```
%syms M m1 m2 l1 l2 g
```

```
M=1000;
m1=100;
m2=100;
l1=20;
l2=10;
g=9.81;
```

A_f and B_f matrices

```
A_f =[0 1 0 0 0 0;0 0 (-m1*g)/M 0 (-m2*g)/M 0;0 0 0 1 0 0;0 0 -(g/l1)*((m1+M)/M) 0 (-m2*g)/M*l1
```

```
A_f = 6×6
         0    1.0000         0         0         0         0
         0         0   -0.9810         0   -0.9810         0
         0         0         0    1.0000         0         0
         0         0   -0.5396         0  -19.6200         0
         0         0         0         0         0    1.0000
         0         0   -9.8100         0   -1.0791         0
```

```
B_f =[0;1/M;0;1/(M*l1);0;1/(M*l2)]
```

```
B_f = 6×1
10^-3 ×
         0
    1.0000
         0
    0.0500
         0
    0.1000
```

```
C = [1,0,0,0,0,0]
```

```
C = 1×6
     1     0     0     0     0     0
```

```
D = [0]
```

```
D = 0
```

```
eigs(A_f);

plant = ss(A_f,B_f,C,D)
```

```
plant =

  A =
            x1       x2       x3       x4       x5       x6
   x1        0        1        0        0        0        0
   x2        0        0   -0.981        0   -0.981        0
```

```
  x3        0        0        0        1        0        0
  x4        0        0  -0.5396        0   -19.62        0
  x5        0        0        0        0        0        1
  x6        0        0    -9.81        0   -1.079        0

 B =
          u1
  x1       0
  x2   0.001
  x3       0
  x4   5e-05
  x5       0
  x6  0.0001

 C =
      x1  x2  x3  x4  x5  x6
  y1   1   0   0   0   0   0

 D =
      u1
  y1   0

Continuous-time state-space model.
```

```
Plant_poles = pole(plant)
```

```
Plant_poles = 6×1 complex
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -3.6148 + 0.0000i
  -0.0000 + 3.8322i
  -0.0000 - 3.8322i
   3.6148 + 0.0000i
```

Controllability Matrix and rank condition

```
Co = ctrb(plant)
```

```
Co = 6×6
         0    0.0010         0   -0.0001         0    0.0025
    0.0010         0   -0.0001         0    0.0025         0
         0    0.0001         0   -0.0020         0    0.0128
    0.0001         0   -0.0020         0    0.0128         0
         0    0.0001         0   -0.0006         0    0.0202
    0.0001         0   -0.0006         0    0.0202         0
```

```
Ob = obsv(plant)
```

```
Ob = 6×6
    1.0000         0         0         0         0         0
         0    1.0000         0         0         0         0
         0         0   -0.9810         0   -0.9810         0
         0         0         0   -0.9810         0   -0.9810
         0         0   10.1529         0   20.3058         0
         0         0         0   10.1529         0   20.3058
```

```
rank_Co = rank(Co)
```

```
rank_Co = 6
```

```
rank_Ob = rank(Ob)
```

```
rank_Ob = 6
```

Position of poles

```
rlocus(plant)
```

Initial conditions

```
I = [0;0;0;0;0;0];
```

LQR Controller Design

```
Q = [1200000000000 10 0 0 0 0;10 2 0 0 0 0;0 0 11000000000000 0 0 0 ; 0 0 0 0.1 0 0;0 0 0 0 140
R = 10;
eig_Q = eigs(Q);
N = [1;2;3;1;5;2];

Positive_Semi_Definite = [Q N;N' R] %this matrix must be positive-semi definite
```

```
Positive_Semi_Definite = 7×7
10^13 ×
    0.1200    0.0000         0         0         0         0    0.0000
    0.0000    0.0000         0         0         0         0    0.0000
         0         0    1.1000         0         0         0    0.0000
         0         0         0    0.0000         0         0    0.0000
         0         0         0         0    1.4000         0    0.0000
         0         0         0         0         0    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
```

```
[K2,S2,e2] = lqr(A_f+eye(size(A_f)),B_f,Q,R,N)
```

```
K2 = 1×6
10^7 ×
   -0.0763   -0.0589   -2.8052   -0.6839    3.0355    0.9735
S2 = 6×6
10^15 ×
    0.0023    0.0011    0.0360    0.0081   -0.0442   -0.0150
    0.0011    0.0006    0.0226    0.0048   -0.0245   -0.0089
    0.0360    0.0226    0.8753    0.1910   -0.9032   -0.3241
    0.0081    0.0048    0.1910    0.0429   -0.2012   -0.0703
   -0.0442   -0.0245   -0.9032   -0.2012    1.0050    0.3488
   -0.0150   -0.0089   -0.3241   -0.0703    0.3488    0.1250
e2 = 6×1 complex
 -13.5859 +13.6057i
```

```
  -13.5859 -13.6057i
   -4.5397 + 0.0000i
   -1.1921 + 3.6421i
   -1.1921 - 3.6421i
   -2.6311 + 0.0000i
```

