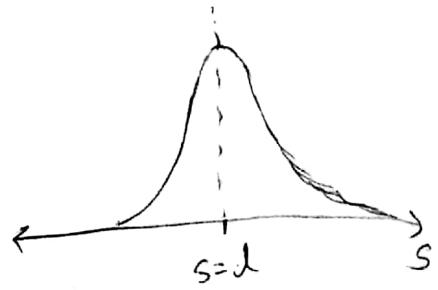
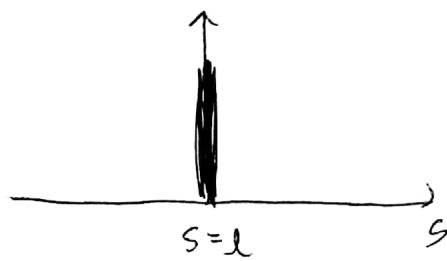


5) a)



N steps

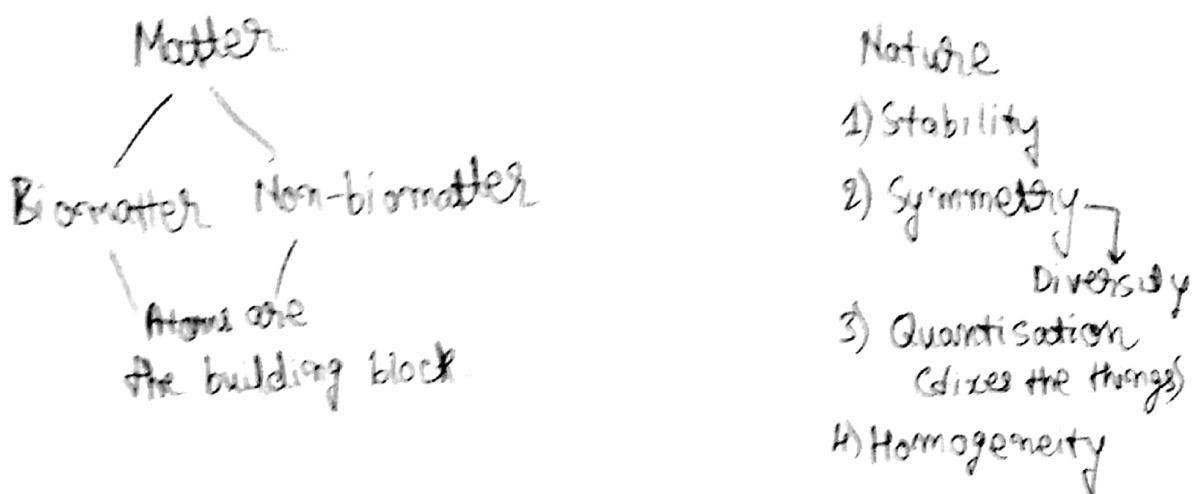
$$x = s_1 + s_2 + \dots + s_N$$
$$\langle x \rangle = \sum_{i=1}^N \langle s_i \rangle$$

$$s_i = s_j \quad \forall i, j$$

$$\Rightarrow \langle x \rangle = N \langle s \rangle \longrightarrow \int_{-\infty}^{\infty} s w(s) ds$$
$$= \underline{Nl}$$

$$s e^{-\frac{(s-1)^2}{2\sigma^2}}$$

Chemistry & Biology



Temperature, Volume can be measured for bulk.
These are known as macroscopic properties.

Microscopic → can't see/measure directly
e.g. size of atom, bond strength b/w atoms

Model → simplest representation of complex phenomena

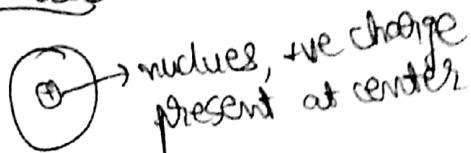
```
graph TD; Model --> Hypothesis; Model --> Theory; Hypothesis --> Theory; Theory --> Law
```

Hypothesis → Theory → Law

Hypothesis is made. If it follows nature,
it becomes theory

- Plum pudding model
- He said positive & negative charge is uniformly distributed.
- Resultant charge is zero.

Rutherford model



Drawbacks:



electrons were revolving around the nucleus.

But electromagnetic theory proved it wrong. No stability.

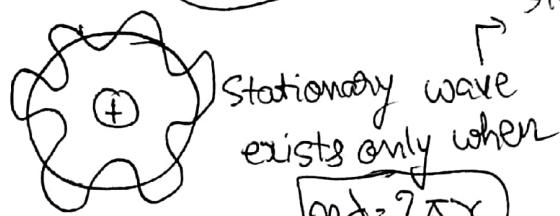
Bohr model:

Some fixed orbitals are allowed for an e⁻.

$$mv\tau = n \left(\frac{h}{2\pi}\right)$$

1, 2, 3, ...

Stationary orbitals
no loss of energy.



$$n\lambda = 2\pi r$$

$$\text{De Broglie} \rightarrow \lambda = \frac{h}{mv}$$

Hence stability & quantization
both are related.

$$\rightarrow \frac{nh}{mv} = 2\pi r$$

Explained 's' spectral lines for hydrogen.

$$\text{wave no. } \bar{v} = R \left[\frac{1}{m_1^2} - \frac{1}{m_2^2} \right]$$

↓ ↓
Rydberg's const.

