AP® Physics C: Electricity and Magnetism
2006 Free-Response Questions

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TABLE OF INFORMATION FOR 2006 and 2007

CONSTANTS AND CONVERSION FACTORS

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}$ kg
	$= 931\text{ MeV}/c^2$
Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg
Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Avogadro's number,	$N_0 = 6.02 \times 10^{23}\text{ mol}^{-1}$
Universal gas constant,	$R = 8.31$ J/(mol•K)
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K
Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Planck's constant,	$h = 6.63 \times 10^{-34}$ J•s
	$= 4.14 \times 10^{-15}$ eV•s
	$hc = 1.99 \times 10^{-25}$ J•m
	$= 1.24 \times 10^{3}$ eV•nm
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N•m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9}\text{ N•m}^2/\text{C}^2$
Vacuum permeability,	$_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = {}_0/4\pi = 10^{-7}$ (T·m)/A
Universal gravitational constant,	$G = 6.67 \times 10^{-11}\text{ m}^3/\text{kg•s}^2$
Acceleration due to gravity at Earth's surface,	$g = 9.8\text{ m/s}^2$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\text{ N/m}^2$
	$= 1.0 \times 10^{5}$ Pa
1 electron volt,	$1\text{ eV} = 1.60 \times 10^{-19}$ J

UNITS

Name	Symbol
meter	m
kilogram	kg
second	s
ampere	A
kelvin	K
mole	mol
hertz	Hz
newton	N
pascal	Pa
joule	J
watt	W
coulomb	C
volt	V
ohm	Ω
henry	H
farad	F
tesla	T
degree Celsius	°C
electron-volt	eV

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
37°	3/5	4/5	3/4
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
53°	4/5	3/5	4/3
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	1	0	∞

The following conventions are used in this examination.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.
II. The direction of any electric current is the direction of flow of positive charge (conventional current).
III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2006 and 2007

MECHANICS

$$\upsilon = \upsilon_0 + at$$

$$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2$$

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0)$$

$$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$$

$$\mathbf{p} = m\mathbf{v}$$

$$F_{fric} \le \mu N$$

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

$$K = \frac{1}{2}m\upsilon^2$$

$$P = \frac{dW}{dt}$$

$$P = \mathbf{F} \cdot \mathbf{v}$$

$$\Delta U_g = mgh$$

$$a_c = \frac{\upsilon^2}{r} = \omega^2 r$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

$$I = \int r^2 dm = \sum mr^2$$

$$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$$

$$\upsilon = r\omega$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$$

$$K = \frac{1}{2}I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
υ = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$E = -\frac{dV}{dr}$$

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$R = \frac{\rho\ell}{A}$$

$$\mathbf{E} = \rho\mathbf{J}$$

$$I = Ne\upsilon_d A$$

$$V = IR$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = IV$$

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
υ = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$$

$$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$$

$$B_s = \mu_0 nI$$

$$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$U_L = \frac{1}{2}LI^2$$

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2006 and 2007

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Parallelepiped

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

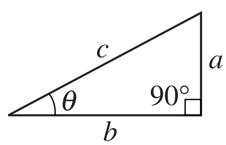
$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

$$\int \frac{dx}{x} = \ln|x|$$

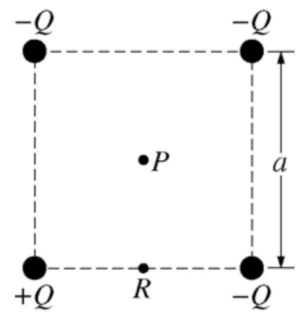
$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



E&M 1.

The square of side a above contains a positive point charge $+Q$ fixed at the lower left corner and negative point charges $-Q$ fixed at the other three corners of the square. Point P is located at the center of the square.

(a) On the diagram, indicate with an arrow the direction of the net electric field at point P.

(b) Derive expressions for each of the following in terms of the given quantities and fundamental constants.

i. The magnitude of the electric field at point P

ii. The electric potential at point P

(c) A positive charge is placed at point P. It is then moved from point P to point R, which is at the midpoint of the bottom side of the square. As the charge is moved, is the work done on it by the electric field positive, negative, or zero?

