

# Software Requirements Specification for Solar Water Heating Systems Incorporating Phase Change Material

Thulasi Jegatheesan, Brooks MacLachlan, and Spencer Smith

May 30, 2016

## Contents

<b>1</b>	<b>Reference Material</b>	<b>2</b>
1.1	Table of Units . . . . .	2
1.2	Table of Symbols . . . . .	2
1.3	Abbreviations and Acronyms . . . . .	4
<b>2</b>	<b>Introduction</b>	<b>5</b>
2.1	Purpose of Document . . . . .	5
2.2	Scope of Requirements . . . . .	5
2.3	Organization of Document . . . . .	6
<b>3</b>	<b>General System Description</b>	<b>6</b>
3.1	User Characteristics . . . . .	6
3.2	System Constraints . . . . .	6
<b>4</b>	<b>Specific System Description</b>	<b>6</b>
4.1	Problem Description . . . . .	6
4.1.1	Terminology and Definitions . . . . .	7
4.1.2	Physical System Description . . . . .	7
4.1.3	Goal Statements . . . . .	7
4.2	Solution Characteristics Specification . . . . .	7
4.2.1	Assumptions . . . . .	8
4.2.2	Theoretical Models . . . . .	9
4.2.3	General Definitions . . . . .	11
4.2.4	Data Definitions . . . . .	13
4.2.5	Instance Models . . . . .	15
4.2.6	Data Constraints . . . . .	21
4.2.7	Properties of a Correct Solution . . . . .	22

<b>5</b>	<b>Requirements</b>	<b>23</b>
5.1	Functional Requirements . . . . .	23
5.2	Nonfunctional Requirements . . . . .	25
<b>6</b>	<b>Likely Changes</b>	<b>25</b>

# 1 Reference Material

This section records information for easy reference.

## 1.1 Table of Units

Throughout this document SI (Système International d’Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit with the SI name in parentheses.

symbol	unit	SI
m	length	metre
kg	mass	kilogram
s	time	second
°C	temperature	centigrade
J	energy	Joule
W	power	Watt ( $W = J s^{-1}$ )

## 1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the heat transfer literature and with existing documentation for solar water heating systems. The symbols are listed in alphabetical order.

symbol	unit	description
$A_C$	$m^2$	coil surface area
$A_{in}$	$m^2$	surface area over which heat is transferred in
$A_{out}$	$m^2$	surface area over which heat is transferred out
$A_P$	$m^2$	phase change material surface area
$C$	$J/(kg\ ^\circ C)$	specific heat capacity
$C^L$	$J/(kg\ ^\circ C)$	specific heat capacity of a liquid

$C_P^L$	J/(kg °C)	specific heat capacity of PCM as a liquid
$C^S$	J/(kg °C)	specific heat capacity of a solid
$C_P^S$	J/(kg °C)	specific heat capacity of PCM as a solid
$C^V$	J/(kg °C)	specific heat capacity of a vapour
$C_W$	J/(kg °C)	specific heat capacity of water
$D$	m	diameter of tank
$E$	J	sensible heat energy
$E_{P\text{melt}}^{\text{init}}$	J	heat energy in the PCM at the instant when melting begins
$E_P$	J	heat energy in the PCM
$E_W$	J	heat energy in the water
$g$	W/m <sup>2</sup>	volumetric heat generation per unit volume
$h$	W/(m <sup>2</sup> °C)	convective heat transfer coefficient
$h_C$	W/(m <sup>2</sup> °C)	convective heat transfer coefficient between coil and water
$H_f$	J/kg	specific latent heat of fusion
$h_P$	W/(m <sup>2</sup> °C)	convective heat transfer coefficient between water and PCM
$L$	m	length of tank
$m$	kg	mass
$m_P$	kg	mass of phase change material
$m_W$	kg	mass of water
$\hat{\mathbf{n}}$	unitless	unit outward normal vector for a surface
$q$	W/(m <sup>2</sup> °C)	heat flux
$Q$	J	latent heat energy
$\mathbf{q}$	W/m <sup>2</sup>	thermal flux vector
$q_C$	W/m <sup>2</sup>	heat flux from coil
$q_{\text{in}}$	W/m <sup>2</sup>	heat flux in
$q_{\text{out}}$	W/m <sup>2</sup>	heat flux out
$q_P$	W/m <sup>2</sup>	heat flux into phase change material
$Q_P$	J	latent heat energy added to PCM
$t$	s	time
$T$	°C	temperature
$T_{\text{boil}}$	°C	temperature at boiling point
$T_C$	°C	temperature of coil
$T_{\text{env}}$	°C	temperature of environment
$t_{\text{final}}$	s	final time
$T_{\text{init}}$	°C	initial temperature

$T_{\text{melt}}$	°C	temperature at melting point
$t_{\text{melt}}^{\text{init}}$	s	time at which melting of PCM begins
$t_{\text{melt}}^{\text{final}}$	s	time at which melting of PCM ends
$T_{\text{melt}}^P$	°C	temperature at melting point for PCM
$T_P$	°C	temperature of phase change material
$T_W$	°C	temperature of water
$V$	m <sup>3</sup>	volume
$V_P$	m <sup>3</sup>	volume of PCM
$V_{\text{tank}}$	m <sup>3</sup>	volume of the cylindrical tank
$V_W$	m <sup>3</sup>	volume of water
$\Delta T$	°C	temperature difference
$\rho$	kg/m <sup>3</sup>	density, mass per unit volume
$\rho_P$	kg/m <sup>3</sup>	density of PCM
$\rho_W$	kg/m <sup>3</sup>	density of water
$\tau$	s	dummy variable for integration over time
$\phi$	unitless	melt fraction

---

### 1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
ODE	Ordinary Differential Equation
PCM	Phase Change Material
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
SWHS	Solar Water Heating System
T	Theoretical Model

---

## 2 Introduction

Due to the increasing cost, diminishing availability, and negative environmental impact of fossil fuels, there is a higher demand for renewable energy sources and energy storage technology. Solar water heating systems incorporating Phase Change Material (PCM) use a renewable energy source and provide a novel way of storing energy. Solar water heating systems with PCM improve over the traditional solar heating systems because of their smaller size. The smaller size is possible because of the ability of PCM to store thermal energy as latent heat, which allows higher thermal energy storage capacity per unit weight.