Lyman	UV
Balmer	Visible
Pachmen	IR
Brackett	IR
Pfund	IR

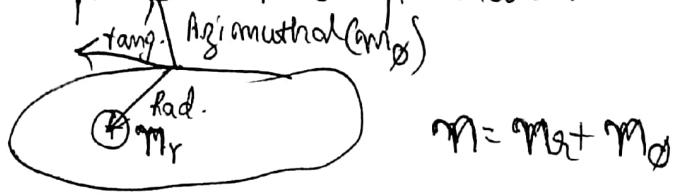
Disadvantages:

when spectral lines placed in magnetic & electric field, splitting of spectral lines couldn't be explained.

Sommerfeld Model:

e^- 'may' have elliptical orbitals.

to prove fine spectral lines.



	m_r	m_ϕ		
$n=1$	1	0	X	can't go through nucleus
	0	1	✓	spherical shape
2	2	0	X	
	1	1	✓	elliptical (s, p orbital)
	0	2	✓	spherical (s orbital)

Couldn't explain fine spectral lines.

Relativistic correction

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

using Bohr, calculate velocity of e^- ($\approx 10^6$)
 & calculated mass of e^- using $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

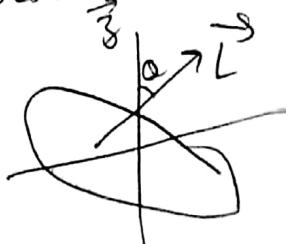
Vector Atom Model

Explained fine spectral lines

In Bohr theory, 1 degree of freedom

In Sommerfeld, 2 "

Quantizing dirⁿ of orbital



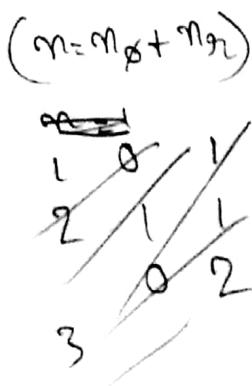
α specifies orientation
of orbital

$$L_z = L \cos \alpha \quad \cos \alpha = \frac{L_z}{L}$$

$$mv\Omega = L = \frac{nh}{2\pi}$$

Principal quantum no.
 $L = \frac{\lambda h}{2\pi}$

$m=1$	$l=0$
2	0, 1
3	0, 1, 2



$$(m = m_x + m_y)$$

n	m_x	m_y
1	0	1
2	1	1
	0	2
3	0	
	1	
	2	

$$\cos\theta = \frac{L_z + m_l h}{L} = \frac{m_l h}{\sqrt{3(3k)} h} = \frac{m_l}{\sqrt{3(3k)}}$$

$$m_l = (\cos\theta) \sqrt{3k} \approx 1$$

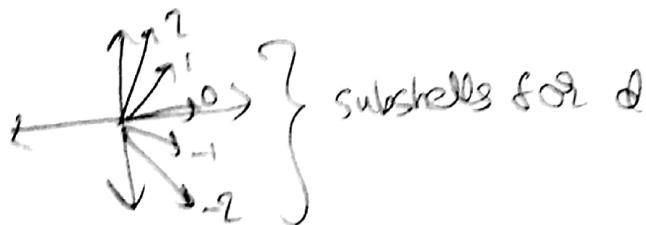
$$= [-1, 1] \text{ A}$$

$$= [1, 1]$$

$L_z = \sqrt{3(k+1)}$
 $L_z = m_l \sqrt{\frac{h}{2\pi}}$
 magnetic \rightarrow h
 quantum no

for $l=2$

$$\cos\theta = \frac{m_l}{\sqrt{3(k+1)}} = \frac{-2, -1, 0, +1, 2}{\sqrt{6}} = \pm \frac{\sqrt{2}}{\sqrt{3}}, \pm \frac{\sqrt{1}}{\sqrt{6}}, 0$$



- Using Bohr theory \rightarrow find freq. all spectral lines
- Allowed/Possible orientations of 's' orbital using vector atom model, identify (θ values).

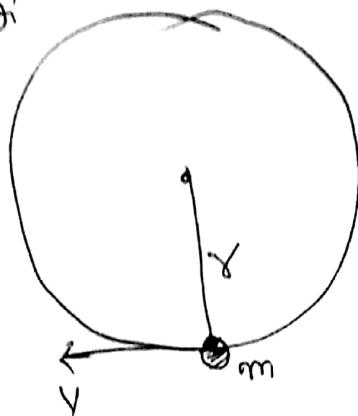
- 1) Sommerfeld model
- 2) Vector atom model

\rightarrow spatial quantization

- 3) Bohr's correspondence model

1)

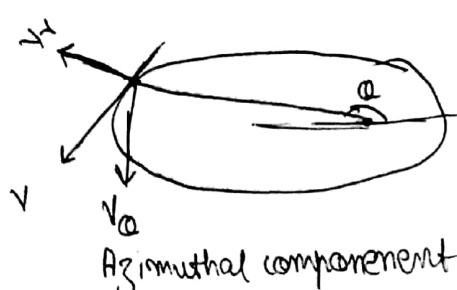
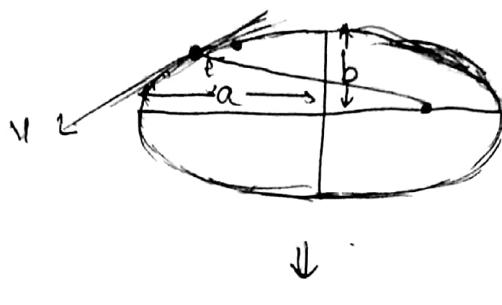
Bohr'



$$mv\vartheta = \frac{nh}{2\pi}$$

only certain orbits are allowed depending on n.

