Calculus II
Maple homework 2

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Directions

- You should save this file using your name: hmwk1_JoeSmith.mw and of course enter your full name in the space above.
- Your work is to be done in line after each provided Maple prompt, [> . Except for #7, you are to type in text.
- I started this for you, so place your cursor in my code provided for #1-4 (I also did problem #15 for you) and hit enter for each line to run it first.
- Any questions you have while working on this should be posted to the ***Maple questions*** discussion forum found in our course on D2L.
- One finished, I want you to go to File and then Print... and select "Microsoft Print to PDF" and **upload that pdf file** (with your name entered again as the file name) to the **Dropbox** on D2L.

Integrating $\int \frac{1}{\sqrt{x^2+81}}\,dx$ using two methods

Part one - trig substitution to get the antiderivative

1.) We want to let $\tan(\theta) = \frac{x}{9}$, so define $x := 9\tan(\theta)$

```
> x := 9* tan(theta)
```

$$x := 9\tan(\theta) \qquad \textbf{(1.1)}$$

2.) Now we want to rewrite the integrand, so find $\sqrt{x^2+81}$:

```
> sqrt( x^2 +81)
```

$$9\sqrt{\tan(\theta)^2+1} \qquad \textbf{(1.2)}$$

3.) We will need to find dx, so differentiate our x which is $9\tan(\theta)$ with respect to θ:

```
> diff( x, theta)
```

$$9\tan(\theta)^2+9 \qquad \textbf{(1.3)}$$

4.) Now what would our integral be in terms of theta rather than x?

```
>  Int(    1/((1.2))*(1.3)   , theta)
```

$$\int \frac{9\tan(\theta)^2 + 9}{9\sqrt{\tan(\theta)^2 + 1}}\, d\theta \qquad \textbf{(1.4)}$$

5.) Well hold on, let's use some trig identities to simplify that integrand in 4. so if $\tan^2(\theta) + 1 = \sec^2(\theta)$ what would our integral now be completely simplifed? Type the simplified integral, using the Maple command `Int( )` :

```
> Int( sec(theta), theta)
```

$$\int \sec(\theta)\, d\theta \qquad \textbf{(1.5)}$$

6.) Okay now let's integrate, take your code from 5. and change the capital I to a small i so you'll be using the Maple command `int( )` :

```
> int( sec(theta), theta)
```

$$\ln(\sec(\theta) + \tan(\theta)) \qquad \textbf{(1.6)}$$

7.) Taking your result from 6.) and subbing back that $\tan(\theta) = \frac{x}{9}$ what do we get as our answer? Type below using C as a constant of integration *answer* :

$$\int \frac{1}{\sqrt{x^2 + 81}}\, dx = \ln\left(\frac{\sqrt{x^2 + 81}}{9} + \frac{x}{9}\right) + C$$

Run the following code (place your cursor in the line and hit enter) to free up the variable *x* before doing parts 2 and 3:

```
> unassign( 'x' )
```

Part two - use of a formula to get the antiderivative

8.) Integrate $\int \frac{1}{\sqrt{x^2 + 81}}\, dx$ using the Maple command `int( )` :

```
> int( 1/sqrt(x^2 + 81), x)
```

$$\operatorname{arcsinh}\left(\frac{x}{9}\right) \qquad \textbf{(2.1)}$$

9.) Convert your result from 8.) to log form using the command `convert( , ln)`:

```
> convert( (2.1), ln)
```

