Permutations

STAT 330 - Iowa State University

Outline

In this lecture students will learn about permutations. The permutation number gives us the number of ways to select k objects from n when we have an ordered sample, without replacement. We will use the permutation number to find probabilities.

Ordered With Replacement

A box has n items numbered $1, \ldots, n$. Draw k items with replacement. (A number can be drawn twice).

Sample Space: $\Omega = \{(x_1, \ldots, x_k) : x_i \in \{1, \ldots, n\}\}$

What is $|\Omega|$?

Break complex action into a series of k single draws.

1. n possibilities for x_1
2. n possibilities for x_2

   $\vdots$

k. n possibilities for x_k

Multiplication principle: $|\Omega| = n \cdot n \cdot n \cdots n = n^k$

Permutation

Ordered Without Replacement

A box has n items numbered $1, \ldots, n$. Select k items without replacement. This means once a number is chosen, it can't be selected again.

Sample Space: $\Omega = \{(x_1, \ldots, x_k) : x_i \in \{1, \ldots, n\}, x_i \neq x_j\}$

What is $|\Omega|$?

Break complex action into a series of k single draws.

1. n possibilities for x_1
2. $n - 1$ possibilities for x_2
3. $n - 2$ possibilities for x_2

$\vdots$

k. $n - (k - 1)$ possibilities for x_k

Multiplication principle: $|\Omega| = n \cdot (n-1) \cdot (n-2) \cdots (n-(k-1))$

This is equivalent to $\frac{n!}{(n-k)!}$

Permutation

Definition

A *permutation* is an ordering of k distinct objects chosen from n objects. This is another name for the *ordered without replacement* scenario.

Theorem

$P(n, k)$, called the *permutation number*, is the number of permutations of k distinct objects out of n objects.

$$P(n, k) = \frac{n!}{(n-k)!}$$

Note (factorials): $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$

$0! = 1$

Ex. $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Permutation Example

Example 5:

Out of a group of 10 students, I choose 3 distinct students to give prizes to. How many ways can I select 3 students?

$n = 10 \qquad k = 3$

$$
\begin{aligned}
P(n,k) &= \frac{n!}{(n-k)!} \\
P(10,3) &= \frac{10!}{(10-3)!} \\
&= \frac{10!}{7!} \\
&= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} \\
&= 10 \cdot 9 \cdot 8 = 720
\end{aligned}
$$

Permutation Example

Example 6:

University phone exchange starts with $641 - \_\ \_\ \_\ \_$

What is the probability that a randomly selected phone number contains 7 distinct digits?

Sample space: (All possibilities for 4 chosen numbers)

$|\Omega| =$

Event: (4 chosen numbers are distinct - no repeats!)

$|A| =$

$\mathbb{P}(A) = \frac{|A|}{|\Omega|} =$

Recap

Students should be familiar with permutations and the permutation number. They should be able to apply it to probability questions under ordered without replacement samples.