2+2\lambda]$

$1-\lambda[1-\lambda-\lambda+\lambda^2-1]+3[2\lambda-2]$

$$\begin{bmatrix} 4 & 9 & 21 \\ 11 & -2 & 11 \\ 1 & -7 & 7 \end{bmatrix} - \begin{bmatrix} 12 & -9 & 18 \\ 9 & 6 & 12 \\ 0 & -6 & 15 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 6 \\ 4 & 2 & -2 \\ 2 & -2 & 2 \end{bmatrix}$$

$$+ \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 4-12-2+9 & 9+9-0+0 & 21-18-6+0 \\ 11-9-4+0 & -2-6-2+9 & 11-12+2+0 \\ 1-0-2+0 & -7-6-2+0 & 7-15-2+9 \end{bmatrix}$$

$$-1 \quad [0] = (0).$$

To find Inverse, Multiply A^{-1} O.B.S. $\quad AA^{-1} = I$

$$A^{-1}.A^3 - 3A^{-1}A^2 - A^{-1}A + 9A^{-1}I = 0.$$

$$A^2 - 3A + I + 9A^{-1} = 0.$$

$$9A^{-1} = -A^2 + 3A + I.$$

$$9A^{-1} = -A^2 + 3A + I.$$

$$A^{-1} = \frac{1}{9}\Big[$$

$$\boxed{A^{-1} = \frac{1}{9}\left\{A^2 + 3A + I\right\}}$$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 8 & 0 & 12 \\ 5 & 6 & 1 \\ 3 & -5 & 8 \end{bmatrix}$$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 4 & -3 & 6 \\ 3 & 2 & 7 \\ 0 & -2 & 5 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solve the given linear equation by Gauss Seidal Method.

$10x + y + z = 12$.
$2x + 10y + z = 13$.
$2x + 2y + 10z = 14$.

$x = \frac{1}{10}[12 - y - z]$ — (1)

$y = \frac{1}{10}[13 - 2x - z]$. — (2)

$z = \frac{1}{10}[14 - 2x - 2y]$. — (3)

By Gauss Seidal Method.

Step 1. 1st Iteration

$\begin{array}{r} 13.00 \\ 2.4 \\ \hline 10.6 \end{array}$

Initially take $y = 0$, $z = 0$.

$x^{(1)} = \frac{1}{10}[12 - y - z]$

$= \frac{1}{10}[12 - 0 - 0]$

$x^1 = \frac{12}{10} = 1.2$.

$\boxed{x^1 = 1.2}$

$y^1 = \frac{1}{10}[13 - 2(1.2) - 0]$.

$y^1 = \frac{10.6}{10}$.

$\boxed{y^1 = 1.06}$

$z^1 = \frac{1}{10}[14 - 2(1.2) - 2(1.06)]$.

$\boxed{z^1 = 0.948}$

$\frac{4}{10}$

Step 2 2nd Iteration

put $x' = 1.2$, $y' = 1.06$, $z' = 0.948$ in using apn

Sol:

$$x^{(11)} = \frac{1}{10}\left[12 - y^{(1)} - z^{(1)}\right]$$

$$= \frac{1}{10}\left[12 - 1.06 - 0.948\right]$$

(1) $\boxed{x^{(11)} = 0.9992}$

$$y^{(11)} = \frac{1}{10}\left[13 - 2x^{(11)} - z^{(1)}\right]$$

$$= \frac{1}{10}\left[13 - 2(0.9992) - 0.948\right]$$

$\boxed{y^{(11)} = 1.\cancel{2897}\,005}$

$$z^{(11)} = \frac{1}{10}\left[14 - 2x^{(11)} - 2y^{(11)}\right]$$

$$= \frac{1}{10}\left[14 - 2(0.9992) - 2(1.2897)\right]$$

$\boxed{z^{(11)} = 1.7160}$

$\boxed{z'' = 0.999}$

Step 3 3rd Iteration

put $x^{(11)} = 0.9992$, $y^{(11)} = 1.005$, $z^{(11)} = 0.999$ in eqn

(2) $$x^{(111)} = \frac{1}{10}\left[12 - y^{(11)} - z^{(11)}\right]$$

$$= \frac{1}{10}\left[12 - 1.005 - 0.999\right]$$

$\boxed{x^{(111)} = 0.999}$

$$y^{(111)} = \frac{1}{10}\left(13 - 2x^{(111)} - z^{(11)}\right)$$

$$= \frac{1}{10}\left[13 - 2(0.999) - 0.999\right]$$

$y^{(111)} = \boxed{1.000}$

$$z^{(111)} = \frac{1}{10}\left[14 - 2x^{(111)} - 2y^{(111)}\right]$$

$$= \frac{1}{10}\left[14 - 2(\cancel{1}\,0.999) - 2(1)\right]$$

$\boxed{z^{(111)} = [1.000]}$

Step 4 4th Iteration

put $x^{(111)} = 0.999$, $y''' = 1$, $z''' = 1$ in eng approximation

(3) $$x^{(IV)} = \frac{1}{10}\left[12 - y^{(111)} - z^{(111)}\right]$$

$$= \frac{1}{10}\left[12 - 1 - 1\right]$$

$\boxed{x^{(IV)} = 1}$

$$y^{(IV)} = \frac{1}{10}\left[13 - 2x^{(IV)} - z^{(111)}\right]$$

$$= \frac{1}{10}\left[13 - 2 - 1\right]$$

$\boxed{y^{(IV)} = 1}$

$$z^{(IV)} = \frac{1}{10}\left[14 - 2x^{(IV)} - 2y^{(IV)}\right]$$

$$= \frac{1}{10}\left\{14 - 2 - 2\right]$$

$\boxed{z^{(IV)} = 1}$

Hence from step 3 & 4 we get $x = 1$, $y = 1$ & $z = 1$ by Gauss Seidel

Applications of Rolles Theorem.

