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Charge the Circuit: Modelling Public Charging With An Electric Circuit Model

Summary Sheet

In this essay, we built an **Electric Circuit Model** to propose an answer to the need for public charging, where we optimized the deployment of outlets to maximize the net profit of the venues. We first found the electricity consumption due to public charging and traced its evolution over recent years through logistic regression and found that **the consumption of public charging would continue to increase until about 2030, when the consumption would amount to 8.5×10^9kWh; besides, the rate of the growth would slow down since 2021.** This discussion helps us determine the time frame in which we would study the problem, and also influences a vital variable k in our modelling.

Then we **compared the public charging system in a venue to an electrical circuit**. The electronics to be charged are the electrons, which carry charges, or the satisfaction of the users. The electricity consumption due to public charging serves as the battery voltage that forces the electronics into a directional migration to the electrical outlets, creating a level of satisfaction that serves as the current in this circuit. The satisfaction and consumption are affected by factors that create inconvenience, including distances between the electronics and the outlet, the number of electronics to be charged, the charging power, and the consumption rate. These variables are integrated into the three resistors in the circuit.

We devised a means to simplify and calculate the overall resistance in the circuit. We compute the total resistance of the circuit by **calculating the weighted sum of the results of two extreme scenarios**, which are easier to evaluate. With the overall resistance, we were able to obtain the cost of the system. Another sub-model is devised to **calculate the potential benefits using another logistic fuction.** In addition, according to the evolution of public charging obtained in the pre-modelling discussion, the growth of the demand would stop at 2030. Hence, we took different demands for different years into consideration and attained to optimal solution on a ten-year basis. **The net cost is consequently acquired.**

Then we **tested the model upon a couple of typical cases**. We reasoned that for venues that place different levels of emphasis on profiting, **the objective function would differ from maximizing the net profit to maximizing the satisfaction raised by a unit amount of expenditure.** We optimized the public charging system at the waiting hall of Shanghai Hongqiao Railway Station, where we computed a deployment consisting of 16 outlets that generates a total cost of \$1445 and a net profit of \$2749 on a 10 year basis; in addition, since the requirements of the non-technical letter asks us to write the our school newspaper and offer recommendations, we also conducted a case study upon our own school, where the deployment of 40 outlets results in a total electricity fee of \$62151 and an installation and maintenance fee of \$28571, amounting to a total of \$90722. For every dollar spent, the satisfaction of the users is raised by 0.009926. [1]

We also provided a few plausible initiatives to benefit both the venues and their users, including providing subsidies for venues that provide public charging and cooperation that develop electric cars, as well as governmental regulation of license plate issuance and oil prices. We altered corresponding values in our model to quantify the impact these initiatives have on our model, which turned out to be profound.

Finally, we conducted a sensitivity analysis upon a coefficient k, as well as a discussion of the strengths and weaknesses of our model. The results of four problems are presented at the end of the essay, and an article about our findings is written for our school newspaper.

[1] However, to preserve the anonymity of our team, we didn't include detailed floor plans and information including school names, locations, etc.

Contents

1 Introduction

1.1 Background

Ever since the third revolution of science and technology, electronic devices have undergone a rapid course of evolution into an essential part of most people of this age of information, convenience and consumerism. These gadget range from cell phones to electric automobiles, and play irreplaceable parts in our life and work. The world electricity consumption per capita has soared from 1200.059 kWh in 1971 to a breath-taking 3130.71 in 2014.[2]

In response to these devices' huge demand of electricity and short charging intervals, public venues are beginning to provide their users with free electricity outlets to charge their phones, laptops, automobiles and other electronics. By providing a more convenient experience for its users, public venues are rewarded with better a reputation and hence a greater influx of customers and other consequent interests.

However, since the usage of electronics are expanding and diversifying at an unprecedented pace, these venues find it necessary to update their charging facilities to keep pace with the ever increasing demands and new requirements. Our main objective in this essay, then, would be to provide plausible solutions for public venues to install and update their charging facilities in order to guarantee a more agreeable experience for their users, and hence greater interests.

1.2 Literature Review

Indeed, much work has been done in the field of predicting electricity demand in a given region. *Forecasting electricity demand for Turkey: Modeling periodic variations and demand segregation*[3] devised a linear model to predict the hourly electricity demands in Turkey with periodic variations taken into account; while in *Multiscale stochastic prediction of electricity demand in smart grids using Bayesian networks*[4], machine learning has been employed as a means to model electricity demands in urban residential areas at a greater degree of precision. In addition, a more comprehensive and innovative approach involving complex network analyses is discussed in *Predicting sectoral electricity consumption based on complex network analysis*[5].

However, despite the abundance of previous research in the overall or residential electricity consumption, there is a considerable lack of literature specifically featuring public charging energy consumption, which is one of the core variables of our task. Also, although various charging gadgets and installation plans have already appeared on the paper or even in the market, a comprehensive research into the deployment of public charging outlets is virtually nonexistent. Then it would remain our responsibility and privilege to conceive and create a study of such practical significance.

1.3 Problem Restatement

According to the problem, we are required to accomplish the following:

1. **Trace the trend of consumption resulting from public charging.** This trend should be discussed from a macroscopic standpoint, since consumption of public charging in individual venues is hard to separate from the total consumption of the venue. This discussion helps justify the importance of research in such fields and sets the backdrop of our study.

2. **Discuss the impacts and requirements this increasing public charging has on public venues.** In our perception, the requirements the problem refers to should be the deployment of charging facilities that meets the public demand for charging, which in turn maximizes the net interest of the venue itself; on the other hand , the impacts would be the cost and benefits brought to the venue.

[2]World Bank Open Data,
https://data.worldbank.org/indicator/EG.USE.ELEC.KH.PC?end=2018&start=1960&view=chart

[3]Yukseltan E, Yucekaya A, Bilge AH. Forecasting electricity demand for Turkey: Modeling periodic variations and demand segregation

[4]Bassamzadeh N,Ghanem R.Multiscale stochastic prediction of electricity demand in smart grids using Bayesian networks

[5]Yang Z, Shuaishuai Z, Libo W, Yingjie T. Predicting sectoral electricity consumption based on complex network analysis

3. **Devise a model to calculate the cost for public charging resulting from a given deployment of outlets.** This model should be able to compute the cost consisting of electricity fees and installation expenses based on the deployment of charging facilities. It should also be able to evaluate the potential benefits resulting from the charging service.
4. **Evaluate how this cost can be paid.** The payer can involve public venues themselves and government funding and the charged fees of public charging, but we believe the revenues acquired from the increased influx of customers and advertising consequent to the satisfactory charging service is another key factor, on which we would discuss and model in detail.
5. **Modify and apply the model to different public venues.** We conceive that different venues can influence our models by the difference in their topographical structures, different types of users and their demands for charging, and different interests of the venue (profit or non-profit)
6. **Propose plausible initiatives to reduce the net cost of public venues.** This should include approaches to maximize the net interest of public venues other than optimizing the deployment of charging facilities, which is already achieved in previous steps. The initiatives to be proposed here should focus on funding public venues so that they can maximize the satisfaction of users while still reducing the cost of the charging service.
7. **Adapt the cost models to the public venues under the implementation of such initiatives.** The initiatives should affect the optimal deployment of charging facilities since the structure of income and expenditure of the venues is altered by the initiatives. The optimal deployment should be reevaluated.

1.4 Our Work

With respect to the requirements stated in the problem, we would tackle the problem in the procedure as shown below:

1. Before constructing the main model, obtain data concerning the public charging electricity consumption in previous years and predict future trends;
2. Estimate the possible impacts and requirements this trend imposes on public venues to guide the following modelling procedures;
3. Devise a electric circuit model to evaluate the effectiveness of the arrangements of charging facilities in a given venue. This model would be the core of our modelling process and serves as a bridge between such concepts as demand, consumption, charging facilities, potential users, satisfaction and potential revenues;
4. Devise series of sub-models to correlate the electric circuit model with important variables. These sub-models would contain variables that are to be altered in the proposed initatives:
   - Construct a sub-model to model the effects of different charging equipments and their distribution on the electric circuit model;
   - Construct a sub-model to take users of different consumption habits into account;
   - Construct a sub-model to evaluate the influx of users and other interests based on the effectiveness of charging facilities;
5. Pair different types of public venues with their corresponding distribution of users and charging facilities;
6. Conduct a case study of the electric circuit model upon different public venues;
7. Propose plausible initiatives to reduce the net cost of public devices;
8. Alter corresponding variables in the original electric circuit model with respect to the proposed initiatives and reevaluate the resulting net cost;
9. Conduct a sensitivity analysis upon the model and discuss its strengths and weaknesses.

2 General Assumptions

1. **The primary incentive of public venues to install and update charging devices is their own interests.**
Justification: The problem asks us to minimize the cost to the venue and to discuss the impacts and requirements on venues, hence the perspective from which we construct our model should be from that of the venues.

