Basic Electronic Circuits (IEC-103)

Lecture-02

Frequency Response Analysis

Frequency Response

In AC steady state analysis the frequency is assumed constant (e.g., 50Hz). Here we consider the frequency as a variable and examine how the performance varies with the frequency.

Frequency Response

In AC steady state analysis the frequency is assumed constant (e.g., 50Hz). Here we consider the frequency as a variable and examine how the performance varies with the frequency.

Frequency response of a circuit is steady state response of the circuit when the excited by sinusoidal signals over a frequency range.

Frequency Response

In AC steady state analysis the frequency is assumed constant (e.g., 50Hz). Here we consider the frequency as a variable and examine how the performance varies with the frequency.

Frequency response of a circuit is steady state response of the circuit when the excited by sinusoidal signals over a frequency range.

Frequency response is the variation in a circuit's behavior with change in source signal frequency.

Applications

This is significant for applications involving filters.

Filters play critical roles in blocking or passing specific frequencies or ranges of frequencies.

Without them, it would be impossible to have multiple channels of data in radio communications.

Frequency Response

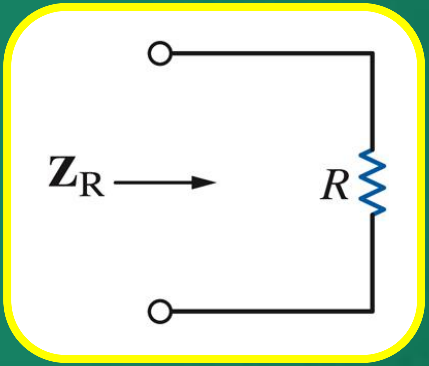
A circuit that is intended to amplify human speech must treat all the sinusoids with frequencies in the range 20 Hz to 20 KHz in the same way or the amplified signal will be distorted.

The amplifier output will be an undistorted copy of input voltage only if the gain of the circuit is constant over the entire frequency range and the phase shift is proportional to the input frequency.