Verify Rolles Theorem for.

$f(x) e^x \sin x$ defined in $[0,\pi]$.

Sol). The given $f(x) e^x \sin x$ is defined in $[0,\pi]$.

$f(0) e^x$
e^x is cont funct except $x = \infty$

① $f(x) e^x \sin x$.

Here $\sin x$ is always cont in $[0,\pi]$.
e^x is also cont in $[0,\pi]$

then $f(x) = e^x \cdot \sin x$ is also cont on $[0,\pi]$

∞ is not present in b/w $0,\pi$.
$\sin \neq \infty$.

② $f(x) = e^x \sin x$.

$$f'(x) = \frac{d}{dx}\left[e^x \sin x\right] = e^x \cdot \cos x + \sin x \cdot e^x.$$

$\boxed{U \times V}$

$$f'(x) = e^x \left\{\cos x + \sin x\right] . -①$$

Hence e^x, $\cos x$, $\sin x$ are derivable functions then

$f'(x)$ is derivable on $(0,\pi)$.

$\cos x \neq \infty$.
$\sin x \neq \infty$.
$\cos x$ & $\sin x$. are derivable
e^x is also der

③ $f(a) = f(b)$
$f(0) = f(\pi)$.

$f(0) = e^0 \sin 0 = 1 \cdot 0 = 0$.

$f(\pi) = e^\pi \sin \pi = e^\pi \cdot 0 = 1 \cdot 0 = 0$.

$\therefore f(a) = f(b)$.

$\therefore$ 3 conditions are satisfied

(4) To find c value.

There exist atleast 1 value $c \in (0,\pi)$ s.t. $\boxed{f'(c)=0}$

put $x=c$ in eq(1) then equate it to 0.

$$f'(c) = e^c [\cos c + \sin c] = 0.$$

$e^c = 0.$

$\cos c + \sin c = 0$

$$-1 = \frac{\sin c}{\cos c}.$$

$$-1 = \tan c.$$

$$= \tan(-45)$$

$$\tan(-45) = \frac{\pi}{4}.$$

$\boxed{\tan(-\theta) = \tan\theta}$

$$\therefore \tan c = \tan\frac{\pi}{4}$$

$$\frac{\pi}{4} \in (0,\pi).$$

(2) $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a,b]$.

$a>0, b>0.$

$\log\frac{a}{b}$

$= \log a - \log b$

$$f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right].$$

unknown

$$f(x) = \log(x^2+ab) - \log[x(a+b)] \text{ in } [a,b].$$

If f(x) is present in any unknown then it is continuous.

① $f(x) = \log(x^2+ab) - \log\{x(a+b)\}$
is a cont func.

② $f(x) = \log(x^2+ab) - \log\{x(a+b)\}$

$$f'(x) = \frac{d}{dx}\left[\log(x^2+ab) - \log\{x(a+b)\}\right]$$

$$f'(x) = \frac{1}{x^2+ab}\frac{d}{dx}(x^2+ab) - \frac{1}{x(a+b)}\frac{d}{dx}(x(a+b))$$

$$= \frac{1}{x^2+ab}\cdot 2x - \frac{1}{x(\cancel{a+b})}\,1(\cancel{a+b})$$

$$\boxed{f'(x) = \frac{2x}{x^2+ab} - \frac{1}{x}} \quad — ①$$

$\therefore$ $f'(x)$ is derivable function in (a,b)

③ $f(a) = f(b)$.

$f(a) = \log(a^2+ab) - \log[a(a+b)]$.

$f(b) = \log(b^2+ab) - \log[b(a+b)]$.

$$\log\left[\frac{a^2+ab}{a(a+b)}\right] = \log\left[\frac{b^2+ab}{b(a+b)}\right]$$

$$\frac{a(a+b)}{a(a+b)} = \frac{b(a+b)}{b(a+b)} \qquad = 1 = 1.$$

$\therefore f(a) = f(b)$.

All conditions satisfied

(9) c such that $c \in (a,b)$, $\underline{f'(c) = 0}$.

put ~~f(x)~~ c in eq(1).

$$f'(c) = \frac{2c}{c^2+ab} - \frac{1}{c}.$$

$$\frac{2c^2 - c^2+ab}{c^3+abc} = 0 = \frac{\cancel{c^2+ab}}{\cancel{c^3+abc}} \quad = \frac{\cancel{c^2+ab}(1)}{\cancel{c^2+ab}(c+c)}$$

$$c^2 - ab = 0.$$

$$c^2 = ab.$$

$$\boxed{c = \sqrt{ab}} \in (a,b).$$

$\therefore$ Rolle's theorem is verified.

$$\frac{1}{2c} = 0.$$

$$\cancel{\frac{1}{2c} = 0}$$

$$0 = \frac{1}{2c}.$$

$$2c = 1.$$