1) $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots$

Assume $e = \frac{p}{q} \Rightarrow p = qe$ (p, q are +ve integers & $GCD(p,q) = 1$)

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots$

$q!e = q! + \frac{q!}{1!} + \frac{q!}{2!} + \cdots \frac{q!}{q!} + \underbrace{\frac{q!}{(q+1)!} + \cdots}_{R}$

Consider $q! + \frac{q!}{1!} + \frac{q!}{2!} + \cdots + \frac{q!}{q!}$ This sum is an integer and since e is assumed to be rational, $q!e$ is also an integer. So, R must be an integer (+ve integer)

$$R = q!\left(\frac{1}{(q+1)!} + \frac{1}{(q+2)!} + \cdots\right)$$

$$= \frac{1}{q+1} + \frac{1}{(q+1)(q+2)} + \cdots$$

$$< \frac{1}{q+1} + \frac{1}{(q+1)^2} + \frac{1}{(q+1)^3} + \cdots$$

$$< \frac{1}{q+1}\left(1 + \frac{1}{1+q} + \frac{1}{(1+q)^2} + \cdots\right)$$

$$< \frac{1}{q+1}\left(\frac{1}{1 - \frac{1}{1+q}}\right) < \frac{1}{\cancel{q+1}}\left(\frac{\cancel{q+1}}{q}\right) < \frac{1}{q}$$

$\therefore R < \frac{1}{q}$

So, now R is positive integer between 0 and $\frac{1}{q}$. But $\frac{1}{q} < 1$. So, R can't be an integer $\Rightarrow$ Our assumption is wrong. Thus, e is irrational

2) $S(n) = 1+2+3+\dots = \frac{n(n+1)}{2}$

$C(n) = \frac{n^4+2n^3+n^2}{4}$

Prove $(S(n))^2 = C(n)$

Basic case: $n=1$. $S(1) = 1$

$C(1) = \frac{1+2+1}{4} = 1$

$(S(1))^2 = C(1)$

$\therefore (S(n))^2 = C(n)$ is true for $n=1$.

Let it be true for $n=k$ i.e. $(S(k))^2 = C(k)$

$S(k+1) = \frac{(k+1)(k+2)}{2}$

$(S(k+1))^2 = \frac{(k+1)^2(k+2)^2}{4} = \frac{1}{4}(k+1)^2((k+1+1)^2)$

$= \frac{1}{4}(k+1)^2((k+1)^2+2(k+1)+1)$

$= \frac{1}{4}((k+1)^4+2(k+1)^3+(k+1)^2)$

$= \frac{1}{4}(p^4+2p^3+p^2) = \frac{p^4+2p^3+p^2}{4}$ $(p = k+1)$

$\frac{p^4+2p^3+p^2}{4} = C(p) = C(k+1)$

$\therefore (S(k+1))^2 = C(k+1)$

Hence proved.

3) If 2^n-1 is prime, then n is prime.

Ans: Let n be composite $\Rightarrow n = a \times b$ where $a>1$ & $b>1$ and $a,b \in N$

$2^n - 1 = 2^{a\times b} - 1 = (2^b)^a - 1$

$= (2^b-1)(2^{b(a-1)} + 2^{b(a-2)} + \dots + 2^{b.1} + 2^{b.0})$

Since $b>1$, $2^b-1>1$ and

Since $a>1$, $2^b-1 < 2^n-1$.

So, 2^n-1 has a proper divisor 2^b-1, 2^n-1 is composite.

But 2^n-1 is prime $\Rightarrow$ Our assumption is false

$\Rightarrow n$ is not composite

$\therefore n$ is prime

4) i, Consider 5n+4 is even.

$5\cancel{n+4} = 2k \quad (k \in N)$

$5\cancel{n} = 2(\cancel{k-2})$

$\cancel{n} = \frac{2(\cancel{k-2})}{5}$

$5n+4$ is even $\Rightarrow$ $5n$ is even ($\because$ $a+b$ is even if $a=b=$ even / $a=b=$ odd

$\underbrace{5}_{odd} \times n =$ even $\Rightarrow$ n is even. But 4 is even. So, $5n$ is even

But given n is odd. So, our assumption is false

$\Rightarrow$ $5n+4$ is odd

whenever n is odd

$S(n) = \cancel{n^3} \; 3$ divides n^3+2n

ii, Basic case: $n=1$. $n^3+2n = 1+2=3$

3 divides 3.

So, $S(n)$ is true for $n=1$

Let $S(n)$ be true for $n=k$.

Consider $S(k+1)$.

$$(k+1)^3 + 2(k+1) = k^3+1+3k^2+3k+2k+2$$
$$= k^3 + \cancel{3}2k^2 + 3k^2 + 3k + 3$$
$$= \underbrace{k^3+2k}_{S(k)} + 3(\underbrace{k^2+k+1}_{})$$

3p is divisible by 3

So, $(k+1)^3 + 2(k+1)$ is divisible by 3.

Thus, $S(k+1)$ is true

$\therefore$ $S(n)$ is true.

Hence proved

5) $S(n) = 2^{3n+1} + 5$ is a multiple of 7

Base case: $n=1$

$2^{3n+1} + 5 = 2^4 + 5 = 16+5 = 21$

$21 \% 7 = 0$.

$\therefore$ $S(1)$ is true

Assume $S(n)$ is true for $n=k$.

Consider $2^{3(k+1)+1} + 5$

$= 2^{3k+4} + 5$

$= 2^{3k+1} \times 2^3 + 5$

$= 2^{3k+1} \times 8 + 5$

$= 2^{3k+1} (7+1) + 5$

$= \underbrace{7 \times 2^{3k+1}}_{\text{multiple of } 7.} + \underbrace{2^{3k+1} + 5}_{\text{divisible by } 7}$

$\therefore 2^{3(k+1)+1} + 5$ is multiple of 7

$\therefore S(k+1)$ is true

Hence, $S(n)$ is true.

6) This can be solved using pigeonhole principle

There are n people. Each person can have friends count between 0 and $n-1$. (both 0 & $n-1$ included).

But, suppose ~~to~~ a person has $n-1$ friends

$\Rightarrow$ Remaining persons have atleast 1 friend

$\Rightarrow$ 0 is not possible.

Similarly, if 0 is present, $n-1$ is not possible.

So, there can be only $n-1$ distinct values.

According to Pigeonhole principle, there are n friends & $n-1$ distinct values $\Rightarrow$ atleast two of them have same number of friends.

7) i) An integer n is even iff $n+1$ is odd

Contrapositive : $n+1$ is even iff n is odd

(a) $n+1$ is even if n is odd.

n is odd $\Rightarrow n = 2k+1$ $(k \in N \cup \{0\})$

$\Rightarrow n+1 = 2k+1+1$

$= 2(k+1)$

$= 2p$ (let $p = k+1 \in N$)

$\therefore n+1$ is even

(b) n is odd if $n+1$ is even

$n+1$ is even $\Rightarrow n+1 = 2k (k \in N)$

$\Rightarrow n = 2k-1$

$= 2p+1$ $(p \in W)$ $(p = k-1)$

$\therefore$ n is odd.

Hence proved.

ii) If n and m have same parity, then $n+m$ is even.

Contrapositive : If $n+m$ is odd, then n and m have different parity.

$n+m = 2k+1$ $(k \in W)$

$= 2(p+q)+1$ $(p, q \in W)$

$= 2p + 2q + 1$

One of them should be $\underset{\text{even}}{2p}$ and other is $\underset{\text{odd}}{2q+1}$

So, n and m have different parity.

iii) If x^2 is even then x is even

Contrapositive: If x is odd, then x^2 is odd

Let $x = 2p+1$ $(p \in \omega)$

$$\Rightarrow x^2 = (2p+1)^2$$
$$= 4p^2 + 1 + 4p$$
$$= 2(2p^2+2p) + 1$$
$$= \underbrace{4(p^2+1)}_{\text{even}} + \underbrace{1}_{\text{odd}} \; : \text{odd}$$

$\Rightarrow x^2$ is odd.

$\therefore$ If x^2 is even, then x is even.

Hence proved.