Sommerf'



Azimuthal component

$$p_r = mv\vartheta$$

$p_\phi = mv\varphi$ azimuthal moment of component

$$= m\vartheta^2 r$$

$$\dot{\varphi} = \frac{d\varphi}{dt} = \omega$$

$$v = \vartheta \times w$$

$$L = \vartheta \times (mv)$$

if spherical, $L = \vartheta mr$.

$$\oint p_r dr = rh \quad n=1, 2, 3, \dots$$

$$\oint p_\theta d\theta = mh \quad m=1, 2, 3, \dots$$

↓
constant by Kepler law

$$p_\theta(2\pi) = mh$$

$$p_\theta = \frac{mh}{2\pi}$$

$$m=1, \quad p_\theta = \frac{h}{2\pi}$$

$$m=2, \quad p_\theta = \frac{2h}{2\pi}$$

$$m=3, \quad p_\theta = \frac{3h}{2\pi}$$

$$\frac{m}{r+m} = \frac{b}{a} \quad r+m=n$$

$$\frac{m}{n} = \frac{b}{a}$$

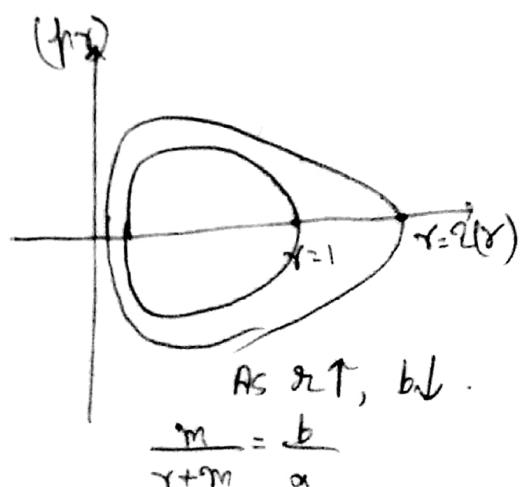
$$\text{if } m=0, \quad p_\theta = 0 \text{ & } b=0$$

only radial part, hence electron will collide with nucleus.

$\Rightarrow m \neq 0$.

$$m=1, 2, 3$$

	m	r
$m=1$	1	0
$m=2$	2, 1	1, 0



$$a = a_1 \frac{m^2}{Z}$$

↓
bohr radius

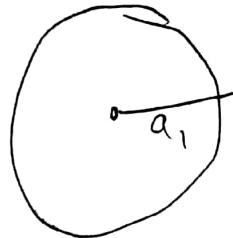
$$b = a_1 \frac{m m}{Z}$$

$Z = 1$

$$n=1, m=1, r=0$$

$$a = a_1$$

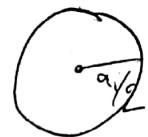
$$b = a_1$$



$$Z=2$$

$$n=1, m=1, r=0$$

$$a = \frac{a_1}{2}, b = \frac{a_1}{2}$$



$$\text{As } Z \uparrow, r \downarrow \propto \frac{1}{Z}$$

$$Z=1, m=2, m=2, 1 \quad r=0, 1$$

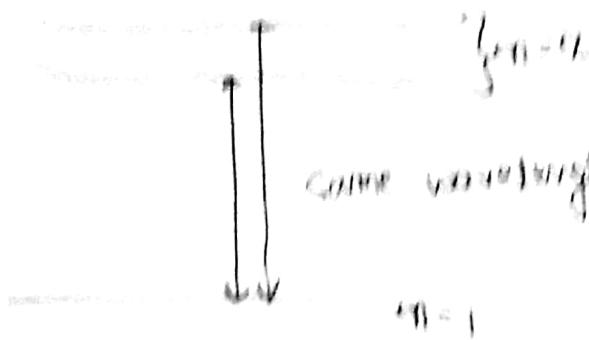
$$r=0, a=k a_1 \quad b=k a_1$$

$$m=9, m=1$$

$$r=1 \quad a=k a_1, b=9 a_1$$

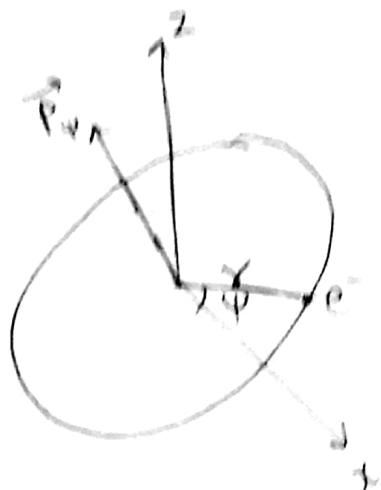
$r \uparrow, b \downarrow$		
$m=3$	$m=2$	$m=1$
$r=0$	a_1	$9 a_1$
$9 a_1$	$9 a_1$	$9 a_1$
$9 a_1$	$6 a_1$	$3 a_1$
d	μ	s