The following section provides an overview of the Software Requirements Specification (SRS) for a solar water heating system that incorporates PCM. The developed program will be referred to as Solar Water Heating System (SWHS). This section explains the purpose of this document, the scope of the system, the organization of the document and the characteristics of the intended readers.

["Characteristics of intended readers" is listed as something that will appear in this section, but it never does appear. —BM]

### 2.1 Purpose of Document

The main purpose of this document is to describe the modelling of solar water heating systems incorporating PCM. The goals and theoretical models used in the SWHS code are provided, with an emphasis on explicitly identifying assumptions and unambiguous definitions. This document is intended to be used as a reference to provide ad hoc access to all information necessary to understand and verify the model. The SRS is abstract because the contents say *what* problem is being solved, but do not say *how* to solve it.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the process is not waterfall, as Parnas and Clements [5] point out, the most logical way to present the documentation is still to “fake” a rational design process.

### 2.2 Scope of Requirements

The scope of the requirements is limited to thermal analysis of a single solar water heating tank incorporating PCM. Given the appropriate inputs, the code for SWHS is intended to predict the temperature and energy histories for the water and the PCM. This entire document is written assuming that the substances inside the heating tank are water and PCM.

## 2.3 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [3] and [6]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section 4.2.5 and trace back to find any additional information they require. The instance models provide the Ordinary Differential Equations (ODEs) and algebraic equations that model the solar water heating system with PCM. SWHS solves these ODEs.

The goal statements are refined to the theoretical models, and theoretical models to the instance models. The instance models (Section 4.2.5) to be solved are referred to as IM1 to IM4.

## 3 General System Description

This section provides general information about the system, identifies the interfaces between the system and its environment, and describes the user characteristics and the system constraints.

[”general information about the system” and ”interfaces between the system and its environment” are not actually provided in this section. —BM]

### 3.1 User Characteristics

The end user of SWHS should have an understanding of undergraduate Level 1 Calculus and Physics.

### 3.2 System Constraints

There are no system constraints.

## 4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models (ODEs) that model the solar water heating tank with PCM.

### 4.1 Problem Description

SWHS is a computer program developed to investigate the effect of employing PCM within a solar water heating tank.

### 4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- Heat Flux: The rate of heat energy transfer per unit area.
- Phase Change Material (PCM): A substance that uses phase changes (melting) to absorb or release large amounts of heat at a constant temperature.
- Specific Heat: heat capacity per unit mass.
- Thermal Conduction: the transfer of heat energy through a substance.
- Transient: Changing with time.

[The definition for PCM makes it seem like melting is the only phase change that can occur. —BM]

### 4.1.2 Physical System Description

The physical system of SWHS, as shown in Figure 1, includes the following elements:

PS1: Tank containing water.

PS2: Heating coil at bottom of tank. ( $q_C$  represents the heat flux from the coil into the water.)

PS3: PCM suspended in tank. ( $q_P$  represents the heat flux from the water into the PCM.)

### 4.1.3 Goal Statements

Given the temperature of the coil, initial conditions for the temperature of the water and the PCM, and material properties, the goal statements are:

GS1: predict the water temperature over time;

GS2: predict the PCM temperature over time;

GS3: predict the change in the energy of the water over time;

GS4: predict the change in the energy of the PCM over time.

## 4.2 Solution Characteristics Specification

The instance models (ODEs) that govern SWHS are presented in Subsection 4.2.5. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

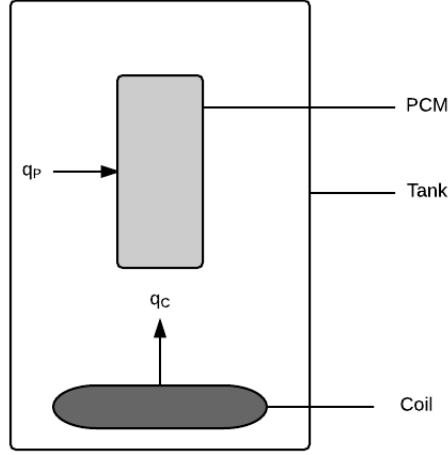


Figure 1: Solar water heating tank, with heat flux from coil and to the PCM of  $q_C$  and  $q_P$ , respectively

#### 4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

- A1: The only form of energy that is relevant for this problem is thermal energy. All other forms of energy, such as mechanical energy, are assumed to be negligible [T1].
- A2: All heat transfer coefficients are constant over time [GD1].
- A3: The water in the tank is fully mixed, so the temperature is the same throughout the entire tank [GD2, DD2].
- A4: The PCM has the same temperature throughout [GD2, DD2, LC1].
- A5: Density of the water and PCM have no spatial variation; that is, they are each constant over their entire volume [GD2].
- A6: Specific heat capacity of the water and PCM have no spatial variation; that is, they are each constant over their entire volume [GD2].
- A7: Newton's law of convective cooling applies between the coil and the water [DD1].



- A8: The temperature of the heating coil is constant over time [DD1, LC2].
- A9: The temperature of the heating coil does not vary along its length [DD1, LC3].
- A10: Newton's law of convective cooling applies between the water and the PCM [DD2].
- A11: The model only accounts for charging of the tank, not discharging. The temperature of the water and PCM can only increase, or remain constant; they do not decrease. This implies that the initial temperature (A12) is less than (or equal) to the temperature of the coil [IM1, LC4].
- A12: The initial temperature of the water and the PCM is the same [IM1, IM2, LC5].
- A13: The simulation will start with the PCM in solid form [IM2, IM4].
- A14: The operating temperature range of the system is such that the water is always in liquid form. That is, the temperature will not drop below the melting point of water, or rise above its boiling point [IM1, IM3].
- A15: The tank is perfectly insulated so that there is no heat loss from the tank [IM1, LC6].
- A16: No internal heat is generated by either the water or the PCM; therefore, the volumetric heat generation is zero [IM1, IM2].
- A17: The volume change of the PCM due to melting is negligible [IM2].
- A18: The PCM is either in a liquid or solid state, but not a gas [IM2, IM4].