8) i) $\Sigma = \{0, 1\}$

ii) $\Sigma = \{a\}$

iii) $\Sigma = \{\epsilon\}$

9) i) $\Sigma = \{a, b\}$

$L = \Sigma^* a \, \Sigma^* b \, \Sigma^* a \, \Sigma^*$

String 1: aaabaaa
String 2: babbbabab
String 3: aba
$\} \in L$

String 1: ab
String 2: a
String 3: b
$\} \notin L$

ii) $\Sigma = \{a, b\}$

$L = (\epsilon \cup a)b$

String 1: ab
String 2: b
String 3: X
$\} \in L$

String 1: a
String 2: ϵ
String 3: aa
$\} \notin L$

10) $(L^*)^* = L^*$ for any language L

Ans: For any language L, $L \subseteq L^*$

$$\Rightarrow L^* \subseteq (L^*)^*$$

Now, suppose $w \in (L^*)^*$

$\Rightarrow w = w_1 w_2, \dots w_l$ for some $w_1, w_2, \dots, w_l \in L^*$

Then $w_i = w_{i,1}\, w_{i,2} \dots, w_{i,l}$ where

$w_{i,j} \in L \;\forall j = 1$ to i

So, $w = w_{1,1}\, w_{1,2} \dots w_{1,l_1}\, w_{2,1}, w_{2,2} \dots w_{2,l_2} \dots$

$w_{l,1}\, w_{l,2} \dots w_{l,l_l} \in L^*$

$\Rightarrow (L^*)^* \subseteq L^*$ (Since $w \in (L^*)^*$ & $w \in L^*$)

$\therefore L^* = (L^*)^*$ ($\because L^* \subseteq (L^*)^*$ and $(L^*)^* \subseteq L^*$)

11) $\Sigma = \{0, 1\}$

i) $(0 \cup 10)^* (1 \cup \epsilon)$

$\{0, 10\}^* . (\{1, \epsilon\})$

Doesn't contain pair of consecutive 1's.

ii) $(0 \cup 10)^* \underbrace{(1 \cup \epsilon)}^*$

Contains consecutive 1's

iii) $(0 \cup 101)^* (0 \cup \epsilon)^*$

$(0 \cup 101)^* = \{\underbrace{0, 101}\}^*$

Contains consecutive 1's

Eg: $10\underline{11}01$

iv) $(1 \cup 010)^* \underbrace{(1 \cup \epsilon)}^*$

Contains consecutive 1's

12) Kleene closure : Σ^+

i) $L = \{0,1\}$

$\Sigma^+ = \{0, 1, 00, 01, 10, 11, 001, \ldots\}$

ii) $L = \{\epsilon\}$

$\Sigma^+ = \{\epsilon\}$

13) i) $(S \cup T)^R = S^R \cup T^R$

Let $P \in (S \cup T)^R$

$\Rightarrow P \in S^R$ or $P \in T^R$

$\Rightarrow P \in S^R \cup T^R$

$(S \cup T)^R \subset S^R \cup T^R$

Let $P \in S^R \cup T^R$

$\Rightarrow P \in S^R$ or $P \in T^R$

$\Rightarrow P \in (S \cup T)^R$

$S^R \cup T^R \subset (S \cup T)^R$

$\therefore (S \cup T)^R = S^R \cup T^R$

ii) $(ST)^R = T^R . S^R$

Consider $x = x_1 x_2 x_3 \ldots x_n \in (ST)^R$

$\Leftrightarrow y = x_n x_{n-1} \ldots x_1 \in ST$

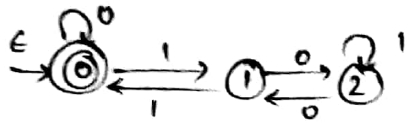
$\Leftrightarrow y = x_n x_{n-1} \ldots x_k \in S$ and $z = x_{k-1} x_{k-2} \ldots x_1 \in T$

$\Leftrightarrow x_k x_{k+1} \ldots x_n \in S^R$ and $x_1 x_2 x_3 \ldots x_{k-1} \in T^R$

$\Leftrightarrow x_1 x_2 x_3 \ldots x_{k-1} \in T^R$ and $x_k x_{k+1} \ldots x_n \in S^R$

$\Leftrightarrow x_1 x_2 x_3 \ldots x_n \in T^R . S^R$

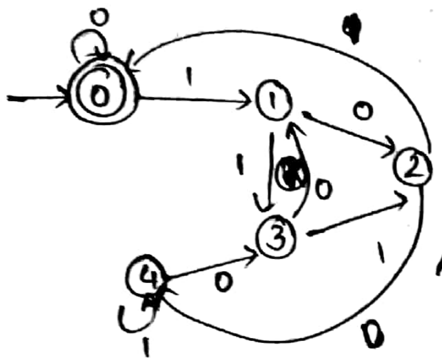
$\therefore (ST)^R = T^R . S^R$

14) 

i) 0^p $(p \in N)$

ii) $0^p 1 0 1^p 0 1 0^p$

$0^* 1 0 1^* 0 1 0^*$ and 0^*

15) 

$\Sigma = \{0, 1\}$

Accept / Final state : q_0

Start state : q_0

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

δ:

	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

16) $\Sigma = \{0, 1\}$

	0	1
q_0	q_2	q_1
q_1	q_2	q_3
q_2	q_6	q_1
q_3	q_2	q_3
q_5	q_5	q_1

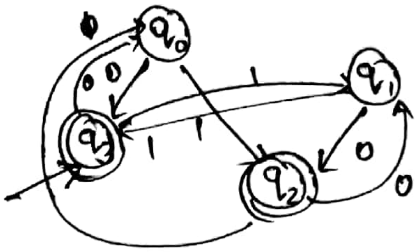
Transition table

$L = \{w \mid w$ contains 2 consecutive 0's or 2 consecutive 1's$\}$

17) i) $\Sigma = \{0, 1\}$

$Q = \{q_0, q_1, q_2, q_3\}$

	1's	0's	
q_0	E	O	
q_1	O	E	
q_2	O	O	✓
q_3	E	E	✓



δ:

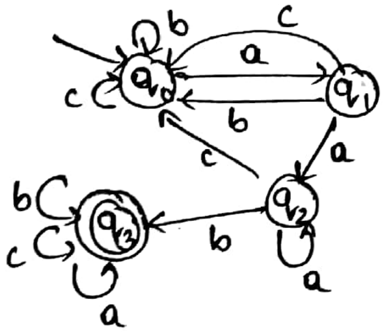
	0	1
q_0	q_3	q_2
q_1	q_2	q_3
q_2	q_1	q_0
q_3	q_0	q_1

Final states : $\{q_2, q_3\}$

Initial state : q_9

ii) $\Sigma = \{a, b, c\}$

$Q = \{q_0, q_1, q_2, q_3\}$



Final states = $\{q_3\}$

Initial state = q_0

δ:

	a	b	c
q_0	q_1	q_0	q_0
q_1	q_2	q_0	q_0
q_2	q_2	q_3	q_0
q_3	q_3	q_3	q_3

18) $\Sigma = \{a, b\}$

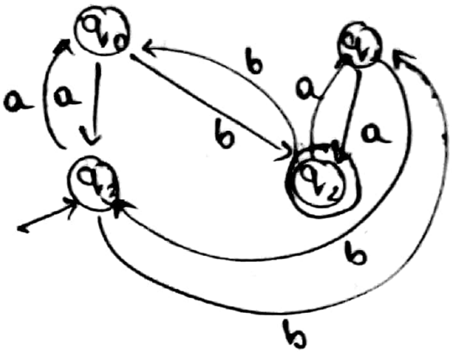
i) $\{w \mid w$ has even length and odd no. of a's$\}$

$q_0 \rightarrow$ odd length & odd a's

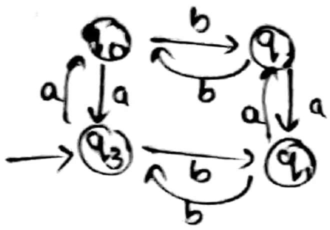
$q_1 \rightarrow$ odd " even "

$q_2 \rightarrow$ even " & odd "

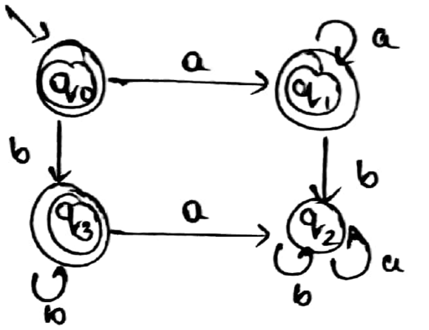
$q_3 \rightarrow$ even " & even "



(or)



ii)



q_2 : Contains either ab or ba

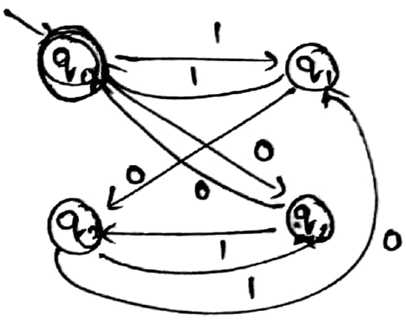
q_3 : Only b's

q_1 : Only a's

19) $L = \{w \mid w$ has exactly even no. of 0's & 1's$\}$

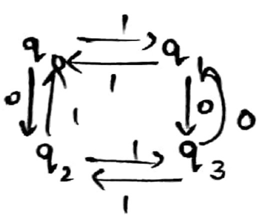
$\Sigma = \{0, 1\}$

	0's	1's
q_0	E	E
q_1	E	O
q_2	O	E
q_3	O	O



δ :

