ASSIGNED FORUM PROBLEM 4

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Chapter 8.8: *Improper Integrals*

$$\textbf{33.} \quad \int_{-1}^{\infty} \frac{d\theta}{\theta^2 + 5\theta + 6} \tag{1}$$

Solution

The first thing that comes to my mind when I look at this problem is that the denominator can be factored into $(\theta + 3)(\theta + 2)$ which means our problem becomes

$$\textbf{33.} \quad \int_{-1}^{\infty} \frac{d\theta}{\theta^2 + 5\theta + 6} = \int_{-1}^{\infty} \frac{d\theta}{(\theta + 3)(\theta + 2)} \tag{2}$$

After that, we can use partial fraction decomposition to make the integrand easier to integrate.

$$\int_{-1}^{\infty} \frac{d\theta}{(\theta + 3)(\theta + 2)} = \int_{-1}^{\infty} \frac{A}{\theta + 3} + \frac{B}{\theta + 2} d\theta \tag{3}$$

We can then multiply both sides of the equation with $(\theta+3)(\theta+2)$. For the sake of readability, let's skip the integral notation for now and look at only the coefficients now.

$$1 = A(\theta + 2) + B(\theta + 3) \tag{4}$$

$$(0)\theta + (1) = (A + B)\theta + (3A + 2B) \tag{5}$$

Solving for these two equations $A + B = 0$ and $3A + 2B = 1$ gives us

$$A = 1 \tag{6}$$

$$B = -1 \tag{7}$$

In other words, if we put everything back to the original equation together, we get something that's much more easier to integrate:

$$\int_{-1}^{\infty} \frac{d\theta}{(\theta + 3)(\theta + 2)} = \int_{-1}^{\infty} \frac{1}{\theta + 3} + \frac{-1}{\theta + 2} d\theta$$

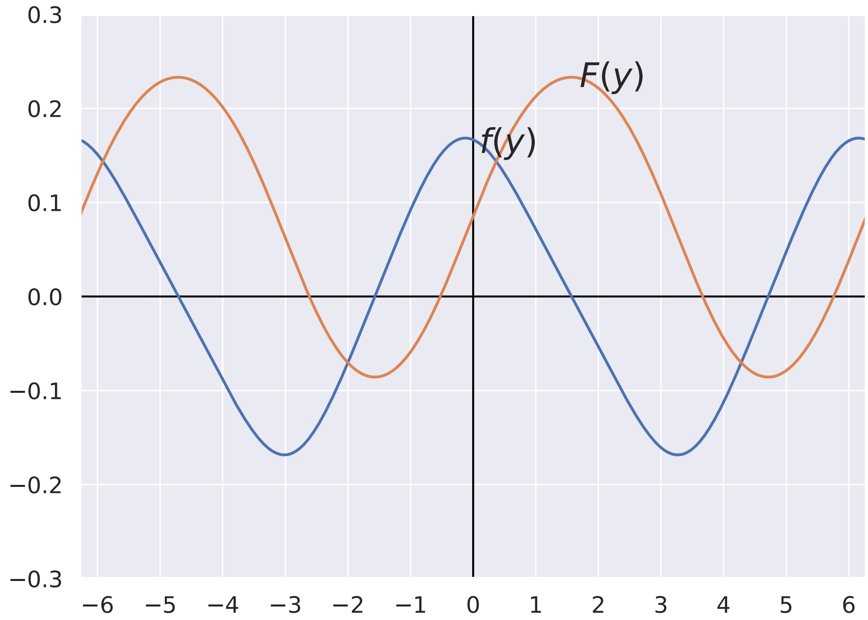
Well, why is it easier to integrate? It's because $\int \frac{1}{x}$ is elegantly $ln|x| + C$.

$$\int_{-1}^{\infty} \frac{d\theta}{(\theta+3)(\theta+2)} = \Big[ln|\theta+2| - ln|\theta+3| \Big]_{-1}^{\infty} \tag{8}$$

$$= \Big[0\Big] - \Big[-ln2\Big] \tag{9}$$

$$= ln2 \tag{10}$$

Verifying the Integral



Let's see if we got the correct integral by graphing it. One thing I've noticed from this graph is that whenever $f(y)$ is at 0, $F(y)$ is either at its minimum or maximum. For instance, at $f(y) \approx -1.6$, $F(y)$ is at its minimum. Notice that this is one of the properties we learned in Calculus I.

The integrand of our original problem $f(y)$ seems to be the derivative of the integral we got, which is $F(y)$.