$$\ln\left(\frac{\sqrt{x^2+81}}{9}+\frac{x}{9}\right) \qquad \textbf{(2.2)}$$

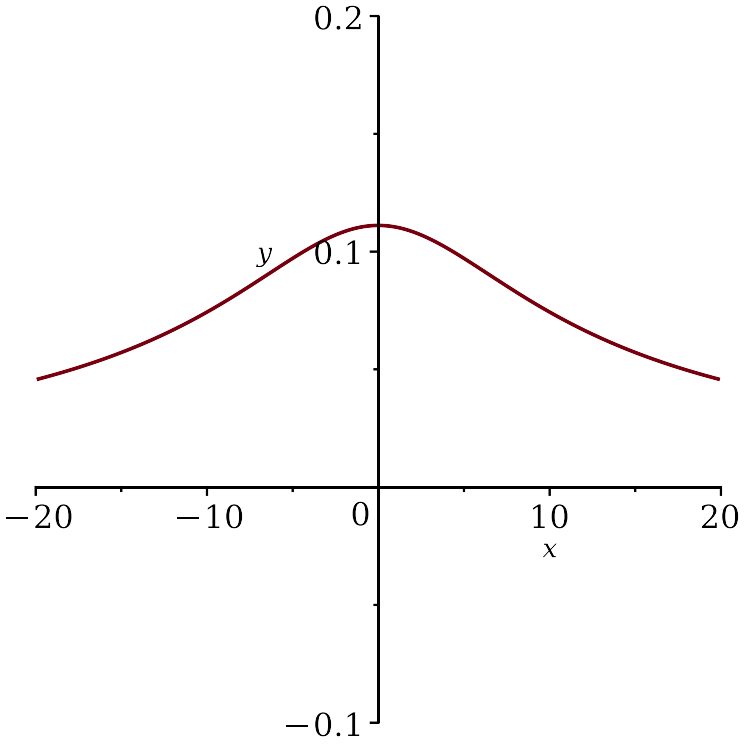
Hint - does your result from 7.) match your answer for 9.)? if not you need to fix something!

Part three - as a definite integral

$$\int_5^{10} \frac{1}{\sqrt{x^2+81}}\,dx$$

10.) Plot the curve $\frac{1}{\sqrt{x^2+81}}$ with $-20 < x < 20$ and $-0.10 < y < 0.20$

```
> plot( 1/(x^2+81)^(1/2), x=-20..20, y=-0.1..0.2);
```



Let's do a comparison of the area values from x=5 to 10 under the curve using the two different versions of the indefinite integral we found above along with Maple's direct calculation:

Using your antiderivative from problem 8.) evaluate the definite integral:
11.) Find `eval( (2.1), x=10) - eval( (2.1), x=5)` <- *write this code in command line below, but be sure to use "CTRL L" to reference equation labels in black)*

```
> eval( (2.1), x=10) - eval( (2.1), x=5)
```

$$-\operatorname{arcsinh}\left(\frac{5}{9}\right)+\operatorname{arcsinh}\left(\frac{10}{9}\right) \quad \textbf{(3.1)}$$

12.) Get a decimal approximation of your result from 11.) using `evalf( (3.2) )`
Student Note: My equation number for question 11.) happens to be **(3.1)** So, I switched it to **(3.1)**

```
> evalf( (3.1))
```

$$0.4274578505 \quad \textbf{(3.2)}$$

Now using your antiderivative from problem 7.) evaluate the definite integral:
13.) Find `eval( (2.2) , x=10) - eval( (2.2), x=5)`

```
> eval( (2.2), x=10) - eval( (2.2), x=5)
```

$$\ln\left(\frac{10}{9}+\frac{\sqrt{181}}{9}\right)-\ln\left(\frac{5}{9}+\frac{\sqrt{106}}{9}\right) \quad \textbf{(3.3)}$$

14.) Get a decimal approximation of your result from 13.) using `evalf( (3.4) )`
Student Note: Here, my equation number happns to be **(3.3)**

```
> evalf( (3.3))
```

$$0.4274578509 \quad \textbf{(3.4)}$$

Lastly, compare with Maple:
15.) Using the Maple command `evalf( int(  , x=  ..  ) )` to directly determine $\int_5^{10} \frac{1}{\sqrt{x^2+81}}\,dx$

```
> evalf( int( 1/sqrt(x^2+81), x=5..10) )
```

$$0.4274578505 \quad \textbf{(3.5)}$$

Hint - Your results from 12.) and 14.) and 15.) should all match up to 5 decimal places. (They may not past that due to the tools Maple uses when approximating.)