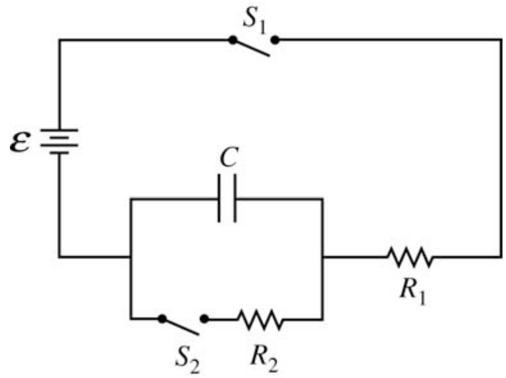
____ Positive ____ Negative ____ Zero

Explain your reasoning.

(d)

i. Describe one way to replace a single charge in this configuration that would make the electric field at the center of the square equal to zero. Justify your answer.

ii. Describe one way to replace a single charge in this configuration such that the electric potential at the center of the square is zero but the electric field is not zero. Justify your answer.



E&M 2.

The circuit above contains a capacitor of capacitance C, a power supply of emf $\mathcal{E}$, two resistors of resistances R_1 and R_2, and two switches, S_1 and S_2. Initially, the capacitor is uncharged and both switches are open. Switch S_1 then gets closed at time $t = 0$.

(a) Write a differential equation that can be solved to obtain the charge on the capacitor as a function of time t.

(b) Solve the differential equation in part (a) to determine the charge on the capacitor as a function of time t.

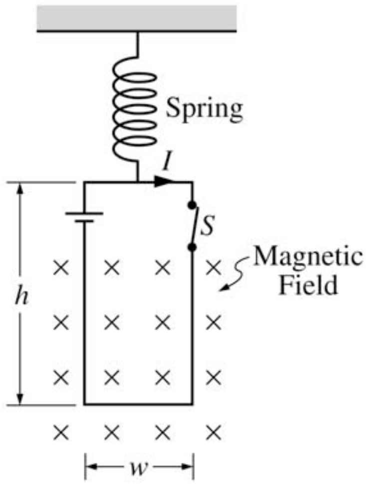
Numerical values for the components are given as follows:

$\mathcal{E} = 12$ V
$C = 0.060$ F
$R_1 = R_2 = 4700\ \Omega$

(c) Determine the time at which the capacitor has a voltage 4.0 V across it.

After switch S_1 has been closed for a long time, switch S_2 gets closed at a new time $t = 0$.

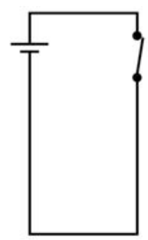
(d) On the axes below, sketch graphs of the current I_1 in R_1 versus time and of the current I_2 in R_2 versus time, beginning when switch S_2 is closed at new time $t = 0$. Clearly label which graph is I_1 and which is I_2.



E&M 3.

A loop of wire of width w and height h contains a switch and a battery and is connected to a spring of force constant k, as shown above. The loop carries a current I in a clockwise direction, and its bottom is in a constant, uniform magnetic field directed into the plane of the page.

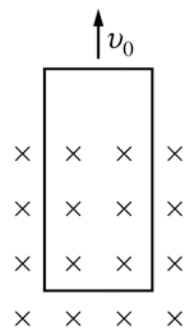
(a) On the diagram of the loop below, indicate the directions of the magnetic forces, if any, that act on each side of the loop.



(b) The switch S is opened, and the loop eventually comes to rest at a new equilibrium position that is a distance x from its former position. Derive an expression for the magnitude B_0 of the uniform magnetic field in terms of the given quantities and fundamental constants.

The spring and loop are replaced with a loop of the same dimensions and resistance R but without the battery and switch. The new loop is pulled upward, out of the magnetic field, at constant speed v_0. Express algebraic answers to the following questions in terms of B_0, v_0, R, and the dimensions of the loop.

(c)
i. On the diagram of the new loop below, indicate the direction of the induced current in the loop as the loop moves upward.



ii. Derive an expression for the magnitude of this current.

(d) Derive an expression for the power dissipated in the loop as the loop is pulled at constant speed out of the field.

(e) Suppose the magnitude of the magnetic field is increased. Does the external force required to pull the loop at speed v_0 increase, decrease, or remain the same?

_____ Increases _____ Decreases _____ Remains the same

Justify your answer.

END OF EXAM