

By Nick Werblun

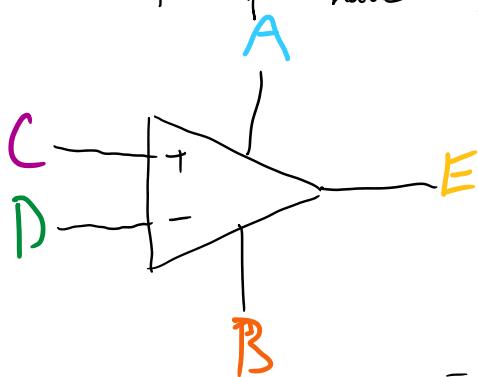
Intro:

- Why?

- Op amps (short for "operational amplifier") were used for doing math operations, hence the name. They were even used in the analog computers that guided some of the Apollo missions in space.
- They can also do a lot more, but some of it is outside the scope of this course.
- They have a very complicated internal structure, which is why we abstract it using the models from this class.

- What?

- Op Amps have 5 terminals, EVEN WHEN THEY ARE NOT ALL DRAWN



A : power terminal. Often labeled V_{DD}
 (should be greater than the voltage on B
 or else you'll break it)

B : another power terminal. Often labeled V_{SS} (usually 0 or negative)

C : "Non-inverting" input. An input that takes an arbitrary voltage

D : "Inverting" input. An input that takes an arbitrary voltage

E : Output

The power rails are always present, even if not drawn. When not drawn, it means you can assume the values at the power terminals are adequate for the circuit.

↑ more later (see negative feedback)

Inverting/non-inverting are called so for the internal structure, ignore the names.

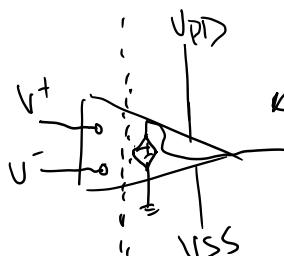
The +/- do NOT mean pos/neg, this again has to do with the internal structure, and has nothing to do with polarity. You can put ANY voltage on either input.

The word "rails" corresponds

Speaking of power, op amps are great, but not

~~extra inputs~~

- The word "rails" corresponds to the power terminals.

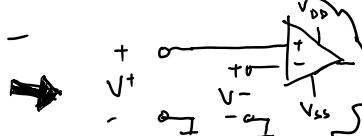


two stages are isolated. Think of it as the first half "reads" the inputs, and the second half creates an output

Speaking of power, op amps are great, but not magic. If I ask you to go into the lab and set the power supply to 638 Volts, could you do it? Probably not. They have a max output of 25V. You can't output more than what you are given to work with. If your op amp is powered with $V_{DD} = 12V$, how can it output 32V? ∞V ? If $V_{SS} = -10V$, how can the output go lower than that? It can't. There is no connection from the inputs to the output, so the output can only rely on the power supplies for its limits

- How,
in code

Note:
I will hereby use V^+ and V^- as such
op-amp(V^+ , V^- , V_{DD} , V_{SS}):



if $V^+ > V^-$:

return V_{DD}

elif $V^- > V^+$:

return V_{SS}

For all op amps, $U_o = A(V^+ - V^-)$ Where we assume $A = \infty$

In reality $A \neq \infty$ but is very large. That's what is important here.
More later

more out of scope internal circuitry stuff

→ This means if $V^+ > V^- \rightarrow V^+ - V^- > 0 \rightarrow A(V^+ - V^-) \xrightarrow[\infty]{\sim} \infty$ positive

BUT, can you output ∞ volts? NO! only what you are given. The op amp tries to push the output to ∞ , but upon reaching its limit at V_{DD} , it can't go any higher (see above).