Impedance Functions

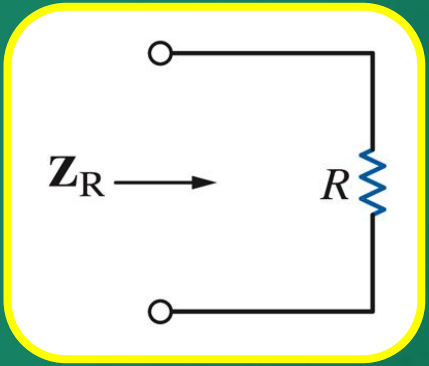
Impedance of a Resistor

Resistor



Impedance of a Resistor

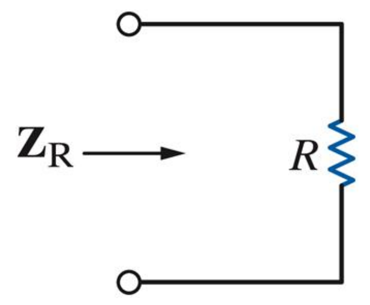
Resistor



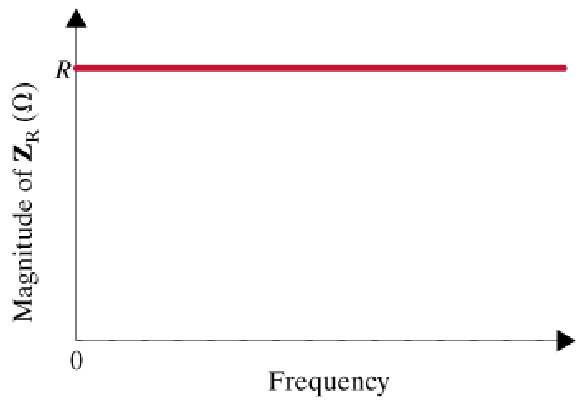
$$Z_R = R = R\angle 0^\circ$$

Impedance of a Resistor

Resistor

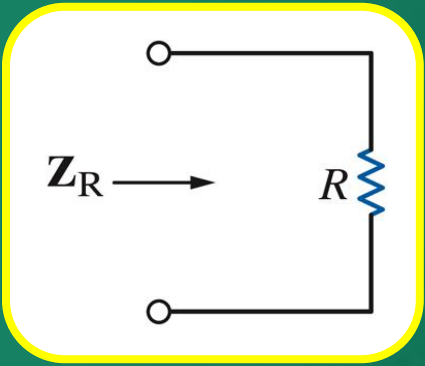


$$Z_R = R = R\angle 0°$$

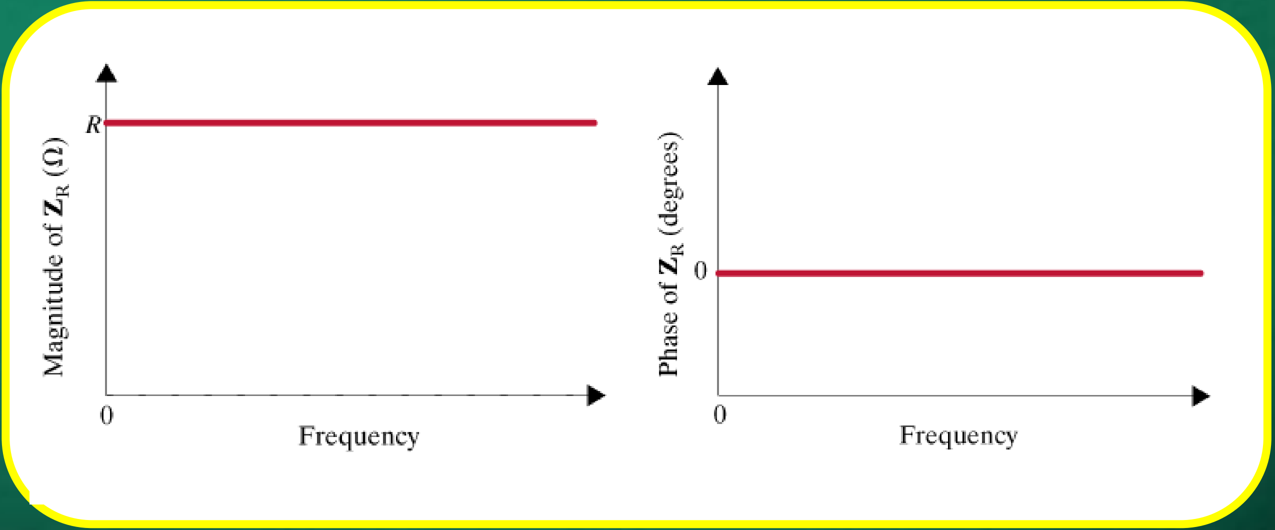


Impedance of a Resistor

Resistor

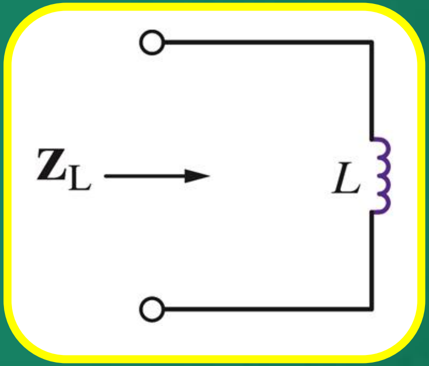


$$Z_R = R = R\angle 0^\circ$$



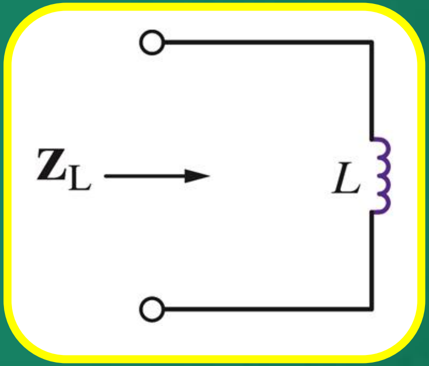
Impedance of an Inductor

Inductor



Impedance of an Inductor

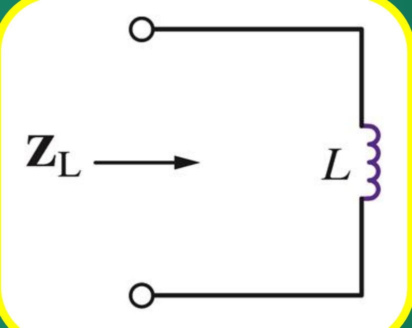
Inductor



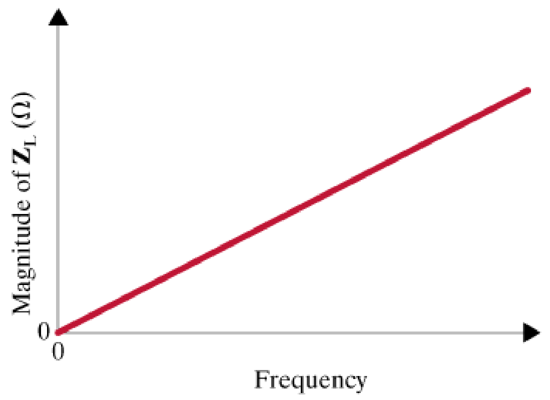
$$Z_L = j\omega L = \omega L \angle 90^\circ$$

Impedance of an Inductor

Inductor

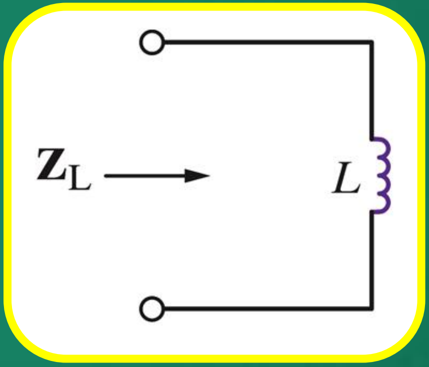


$$Z_L = j\omega L = \omega L\angle 90^\circ$$

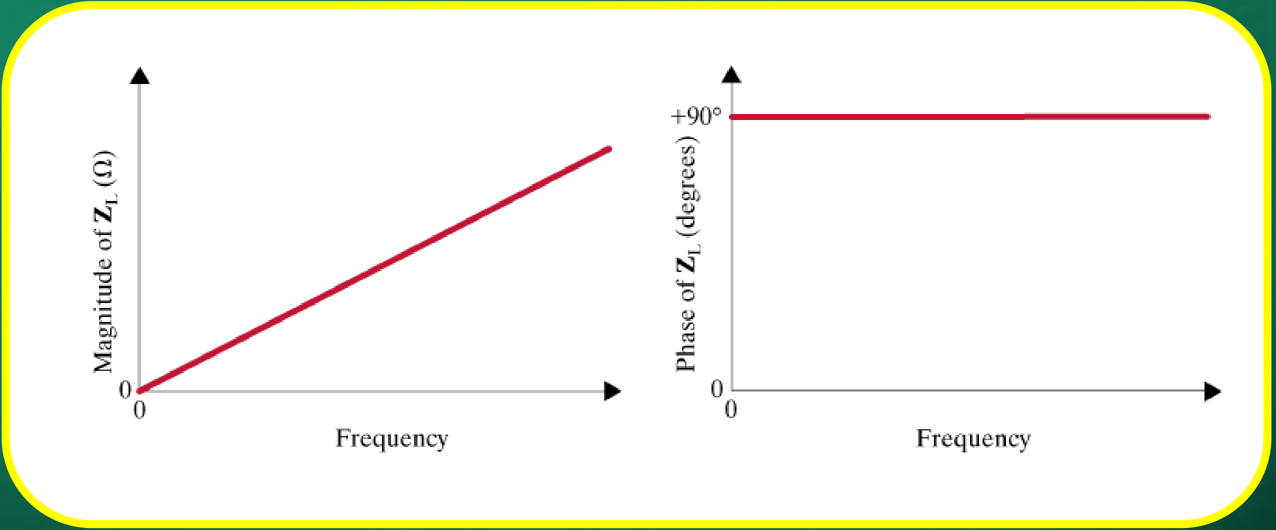


Impedance of an Inductor

Inductor

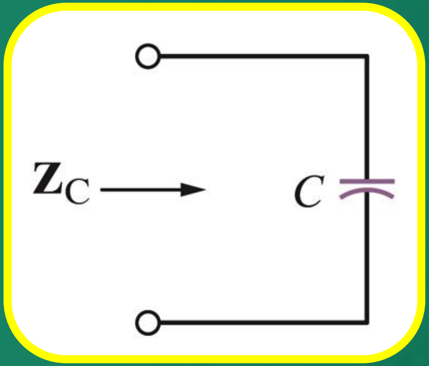


$$\boldsymbol{Z}_L = \boldsymbol{j}\omega \boldsymbol{L} = \omega \boldsymbol{L}\angle 90°$$



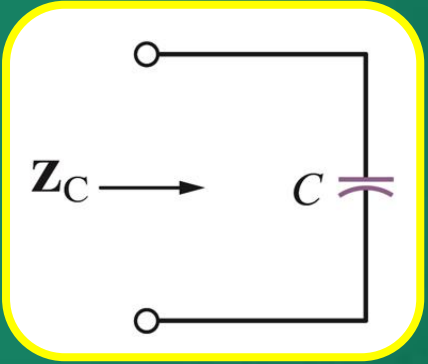
Impedance of a Capacitor

Capacitor



Impedance of a Capacitor

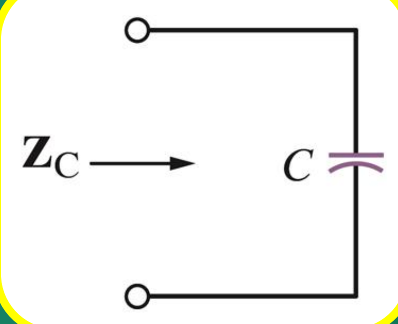
Capacitor



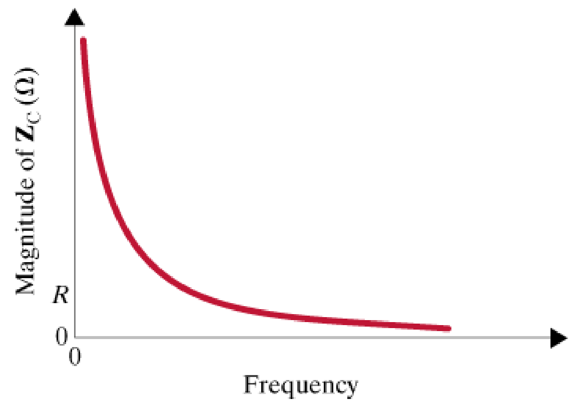
$$\boldsymbol{Z_c} = \frac{1}{\boldsymbol{j\omega C}} = \frac{1}{\boldsymbol{\omega C}} \angle -90^{\circ}$$

