HOMEWORK 3.4

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Proofs by Contradiction or Contrapositive

B3-B9. Do proofs by contrapositive or contradiction.

B3. $Conjecture: \;\; If \;\; for \;\; all \;\; \epsilon > 0, \; x < c + \epsilon, \; then \;\; x \leq c.$

This can be proven with proof by contrapositive $\forall x, c \in \mathbb{R}, \;\; x > c \Rightarrow \exists \, \epsilon > 0 \;\; s.t. \;\; x \geq c + \epsilon$

$$\begin{aligned}
&Let\ x, c \in \mathbb{R}, \; where \;\; x > c \\
&Choose \;\; \epsilon = x - c \\
&Since \;\; x > c, \; x - c > 0 && \text{Theorem 3.1.3} \\
&Also, \;\; c + \epsilon = c + (x - c) = x \\
&Therefore, \;\; c + \epsilon \leq x
\end{aligned}$$

□

B4. $Prove: \;\; There \;\; are \;\; no \;\; integers \;\; a \;\; and \;\; b \;\; such \;\; that \;\; b^2 = 4a + 2.$

$$\begin{gathered}
Filler \\
Filler \\
Filler \\
Filler \\
Filler \\
Filler
\end{gathered}$$

□

B6. $Prove: \ If \ n^3 \ is \ even, \ so \ is \ n.$

$$Filler$$
$$Filler$$
$$Filler$$
$$Filler$$
$$Filler$$
$$Filler$$

□

B7. $Prove: \ Let \ a \ and \ b \ be \ real \ numbers. \ If \ a > 0 \ and \ b > 0, \ then \ \frac{2}{a} + \frac{2}{b} \neq \frac{4}{a+b}$

$$Filler$$
$$Filler$$
$$Filler$$
$$Filler$$
$$Filler$$
$$Filler$$

□

B9. $Prove: \ \sqrt{3} \ is \ not \ a \ rational \ number.$

$$Filler$$
$$Filler$$
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$$Filler$$
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$$Filler$$

□

B8-B14. Resolve the conjecture of the same number.

$$
\begin{array}{ll}
Let\ \ x \in \mathbb{R}\ \ where\ \ x \neq 0 & \\
Choose\ \ \delta = \dfrac{|x|}{2} & \\
Since\ \ x \neq 0,\ |x| > 0 & \text{Axiom 3.1.1A, Thorem 3.2.2A} \\
Since\ \ \dfrac{1}{2} > 0,\ \epsilon = \dfrac{|x|}{2} > 0 & \text{Axiom 3.1.1C} \\
Since\ \ \dfrac{1}{2} \leq 1\ \ and\ \ |x| > 0, & \text{Theorem 3.1.13A} \\
\epsilon = \dfrac{|x|}{2} \leq |x| &
\end{array}
$$

$\square$

SECTION 3.2 B3. *With the list approach, a statement is not regarded as true until it is on the list. With this approach, when can a statement be regarded as fales?*