2. **The total energy consumption in a country can be categorized into civilian consumption and non-civilian consumption, the latter mainly consisting of agricultural and industrial production and transportation and the service industry.**
Justification: Though the structure of the non-civilian consumption is quite complicated, the consumption of activities other than agriculture, industry and services amount to a comparatively small value.

3. **The civilian energy consumption in a country can be categorized into household consumption and public charging consumption.**
Justification: A civilian either consumes energy by using outlets in his own residence, e.g. using an air-conditioner or charging a portable battery to be used outside of the residence, or by consuming energy in public places, which is public charging. All the other consumption activities committed in public venues are already calculated in the non-civilian consumption.

4. **When the satisfaction of customers is large enough, the income of a venue is proportional to monetary value of the service it provides.**
Justification: When the satisfaction of customers is not large enough, there will most probably be a portion of service that is not consumed by customers due to the inadequate popularity. However, when the satisfaction is large enough, nearly all of the service would be consumed, and hence the income is proportional to the monetary value of the service, e.g. the price of a train ticket.

3 Pre-Modelling Discussion

This sub-model aims to find the evolution of energy consumption over time and its possible impacts and requirements on the public venues we are about to explore, namely to solve part 1 and 2 of the restated problem. This quantity is essential to our electric circuit model in that it provides the users' original incentives to charge their electronics in public venues, which acts as the electric potential in the circuit. Since the electricity consumed in public electricity outlets is not available, we have to derive this quantity from other data at hand concerning energy consumption. Then a logistic regression is exerted to predict the evolution of this quantity in the future.

3.1 Nomenclature

Table 1: Nomenclature for Sub-Model A

Symbol	Definition
$ET_i(t)$	The total electrical energy consumption in country i at year t
$EC_i(t)$	The electrical energy consumption for civilian usages in country i at year t
$EP_i(t)$	The electrical energy consumption for non-civilian usages in country i at year t
$EH_i(t)$	The household electrical energy consumption in country i at year t
$EA_i(t)$	The electrical energy consumption for agricultural production and transportation in country i at year t
$EI_i(t)$	The electrical energy consumption for industrial production and transportation in country i at year t
$ES_i(t)$	The electrical energy consumption for the service industry i at year t
$E_i(t)$	The public charging electrical energy consumption in country i at year t

3.2 Calculation of the Public Charging Electrical Energy Consumption

The main difficulty in obtaining the value of E, or the public charging electrical energy consumption, lies in that this value is often accounted for in the electricity consumption of public venues and not as a part of household electrical energy consumption. Hence, in resolving this quantity, we have to switch our perspective from public venues to users. By identifying the contents of users' electricity consumption and the contribution of the users' consumption in a country's overall energy consumption, we are then able to settle this value.

First of all, as we have assumed in the model assumptions, the total energy consumption $ET_i(t)$ in a country can be categorized into civilian consumption $EC_i(t)$ and non-civilian consumption $EP_i(t)$, the former further categorized into household consumption $EC_i(t)$ and public charging consumption $E_i(t)$, the last of which being the quantity we are trying to acquire in this model.

Hence we can get

$$ET_i(t) = EC_i(t) + EP_i(t) \tag{1}$$

$$EC_i(t) = EH_i(t) + E_i(t) \tag{2}$$

$$EP_i(t) = EA_i(t) + EI_i(t) \tag{3}$$

Consequently

$$E_i(t) = ET_i(t) - EA_i(t) - EI_i(t) - ES_i(t) - EH_i(t) \tag{4}$$

3.3 Logistic Regression to Predict Future Trends

In order to predict future trends in the public charging electricity consumption, we decided that curve fitting is expedient. By identifying a plausible type of function governing the evolution of this consumption, we would then be able to estimate its future trends. The function we exerted is the logistic model.

The principles of the logistic model are:

- A quantity experiencing growth;
- A factor or a group of factors facilitating this growth;
- A factor or a group of factors restraining this growth.

The differential equation of this model is

$$\frac{dP}{dt} = kP(b - P) \tag{5}$$

which is equivalent to

$$P = a \cdot (1 - \frac{1}{1 + e^{b\cdot(x-c)}}) \tag{6}$$

The characteristic of this function is that it experiences an accelerated growth ($\frac{d^2y}{dx^2} > 0$) during the first half of time, and a decelerated one ($\frac{d^2y}{dx^2} < 0$). The derivative of the function is at global maximum when the population is half of its carrying capacity.

Briefly analyzing the scenario of public charging, we found that the resultant consumption satisfies this function since

1. The public charging consumption experiences a growth as it is evolving into a major social trend;
2. The growth of this consumption is facilitated by the convenience it provides and potential profits seen by public venues;
3. The growth of this consumption is restricted by the cost it brings along and other alternative means of charging and resource sharing.

Hence, the logistic regression can be applied to our prediction of future levels of public charging electricity consumption.

3.4 Case Study upon the European Union

In order to illustrate the feasibility of this sub-model, we conducted a case study upon data regarding the European Union in years 2015 to 2019.[6] We calculated that

Table 2: Eurostat Data

Variable	2015	2016	2017	2018	2019
ET	3.135×10^{12}	3.143×10^{12}	3.147×10^{12}	3.152×10^{12}	3.158×10^{12}
EP	2.382×10^{12}	2.376×10^{12}	2.361×10^{12}	2.348×10^{12}	2.343×10^{12}
EH	0.752×10^{12}	0.766×10^{12}	0.785×10^{12}	0.802×10^{12}	0.812×10^{12}
E	0.005×10^{12}	0.008×10^{12}	0.011×10^{12}	0.015×10^{12}	0.023×10^{12}

With these data points, we conducted a logistic regression and fitted the following function:

$$E = 7.627 \cdot 10^9 \cdot \left(1 - \frac{1}{1 + e^{0.4615 \cdot (x - 2021)}}\right) \tag{7}$$

Which generates the following graph:

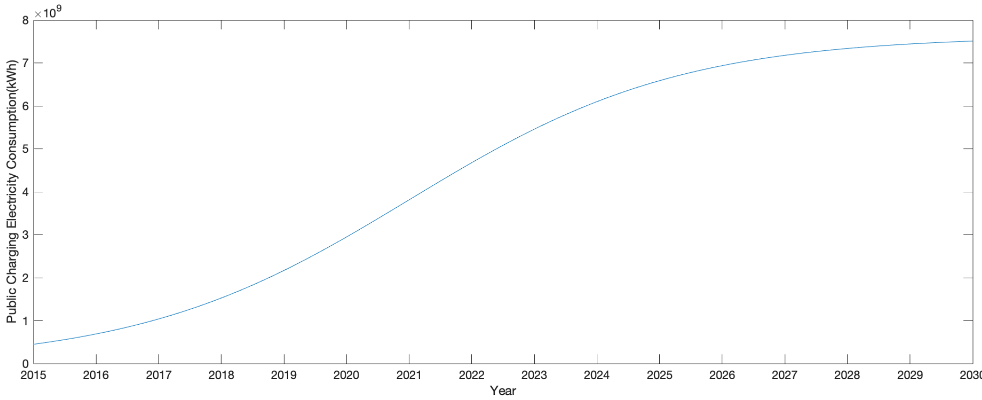


Figure 1: Logistic Regression Based on Eurostat Data

Observing this graph, we can conclude that

- The electricity consumption of public charging will continue to increase until it reaches a level of approximately 8.5×10^9kWh;
- The public charging electricity consumption would largely remain constant after 2030;
- The rate at which this consumption expands would accelerate until approximately 2021, during which this expansion would slow down.

3.5 Impacts and Requirements Imposed on Public Venues

Based on the analysis upon the trends of public charging, some possible conclusions can be drawn:

- The market for public charging would continue to expand considerably in the next ten years;
- Effectively deploying public charging facilities can contribute to increase the influx of customers and other interests such as increased income due to increased advertising;
- The increasing demand for public charging would require the venues to constantly update their facilities in align with the demand;
- The updated facilities could take the form of new charging devices and/or new deployment of charging devices.

[6] Eurostat,
http://appsso.eurostat.ec.europa.eu/nui/submitViewTableAction.do

4 The Electric Circuit Model

4.1 Model Overview

The Electric Circuit Model is the main model of our modelling process and is responsible for all the requirements in the restated problem except parts 1 and 2. It compares a public venue to an electric circuit, and compares such quantities as demand, satisfaction and the distribution of electric outlets. Hence, exerting basic theorems governing an electric circuit, we were able to establish a method to link these important values together. Consequently, we would be able to calculate the cost resulting from the increasing demands as well as to provide specific courses of action concerning the optimization of outlet deployment.

According to the problem, we are to model the cost that maximizes the interest of the public venues under the increasing demand. The most practical and relevant way to maximize this interest would be to optimize the deployment of electricity outlets. Therefore, the main task of this model would be to output the optimal arrangement of such outlets that exactly meets the demand for public charging. This interest would be affected by

1. The cost of public charging, which consists of electricity fees and expenses of purchasing, installing and maintaining electricity outlets;

2. The potential revenues gained from offering convenience to users, including a greater influx of customers and more advertising.

Hence, in this model we would first need to calculate the cost and potential revenue of a given plan of deployment, and then use an algorithm to optimize these two outcomes.