Impedance of a Capacitor

Capacitor

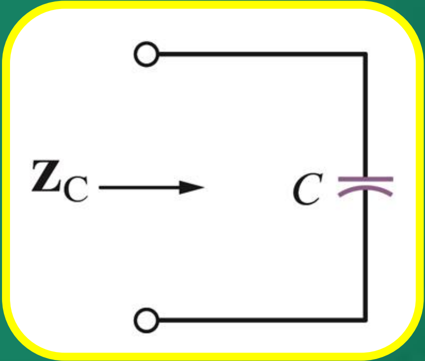


$$Z_c = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

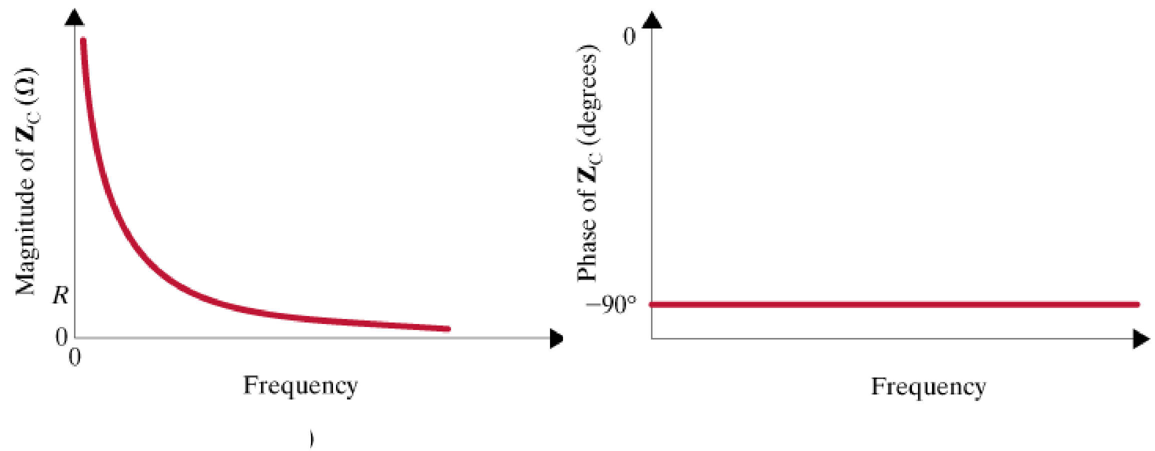


Impedance of a Capacitor

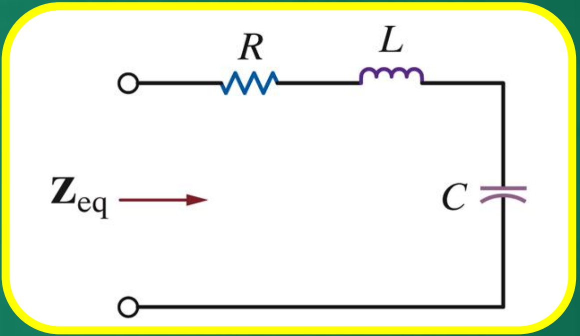
Capacitor



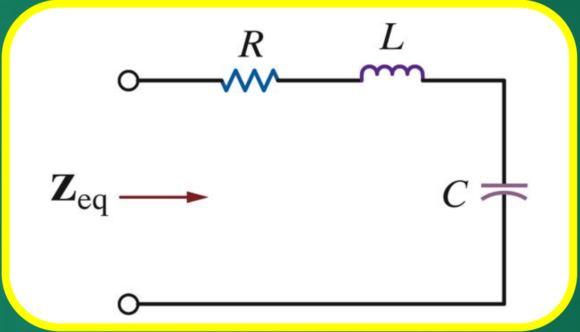
$$Z_c = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$



Series RLC Circuit

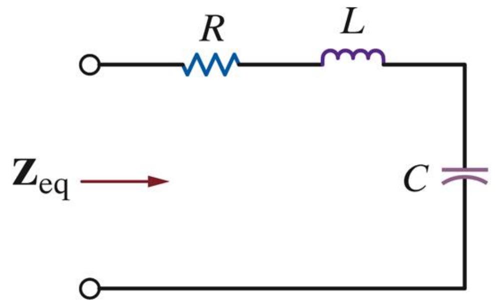


Series RLC Circuit



$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} = \frac{(j\omega)^2 LC + j\omega RC + 1}{j\omega C} \times \frac{-j}{-j} = \frac{\omega RC + j(\omega^2 LC - 1)}{\omega C}$$

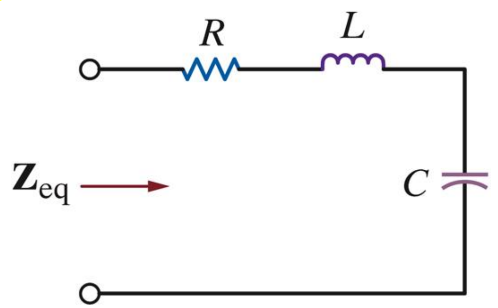
Series RLC Circuit



$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} = \frac{(j\omega)^2 LC + j\omega RC + 1}{j\omega C} \times \frac{-j}{-j} = \frac{\omega RC + j(\omega^2 LC - 1)}{\omega C}$$

$$|Z_{eq}| = \frac{\sqrt{(\omega RC)^2 + (1 - \omega^2 LC)^2}}{\omega C}$$

Series RLC Circuit

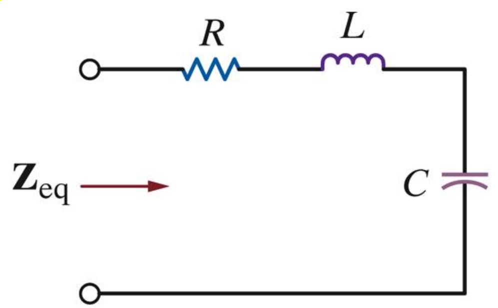


$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} = \frac{(j\omega)^2 LC + j\omega RC + 1}{j\omega C} \times \frac{-j}{-j} = \frac{\omega RC + j(\omega^2 LC - 1)}{\omega C}$$

$$|Z_{eq}| = \frac{\sqrt{(\omega RC)^2 + (1-\omega^2 LC)^2}}{\omega C}$$

$$\angle Z_{eq} = \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$

Series RLC Circuit



$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} = \frac{(j\omega)^2 LC + j\omega RC + 1}{j\omega C} \times \frac{-j}{-j} = \frac{\omega RC + j(\omega^2 LC - 1)}{\omega C}$$

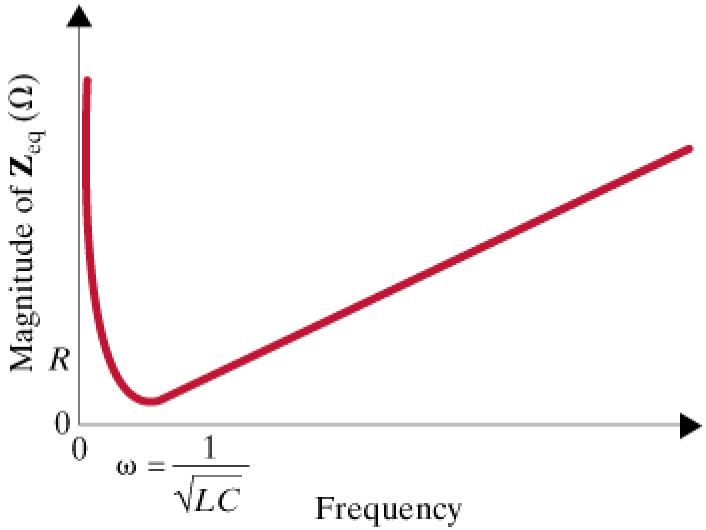
$$|Z_{eq}| = \frac{\sqrt{(\omega RC)^2 + (1-\omega^2 LC)^2}}{\omega C}$$

