AP® Physics C: Electricity and Magnetism
2011 Free-Response Questions

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TABLE OF INFORMATION FOR 2010 and 2011

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^3/kg$\cdot$s^2
Universal gas constant, $R = 8.31$ J/(mol$\cdot$K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\text{ J}\cdot\text{s} = 4.14 \times 10^{-15}\text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\text{ J}\cdot\text{m} = 1.24 \times 10^{3}\text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\text{ (T}\cdot\text{m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}\text{ (T}\cdot\text{m)/A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\text{ N/m}^2 = 1.0 \times 10^{5}\text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2010 and 2011

MECHANICS

$$\upsilon = \upsilon_0 + at$$

$$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2$$

$$\upsilon^2 = \upsilon_0^{\,2} + 2a(x - x_0)$$

$$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$$

$$\mathbf{p} = m\mathbf{v}$$

$$F_{fric} \le \mu N$$

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

$$K = \frac{1}{2}m\upsilon^2$$

$$P = \frac{dW}{dt}$$

$$P = \mathbf{F} \cdot \mathbf{v}$$

$$\Delta U_g = mgh$$

$$a_c = \frac{\upsilon^2}{r} = \omega^2 r$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

$$I = \int r^2 dm = \sum mr^2$$

$$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$$

$$\upsilon = r\omega$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$$

$$K = \frac{1}{2}I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
υ = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$E = -\frac{dV}{dr}$$

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$R = \frac{\rho\ell}{A}$$

$$\mathbf{E} = \rho\mathbf{J}$$

$$I = Nev_dA$$

$$V = IR$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = IV$$

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
υ = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$$

$$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$$

$$B_s = \mu_0 nI$$

$$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$U_L = \frac{1}{2}LI^2$$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Parallelepiped

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

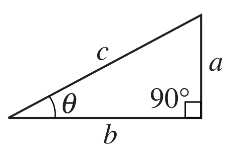
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.

E&M. 1.

A nonconducting, thin, spherical shell has a uniform surface charge density σ on its outside surface and no charge anywhere else inside.

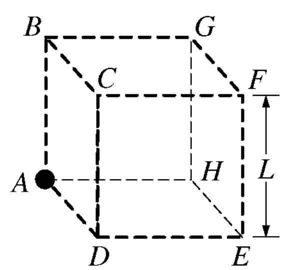
(a) Use Gauss's law to prove that the electric field inside the shell is zero everywhere. Describe the Gaussian surface that you use.

(b) The charges are now redistributed so that the surface charge density is no longer uniform. Is the electric field still zero everywhere inside the shell?

____ Yes ____ No ____ It cannot be determined from the information given.

Justify your answer.

Now consider a small conducting sphere with charge $+Q$ whose center is at corner A of a cubical surface, as shown below.



(c) For which faces of the surface, if any, is the electric flux through that face equal to zero?

____ *ABCD* ____ *CDEF* ____ *EFGH* ____ *ABGH* ____ *BCFG* ____ *ADEH*

Explain your reasoning.

(d) At which corner(s) of the surface does the electric field have the least magnitude?

(e) Determine the electric field strength at the position(s) you have indicated in part (d) in terms of Q, L, and fundamental constants, as appropriate.

(f) Given that one-eighth of the sphere at point A is inside the surface, calculate the electric flux through face $CDEF$.

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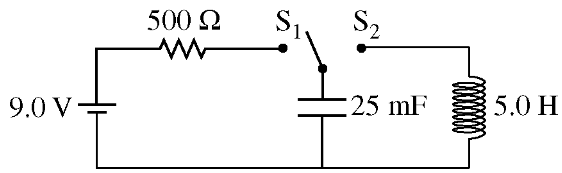


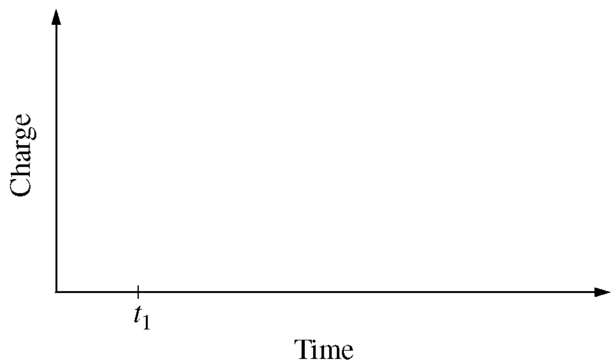
E&M. 2.

The circuit represented above contains a 9.0 V battery, a 25 mF capacitor, a 5.0 H inductor, a 500 Ω resistor, and a switch with two positions, S_1 and S_2. Initially the capacitor is uncharged and the switch is open.

