Assigning Probabilities

STAT 330 - Iowa State University

Outline

In this lecture students will learn two ways to begin assigning probabilities to events using the probability axioms.

Assigning Probabilities

There are 2 main approaches to assign probabilities to events at this point.

1. When we know events are disjoint (easy!).
    - Let A be a collection of of k outcomes $(\omega_1, \ldots, \omega_k)$ that are all *pairwise disjoint*.
    - Use Kolmogorov's axiom 3: $\mathbb{P}(A) = \mathbb{P}(\cup_{i=1}^{k} \omega_i) = \sum_{i=1}^{k} \mathbb{P}(\omega_i)$.

Example 9: Roll a die. Suppose event A is rolling an even number. (Assume all numbers are equally likely $\rightarrow \mathbb{P}(\omega) = \frac{1}{6}$ for all ω)

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$

$$\begin{aligned}\mathbb{P}(A) &= \mathbb{P}(\text{"2" or "4" or "6"}) \\ &= \mathbb{P}(\text{"2"} \cup \text{"4"} \cup \text{"6"}) \\ &= \mathbb{P}(2) + \mathbb{P}(4) + \mathbb{P}(6) = 3/6 = 1/2\end{aligned}$$

Assigning Probabilities cont.

2. When events may or may not be disjoint (harder).
   - Start with known probability of some of the events.
   - Use this information and Kolmogorov's axioms to deduce probabilities of other events.
   - Drawing Venn diagrams will simplify the problem

Example 10: Suppose in a small college of 1000 students, 650 students own Iphones, 400 students own MacBooks, and 300 students own both.
Define events: I = "owns Iphone", and M = "owns MacBook".

Known

$\mathbb{P}(I) = 0.65$

$\mathbb{P}(M) = 0.40$

$\mathbb{P}(I \cap M) = 0.30$

Assigning Probabilities cont.

a. What is the probability of owning an Iphone or a MacBook?

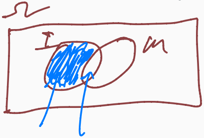
$$\mathbb{P}(I \cup M) = \mathbb{P}(I) + \mathbb{P}(M) - \mathbb{P}(I \cap M)$$
$$= .65 + .4 - .3$$
$$= .75$$

b. What is the probability of owning neither an Iphone nor a MacBook?

$$\mathbb{P}(\bar{I} \cap \bar{M}) = \mathbb{P}(\overline{I \cup M}) = 1 - \mathbb{P}(I \cup M)$$
$$= 1 - .75$$
$$= .25$$

c. What is the probability of owning only an Iphone? (ie. owning an iphone and no MacBook)

$\mathbb{P}(I \cap \overline{m})$



$I = (I \cap \overline{m}) \cup (I \cap m)$

$\mathbb{P}(I) = \mathbb{P}(I \cap \overline{m}) + \mathbb{P}(I \cap m) \Rightarrow \mathbb{P}(I \cap \overline{m}) = .65 - .30$

$= \boxed{.35}$

d. What is the probability of not owning an Iphone?

$\mathbb{P}(\overline{I}) = 1 - \mathbb{P}(I)$

$= 1 - .65$

$= .35$

$I \cap M$ $\qquad\qquad\qquad\qquad$ $I \cap \bar{M}$

	M	$\bar{M}$	
I	.3	.35	.65
$\bar{I}$	.1	.25	.35
	.4	.6	

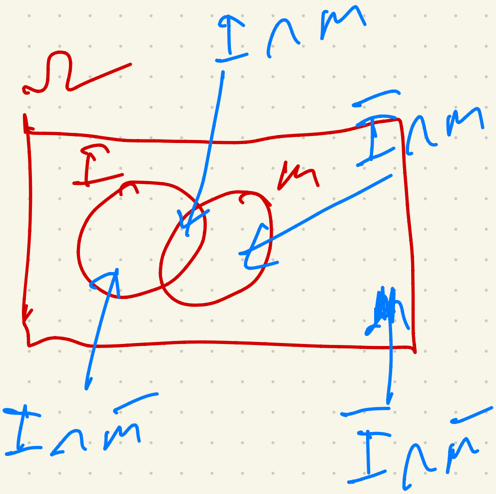
$\bar{I} \cap M$ $\qquad\qquad\qquad\qquad$ $\bar{I} \cap \bar{M}$

a.) .75

b.) .25

c.) .35

d.) .35



Recap

Students should now be comfortable using the probability axioms to begin assigning probabilities to simple and more complex events.