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Auc2Reserve: A Differentially Private Auction for Electric Vehicle Fast Charging Reservation

Invited Paper

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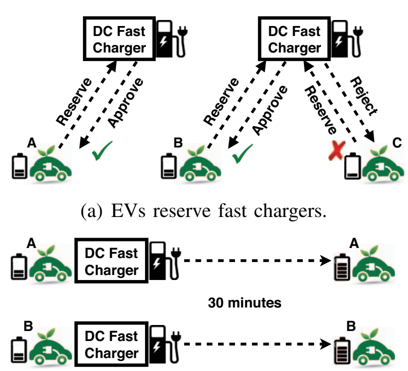
***Abstract*—The increasing market share of electric vehicles (EVs) makes charging facilities indispensable infrastructure for integrating EVs into the future intelligent transportation systems and smart grid. One promising facility called fast charging reservation(FCR) system was recently proposed. It allows people to reserve fast chargers ahead of time. In this system, fast chargers are the most scarce resource instead of electricity. Thus how to allocate these charging points requires careful designing. A good allocation policy should 1) ensure charging points to be allocated to EV users who really value them, and 2) prevent users' private information, e.g., identity, personal agenda, residing area and etc., from being inferred. A simple combination of classic multi-item auction and user identity anonymization cannot satisfy both criteria simultaneously. To find such an allocation, in this paper we investigate the design of privacy-preserving auctions in FCR systems. Traditional privacy-preserving strategies such as cryptography could incur high computation and communication overhead and hence jeopardize the efficiency of allocation. To this end, we propose *Auc2Reserve*, a differentially private randomized auction. *Auc2Reserve* applies an improved approximate sampler and the belief propagation(BP) technique to accelerate the resource allocation and pricing process. As a result, it is much more computationally efficient than generic exponential differentially private mechanisms and other theoretical approximate implementations. Through theoretical analysis, we show that *Auc2Reserve* is γ-incentive compatible, individual rational and ϵ-differentially private. And it provides a close-form approximation ratio in social welfare of FCR systems. In addition, we also demonstrate the efficacy of *Auc2Reserve* in terms of social welfare and privacy leakage via numerical simulation.**

I. Introduction

The electric vehicle (EV) is visioned as a crucial component of intelligent transportation systems (ITS) [5]. Compared with gasoline-powered vehicles, EVs have the potential benefits of a lower carbon emission, a lower powering cost and a higher power efficiency. With these promising benefits, however, they also introduce a high penetration into the power grid by shifting the energy load from gasoline to electricity. As EVs' market share is increasing, the large-scale integration of EVs into the future smart grid has drawn great attention from both academia and industry. And charging facilities have become indispensable infrastructure to support such integration[10][13].

Among various charging facilities have been designed and studied, e.g., home charging point [9] [24], workplace charging facility [10] [23] and etc., one up-and-coming facility called *fast charging reservation* (FCR) system was recently proposed. In an FCR system, EV users can send requests to reserve Direct Current (DC) fast charging points at different locations, which are capable of charging the battery of EV to 80% capacity, i.e., a 0.8 state of charge(SOC), in half an hour. Figure 1 gives an overview of the FCR system. FCR systems facilitate users to charge EVs during a long distance trip without experiencing long-time charging delay. Several FCR systems have been deployed in major automobile markets [4], [1], [2]. For instance, Tesla has deployed over 400 Supercharger stations across the United States [4]. And China has initiated a project to develop a FCR system with over 600 fast chargers along major highways across the country by 2020[2].



(a) EVs reserve fast chargers.

(b) EVs get charged within 30 minutes.

Fig. 1: An overview of fast charging reservation system.

One important observation we can get from these FCR systems is that the main principle when deploying a FCR system is to ensure coverage, i.e. distribute DC fast chargers across a large area. This is because 1) the hardware cost of

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DC fast charger is high; and 2) the short charging time of fast charger determines that a 1:1 ratio between the number of fast chargers and EV is unnecessary. As a result, in the FCR system, the number of fast charging points is much less than that of EVs. This means when EV users send reservation request to the FCR system, they are competing for not only the electricity, but also the fast charging points. It is shown in recent study [3] [35] that in an FCR system where there are more EVs than fast charging points, fast chargers are *the most scarce resource*, instead of the commonly assumed electricity. Therefore, *how to allocate fast chargers* in FCR systems requires careful designing.

Regardless of the differences on hardware and charging schedule, current FCR systems usually adopt a first-reserve-first-serve approach with fixed pricing policies, e.g., pay-per-use or flat-rate, to allocate fast charging points to EV users. Though this strategy is simple and could help the market expanding of EVs, they are not efficient allocation mechanisms in that 1) fast charging points may not be assigned to EV users who really value them, i.e., EVs with a lower state of charge (SOC); and 2) overpricing and underpricing could happen due to the fluctuation of electricity price, thus impairing the benefit of both EV users and fast charging stations. These deficiencies have also been identified in other charging facilities. Recently researchers propose to tackle these drawbacks using auctions as resource allocation strategy for EV charging. Different auctions have been proposed to compute an efficient allocation of electricity and charging points for EV users so that the system social welfare can be maximized. And social welfare is the monetary sum of the revenue of charging facilities and the utility gained by EV users. Exemplary studies in this area include auctions for residential charging [32], park-and-charge [29] and FCR systems [35].

As a powerful tool for resource allocation in EV charging, auctions in current studies are designed to incentivize EV users to truthfully report their valuation on different sets of resources, i.e., incentive compatible, so that social-welfare-maximization allocation and pricing decisions can be derived. However, forcing EV users to reveal their real valuation profile during the auction put users at the risk of exposing their privacy. These real valuation contains users' preferences on different charging points and different amount of electricity. Adversaries may use these information to infer users' personal information, such as transportation agenda, residing area and etc. These information have high commercial value. More importantly, they are also crucial for EV users' personal safety. And due to the fact that EVs need to be charged every one or two days, users may participate EV charging auction frequently, which makes inferences on these information even easier. Though anonymizing users' valuation profile appears to be an efficient approach for protecting these information and users' identity, recent studies [27], [15] show that adversaries can easily deanonymize users' identity and expose all such private profiles by linking two or more separate sets of users' profiles. Such privacy vulnerabilities is a major barrier preventing the large scale deployment of auction-based EV charging resource allocation. In this paper, we aim to find solutions to overcome this obstacle and hence advocate the further development of electric vehicles. In particular, we take FCR systems as an example and explore the feasibility and benefits of designing *an efficient privacy-preserving auction* to allocate fast chargers, the most scarce resource in FCR systems, between EV users. Designing such a mechanism requires us to address a series of challenges:

Challenge 1. The proposed auction should preserve user privacy while satisfying other requirements of mechanism design, i.e., (approximate) incentive compatibility and individual rationality.

Challenge 2. The proposed auction should provide an explicit guarantee on social welfare of FCR systems.