Spatial quantization



Same quantity is measured, first spatial mode

$$m_l = 1$$



$$\vec{p}_r + \vec{p}_\theta$$

In spherical polar co-ordinates
it is p_r, p_θ, p_ϕ

$$\oint p_r dr = rh$$

$$\oint p_\theta d\theta = kh$$

$$+ \oint p_\phi d\phi = mh$$

$$+ \oint p_\phi d\phi = mh$$

$$+ \oint p_\phi d\phi = mh$$

~~$$\cancel{\oint} K + kh = \cancel{\oint} rh + mh$$~~

$$\Rightarrow K = t + mh$$

$$\cos \alpha = \frac{p_\phi}{p_\psi} = \frac{m}{t + mh} = \frac{m}{K}$$

Heat Equation

$u(x,t)$ Temperature



$$\frac{\partial u}{\partial t} = -\alpha \frac{\partial^2 u}{\partial x^2}$$

Fourier's law of heat transfer

$$\underline{\text{Rate of heat transfer}} = -k_0 \frac{\partial u}{\partial t}$$

$$\frac{\Delta Q}{\Delta t} = \text{Rate of heat transfer} = -A \times k_0 \times \frac{\partial u}{\partial t}$$

Heat transfer = $\Delta Q = \left[\Delta t \left(\frac{\partial u}{\partial t} \right)_{x+\Delta x} - \Delta t \left(\frac{\partial u}{\partial t} \right)_x \right] k_0 A$

$$\cancel{\Delta t}(k_0) \left(\frac{\partial u}{\partial t} \right)_{x+\Delta x} - \left(\frac{\partial u}{\partial t} \right)_x = \frac{\cancel{\Delta t} C_p (u(x,t+\Delta t) - u(x,t))}{\Delta t}$$

$$\Rightarrow \frac{\partial u}{\partial t} = -\alpha \frac{\partial^2 u}{\partial x^2}$$

Conservation of energy

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = \Delta U$$

In a isochoric process (isolated system)

$$U = m C_p \times U \xrightarrow{\text{Temp}} (\Delta U = m C_p \Delta T) \xrightarrow{\text{Temp}}$$