	0	1
q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2



Initial state : q_0

Accept states = $\{q_0\}$

$\Sigma = \{0, 1\}$

$Q = \{q_0, q_1, q_2, q_3\}$

20) $(111 \cup 11111)^*$

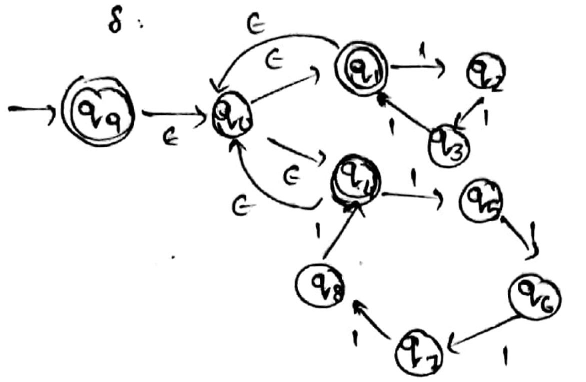
$\Sigma = \{1\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}$

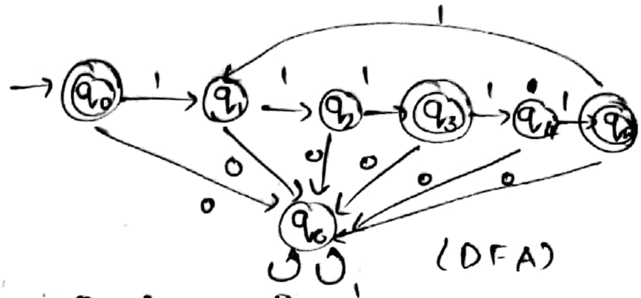
Start state = q_9

Accept/Final states = $\{q_1, q_4, q_9\}$

δ :



(NFA)



(DFA)

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$

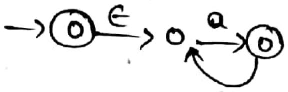
Start state = q_0

Accept states = $\{q_0, q_3, q_5\}$

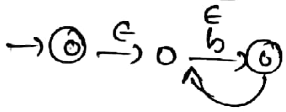
21) i) $\Sigma = \{a, b\}$

$a^* (ab)^* b^*$

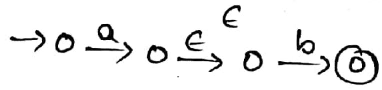
a^* ⇒



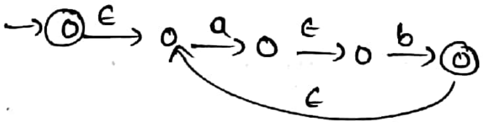
b^* ⇒



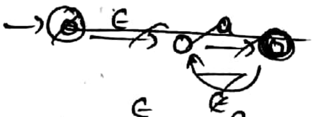
ab ⇒



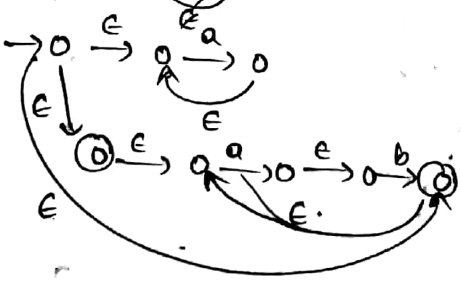
$(ab)^*$ ⇒



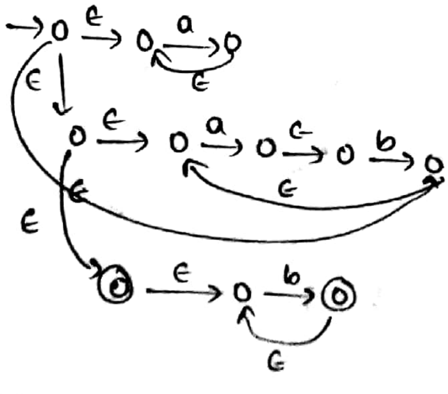
~~$a^*(ab)^*$~~ ~~⇒~~



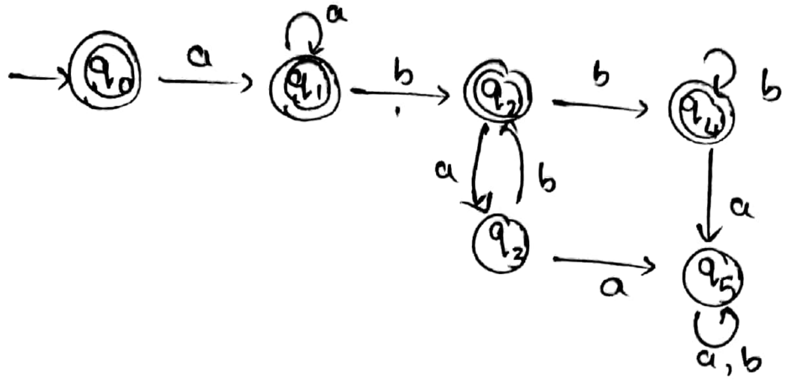
$a^*(ab)^*$ ⇒



$a^*(ab)^* b^*$ ⇒



21) i, $a^*(ab)^*b^*$ $\Sigma = \{a, b\}$

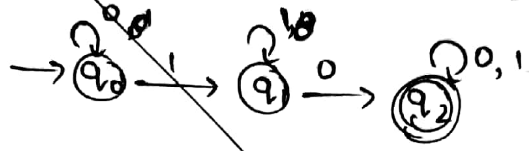


$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

Start state $= q_0$

Accept states $= \{q_0, q_1, q_2, q_4\}$

~~(ii, $(0 \cup 1)^* 10 (0 \cup 1)^*$~~ ~~$\Sigma = \{0, 1\}$~~



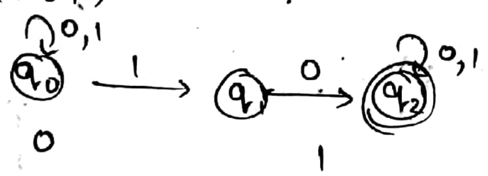
~~$Q = \{q_0, q_1, q_2\}$~~

~~Start state $= q_0$~~

~~Accept states $= \{q_2\}$)~~ *

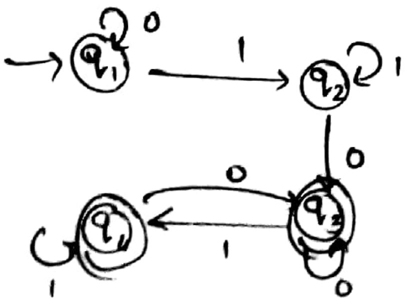
ii, $(0 \cup 1)^* 10 (0 \cup 1)^*$

NFA :



		0	1
q_1	$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
q_2	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
q_3	$\{q_0, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
q_4	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$

DFA :

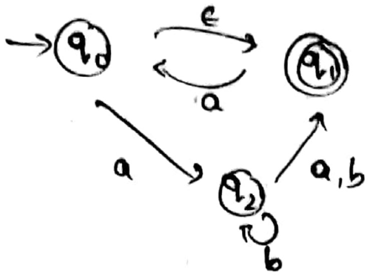


$Q = \{q_1, q_2, q_3, q_4\}$

Start state $= q_1$

Accept states $= \{q_3, q_4\}$

22)

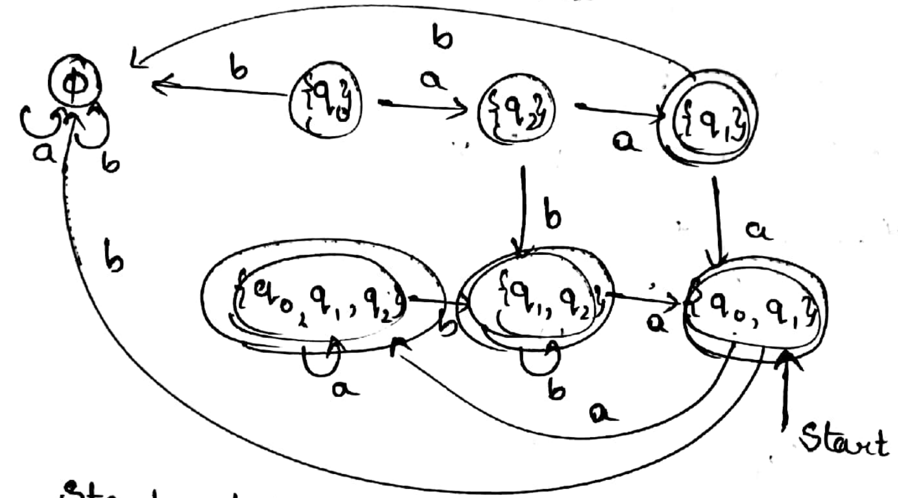


	a	b
q_0	q_2	ϕ
q_1	$\{q_0, q_1\}$	ϕ
q_2	$\{q_1\}$	$\{q_1, q_2\}$

Ans:

	a	b
$\{q_0\}$	$\{q_2\}$	ϕ
$\{q_2\}$	$\{q_1\}$	$\{q_1, q_2\}$
$\{q_1\}$	$\{q_0, q_1\}$	ϕ
$\{q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	ϕ
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$