Root Locus with LQR Controller

```
rlocus(Plant_LQR)
```

```
A_cl = A_f-(B_f*K2);
Plant_LQR = ss(A_cl,B_f,C,D)
```

```
Plant_LQR =

  A = 
              x1          x2          x3          x4          x5          x6
   x1          0           1           0           0           0           0
   x2      762.8       588.8   2.805e+04        6839  -3.036e+04       -9735
   x3          0           0           0           1           0           0
   x4      38.14       29.44        1402         342       -1537      -486.7
   x5          0           0           0           0           0           1
   x6      76.28       58.88        2795       683.9       -3037      -973.5

  B = 
           u1
   x1       0
   x2   0.001
   x3       0
   x4   5e-05
   x5       0
   x6  0.0001

  C = 
       x1  x2  x3  x4  x5  x6
   y1   1   0   0   0   0   0

  D = 
       u1
   y1   0

Continuous-time state-space model.
```

```
step(Plant_LQR)
```

Luenberger Observer Design

```
%Desired Location of Poles
```

```
Poles_L_Obs = [-28;-30;-10;-6;-2;-4];
L = place(A_f',C',Poles_L_Obs)'
```

```
L = 6×1
10^5 ×
    0.0008
    0.0228
   -0.9791
   -3.8919
    0.6901
    2.2299
```

```
Luen_SYS = ss(A_f-L*C,B_f,C,D)
```

```
Luen_SYS =

  A =
                 x1          x2          x3          x4          x5          x6
   x1           -80           1           0           0           0           0
   x2         -2278           0      -0.981           0      -0.981           0
   x3     9.791e+04           0           0           1           0           0
   x4     3.892e+05           0     -0.5396           0      -19.62           0
   x5    -6.901e+04           0           0           0           0           1
   x6     -2.23e+05           0       -9.81           0      -1.079           0

  B =
           u1
   x1       0
   x2   0.001
   x3       0
   x4   5e-05
   x5       0
   x6  0.0001

  C =
       x1  x2  x3  x4  x5  x6
   y1   1   0   0   0   0   0

  D =
       u1
   y1   0

Continuous-time state-space model.
```

```
step(Luen_SYS)
```

Check for obervability

```
C_Lo = ctrb(A_f',C')
```

```
C_Lo = 6×6
    1.0000         0         0         0         0         0
         0    1.0000         0         0         0         0
```

```
         0         0   -0.9810         0   10.1529         0
         0         0         0   -0.9810         0   10.1529
         0         0   -0.9810         0   20.3058         0
         0         0         0   -0.9810         0   20.3058
```

```
rank(C_Lo)
```

```
ans = 6
```

```
Ob_Lo = obsv(A_f,C)
```

```
Ob_Lo = 6×6
    1.0000         0         0         0         0         0
         0    1.0000         0         0         0         0
         0         0   -0.9810         0   -0.9810         0
         0         0         0   -0.9810         0   -0.9810
         0         0   10.1529         0   20.3058         0
         0         0         0   10.1529         0   20.3058
```

```
rank(Ob_Lo)
```

```
ans = 6
```

LQG Controller Design

```
At = [ A_f-B_f*K2             B_f*K2
       zeros(size(A_f))    A_f-L*C ];

Bt = [    B_f
       zeros(size(B_f)) ];

Ct = [ C    zeros(size(C)) ];
```

LQG Step Response

```
step(ss(At,Bt,Ct,0))
```