$$\Delta Q = (P \cdot A \Delta x) C_p (U(x, t + \Delta t) - U(x, t))$$

$$u = F(x) \cdot G(t) \equiv F \cdot G$$

$$\frac{\partial u}{\partial t} = -C^2 \frac{\partial^2 u}{\partial x^2}$$

$$F \dot{G} = -C^2 \ddot{F} G$$

$$\frac{F}{\dot{F}} = \frac{\dot{G}}{-C^2 G} = R$$

$$\dot{G} + R^2 c G = 0$$

$$\dot{F} - RF = 0$$

$$R = 0$$

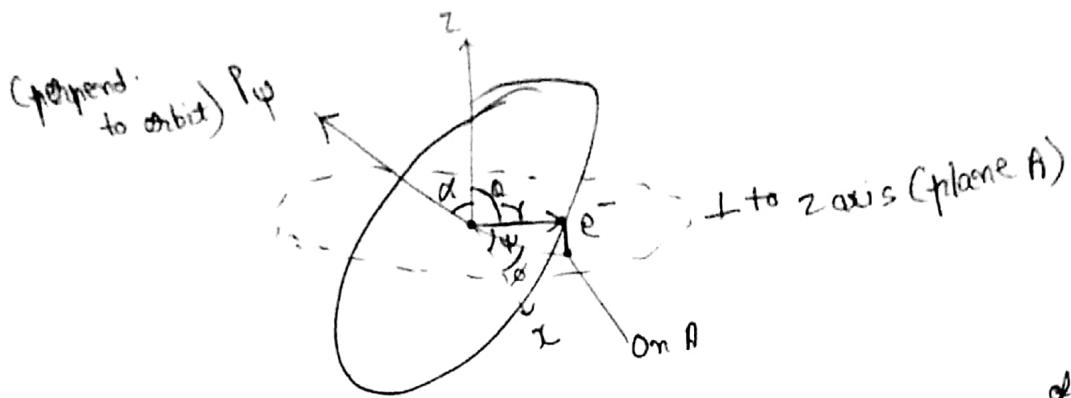
$$R < 0, R = f^2$$

$$F = ax + b$$

$$F = A \cos px + B \sin px$$

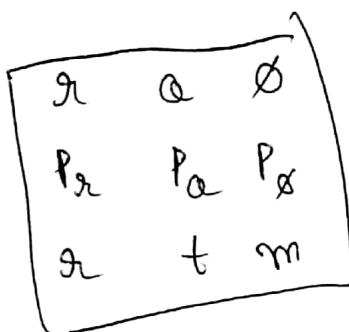
$$R > 0, R = u^2$$

$$F = A e^{ut} + B e^{-ut}$$



ϕ b/w x axis and ft. of impression on A

α b/w r & z axis

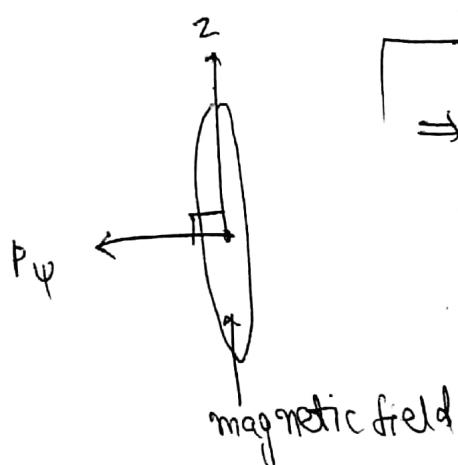


$$\cos \alpha = \frac{P_\phi}{P_\psi}$$

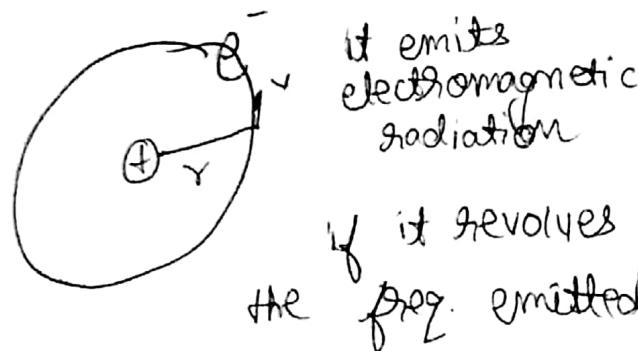
$$= \frac{m}{(t+m)} \Rightarrow k$$

if $k=1$, $k=2$,
 $m=\pm 1$ $m=\pm 1, \pm 2$
 $\alpha=0, \pi$ $\alpha=0, \frac{\pi}{3}, \frac{2\pi}{3}, \dots, 2\pi$

m can't be 0
 $\Rightarrow \alpha=\pi/2$ which means magnetic field will destabilize the electron orbit



Classical electromagnetic theory



$$v = \frac{2\pi r}{T}$$

$$\frac{1}{T} = \boxed{\frac{v}{2\pi r} = f}$$

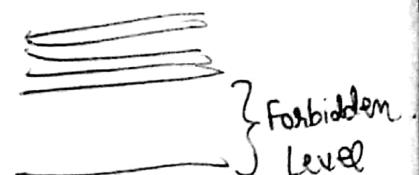
$$\rightarrow \frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$mv^2 = \frac{nh}{2\pi}$$

$$\epsilon_{em} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = h\nu$$

$$\nu = \frac{R}{h} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

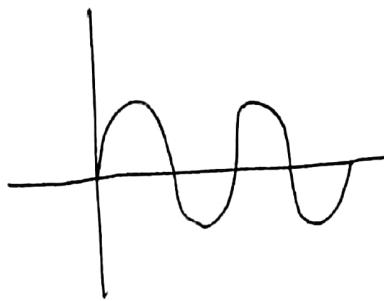
$\downarrow m$



Wave Mechanical Model (Wave Mechanics)

$$\gamma = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{mv} \text{ J-sec}$$

$$y = A \sin(\omega t - kx) \text{ or } A \cos(\omega t - kx)$$



$$\frac{2\pi}{\lambda} = K \left(\frac{\text{Phase variation}}{\text{Distance travelled}} \right)$$

Matter Wave: Periodic variation of wave funct.

↓ Not constituted of single wave.

Composed of multiple waves with superimposing wavelengths -

$$\begin{cases} Y_1 = A_1 \cos(\omega t - kx) \\ Y_2 = A_2 \cos(\omega t + dx) + -(k + dk)x \end{cases}$$

Superimposition

motion of matter wave.

Phase Velocity and Group Velocity

$$Y_{\text{total}} = Y_1 + Y_2$$



Wave funcⁿ:

$$\psi, \psi^2$$

Actual significance

Probability
of finding an
 e^- (0 to i)

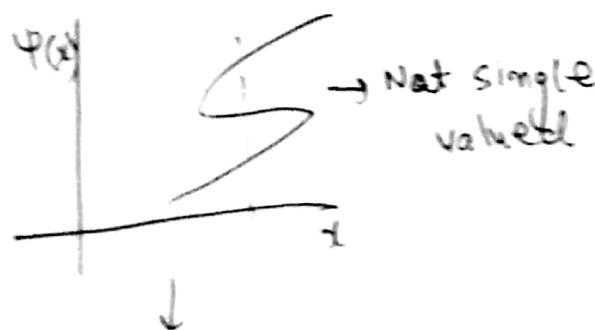
but ψ gets negative values sometimes.
so ψ^2 to avoid -ve value

Ψ^2 : Probability Density = Prob. of finding particle
in a volume in unit
volume.

$$\int \Psi^2 dV = \iiint_{-\infty}^{\infty} \Psi^2 dx dy dz = 1 \quad (\text{can find anywhere in universe})$$

$$|\Psi|^2 = \Psi \Psi^* \text{conjugate}$$

→ Wave functⁿ (Ψ) must be single valued at given point/location.