Likewise,

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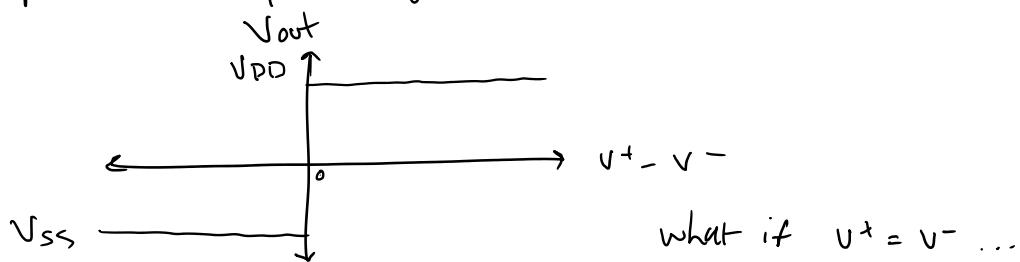
If $V^- > V^+ \rightarrow V^+ - V^- < 0 \rightarrow A(V^+ - V^-) = -\infty$

∞ negative

Same problem. How do you hit $-\infty$ when V_{SS} is only $0V$?
You don't. You hit V_{SS} and give up.

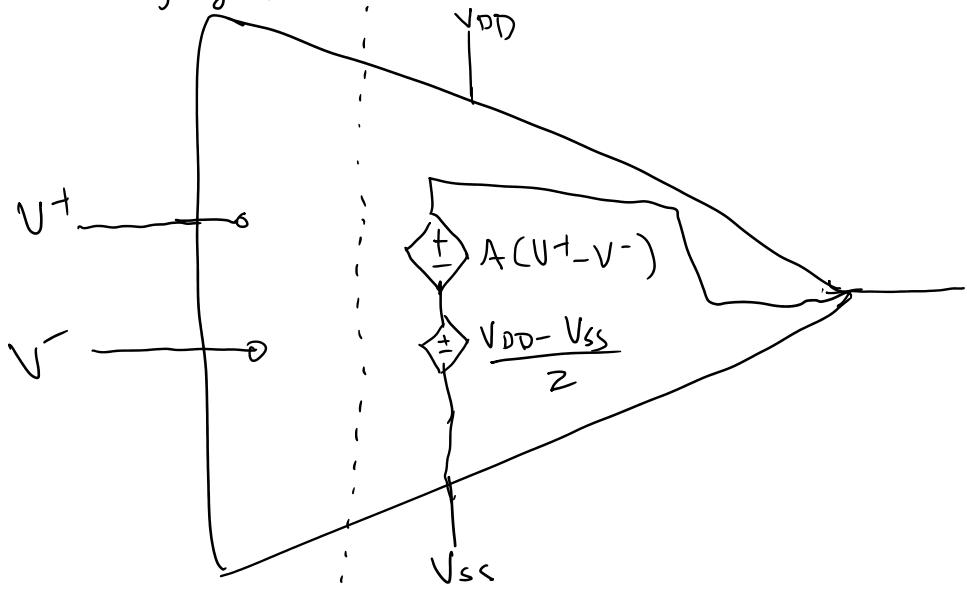
- This is why you may have seen that V^+ is "connected" to V_{DD} . It's not that there's any physical connection, just that if $V^+ > V^-$ then the output ends up being V_{DD} . Same with V^- and V_{SS} .

Graphically



what if $V^+ = V^- \dots$

A slightly better model:



Ideal:

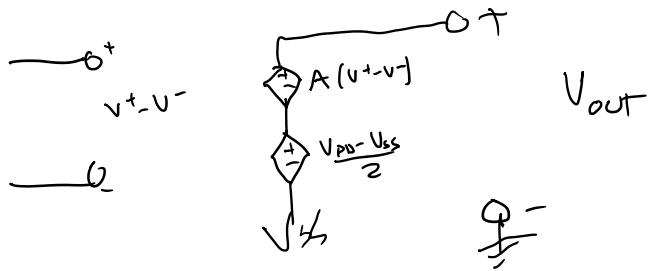
- $A = \infty$

still isolated

if you ignore the triangle, then it's basically



Notice that if $V_{DD} = -V_{SS}$



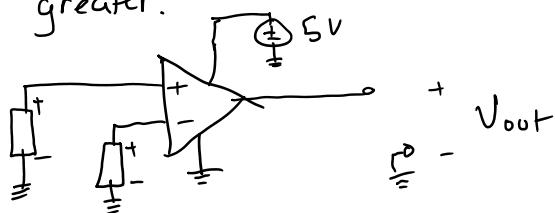
- Notice that if $V_{DD} = -V_{SS}$ then the bottom source is 0 and can be ignored.