$$\angle Z_{eq} = \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$

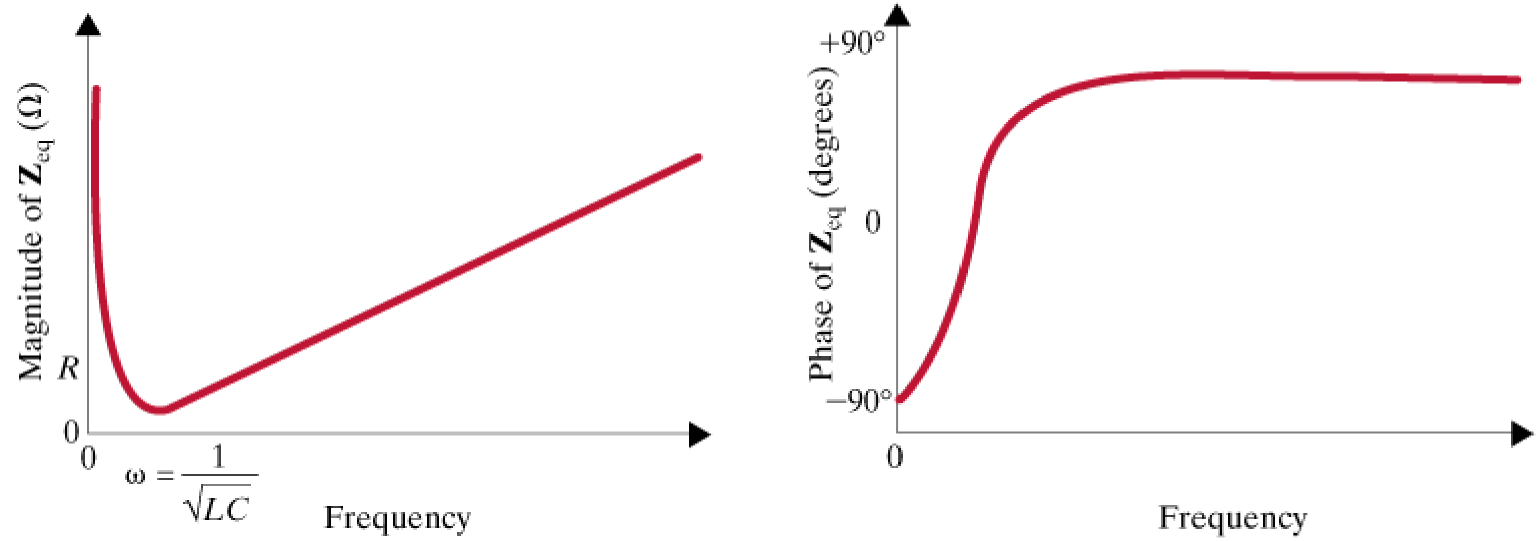
"Simplification in notation" $j\omega \approx s$

$$Z_{eq}(s) = \frac{s^2 LC + sRC + 1}{sC}$$

Series RLC Circuit



Series RLC Circuit



Impedance Function

Simplified notation for basic components

$$Z_R(s) = R,\ \ Z_L(s) = sL,\ \ Z_C = \frac{1}{sC}$$

Impedance Function

Simplified notation for basic components

$$Z_R(s) = R,\ \ Z_L(s) = sL,\ \ Z_C = \frac{1}{sC}$$

For all cases seen, and all cases to be studied, the impedance is of the form

$$Z(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

Impedance Function

Simplified notation for basic components

$$Z_R(s) = R,\ \ Z_L(s) = sL,\ \ Z_C = \frac{1}{sC}$$

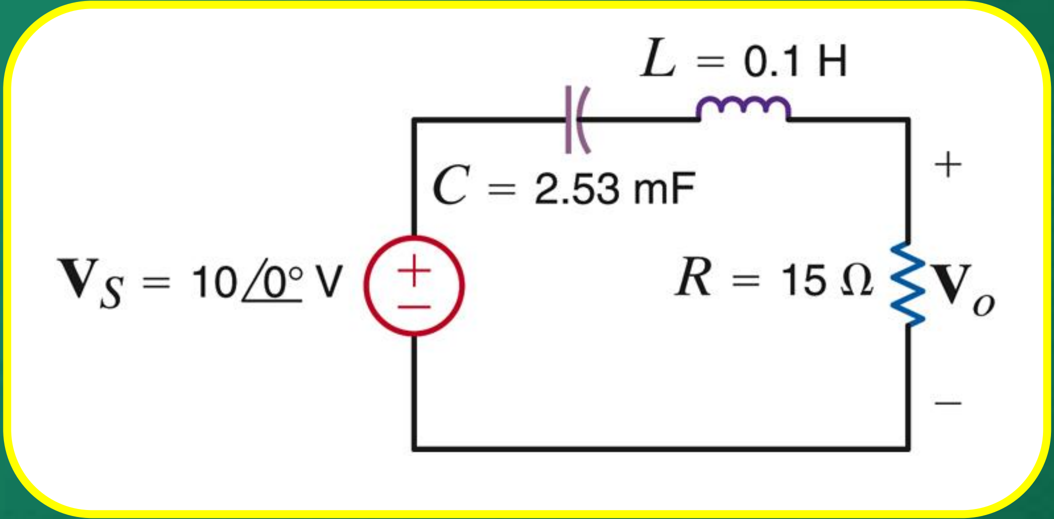
For all cases seen, and all cases to be studied, the impedance is of the form

$$Z(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

Moreover, if the circuit elements (L, R, C, dependent sources) are real then the expression for any voltage or current will also be a rational function in s.

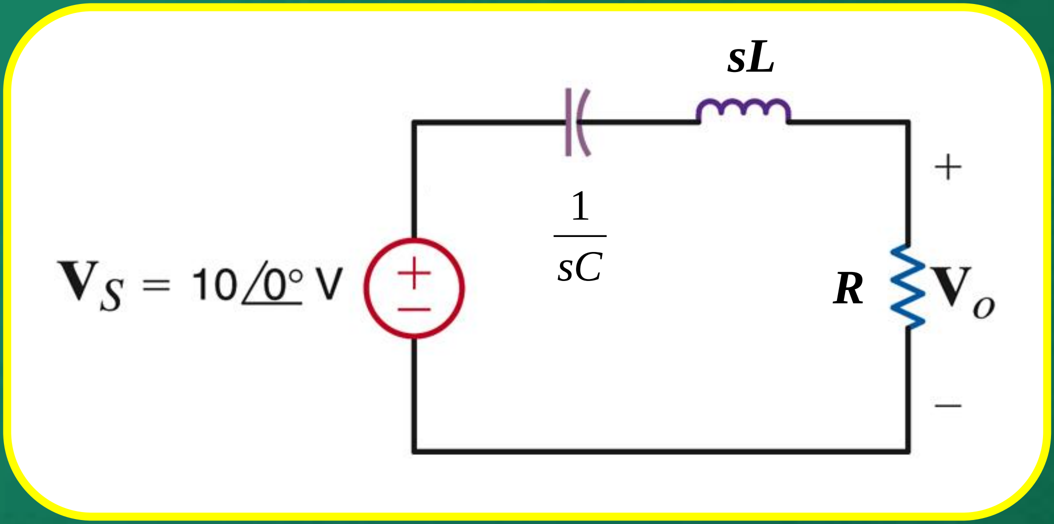
Example

Find the expression for magnitude and Phase of V_o as a function of frequency ω and plot the response as ω is varied from 0.1 Hz to 1 KHz.



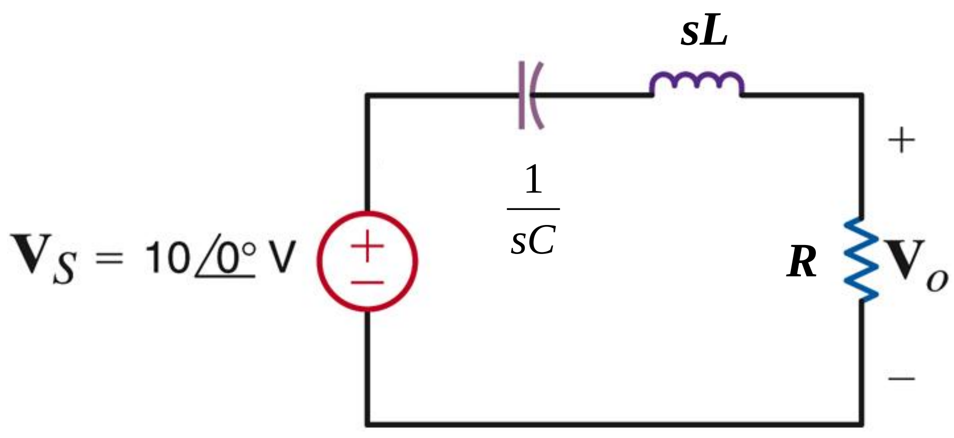
Example

Find the expression for magnitude and Phase of V_o as a function of frequency ω and plot the response as ω is varied from 0.1 Hz to 1 KHz.