#### 4.2.2 Theoretical Models

This section focuses on the general equations and laws that SWHS is based on.

Number	T1
Label	<b>Conservation of thermal energy</b>
Equation	$-\nabla \cdot \mathbf{q} + g = \rho C \frac{\partial T}{\partial t}$
Description	The above equation gives the conservation of energy for transient heat transfer in a material of specific heat capacity $C$ ( $\text{J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ ) and density $\rho$ ( $\text{kg m}^{-3}$ ), where $\mathbf{q}$ is the thermal flux vector ( $\text{W m}^{-2}$ ), $g$ is the volumetric heat generation ( $\text{W m}^{-3}$ ), $T$ is the temperature ( $^\circ\text{C}$ ), $t$ is time (s), and $\nabla$ is the gradient operator. For this equation to apply, other forms of energy, such as mechanical energy, are assumed to be negligible in the system (A1). In general, the material properties ( $\rho$ and $C$ ) depend on temperature.
Source	<a href="http://www.efunda.com/formulae/heat_transfer/conduction/overview_cond.cfm">http://www.efunda.com/formulae/heat_transfer/conduction/overview_cond.cfm</a>
Ref. By	GD2

Number	T2
Label	<b>Sensible heat energy</b>
Equation	$E = \begin{cases} C^S m \Delta T & \text{if } T < T_{\text{melt}} \\ C^L m \Delta T & \text{if } T_{\text{melt}} < T < T_{\text{boil}} \\ C^V m \Delta T & \text{if } T_{\text{boil}} > T \end{cases}$ <p>See T3, Latent heat energy, if <math>T = T_{\text{boil}}</math> or <math>T = T_{\text{melt}}</math>.</p>
Description	<p><math>E</math> is the change in sensible heat energy (J).</p> <p><math>C^S</math>, <math>C^L</math>, <math>C^V</math> are the specific heat capacities of a solid, liquid, and vapour, respectively (<math>\text{J kg}^{-1} \text{ }^\circ\text{C}^{-1}</math>).</p> <p><math>m</math> is the mass (kg).</p> <p><math>T</math> is the temperature (<math>^\circ\text{C}</math>), and <math>\Delta T</math> is the change in temperature (<math>^\circ\text{C}</math>).</p> <p><math>T_{\text{melt}}</math> and <math>T_{\text{boil}}</math> are the melting and boiling points, respectively (<math>^\circ\text{C}</math>).</p> <p>Sensible heating occurs as long as the material does not reach a temperature where a phase change occurs.</p> <p>A phase change occurs if <math>T = T_{\text{boil}}</math> or <math>T = T_{\text{melt}}</math>. If this is the case, refer to T3, Latent heat energy.</p>
Source	<a href="http://en.wikipedia.org/wiki/Sensible_heat">http://en.wikipedia.org/wiki/Sensible_heat</a>
Ref. By	IM3, IM4

Number	T3
Label	<b>Latent heat energy</b>
Equation	$T = T_{\text{melt}}$ or $T = T_{\text{boil}}$ , $Q(t) = \int_0^t \frac{dQ(\tau)}{d\tau} d\tau$ , with $Q(0) = 0$
Description	<p><math>Q</math> is the change in thermal energy (J), latent heat energy.</p> <p><math>\int_0^t \frac{dQ(\tau)}{d\tau} d\tau</math> is the rate of change of <math>Q</math> with respect to time <math>\tau</math> (s). <math>t</math> is the time (s) elapsed, as long as the phase change is not complete. The status of the phase change depends on the melt fraction DD4.</p> <p><math>T_{\text{melt}}</math> and <math>T_{\text{boil}}</math> are the melting and boiling points, respectively (<math>^{\circ}\text{C}</math>).</p> <p>Latent heating stops when all material has changed to the new phase.</p>
Source	<a href="http://en.wikipedia.org/wiki/Latent_heat">http://en.wikipedia.org/wiki/Latent_heat</a>
Ref. By	T2, IM4

#### 4.2.3 General Definitions

This section collects the laws and equations that will be used in deriving the data definitions, which in turn are used to build the instance models.

Number	GD1
Label	<b>Newton's law of cooling</b>
SI Units	$\text{W m}^{-2}$
Equation	$q(t) = h\Delta T(t)$
Description	<p>Newton's law of cooling describes convective cooling from a surface. The law is stated as: the rate of heat loss from a body is proportional to the difference in temperatures between the body and its surroundings.</p> <p><math>q(t)</math> is the thermal flux (<math>\text{W m}^{-2}</math>).</p> <p><math>h</math> is the heat transfer coefficient, assumed independent of <math>T</math> (A2) (<math>\text{W m}^{-2} \text{ }^{\circ}\text{C}^{-1}</math>).</p> <p><math>\Delta T(t) = T(t) - T_{\text{env}}(t)</math> is the time-dependent thermal gradient between the environment and the object (<math>^{\circ}\text{C}</math>).</p>
Source	[2, p. 8]
Ref. By	DD1, DD2

Number	GD2
Label	<b>Simplified rate of change of temperature</b>
Equation	$mC \frac{dT}{dt} = q_{\text{in}}A_{\text{in}} - q_{\text{out}}A_{\text{out}} + gV$
Description	<p>The basic equation governing the rate of change of temperature, for a given volume <math>V</math>, with time.</p> <p><math>m</math> is the mass (kg).</p> <p><math>C</math> is the specific heat capacity (<math>\text{J kg}^{-1} \text{ }^{\circ}\text{C}^{-1}</math>).</p> <p><math>T</math> is the temperature (<math>^{\circ}\text{C}</math>) and <math>t</math> is the time (s).</p> <p><math>q_{\text{in}}</math> and <math>q_{\text{out}}</math> are the in and out heat transfer rates, respectively (<math>\text{W m}^{-2}</math>).</p> <p><math>A_{\text{in}}</math> and <math>A_{\text{out}}</math> are the surface areas over which the heat is being transferred in and out, respectively (<math>\text{m}^2</math>).</p> <p><math>g</math> is the volumetric heat generated (<math>\text{W m}^{-3}</math>).</p> <p><math>V</math> is the volume (<math>\text{m}^3</math>).</p>
Ref. By	IM1, IM2