4.2 The Public Charging Process

To successfully set up this rather complicated model, we have to clarify the object we are analyzing, public charging. It has to be simplified into a collection of entities and interactions between them. First of all, we identified three key entities in this process:

1. **The venue.** The venue is the environment that houses all the other characters and interactions. It provides a certain topographical structure that restrains how characters can interact with each other, namely the way in which different architectural components, including corridors, halls, aisles, open spaces, etc., affect the convenience in charging a certain device.

2. **The electronics.** The electronics are the devices to be charged in this venue. We use them to instead of different users since they are more intimately related to the charging devices; if a user happens to have in his possession two or more electronics to be charged, we would still view these electronics as independent with each other. The types of electronics and the habits of how the user utilizes them affect the frequency of charging as well as the outlets required.

3. **The outlets.** The outlets are the fixed devices by which the electronics can be charged. The distribution of outlets, which includes numbers and deployment patterns of different types of outlets, also affect the overall efficiency of the venue's public charging system, and consequently the users' satisfaction, which in turn determines the costs of the system and the potential profits that can be generated.

Then we discussed how these entities would interact with each other. First of all, the users would carry their devices to the outlets, hoping to charge the electronics. In this procedure, they have to travel in the venue and possibly compete with other users for the outlets. This trouble on the way to the outlet negatively affects the satisfaction of the users and would possibly discourage users from charging, hence failing to meet their demands. Therefore, we would have to minimize this hampering effect if the interest is to be maximized. The key to this minimization lies in the optimal deployment of electronics in the venue and possible changes of the physical topography of the venue.

4.3 Nomenclature

Table 3: Nomenclature for the Electric Circuit Model

Symbol	Definition
	Constants
k_1	The coefficient of resistivity of the electronics resistor
k_2	The coefficient of resistivity of the venue resistor
k_3	The coefficient of resistivity of the outlet resistor
d_0	The minimum width of a passageway for a person to comfortably walk through
v_0	The average walking speed of users in a venue
f	The installation and maintenance fee of the charging devices
k_i	The stretching coefficient of the revenue function in year i
	Variables
U_0	The total electricity consumption of public charging in the venue
I_0	The overall user satisfaction of the public charging system the venue
R_0	The overall inconvenience of the public charging system in the venue
RE_i	The inconvenience caused by properties of electronics
RV_i	The inconvenience caused by the topography of the venue
RO_i	The inconvenience caused by properties of outlets
v	The rate at which electronics consume electricity
Q	The capacity of a type of electronics
L_i	The length of passageway i in the venue
d_i	The width of passageway i in the venue
P	The charging power of an outlet
E_1	The demand for electricity
E_2	The maximum electricity output of the outlets
w_1	The weight on the electronics-directed evaluation
w_2	The weight on the outlets-directed evaluation
m	The volume of users in the venue
n	The number of outlets
t	The time length that the venue is open
CO	The cost to install and maintain an electrical outlet
CE	The cost of the public charging consumption
CS	The original cost of the service of the venue
C	The revenue of the venue

4.4 The Simplification of Electronics Distribution

If we view all electronics in this venue as discrete individuals, the model would be overly complicated and sensitive. Hence, the electronics should be viewed as a probability distribution of different quantities in the venue, among which the most prevalent would include the pace at which the electronics are charged and the density of the electronics. This quantity would affect the convenience of the entire system.

Nevertheless, if viewed as a continuous distribution, the model would still lack simplicity as the differential equations in the context of a electrical circuit requires pain-staking calculations. Consequently, we further simplified the distribution of electronics into an equivalent distribution of clusters of electronics at nodes of the topographical structure of the venue. The reason we chose to cluster the electronics at node locations is because these locations are fixed given the floor plan of the venue, and hence simplicity is guaranteed.

As shown in the figure above, the circuit consists of 8 nodes with 10 wires connecting them. The electronics on wires are clustered into their adjacent nodes, for example, electronics on wire 2 are clustered into nodes B and C. In order to maintain the overall population of electronics, only half of the electronics on one line would be clustered into one node, e.g. half of the electronics on wire 2 will be clustered onto node B while the other half onto node C.

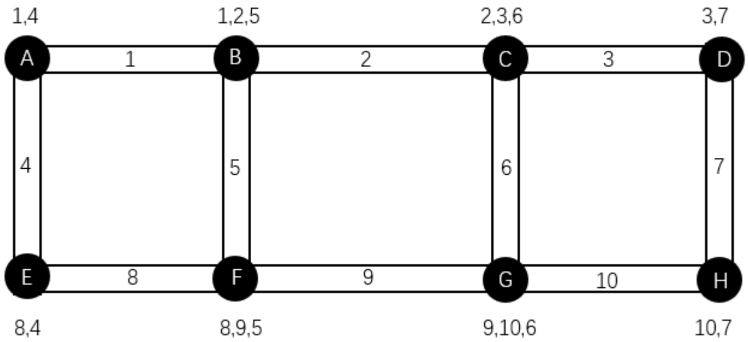


Figure 2: The Simplification of Electronics Distribution

4.5 Justification for the Electric Circuit Model

Our model is based on a comparison between the public venues in reality and electrical circuits. To justify this notion, we have to explore the basics of an electrical circuit and whether they are fully represented in the affair of public charging.

First of all, the fundamental factors of an electrical circuit are:

- A system of closed conducting wires;
- A multitude of free electrons in the wires;
- A voltage at the ends of the wires to force the electrons into a directional migration.

Examining the public charging process as discussed in the previous subsection, we found the literal meaning of each component of the electrical circuit:

1. The passageways in the venue serve as the electrical circuit;
2. The electronics in the venue serve as the electrons able to move freely in the circuit (passageways);
3. The lengths, widths, and other electronics create inconvenience that serves as a system of resistors that restrain the movements of the electrons (electronics);
4. The users of electronics have a certain degree of satisfaction which increases as the electronics move through the venue to be charged at outlets;
5. The public charging consumption serves as the voltage that drives the electrons (electronics) to move through the venue to the outlets; in other words, it drives the satisfaction of the electronics to form a directional migration through the circuit to the outlets;
6. The overall satisfaction of the public charging system serves as the total current through the circuit of passageways, which is the result of the voltage (consumption) forcing electrons (electronics) into a directional migration.

To further justify this comparison, we provided a couple of fundamental theorems governing electrical circuits. Through analyzing these scenarios, we found that our interpretation of the public charging fits these theorems:

4.5.1 Ohm's Law

By Ohm's Law, the current through a resistor can be calculated by

$$I = \frac{U}{R} \tag{8}$$

where U denotes the voltage between the ends of a resistor and R represents the resistance of that resistor. When applied to the entire circuit, this equation then becomes

$$I_0 = \frac{U_0}{R_0} \tag{9}$$

where U_0 denotes the battery voltage (total consumption) and R_0 represents the resistance of the entire circuit (total hinderance).

The resistance of the circuit can be treated as a coefficient, and when holding this equal the total current (satisfaction) is directly proportional to the battery voltage (consumption). This relationship is reasonable, since the more electricity is charged, the more satisfied the user would be. Also, when the total consumption of electricity is held equal, the more resistant the circuit is (the less convenient the system is), the less current (satisfaction) would pass through. Hence the electricity model is well justified.

4.5.2 Expression of Current

The expression of current is that

$$I = \frac{Q}{t} \tag{10}$$

Hence, the charge through a cross section can be described as the integral of current over time. The charge can be understood as the total capacity for satisfaction in the users. The more rapidly this capacity is filled, the more satisfaction is aroused. Hence, the electricity model is well justified.

4.5.3 Mapping between the Public Charging System and the Electrical Circuit

Table 4: Mapping between the Public Charging System and the Electrical Circuit

Electrical Circuit	Public Charging
Battery Voltage	Total Electricity Consumption
Total Current	Overall Satisfaction
Circuit Resistance	Overall Inconvenience
Electrons	Electronics to be Charged
Charge	Satisfaction of a user towards one of his electronics

4.6 Construction of the Circuit

With the Electrical Circuit Model justified, we can now move on to construct this model under the comparison of the public charging system to an electrical circuit. Since our model mainly considers the topographical features of a venue and its corresponding spatial deployment of electrical outlets, our model would have to operate upon a floor plan of a given venue. Suppose the floor plan is the figure shown below:

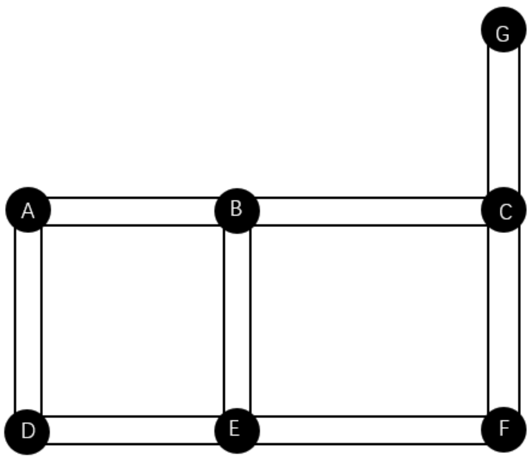


Figure 3: Demonstrative Floor Plan

With the venue at hand, we then place a set of electrical outlets on the floor plan. Note that this deployment is only one of the possible solutions from which the optimal scheme would emerge. Next, with the data concerning the distribution of electronics to be charged in the venue, we cluster these electronics at the locations of nodes as explained beforehand. Laying out all the clustered nodes and the outlets, we updated the floor plan as follows:

