Reliability

STAT 330 - Iowa State University

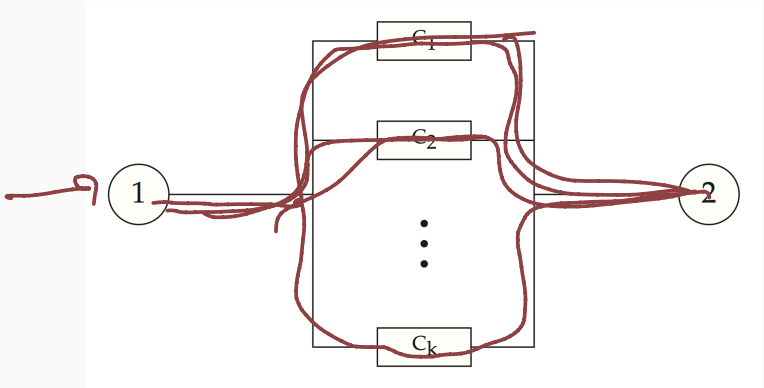
Outline

In this lecture students will be introduce to an application that uses topics from previous lectures. We will calculate system reliability for systems connected in parallel or in series.

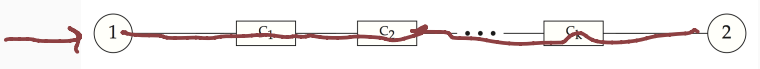
System Reliability

Application: System Reliability

Parallel: A parallel system consists of k components $(c_1, \ldots, c_k)$ arranged such that the system works if and only if at least one of the k components functions properly.



Series: A series system consists of k components $(c_1, \ldots, c_k)$ arranged such that the system works if and only if ALL components function properly.



Reliability: Reliability of a system is the probability that the system works.

Reliability of Parallel System

Example 4:

Let $c_1, \ldots, c_k$ denote the k components in a *parallel* system. Assume the k components operate independently, and $\mathbb{P}(c_j \text{ works }) = p_j$. What is the reliability of the system?

$$\begin{aligned}
\mathbb{P}(\text{system works}) &= \mathbb{P}(\text{at least one component works}) \\
&= 1 - \mathbb{P}(\text{all components fail}) \\
&= 1 - \mathbb{P}(c_1 \text{ fails} \cap c_2 \text{ fails} \cap \cdots \cap c_k \text{ fails}) \\
&= 1 - \prod_{j=1}^{k} \mathbb{P}(c_j \text{ fails}) \\
&= 1 - \prod_{j=1}^{k} (1 - p_j)
\end{aligned}$$

Reliability

Reliability of Series System

Example 5:

Let $c_1, \ldots, c_k$ denote the k components in a *series* system. Assume the k components operate independently, and $\mathbb{P}(c_j \text{ works }) = p_j$. What is the reliability of the system?

$$\begin{aligned}
\mathbb{P}(\text{system works}) &= \mathbb{P}(\text{all components work}) \\
&= \mathbb{P}(c_1 \text{ works} \cap c_2 \text{ works} \cap \cdots \cap c_k \text{ works}) \\
&= \prod_{j=1}^{k} \mathbb{P}(c_j \text{ works}) \\
&= \prod_{j=1}^{k} p_j
\end{aligned}$$

Reliability Example

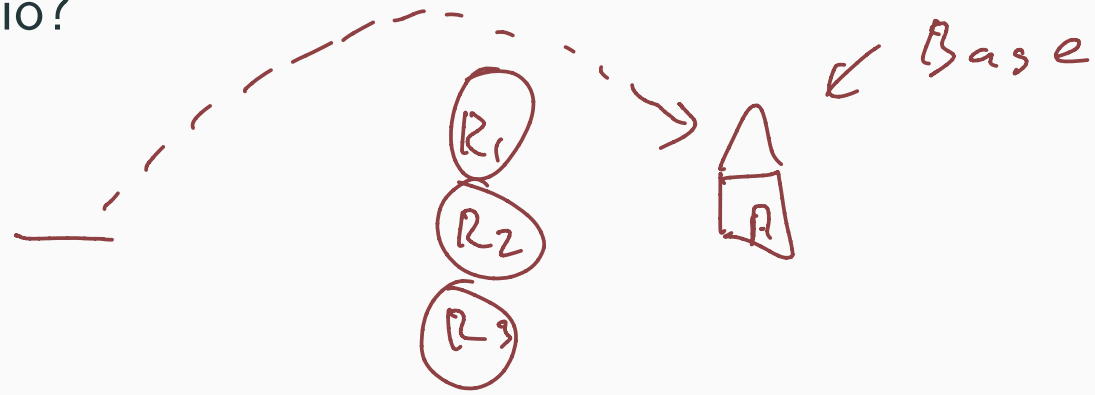
Example 6: Suppose a base is guarded by 3 radars (R_1, R_2, R_3), and the radars are independent of each other. The detection probability are ...

$\mathbb{P}(R_1 \text{ detects}) = 0.95$

$\mathbb{P}(R_2 \text{ detects}) = 0.98$

$\mathbb{P}(R_3 \text{ detects}) = 0.99$

Does a system in *parallel* or *series* have higher reliability for this scenario?



Reliability Example

Parallel

$$P(\text{Alarm}) = P(\text{At least 1 R detects})$$

$$= 1 - P(\text{None detect})$$

$$= 1 - [(.05)(.02)(.01)]$$

$$= 1 - .00001$$

$$= \boxed{.99999}$$

Reliability Example

Series

$$P(\text{Alarm}) = P(\text{All R's detect})$$

$$= (.95)(.98)(.99)$$

$$= \boxed{.922}$$

Parallel system much more reliable

Recap

Students should now be able to apply topics from probability to find the reliability of systems connected in parallel or in series.