Challenge 3. The proposed auction should be computationally efficient so that the allocation and pricing decisions can be quickly made in large-scale FCR systems.

Dealing with these challenges is a non-trivial mission. Traditional privacy-preserving mechanisms use cryptosystems to protect users' privacy [26][28]. However, the high computation and communication overhead in such cryptosystems often compromise the performance of corresponding mechanisms such as social welfare and incentive compatibility. McSherry *et al.* [25] proposed to incorporate differential privacy in mechanism design. In a differentially private mechanism, it is hard to infer users' personal information as a single change in users' reported valuation has very limited impact on the outcome of the auction. And it is also proved that differential privacy implies approximate incentive compatibility [25]. Nonetheless, the generic differentially private mechanism has an exponential computational complexity. Though some polynomial-time approximate implementation was proposed afterwards [19], it is only theoretical and impractical in real-world due to its $O(n^{13})$ complexity with a large implied constant.

In this paper we cope with the aforementioned challenges by designing *Auc2Reserve*, a computationally efficient differentially private auction. For an FCR system with M EV users competing for N fast chargers, we developed an improved randomized approximate sampler in *Auc2Reserve* to iteratively allocate fast charging points to EV users. Leveraging the fact that there are usually more EV users than fast chargers in FCR system, *Auc2Reserve* randomly selects an EV user (agent) for receiving a given fast charging point (item) at every iteration. In this way, it reduces the sampling overhead by $\frac{M-N}{M}$ times than that of the original sampler [19]. By integrating this allocation process with an approximate pricing function in generic differentially private mechanisms [19] [25], *Auc2Reserve* successfully address Challenges **1** and **2**. Both the allocation and pricing policies in *Auc2Reserve* involve computing the permanent of non-negative matrix, which is #P-complete. To resolve Challenge **3**, we apply the belief propagation technique [11][12] for permanent approximation in *Auc2Reserve*. As a result, not only does *Auc2Reserve* ensure incentive compatibility, individual rationality and differential privacy, it is also computationally efficient in making social-welfare-guaranteed allocation and pricing decisions for FCR systems.

Our **main contributions** in this paper are as follow:

• We study the novel problem of designing privacy-preserving auction for EV fast charging reservation systems. In particular, We propose *Auc2Reserve*, a differentially private randomized auction. Compared with generic exponential differentially private mechanisms and other approximate implementations, *Auc2Reserve* is much more computationally efficient in making allocation and pricing decisions. In addition, *Auc2Reserve* can also be generalized for different scenarios in FCR systems.

• Through theoretical analysis, we show that *Auc2Reserve* is γ-incentive compatible, individual rational and ϵ-differentially private. It also provides a close-form approximation ratio on social welfare of FCR system. We also demonstrate the efficacy of *Auc2Reserve* under various settings of FCR systems in terms of social welfare and privacy leakage via numerical simulation.

The remaining of this paper is organized as follows. We introduce system settings, related solution concepts and present a formal problem definition in Section II. We give an exponential generic differentially private mechanism in Section III. We develop the *Auc2Reserve* differentially private auction for FCR systems and analyze its performance in Section IV. We demonstrate the efficacy of *Auc2Reserve* via simulation in Section V. We discuss related work on EV charging facilities and auction theory in Section VI, and conclude our paper in Section VII.

II. System Settings and Problem Formulation

In this section, we present the settings of EV fast charging reservation systems, discuss related solution concepts and formally define the auction problem in FCR system.

A. System Description

We consider a fast charging reservation system composed of a set of M EV users, indexed by $i = 1, 2, \ldots, M$, and N DC fast charging points, indexed by $j = 1, 2, \ldots, N$. We assume that $M > N$ as the number of EVs is usually higher than that of DC fast charging points in real world systems. In our model we divide time into slots with a equal length of 30-minute. Thus every fast charging can be finished within one time-slot. To keep a concise presentation, we focus on the auction scenario where all EV users submit bids to reserve fast charging points and electricity for the same future time slot. Our solution to this simpler scenario can be easily generalized to the reservation auction in multiple time slots, as will be discussed in Section IV-E.

In the auction, every EV user i can submit multiple bids to the system central controller to reserve fast charging point. These bids are sent via mobile devices, personal computers or a reliable vehicle-to-infrastructure communication system, e.g., OnStar. Every EV user is unit-demand, i.e., she only needs at most one charging point. Thus for every EV user, no duplicate bids on the same charging point is allowed. We use an M-by-N matrix $\mathbf{B}$ to denoted the bids submitted by EV users. The bids submitted by EV user i are denoted by a row vector $\mathbf{b_i}$. This vector is composed of $b_{ij}, j = 1, 2, \ldots, N$, where $b_{ij} \geq 0$ represents user i's *reported valuation* on getting charging point j in a monetary form. A $b_{ij} > 0$ means that user i submitted a bid of value to reserve the charging point j, and a $b_{ij} = 0$ means that user i does not submit any bid to reserve charging point j in this auction. Other than the reported valuation, every user i also has a *real valuation* v_{ij} on reserving every charging point j. This real valuation is private to user i, which can be affected by many factors, e.g., personal agenda, distance to a certain charging station, risk preference and etc., and is also expressed in monetary form. In addition, given an EV user i, we use $\mathbf{b_{-i}}$ and $\mathbf{v_{-i}}$ to represent the reported valuation and real valuation of all EV users other than i, respectively.

After collecting bids from all EV users, a central controller of the FCR system makes allocation decision on fast charging points, and the corresponding pricing decision. We use a set of binary decision variables $y_{ij} \in \{0, 1\}$ to denote the allocation decision for every bid b_{ij}. A $y_{ij} = 1$ means the bid b_{ij} is a winning bid and user i will get a reservation charging point j. And user i need to pay Γ_i for this reservation. A $y_{ij} = 0$ means user i does not win the bid b_{ij} and will pay nothing for this bid. Because every user is unit-demand, we have $\sum_j^N y_{ij} = 1$ for any EV user i. We define u_i, the utility for user i as follows:

$$u_i = \sum_{j=1}^{N} v_{ij} y_{ij} - \Gamma_i$$

In the auction, every user is selfish and aims to maximize her own utility. We use SW to denote the social welfare of FCR system, which is calculated as the sum of the revenue made by fast charging points and the utility of all users. And we can express it as:

$$SW = \sum_i^M \Gamma_i + \sum_i^M u_i = \sum_i^M \sum_j^N v_{ij} y_{ij}. \tag{1}$$

We see that the EV fast charging reservation auction falls into the category of multi-item auction[7][8]. We use $r = \{r[1], r[2], \ldots, r[M]\}$ to denote an allocation outcome, where $r[i]$ records which charging point is assigned to user i. If $r[i] \in [1, N]$, we have $y_{ir[i]} = 1$, otherwise $y_{ir[i]} = 0$. We denote the set of all its allocation outcomes as R. And the number of possible allocation outcomes in the auction, i.e., the cardinality of R, can be expressed as $|R| = \frac{M!}{(M-N)!}$.