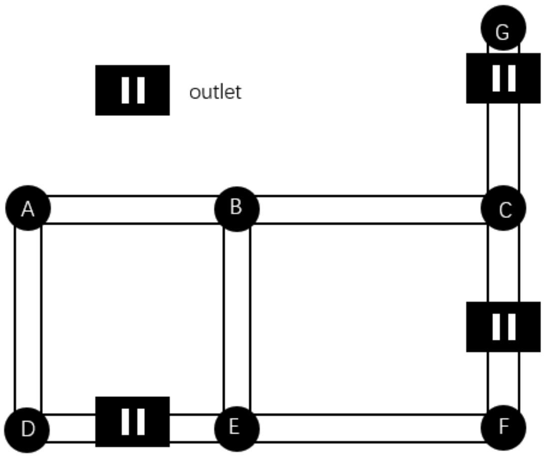


Figure 4: A Typical Floor Plan with Electronics and Outlets

Notice that in the graph above, each node of electronics are linked with at least one of the outlets. However, to form a closed circuit, extra wires are needed. Therefore, we attach one wire to each node/outlet perpendicular to the floor plane. For each pair of node and outlet, connect the added wires. Consequently, a closed circuit now connects each pair of node and outlet. According to the direction of the current, the electronics flow inside the venue directs at the outlets, while the flow in the added lines directs at the electronics. Therefore, the current flows reversely. The current flows out of the positive pole of the battery and into the outlet, and out of the electronics into the negative pole of the battery.

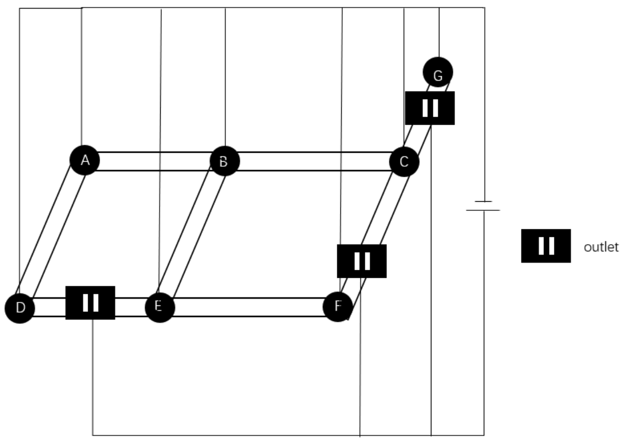


Figure 5: The closed circuits

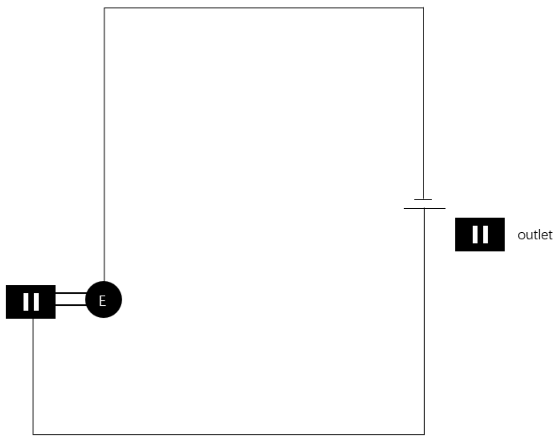


Figure 6: The closed circuit between one pair of node and outlet

Now, it's obvious that in each closed circuit, the only resistor is the venue itself. However, we also need to take other quantities into account, e.g. the number of electronics at a node, the number of adjacent outlets, the electronics' power consumption rates and the outlets' charging rates. These variables either facilitate or restrain public charging, in other words serve as a resistor or counter the effect of a resistor. Hence, to account for the effects of these variables on the model, we added resistors on both ends of the added wire:

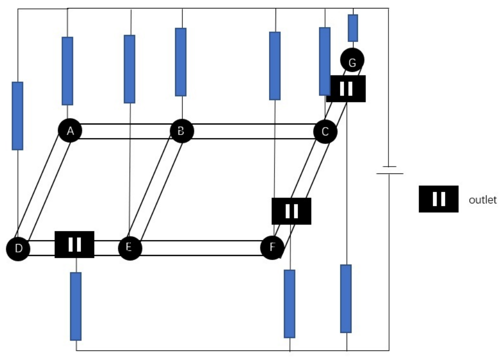


Figure 7: Additional resistors on the ends of added wires

Now we have completed the electrical circuit, in which we take a wide range of relevant variables into account. However, all variables except the topographical structure of the venue are embodied by the resistors. Thus, we would focus on these modification factors in the next subsection.

5 Sub-Models

5.1 Sub-Model A: Modification Factors of Resistances

The density of electronics and outlets and the rate at which electricity is consumed and charged are the modification factors that will have a great impact upon the electrical circuit model, and we have decided to account for them by adding resistors to the circuit. The question is, how much resistance will each of them add to the circuit?

We categorize all the resistors in this circuit as follows:

1. **The resistor created by the properties of the electronics**
   We identified one variable that determines this resistance: the rate at which the electronics at the same node consume electricity. A competitive relationship exists between electronics at the same node in that they are to compete for the same set of outlets. From the perspective of one of the electronics, the faster the other electronics consume electricity, the greater the frequency they compete for an outlet, and the less the convenience felt by this electronic, consequently the greater the resistance. Therefore, the resistance of the resistance of node i is

$$RE_i = k_1 v \tag{11}$$

   where v is the rate at which electronics at this node consume electricity, and k_1 is a coefficient. Note that the consumption rate reflects a number of factors:

   - The number of electronics clustered at this node. The more electronics there are, the fiercer the competition is.
   - The type of the electronic to be charged. Different appliances have different powers.
   - The habit of the user. Users of different occupations use electronics in different manners, and hence different consumption rates.
   - The overall demand for public charging as calculated in the pre-modelling discussion.

2. **The resistor created by passageways on the venue**
   The impact of the relative distances on the venue on the inconvenience of the public charging system is also considerable. Users don't prefer to travel long distances to charge their devices. In addition, the width of a passageway influences the density of users, so similar to the resistance of electronics, the narrower the passageway is, the denser the electronics, and hence greater competition and inconvenience. According to the original expression of resistance,

$$R = \frac{\rho L}{S} \tag{12}$$

Likewise, our expression for the venue resistance at passageway i can be calculated by

$$RV_i = \frac{k_2 L_i}{d_i} \tag{13}$$

where L_i and d_i are the length and width of this passageway, and k_2 is yet another coefficient.

3. **The resistor created by the properties of the outlets**
The main variable that affects this resistor is the charging power of the outlets. The faster the outlets charge the electronics, the more electronics this outlet can charge in a given time period, and the less the competition and inconvenience. The resistance of the outlet resistor is:

$$RO_i = \frac{k_3}{P_i} \tag{14}$$

where P_i is the charging power of outlets and k_3 is another coefficient.

Considering all three resistances, we found that they mainly represent two factors:

- The competition involved in charging. This is accounted for in the electronics resistance and the venue resistance. The electronics resistance is dependent on the competition between electronics for outlets, while the venue resistance is dependent on the competition felt on the way to outlets.

- The time required to get charged. This is accounted for in the venue resistance and the outlet resistance. The venue resistance is dependent on the travelling time when searching for an outlet, while the outlet resistance is dependent on the charging time at the outlet.

When the time required in travelling and charging is the same, the inconvenience felt by the user should also be the same. Similarly, when the competition between electronics and the competition on the way is the same, the inconvenience felt by the user should also be the same. Hence, we can eliminate two of the coefficients by solving these equations

$$\begin{cases} \frac{v}{Q} = \frac{d_0}{d_i} \Rightarrow k_1 v = \frac{k_1}{d} \\ \frac{L_i}{v_0} = \frac{Q}{P_i} \Rightarrow k_2 L_0 = \frac{k_3}{P_i} \end{cases} \tag{15}$$

where v, Q, d_0 and d_i denote the electricity consumption rate, capacity of electronics, minimum width of passageway for a person to pass through, and the width of the passageway being studied upon respectively; L_i, v_0 and P_i represent the length of the passageway, walking speed of the person, and the charging power of the outlet respectively.

Thus

$$\begin{cases} \frac{k_1}{k_2} = \frac{1}{Q \cdot d_0} \\ \frac{k_2}{k_3} = \frac{1}{Q \cdot v_0} \end{cases} \tag{16}$$

5.2 Sub-Model B: Net Cost to Public Venues

As has been discussed in previous subsections, the battery voltage and the total current represent the total electrical consumption and the overall satisfaction respectively, the latter in turn affecting the potential revenues of the venue. Therefore the net cost can be regarded as a function of the battery voltage and the total current. In order to minimize the net cost, an equation has to be established between the voltage and the current. Hence, the overall resistance of the circuit becomes the key to modelling the cost.