• Also note that even if $V_{DD} \neq -V_{SS}$, the bottom source only matters when $V^+ = V^-$. otherwise the output wants to go to ∞ or $-\infty$. $\infty \pm \frac{V_{DD}-V_{SS}}{2} = \infty$ and $-\infty \pm \frac{V_{DD}-V_{SS}}{2} = -\infty$, so it all stops at V_{DD} or V_{SS} anyways

Usage:

- Comparator

- Compare two voltages.
- Problem: I have two black boxes that output a constant voltage, but I can't measure them. I want to know which is greater.
- Solution: Use an op amp. Recall: if $V^+ > V^- \rightarrow V_{out} = V_{DD}$, otherwise $V_{out} = V_{SS}$. Let's pick $V_{DD} = 5V$ and $V_{SS} = 0V$ (arbitrary). If we see 5V at V_{out} , V^+ was greater. If we see 0V, V^- was greater.



if you replace the box on V^+ with your touchscreen and the box on V^- with a voltage source of a known value, then you have created the Touchscreen 3+4 labs (shoutout Thomas Rembert)



1 | F. (negative) FEEDBACK

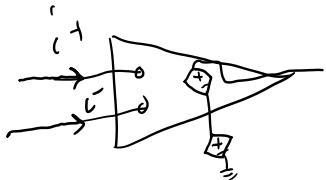
NEGATIVE FEEDBACK

Q.R. cool. Don't care, what else can they do?

WATCH THIS

wait actually we need the golden rules first. S A D.

Golden Rule ONE:

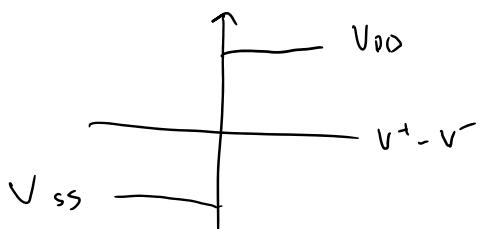


using the model above, we know that there is an open circuit at the inputs. Where can the currents i^+ and i^- go?

Nowhere. No current can enter the op amp because of the open circuit. This is Always true

Golden Rule TWO:

Remember this curve from above?



well it's only true if $A = \infty$

In a real op amp, $A \approx 1000$ to $100,000$ or so

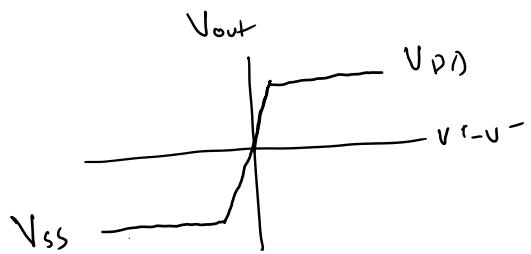
If we assume $A = 10,000$ (which is reasonable), then what happens when $V^+ - V^- = 0.0001$? $V_{out} = A(V^+ - V^-) = (10^4)(10^{-4}) = 1$

If we assume $A = 10,000$ (which is reasonable), then what happens when $V^+ - V^- = 0.0001$? $V_{out} = A(V^+ - V^-) = (10^4)(10^{-4}) = 1$

Whoa... a value that isn't V_{DD} or V_{SS} ...