## Detailed derivation of simplified rate of change of temperature

Integrating (T1) over a volume ( $V$ ), we have

$$-\int_V \nabla \mathbf{q} dV + \int_V g dV = \int_V \rho C \frac{\partial T}{\partial t} dV.$$

Applying Gauss's Divergence theorem to the first term over the surface  $S$  of the volume, with  $\mathbf{q}$  as the thermal flux vector for the surface, and  $\hat{\mathbf{n}}$  as a unit outward normal for the surface,

$$-\int_S \mathbf{q} \cdot \hat{\mathbf{n}} dS + \int_V g dV = \int_V \rho C \frac{\partial T}{\partial t} dV. \quad (1)$$

We consider an arbitrary volume. The volumetric heat generation is assumed constant. Then (1) can be written as

$$q_{\text{in}} A_{\text{in}} - q_{\text{out}} A_{\text{out}} + gV = \int_V \rho C \frac{\partial T}{\partial t} dV,$$

where  $q_{\text{in}}$ ,  $q_{\text{out}}$ ,  $A_{\text{in}}$ , and  $A_{\text{out}}$  are explained in GD2. Assuming  $\rho$ ,  $C$  and  $T$  are constant over the volume, which is true in our case by assumptions (A3), (A4), (A5), and (A6), we have

$$\rho C V \frac{dT}{dt} = q_{\text{in}} A_{\text{in}} - q_{\text{out}} A_{\text{out}} + gV. \quad (2)$$

Using the fact that  $\rho = m/V$ , (2) can be written as

$$mC \frac{dT}{dt} = q_{\text{in}} A_{\text{in}} - q_{\text{out}} A_{\text{out}} + gV.$$

### 4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models. The dimension of each quantity is also given.

Number	DD1
Label	<b>Heat flux out of coil</b>
Symbol	$q_C$
SI Units	$\text{W m}^{-2}$
Equation	$q_C(t) = h_C(T_C - T_W(t))$ , over area $A_C$
Description	$T_C$ is the temperature of the coil ( $^{\circ}\text{C}$ ). $T_W$ is the temperature of the water ( $^{\circ}\text{C}$ ). The heat flux out of the coil, $q_C$ ( $\text{W m}^{-2}$ ), is found by assuming that Newton's Law of Cooling applies (A7). This law (GD1) is used on the surface of the coil, which has area $A_C$ ( $\text{m}^2$ ) and heat transfer coefficient $h_C$ ( $\text{W m}^{-2} ^{\circ}\text{C}^{-1}$ ). This equation assumes that the temperature of the coil is constant over time (A8) and that it does not vary along the length of the coil (A9).
Sources	[4]
Ref. By	IM1

Number	DD2
Label	<b>Heat flux into PCM</b>
Symbol	$q_P$
SI Units	$\text{W m}^{-2}$
Equation	$q_P(t) = h_P(T_W(t) - T_P(t))$ , over area $A_P$
Description	$T_W$ is the temperature of the water ( $^{\circ}\text{C}$ ). $T_P$ is the temperature of the PCM ( $^{\circ}\text{C}$ ). The heat flux into the PCM, $q_P$ ( $\text{W m}^{-2}$ ), is found by assuming that Newton's Law of Cooling applies (A10). This law (GD1) is used on the surface of the PCM, which has area $A_P$ ( $\text{m}^2$ ) and heat transfer coefficient $h_P$ ( $\text{W m}^{-2} ^{\circ}\text{C}^{-1}$ ). This equation assumes that the temperature of the PCM and water each have constant values over the surface area over which they interact (A3 and A4).
Sources	[4]
Ref. By	IM1, IM2, IM4

Number	DD3
Label	<b>Specific latent heat of fusion</b>
Symbol	$H_f$
SI Units	$\text{J kg}^{-1}$
Equation	$H_f = \frac{Q}{m}$
Description	<p><math>H_f</math> is the amount of heat energy required to completely melt a unit mass of a substance (<math>\text{J kg}^{-1}</math>).</p> <p><math>Q</math> is the heat energy required for melting (J).</p> <p><math>m</math> is the mass being melted (kg).</p>
Sources	[1, p. 282]
Ref. By	DD4, IM4

Number	DD4
Label	<b>Melt fraction</b>
Symbol	$\phi$
SI Units	dimensionless
Equation	$\phi = \frac{Q_P}{H_f m_P}, (0 \leq \phi \leq 1)$
Description	<p><math>\phi</math> is the fraction of the PCM that is liquid (dimensionless).</p> <p><math>Q_P</math> is the latent heat energy added to the PCM (J).</p> <p><math>H_f</math> is the latent heat of fusion of the PCM (<math>\text{J kg}^{-1}</math>) (DD3).</p> <p><math>m_P</math> is the mass of the PCM (kg).</p>
Sources	[4]
Ref. By	T3, IM2, IM4

#### 4.2.5 Instance Models

This section transforms the problem defined in Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.4 to replace the abstract symbols in the models identified in Sections 4.2.2 and 4.2.3.

The goals GS1 to GS4 are solved by IM1 to IM4. The solutions for IM1 and IM2 are

coupled since the solution for  $T_W$  and  $T_P$  depend on one another. IM3 can be solved once IM1 has been solved. The solution of IM2 and IM4 are also coupled, since the temperature and energy of the PCM depend on the phase change.

