AFP1

`8.2.80` `Calculus II` `June 8, 2022` `Soobin Rho`

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80. $\displaystyle\int \log_2 x \, dx$

Figure 1. Problem 8.2.80

Steps

At first, I had no idea how to begin this problem. Especially, I was not sure about the instruction right above the question, which was this:

Use the formula

$$\int f^{-1}(x)\,dx = xf^{-1}(x) - \int f(y)\,dy \qquad y = f^{-1}(x) \qquad (4)$$

to evaluate the integrals in Exercises 77–80. Express your answers in terms of x.

Figure 2. This description was right above the question.

I thought to myself, I cannot understand how something for inverse functions can be used for integrating $log_2(x)$. So, here's what I did. I skipped over this formula; instead, I started somewhere else.

First, I evaluated this function on Maple -- I started backwards. I got the answer first, and then I worked backwards so that I can understand the problem and become able to solve the problem without Maple next time.

On Maple, I wrote `integrate( log[2](x), x )` and it gave me the following answer:

$$\frac{x\ln(x)}{\ln(2)} - \frac{x}{\ln(2)}$$

Figure 3. I evaluated the problem on Maple first.

This gave me a random idea. Is this something to do with converting log_2 with a different base to ln first? So, I tried converting $log_2(x)$ to ln. I did this by using the fact that

$log_b(x) = \frac{log_d(x)}{log_d(b)}$. This gave me:

$$log_2(x) = \frac{ln(x)}{ln(2)}$$

To check if I am going in the right direction, I tried evaluating this on Maple. I wrote `integrate( ln(x)/ln(2), x )` which gave me:

$$\frac{x \ln(x)}{\ln(2)} - \frac{x}{\ln(2)}$$

Figure 4. After I converted the function from log base 2 to the natural logarithm ln, it was evaluated again on Maple to check if I am headed towards the right direction.

It turns out that `integrate( log[2](x), x )` and `integrate( ln(x)/ln(2), x )` give the same answer! Well, this may sound obvious to you, but it was not for me :) Anyways, what I did now was to change the original questoin from $\int log_2(x)\, dx$ into $\int \frac{ln(x)}{ln(2)}\, dx$

What, then, is the answer to this?

$$\int \frac{ln(x)}{ln(2)}\, dx$$

Notice that $ln(2)$ is a constant. Therefore, it is possible to change the above to this:

$$\frac{1}{ln(2)} \int ln(x)\, dx$$

Then, how can I integrate $ln(x)$? I used the intergration by parts method. Suppose that

$$f(x) = ln(x) \qquad g(x) = x$$

$$f'(x) = \frac{1}{x} \qquad g'(x) = 1$$

Then, $\int f(x)g'(x)\, dx = f(x)g(x) - \int f'(x)g(x)\, dx$, so

$$\frac{1}{ln(2)} \int ln(x)\, dx = \frac{1}{ln(2)}[xln(x) - \int \frac{1}{x} x\, dx]$$

$$= \frac{1}{ln(2)}[xln(x) - \int \frac{x}{x}\, dx]$$

$$= \frac{1}{ln(2)}[xln(x) - \int 1\, dx]$$

$$= \frac{1}{ln(2)}[xln(x) - x + C]$$

$$= \frac{xln(x) - x + C}{ln(2)}$$

The answer I got here is the same as the one Maple gave us at first -- they look different, but they are just in different forms:

$$\int log_2(x)\,dx = \frac{xln(x) - x + C}{ln(2)} \approx \frac{xln(x)}{ln(2)} - \frac{x}{ln(2)}$$

By the way, I used the approximate equal sign because the answer given by Maple did not include the C.

Testing if it's correct

I tested if my answer is correct by using two methods. One, I just differentiated it to see if it gave us the original question. Two, I graphed both the original problem and the solution to see if they make sense.

On Maple, I typed `diff( x*ln(x)/ln(2) - x/ln(2), x )` which gave me the function in the original problem:

$$\frac{\ln(x)}{\ln(2)}$$

Figure 5. Differentiating the answer I got gives us the function in the original problem.

Next, I plotted both $\frac{xln(x)-x+C}{ln(2)}$ and $\frac{ln(x)}{ln(2)}$ on Maple to see if they make sense, using `plot( [ln(x)/ln(2), (x*ln(x)-x)/(ln(2))] )`:

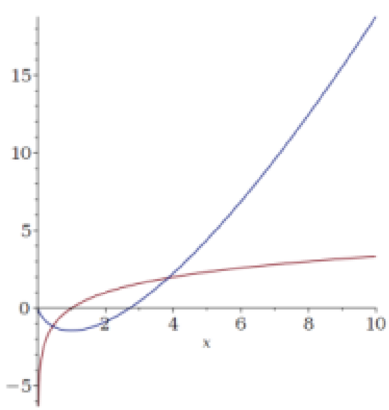

Figure 6. They look right.

Thank you for reading my AFP! Hope to see you all again.