B. Solution Concepts

To avoid overpricing and underpricing, and to allocate the fast charging point to EV users who really value it, a good auction mechanism for FCR system should possess the following properties:

Incentive Compatibility. An auction achieves *incentive compatibility* if every user i can always maximize her utility by truthfully reporting her *real valuation* as the *reported valuation* no matter what strategies are adopted by other users, i.e., $u_i(\mathbf{v_i}, \mathbf{b_{-i}}) \geq u_i(\mathbf{b_i}, \mathbf{b_{-i}})$ for any $\mathbf{b_i}$. Incentive compatibility saves users the trouble to perform complex strategic calculations[22]. In addition, we also consider an approximate form of incentive compatibility, called γ-*incentive compatibility*. i.e., $u_i(\mathbf{v_i}, \mathbf{b_{-i}}) \geq u_i(\mathbf{b_i}, \mathbf{b_{-i}}) - \gamma$, where $\gamma \geq 0$ is a small constant. This relaxed definition further simplifies the design and analysis of mechanism. When $\gamma = 0$, we see that it reduces to the original definition of incentive compatibility.

Individual Rationality. An auction achieves *individual rationally* if every participating user always gets non-negative utility regardless what strategy is adopted by her and other users, i.e., $u_i(\mathbf{b_i}, \mathbf{b_{-i}}) \geq 0$, for any $\mathbf{b_i}$. This property is also known as the "participation constraint" [22] [19].

Social Welfare Maximization. As shown in Equation (1), the auction maximizes social welfare by maximizing the total *real valuation* of all winning bids. However, the real valuation v of every bid is private information to EV user, and is unknown to the charging reservation system. When the auction is incentive compatible, the expression of social welfare in Equation (1) can be rewritten as $SW = \sum_i^M \sum_j^N b_{ij} y_{ij}$ since the reported valuation for each bid equals to the corresponding real valuation [22].

Other than incentive compatibility, individual rationality and social welfare maximization, the auction for EV FCR systems also needs to ensure the privacy of EV users. This is because users need to charge their EVs every one or two days. As a result, they would participate fast charger reservation auction frequently, which makes the inference on EV users' personal preferences much easier. Not only do such information have high commercial value, they are also crucial in protecting EV users personal safety. Therefore, it is important to design a privacy-preserving auction for FCR systems. Among various privacy standards, in this paper we focus on *differential privacy*, a paradigm for private data analysis that has drawn much attention in the past decade [15] [36].

Differential Privacy. Given a small constant $\epsilon > 0$, an auction *Auc* is ϵ-differentially private if for any two sets of reported valuation $\mathbf{b}$ and $\mathbf{b}'$ that only differ in one reported valuation, and for any set of allocations $S \subset R(Auc)$, we have

$$\mathbf{Pr}[Auc(\mathbf{b}) \in S] \leq \mathbf{Pr}[Auc(\mathbf{b}') \in S] \cdot exp(\epsilon).$$

Differential privacy has many elegant theoretical properties as well as useful applications [16]. And its feasibility and potentials in mechanism design have been studied under different scenarios, such as digital good auction[22] [19], spectrum auction [37] [36] and etc. Having reviewed related concepts in mechanism design and privacy, we are able to formally define the auction design problem for FCR systems.

FCR-Auc Problem: Given the aforementioned settings of FCR system, design an auction that is γ-incentive compatible, individual rational, ϵ-differentially private, and maximizes the social welfare in a computationally efficient way.

III. Exponential Differential Private Mechanism

One general technique in designing differentially private auction is the exponential mechanism [25] [19]. The basic idea of exponential mechanism, denoted as EXP, is to first compute a probability for every feasible allocation outcome $r \in R$ as follows:

$$\mathbf{Pr}[EXP(R, Q, D, \epsilon) = r] = exp(\frac{\epsilon}{2\Delta} Q(D, r)). \quad (2)$$

In Equation (2), Q is the *quality function* in the differential privacy literature [16]. It takes a data set D and a feasible allocation outcome r as the input and compute a real-valued score as the output. When designing differentially private mechanisms, D is usually the set of reported valuation $\mathbf{b}$, and $Q(\mathbf{b}, r) = \sum_i^M \sum_j^N v_{ij} y_{ij}(r)$ is the social welfare of allocation r. In addition, Δ is the Lipschitz constant. Without loss of generality, we set it to be 1.

With the probability for every feasible outcome r, the exponential mechanism randomly selects one outcome based on the computed probability in Equation (2) as the final allocation decision in the auction, and then makes corresponding pricing decisions. We present the exponential differential private mechanism EXP_ϵ^R for the **FCR-Auc** problem in Algorithm 1. Among different pricing policies, we use the well-studied policy proposed in [19] for EXP_ϵ^R.

EXP_ϵ^R has many nice properties, applying the results in [25] we can have the following theorem on the social welfare of EXP_ϵ^R.

Algorithm 1 EXP_ϵ^R: An Exponential Differentially Private Mechanism for **FCR-Auc** Problem

1: INPUT: An M-by-N bidding matrix $\mathbf{B}$
2: **for** every feasible allocation outcome $r \in R$ **do**
3: $\quad \mathbf{Pr}[r] \rightarrow exp(\frac{\epsilon}{2} \sum_i^M b_{ir[i]})$
4: **end for**
5: **for** every feasible allocation outcome $r \in R$ **do**
6: $\quad \mathbf{Pr}[r] \rightarrow \frac{\mathbf{Pr}[r]}{\sum_{r \in R} \mathbf{Pr}[r]}$
7: **end for**
8: Select an allocation outcome r with probability $Pr[r]$
9: **for** $i \rightarrow 1, 2, \ldots, M$ **do**
10: $\quad \Gamma_i = \frac{2}{\epsilon} \ln\left(\sum_{r \in R} exp\left(\frac{\epsilon}{2} \sum_{k \neq i} b_{kr[k]}\right)\right) - \frac{2}{\epsilon} \cdot S\left(EXP_\epsilon^R(\mathbf{b_i}, \mathbf{b_{-i}})\right) - \underset{r \sim EXP_\epsilon^R(\mathbf{b_i}, \mathbf{b_{-i}})}{\mathbf{E}} \left[\sum_{k \neq i} b_{kr[k]}\right]$
11: **end for**

Theorem 1: EXP_ϵ^R is ϵ-differential private and ensures that

$$\mathbf{Pr}\left[SW_{EXP_\epsilon} < SW_{opt} - \ln\frac{|R|}{\epsilon} - \frac{t}{\epsilon}\right] \leq exp(-t). \quad (3)$$

for any $t > 0$.

Furthermore, in [19] Huang *et al.* proved the following theorem regarding the incentive compatibility and individual rationality of EXP_ϵ^R .

Theorem 2: With the pricing policy in Line 9-11, the exponential mechanism EXP_ϵ^R is incentive compatible and individual rational.