Example

Find the expression for magnitude and Phase of V_o as a function of frequency ω and plot the response as ω is varied from 0.1 Hz to 1 KHz.



$$V_o(s) = \frac{R}{R + sL + 1/sC} V_S = \frac{sRC}{s^2LC + sRC + 1} V_S$$

Example

$$V_o(s) = \frac{R}{R + sL + 1/sC} V_S = \frac{sRC}{s^2 LC + sRC + 1} V_S$$

Example

$$V_o(s) = \frac{R}{R + sL + 1/sC} V_S = \frac{sRC}{s^2 LC + sRC + 1} V_S$$

$$s \approx j\omega$$

$$V_o = \frac{j\omega RC}{(j\omega)^2 LC + j\omega RC + 1} V_S$$

Example

$$V_o(s) = \frac{R}{R + sL + 1/sC} V_S = \frac{sRC}{s^2LC + sRC + 1} V_S$$

$$s \approx j\omega$$

$$V_o = \frac{j\omega RC}{(j\omega)^2 LC + j\omega RC + 1} V_S$$

$$V_o = \frac{j\omega(15 \times 2.53 \times 10^{-3})}{(j\omega)^2(0.1 \times 2.53 \times 10^{-3}) + j\omega(15 \times 2.53 \times 10^{-3}) + 1} 10\angle 0^\circ$$

Example

$$V_o(s) = \frac{R}{R + sL + 1/sC} V_S = \frac{sRC}{s^2LC + sRC + 1} V_S$$

$$s \approx j\omega$$

$$V_o = \frac{j\omega RC}{(j\omega)^2 LC + j\omega RC + 1} V_S$$

$$V_o = \frac{j\omega(15\times 2.53\times 10^{-3})}{(j\omega)^2(0.1\times 2.53\times 10^{-3}) + j\omega(15\times 2.53\times 10^{-3}) + 1} 10\angle 0^\circ$$

$$V_o = \frac{0.3795\omega j}{(1 - 2.53\times 10^{-4})\omega^2 + 0.03795\omega j}$$

Example

$$\left|V_o\right| = \frac{0.3795\omega}{\sqrt{\left((1-2.53\times10^{-4})\omega^2\right)^2 + \left(0.03795\omega\right)^2}}$$

Example

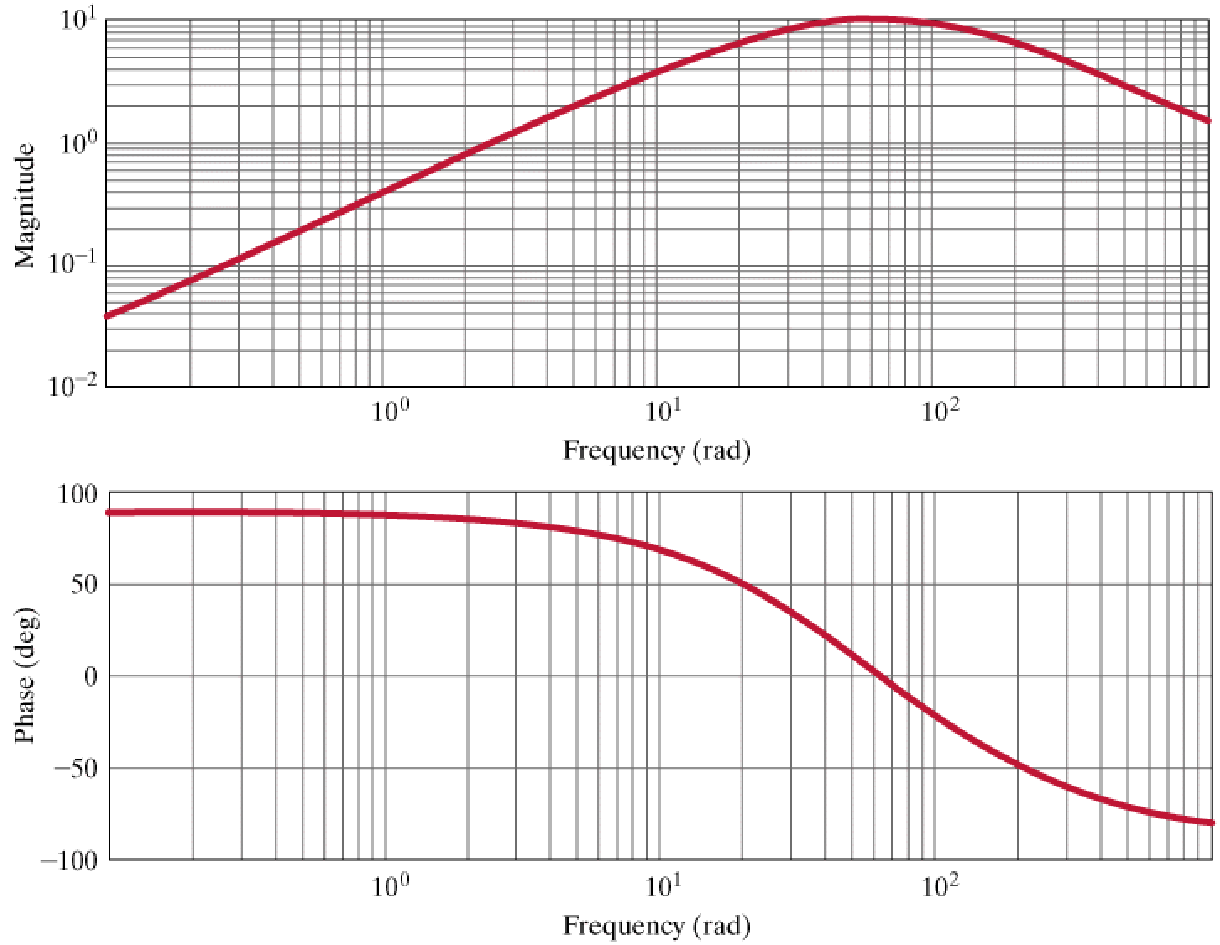
$$|V_o| = \frac{0.3795\omega}{\sqrt{\left((1-2.53\times10^{-4})\omega^2\right)^2 + (0.03795\omega)^2}}$$

$$\angle V_o = \frac{\pi}{2} - \tan^{-1}\left(\frac{0.03795\omega}{(1-2.53\times10^{-4})\omega^2}\right)$$

Frequency Response

Log-log plot

Semi-log plot



(b)

Frequency in log scale

Use of logarithms expands the range of frequencies portrayed on the horizontal axis.

Gain in bels

In communication systems, gain is measured in bels. The bel is used to measure the ration of two levels of power or power gain G, that is

$$G = \text{Number of bels} = \log_{10} \frac{P_2}{P_1}$$

Gain in bels

In communication systems, gain is measured in bels. The bel is used to measure the ration of two levels of power or power gain G, that is

$$G = \text{Number of bels} = \log_{10} \frac{P_2}{P_1}$$

The ear, responds in a logarithmic fashion to changes in audio power levels.