δ:



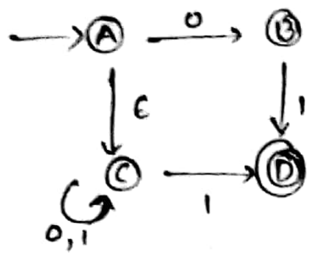
Start state = $\{q_0, q_1\}$

Accept states = $\{ \{q_0, q_1\}, \{q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

$\Sigma = \{a, b\}$

$Q = \{\phi, \{q_0\}, \{q_2\}, \{q_1\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

24)



δ for NFA:

	0	1
A	{B}	ϕ
B	ϕ	{D}
C	{C}	{C, D}
D	ϕ	ϕ

Ans: δ for DFA:

	0	1
{A}	{B}	ϕ
{B}	ϕ	{D}
{D}	ϕ	ϕ
{C}	{C}	{C, D}
{C, D}	{C}	{C, D}

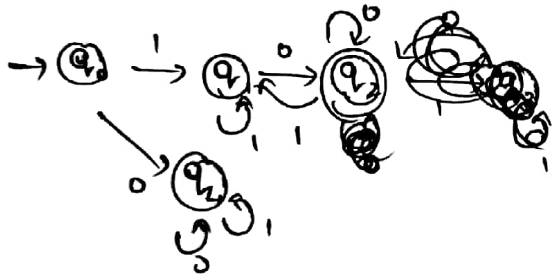
Q = { ϕ, {A}, {B}, {C}, {D}, {C, D}}

Start states = { {A}, {C} }

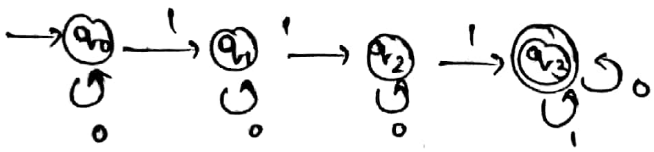
Accept states = { {D}, {C, D} }

25) i, {w | w begins with 1 & ends with 0}

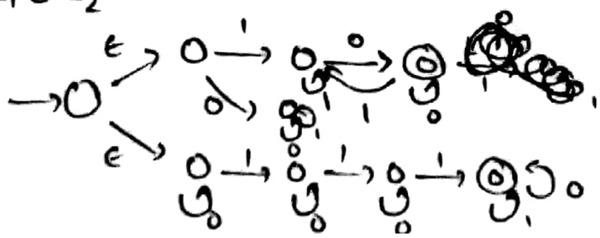
Ans:

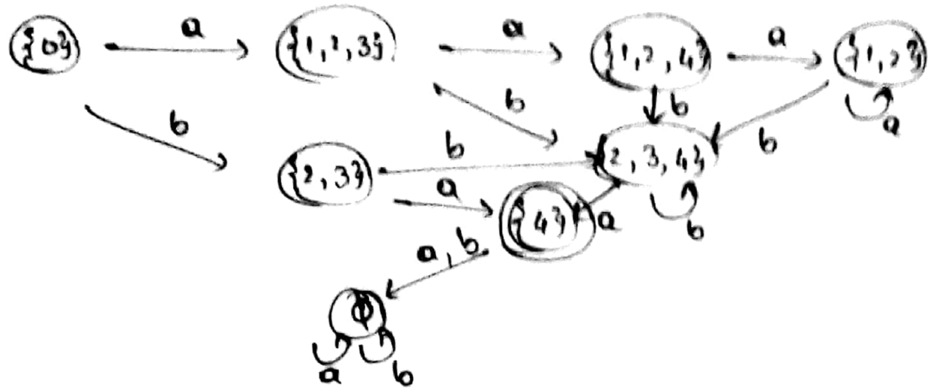


ii, {w | w contains at least three 1's}



iii, $L_1 \cup L_2$





28) δ for NFA :

	a	b
1	3	ϕ
2	{4, 5}	ϕ
3	ϕ	{4}
4	{5}	{5}
5	ϕ	ϕ

$\Sigma = \{a, b\}$

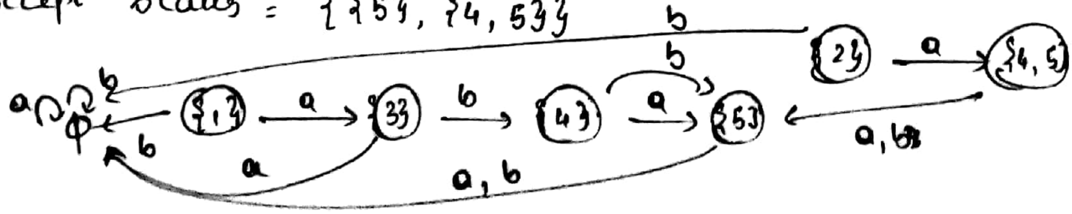
δ for DFA :

	a	b
{1}	{3}	ϕ
{3}	ϕ	{4}
{4}	{5}	{5}
{5}	ϕ	ϕ
{2}	{4, 5}	ϕ
{4, 5}	{5}	{5}

$Q = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{4, 5\}\}$

Start states = {{1},{2}}

Accept states = {{5}, {4, 5}}



29) $L = \{w \mid w$ has odd no. of a's and even no of b's$\}$

Ans: To prove that L is regular, an DFA N is to be constructed.

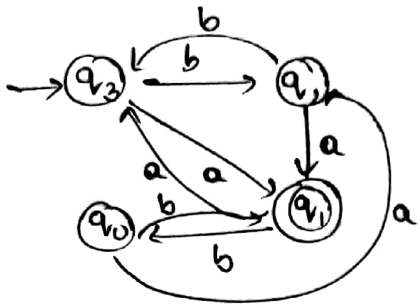
q_0 : Odd a's and Odd b's

q_1 : Odd " " Even "

q_2 : Even " " Odd "

q_3 : Even " " Even "

δ :



$\Sigma = \{a, b\}$

$Q = \{q_0, q_1, q_2, q_3\}$

Start state $= \{q_3\}$

Accept state $= \{q_1\}$

30) L_1 and L_2 are regular

$\Rightarrow L_1^c$ and L_2^c are regular $\} \rightarrow$ ①

$L_1 \cap L_2 = (L_1^c \cup L_2^c)^c$

$\Rightarrow L_1^c$ and L_2^c are regular

$\Rightarrow L_1^c \cup L_2^c$ is regular

$\Rightarrow (L_1^c \cup L_2^c)^c$ is regular (From ①)

$\therefore L_1 \cap L_2$ is regular.

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31) i) Let M be DFA that recognizes L. Let M' be the DFA obtained from M by making all states that are not final states of M final states of M' and vice versa. Then M' recognises L^c.

M' accepts string x

<=> Computation of M' on x ends in final state of M'

<=> Computation of M' on x doesn't end in final state of M

<=> Computation of M on x doesn't end in final state of M

<=> M doesn't accept x

<=> $x \in L^c$

So, if L is regular, L^c is regular.

ii) $L_1 - L_2 = L_1 \cap L_2^c$ $\quad$ L_1 and L_2 are regular languages

$= (L_1^c \cup L_2)^c$

L_1 is regular $\Rightarrow$ L_1^c is regular

L_1^c and L_2 are regular $\Rightarrow$ $L_1^c \cup L_2$ is regular

$\Rightarrow (L_1^c \cup L_2)^c$ is also regular

$\Rightarrow L_1 - L_2$ is regular

$\therefore$ $L_1 - L_2$ is regular.

