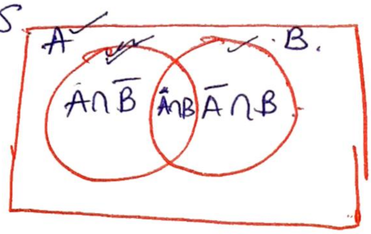
Additive Rule - Law of Addition of Prob.

3 Axioms
1) $P(A) > 0$
2) $P(S) = 1$.
3) If $A_1, A_2 \dots A_n$ are disjoint subsets of S, then
$P(A_1 \cup A_2 \cup \dots A_n) =$
$P(A_1) + P(A_2) + \dots P(A_n)$

(Union = +].

Theorem:- (Additive Theorem).
If A & B are any 2 events, subsets of Sample Space S, & not disjoint
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

* The three sets are disjoint sets.




$A \cup B = A \cup (\bar{A} \cap B)$.

PROOF:- $P(A \cup B) = P[A \cup (\bar{A} \cap B)]$.

$= P(A) + P(\bar{A} \cap B)$ using axiom 3.

$A \cup B =$
$P(A \cup (\bar{A} \cap$

Add & sub $P(A \cap B)$.

$= P(A) + \underbrace{P(\bar{A} \cap B) + P(A \cap B)} - P(A \cap B)$

$= P(A) + P[(\bar{A} \cap B) \cup (A \cap B)] - P(A \cap B)$ axiom 3

$= P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

## Extended Theorem

If A, B, C are 3 events then:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

**Proof:-**

Consider,

$$P[(A \cup B \cup C)] = P[\overbrace{(A \cup B)}^{A} \cup \overbrace{C}^{B}]$$

$$= P(A \cup B) + P(C) - P[(A \cup B) \cap C]$$

$$= P(A) + P(B) - P(A \cap B) + P(C) - P[\underbrace{(\overset{A}{(A \cap C)}) \cup P(\overset{B}{B \cap C})}_{\text{distributive law}}].$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - [P(A \cap C) + P(B \cap C) - P(A \cap C \cap B \cap C)]$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Collory 1

If A && B are mutually exclusive then
$P(A \cup B) = P(A) + P(B)$

Collory 2 (extend)

If $A_1, A_2, \ldots A_n$ are ME, then
$P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = P(A_1) + P(A_2) \ldots + P(A_n)$

Collory 3 (S)

If $A_1, A_2 \ldots A_n$ is a partition of S, then
$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots P(A_n)$
$= \boxed{P(S) = 1}$

events → Mutually exclusive } consider
sets → disjoint }

* Conditional Probability (depends)

* The prob of an event B occuring when it is known that some event A has occured
* Denoted by $P(B/A)$
* read as prob of B given A

Defination :- The Prob of B given A denoted by $P(B/A)$ is ~~given~~ defined by

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \text{if } P(A) > 0$$

$$P(A/B) = \frac{P(B \cap A)}{P(B)} \quad \text{if } P(B) > 0.$$

Note :- when $P(B/A) = P(B)$ then the events A & B are said to be Independ.
Similarly $P(A/B) = P(A)$ then the events A & B are said to be Independent

Multiplicative Rules.

Theorem:

In an experiment, the events A & B.
both can occur, then.

$$P(A \cap B) = P(A)\,P(B/A) \rightarrow \boxed{\text{cond prob. } P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

Also $P(A \cap B) = P(B)\,P(A/B)$

Note :- Independent.

$$P(A \cap B) = P(A) \cdot P(B).$$

$P(A \cap B) = P(A)\,P(B)$

# Baye's Rule.

If events $B_1, B_2 \ldots B_n$ constitute a partition in 'S', where $P(B_i) \neq 0$. for $i = 1, 2 \ldots n$, then for any event A in 'S' such that $P(A) \neq 0$.

$$\boxed{\begin{aligned} P(B_k/A) &= \frac{P(B_k \cap A)}{\sum_{i=1}^{n} P(B_i \cap A)} \\ &= \frac{P(B_k)\,(B(A/B_k))}{\sum_{i=1}^{n} P(B_i)\,(B(A/B_i))} \quad \text{for } r = 1, 2 \ldots n. \end{aligned}}$$

Multip theor

Note :- Bayes Theorem is also known as formula for the probability of causes.