Bels & Decibels

The decibel (dB) provides us with a unit of less magnitude. It is 1/10th of a bel and is given by

$$G_{\text{db}} = 10 \log_{10} \frac{P_2}{P_1}$$

Bels & Decibels

The decibel (dB) provides us with a unit of less magnitude. It is 1/10th of a bel and is given by

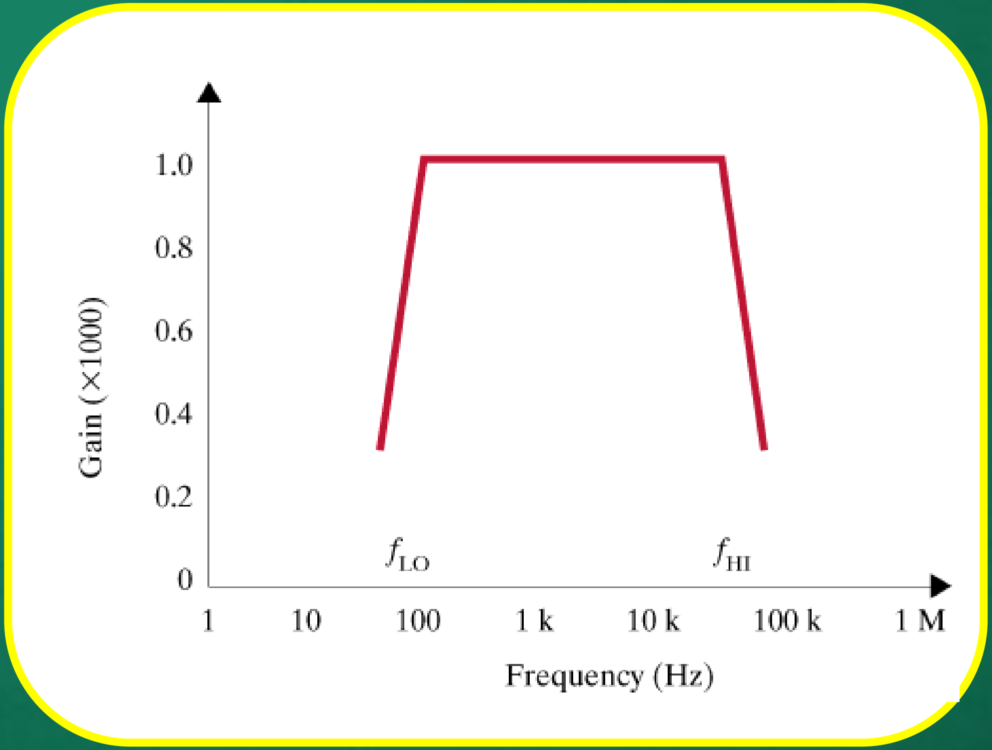
$$G_{db} = 10\log_{10}\frac{P_2}{P_1}$$

For comparing voltages or current levels, we use

$$G_{db} = 20\log_{10}\frac{V_2}{V_1} \text{ or } G_{db} = 20\log_{10}\frac{I_2}{I_1}$$

Specs of an Amplifier

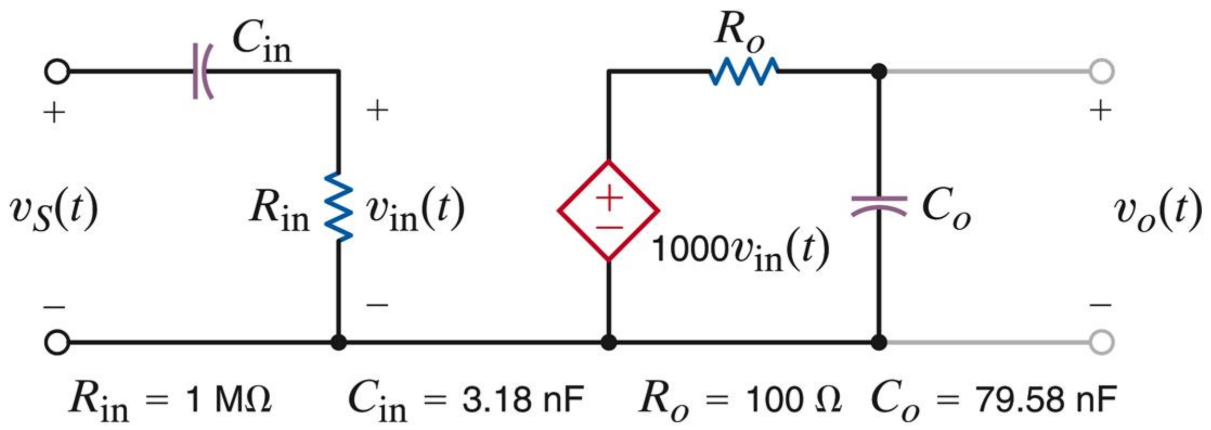
Desired frequency characteristic (flat between 50 Hz and 15 KHz)



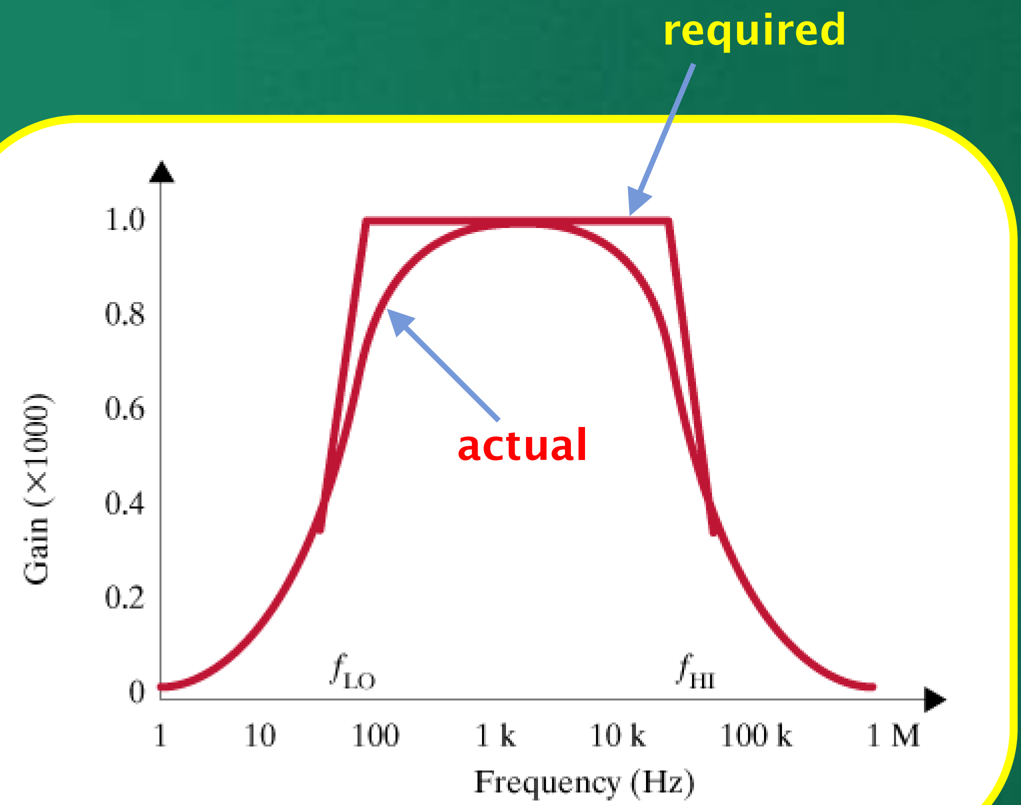
Log scale (frequency)

Synthesis of an Amplifier

Postulated amplifier



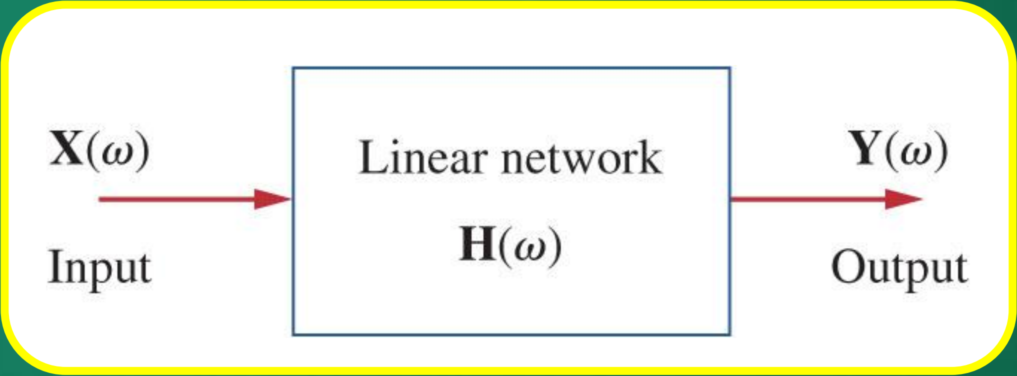
Specs and actual response



Transfer Function

Definition

One useful way to analyze the frequency response of a circuit is the concept of the transfer function $H(\omega)$.