32) If L^* is regular, then L is not necessarily regular

Example: $\Sigma = \{a\}$

and $L = \{a^{2^n} \mid n \in N\}$

L is not regular.

x ($w = a^{p-1}$

$x \to r \quad y \to s \quad z \to p-r-s-1$

$xy^2z = \cancel{r} + 2s + p - \cancel{r} - s - 1$

$= p + s - 1$

$p = 2^k$

$-1 + p + s \geq 2^{k+1}$

$s \geq 2^k + 1$ But $s \leq 2^k$) X

$w = a^{2^p}$

$x \to r \quad y \to s \quad z \to 2^p - r - s$

$xy^2z = \cancel{r} + 2^p - \cancel{r} - s + 2s$

$= 2^p + s \leq 2^p + p < 2^p + 2^p$

$< 2^{p+1}$

$xy^2z > 2^p$

So, $xy^2z \notin L$

$\Rightarrow L$ is not regular

But L^*

32) Let $\Sigma = \{a\}$ and

$L = \{a^{n^2} \mid n \in N\}$

Let $s = a^{p^2}$

$x \to r \quad y \to s \quad z \to p^2 - r - s$

$xy^2z = \cancel{r} + 2s + p^2 - \cancel{r} - s$

$= p^2 + s \leq p^2 + p < (p+1)^2$

and $xy^2z > p^2$

So, $xy^2z \notin L$

$\therefore$ L is not regular

But $L^* = \{a^*\}$. So, L^* is regular.

$\Rightarrow L^*$ is regular but L is not

$\therefore$ If L^* is regular, then L is not necessarily regular.

33) . L, M and N are languages

$L(M \cup N) = LM \cup LN$

Let $w \in L(M \cup N)$

$\Leftrightarrow \exists u \exists v$ st $(w = uv)$ and $u \in L \wedge v \in (M \cup N)$

$\Leftrightarrow \exists u \exists v$ st $(w = uv)$ and $u \in L \wedge (v \in M \vee v \in N)$

$\Leftrightarrow \exists u \exists v$ st $(w = uv)$ and $((u \in L \wedge v \in M) \vee (u \in L \wedge v \in N))$

$\Leftrightarrow (\exists u \exists v$ st $(w = uv)$ and $u \in L \wedge v \in M) \vee$

$(\exists u \exists v$ st $(w = uv)$ and $u \in L \wedge v \in N)$

$\Leftrightarrow \quad w \in LM \vee w \in LN$

$\Leftrightarrow \quad w \in (LM \cup LN)$

$\therefore L(M \cup N) = LM \cup LN$

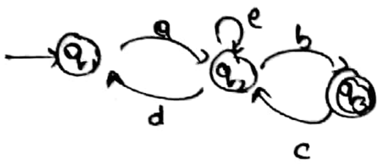
36) $\Sigma = \{0, 1\}$

i) $0 \Sigma^* 1$

ii) $(0 \cup 1) 1 \Sigma^*$

iii) $\underbrace{(0 \cup 1)0 (0 \cup 1)0 \ldots\ldots}_{k-1 \text{ times}} 1 \Sigma^*$ (or) $(0 \cup 1)^{k-1} 1 \Sigma^*$

37) $\Sigma = \{a, b, c, d, e\}$



Regular expression : $a(da)^* e^* b(cb)^* (cdae^*b)^*$

38) $K = \{w \mid w \in \{0,1\}^*, w = \langle n \rangle, n \in N, n \equiv 4 \bmod 5\}$

39) $L = \{w \mid w \in \{0,1\}^*, w = \langle n \rangle, n \in N, n \equiv 4 \bmod 5\}$

Regular expression

$= 0^* 1 1^* 0 (10)^* 0 0^* (1 1^* 0 (10)^* 0 0^*)^*$

40) i) $\Sigma = \{a, b, c\}$

Regular expression = $\Sigma^* a \Sigma^* b \Sigma^* \cup \Sigma^* b \Sigma^* a \Sigma^*$

ii) $\Sigma = \{0, 1\}$

Regular expression = $\Sigma^* 1 (0 \cup 1)^9$

iii) $\Sigma = \{0, 1\}$

Regular expression = $(0 \cup (10))^* (11 \cup \epsilon) (0 \cup 10)^*$

41) $S = (a \cup b^*)^*$

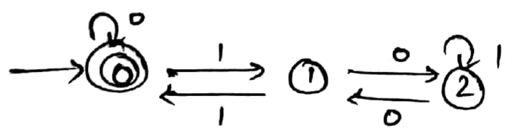
$T = (a \cup b)^*$

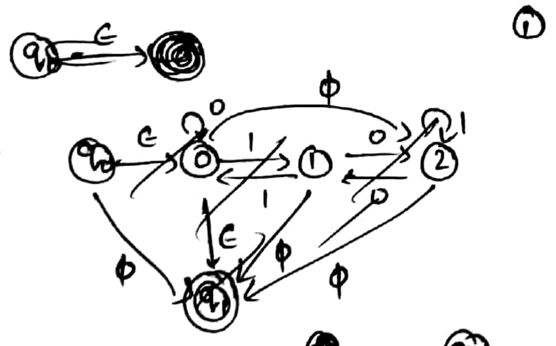
$S \subset T \,;\, T \subset S \Rightarrow S = T$

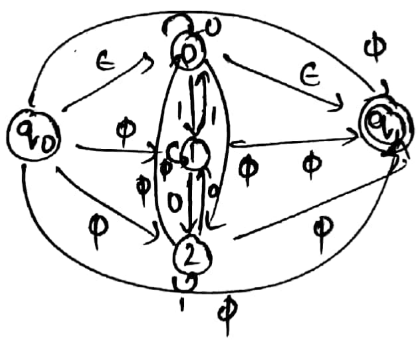
$S \cap T \neq \phi$

$\therefore$ i, ii, iii are correct

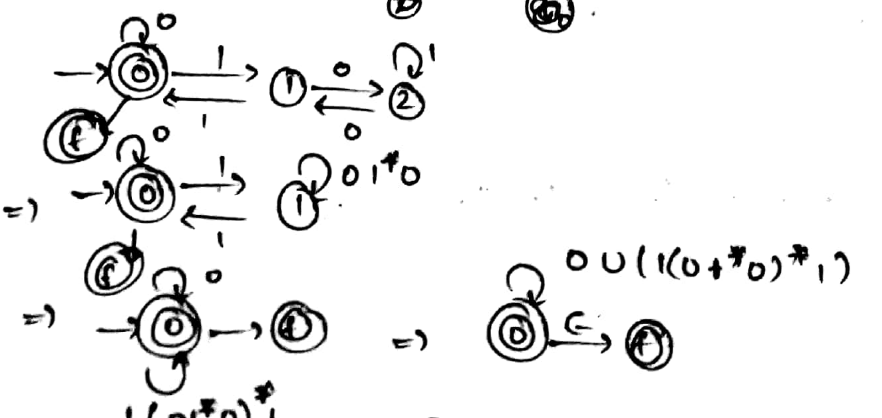
42)







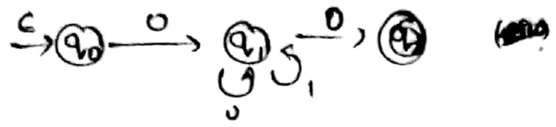
42)



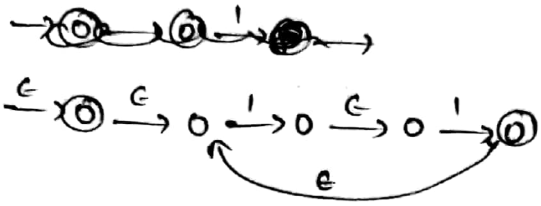
Regular expression = $(0 \cup (1(01^*0)^*1))^*$

43) i) $0\Sigma^*0$

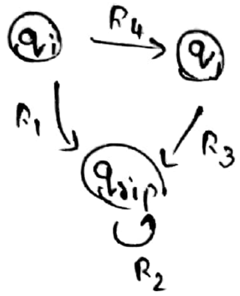
Ans:

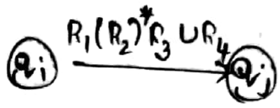


ii) $(11)^*$

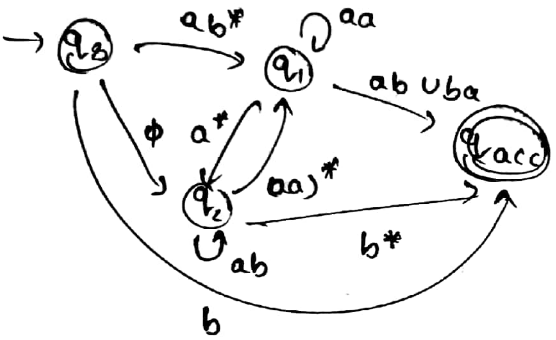


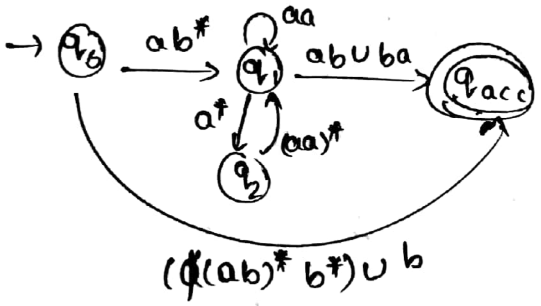
44) i)