Both theorems have been proved in an elegant way. For instance, the proof of Theorem 2 relies on the connection between exponential mechanism and a well-known probability measure called the Gibbs measure. Interested readers may refer to [25] [19] for more details.

Although with the appealing feature in satisfying all four requirement in **FCR-Auc** problem, EXP_ϵ^R has an important drawback in that it needs to traverse all $\frac{M!}{(M-N)!}$ feasible allocation outcomes and compute a probability to each of them. As a result, this high computational complexity makes EXP_ϵ^R inapplicable in large-scale FCR systems. Therefore, in the next section, we propose $Auc2Reserve$, a computationally efficient differentially private auction as the solution to the **FCR-Auc** problem.

IV. Auc2Reserve: A Differentially Private Auction

In this section, we propose *Auc2Reserve*, a computationally efficient differentially private auction for EV fast charging reservation system. *Auc2Reserve* adopts an approximate random sampler to iteratively allocate fast charing point to EV user. In each iteration, it applies a belief-propagation-based algorithm for matrix permanent approximation. In this way, *Auc2Reserve* is significantly more computationally efficient than exponential differentially private auction EXP_ϵ^R and other theoretical approximate implementations [19]. We show that *Auc2Reserve* is γ-incentive compatible, individual rational, ϵ-differentially private and provides an explicit guarantee on social welfare of FCR systems. We also discuss the generalization of *Auc2Reserve* in other scenarios of FCR systems.

A. Preliminary

Before we present the design of *Auc2Charge*, we first review an important concept in linear algebra called *permanent*.

Definition 1: Given a K-by-K matrix $\mathbf{W}$, $\mathbf{W} = (W_{ij}|i, j = 1, 2, \dots, K)$, its permanent is defined as

$$perm(\mathbf{W}) = \sum_{\pi \in \Pi(K)} \prod_{i=1}^{K} W_{i\pi[i]}, \tag{4}$$

where $\Pi(K)$ is the set of all permutations of set $\{1, 2, \dots, n\}$. One important property of permanent we leverage in the design of *Auc2Charge* is that $perm(\mathbf{W}) = perm(\mathbf{W}^T)$, where $\mathbf{W}^T$ is the transpose of $\mathbf{W}$. Permanent has many important applications, i.e., the permanent of a 0-1 matrix is equivalent to the number of perfect matchings of a bipartite undirected graph. However, computing the permanent is a complex problem. The fastest known general algorithm in computing the permanent of matrix is the Ryser Algorithm, which requires $O(n2^n)$ operations. Even for the case of non-negative matrix, i.e., $W_{ij} \geq 0, \forall i, j$, finding its permanent is a #P-complete problem. Therefore, people often resort to different approaches to compute the approximate permanent, denoted as $perm_a$, which is a challenging task as well. To keep the intactness of the presentation, we leave the discussion on how to approximate matrix permanent later till Section IV-C.

B. Auc2Reserve in a NutShell

We present the design of *Auc2Resrve* auction in Algorithm 2. In the **FCR-Auc** problem, the bidding matrix $\mathbf{B}$ is of size M-by-N, where $M > N$. Other than N actual fast charging points, we add another $M - N$ dummy charging points into the auction. And the reported valuation from all EV users on these dummy charging points are all zeros. In this way, *Auc2Reserve* creates a M-by-M square bidding matrix $\mathbf{B_{sq}}$ by concatenating a M-by-$(M - N)$ zero matrix to $\mathbf{B}$ (Line 2). After the transformation, we construct an *auxiliary bidding matrix* D, which is the transpose of $\mathbf{B_{sq}}$ (Line 3). For any square matrix $\mathbf{W}$, we define a function G: $G(\mathbf{W}) = \{exp(\frac{\epsilon}{2} w_{ij})\}, \forall i, j$. And we also use $\mathbf{W}_{-i,-j}$ to denote the matrix obtained by removing the ith row and the jth column of suqare matrix $\mathbf{W}$.

In *Auc2Reserve*, we developed an improved approximate sampler that iteratively allocates charging points to EV users, one at a time. Given an actual charging point $j = 1, 2, \dots, N$, we first compute x_i, the probability that EV user i wins the reservation of charging point j, for all the users who have not won any charging point yet (Line 6-15). *Auc2Reserve* then randomly selects an EV user i with the normalized probability x_i as the winner of charging point j (Line 16). Once a charging point j is allocated to a winning user i_c, all bids submitted to reserve j or submitted by user i_c are removed from the auxiliary matrix $\mathbf{D}$ (Line 19). We then repeat the same allocation process using the updated matrix $\mathbf{D}$ until all N actual charging points are allocated.

The allocation process in *Auc2Reserve* is an item-oriented randomized allocation, i.e., randomly select a user (agent) to receive a given charging point (item). It differs from the original sampler in [19], which uses an agent-oriented randomized allocation, i.e., randomly select an item to assign to a given agent. With the property that $perm(\mathbf{W} = perm(\mathbf{W}^T)$, both allocation policies would yield the same output in expectation. However, given the fact that there are more EV users than fast charging points in the FCR system, i.e., $M > N$, the item-oriented allocation used in *Auc2Reserve* is more computationally efficient in that it reduces the computation overhead by $\frac{M-N}{M}$ times than that of agent-oriented allocation. This reduction is significant due to the involvement of matrix permanent computing during the allocation process.

Algorithm 2 *Auc2Reserve*: A Differentially Private Auction for **FCR-Auc** Problem

1: INPUT: An M-by-N bidding matrix $\mathbf{B}$
2: $\mathbf{B_{sq}} \rightarrow \mathbf{B}||\mathbf{0}_{M\times(M-N)}$
3: $\mathbf{D} \rightarrow \mathbf{B_{sq}}^T$
4: $I \rightarrow \{1, 2, \dots, M\}$
5: **for** $j \rightarrow 1, 2, \dots, N$ **do**
6:   **for** $i \rightarrow 1, 2, \dots, M - j + 1$ **do**
7:     **if** $d_{1i} == 0$ **then**
8:       $x_i \rightarrow 0$
9:     **else**
10:       $x_i \rightarrow perm_a(G(\mathbf{D}_{-1,-i}))$
11:     **end if**
12:   **end for**
13:   **for** $i \rightarrow 1, 2, \dots, M$ **do**
14:     $x_i \rightarrow \frac{x_i}{\sum_i x_i}$
15:   **end for**
16:   Randomly allocate charging point j to user i with probability x_i, denote the assigned user as i_c
17:   $y_{I[i_c]j} \rightarrow 1$
18:   $\Gamma_{I[i_c]} \rightarrow b_{I[i_c]j} + \frac{2}{\epsilon} \ln\Big(perm_a\big(G(\mathbf{B}(\mathbf{b_i} = \mathbf{0}, \mathbf{b_{-i}}))\big)\Big) - \frac{2}{\epsilon} \ln\Big(perm_a\big(G(\mathbf{B})\big)\Big)$
19:   $\mathbf{D} \rightarrow \mathbf{D}_{-1,-i_c}$
20:   $I \rightarrow \{1, \dots, I[i_c] - 1, I[i_c] + 1, \dots, M\}$
21: **end for**
22: **for** $i \rightarrow 1, 2, \dots M$ **do**
23:   **if** $\sum_j y_{ij} == 0$ **then**
24:     $\Gamma_i \rightarrow 0$
25:   **end if**
26: **end for**

After an actual fast charging point j is allocated to a winning user i_c, *Auc2Reserve* computes the price i_c needs to pay in Line 18 of Algorithm 2. This pricing policy is an approximated price of that in the exponential mechanism EXP_ϵ^R. After all actual charging points have been assigned, users who do not win any charging point do not need to pay anything (Line 22-26).