$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$H(\omega)$ is the frequency dependent ratio of a forced function or response $Y(\omega)$ to the forcing function $X(\omega)$.

Terminology

One useful way to analyze the frequency response of a circuit is the concept of the transfer function $H(\omega)$.

$$H(\omega) = \text{Voltage gain} = \frac{V_o(\omega)}{V_i(\omega)}$$

$$H(\omega) = \text{Current gain} = \frac{I_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Impedance} = \frac{V_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Admittance} = \frac{I_o(\omega)}{V_i(\omega)}$$

Zeros & Poles

To obtain $H(\omega)$, first convert to frequency domain equivalent components in the circuit.

$H(\omega)$ can be expressed as the ratio of numerator $N(\omega)$ and denominator $D(\omega)$ polynomials.

$$H(\omega) = \frac{N(\omega)}{D(\omega)}$$

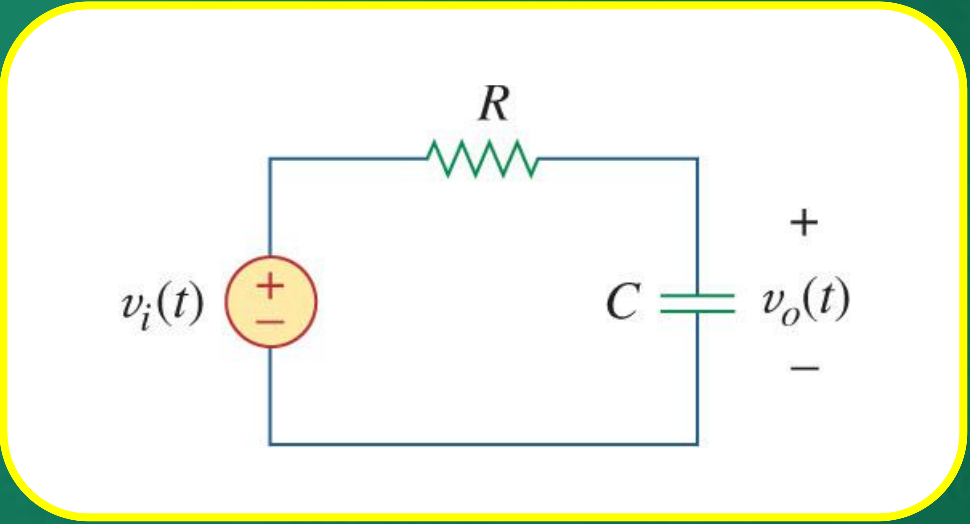
Zeros are where the transfer function goes to zero.

Poles are where it goes to infinity.

They can be related to the roots of $N(\omega)$ and $D(\omega)$.

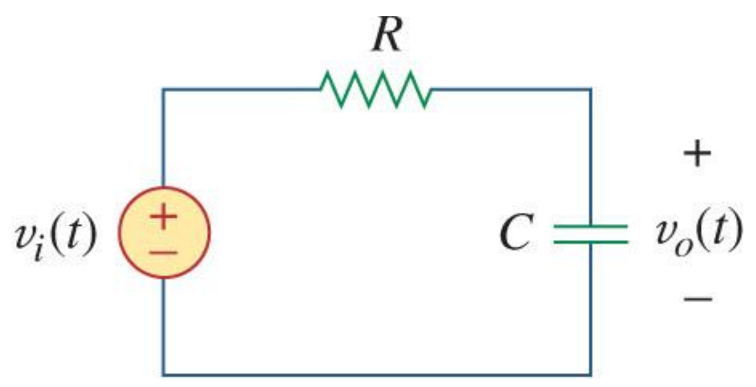
Example

Obtain the transfer function V_o/V_s of RC network shown below and plot the frequency response



Example

Obtain the transfer function V_o/V_s of RC network shown below and plot the frequency response



Answer:

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Example

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Example

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Example

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle H(\omega) = -\tan^{-1}(\omega RC)$$

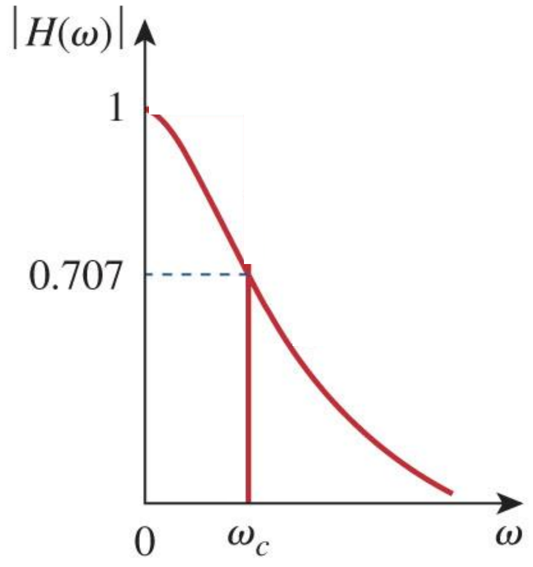
Example

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle H(\omega) = -\tan^{-1}(\omega RC)$$

Frequency Response



Bode Plots

Bode Plots

One problem with the transfer function is that it needs to cover a large range in frequency.

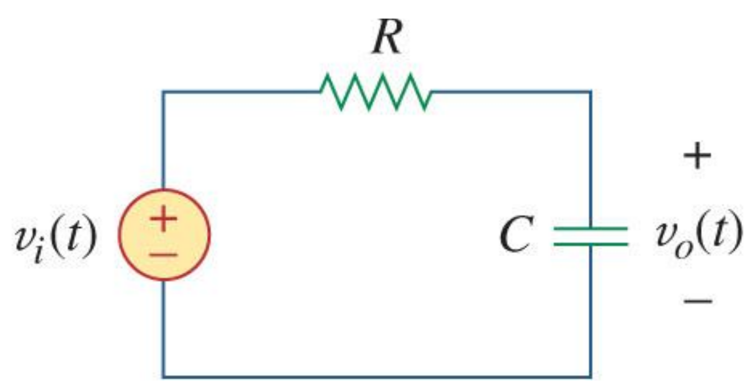
Plotting the frequency response on a semi log plot (where the X axis is plotted in log form) makes the task easier.

These plots are referred to as Bode plots.

Bode plots either show magnitude (in decibels) or phase (in degrees) as a function of frequency.

Example

Draw the Bode plot of V_o/V_s of the RC network shown below .

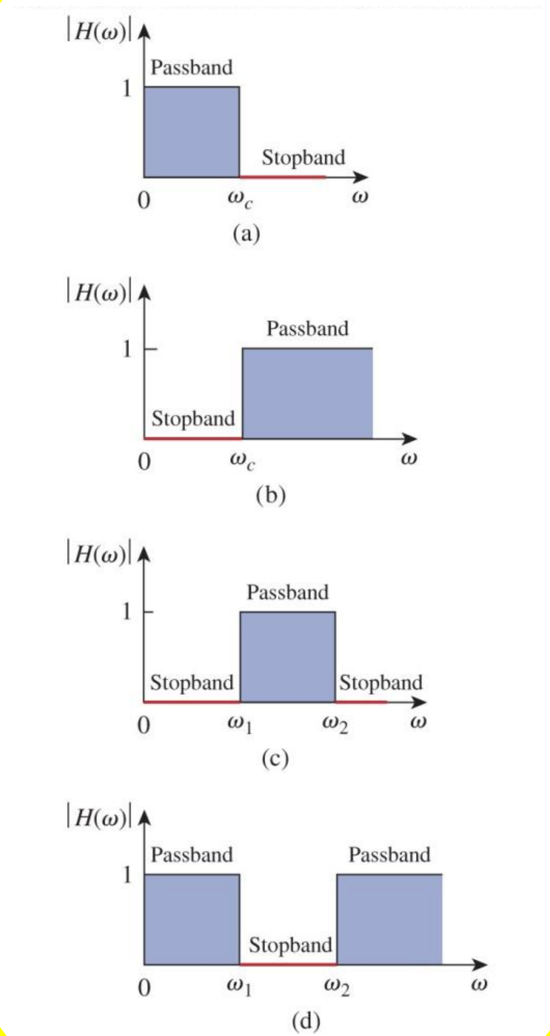


Passive Filters

Passive Filters

There are four types of filters:

- **Lowpass** passes only low frequencies and blocks high frequencies.
- **Highpass** does the opposite of lowpass
- **Bandpass** only allows a range of frequency to pass through
- **Bandstop** does the opposite of bandpass

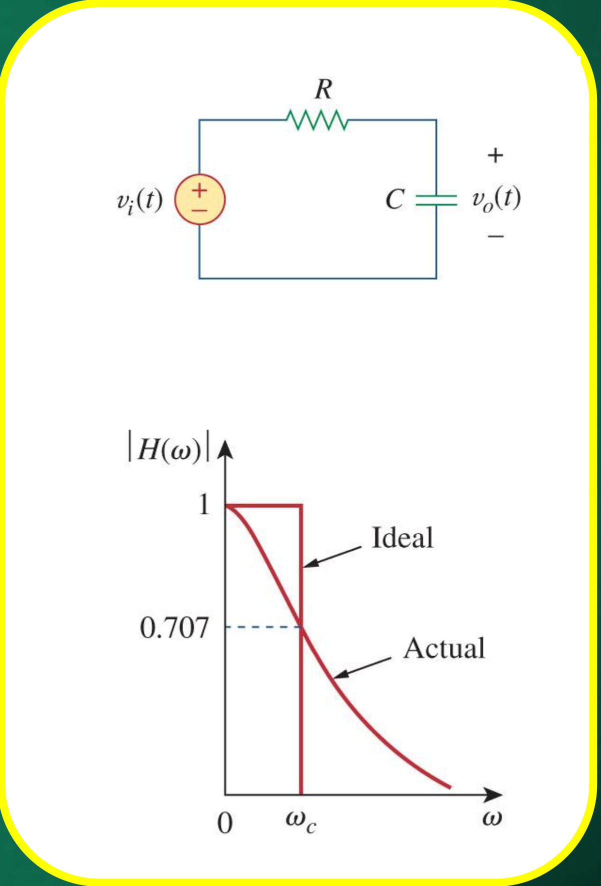


Lowpass Filter

- A typical lowpass filter is formed when the output of a RC circuit is taken off the capacitor.
- The half power frequency is:

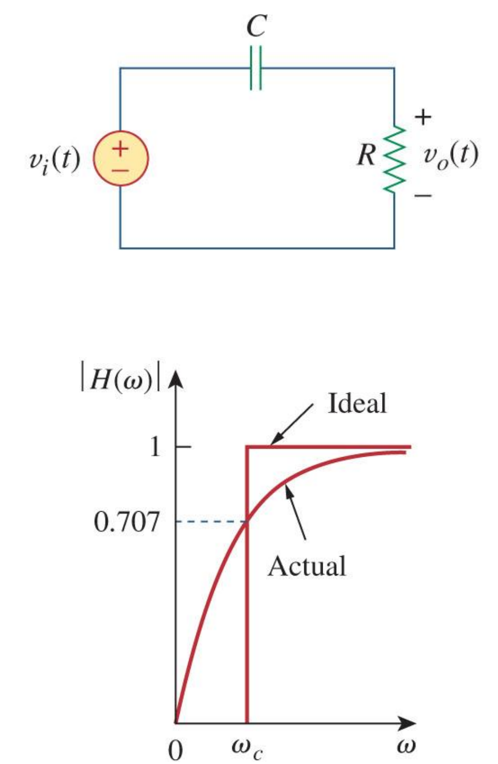
$$\boxed{\omega_c = \frac{1}{RC}}$$

- This is also referred to as the **cutoff** frequency.
- The filter is designed to pass from DC up to ω_c



Lowpass Filter

- A highpass filter is also made of a RC circuit, with the output taken off the resistor.
- The cutoff frequency will be the same as the lowpass filter.
- The difference being that the frequencies passed go from ω_c to infinity.

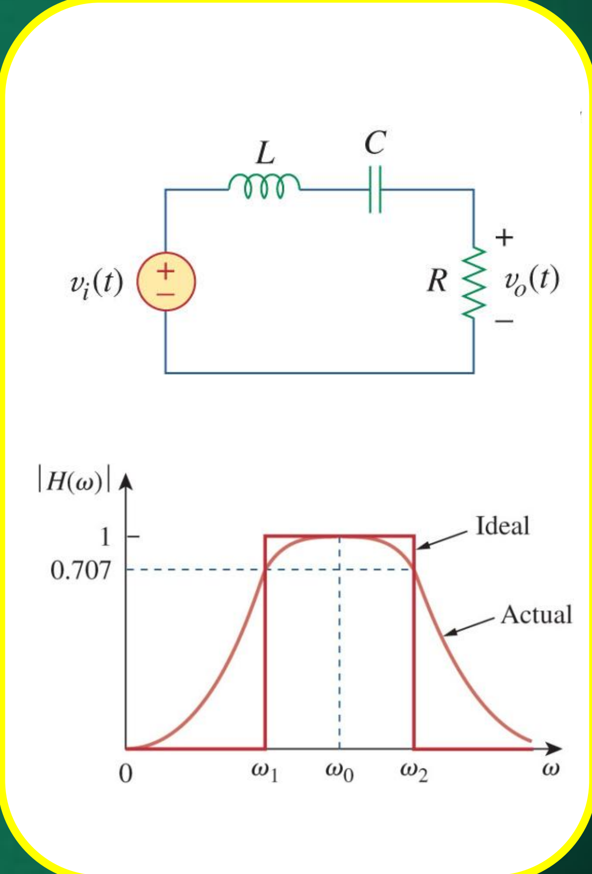


Bandpass Filter

- The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor.
- The center frequency is:

$$\omega_c = \frac{1}{\sqrt{LC}}$$

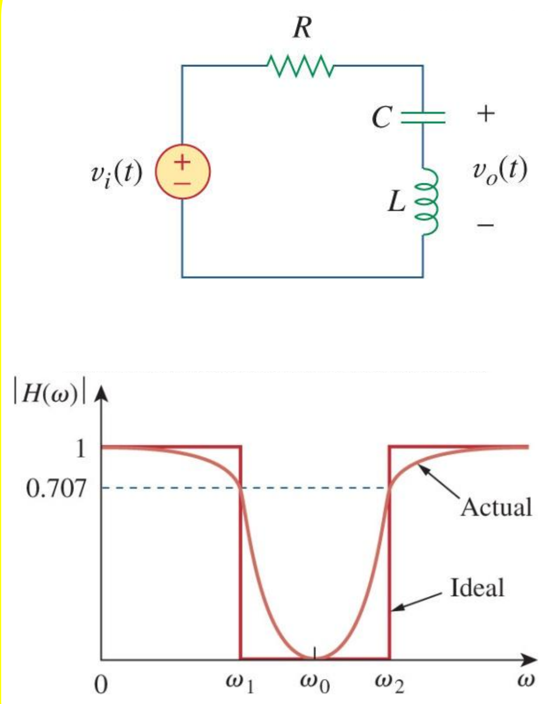
- The filter will pass frequencies from ω_1 to ω_2.
- It can also be made by feeding the output from a lowpass to a highpass filter.



Bandstop Filter

- A bandstop filter can be created from a RLC circuit by taking the output from the LC series combination.

- The range of frequencies (from ω_1 and ω_2) are blocked.



Active Filters

- Passive filters have a few drawbacks.
  - They cannot create gain greater than 1.
  - They do not work well for frequencies below the audio range.
  - They require inductors, which tend to be bulky and more expensive than other components.
- It is possible, using op-amps, to create all the common filters (active filters).
- Their ability to isolate input and output also makes them very desirable.