Proof.

Let $B_1, B_2, \ldots B_n$ are events disjoint such that $P(B_i) > 0$ & $B_i \cap B_j \neq 0$. for $i \neq j$ where $i \,\&\, j = 1, 2, \ldots n$.

Also $B_1, B_2 \ldots B_n$ are exhaustive events set of possible solution of S & A is any other event of S where $P(A) > 0$

consider, $S = B_1 \cup B_2 \cup \cdots \cup B_n$.

$A = A \cap S$.

$A = A \cap (B_1 \cup B_2 \cup \cdots \cup B_n)$

$A = (A \cap B_1) \cup (A \cap B_2) \cup \cdots \cup (A \cap B_n)$

consider, CP. $P(B_k/A) = \dfrac{P(B_k \cap A)}{P(A)}$

$$= \frac{P(B_k \cap A)}{P[(A \cap B_1) \cup (A \cap B_2) \cup \cdots \cup (A \cap B_n)]}$$

$\Rightarrow \cancel{P(B_k)\, P(A/B_k)}$

by using Mul Rule

$$\frac{P(B_k)\, P(A/B_k)}{P(B_1)\, P(A/B_1) + P(B_2)\, P(A/B_2) + \cdots + P(B_n)\, P(A/B_n)}$$

sol on CP

aw $y = 1, 2, 3, 4, 5, 6.$
$x =$ raw no.

7, 11
pair of die = 36.

S.

(1,1) (1,2). (1,3) (1,4) (1,5) 7 (1,6) 7.
(2,1) (2,2) (2,3) (2,4) 7 (2,5) (2,6).
(3,1) (3,2) (3,3) 7 (3,4) (3,5) (3,6)
(4,1) (4,2) 7 (4,3) (4,4) (4,5) (4,6) 11
(5,1) (5,2). (5,3) (5,4) (5,5) (5,6)
7 (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)
11.

$\boxed{\text{OR} = \text{Union} + }$

Req prob = $P(A \cup B)$.
but undependent: $P(A) + P(B) = \frac{6}{36} + \frac{2}{36}$

$= \frac{8}{36} = \frac{2}{9}.$

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dep time = 0.83. = $P(A)$.
arr time = 0.82. = $P(B)$.
both = 0.78 = $P(A \cap B)$.

a). $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.78}{0.83} = 0.93$

b). $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.78}{0.82} = 0.95$

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$\boxed{20}$ 5 def
pick 2.
Both defective

$$P(A) = \frac{5C_1}{20C_1} = \frac{1}{4}$$

$$P(B/A) = \frac{4C_1}{19C_1}$$

$\boxed{\text{without replacement. } \boxed{S-1}}$
ie $-1$

MUL LAW.

$$P(A \cap B) = P(A) \cdot P(B/A)$$

$$= \frac{1}{4} \times \frac{4}{19} = \frac{1}{19}.$$

a) $P(A) = \frac{5C1}{20C1}$ = ∴

b) { Selected at Ran & removed without Rep. (Succession)

{ $P(B/A) = \frac{4C1}{19C1}$ — —

↳ remove a card from S.

Both $A \cap B = P(A) \cdot P(B/A)$

↳ after one anothr without replacmt. ↳ card removed from SS.

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3 cards

$P(A) = \frac{3}{52}$ $\frac{2}{52}$

$A_1$ = red ace. $8/51$

$A_2$ = 10 or jack. $\frac{12}{50}$, etc.

$A_3$ = >3 but less <7.

$A_1 \cap A_2 \cap A_3$ given

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_1 \cap A_2)$$

$$\frac{2}{52} \times \frac{8}{51} \times \frac{12}{50}$$

with replacmet – card kept back again (sampl space – same).

$$\frac{2}{52} \times \frac{8}{52} \times \frac{12}{52}.$$

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S = 16. 3 students

B = 12 }

G = 4 }

~~$P(EA) = \frac{3C1}{16C1}$ (Ev.)~~

PM

~~16. (3)~~

X

16 Student

Boys

$A_1 \cap A_2 \cap A_3$

$\frac{12C_1}{16C_1} \times \frac{11C_1}{15C_1} \times \frac{10C_1}{14C_1}$

| | A | B | C |
|---|---|---|---|
| | | | |
| | | | |

10 B

5 G

$\boxed{15\ S}$

a) 1st 2 — B

2nd — G

$15C_r \quad \frac{15}{91}$

$\frac{10}{15} \times \frac{9}{14} \times \frac{5}{13}$

50

b). 1st & 3rd BB or GG.