Observe the structure of Algorithm 2, we see that the key step in determining the computational efficiency of *Auc2Reserve* is to compute the permanent of non-negative matrix. This step is needed in both allocation and pricing process. Due to the #P-completeness of this task[33] , in the following we review possible approaches in approximating matrix permanent, and apply a newly developed belief propagation approximation technique in *Auc2Reserve*.

C. Approximating Matrix Permanent Using Belief Propagation

Researchers have explored different approaches in developing approximate algorithms for matrix permanent. In the seminal paper [20], Jerrum *et al.* proposed a fully polynomial randomized approximation scheme (FPRAS) for computing the permanent of non-negative matrix using Monte-Carlo sampling. However, its complexity is $O(n^{11})$ in the general case. Even though the complexity of this approach was later improved to $O(n^7)$ in [6], it is still impractical for most realistic applications.

Recently, belief propagation heuristics is applied to computing the matrix permanent and showed surprisingly good performance[11][12]. The basic idea of this approach is that given a non-negative K-by-K matrix $\mathbf{W}$, we can build a graphical model with a bipartite undirected graph $G = (V_1, V_2, E)$. In this graph, V_1 and V_2 are two sets of K vertices. And binary variables $\sigma_{ij} = 0, 1$ are assigned to edges (i, j) with the constraints $\forall i \in V_1, \sum_{j \in V_2} \sigma_{ij} = 1$ and $\forall j \in V_2, \sum_{i \in V_1} \sigma_{ij} = 1$. And W_{ij} is assigned as the weight of edge (i, j) in graph G. In this way, the graphic model can express the permanent of matrix W as:

$$perm(\mathbf{W}) = \sum_{\sigma} \prod_{(i,j) \in E} (W_{ij})^{\sigma_{ij}}. \tag{5}$$

With this graphical model, we define a K-by-K matrix β where β_{ij} is the marginal belief corresponding to finding edge (i, j) in the matching of G. And the following theorem on BP-based permanent approximation was proposed in [11].

Theorem 3: Given a non-negative K-by-K matrix $\mathbf{W}$ with corresponding graph G and marginal belief matrix β. A belief propagation function can be developed as $perm(\mathbf{W})$ as

$$F_{BP}(\beta|\mathbf{W}) = \sum_i \sum_j \Big(\beta_{ij} \log(\frac{\beta_{ij}}{W_{ij}}) - (1-\beta ij) \log(1-\beta_{ij})\Big), \tag{6}$$

and the permanent of $\mathbf{W}$ can be approximated as:

$$perm_{BP}(\mathbf{W}) = exp\Big(-F_{BP}(\beta|\mathbf{W})\Big). \tag{7}$$

To compute the marginal belief matrix β, standard belief propagation technique can be applied, and an iterative algorithm is developed and presented as Algorithm 3. The convergence of this algorithm is proved in [11]. However, its convergence speed highly depends on the initial value of matrix $\beta(0)$. To ensure a fast convergence and hence the computational efficiency of *Auc2Charge*, we use a mean-field heuristic algorithm, presented as Algorithm 4, to precompute a matrix ϕ as the initial value $\beta(0)$. Extensive empirical experiments in [11] [12] show that using ϕ computed by Algorithm 4 as the initial value of marginal belief matrix β ensures the fast convergence of Algorithm 3 with arbitrary input matrix $\mathbf{W}$. In addition, the following Theorem 4 provides a close-form approximation ratio on this BP-based method [12].

Theorem 4: Given a K-by-K matrix $\mathbf{W}$, compute the doubly stochastic matrix β using Algorithm 3, and define

$$\alpha_{\mathbf{W}} = \frac{\prod_{i,j}(1-\beta_{ij})}{perm\Big(\beta \cdot (1-\beta)\Big)}. \tag{8}$$

Then the approximate permanent computed from Theorem 3 satisfies:

$$perm_{BP}(\mathbf{W}) = \alpha_{\mathbf{W}} \cdot perm(\mathbf{W}). \tag{9}$$

To summarize, by transforming a given matrix $\mathbf{W}$ to a graphical model and applying belief propagation technique

Algorithm 3 An Iterative Algorithm for Computing Marginal Belief Matrix β

INPUT: a K-by-K non-negative matrix $\mathbf{W}$
$n \to 0$
$u_i(0) = u^j(0) = 1, \forall i, j \to 1, 2, \ldots, K$
Initialize $\beta_{ij}(0)$, $\forall i, j \to 1, 2, \ldots, K$
while 1 **do**
  for $i, j \to 1, 2, \ldots K$ **do**
    $\beta_{ij}(n+1) \to \lambda \beta_{ij}(n)$
    $+\dfrac{(1-\lambda)W_{ij}}{W_{ij} + (\frac{1}{2}\sum_s \beta_{is}(n) + \frac{1}{2}\sum_s \beta_{sj}(n) - \beta_{ij}(n))^2 u_i(n) u^j(n)}$
  end for
  for $i, j \to 1, 2, \ldots K$ **do**
    $\beta_{ij}(n) \to \beta_{ij}(n) / \sum_s \beta_{is}(n)$
  end for
  for $i, j \to 1, 2, \ldots K$ **do**
    $\beta_{ij}(n) \to \beta_{ij}(n) / \sum_s \beta_{sj}(n)$
  end for
  for $i \to 1, 2, \ldots K$ **do**
    $u_i(n+1) \to \frac{\sum_s (W_{is}/u^s(n))}{1-\sum_j (\beta_{ij}(n))^2}$
  end for
  for $j \to 1, 2, \ldots K$ **do**
    $u^j(n+1) \to \frac{\sum_s (W_{sj}/u_s(n))}{1-\sum_i (\beta_{ij}(n))^2}$
  end for
  if $max_{i,j}\{|\beta_{ij}(n+1) - \beta_{ij}(n)|\} > \delta$ **then**
    $n \to n+1$
  else
    for $i, j \to 1, 2, \ldots K$ **do**
      $\beta_{ij} \to \beta_{ij}(n)$
    end for
    break
  end if
end while

to computing its marginal belief matrix, $perm(W)$ can be approximated very efficiently with a close-form approximation ratio. We leave the theoretical proof on convergence speed of this BP approach in future work. In this way, *Auc2Reserve* is significantly more computationally efficient than the generic exponential differentially private mechanism EXP_{ϵ}^{R} and other theoretical approximate implementations[19].