Number	IM1
Label	<b>Energy balance on water to find <math>T_W</math></b>
Input	$m_W, C_W, h_C, A_C, h_P, A_P, t_{\text{final}}, T_C, T_{\text{init}}, T_P(t)$ from IM2 The input is constrained so that $T_{\text{init}} \leq T_C$ (A11)
Output	$T_W(t), 0 \leq t \leq t_{\text{final}}$ , such that $\frac{dT_W}{dt} = \frac{1}{\tau_W}[(T_C - T_W(t)) + \eta(T_P(t) - T_W(t))]$ , $T_W(0) = T_P(0) = T_{\text{init}}$ (A12) and $T_P(t)$ from IM2
Description	$T_W$ is the water temperature ( $^{\circ}\text{C}$ ). $T_P$ is the PCM temperature ( $^{\circ}\text{C}$ ). $T_C$ is the coil temperature ( $^{\circ}\text{C}$ ). $\tau_W = \frac{m_W C_W}{h_C A_C}$ is a constant (s). $\eta = \frac{h_P A_P}{h_C A_C}$ is a constant (dimensionless). The above equation applies as long as the water is in liquid form, $0 < T_W < 100^{\circ}\text{C}$ , where $0^{\circ}\text{C}$ and $100^{\circ}\text{C}$ are the melting and boiling points of water, respectively (A14).
Sources	[4]
Ref. By	IM2

[The description specifies that the water temperature must be between 0 and 100 degrees Celsius. For this to be true, there should be an assumption that the pressure is atmospheric. —BM]

### Derivation of the energy balance on water

To find the rate of change of  $T_W$ , we look at the energy balance on water. The volume being considered is the volume of water in the tank  $V_W$ , which has mass  $m_W$  and specific heat capacity,  $C_W$ . Heat transfer occurs in the water from the coil as  $q_C$  and from the water into the PCM as  $q_p$ , over areas  $A_C$  and  $A_P$ , respectively. No heat transfer occurs to the outside of the tank, since it has been assumed to be perfectly insulated (A15). Assuming no internal heat is generated (A16),  $g = 0$ . Therefore, the equation for GD2 can be written as:



$$m_W C_W \frac{dT_W}{dt} = q_C A_C - q_P A_P.$$

Using DD1 and DD2 for  $q_C$  and  $q_P$  respectively, this can be written as

$$m_W C_W \frac{dT_W}{dt} = h_C A_C (T_C - T_W) - h_P A_P (T_W - T_P). \quad (3)$$

Dividing (3) by  $m_W C_W$ , we obtain

$$\frac{dT_W}{dt} = \frac{h_C A_C}{m_W C_W} (T_C - T_W) - \frac{h_P A_P}{m_W C_W} (T_W - T_P). \quad (4)$$

Factoring the negative sign out of the second term of the RHS of Equation (4) and multiplying it by  $\frac{h_C A_C}{h_C A_C}$  yields,

$$\frac{dT_W}{dt} = \frac{h_C A_C}{m_W C_W} (T_C - T_W) + \frac{h_C A_C}{h_C A_C} \frac{h_P A_P}{m_W C_W} (T_P - T_W),$$

which simplifies to

$$\frac{dT_W}{dt} = \frac{h_C A_C}{m_W C_W} (T_C - T_W) + \frac{h_P A_P}{h_C A_C} \frac{h_C A_C}{m_W C_W} (T_P - T_W). \quad (5)$$

Setting  $\tau_W = \frac{m_W C_W}{h_C A_C}$  and  $\eta = \frac{h_P A_P}{h_C A_C}$ , Equation (5) can be written as

$$\frac{dT_W}{dt} = \frac{1}{\tau_W} (T_C - T_W) + \frac{\eta}{\tau_W} (T_P - T_W).$$

Finally, factoring out  $\frac{1}{\tau_W}$ , we are left with the governing ODE for IM1:

$$\frac{dT_W}{dt} = \frac{1}{\tau_W} [(T_C - T_W) + \eta(T_P - T_W)].$$

Number	IM2
Label	<b>Energy balance on PCM to find <math>T_P</math></b>
Input	$m_P, C_P^S, C_P^L, h_P, A_P, t_{\text{final}}, T_{\text{init}}, T_{\text{melt}}^P, T_W(t)$ from IM1 The input is constrained so that $T_{\text{init}} < T_{\text{melt}}^P$ (A13)
Output	$T_P(t), 0 \leq t \leq t_{\text{final}}$ , with initial conditions, $T_W(0) = T_P(0) = T_{\text{init}}$ (A12), and $T_W(t)$ from IM1, such that the following governing ODE is satisfied. The specific ODE depends on $T_P$ as follows: $\frac{dT_P}{dt} = \begin{cases} \frac{dT_P}{dt} = \frac{1}{\tau_P^S}(T_W(t) - T_P(t)) & \text{if } T_P < T_{\text{melt}}^P \\ \frac{dT_P}{dt} = \frac{1}{\tau_P^L}(T_W(t) - T_P(t)) & \text{if } T_P > T_{\text{melt}}^P \\ 0 & \text{if } T_P = T_{\text{melt}}^P \text{ and } 0 < \phi < 1 \end{cases}$ The temperature remains constant at $T_{\text{melt}}^P$ , even with the heating (or cooling), until the phase change has occurred for all of the material; that is as long as $0 < \phi < 1$ . $\phi$ (from DD4) is determined as part of the heat energy in the PCM, as given in IM4 $t_{\text{melt}}^{\text{init}}$ , the time at which melting begins. $t_{\text{melt}}^{\text{final}}$ , the time at which melting ends.
Description	$T_W$ is water temperature ( $^{\circ}\text{C}$ ). $T_P$ is the PCM temperature ( $^{\circ}\text{C}$ ). $\tau_P^S = \frac{m_P C_P^S}{h_P A_P}$ is a constant (s). $\tau_P^L = \frac{m_P C_P^L}{h_P A_P}$ is a constant (s).
Sources	[4]
Ref. By	IM1, IM4

