Discrete Distributions: Poisson Distribution

STAT 330 - Iowa State University

Outline

In this lecture, students will learn about the Poisson distribution. We will see properties of this distribution and how to calculate probabilities for a Poisson random variable.

Poisson Distribution

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Set up: The Poisson distribution is used to model the number of ("rare") events occurring in a fixed interval of time.

Examples of Poisson R.Vs

- $Y = \#$ of meteorites that strike Earth in a year
- $Z = \#$ of patients arriving to emergency room from 10-11 pm

Define the random variable

$$X = \text{"\# of events occurring during an interval"}$$

This random variable X follows a *Poisson Distribution*

$$X \sim Pois(\lambda)$$

where $\lambda > 0$ is called the rate parameter

Poisson R.V. Summary

- Probability Mass Function (pmf)

$$p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x = 0, 1, 2, 3, \ldots$$

  where $\lambda > 0$ is the rate parameter.

- Cumulative Distribution Function (cdf)

$$F_X(t) = \mathbb{P}(X \leq t) = \sum_{x=0}^{\lfloor t \rfloor} p_X(x)$$

- Expected Value: $\mathbb{E}(X) = \lambda$

- Variance: $Var(X) = \lambda$

Poisson Distribution Examples

Poisson Distribution Examples

Example 2: Suppose the number of customers entering West Street Deli can be modeled using a Poisson distribution. Customers enter the deli at an average rate of 10 customers every 10 minutes during the lunch rush.

Between 12pm and 12:10pm today, what is the probability that ...

1. exactly 3 customers enter?
2. at most 3 customers enter?
3. at least 4 customers enter?
4. Between 8 and 10 customers enter? (inclusive)
5. What is the expected value of the random variable?
6. What is the variance of the random variable?

Poisson Distribution Examples

Start by defining the R.V and stating it's distribution.

1. What is the probability that exactly 3 customers enter?

Poisson Distribution Examples

2. What is the probability that at most 3 customers enter

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3. What is the probability that at least 4 customers enter?

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4. What is the probability that between 8 and 10 customers enter (inclusive)

5. What is the expected value of the random variable?

6. What is the variance of the random variable?

Recap

Students should now be familiar with the Poisson distribution. They should know the scenario where the Poisson distribution is used, and how to calculate probabilities for a Poisson random variable.