But here's the thing; it's impossible to generate voltages THAT similar to each other. They're so close that we can say $V^+ \approx V^-$

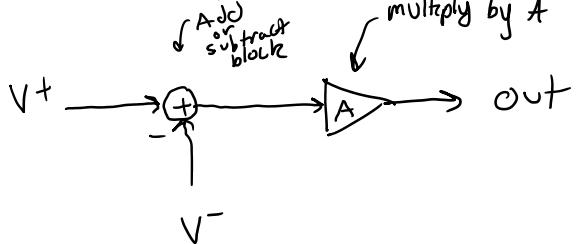
which means the curve is more like:



- At a certain point it still just shoots to V_{DD} or V_{SS} , but there's a range where the op amp is linear! $A(V^+ - V^-)$ is linear, and linear is good!

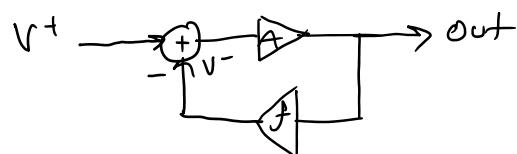
- So we can get a linear output, but only on that sloped part. And we only get that when $V^+ \approx V^-$. So for nice linear output, we have to force V^+ to equal V^-

To do that, let's look at a block diagram of an op amp



this can be read as $out = A(V^+ - V^-)$
just like an op amp.

What if we do this...



$$\text{Now } V^- = f V_{out}$$

$$\text{and } V_{out} = A(V^+ - V^-) = A(V^+ - f V_{out})$$

$$\rightarrow V_{out} = AV^+ - AfV_{out}$$

$$\rightarrow V_{out} + AfV_{out} = AV^+$$

$$\rightarrow V_{out}(1 + Af) = AV^+$$

$$\rightarrow V_{out}(1+A_f) = AV^+$$

$$\rightarrow V_{out} = \frac{V^+}{1+A_f} A$$

from above we know $V^- = f V_{out}$

$$\rightarrow f V_{out} = V^+ \frac{A_f}{1+A_f}$$

$$\rightarrow V^- = V^+ \frac{A_f}{1+A_f}$$

This is where $A = \infty$ comes into play. If $A = \infty$

then $\lim_{A \rightarrow \infty} V^- = V^+ \frac{A_f}{1+A_f} \rightarrow V^+ = V^-$

That's the secret. We can force $V^+ = V^-$ by passing V_{out} through ANY BLOCK f back to the "inverting input" and when $A = \infty$. Note that A being large works too. If $A = 10000$, and we assume $f = 1$, then

$$V^- = V^+ \frac{10000}{10001} = 0.9999V^+ \approx V^+, \text{ but if } A = 10,$$

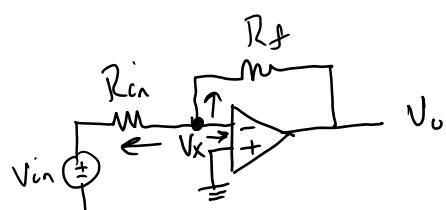
then $V^- = V^+ \frac{10}{11} = 0.91V^+$, which is not close enough.

That's our second golden rule. $V^+ = V^-$ IN NEGATIVE FEEDBACK

We assume $A = \infty$ always in this class, unless stated.

Now we have the tools to make op amps useful.

Ex:



steps:

- 1) Apply KCL
- 2) Use golden rules



- 1) Apply rule
- 2) Use golden rules
- 3) ????
- 4) Vout

1) KCL at V_x :

3 currents. Assume they all leave the node.

we have $\frac{V_x - V_{in}}{R_{in}} + i^- + \frac{V_x - V_o}{R_f} = 0$

golden rule 1: no current enters the op amp $\rightarrow i^- = 0$

GR 2: $V^+ = V^-$ in NFB $\rightarrow V^+ = V^- = V_x = 0$ (grounded)

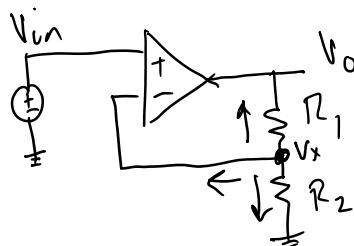
Substitute

$$\frac{0 - V_{in}}{R_{in}} + 0 + \frac{0 - V_o}{R_f} = 0$$

$$\rightarrow -\frac{V_{in}}{R_{in}} = \frac{V_o}{R_f} \rightarrow \boxed{V_o = -\frac{R_f}{R_{in}} V_{in}}$$

Nice . . .