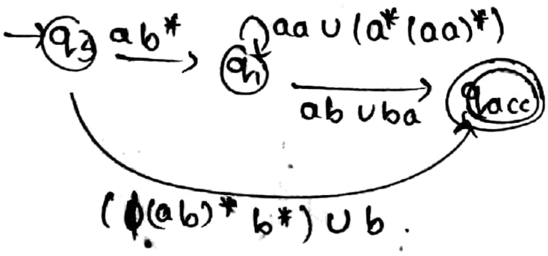




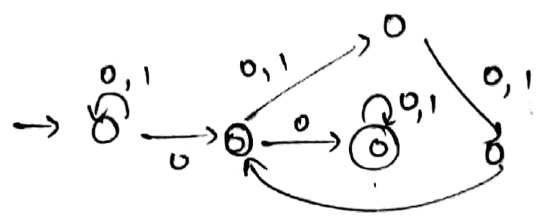
ii)

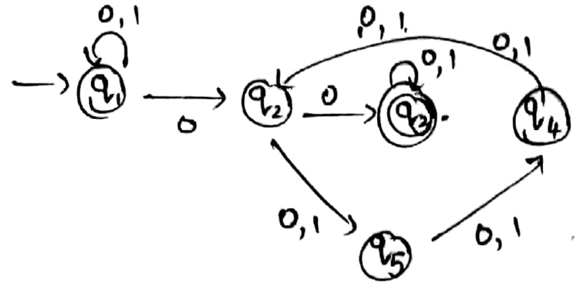






45)

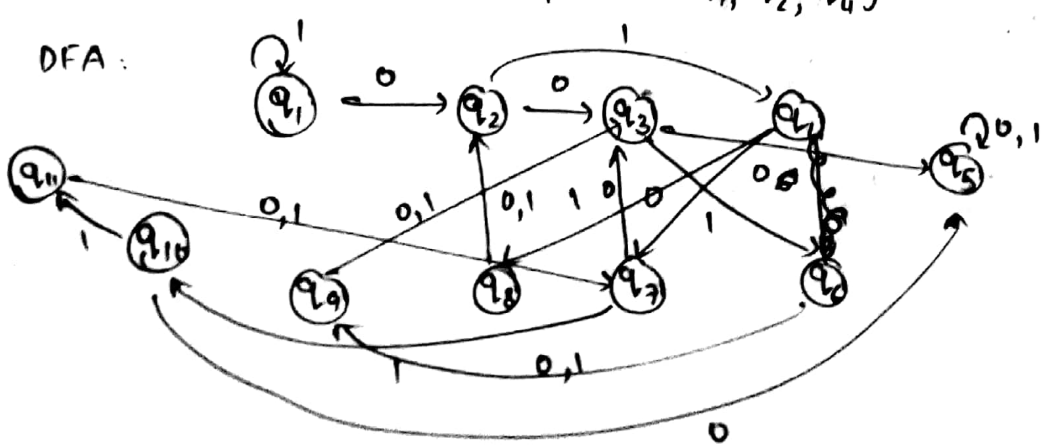




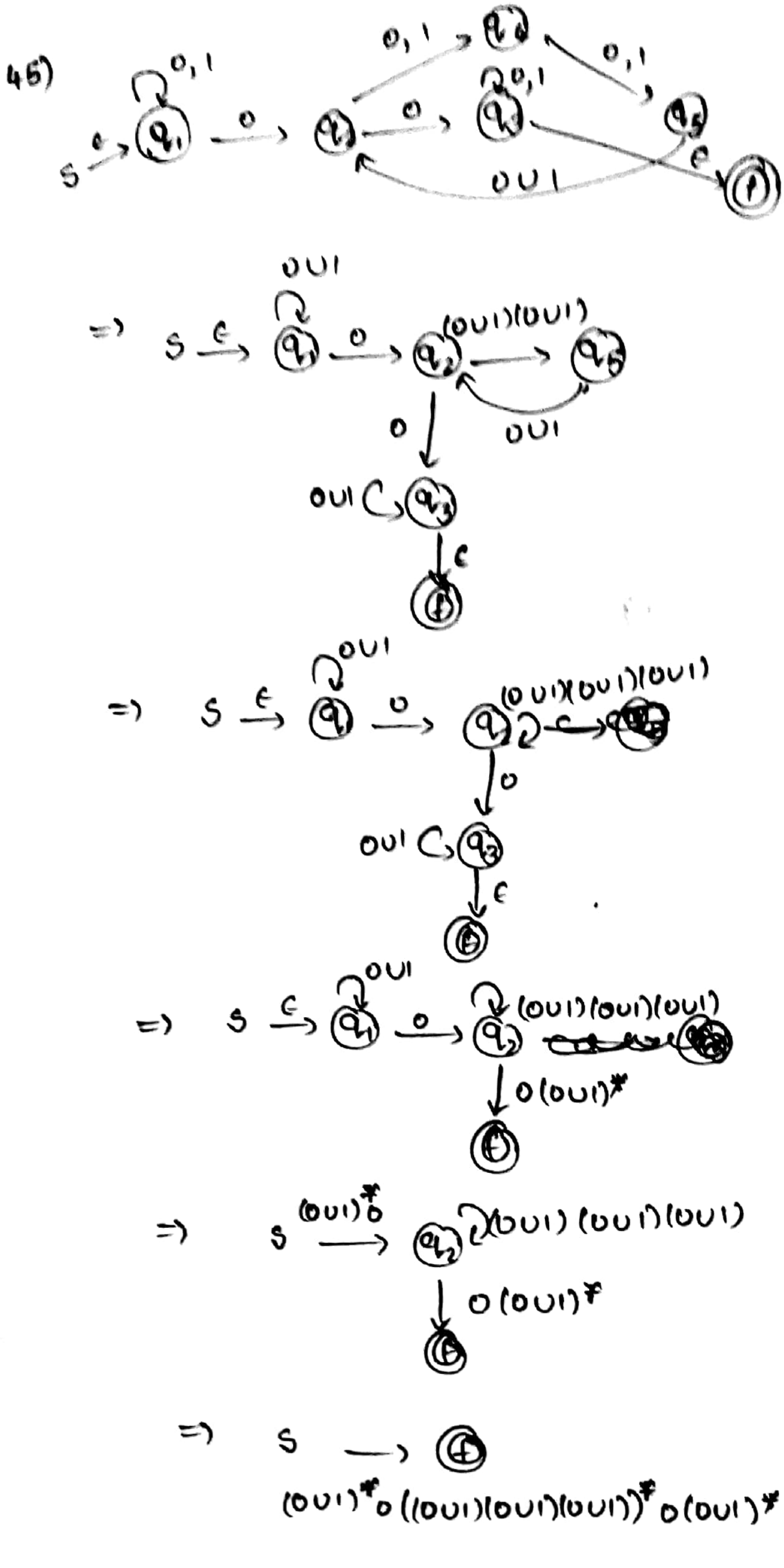
NFA

		0	1
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\{q_1\}$
q_2	$\{q_1, q_2\}$	$\{q_1, q_2, q_3, q_5\}$	$\{q_1, q_5\}$
q_3	$\{q_1, q_2, q_3, q_5\}$	$\{q_1, q_2, q_3, q_5, q_4\}$	$\{q_1, q_5, q_3, q_4\}$
q_4	$\{q_1, q_5\}$	$\{q_1, q_2, q_4\}$	$\{q_1, q_4\}$
q_5	$\{q_1, q_2, q_3, q_4, q_5\}$	$\{q_1, q_2, q_3, q_4, q_5\}$	$\{q_1, q_5, q_3, q_2, q_4\}$
q_6	$\{q_1, q_3, q_4, q_5\}$	$\{q_1, q_2, q_3, q_4\}$	$\{q_1, q_3, q_2, q_4\}$
q_7	$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_3, q_5\}$	$\{q_1, q_5, q_2\}$
q_8	$\{q_1, q_4\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
q_9	$\{q_1, q_2, q_3, q_4\}$	$\{q_1, q_2, q_3, q_5\}$	$\{q_1, q_5, q_3, q_2\}$
q_{10}	$\{q_1, q_2, q_5\}$	$\{q_1, q_2, q_3, q_5, q_4\}$	$\{q_1, \cancel{q_3}, q_5, q_4\}$
q_{11}	$\{q_1, q_4, q_5\}$	$\{q_1, q_2, q_4\}$	$\{q_1, q_2, q_4\}$