D. Properties of Auc2Reserve

We now analyze the performance of *Auc2Reserve* auction in terms of incentive compatibility, individual rationality, differential privacy and social welfare. To this end, we first propose the following theorem.

Theorem 5: Auc2Reserve is an ϵ-differential private and individual rational.

Proof 1: The proof of this theorem follows the sketch in [25] [19]. It relies on the connection between the exponential mechanism and the well-known Gibbs measure, which is also known as the Boltzmann distribution. With the connection established in [19], the correctness of this theorem is a straightforward conclusion.

Next we study the incentive compatibility and social welfare of *Auc2Reserve*. Given a matrix $\mathbf{W}$, we define $\alpha_{\mathbf{W}}^{max} = max_{s(\mathbf{W})}\{\alpha_{s(\mathbf{W})}\}$, where $s(\mathbf{W})$ is any sub-matrix of $\mathbf{W}$. Then we are able to propose the following theorem.

Theorem 6: Given any M-by-N bidding matrix $\mathbf{B}$, the Auc2Reserve mechanism is γ-incentive compatible and the

Algorithm 4 A Mean-Field Algorithm for Initializing Marginal Belief Matrix $\beta(0)$

INPUT: a K-by-K non-negative matrix $\mathbf{W}$
$n \to 0$
$v_i(0) = v^j(0) = 1, \forall i, j \to 1, 2, \ldots, K$
$\phi_{ij}(0) = 1, \forall i, j \to 1, 2, \ldots, K$
while 1 **do**
  for $i, j \to 1, 2, \ldots K$ **do**
    $\phi_{ij}(n+1) \to \dfrac{W_{ij}}{W_{ij} + v_i(n)v^j(n)}$
  end for
  for $i \to 1, 2, \ldots K$ **do**
    $v_i(n+1) \to v_i(n) \sum_j \phi_{ij}(n)$
  end for
  for $j \to 1, 2, \ldots K$ **do**
    $v^j(n+1) \to v^j(n) \sum_i \phi_{ij}(n)$
  end for
  if $max_{i,j}\{|\phi_{ij}(n+1) - \phi_{ij}(n)|\} > \delta$ **then**
    $n \to n+1$
  else
    for $i, j \to 1, 2, \ldots K$ **do**
      $\beta_{ij}(0) \to \phi_{ij}(n)$
    end for
    break
  end if
end while

social welfare computed by Auc2Reserve satisfies:

$$\mathbf{Pr}\left[SW < SW_{opt} - M\gamma - \frac{M!}{\epsilon(M-N)!} - \frac{t}{\epsilon}\right] \leq exp(-t), \quad (10)$$

where $\gamma = M \ln(\alpha^{max}_{G(\mathbf{B_{sq}})})$ and $\mathbf{B_{sq}}$ is defined in Line 2 of Algorithm 2, for any $t > 0$.

Proof 2: The basic idea in this proof is that according to Bayes' rule and the BP-based permanent approximation ratio in Theorem 4, in every iteration of Algorithm 2, the probability that charging point j is allocated to an user i approximates the correct exponential allocation distribution by a multiplicative factor of $\alpha^{max}_{G(\mathbf{B})}$. So after allocating all M fast charging points, the probability that we sample an allocation outcome $r \in R$ differs from the correct exponential allocation distribution by a multiplicative factor of $(\alpha^{max}_{G(\mathbf{B_{sq}})})^M$. Then the correctness of this theorem can be achieved by directly applying the results in [25] [19]. To begin with, we first prove the following lemma:

Lemma 1: Auc2Reserve yields an outcome $r \in R$ differing from the correct distribution by at most $(\alpha^{max}_{G(\mathbf{B_{sq}})})^M$ factor.

We use $j \to i$ to denote that charing point j is assigned to user i. In the exponential mechanism EXP^R_ϵ, for any allocation outcome $r \in R$, using the Bayes' rule, the probability r is chosen as the final allocation outcome can be expressed as follows:

$$\begin{aligned} \mathbf{Pr}[EXP^R_\epsilon(\mathbf{b}) = r] \quad &= \mathbf{Pr}[\text{point 1 is assigned to } r^{-1}[1]] \\ &\cdot \mathbf{Pr}[\text{point 2 is assigned to } r^{-1}[2] | 1 \to r^{-1} \\ &\ldots \cdot \mathbf{Pr}[\text{point } N \text{ is assigned to } r^{-1}[N] \\ &|1 \to r^{-1}[1], \ldots, N-1 \to r^{-1}[N-1]]. \end{aligned} \quad (11)$$

Take the first iteration of our *Auc2Reserve* algorithm as start, we use the distribution

$$\mathbf{Pr}[\text{point 1 is assigned to user } i] = x_i \propto perm_a(G(\mathbf{D}_{1,i})).$$

In the exponential mechanism EXP^R_ϵ,

$$\mathbf{Pr}[\text{point 1 is assigned to user } i] \propto exp(\frac{\epsilon}{2} b_{i1}) perm_a(G(\mathbf{D}_{1,i}))$$

Because x_i approximates $perm_a(G(\mathbf{D}_{1,i}))$ by up to an multiplicative factor of $\alpha^{max}_{G(\mathbf{B_{sq}})}$, we know that the probability that user i gets charging point 1 in *Auc2Reserve* approximates the correct marginal in EXP^R_ϵ up to an $\alpha^{max}_{\mathbf{B_{sq}}}$ multiplicative factor. Not only for the first iteration, we can find this $\alpha^{max}_{G(\mathbf{B_{sq}})}$ multiplicative factor holds for the remaining iterations in *Auc2Reserve*. Therefore, the probability that *Auc2Reserve* yields an outcome $r \in R$ differing from the correct distribution by at most $(\alpha^{max}_{G(\mathbf{B_{sq}})})^M$ factor.

Having proved this lemma, the correctness of this theorem can be achieved by directly applying the results in [25] [19].

Theorem 6 provides an upper bound on the social welfare of *Auc2Reserve*. We observe that this bound decreases as the number of EV users increases, and increases as ϵ and N increases. As the differential privacy factor, a smaller ϵ is the indicator of a stronger differential privacy. Therefore, this upper bound indicates that there exists an explicit tradeoff between maximizing social welfare and providing stronger differential privacy for more EV users. Theorems 5 and 6 together show that *Auc2Reserve* satisfies all four requirements in the **FCR-Auc** problem. Compared to the exponential mechanism EXP^R_ϵ, *Auc2Reserve* achieves a much higher computationally efficiency via a tradeoff on an additional γ factor in incentive compatibility and an additional $M\gamma$ factor in social welfare.

