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COM S 474 - Homework 4 Written Solutions

10/13/20

Question 1:

To estimate the value of $\mathbf{w}$, we will use the formula $\mathbf{w} = \sum_{x_k \in N_s} \lambda_k \mathbf{x_k} y_k$.

$\mathbf{w} = \lambda_1 \mathbf{x_1} y_1 + \lambda_3 \mathbf{x_3} y_3$ since $\lambda_2 = \lambda_4 = 0$.

Plugging in the respective values for all the parameters, we get that the $\mathbf{w} = \begin{pmatrix} 1.8 \\ 0 \\ 0.075 \end{pmatrix}$.

To classify the point $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, we use the formula $y = \mathbf{w^T} * \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + w_b$ which is equal to $2.8 > 0$. Therefore, the point belongs to the $+\mathbf{1}$ class.

Question 2:

Given the following formula for the two gutters: $\mathbf{w^T} x + w_b = \pm 1$.

Therefore, the equations are: $\begin{pmatrix} 1.8 & 0 & 0.075 \end{pmatrix} * \begin{pmatrix} x_a \\ x_b \\ x_c \end{pmatrix} + 1 = \pm 1$

or

$1.8x_a + 0.075x_c + 1 = +1$ and $1.8x_a + 0.075x_c + 1 = -1$.

Finally, the equations for the gutters are:

$1.8x_a + 0.075x_c = 0$ and $1.8x_a + 0.075x_c + 2 = 0$

Question 3:

We are going to check which of the given samples fall into the margin by using the following equation:

$$-1 < \mathbf{w}^T \mathbf{x} + w_b < 1$$

For sample 1: $x_1 = \begin{pmatrix} 0.5 \\ 0.25 \\ 0.125 \end{pmatrix}$

We evaluate $\mathbf{w}^T\mathbf{x_1}+w_b$ which is equal to $\begin{pmatrix}1.8 & 0 & 0.075\end{pmatrix} * \begin{pmatrix}0.5\\0.25\\0.125\end{pmatrix} + 1 = \mathbf{1.909}$.

Since $-1 < \mathbf{0.909} < 1$, this falls outside of the gutter area.

For sample 2: $x_2 = \begin{pmatrix}0.4\\0.15\\0.225\end{pmatrix}$

We evaluate $\mathbf{w}^T\mathbf{x_2}+w_b$ which is equal to $\begin{pmatrix}1.8 & 0 & 0.075\end{pmatrix} * \begin{pmatrix}0.4\\0.15\\0.225\end{pmatrix} + 1 = \mathbf{1.736}$.

Since $-1 < \mathbf{0.736} < 1$, this falls outside of the gutter area.

For sample 3, $x_3 = \begin{pmatrix}0.3\\0.75\\0.325\end{pmatrix}$

We evaluate $\mathbf{w}^T\mathbf{x_3}+w_b$ which is equal to $\begin{pmatrix}1.8 & 0 & 0.075\end{pmatrix} * \begin{pmatrix}0.3\\0.75\\0.325\end{pmatrix} + 1 = \mathbf{1.564}$.

Since $-1 < \mathbf{0.564} < 1$, this falls outside of the gutter area.

For sample 3: $x_4 = \begin{pmatrix}0.2\\0.65\\0.425\end{pmatrix}$

We evaluate $\mathbf{w}^T\mathbf{x_4}+w_b$ which is equal to $\begin{pmatrix}1.8 & 0 & 0.075\end{pmatrix} * \begin{pmatrix}0.2\\0.65\\0.425\end{pmatrix} + 1 = \mathbf{1.391}$.

Since $-1 < \mathbf{0.391} < 1$, this falls outside of the gutter area.

All of the provided samples fall outside of the gutter margin.

Question 4:

1. The condition $y_i(\mathbf{w}^T\mathbf{x} + w_b) \geq -1$ represents **misclassified** samples falling in the gutter area.
2. The condition $y_i(\mathbf{w}^T\mathbf{x}+w_b) \leq -1$ represents **misclassified** samples falling on the outer sides of the gutter.
3. The condition $y_i(\mathbf{w}^T\mathbf{x} + w_b) \geq 1$ represents **correctly** classified samples.
4. The condition $y_i(\mathbf{w}^T\mathbf{x} + w_b) \leq 1$ represents **correctly** classified samples falling in the gutter area.
5. The condition $y_i(\mathbf{w}^T\mathbf{x} + w_b) \geq 0$ represents **correctly** classified samples falling in the gutter area.
6. The condition $y_i(\mathbf{w}^T\mathbf{x} + w_b) \leq 0$ represents **misclassified** samples falling in the gutter area.