DFA :



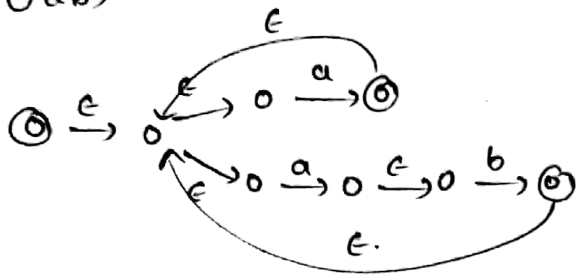
45)



Regular exp = $(0\cup1)^*0((0\cup1)(0\cup1)(0\cup1))^*0(0\cup1)^*$

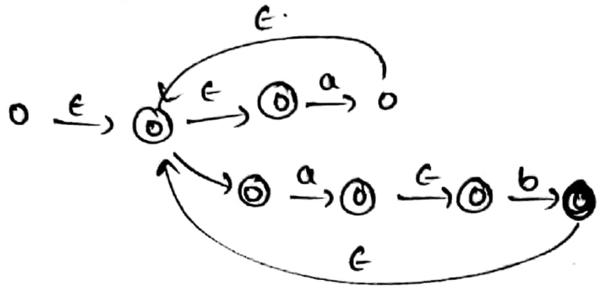
46) $(a \cup ab)^*$

$\Rightarrow$

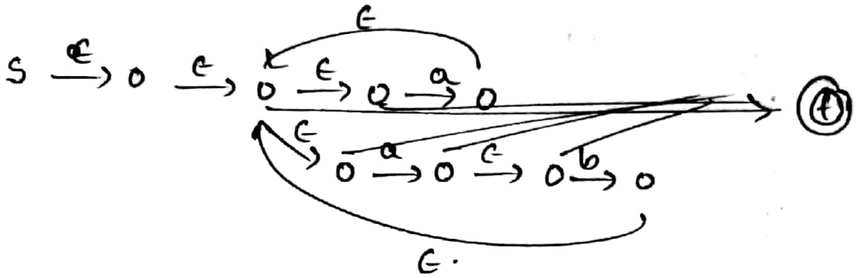


$((a \cup ab)^*)^c$

$\Rightarrow$



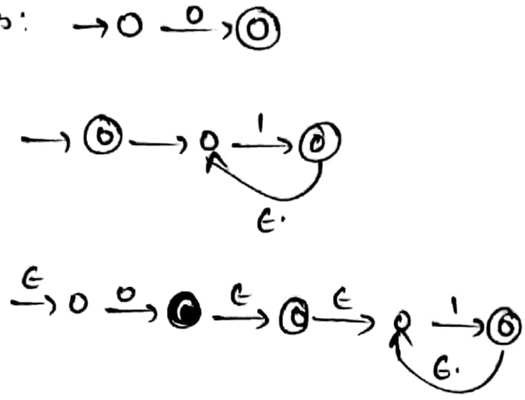
$\Rightarrow$



$\Rightarrow$

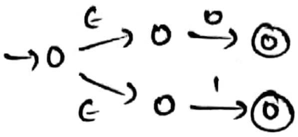
47) i, 01^*

Ans:

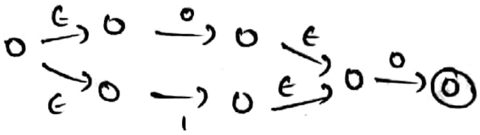


ii, $(0\cup1)01$

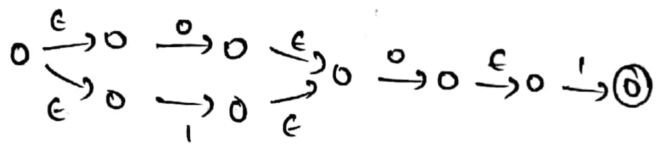
Ans: $0\cup1$ ⇒



$(0\cup1)0$ ⇒

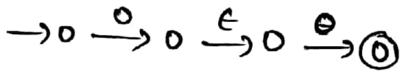


$(0\cup1)01$ ⇒

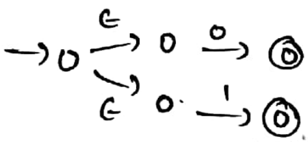


iii, $00(0\cup1)^*$

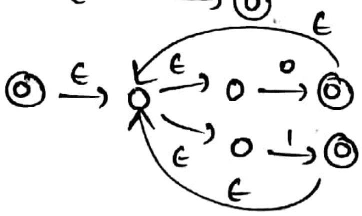
Ans: 00 ⇒



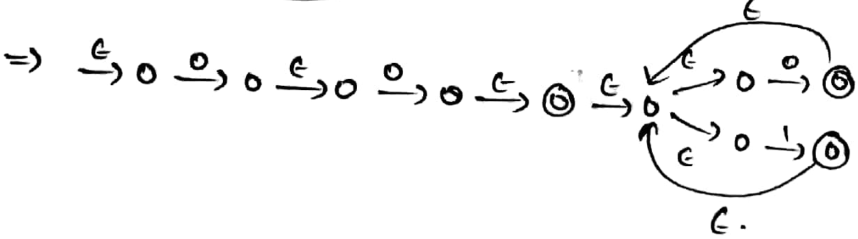
$(0\cup1)^*$ ⇒



$(0\cup1)^*$ ⇒



$00(0\cup1)^*$ ⇒



48) i, $(110 \cup 1)^* 0 \Rightarrow 0$ ✓

$\Rightarrow (110 \cup 1) 0$ ✗

ii, $(11 \cup 110)^* 1 \Rightarrow 1$ ✗

iii, $(110 \cup 11)^* 0 \Rightarrow 0$ ✓

$\Rightarrow (110 \cup 11) 0$

Either 1100 or 110

$1100 \Rightarrow q_1 q_3 q_4 q_5$

q_5 is accept state

$110 \Rightarrow q_1 q_3 q_4$

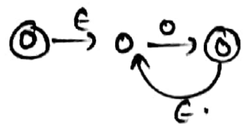
q_4 is accept state

$\therefore (110 \cup 11)^* 0$ is regular expression of given FA.

