ASSIGNED FORUM PROBLEM 3

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Chapter 8.5: *Integration of Rational Functions by Partial Fractions*

$$\textbf{41.} \quad \int \frac{cosy}{sin^2y + siny - 6} dy \tag{1}$$

How Can We Solve This Problem?

Since this problem is on Chapter 8.5: *Integration of Rational Functions by Partial Fractions*, I thought we'd have to use an integration technique involving partial fractions, which we will learn in *Lesson 14*. However, since we haven't learned about partial fractions yet - as of June 28, 2022 - I learned by trial and error that this problem can be solved by using u-substitution.

Solution

Let's start backwards. Here's the solution:

$$\textbf{41.} \quad \int \frac{cosy}{sin^2y + siny - 6} dy = \frac{2}{\sqrt{23}} tan^{-1}\left(\frac{2}{\sqrt{23}}(siny + \frac{1}{2})\right) + C \tag{2}$$

As you can see, the integral - or the solution - of this problem is in the form of $tan^{-1}x + C$. This is because it turns out that the problem can be rewritten in what we learned in *Lesson 5* as the integrand of an inverse trigonometric function. We learned in *Lesson 5* that if we differentiate an inverse trigonometric function like $tan^{-1}x$, we get $\frac{1}{1+x^2}$. In other words, $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$. Now, what this means is that $\tan^{-1}x$ should also be the integral of $\frac{1}{1+x^2}$.

This is the reason why the following equation, which I got from from *Lesson 5*, holds true:

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C \tag{3}$$

Now, let's go back to our problem. What we'll see is that **41.** $\int \frac{cosy}{sin^2y+siny-6} dy$ can be rewritten with u-substitution and with completing the square. It can eventually be rewritten in the form of $\int \frac{1}{1+u^2} du$; hence, the solution is in the form of $tan^{-1}u + C$.

Steps

Let's do u-substitution and complete the square:

41.
$$\int \frac{cosy}{sin^2y + siny - 6}dy = \int \frac{cosy}{u^2+u+6}dy \qquad (u = siny)$$
$$= \int \frac{cosy}{u^2+u+6}\cdot\frac{du}{cosy} \qquad (dy = \frac{du}{cosy})$$
$$= \int \frac{1}{u^2+u+6}du$$
$$= \int \frac{1}{(u+\frac{1}{2})^2+\frac{23}{4}}du \qquad \text{(completing the square)}$$
$$= \int \frac{1}{w^2+\frac{23}{4}}du \qquad (w = u+\frac{1}{2})$$
$$= \int \frac{1}{w^2+\frac{23}{4}}dw \qquad (du = dw)$$
$$= \int \frac{1}{\frac{23}{4}(\frac{4}{23}w^2+1)}dw \qquad \text{(algebraically rewriting)}$$
$$= \int \frac{4}{23}\cdot\frac{1}{\frac{4}{23}w^2+1}dw$$
$$= \frac{4}{23}\int \frac{1}{\frac{4}{23}w^2+1}dw$$
$$= \frac{4}{23}\int \frac{1}{z^2+1}dw \qquad (z = \frac{2}{\sqrt{23}}w)$$
$$= \frac{4}{23}\int \frac{1}{z^2+1}\cdot\frac{\sqrt{23}}{2}dz \qquad (dw = \frac{\sqrt{23}}{2}dz)$$
$$= \frac{4}{23}\cdot\frac{\sqrt{23}}{2}\int \frac{1}{z^2+1}dz$$
$$= 2\cdot\frac{23^{\frac{1}{2}}}{23^1}\int \frac{1}{z^2+1}dz$$
$$= 2\cdot 23^{\frac{1}{2}-1}\int \frac{1}{z^2+1}dz$$
$$= 2\cdot 23^{-\frac{1}{2}}\int \frac{1}{z^2+1}dz$$
$$= \frac{2}{\sqrt{23}}\int \frac{1}{z^2+1}dz$$
$$= \frac{2}{\sqrt{23}}tan^{-1}z + C \qquad \left(\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C\right)$$

Finally, we can substitute back $u = siny$, $w = u + \frac{1}{2}$, and $z = \frac{2}{\sqrt{23}}$.

41. $$\int \frac{cosy}{sin^2y + siny - 6}dy = \frac{2}{\sqrt{23}}tan^{-1}z + C$$
$$= \frac{2}{\sqrt{23}}tan^{-1}\left(\frac{2}{\sqrt{23}}w\right) + C$$
$$= \frac{2}{\sqrt{23}}tan^{-1}\left(\frac{2}{\sqrt{23}}(u + \frac{1}{2})\right) + C$$
$$= \frac{2}{\sqrt{23}}tan^{-1}\left(\frac{2}{\sqrt{23}}(siny + \frac{1}{2})\right) + C$$

Geometrically Verifying the Integral

You can insert Jupyter Lab style code blocks. First, write your code on Jupyter Lab, export it as LaTeX by clicking `File`, `Export ... As`, and `LaTeX`.

When you export your Jupyter Lab file into a LaTeXfile, you'll see a lot of preambles on the top. These preambles are what makes the code cells look the way they do. You don't need to copy any of those preambles because they have already been cloned into `coding.tex` and are automatically loaded by `main.tex`. Anyway, skip to the bottom of the exported file, where your code parts are. Your codes will start with `begin{tcolorbox}` and end with `end{Verbatim}`.

Copy and paste these into anywhere on `body.tex`. Now, when you compile `main.tex`, every content on `body.tex` - including your codes - will render like this example pdf.

[3]:
```
# Comments
print('Hello world')
```

```
Hello world
```

Inserting Inline Codes

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1 General Guidelines

Please read these instructions carefully. The objective of this template is to enable you in an easy way to style your article attractively in a style similar to that of the typeset journal. It should be emphasized, however, that the final appearance of your paper in print and in electronic media may likely vary to some extent from the presentation achieved in this template.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4}$$

has all non-zero principle minors, A_1, A_2 and A_3. Therefore, there is a unique LU factorization with both L and U nonsingular given by

2 Definition, Theorem, Corollary

Definition 1. *Definitions are if and only if statements.*

```
\begin{definition}
Definitions are if and only if statements.
\end{definition}
```

Theorem 2 (Matrices with Infinitely Many LU Factorizations)**.** *For $A \in M_n$, if two or more of any first $(n-1)$ columns are linearly dependent or any of the first $(n-1)$ columns are 0, then A has infinitely many LU factorizations.*

Proof. We will prove only for the the case when $A \in M_3$.

$$dm + r = e \Rightarrow r = e - dm \tag{5}$$
$$dn + rp = f \Rightarrow p = \frac{f - dn}{r} \tag{6}$$
$$gm + s = h \Rightarrow s = h - gm \tag{7}$$
$$gn + sp + t = i \Rightarrow t = i - sp - gn \tag{8}$$

□

Corollary 3. *If x, then y.*

3 Examples

Here is an example of an example.

Example 1. *Let $\{1, 2, 3\}$ and $\{2, 1, 3\}$ be two lists of integers. Then, to check if the two lists are equal we would have,*

`{1,2,3} == {2,1,3}` .

```
\begin{example}
Let  $\{1,2,3\}$ and $\{2,1,3\}$ be two lists of integers.  Then, to check
\begin{center}
\texttt{ \{1,2,3\} == \{2,1,3\} }.
\end{center}\label{ex:equallists}
\end{example}
```

Figures

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