E. Generalization of Auc2Reserve

We design *Auc2Reserve* under the scenario where all EV users submit bids to reserve fast charging points for the same future time slot. In fact, *Auc2Reserve* can be applied to other generalized scenarios of FCR systems. One scenario is that future time slots are to be available for reservation one by one. Then the central controller in FCR system simply needs to execute *Auc2Reserve* for every future time slot after it becomes available for reservation. Another scenario is that the administrator opens the reservation for N fast charging points at $T > 1$ different time slots at one time, and make the allocation and pricing decisions all at once. In this case, the central controller can construct a bidding matrix of size M-by-NT and then execute *Auc2Reserve* using this matrix as the input. If $M > NT$, *Auc2Reserve* will use the same item-oriented allocation process. If $M \leq NT$, an agent-oriented allocation process will be applied. In both scenarios, *Auc2Reserve* is able to make fast charging point allocation and pricing decisions in a computationally efficient manner, i.e., providing an explicit guarantee on social welfare of FCR system, and ensure γ-incentive compatibility, individual rationality and ϵ-differential privacy simultaneously.

V. PERFORMANCE EVALUATION

[1]] In this section, we demonstrate the efficiency of our proposed *Auc2Reserve* mechanism for EV fast charging reservation via numerical simulation.

Methodology. In our simulation, we assume a virtual traffic network where M electric vehicles and N DC fast charging points are randomly distributed. Because EV users usually want to get fast charging at certain locations, they would only submit bids to a limited number of fast charging points. Therefore, in the simulation, we assume that every EV user will submit at most 8 bids in the form of *(valuation, charging point)* to compete for reservation at these fast charging points. We assume that the budget, i.e., the maximal valuation, of each EV user, follows a uniform distribution between 8 and 12 dollars. For every EV user j, the *valuation* and *charging point* in her bids are separately randomly generated. In our simulation, we set the parameters λ and δ in the BP-based permanent approximation algorithm as 0.7 and 0.1, respectively. We perform simulation of *Auc2Reserve* under the settings of $M = 60, 70, 80, 90, 100$ EVs and $N = 10, 20, 30, 40$ fast charging points. And we set the privacy parameter ϵ in *Auc2Reserve* to be 0.1 and 0.5. For each combination of M, N and ϵ, we repeat the simulation for 20 times and compute the average value.

Results. In what follows, we evaluate the performance of *Auc2Charge* on both social welfare and user privacy. Figure 2 shows the social welfare achieved by *Auc2Reserve* under different numbers of EVs when the number of fast charging points is fixed at 40 in the FCR system. We first see that the social welfare of *Auc2Reserve* with $\epsilon = 0.1$ is smaller than that with $\epsilon = 0.5$ at all cases. This is because a smaller ϵ represents a stronger differential privacy of EV users. And as pointed in Theorem 6, this stronger is achieved through a trade of social welfare loss. The smaller ϵ is, the higher this loss becomes. An interesting observation we find is that when ϵ and number of charging points are fixed, the social welfare of *Auc2Reserve* decreases as the number of EVs increases. The reason of this observation is as follows. In our simulation, the reservation bids submitted by EV users are generated following the same distribution. Thus when the number of fast chargers are fixed, the optimal social welfare SW_{opt} should also be a fixed value in expectation, and independent from the number of EV users. However, when there are more EVs participating the auction, *Auc2Reserve* ensures the differential privacy of EV users through a higher tradeoff of social welfare loss. This observation is consistent with the social welfare bound of *Auc2Reserve* in Equation (10) from Theorem 6. From this equation we see that even if SW_{opt} is fixed, both $M\gamma$ and $\frac{M!}{(M-N)!}$ increase as M increases, which leads to the decrease of upper bound on *Auc2Reserve*'s social welfare.

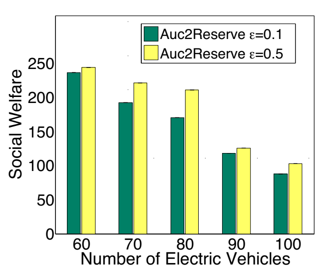


Fig. 2: Social Welfare with $N = 40$ Fast Charging Points

We then plot the social welfare of *Auc2Reserve* under different number of fast chargers when there are 100 EVs in the FCR system in Figure 3. We also observe that the social welfare when $\epsilon = 0.5$ is higher than that of $\epsilon = 0.1$ due to the stronger differential privacy of the latter. And opposite from the trend of varying EV numbers, *Auc2Reserve* yields a higher social welfare when there are more fast charging points in the system. This is because with more resources in the auction, there are more EV users being allocated a fast charger when executing *Auc2Reserve*. And it is also consistent with Equation (10), where the social welfare loss $\frac{M!}{(M-N)!}$ decreases as the number of fast chargers N increases.

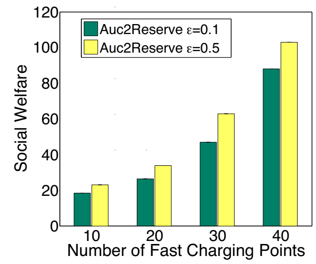


Fig. 3: Social Welfare with $M = 100$ Electric Vehicles

Other than social welfare, we also use the recent proposed notion *privacy leakage* [37] [36] to evaluate the efficacy of *Auc2Charge* in protecting user privacy.

Definition 2: (*Privacy Leakage*[36]) Let $\vec{a}$ and $\vec{a'}$ be probability distributions over a price set P for bidding matrix $\mathbf{B}$ and $\mathbf{B}'$, which only differ in one single bid, respectively. The *privacy leakage* between the two bidding matrices is the maximum of absolute differentces between the logarithmic probabilities of the two distributions, i.e.,

$$max_i |\ln a_i - \ln a'_i|. \tag{12}$$

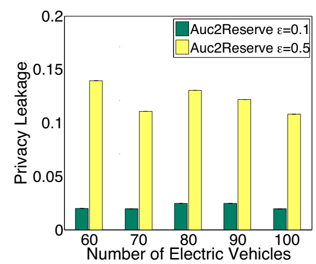


Fig. 4: Privacy Leakage with $N = 40$ Fast Charging Points

Given an auction, a smaller privacy leakage implies that when there is one arbitrary EV user changes one of her bids in the FCR system, the probability distribution over pricing decisions made by this auction would only have a small change as well, which proves the differential privacy of this auction.

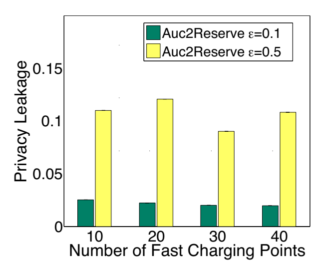
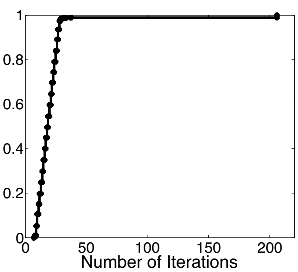
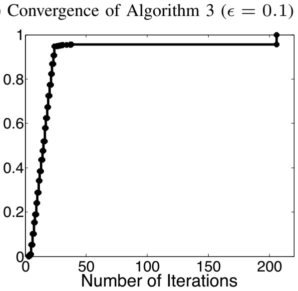


Fig. 5: Privacy Leakage with $M = 100$ Electric Vehicles

Figure 4 plot the privacy leakage of *Auc2Reserve* under different number of EVs with 40 fast chargers in the FCR system. We observe that when $\epsilon = 0.1$, the privacy leakage of *Auc2Reserve* is less than 0.02. And when $\epsilon = 0.5$, this leakage is less than 0.15. This observation implies that it is almost impossible to for an adversary to infer the personal information of EV users, e.g., charging location preference. When fixing the number of EVs as 100, from Figure 5 we also have a similar observation on the privacy leakage of *Auc2Reserve* under different number of fast chargers. Therefore, *Auc2Reserve* is efficient in ensuring EV users' privacy.