5.2.1 Approximation of Total Resistance

In the subsection above, we have already proposed a means to calculate the resistance of different resistors throughout the circuit. The difficulty that lies before us now is the highly complicated calculation of the resistance of a complex circuit, which results from the structure of the circuit. Since this value has to be calculated many times in the optimization algorithm in order to approximate the best deployment of charging facilities, the evaluation has to be highly simplified, in other words approximated to a predictable structure. Hence, we decided to only calculate the most possible pairings between the electronics and outlets, which are the ones with the least resistances. The procedure is as follows:

Procedure	Illustration

1. Compare the number of outlets with the number of nodes.

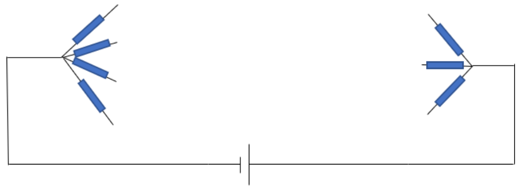


Figure 8: Resistance Calculation Stage 1

2. Start with the type of entities that amounts to a smaller number. For every one of that entity, pair it with the one of the other type of entities that results in the least resistance. This eliminates pairings between outlets and electronics of long and disagreeable distances, which aren't likely to be taken by many users.

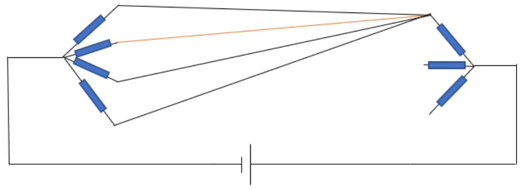


Figure 9: Resistance Calculation Stage 2

3. For each of the established pairings, find the routes with the resistances not more than 200 greater than the least resistance. This is because although users generally choose the outlet that guarantees the most convenience, the routes to this outlet might vary greatly, some of which with similar resistances.

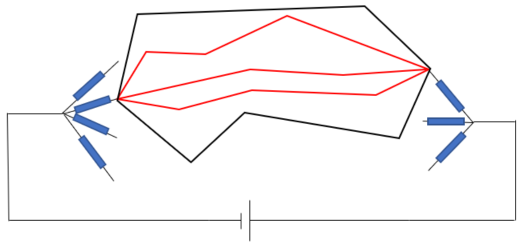


Figure 10: Resistance Calculation Stage 3

4. Each of the identified routes should be a series connection of resistors. Calculate the total resistance on this route.

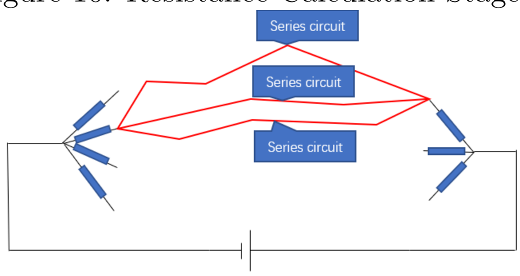


Figure 11: Resistance Calculation Stage 4

5. For each pair of outlets and electronics, the series connection routes should form a larger parallel circuit. Calculate the total resistance of this circuit.

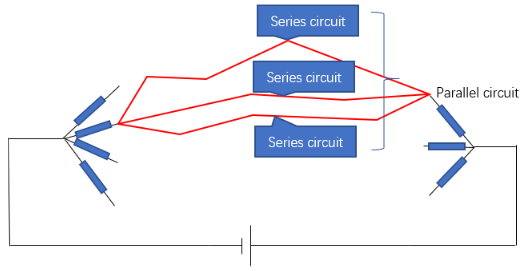


Figure 12: Resistance Calculation Stage 5

6. All the pairs of outlets and electronics should form the whole circuit, which is also a parallel circuit. Calculate the total resistance of the circuit R_0.

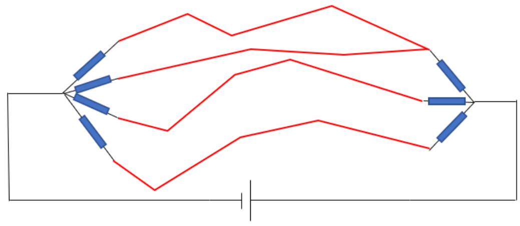


Figure 13: Resistance Calculation Stage 6

This procedure is based on the assumption that there is an overwhelming unbalance between the outlets and the electronics. If the number of users greatly exceeds that of the electronics, the electronics would be distributed to the outlets, and vice versa. However, in reality the situation is usually more mixed and lies in the gray area between the two extremes. Hence in these situations the course of action would be the weighted sum of the two evaluations $R_0(1)$ and $R_0(2)$, the weights being w_1 and w_2,

$$R_0 = w_1 R_0(1) + w_2 R_0(2) \tag{17}$$

where

$$\frac{w_1}{w_2} = \frac{E_1}{E_2} = \frac{m \cdot t \cdot v}{n \cdot P} \tag{18}$$

E_1 and E_2 denote the demand for electricity and the maximum output of outlets, respectively. These two variables are in turn determined by the number of outlets n, the volume of users of the venue m, the time the venue is open t, the charging power of the outlets P, and the consumption rate of the electronics v.

Consequently the electricity fees are obtained. However, there is another part of the cost, the installation and maintenance fee of the charging devices. This can be obtained by multiplying the fee for each outlet f by the number of outlets in each trial.

5.2.2 Potential Profit

The satisfaction that the public charging service generates in turn benefits the venue, which could come in the form of increased influx of customers, better reputation or increased advertising income. The relationship between the benefits and the satisfaction should also satisfy the logistic model, and the facilitating and restraining factors are:

1. The satisfaction arouses the inclination to consume more at the venue, and the growth rate increases in align with the satisfaction as the more ideal the service is, the rarer this kind of service is, and the more appealing the venue will seem to be;

2. The maximum number of potential customers is fixed and it's impossible to attract all the customers to the venue.

Hence, the logistic expression of the consequent benefits is

$$C = a \cdot \left(1 - \frac{1}{1 + e^{b \cdot (I - c)}}\right) \tag{19}$$

where C and I denote the income of a venue and the satisfaction of users, respectively. a, b, and c are three coefficients that determine the shape of the graph. a denotes the higher parallel asymptote of the graph, b represents the degree to which the graph is stretched along the x-axis, and c is the x-coordinate of the point where the derivative is at maximum.

As has been assumed, when the satisfaction is large enough, the income of a venue is proportional to the value of service it provides. In other words, the rate of increase in the income of a venue is proportional to the rate of increase in the value of the services, in our case the ratio of the satisfaction of the users to the original income of the venue. Since the y-value of the asymptote is the limit of the function when the satisfaction approaches infinity, we get

$$a = \lim_{I \to \infty} C = \frac{C_2}{C_1} = \frac{CE}{CS} \tag{20}$$

where C_1 and C_2 represent the income of the venue before and after the improvement of public charging, and CE and CS denote the electricity fees and the original service value of the venue.

Then the stretching coefficient b is left as it a coefficient k and will be discussed in the next sub-model.

In addition, when the satisfaction of the users is 0, the derivative of the function should be at global maximum since when the current satisfaction is low enough, even a tiny amount of improvement would result in a huge increase in revenue; however, when the satisfaction is already large enough, even when the satisfaction is improved many times, the marginal benefit is relatively small. Hence, by the theory of marginal benefit the coefficient c should be 0.

Consequently,

$$C = \frac{CE}{CS} \cdot \left(1 - \frac{1}{1 + e^{k \cdot I}}\right) \tag{21}$$

An illustrative graph of the function is shown below:

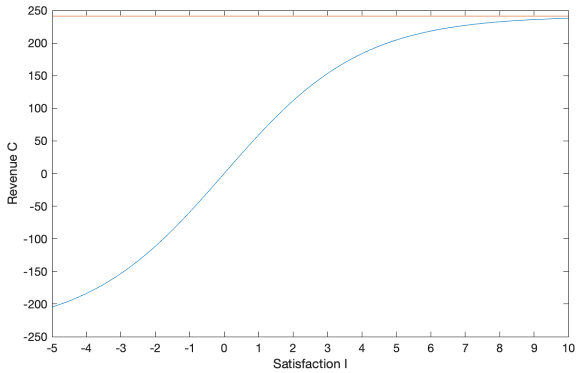


Figure 14: Illustrative Graph of the Satisfaction-Revenue Function

5.3 Sub-Model C: The Implementation plan on a Ten-year Time span

As has been determined in the pre-modelling discussion, the demand for public charging E would continue to grow in the next ten years following a logistic function. The demand would influence the stretching coefficient k in sub-model B. The larger k is, the more squeezed the graph would be, which means that the maximum derivative would be greater. If E is greater, the demand for public charging would be greater, and hence the importance of public charging would be greater. Therefore, for the same amount of satisfaction increase, when E is greater, the increase of corresponding revenue would be greater, and hence the derivative would be greater, consequently k is greater. Hence k is a coefficient corresponding to a certain year. Thus

$$k_i \propto E_i \tag{22}$$

The time now is towards the end of year 2019. Assuming the installation of the charging devices would require at most a year, we need to consider the years since 2021. Since the demand would approximately become constant after 2030, we only need to consider the ten years from 2021 to 2030.

