Applied Linear Algebra in Data Analysis: January 2026

Quiz 1 | Duration: 10min | Marks: 10

1. Consider the following linear equation with 100 unknown variables,

$$a_1x_1 + a_2x_2 + \cdots + a_{100}x_{100} = b$$

   What spaces do the row view and column view of this equation correspond to? [Marks: 1]

   What are the different possible solutions for this equation? [Marks: 1]

   Explain the complete set of conditions for the different types of solutions for this equation? [Marks: 2]

   If the above equation is solvable, explain the procedure for obtaining the solution if it had a unique solution, else explain how you can generate any of the infinitely many solutions. [Marks: 2]

2. Choose the most appropriate answer. What is vector space? [Marks: 1]
   a) A quantity with magnitude and direction.
   b) A set with no elements.
   c) A set containing the zero element.
   d) A set closed under linear combination of its elements.

3. Choose the correct answer: What is linear independence? [Marks: 1]
   a) A set of vectors.
   b) An operation on a set of vectors.
   c) A property of a set of vectors.
   d) A geometric operation on a set of vectors.

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Quiz 2 | Duration: 20min | Marks: 10

1. Consider a system of p equations with q unknowns. What space does the **row view** correspond to? [Marks: 0.5]

   a) $\mathbb{R}^p$

   b) $\mathbb{R}^q$

   c) $\mathbb{R}^1$

   d) $\mathbb{R}^2$

   What space does the **column view** correspond to? [Marks: 0.5]

   a) $\mathbb{R}^p$

   b) $\mathbb{R}^q$

   c) $\mathbb{R}^1$

   d) $\mathbb{R}^2$

2. Consider a system of p equations with q unknowns. In the column view interpretation, what question are we asking in this problem? [Marks: 1]

   a) Where is the intersection of the hyperplanes represented by the p equations?

   b) Where is the intersection of the lines represented by the p unknowns?

   c) What linear combination of the q vectors from $\mathbb{R}^p$ give us the vector $\mathbf{b}$?

   d) What linear combination of the p vectors from $\mathbb{R}^q$ give us the vector $\mathbf{b}$?

3. (True or False) The set of all n unit vectors of $\mathbb{R}^n$ form a vector space. Note: *Explain your answer. Answers without an explanation will not be credited* [Marks: 2]

4. (True or False) The set of all n unit vectors of $\mathbb{R}^n$ form a linearly independent set. Note: *Explain your answer. Answers without an explanation will not be credited* [Marks: 2]

5. (True or False) The set of all points from $\mathbb{R}^n$ satisfying the following equation form a subspace of $\mathbb{R}^n$. Note: *Explain your answer. Answers without an explanation will not be credited* [Marks: 2]

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n, \text{such that } x_1 + x_2 + \cdots + x_n = 0$$

6. Shows that the set of solutions to the following two equations forms a subspace of $\mathbb{R}^3$. [Marks: 1]

$$x_1 + x_2 + x_3 = 0$$
$$x_1 + x_2 - x_3 = 0$$

   What is the dimension of this subspace? [Marks: 1]

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Quiz 3 | Duration: 10min | Marks: 10

1. Consider the following equation,

$$x_1\mathbf{a}_1 + x_1\mathbf{a}_1 + \cdots + x_q\mathbf{a}_q = \mathbf{b}$$

   where, $\mathbf{a}_i, \mathbf{b} \in \mathbb{R}^p$. Answer the following true or false questions and provide and explanation for your answer. Answers without an explanation will not be credited.

   (True or False) If $\text{span}\left(\{\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_q\}\right) = \mathbb{R}^p$, then the equation has a solution for any $\mathbf{b} \in \mathbb{R}^p$. [Marks: 2]

   (True or False) If the set $\{\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_q\}$ is a basis for $\mathbb{R}^p$, then the equation has a unique solution for any $\mathbf{b} \in \mathbb{R}^p$. [Marks: 2]

2. What is a basis for a vector space? [Marks: 1]

   Provide a basis for the following vector spaces:

   a) $\mathbb{R}^2$ [Marks: 1]

   b) $\left\{\begin{bmatrix} \alpha \\ -\alpha \end{bmatrix} \middle| \alpha \in \mathbb{R}\right\}$ [Marks: 1]

   c) $\left\{\begin{bmatrix} \alpha \\ -\alpha \\ \beta \end{bmatrix} \middle| \alpha, \beta \in \mathbb{R}\right\}$ [Marks: 1]

   d) $\left\{\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \middle| \alpha, \beta, \gamma \in \mathbb{R}\right\}$ [Marks: 1]

3. Compute the inner product $\mathbf{u}^\top\mathbf{v}$ and outer product $\mathbf{u}\mathbf{v}^\top$ of the following two vectors [Marks: 1]:

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$