### Detailed derivation of the energy balance on the PCM during sensible heating phase

To find the rate of change of  $T_P$ , we look at the energy balance on the PCM. The volume being considered is the volume of the PCM,  $V_P$ . The derivation that follows is initially for the solid PCM. The PCM in the tank has mass  $m_P$  and specific heat capacity  $C_P^S$ . Heat input from the water to the PCM is  $q_P$  over area  $A_P$ . There is no heat flux output. Assuming no internal heat generated (A16),  $g = 0$ , the equation for GD2 can be written as:

$$m_P C_P^S \frac{dT_P}{dt} = q_P A_P$$

Using DD2 for  $q_P$ , this equation can be written as

$$m_P C_P^S \frac{dT_P}{dt} = h_P A_P (T_W - T_P).$$

Dividing by  $m_P C_P^S$  we obtain,

$$\frac{dT_P}{dt} = \frac{h_P A_P}{m_P C_P^S} (T_W - T_P).$$

Setting  $\tau_P^S = \frac{m_P C_P^S}{h_P A_P}$ , this can be written as

$$\frac{dT_P}{dt} = \frac{1}{\tau_P^S} (T_W - T_P). \quad (6)$$

Equation (6) applies for the solid PCM. In the case where all the PCM is melted, the same derivation applies, except that  $C_P^S$  is replaced by  $C_P^L$ , and thus  $\tau_P^S$  is replaced by  $\tau_P^L$ . Although a small change in surface area would be expected with melting, this is not included, since the volume change of the PCM with melting is assumed to be negligible (A17).

In the case where  $T_P = T_{\text{melt}}^P$  and not all of the PCM is melted, the temperature does not change. Therefore, in this case  $\frac{dT_P}{dt} = 0$ .

This derivation does not consider the boiling of the PCM, as the PCM is assumed to either be in a solid or liquid state (A18).

Number	IM3
Label	<b>Heat energy in the water</b>
Input	$C_W, m_W, T_{\text{init}}, T_W(t)$
Output	$E_W(t), 0 \leq t \leq t_{\text{final}}, \text{ such that}$ $E_W(t) = C_W m_W (T_W(t) - T_{\text{init}})$
Description	The above equation is derived using T2. $E_W$ is the change in thermal energy of the liquid water relative to the energy at the initial temperature ( $T_{\text{init}}$ ) (J). $C_W$ is the specific heat capacity of liquid water ( $\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ ) and $m_W$ is the mass of the water (kg). The change in temperature is the difference between the temperature at time $t$ (s), $T_W$ , and the initial temperature, $T_{\text{init}}$ ( $^\circ\text{C}$ ). This equation applies as long as $0 < T_W < 100^\circ\text{C}$ (A14).
Sources	[4]
Ref. By	—

[In the "Input" section, should it be specified that  $T_W(t)$  comes from IM1? (Similar to how the "Input" sections in IM1 and IM2 specify this.) —BM]

Number	IM4
Label	<b>Heat energy in the PCM</b>
Input	$C_P^S, C_P^L, H_f, T_P(t), T_{\text{init}}, m_P, T_{\text{melt}}^P, h_P, A_P, t_{\text{melt}}^{\text{init}}$ (from IM2), $t_{\text{final}}$ The input is constrained so that $T_{\text{init}} < T_{\text{melt}}^P$ (A13)
Output	$E_P(t), 0 \leq t \leq t_{\text{final}}$ , such that $E_P = \begin{cases} C_P^S m_P (T_P(t) - T_{\text{init}}) & \text{if } T_P < T_{\text{melt}}^P \\ E_{P_{\text{melt}}}^{\text{init}} + H_f m_p + C_P^L m_P (T_P(t) - T_{\text{melt}}^P) & \text{if } T_P > T_{\text{melt}}^P \\ E_{P_{\text{melt}}}^{\text{init}} + Q_P(t) & \text{if } T_P = T_{\text{melt}}^P, (0 < \phi < 1) \end{cases}$ $E_{P_{\text{melt}}}^{\text{init}} = C_P^S m_p (T_{\text{melt}}^P - T_{\text{init}})$ Here $Q_P(t)$ is found with the initial condition that $Q_P(t_{\text{melt}}^{\text{init}}) = 0$ and $\frac{dQ_P(t)}{dt} = q_P(t) A_P = h_P A_P (T_W(t) - T_{\text{melt}}^P),$ with the heat flux $q_P$ from DD2, over area $A_P$ , $\phi = \frac{Q_P}{H_f m_P}$ (DD4)
Description	<p>The above equation is derived using T2 and T3. <math>E_P</math> is the change in thermal energy of the PCM relative to the energy at the initial temperature (<math>T_{\text{init}}</math>) (J).</p> <p><math>E_P</math> for the solid PCM is found using T2 for sensible heating, with the specific heat capacity of the solid PCM, <math>C_P^S</math> (<math>\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}</math>), and the change in the PCM temperature from the initial temperature (<math>^\circ\text{C}</math>).</p> <p><math>E_P</math> for the melted PCM (<math>T_P &gt; T_{\text{melt}}^P</math>) is found using T2 for sensible heating of the liquid PCM, plus the energy when melting starts, plus the energy required to melt all of the PCM. The energy when melting starts is <math>E_{P_{\text{melt}}}^{\text{init}}</math> (J). The energy required to melt all of the PCM is <math>H_f m_p</math> (J) (DD3). The specific heat capacity of the liquid PCM is <math>C_P^L</math> (<math>\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}</math>) and the change in temperature is <math>T_P - T_{\text{melt}}^P</math> (<math>^\circ\text{C}</math>).</p> <p><math>E_P</math> during melting of the PCM is found using the energy required at the instant melting of the PCM begins, <math>E_{P_{\text{melt}}}^{\text{init}}</math>, plus the latent heat energy added to the PCM, <math>Q_P</math> (J), since the time when melting began (<math>t_{\text{melt}}^{\text{init}}</math> (s)).</p> <p>The heat energy for boiling of the PCM is not detailed, since the PCM is assumed to either be in a solid or liquid state (A18).</p>
Sources	[4]
Ref. By	IM2

#### 4.2.6 Data Constraints

Tables 1 and 3 show the data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

The specification parameters in Table 1 are listed in Table 2.

