FINALS OUT-OF-CLASS COMPONENT

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Functions

Prove: There exists a unique function $h : Z \to A \times B$ such that
$f_1(z) = \pi_1 \circ h(z)$ and $f_2(z) = \pi_2 \circ h(z)$.

$[p.f.]$ **Show Existence:**

$$\begin{aligned}
& Choose \;\; Z = A \cap B. \\
& Then, \;\; x \in Z \;\text{ iff }\; x \in (A \cap B) && \text{(By def of set equality)} \\
\Rightarrow \;& x \in Z \;\text{ iff }\; (x \in A \;\; and \;\; x \in B) && \text{(By def of set intersection)} \\
& Since \;\; Definition \;\; 0.2 \;\; suggests \\
& \quad (a, b) \in A \times B \;\text{ iff }\; a \in A \;\; and \;\; b \in B, \\
& \quad the \;\; function \;\; h : Z \to A \times B \;\; exists.
\end{aligned}$$

Show Uniqueness:

$$\begin{aligned}
& Let \;\; f_1 : Z \to A \;\; and \;\; f_2 : Z \to B \;\; be \;\; functions. \\
& Let \;\; \pi_1 : A \times B \to A \;\; and \;\; \pi_2 : A \times B \to B \;\; be \;\; functions \\
& \quad defined \;\; as \;\; \pi_1(a, b) = a \;\; and \;\; \pi_2(a, b) = b, \;\; for \;\; all \;\; (a, b) \in A \times B. \\
& Let \;\; A \;\; and \;\; B \;\; be \;\; nonempty \;\; sets. \\
& Then, \;\; h(z) \;\; has \;\; a \;\; domain \;\; of \;\; Z \;\; and \\
& \quad a \;\; codomain \;\; of \;\; A \times B. && \text{(By def of "Show Existence" part)} \\
& Also, \;\; since \;\; \pi_1(a, b) = a \;\; and \;\; \pi_2(a, b) = b, \\
& \quad \pi_1 \circ h(z) \;\; has \;\; a \;\; domain \;\; of \;\; A \times B \\
& \quad and \;\; a \;\; codomain \;\; of A. && \text{(By def of composite function)} \\
& \quad \pi_2 \circ h(z) \;\; has \;\; a \;\; domain \;\; of \;\; A \times B \\
& \quad and \;\; a \;\; codomain \;\; of B.
\end{aligned}$$

$$\begin{aligned}
& Since \;\; f_1 : Z \to A, \;\; then \;\; f_1(z) = \pi_1 \circ h(z). \\
& Since \;\; f_2 : Z \to B, \;\; then \;\; f_2(z) = \pi_2 \circ h(z).
\end{aligned}$$

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