wrong graph

complex

curvature

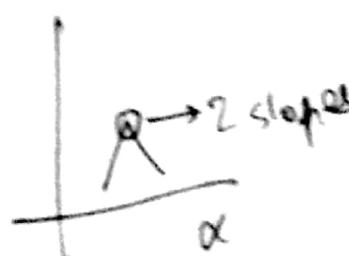
→ Ψ should not be ∞

and

→ Ψ should be continuous & its 1st & 2nd derivative
must also be continuous.

slope

$$N \int_{-a}^b \Psi \Psi^* dV = 1$$



⇒ Normalization

so that the prob.
is 1.

$$y = Ae^{-i(\omega t - kx)}$$

$$\frac{d^2y}{dt^2} = \frac{1}{v^2} \frac{d^2y}{dx^2}$$

$$\begin{aligned}\Psi(x,t) &= Ae^{-i(\omega t - kx)} \\ &= Ae^{-i(2\pi\mu t - \frac{2\pi x}{\lambda})} \\ \boxed{\Psi(x,t) = Ae^{-2\pi i(\mu t - \frac{x}{\lambda})}} &\quad \text{--- } \textcircled{1}\end{aligned}$$

$$E = \hbar\mu = \frac{\hbar}{2\pi} \times 2\pi\mu = \frac{\hbar}{\lambda} (2\pi\mu)$$

$$\mu = \frac{E}{\frac{\hbar}{\lambda} (2\pi)} \quad \text{--- } \textcircled{2}$$

$$\begin{aligned}\lambda &= \frac{\hbar}{p} = \frac{2\pi\hbar}{2\pi p} = \frac{2\pi\hbar}{p} \\ \boxed{\Psi(x,t) = Ae^{-2\pi i \left[\frac{E}{2\pi\hbar} t - \frac{xp}{2\pi\hbar} \right]}} &\quad \text{--- } \textcircled{3} \\ \rightarrow \Psi(x,t) &= Ae^{-2\pi i \left[\frac{E}{2\pi\hbar} t - \frac{xp}{2\pi\hbar} \right]} \\ &= Ae^{-\frac{i}{\hbar} [E(t) - px]}\end{aligned}$$

$$\boxed{E = \frac{p^2}{2m} + V(x) \rightarrow (\text{Potential})} \quad \text{--- } *$$

$$\Psi(x,t) = Ae^{-\frac{i}{\hbar} [E(t) - px]}$$

$$\frac{\partial \Psi(x,t)}{\partial t} = Ae^{-\frac{i}{\hbar} [E(t) - px]} \cdot -\frac{iE}{\hbar}$$

$$= \Psi(x,t) \cdot -\frac{iE}{\hbar}$$

$$\Rightarrow \boxed{E\Psi(x,t) = \frac{\partial \Psi(x,t)}{\partial t} \times \frac{\hbar}{-i}} \quad \text{--- } \textcircled{4}$$

$$\frac{\partial \Psi(x,t)}{\partial x} = \Psi(x,t) \frac{i\hbar}{\hbar}$$

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \left(\frac{i\hbar}{\hbar}\right)^2 \Psi(x,t)$$

$$\Rightarrow p^2(\Psi(x,t)) = \frac{\hbar^2}{i^2} \frac{\partial^2 (\Psi(x,t))}{\partial x^2}$$

$$p^2(\Psi(x,t)) = -\frac{\hbar^2}{2m} \frac{\partial^2 (\Psi(x,t))}{\partial x^2} \quad (5)$$

Now *, from (4) & (5)

$$-\frac{\hbar}{i} \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 (\Psi(x,t))}{\partial x^2} + U(x) \Psi(x,t)$$

$$\downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ E \Psi(x,t) \quad \quad \quad \frac{p^2 \Psi(x,t)}{2m} \quad \quad \quad U(x) \Psi(x,t)$$

If excitation takes place, use this equation as the particle distribution changes with time.

Without excitation, i.e. no change w.r.t to time, only $U(x)$ is important. by removing the time factor in above equation:

$$\rightarrow \Psi(x,t) = A e^{-\frac{i}{\hbar}(Et - px)}$$

$$= A e^{-\frac{iEt}{\hbar}} \cdot e^{-\frac{ipx}{\hbar}}$$

$$\Psi(x,t) = \Psi(x) \cdot e^{-\frac{iEt}{\hbar}}$$

$$\frac{\partial \Psi(x,t)}{\partial t} = \Psi(x) e^{-\frac{iEt}{\hbar}} \cdot -\frac{iE}{\hbar}$$

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{\partial^2 \Psi(x)}{\partial x^2} e^{-\frac{iEt}{\hbar}}$$

into Schrödinger

$$E \psi(x) e^{-iEt/\hbar} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) + U(x) \psi(x)$$