2nd opp

$\frac{650}{2730}$

$\frac{10}{15} \times \frac{5}{14} \times \frac{10}{13}$

$\cap \left( \frac{5}{15} \times \frac{10}{14} \times \frac{5}{13} \right.$

$\frac{250}{2730}$

5 | 65

13 | 13

| 1

## Bayer Theorem

$P(A) = \frac{20}{100}$

$P(B) = \frac{30}{100}$

$P(C) = \frac{50}{100}$

$P\left(\frac{A}{D}\right) = ?$

$P\left(\frac{B}{D}\right) = ?$

$P\left(\frac{C}{D}\right) = ?$

$P\left(\frac{D}{A}\right) = \frac{\frac{6}{100}}{\frac{20}{100}} \rightarrow \frac{3}{10}$

$\frac{P(D)}{P(A)}$

$P\left(\frac{D}{B}\right) = \frac{\frac{3}{100}}{\frac{30}{100}} \rightarrow \frac{1}{10}$

$P\left(\frac{D}{C}\right) = \frac{\frac{2}{100}}{\frac{50}{100}} \rightarrow \frac{1}{25}$

Main $P\left(\frac{A}{D}\right) = \dfrac{P(D/A) \cdot P(A)}{P(D/A) \cdot P(A) + P(D/B) \cdot P(B) + P(D/C) \cdot P(C)}$

Simp. P.T. $\frac{no}{tot}$ ; $\boxed{\text{tot}}$ → 100%
outer (marginal)

inner — $\frac{row}{row\,tot}$ × outer Row Tot
↓
joint

Probability Table.

Set up of probability Table.
$A_1, A_2, \ldots A_m$ are m ME & collectively exhaustive events.

$B_1 B_2 \ldots B_k$ are k ME & collectively exhaustive events.

$$P\left(\frac{E}{O}\right) = \frac{P(E \cap O)}{P(O)} = \frac{0.381}{0.400} = 95\%$$

$$P(O/E) = \frac{P(E \cap O)}{P(E)} = \frac{0.381}{0.842} = 45\%$$

Bayes, Bolts

| | A | B | C | Total |
|---|---|---|---|---|
| Bad | 0.06×0.2 $\boxed{0.012}$ | 0.03×0.3 0.009 | 0.01 | 0.031 |
| Good | 0.94×0.2 $\boxed{0.188}$ | 0.97×0.3 0.291 | 0.49 | 0.969 |
| Total | 0.2 | 0.3 | 0.5 | 1.0 |

Prob table for Bays Problem

$$P(A/D) \quad \frac{P(A \cap D)}{P(D)} = \frac{0.012}{0.031} = 0.387$$

Probability :-

① Classical defination

$$P(A) = \frac{\text{No of basic outcomes that satisfy } A}{\text{Tot no of outcomes in sample space}}$$

② Emperical

$$P(A) = \frac{\text{No of times the event } A \text{ occurs in repeated trials}}{\text{Tot of trials in random exp}}$$

③ Subjective

$P(A)$ = An opinion or belief about the chance of occurrence.

④ Conditional Probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$ , Ind :- $P(A/B) = P(A)$

⑤ Multiplicative theorem / Rule / Product Rule

$$P(A \cap B) = P(A/B) \cdot P(B)$$

Ind :- $P(A \cap B) = P(A) \cdot P(B)$

⑥ Additive Theorem. (not disjoint)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

⑦ Additive Theorem Axioms

a) $P(A) > 0$.

b) $P(S) = 1$.

c). If $A_1, A_2 \dots A_n$ are disjoint sets of S.

$$P(A_1 \cup A_2 \cup A_3 \dots A_n) =$$
$$P(A_1) + P(A_2) + P(A_3) + \dots P(A_n) \dots$$

⑧ Bayes Rule.