Employing an randomized algorithm, we were able to find the deployment of outlets that minimizes the net cost.

6 Case Study

6.1 Variables Influencing the Outputs of Our Model

Part 3 of the problem requires us to discuss how our model changes with respect to different types of venues. First of all, we summarized the variables that impact the outcomes of the model that have been mentioned in previous sections:

Table 5: Influential Variables of Our Model

Variable	Impact
Topographical structure of the venue	Influences the venue resistances of the circuit RV and also the possible locations of outlets
The volume of users of the venue m	Influences the weight of the two extreme scenario results when computing the total resistance
The time length the venue is open t	Influences the weight of the two extreme scenario results when computing the total resistance
The types of users	Different users have electronic using habits, which influences the power consumption rate of the electronics v, in turn determining the electronics resistance RE and the weights
The charging power of outlets P	Influences the outlet resistance RO and the weights
The installation and maintenance fee of an outlet f	Influences the total cost and consequently the optimal deployment plan

6.2 Shanghai Hongqiao Railway Station

Our first case is the waiting hall of Shanghai Hongqiao Railway Station[7]. Some of the values of the influential variables in this case is given below:

Table 6: Influential Variables Values of the Railway Station Case

Variable	Value
The volume of users of the venue m	2000
The time length the venue is open t	24hours
The types of users	All kinds with mean consumption power of 2W
The charging power of outlets P	5W
The installation and maintenance fee of an outlet f	Simple Charging Sockets with price of \$28.6 each

Its topographical floor plan with respective resistance values is shown below.Exerting our model, we found that the optimal deployment of charging facilities would be:

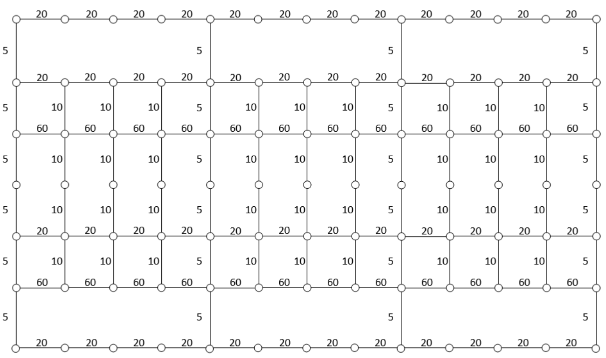


Figure 15: Topographical floor plan of Shanghai Hongqiao Railway Station

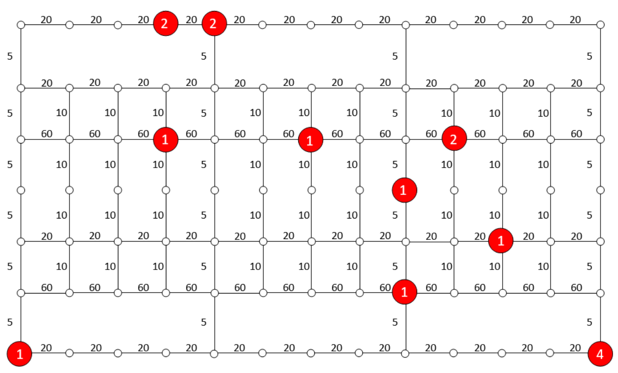


Figure 16: Resulting Deployment of Shanghai Hongqiao Railway Station outlets

Over a time interval of 10 years, this deployment results in a total electricity fee of \$987 and

[7]Introduction to the departure layer of Shanghai Hongqiao Railway Station

an installation and maintenance fee of \$458. The additional revenue due to the installation, however, amounts to \$4194, creating a net profit of \$2749.

6.3 Our Own School

We also conducted a case study upon our own school. **To guarantee the anonymity of our team, the detailed data concerning our school is not given.** We could have studied some other campuses, but since we are required to write to our own school newspaper, we perceive that our own school should be studied. Since our school generally prohibits students from carrying electronics such as cell phones and laptops, the main type of electronics in our study would be the electric cars owned by the staff. Also, since our school is publicly funded, the income wouldn't be a main subject of consideration; rather, the reputation of this venue would be of more importance. Therefore, we wouldn't have to calculate the net profit of this venue, but instead should focus on the satisfaction attained by spending a unit amount of money, the ratio $\frac{I}{CO+CE}$. The values of influential variables are listed below:

Table 7: Influential Variables Values of the School Case

Variable	Value
The volume of users of the venue m	100
The time length the venue is open t	12hours
The types of users	Only teachers and other white-collar adults with mean consumption power of 30kW
The charging power of outlets P	3.52kW
The installation and maintenance fee of an outlet f	Electric car charging piles with price of \$714.3 each

Its topographical floor plan with respective resistance values is shown below.Exerting our model, we found that the optimal deployment of charging facilities would be:

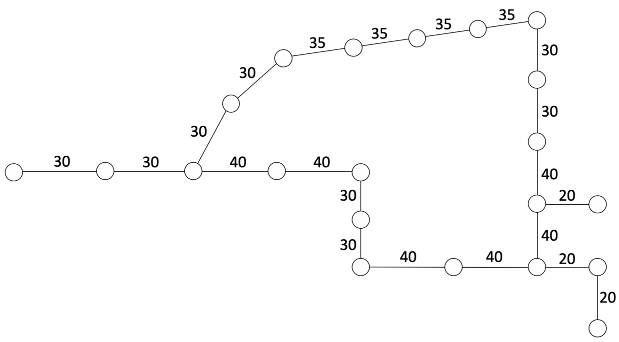


Figure 17: Topographical floor plan of the School

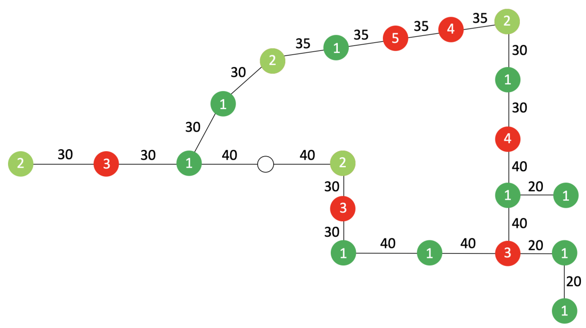


Figure 18: Resulting Deployment of the School

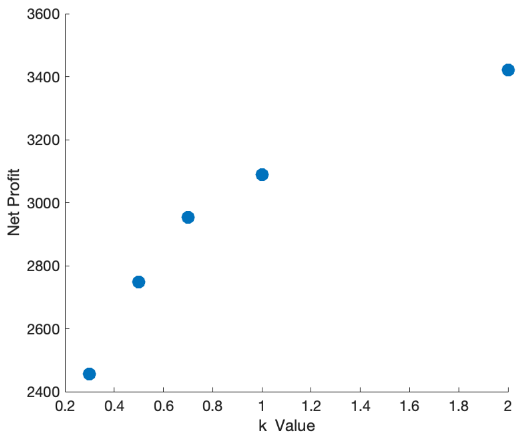
Over a time interval of 10 years, this deployment of 40 outlets results in a total electricity fee of \$62151 and an installation and maintenance fee of \$28571, amounting to a total expense of \$90722. For every dollar spent, the satisfaction of the users is raised by 0.009926.

7 Sensitivity Analysis

In Sub-Model C, where we established a logistic model between the satisfaction level I and the total revenue C, there still exists on key coefficient yet to determine: k, which measures the degree to which the graph stretches along the x-axis. Hence, here we conducted a sensitivity analysis upon this factor to see if its value greatly affects the total net profit.

This analysis shows that the net profit of the charging system is actually quite sensitive to the coefficient k. However, in the possible initiatives we will make use of this sensitivity in order to greatly improve the benefits brought to both the venue and its users.

Graph



Data

Figure 19: Sensitivity Analysis

k	Number of outlets	Net Profit ($)
0.3	19	2456
0.5	16	2749
0.7	15	2954
1.0	13	3089
2.0	8	3421

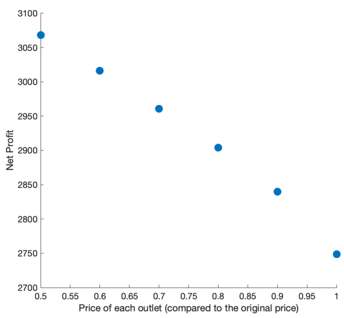
8 Plausible Initiatives and Their Impacts

8.1 Initiative #1: Provide Subsidies for Building Outlets

Providing subsidies for venues that build outlets would be a good way to benefit both the venues and the users. Such an initiative could effectively lower the price of outlet building, and therefore venues can yield more profits; on the other hand, venues could also afford to build more outlets, which in turn benefits the users.