Ex? :



$$KCL \text{ at } V_x: i^- + \frac{V_x - V_o}{R_1} + \frac{V_x - 0}{R_2} = 0$$

$$i^- = 0 \rightarrow \frac{V_x - V_o}{R_1} + \frac{V_x}{R_2} = 0$$

$$\rightarrow V_x \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{V_o}{R_1} = 0$$

$$\rightarrow V_x \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_o}{R_1}$$

$$\rightarrow V_o = V_x \left(\frac{1}{R_1} + \frac{1}{R_2} \right) R_1$$

$$\rightarrow V_o = V_x \left(\frac{R_1}{R_1 + R_2} \right)$$

$$\rightarrow V_o = V_x \left(1 + \frac{R_1}{R_2} \right)$$

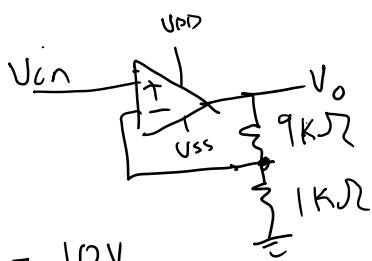
$$GR 2: V^+ = V^- = V_x \rightarrow V_x = V_{in}$$

$$\rightarrow \boxed{V_o = V_{in} \left(1 + \frac{R_1}{R_2} \right)}$$

Practical concerns

upd

$$\dots \text{that } V_o = V_{in} \left(1 + \frac{R_1}{R_2} \right)$$



$$V_{DD} = 10V$$

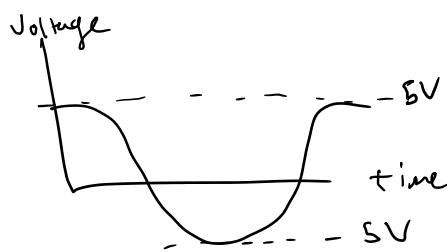
$$V_{SS} = 0V$$

We know that $V_o = V_{in}(1 + R_1/R_2)$
 so $V_o = V_{in} \left(1 + \frac{9k\Omega}{1k\Omega}\right) = 10V_{in}$

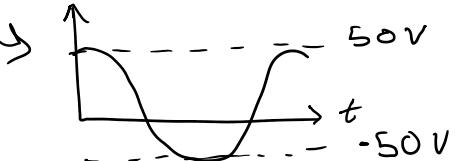
what if $V_{in} = 0.1$? $V_o = 1V$
 but what if $V_{in} = 3V$? $V_o = 30V$

but $V_{DD} = 10$... can't do 30V. caps at 10.