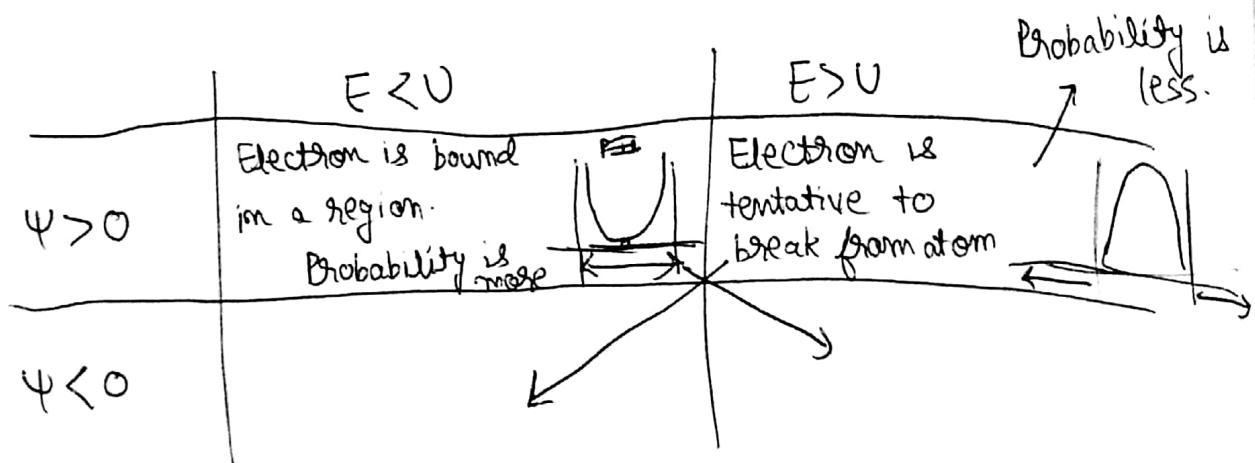
$$E \psi(x) = -\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + U(x) \psi(x)$$

$$\Rightarrow \boxed{\psi(x) [E - U(x)] \frac{2m}{\hbar^2} + \frac{d^2 \psi(x)}{dx^2} = 0}$$

time
independent

if $U(x) > E$, it is tightly bound to nucleus
can't participate in bond formation or transfer to other atom.

if $E > U(x)$, like a free e^- , participates in bond formation.



$$\Psi(x,t) = A e^{-\frac{i}{\hbar}[E(t) - P(x)]}$$

$$(i) \frac{\partial \Psi(x,t)}{\partial t} = A \left(\frac{-iE}{\hbar} \right) e^{-\frac{i}{\hbar}[Et - Px]}$$

$$= \Psi(x,t) - \frac{1}{\hbar} E$$

$$E \Psi(x,t) = -\frac{1}{\hbar} \frac{\partial \Psi(x,t)}{\partial t}$$

this is the corr. value of the operator.

$$O \Psi(x,t) = \left(\frac{m}{\hbar^2} \frac{\partial^2}{\partial x^2} \right) \Psi(x,t) \Rightarrow \text{Eigen func}$$

operator Eigen value

$$\hat{E} = -\frac{\hbar}{i} \frac{\partial}{\partial t}$$

is energy operator

$$(ii) \frac{\partial \Psi(x,t)}{\partial x} = A \left(\frac{i\hbar}{\hbar} \right) e^{-\frac{i}{\hbar}[E(t) - P(x)]}$$

$$\frac{\partial \Psi(x,t)}{\partial x} = \Psi(x,t) \frac{iP}{\hbar}$$

$$\hat{p} \Psi(x,t) = \frac{i}{\hbar} \frac{\partial \Psi(x,t)}{\partial x}$$

$$(iii) \frac{\partial^2 \Psi(x,t)}{\partial x^2} = -\frac{p^2}{\hbar^2} \Psi(x,t)$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{p^2}{2m} \Psi(x,t)$$

$$\boxed{-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}}$$

is K.E operator

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

is momentum operator

From time ind. eq.,

$$E \Psi(x) = \left[\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x)}{\partial x^2} - U(x) \Psi(x) \right] \times t(1)$$

$$= - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x)}{\partial x^2} + U(x) \Psi(x)$$

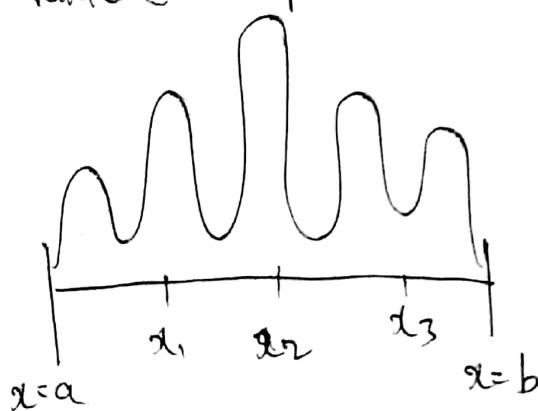
~~KE operator~~

$$E \Psi(x) = \left[- \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x) \right] \Psi(x)$$



Total energy operator
or hamiltonian op-

⇒ Expectation value (Most probable value):



Weighted Average over x is expected value.

$$N_1 \longrightarrow x_1$$

$$N_2 \longrightarrow x_2$$

$$N_3 \longrightarrow x_3$$

,

,

,

$$N_n \longrightarrow x_n$$

$$\bar{x} = \underbrace{N_1 x_1 + N_2 x_2 + \dots + N_n x_n}_{N_1 + N_2 + \dots + N_n}$$

Probability of finding particle at $x_1 \rightarrow |\psi_1|^2$
 $x_2 \rightarrow |\psi_2|^2$
 $x_3 \rightarrow |\psi_3|^2$

$$\langle \bar{x} \rangle = \frac{x_1|\psi_1|^2 + x_2|\psi_2|^2 + \dots + x_n|\psi_n|^2}{|\psi_1|^2 + |\psi_2|^2 + \dots + |\psi_n|^2} = \frac{\int_0^L x_i |\psi_i|^2 dx}{\int_0^L |\psi_i|^2 dx}$$

(Expected/Avg. value)
of i^{th} particle

Momentum operator

$$\langle \bar{P} \rangle = \int_{-\infty}^{\infty} \hat{P} \psi^* \psi dx$$

Expected momentum =

$$\langle \bar{O} \rangle = \int_{-\infty}^{\infty} \psi^* \hat{O} \psi dx \quad (\text{e.g. } dz dy dz)$$

Q) Find expectation value of position of a particle trapped in box of lt. L.

