HOMEWORK 3.2 & 3.3

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Absolute Values & Theory of Proofs

A1-A22. These are conjectures. If the conjecture is true, just say so. However, if it is false, give a counterexample.

A2. $x < -6 \Rightarrow |x| > 6$

Work in progress. Sorry!

A4. $|x| < 4 \Rightarrow x < 10$

A6. $|x| > 9 \;\; and \;\; x < 0 \Rightarrow x < 2$

A14. $a < 0 \;\; and \;\; |a| < |b| \Rightarrow a < b$

A16. $a < 0 \;\; and \;\; b < 0 \;\; and \;\; |a| < |b| \Rightarrow a > b$

A22. $b > 0 \Rightarrow |a - b| < |a|$

A23-A31. Solve for x and answer in the form $a < x < b$.

A26. $|x + 6| < 2$

A28. $|5 - x| < 3$

A32-A35. For the next group, here is a theorem:

Let $a < b$. Then ($a < x < b$ is equivalent to $|x - c| < d$) iff $c = (a + b)/2$ and $d = (b - a)/2$. Read and use the theorem to rewrite the given interval in the form $|x - c| < d$.

A32. $5 < x < 9$

B3. *Prove the $x < 0$ case of Theorem 3.*

B4. *Prove the second half, the $\Leftarrow$ direction, of Theorem 4.*

B6a *Reproduce the given argument for Conjecture 6. Mark each line as okay or not and give its justification. Explain why it is or is not a proof.*

B6b. *Resolve Conjecture 6.*

B8-B14. Resolve the conjecture of the same number.

B11. $|x - y| \leq |x| - |y|$

B12. $|x - y| \leq |x| + |y|$

B13. $|x - y| \geq |x| - |y|$

B18. *Prove Theorem* 18.

SECTION 3.2 B3. *With the list approach, a statement is not regarded as true until it is on the list. With this approach, when can a statement be regarded as fales?*

B4. *To prove For all* x, $H(x) \Rightarrow C(x)$ *is false, we can give a counterexample. What properties must the counterexample have?*

B5. *A student write a proof and objects when the instructor marks it wrong, "But every step is true!" If the student is right, how can the proof still be incorrect?*

B6. *Suppose you state the steps* "$H \Rightarrow A$" *and* "$A \Rightarrow C$" *in an attemp to prove* "$H \Rightarrow C$." *Suppose further that you are informed that those steps do not constitute a proof. How can that be?*

B8. *Prior results are true and hypotheses are not necessarily true. So, why can hypothesis be treated as a prior result?*

B10. *The text notes that, in a proof of* "$H \Rightarrow C$", *the proof may conclude* C, *but it does not prove* C. *Explain why it does not prove* C.

B20. *Prove : If* $x \in (1, 10)$, *then these exists* $y \in (1, 10)$ *such that* $y < x$.

B22. *Suppose we wish to prove* "$H \Rightarrow C$." *Would providing this do it?* (*Answer "Yes" or "No".*)

B22A. *We prove that* C *is true if* H *is.*

B22B. *We prove that* H *is true if* C *is.*

B22C. *We prove that* C *is false if* H *is.*

B22D. *We prove that* H *is false if* C *is.*

B23-B28. True or false? Decide if these conjectures are true or false.

B23. *Conjecture :* *A proof of "$A \Rightarrow C$" proves "A and $B \Rightarrow C$."*

B24. *Conjecture :* *A proof of "$A \Rightarrow C$" proves "A or $B \Rightarrow C$."*

B25. *Conjecture :* *A proof of "not $C \Rightarrow$ not B" proves "A and $B \Rightarrow C$."*

B26. *Conjecture :* *A proof of "not $C \Rightarrow$ not A" proves "A or $B \Rightarrow C$."*

B27. *Conjecture :* *A proof of "(A or B)" $\Rightarrow C$" proves "$A \Rightarrow C$."*

B27. *Conjecture :* *A proof of "(A and B)" $\Rightarrow C$" proves "$A \Rightarrow C$."*

B57. *Conjecture :* *There exists $h > 0$ such that if $10 < x < 10 + h$, then $x^2 < 101$.*

B61. *Let $f(x) = 4x - 2$. Prove : If $c > 0$, then there exists $d > 0$ such that $|f(x) - 10| < c$ if $|x - 3| < d$.*

B69-B76. Definition: The function f is continuous at a iff for each $\epsilon > 0$ there exists $\delta > 0$ such that, if $|x - a| < \delta$, then $|f(x) - f(a)| < \epsilon$.

B69a. *Restarte "f is continuous at 5" when $f(x) = 3x + 2$.*

B69a. *Prove it.*