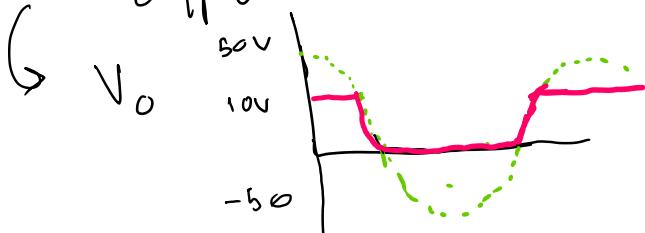
so if $V_{in} \rightarrow$



then V_o should be

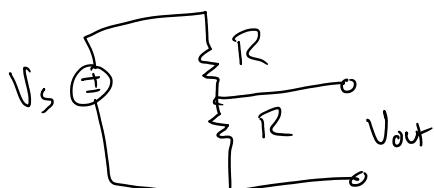


but $V_{DD} = 10$ and $V_{SS} = 0$... anything outside these bounds
 is clipped



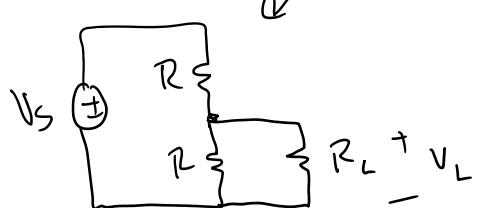
Ideal
REAL

Loading



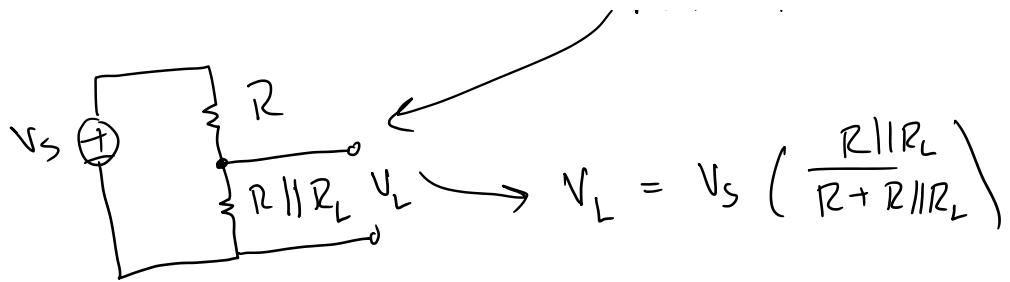
we know $V_o = V_s \left(\frac{R}{R+R_L}\right) = \frac{1}{2} V_s$

but what if we want to use
 this voltage to drive a resistor?



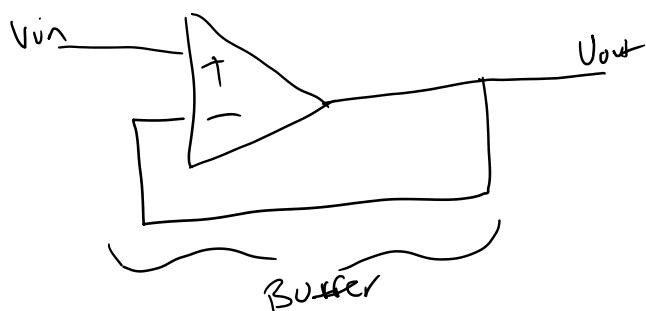
Does R_L have $\frac{1}{2} V_s$ across it?

No. this circuit can be
 redrawn



adding the resistor has "loaded" the circuit and changed its output,

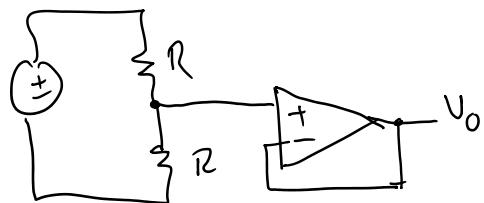
Surprise Example ! !



$$\text{OpAmp 2: } V^+ = V^-$$

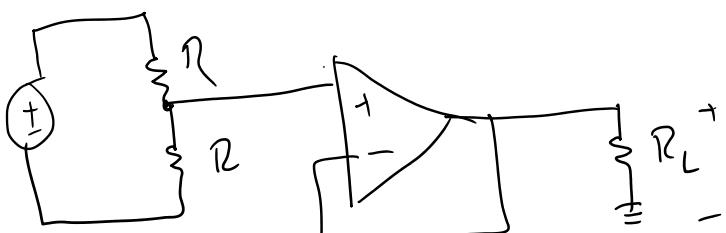
$$\text{and } V^- = V_{\text{out}} \rightarrow V_{\text{out}} = V^+$$

now let's put the two together...



since this "buffer" just copies the voltage on V^+ , then as before, $V_o = \frac{1}{2}V_s$.

and from before, the inputs and outputs are isolated, so adding a resistor does not affect the input!



so now R_L has $\frac{1}{2}V_s$ across it!