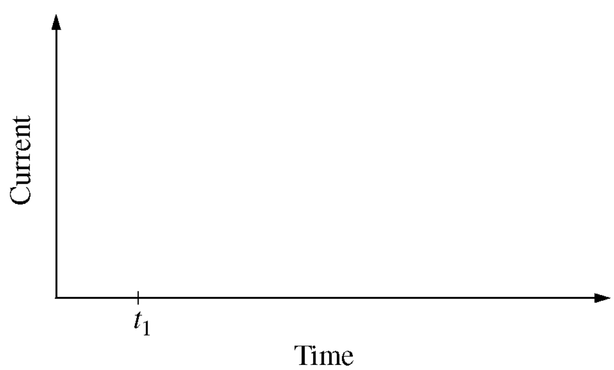
(a) In experiment 1 the switch is closed to position S_1 at time t_1 and left there for a long time.

i. Calculate the value of the charge on the bottom plate of the capacitor a long time after the switch is closed.

ii. On the axes below, sketch a graph of the magnitude of the charge on the bottom plate of the capacitor as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



iii. On the axes below, sketch a graph of the current through the resistor as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



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(b) In experiment 2 the capacitor is again uncharged when the switch is closed to position S_1 at time t_1. The switch is then moved to position S_2 at time t_2 when the magnitude of the charge on the capacitor plate is 105 mC, allowing electromagnetic oscillations in the LC circuit.

i. Calculate the energy stored in the capacitor at time t_2.

ii. Calculate the maximum current that will be present during the oscillations.

iii. Calculate the time rate of change of the current when the charge on the capacitor plate is 50 mC.

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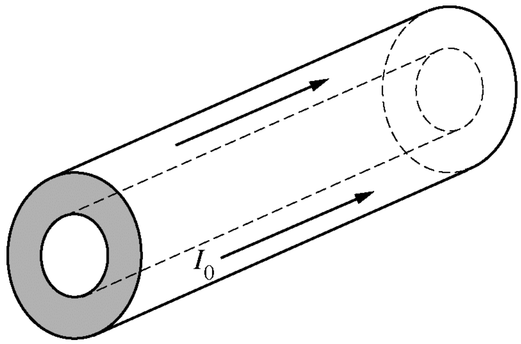

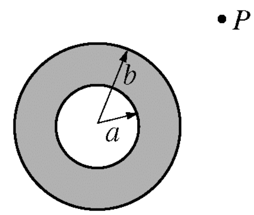

Cross-sectional View
(current into page)

E&M. 3.

A section of a long conducting cylinder with inner radius a and outer radius b carries a current I_0 that has a uniform current density, as shown in the figure above.

(a) Using Ampère's law, derive an expression for the magnitude of the magnetic field in the following regions as a function of the distance r from the central axis.

i. $r < a$

ii. $a < r < b$

iii. $r = 2b$

(b) On the cross-sectional view in the diagram above, indicate the direction of the field at point P, which is at a distance $r = 2b$ from the axis of the cylinder.

(c) An electron is at rest at point P. Describe any electromagnetic forces acting on the electron. Justify your answer.

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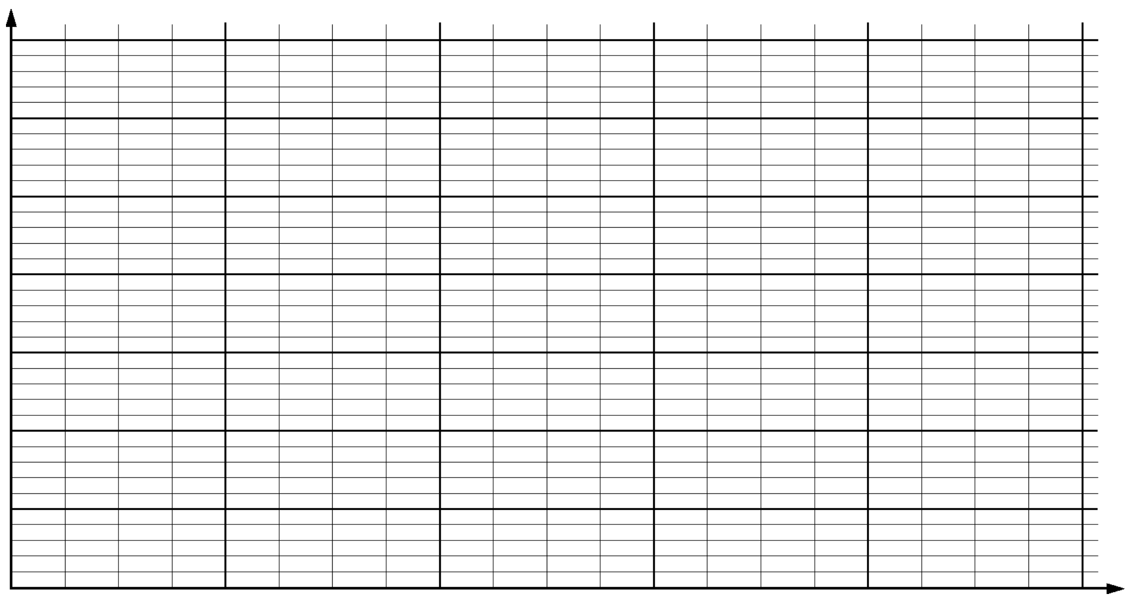
Now consider a long, solid conducting cylinder of radius b carrying a current I_0. The magnitude of the magnetic field inside this cylinder as a function of r is given by $B = \mu_0 I_0 r / 2\pi b^2$. An experiment is conducted using a particular solid cylinder of radius 0.010 m carrying a current of 25 A. The magnetic field inside the cylinder is measured as a function of r, and the data is tabulated below.

Distance r (m)	0.002	0.004	0.006	0.008	0.010
Magnetic Field B (T)	1.2×10^{-4}	2.7×10^{-4}	3.6×10^{-4}	4.7×10^{-4}	6.4×10^{-4}

(d)

i. On the graph below, plot the data points for the magnetic field B as a function of the distance r, and label the scale on both axes. Draw a straight line that best represents the data.



ii. Use the slope of your line to estimate a value of the permeability μ_0.

END OF EXAM

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