$$P(B_k/A) = \frac{P(B_k \cap A)}{\sum_{i=1}^{n} P(B_i \cap A)}$$

$$= \frac{P(A/B_k)\,P(B_k)}{\sum_{i=1}^{n} P(A/B_i)\,P(B_i)}.$$

$\mu-2\sigma \quad \mu-\sigma \quad \mu \quad \mu+\sigma \quad \mu+2\sigma$

$\underbrace{\qquad\qquad\qquad\qquad}_{1-\frac{1}{K^2}}$ $K=2$

# Chebyshev's theorem

If X is RV with $\mu$ & $\sigma^2$, then for any +ve no. K, we have

i.e $P(|X-\mu| \geq K\sigma) \leq \frac{1}{K^2}$.

ie. $P(|X-\mu| < K\sigma) \geq 1-\frac{1}{K^2}$.

i.e The probability that any RV X will assume a value within K. SD of mean is atleast $1-\frac{1}{K^2}$

**Proof**: Let X be CRV.

$$\sigma^2 = E\left[(X-\mu)^2\right]$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx.$$

$$= \int_{-\infty}^{\mu-K\sigma} (x-\mu)^2 f(x)\,dx +$$

$$+ \int_{\mu-K\sigma}^{\mu+K\sigma} (x-\mu)^2 f(x)\,dx +$$

$$\int_{\mu+K\sigma}^{\infty} (x-\mu)^2 f(x)\,dx.$$

$$\int_{-\infty}^{\mu-k\sigma} (x-\mu)^2 f(x)\,dx + \int_{\mu+k\sigma}^{\infty} (x-\mu)^2 f(x)\,dx \quad \text{---}(1)$$

Mea $x \le \mu - k\sigma$ (or) $x \ge \mu + k\sigma$.

$|x-\mu| \ge k\sigma$.

S.O.B.S.

$$\boxed{(x-\mu)^2 \ge k^2\sigma^2} \text{---}(2)$$

Sub (2) in (1)

$$\sigma^2 \ge \int_{-\infty}^{\mu-k\sigma} k^2 \cancel{\sigma^2} f(x)\,dx + \int_{\mu+k\sigma}^{\infty} (k^2 \cancel{\sigma^2} f(x)\,dx$$

$$\Rightarrow 1 \ge k^2 \int_{-\infty}^{\mu-k\sigma} f(x)\,dx + k^2 \int_{\mu+k\sigma}^{\infty} f(x)\,dx$$

$$\Rightarrow \int_{-\infty}^{\mu-k\sigma} f(x)\,dx + \int_{\mu+k\sigma}^{\infty} f(x)\,dx < \frac{1}{k^2}$$

$$\boxed{\begin{aligned} P[X \le a] &= \int_{-\infty}^{a} f(x)\,dx \\ P[X \ge a] &= \int_{a}^{\infty} f(x)\,dx \end{aligned}}$$

$$\Rightarrow P((X \le \mu - k\sigma)] + P[X \ge \mu + k\sigma] \le \frac{1}{k^2}$$

$$\Rightarrow P\{|X-\mu| \ge k\sigma] \le \frac{1}{k^2}$$ Hence proved

ow).

$[|x-\mu| \geq k\sigma] + P[|x-\mu| < k\sigma] = 1.$

$[|x-\mu| < k\sigma] = 1 - P[|x-\mu| \geq k\sigma].$

$[|x-\mu| < k\sigma] \geq 1 - \frac{1}{k^2}.$

Eg:

$y = 0, 1, 2, 3.$

$y = 0, \quad x = 1.$

$y = 0, \quad x = 0.$

$y = 1, \quad x = -1.$

$y = 2, \quad x = -2.$

$y = 3,$

Max of $y = 3$, Mini of $x = -2$.

$x \geq 1 - 3 \geq -2.$

Note: Let $k\sigma = c > 0$.

$$P[|x-\mu| \geq c] \leq \frac{1}{c^2/\sigma^2} = \frac{\sigma^2}{c^2}.$$

(OR) $$P[|x-\mu| < c] \geq 1 - \frac{\sigma^2}{c^2}.$$

$$\Rightarrow P[|x-\mu| \geq c] \leq \frac{Var(X)}{c^2}$$

(OR) $$P[|x-\mu| \leq c] \geq 1 - \frac{Var(x)}{c^2}$$