Table 1: Input Variables

Var	Physical Constraints	Software Constraints	Typical Value	Uncertainty
$L$	$L > 0$	$L_{\min} \leq L \leq L_{\max}$	1.5 m	10%
$D$	$D > 0$	$\frac{D}{L_{\min}} \leq \frac{D}{L} \leq \frac{D}{L_{\max}}$	0.412 m	10%
$V_P$	$V_P > 0$ (*) $V_P < V_{\text{tank}}(D, L)$	$V_P \geq \text{minfract} \cdot V_{\text{tank}}(D, L)$	0.05 m <sup>3</sup>	10%
$A_P$	$A_P > 0$ (*)	$V_P \leq A_P \leq \frac{2}{h_{\min}} V_P$ (#)	1.2 m <sup>2</sup>	10%
$\rho_P$	$\rho_P > 0$	$\rho_P^{\min} < \rho_P < \rho_P^{\max}$	1007 kg/m <sup>3</sup>	10%
$T_{\text{melt}}^P$	$0 < T_{\text{melt}}^P < T_C$		44.2 °C	10%
$C_P^S$	$C_P^S > 0$	$C_{P\min}^S < C_P^S < C_{P\max}^S$	1760 J/(kg °C)	10%
$C_P^L$	$C_P^L > 0$	$C_{P\min}^L < C_P^L < C_{P\max}^L$	2270 J/(kg °C)	10%
$H_f$	$H_f > 0$	$H_f^{\min} < H_f < H_f^{\max}$	211600 J/kg	10%
$A_C$	$A_C > 0$ (*)	$A_C \leq A_C^{\max}$	0.12 m <sup>2</sup>	10%
$T_C$	$0 < T_C < 100$ (+)		50 °C	10%
$\rho_W$	$\rho_W > 0$	$\rho_W^{\min} < \rho_W \leq \rho_W^{\max}$	1000 kg/m <sup>3</sup>	10%
$C_W$	$C_W > 0$	$C_W^{\min} < C_W < C_W^{\max}$	4186 J/(kg °C)	10%
$h_C$	$h_C > 0$	$h_C^{\min} \leq h_C \leq h_C^{\max}$	1000 W/(m <sup>2</sup> °C)	10%
$h_P$	$h_P > 0$	$h_P^{\min} \leq h_P \leq h_P^{\max}$	1000 W/(m <sup>2</sup> °C)	10%
$T_{\text{init}}$	$0 < T_{\text{init}} < T_{\text{melt}}$ (+)		40 °C	10%
$t_{\text{final}}$	$t_{\text{final}} > 0$	$t_{\text{final}} < t_{\text{final}}^{\max}$ (**)	50000 s	10%

(\*) These quantities cannot be equal to zero, or there will be a divide by zero in the model.

- (+) These quantities cannot be zero, or there would be freezing (A14).
- (#) The constraints on the surface area are calculated by considering the surface area to volume ratio. The assumption is that the lowest ratio is 1 and the highest possible is  $2/h_{\min}$ , where  $h_{\min}$  is the thickness of a “sheet” of PCM. A thin sheet has the greatest surface area to volume ratio.
- (\*\*) The constraint on the maximum time at the end of the simulation is the total number of seconds in one day.

[Should the physical constraints column for PCM melting temperature be labelled with a (+)? —BM]

#### 4.2.7 Properties of a Correct Solution

[What we want in this section is something that will lead to test cases. We want something like the energy into the water - energy out of the water equals the energy gain in the water. We also want the energy into the PCM through heat transfer to equal the energy gain in the PCM. We discussed the equations on May 26, 2016. We looked at a spreadsheet using the rectangular rule for integration. After the meeting I tried the trapezoidal rule for the integration and the agreement was dramatically improved. The spreadsheet looks at each time step, but the results are misleading while the PCM is melting. A better summary is to look at the energy added for the total experiment. The total energy added through the coil should equal the total energy in the system. The total energy added through the coil minus the total energy output to the pcm should equal the total energy in the water and the total energy input to the PCM should equal the energy in the PCM. We should turn this into a requirement. As an example, for the calculations that I did, the total energy in the water, calculated using energy input from the coil and output to the coil had a relative error of 0.0005%. This seems reasonable. —SS]

A correct solution must exhibit the law of conservation of energy. This means that the energy change in the water should equal the difference between the total energy input from the coil and the energy output to the PCM. This can be shown as an equation by taking DD1 and DD2, multiplying each by their respective surface area of heat transfer, and integrating each over the simulation time, as follows:

$$E_W = \int_0^t h_C A_C (T_C - T_W(t)) dt - \int_0^t h_P A_P (T_W(t) - T_P(t)) dt \quad (7)$$

In addition, the energy change in the PCM should equal the energy input to the PCM from the water. This can be expressed as

$$E_P = \int_0^t h_P A_P (T_W(t) - T_P(t)) dt. \quad (8)$$

Equations (7) and (8) can be used as “sanity” checks to gain confidence in any solution computed by SWHS. The relative error between the results computed by SWHS and the results calculated from the right side of these equations should be less than 0.001% (R9).

Table 2: Specification Parameter Values

Var	Value
$L_{\min}$	0.1 m
$L_{\max}$	50 m
$\frac{D}{L}_{\min}$	0.002
$\frac{D}{L}_{\max}$	200
minfrac	$10^{-6}$
$h_{\min}$	0.001 m
$\rho_P^{\min}$	500 kg m <sup>-3</sup>
$\rho_P^{\max}$	20000 kg m <sup>-3</sup>
$C_{P\min}^S$	100 J kg <sup>-1</sup> °C <sup>-1</sup>
$C_{P\max}^S$	4000 J kg <sup>-1</sup> °C <sup>-1</sup>
$C_{P\min}^L$	100 J kg <sup>-1</sup> °C <sup>-1</sup>
$C_{P\max}^L$	5000 J kg <sup>-1</sup> °C <sup>-1</sup>
$A_C^{\max}$	$\pi(\frac{D}{2})^2$ m <sup>2</sup>
$\rho_W^{\min}$	950 kg m <sup>-3</sup>
$\rho_W^{\max}$	1000 kg m <sup>-3</sup>
$C_W^{\min}$	4170 J kg <sup>-1</sup> °C <sup>-1</sup>
$C_W^{\max}$	4210 J kg <sup>-1</sup> °C <sup>-1</sup>
$h_C^{\min}$	10 W m <sup>-2</sup> °C <sup>-1</sup>
$h_C^{\max}$	10000 W m <sup>-2</sup> °C <sup>-1</sup>
$h_P^{\min}$	10 W m <sup>-2</sup> °C <sup>-1</sup>
$h_P^{\max}$	10000 W m <sup>-2</sup> °C <sup>-1</sup>
$t_{\text{final}}^{\max}$	86400 s