Thus, we consider that the government provides an amount of subsidy proportional to the venue's expenditure on installing and maintaining the outlets, which is equivalent to lowering the installation and maintenance fee for each outlet f.

Scatterplot



Data

Figure 20: Initiative 1

Magnification	Number of outlets	Net Profit ($)
1	16	2749
0.9	15	2840
0.8	15	2904
0.7	15	2961
0.6	13	3016
0.5	14	3068

8.2 Initiative #2: Encourage Purchase of Electric Vehicles

Through the Case Study, we found that the larger k is, the larger the net profit would be. Therefore, if the government found a way to increase the k in most of the years, the venues and the users would both be greatly benefitted as venues can harvest larger profits, meanwhile building more outlets to increase the satisfaction of the users.

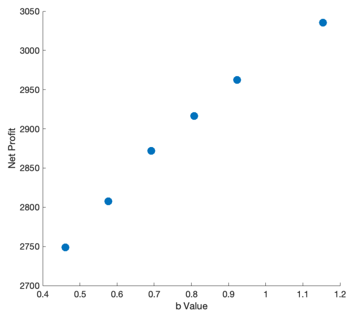
The way to increase k lies in the first logistic model in the pre-modelling discussion. Since $k_i \propto E_i$, by enlarging the demand for public charging k would also expand. By encouraging people to purchase more electric cars and less gasoline ones, the demand for public charging would be enlarged, and the consumption-time logistic function would be more squeezed along the x-axis, and the k would be greater. Therefore, the main adaptation would be on the coefficient b of the first logistic model.

To justify our reasoning, we multiplied b by different values and obtained the corresponding net costs.

Some initiatives are listed below:

- Provide subsidies to cooperations developing new energies and electric cars so that electric cars would be more available at a lower price;
- Regulate the gasoline car licence plates that are issued every year;
- Regulate the oil price so that it does not plummet.

Scatterplot



Data

Figure 21: Initiative 2

b	Magnification	Number of outlets	Net Profit ($)
0.4675	1	16	2749
0.5769	1.25	15	2808
0.6923	1.5	15	2872
0.8076	1.75	14	2916
0.9230	2	14	2962
1.1538	2.25	13	3035

9 Conclusions

9.1 Problem 1

The electricity consumption of public charging satisfies the following function:

$$P = 7.627 \cdot 10^9 \cdot \left(1 - \frac{1}{1 + e^{0.4615 \cdot (x-2021)}}\right) \tag{23}$$

- The electricity consumption of public charging will continue to increase until it reaches a level of approximately 8.5×10^9kWh;
- The public charging electricity consumption would largely remain constant after 2030;
- The rate at which this consumption expands would accelerate until approximately 2021, during which this expansion would slow down.

Based on the analysis upon the trends of public charging, we identified the requirements and impacts on the public venues:

- Public venues should optimize their deployment of public charging in order to increase the satisfaction of users, which can become a major area of competition between different venues as the demand for such services will continue to expand rapidly;
- While satisfying their users, the venues can also gain benefits to themselves including better reputations, more customers and advertising. By balancing their interests and the cost, the profit of the venues can be optimized, resulting in a more reciprocal relationship between the venue and its users.

9.2 Problem 2

The cost model is discussed in detail in sections 4 and 5. In the case study of Hongqiao Railway Station, the cost of the optimal public charging system is $1445 over three years with a total of 16 plugs, but this expense is fully covered by the revenue the charging system generates. By providing a more agreeable environment, the station attracts more users and advertising, which generate a revenue of $4194, amounting to a net profit of $2749. In the school case, the optimal deployment of 40 outlets results in a total electricity fee of $62151 and an installation and maintenance fee of $28571, amounting to a total expense of $90722.

9.3 Problem 3

Through the different cases we have studies, we found that the main structure of our model remains exactly the same. The only modification would be the objective function we are to maximize. For a venue that places profit at a high priority, the net profit $C - CO - CE$ should be maximized; however, for a venue to which the profit is of lower significance, the ratio $\frac{I}{CO+CE}$ should be maximized. Also, different values of data are input, including different opening times, different consumption rates, different volumes of users, different electronics to be charged, and different topographical structures.

9.4 Problem 4

In order to benefit both the venues and its users, we proposed four initiatives:

- Provide subsidies to venues with respect to the number of outlets they plan to build;
- Provide subsidies to cooperations developing new energies and electric cars so that electric cars would be more available at a lower price;
- Regulate the gasoline car licence plates that are issued every year;
- Regulate the oil price so that it does not plummet.

The significance of their impacts on the net profit of the venues is justified in the previous section.

10 Strengths and Weaknesses

10.1 Strengths

1. Our models are only dependent on easy-to-obtain data, including the installation and maintenance fee of an outlet, power consumption rate of electronics, charging power of outlets, the opening time of the venue, the volume of the users, and the floor plan of the venue. All four are readily available and hence our model is easy to implement.
2. Our models took a multitude of factors into account, including the physical structure and opening times of the venue, the volume of visitors and their habits of using electronics, and different types of electrical outlets. This helps make our model more realistic and accurate.
3. We built our model upon the electromagnetic model of electric circuit and this comparison is well justified. With this innovative approach, our models are very simple and comprehensible; furthermore, since the fundamental comparison is solid and sound, our model is built upon a firm foundation, which guarantees the accurateness of its details, including calculations and correlations between variables whose relationships are initially obscure.
4. Our model uses a greatly simplified procedure of calculation the total resistance of the circuit, where we pick out the most influential circuit branches and calculate the weighted sum of the two extreme scenarios. This means of evaluation greatly enhances the simplicity of our models, making it much more practical when applying to real world cases.
5. We conducted multiple case studies to justify our models, where we selected two different types of venues in terms of intended users, ways of income, topographical structure and opening times. This testing stage clearly illustrates the feasibility of our models.

10.2 Weaknesses

1. Our model did not include a discussion of the interaction between the venue we are planning the deployment and other potential competitors. Theoretically, the joint efforts of different venues could enhance the overall quality of the public charging service, while on the other hand, different venues could affect each others' profits by forming rivalry. If these are considered, our model would be even more complete.
2. Although the feasibility of our models are well justified through the case studies, they can not well guarantee the accuracy of their results since there's virtually no way to test the deployment in a real world scenario.
3. Though we made use of the logistic regression to model the demand for public charging over the next ten years and this model is well justified, we did not take a further look on other variables that could plausibly impact the demand, including the introduction of new cell phones, the government energy policies, and the situation of other energies such as petroleum, whose fluctuation of price has caused profound effects. With these modification factors taken into consideration, our model can be more realistic; however, in doing this we would have to trade off some robustness.

11 An Article for the School Newspaper

Optimization of Public Charging and its Applications to our Campus

If you take a walk along the track beside our school canteen, you'll see a row of charging piles pumping electricity into the vehicles parked beside them. This is public charging, where public places offer electrical outlets to its users to charge their electronic devices with. The devices to be charged can range from cell phones to electric cars, and the outlets can take the form of simple plugs or bulky charging piles. They offer great convenience to users like us, but in fact this convenience can in turn bring great benefits to the venue itself. Through our modelling process, we found an approach to maximize the interest of a public venue by optimizing the deployment of public charging services.

The public charging business will continue to expand in the next ten years, reaching 8.5×10^{29} by 2030; however, its growth will begin to slow down by 2021. This trend of a rapidly increasing demand makes it necessary for public venues to take advantage of this service for its own benefits as well as for the satisfaction of its customers. By optimizing this service, the relationship between the venues and its users become more reciprocal and constructive.

The optimization of the charging service is complex and requires detailed mathematical modelling to realize. Fortunately, our school is home to the superb mathematical modelling team, Team #10005, who devised an Electrical Circuit Model to propose an answer to this task. Through our modelling, we were able to generate the optimal deployment of charging facilities in a given venue which maximizes its net profit. We applied our model to the waiting hall of Shanghai Hongqiao Railway Station, and returned a deployment of 16 outlets with a total cost of \$1445 and net profit of \$2749 over 10 years.

More importantly, they proposed an optimal solution for our school's electric car charging system, where over a time interval of 10 years a total fee of \$90722 will be spent. For every dollar spent, the satisfaction of the users is raised by 0.009926, which means that our school might have a better reputation, and consequently be more attractive to potential teachers. With better teachers attracted to our school by its better charging service, we students can also expect to see a boost in our academic performances. The plan is shown below:

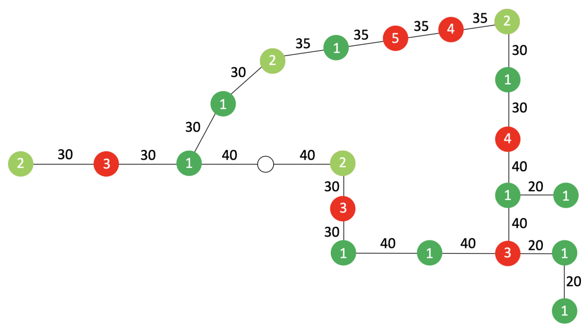


Figure 22: Installation Plan of Charging Piles

In addition, the team has also provided a few initiatives that can be implemented to benefit both the venues offering public charging and their users, including

- Provide subsidies to venues with respect to the number of outlets they plan to build;
- Provide subsidies to cooperations developing new energies and electric cars so that electric cars would be more available at a lower price;
- Regulate the gasoline car licence plates that are issued every year;
- Regulate the oil price so that it does not plummet.