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

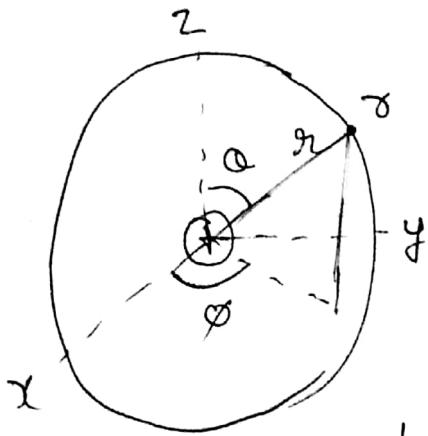
$n = 1, 2, 3, \dots$ energy level.

$$\langle \bar{x} \rangle = \int_{-\infty}^{\infty} \bar{x} |\psi|^2 dx \quad |\psi|^2 = \frac{2}{L} \sin^2 \frac{n\pi x}{L}$$

$$\langle \bar{x} \rangle = \int_0^L \frac{2x}{L} \sin^2 \frac{n\pi x}{L} dx = \frac{2x}{L} \left(1 - \cos \frac{2n\pi x}{L} \right) dx$$

$$\langle \bar{x} \rangle = \frac{x^2}{2L} - \frac{x \cos \frac{2n\pi x}{L}}{L} \rightarrow 0 \quad \text{after integ.}$$

Quantum theory of H-atom



$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2} [E - \mu] \psi (xyz) = 0$$

For each value of α , there are many values of ϕ . \Rightarrow quantum no. ass. with α is more than $\phi(1)$ & that with R is more than ℓ .

$$\frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial \psi}{\partial R} \right) + \frac{1}{R^2 \sin \alpha} \frac{\partial}{\partial \alpha} \left(\sin \alpha \frac{\partial \psi}{\partial \alpha} \right) + \frac{1}{R^2 \sin^2 \alpha} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{2m}{\hbar^2} \left[E + \frac{e^2}{4\pi \epsilon_0 R} \right] = 0$$

$$\Rightarrow \frac{1}{\sin \alpha} \frac{\partial}{\partial \alpha} \left(\sin \alpha \frac{\partial \phi(\alpha)}{\partial \alpha} \right) + \left[\ell(\ell+1) - \frac{m_\ell^2}{\sin^2 \alpha} \right] \phi(\alpha) = 0$$

$$\Rightarrow \frac{\partial^2 \phi(\alpha)}{\partial \phi^2} + m_\ell^2 \phi(\alpha) = 0$$

$$\Rightarrow \frac{1}{R} \frac{\partial}{\partial R} \left(R^2 \frac{\partial R}{\partial R} \right) + \frac{2m e^2}{K^2} \left[\frac{e^2}{4\pi \epsilon_0 R} + E \right] = \ell(\ell+1)$$

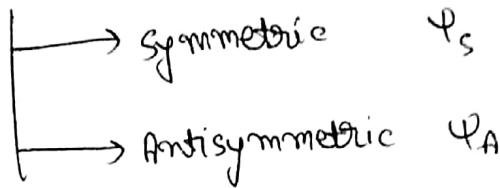
$$\phi(\alpha) = A e^{im_\ell \alpha}$$

$$\phi(\alpha) = \phi(\alpha + 2\pi)$$

$$\Rightarrow A e^{im_\ell \alpha} = A e^{im_\ell \alpha} \cdot A e^{im_\ell 2\pi}$$

$$\Rightarrow m_\ell = 0, \pm 1, \pm 2, \dots$$

Types of wave functions



if ψ_1^{\uparrow} , ψ_2^{\uparrow} , ..., ψ_n^{\uparrow} are independent.

$$\psi = \psi_1 \cdot \psi_2 \cdot \dots \cdot \psi_n$$

$= 0$

1 - 'a' state	it has its own n, l, m_l
2 - 'b' state	same here

$$\psi_I = \psi_a(1)\psi_b(2) \rightarrow \text{Both are equal as } \bar{e} \text{ are identical}$$

$$\psi_{II} = \psi_a(2)\psi_b(1)$$

~~This is called~~

$$\psi_s = \frac{1}{\sqrt{2}} [\psi_a(1)\psi_b(2) + \psi_b(1)\psi_a(2)]$$

$$\psi_A = \frac{1}{\sqrt{2}} [\psi_a(1)\psi_b(2) - \psi_b(1)\psi_a(2)]$$

↓
Normalization factor

If 2 es have same state, $\psi_A = 0$ which isn't possible and this is Pauli's exclusion principle.