## 5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

### 5.1 Functional Requirements

R1: Input the following quantities, which define the tank parameters, material properties

Table 3: Output Variables

Var	Physical Constraints
$T_W$	$T_{\text{init}} \leq T_W \leq T_C$ (by A11)
$T_P$	$T_{\text{init}} \leq T_P \leq T_C$ (by A11)
$E_W$	$E_W \geq 0$
$E_P$	$E_P \geq 0$

and initial conditions:

symbol	unit	description
$L$	m	length of tank
$D$	m	diameter of tank
$V_P$	m <sup>3</sup>	volume of PCM
$A_P$	m <sup>2</sup>	phase change material surface area
$\rho_P$	kg/m <sup>3</sup>	density of PCM
$T_{\text{melt}}^P$	°C	temperature at melting point for PCM
$C_P^S$	J/(kg °C)	specific heat capacity of PCM during sensible heating phase
$C_P^L$	J/(kg °C)	specific heat capacity of PCM during latent heating phase
$H_f$	J/kg	specific latent heat of fusion
$A_C$	m <sup>2</sup>	coil surface area
$T_C$	°C	temperature of coil
$\rho_W$	kg/m <sup>3</sup>	density of water
$C_W$	J/(kg °C)	specific heat capacity of water
$h_C$	W/(m <sup>2</sup> °C)	convective heat transfer coefficient between coil and water
$h_P$	W/(m <sup>2</sup> °C)	convective heat transfer coefficient between water and PCM
$T_{\text{init}}$	°C	initial temperature of water and PCM
$t_{\text{final}}$	s	time at end of simulation

R2: Use the inputs in R1 to find the mass needed for IM1 to IM4, as follows:

$$m_W = V_W \rho_W = (V_{\text{tank}} - V_P) \rho_W = (\pi(D/2)^2 L - V_P) \rho_W$$

$$m_P = V_P \rho_P,$$

where  $V_W$  is the volume of water in the tank and  $V_{\text{tank}}$  is the volume inside of the cylindrical tank.

R3: Verify that the inputs satisfy the required physical constraints shown in Table 1.

R4: Output the input quantities and derived quantities in the following list: the quantities from R1, the masses from R2,  $\tau_W$  (from IM1),  $\eta$  (from IM1),  $\tau_P^S$  (from IM2) and  $\tau_P^L$  (from IM2).



- R6: Calculate and output the temperature of the PCM ( $T_P(t)$ ) over the simulation time (from IM2).
- R7: Calculate and output the energy in the water ( $E_W(t)$ ) over the simulation time (from IM3).
- R8: Calculate and output the energy in the PCM ( $E_P(t)$ ) over the simulation time (from IM4).
- R9: Verify that the energy outputs ( $E_W(t)$  and  $E_P(t)$ ) follow the law of conservation of energy, as outlined in Section 4.2.7, with relative error no greater than 0.001%.
- R10: Calculate and output the time at which the PCM begins to melt  $t_{\text{melt}}^{\text{init}}$  (from IM2).
- R11: Calculate and output the time at which the PCM stops melting  $t_{\text{melt}}^{\text{final}}$  (from IM2).

[For R2, why isn't the volume of coil considered when calculating the volume of water? Is the volume of coil assumed negligible? —BM]

## 5.2 Nonfunctional Requirements

Given the small size, and relative simplicity, of this problem, performance is not a priority. Any reasonable implementation will be very quick and use minimal storage. Rather than performance, the priority nonfunctional requirements are correctness, verifiability, understandability, reusability and maintainability.

## 6 Likely Changes

- LC1: A4 - PCM is actually a poor thermal conductor, so the assumption of uniform PCM temperature is not likely.
- LC2: A8 - The temperature of the heating coil will change over the course of the day, depending on the energy received from the sun.
- LC3: A9 - The temperature of the water in the coil will actually change along its length as the water cools.
- LC4: A11 - The model currently only accounts for charging of the tank. A more complete model would also account for discharging of the tank.
- LC5: A12 - To add more flexibility to the simulation, the initial temperature of the water and the PCM could be allowed to have different values.
- LC6: A15 - Any real tank cannot be perfectly insulated and will lose heat.

## References

- [1] J. Frederick Bueche. *Introduction to Physics for Scientists*. McGraw Hill, United States, fourth edition edition, 1986.
- [2] F. P. Incropera, D. P. Dewitt, T. L. Bergman, and A. S. Lavine. *Fundamentals of Heat and Mass Transfer*. John Wiley and Sons, United States, sixth edition edition, 2007.
- [3] Nirmitha Koothoor. A document drive approach to certifying scientific computing software. Master’s thesis, McMaster University, Hamilton, Ontario, Canada, 2013.
- [4] Marilyn Lightstone. Derivation of tank/pcm model. Personal Notes, 2012.
- [5] David L. Parnas and P.C. Clements. A rational design process: How and why to fake it. *IEEE Transactions on Software Engineering*, 12(2):251–257, February 1986.
- [6] W. Spencer Smith and Lei Lai. A new requirements template for scientific computing. In J. Ralyté, P. Ågerfalk, and N. Kraiem, editors, *Proceedings of the First International Workshop on Situational Requirements Engineering Processes – Methods, Techniques and Tools to Support Situation-Specific Requirements Engineering Processes, SREP’05*, pages 107–121, Paris, France, 2005. In conjunction with 13th IEEE International Requirements Engineering Conference.