(a) Convergence of Algorithm 3 ($\epsilon = 0.1$)



(b) Convergence of Algorithm 3 ($\epsilon = 0.5$)

Fig. 6: Computational Efficiency of *Auc2Reserve* with $M = 100$ Electric Vehicles

Furthermore, we demonstrate the computational efficiency of *Auc2Reserve* by plotting the cumulative distribution function on number of iterations in Algorithm 3 when it converges. It is shown in Figure 6 that in *Auc2Reserve*, the permanent of bidding matrices can be approximated very fast under different values of ϵ, i.e., converging within 30 iterations in 99% cases. With this fast convergence, *Auc2Reserve* is highly computationally efficient in making fast charger allocation and pricing decisions for FCR systems.

VI. Related Work

EV Charging Facilities. EV charging facilities are indispensable infrastructure of both intelligent transportation systems and smart grid, and have drawn great attention from both academia and industry. There has been a growing literature on various EV charging facilities [23], [10], [5], [17], [9], [4], [1], [2]. Ardakanian *et al.* [5] designed a distributed charging algorithm to adjust EV charging rate for residential chargers. Lopes *et al.* [23] designed a framework to integrate EVs into power system. Chen *et al.*[10] designed a central controller to schedule the EV charging using renewable energy. Chen *et al.* [9] studied a joint optimal power flow and EV charging problem, and built an online controller to enable efficient EV charging. Jin *et al.* [21] built a stochastic optimization framework to minimize the cost of single charging station.

Recently, the fast charging reservation (FCR) system, an innovative charging facility, was developed and has drawn special attention form both academia and industry. In this system, EV users can reserve DC fast chargers at different locations ahead of time. which charge the battery of EV to 80% capacity within 30 minutes. The FCR system facilitate people to charge EVs during a long distance trip with a short time delay, and thus are welcomed by EV users. In major automobile markets, several FCR systems have been developed. Tesla has deployed a Supercharger network with over 400 Supercharger stations across the United State s[4]. BMW and ChargePoint develop the ChargeNow program, in which BMW EV users can reserve public fast chargers via mobile devices [1]. China has initiated a project to develop a fast charging reservation system with over 600 fast chargers along major highways across the country by 2020 [2]. And over 100 fast chargers have been deployed by early 2015. In FCR systems, there are usually more EVs than fast charging points. And recent studies [3] [35] show that fast chargers are in fact the most scarce resource in FCR systems, instead of the commonly assumed electricity. Thus *how to allocate fast chargers* between EV users requires careful study.

Auction Theory. Auction allocates resources to buyers who value them most, reduces the chance of overpricing and underpricing, and thus improves social welfare. It has been widely used in Internet advertisement [14], wholesale electricity market [30] and cloud computing [31]. Recently researchers propose to utilize auction to improve resource allocation efficiency for EV charging, and different auctions are designed for different scenarios [18], [34], [29]. Gerding *et al.* [18] proposed a two-side truthful online auction with advanced reservation, in which EV users and the charging station can exchange their charging preference and cost. Robu *et al.* [29] designed an online mechanism, in which EV users bid for different charging speeds based on their arrival time, and cancel the charging allocation on departure. Xiang *et al.* [34] proposed an online auction framework for EV park-and-charge. However, these auctions achieve social welfare maximization by incentivizing EV users to truthfully report their valuation on different set of resources, e.g., electricity and charging points, putting EV users at the risk of exposing their privacy. Adversaries may use these real valuations to infer EV users' personal information. And this inference becomes even easier when EV users participate charging auctions frequently. Therefore, a privacy-preserving auction is desired to protect the privacy of EV users against such inference.

McSheery *et al.* first proposed to use differentially private mechanisms in auction design in [25]. They showed that differential privacy implies approximate incentive compatibility, and designed exponential mechanism for differentially private digital auctions and attribute auctions. Huang *et al.* [19] instantiate the principle in [25], and develop an approximate implementation for generic differentially private auctions. Though the implementation has a polynomial-time complexity, i.e., $O(n^{13})$. It is impractical in real-world as the implied constant in the O function is very large. Zhu *et al.* designed differentially private mechanisms for spectrum auction in [37] [36]. The proposed algorithms leverage unique characteristics in spectrum auctions and achieves approximate revenue maximization. In our *Auc2Reserve*, we developed an improved approximate sampler for fast charger allocation. To accelerate the allocation, we applies the belief-propagation technique in matrix permanent approximation. Therefore, not only is *Auc2Reserve* γ-incentive compatible, individual rational, ϵ-differentially private, it also makes fast charging points allocation and pricing decisions with a close-form social welfare guarantee in a computational efficient way. To the best of our knowledge, *Auc2Reserve* is the first differentially private auction for EV fast charging reservation systems.

VII. Conclusion

The FCR system allows EVs to send requests to reserve DC fast chargers ahead of time. In this system, the allocation of fast charging points requires careful design because they are the most scarce resource instead of electricity. Not only charging points should be allocated to EV users who really values them, the allocation and the corresponding pricing policies should also prevent users' private information from being inferred. In this paper, we explore the feasibility and benefits of differentially private auction in FCR systems. We design *Auc2Reserve*, a differentially private randomized auction for FCR system. *Auc2Reserve* applies the belief propagation technique to accelerate the randomized allocation process. Thus it is significantly more computationally efficient than generic differentially private mechanisms. To the best of our knowledge, we are the first to apply belief-propagation in designing computationally efficient differentially private mechanisms. *Auc2Reserve* is γ-incentive compatible, individual rational, ϵ-differentially private and provides an explicit approximation ratio on the social welfare of FCR systems. Using simulation, we further demonstrate the efficiency of *Auc2Reserve* on social welfare and privacy leakage under various settings of FCR systems. In future work, we plan to extend the *Auc2Reserve* mechanism by including other realistic constraints in both the electricity market, e.g., electricity allocation, vehicle-to-grid transmission and ramp-up/ramp-down cost of electricity generation, and intelligent transportation systems, e.g., the uncertainty of EV's mobility. We will also explore the feasibility of computationally efficient differentially private auctions for other charging facilities, e.g., EV park-and-charge lot.

VIII. Acknowledgment

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