Besides, since Team#10005 has submitted their study to the 22nd High school Mathematical Contest for Modelling, let's hope that they receive good results for their excellent performance and win honor for our beloved school!

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6. **Introduction to the departure layer of Shanghai Hongqiao Railway Station**

Appendix A Code for Case Study 1

```
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
const int maxn = 1000;
//var
double k = 0.1;
int t00 = 1000;
//end
int n0 = 7, m0 = 13, n = 7*13;
int dist[maxn][maxn], G[maxn][maxn];
vector<int> d[maxn][maxn];
int xb, xe, vis[maxn];
void go(int x,int num) {
if(num>dist[xb][x]+50) return;
if(x==xe) {
d[xb][xe].push_back(num);
// d[xe][xb].push_back(num);
}
for(int i=1;i<=n;i++) {
if(G[x][i]>0 && vis[i]==0) {
vis[i]=1;
go(i,num+G[x][i]);
vis[i]=0;
}
}
}
void work1() {
for(xb=1;xb<=n;xb++)
for(xe=1;xe<=n;xe++) {
vis[xb]=1;
go(xb,0);
vis[xb]=0;
}
for(int i=1;i<=n;i++)
for(int j=1;j<=n;j++) {
sort(d[i][j].begin(),d[i][j].end());
}
}
void addedge(int x1, int x2, int c) {
```

```
dist[x1][x2] = dist[x2][x1] = c;
G[x1][x2] = G[x2][x1] = c;
}
void floyed() {
for(int k = 1; k <= n; k++)
for(int i = 1; i <= n; i++)
for(int j = 1; j <= n; j++) {
// printf("%d - %d\n",dist[i][k],dist[k][j]);
dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
}
}
int num(int x,int y){
return (x-1)*m0+y;
}
void build() {
memset(dist,63,sizeof(dist));
for(int i=1;i<=n;i++)
dist[i][i] = 0;
for(int i=2;i<=5;i++) {
for(int j=2;j<=m0;j++) {
int p1 = num(i,j), p2 = num(i+1,j);
// printf("%d-%d\n",p1,p2);
addedge(p1,p2,10);
}
}
// puts("---------");
int y1[] = {0,1,5,9,13};
for(int i=1;i<n0;i++) {
for(int j=1;j<=4;j++) {
int p1 = num(i,y1[j]), p2 = num(i+1,y1[j]);
// printf("%d-%d\n",p1,p2);
addedge(p1,p2,5);
}
}
// puts("---------");
int x2[10] = {0,1,2,6,7}, x3[10] = {0,3,5};
for(int i=1;i<=4;i++)
for(int j=1;j<m0;j++) {
int p1 = num(x2[i],j), p2 = num(x2[i],j+1);
// printf("%d-%d\n",p1,p2);
addedge(p1,p2,20);
}
// puts("---------");
for(int i=1;i<=2;i++)
for(int j=1;j<m0;j++) {
int p1 = num(x3[i],j), p2 = num(x3[i],j+1);
// printf("%d-%d\n",p1,p2);
addedge(p1,p2,60);
}
}
double cal(int a,int b,int c,int dd) {
int x = num(a,b), y = num(c,dd);
double ans=0;
for(int i=0;i<d[x][y].size();i++) {
ans+=1.0/d[x][y][i];
```

```
}
ans = 1.0/ans;
}
void debug1() {
// for(int i=1;i<=n;i++,puts(""))
// for(int j=1;j<=n;j++) {
// printf("%d ",dist[i][j]);
// }
int a, b, c, dd;
while(scanf("%d%d%d%d",&a,&b,&c,&dd)==4) {
printf(": %d\n",dist[num(a,b)][num(c,dd)]);
}
}
void debug2() {
int a, b, c, dd;
while(scanf("%d%d%d%d",&a,&b,&c,&dd)==4) {
int x = num(a,b), y = num(c,dd);
for(int i=0;i<d[x][y].size();i++) {
printf("%d ",d[x][y][i]);
}
puts("");
// printf(": %f\n",cal(a,b,c,d));
}
}
void print() {
for(int i=1;i<=n0;i++)
for(int j=1;j<=m0;j++)
for(int k=1;k<=n0;k++)
for(int l=1;l<=n0;l++) {
printf("%d %d %d %d %f\n",i,j,k,l,cal(i,j,k,l));
}
}
int people[maxn], app[maxn], sizep, sizea;
void init() {
sizep = n;
for(int i=1;i<=n;i++) people[i] = i;
scanf("%d",&sizea);
int xx,yy;
for(int i=1;i<=sizea;i++) {
scanf("%d%d",&xx,&yy);
app[i] = num(xx,yy);
}
}
double find1(){ //extreme 1
vector<int> p1[maxn];
for(int i=1;i<=sizep;i++) {
int qmin=1e7, qpos;
for(int j=1;j<=sizea;j++) {
if(qmin>dist[people[i]][app[j]]) {
qmin = dist[people[i]][app[j]];
qpos = app[j];
}
}
p1[qpos].push_back(people[i]);
}
```

```
double ans=0;
for(int i=1;i<=sizea;i++) {
double res=0;
for(int j=0;j<p1[people[i]].size();j++) {
res+=1.0/(68.25+dist[people[i]][p1[people[i]][j]]);
}
res = 1.0/res + 450;
ans+=1.0/res;
}
ans = 1.0/ans;
return ans;
}
double find2() { // extreme 2
vector<int> p1[maxn];
for(int i=1;i<=sizea;i++) {
int qmin=1e7, qpos;
for(int j=1;j<=sizep;j++) {
if(qmin>dist[people[j]][app[i]]) {
qmin = dist[people[j]][app[i]];
qpos = people[j];
}
}
p1[qpos].push_back(app[i]);
}
double ans=0;
for(int i=1;i<=sizep;i++) {
double res=0;
for(int j=0;j<p1[app[i]].size();j++) {
res+=1.0/(450+dist[app[i]][p1[app[i]][j]]);
}
res = 1.0/res + 68.25;
ans+=1.0/res;
}
ans = 1.0/ans;
return ans;
}
double getans() {
double r1 = find2(), r2 = find1(), u = 3.6*sizea;
// if(isinf(r1)——isinf(r2)) return 1e16;
double i1 = u/r1, i2 = u/r2, p1 = 1000, p2 = 5.0*sizea;
double I = (i1*p1+i2*p2)/(p1+p2);
double ss[10]={1.7584,2.1571,2.5168,2.8125,3.0373,3.1985,3.3093,3.3831,3.4313,3.4624};
double f0 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[0]))) - 241.1929;
double f1 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[1]))) - 241.1929;
double f2 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[2]))) - 241.1929;
double f3 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[3]))) - 241.1929;
double f4 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[4]))) - 241.1929;
double f5 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[5]))) - 241.1929;
double f6 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[6]))) - 241.1929;
double f7 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[7]))) - 241.1929;
double f8 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[8]))) - 241.1929;
double f9 = 2*241.1929*(1-1.0/(1+exp(k*I*ss[9]))) - 241.1929;
double f = (f0-u)*12+(f1-u)*12+(f2-u)*12+(f3-u)*12+(f4-u)*12+(f5-u)*12+(f6-u)*12+(f7-u)*12+(f8-
u)*12+(f9-u)*12- 100*sizea;
// printf(”r1:%f r2:%f I:%f f1i:%f f:%f\n”,r1,r2,I,f1i,f);
return f;
}
void f() {
```

```
double ans = -1e15;
int ans_siz,ans_app[maxn];
sizep = n;
for(int i=1;i<=n;i++) people[i] = i;
for(sizea=0;sizea<=50;sizea++) {
int t=t00;
int t1 = t;
while(t–) {
for(int i=1;i<=sizea;i++) {
app[i] = rand()%n+1;
}
double ff = getans();
// cout<<ff<<endl;
if(ans<ff) {
ans = ff;
ans_siz = sizea;
for(int i=1;i<=sizea;i++) ans_app[i] = app[i];
}
//cout<<((sizea-1)/50+(1/50)*(1-t/t1))*100<<"%"<<endl;
cout<<float(2.0*sizea-float(t/t1)*2.0)<<"%"<<endl;
}
}
printf("%f %d\n",ans,ans_siz);
for(int i=1;i<=ans_siz;i++) {
printf("%d ",ans_app[i]);
}
std::cout<<k;
}


int main() {
srand(time(NULL));
clock_t startTime,endTime;
startTime = clock();
build();
floyed();
// debug1();
work1();
// init();
// printf("%f %f\n",find1(),find2());
// debug2();
// print();
f();
puts("");
puts("done");
endTime = clock();//end timing
cout << "The run time is: " <<(double)(endTime - startTime) / CLOCKS_PER_SEC <<
"s" << endl;

}
```