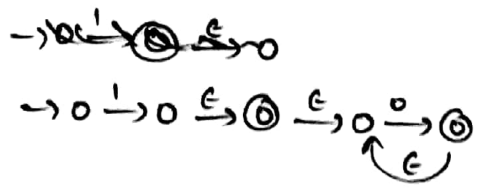
iv, $(1 \cup 110)^* 1 \Rightarrow 1$ ✗

49) i, $0^* (10^*)^*$

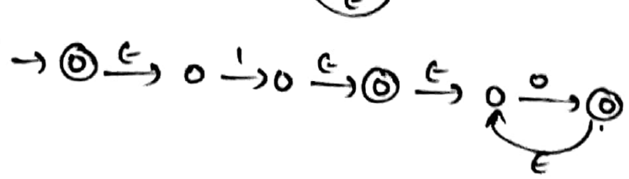
Ans: $0^* \Rightarrow$



$10^* \Rightarrow$

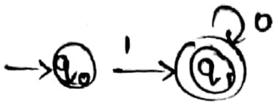


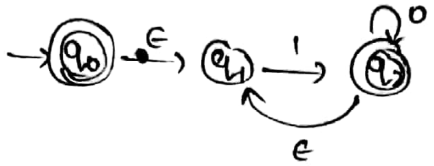
$(10^*)^* \Rightarrow$

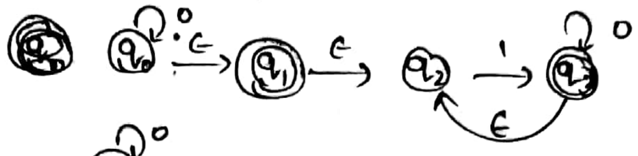


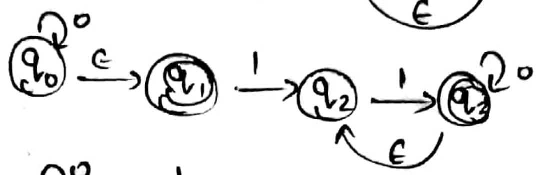
49) i, $0^*(10^*)^*$

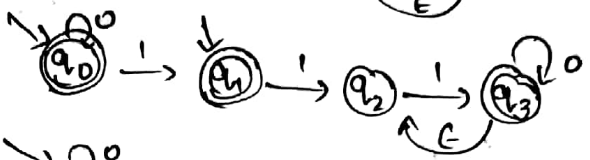
$0^* \Rightarrow$ 

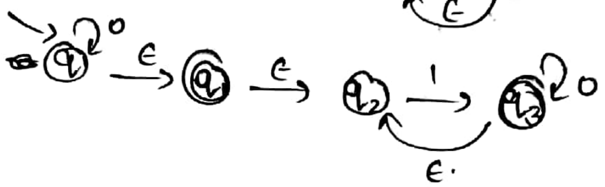
$10^* \Rightarrow$ 

$0^*(10^*)^* \Rightarrow$ 

$0^*(10^*)^* \Rightarrow$ 

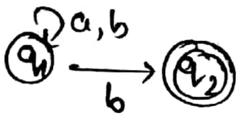


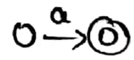


$0^*(10^*)^* \Rightarrow$ 

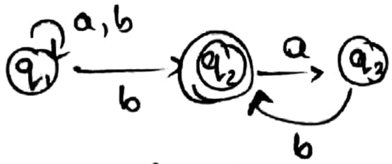
ii, $a \cup (a \cup b)^* b(ab)^*$

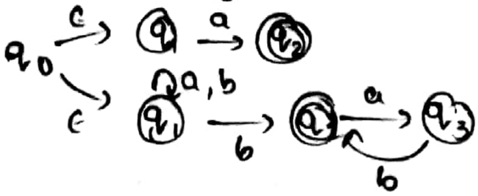
$(a \cup b)^* \Rightarrow$ 

$(a \cup b)^* b \Rightarrow$ 

$a \Rightarrow$ 

$(ab)^* \Rightarrow$ 

$(a \cup b)^* b(ab)^* \Rightarrow$ 

$a \cup (a \cup b)^* b(ab)^* \Rightarrow$ 

50) $L = \{1^i \# 1^j \# 1^{i+j}\}$

Ans: $\Sigma = \{1, \#\}$

$w = 1^{P/3} \# 1^{P/3} \# 1^{2P/3}$

$P \Rightarrow$ Pumping length

$x \rightarrow 1^{P/3} \# 1^{P/3} \#$

$yz \Rightarrow 1^{2P/3}$

$xy^2z = 1^{P/3} \# 1^{P/3} \# 1^k$ where $k > 2P/3$ because

let $|y| = q$, $|z| = \frac{2P}{3} - q$

Now, $|y^2| = 2q$, $|z| = \frac{2P}{3} - q$

$|y^2z| = q + \frac{2P}{3} > \frac{2P}{3}$ ($\because q > 0$)

$xy^2z \notin L$

$\therefore$ L is not regular.

(or)

$w = 1^P \# 1^P \# 1^{2P}$

$|xy| \le P$

So, y contains only 1's.

$xy^2z \notin L$ ~~i.e. last 1's~~

Last 1's = First 1's + Second 1's

Last 1's $= 2P$.

First 1's $> P$. $\Rightarrow$ So, First 1's + Second 1's $> 2P$

$\Rightarrow xy^2z \notin L$

$\therefore$ L is not regular

51) i) $L = \{w \mid w = w^R, w \in \{0,1\}^*\}$ $\Rightarrow$ Palindrome language

Ans: $\Sigma = \{0, 1\}^*$

$w = 10^{P^2}\cancel{10}$ P = pumping length

$w = 10^{P^2}1$ $P \in N$

$|xy^2z| = \cancel{l + 2(p^2+2-l)} = \cancel{l} + 2m + p^2 + 2 - \cancel{l} - m$

$= p^2 + 2 + m$

$< p^2 + 2 + P < (P+1)^2 + 2$

$|xy^2z| > p^2 + 2$

So, $xy^2z \notin L$

$\therefore$ L is not regular.

ii) $L = \{ww^R \mid w \in \{0,1\}^*\}$

Let s be a string in L

$s = 0^p 1 1 0^p$.

Let $x = 0^k$, $y = 0^l$ st. $k + l \le p$ & $l > 0$

$\Rightarrow z = 0^{p-k-l} \cdot 1 1 0^p$

$xz = 0^{p-l} 1 1 0^p \notin L$.

Which is contradiction

$\therefore L$ is not regular.

5 2) $L = \{0^m 1^n 0^{m+n} : m, n > 0\}$

Let w be a string in L st.

$w = 0^p 1^p 0^{2p}$ where p = Pumping length

Let $x = 0^k$ and $y = 0^l$ where

$k + l \le p$ and $l > 0$.

Then, $z = 0^{p-k-l} 1^p 0^{2p}$

Consider $xy^2z = 0^{k+2l} \cdot 0^{p-k-l} \cdot 1^p \cdot 0^{2p}$

$= 0^{p+l} \cdot 1^p 0^{2p} \notin L$ $(l > 0)$

$\therefore L$ is not regular.

5 3) $L = \{w \mid w \in \{0,1\}^*$ is not palindrome$\}$

Let s be a string in L

$s = 0^p 1^p$ where p = Pumping length

Let $x = 0^k$ and $y = 0^l$ and $k + l \le p$ & $l > 0$

$xy^2z = 0^{k+2l} \cdot 0^{p-k-l} 1^p$

$= 0^{p+l} \cdot 1^p$

53) $L = \{w \mid w$ is not a palindrome$\}$

Let us suppose that L is regular.

Then L^c should also be regular.

But $L^c = \{w \mid w$ is a palindrome$\}$ is not regular. {Proved in 51 (i)}

So, our assumption is wrong

$\therefore$ L is not regular.

54) i) $L = \{0^n \mid n$ is perfect square$\}$

Ans: Let w be a string in L st

$w = 0^{p^2}$ P = pumping length

$|x| = k$ $|y| = l$ $|z| = p^2 - k - l$

$|xy^2z| = \cancel{k} + 2l + p^2 - \cancel{k} - l$

$= p^2 + l > p^2 \quad (\because l > 0)$

$p^2 + l < p^2 + p < (p+1)^2$

So, $xy^2z \notin L$

$\therefore$ L is not regular.

ii) $L = \{0^n \mid n$ is power of 2$\}$

Let w be a string in L st.

$w = 0^{2^P}$ P = pumping length $|x| = k$, $|y| = l$

Consider $|xy^2z| = k + 2l + 2^P - k - l$

$= 2^P + l$

If $l < 2^P \Rightarrow xy^2z \notin L \Rightarrow L$ is not regular

If $l = 2^P \Rightarrow xy^2z \in L$

Now, consider xz $(|x| = 0, |z| = 0)$ So, $xz = \phi$

But $xz \notin L \Rightarrow L$ is not regular

$\therefore L$ is not regular.

55) i) $L = \{w0w \mid w \in \{0,1\}^*\}$

Let s be a string in L st.

$s = 0^p 1 0^p 1$

$|x| = l \quad |y| = k \quad |z| = 2p+2-k-l$

$x = 0^l$ st. $y = 0^k$ $l+k \leq p$ & $k > 0$

$xy^2z = 0^l . 0^{2k} . 0^{p-l-k} . 1 . 0^p . 1$

$= 0^{p+k} . 1 . 0^p . 1 \notin L$

$\therefore L$ is not regular.

ii) $L = \{w \mid$ No. of 0's in $w \neq$ No. of 1's in $w\}$

Consider $L' = \{w \mid$ No. of 0's in $w =$ No. of 1's in $w\}$

Let s be a string in L' st.

$s = 0^p 1^p$ where p = Pumping length

$xy^2z = 0^k . 0^{2l} . 0^{p-k-l} . 1^p$

$= 0^{p+l} . 1^p \notin L'$

$x = 0^k$ where $y = 0^l$ $k+l \leq p$ & $l > 0$

$\therefore L'$ is not regular.

Lets suppose L is regular, then L' should be regular

But L' is not regular

$\Rightarrow L$ is not regular.

56) i) $L = \{1^n \mid n$ is prime no.$\}$

Ans: Let w be a string in L st.

$w =$

ii) $L = \{0^i 1^j \mid i > j\}$

Let w be a string in L s.t.

$w = 0^{p+1} 1^p$ where p = Pumping length

Let $x = 0^k$, $y = 0^l$ where $k + l \leq p$ & $l \geq 1$

xy^iz for $i \geq 1$ $= 0^k . 0^{il} . 0^{p+1-k-l} . 1^p$

$= 0^{p+1+(i-1)l} . 1^p$

$i - 1 \geq 0$

$(i-1)l \geq 0$

$p + 1 + (i-1)l > p$

So, $xy^iz \in L$ for $i \geq 0$

Consider $i = 0 \Rightarrow xz = 0^k . 0^{p+1-k-l} . 1^p$.

$= 0^{p+1-l} . 1^p$

$l \geq 1$

$1 - l \leq 0$

$p + 1 - l \leq p$.

So, $xz \notin L$

$\therefore L$ is not regular