

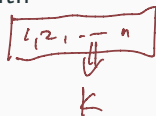
Permutations

STAT 330 - Iowa State University

In this lecture students will learn about permutations. The permutation number gives us the number of ways to select k objects from n when we have an ordered sample, without replacement. We will use the permutation number to find probabilities.

Ordered With Replacement

A box has n items numbered $1, \dots, n$. Draw k items with replacement. (A number can be drawn twice).



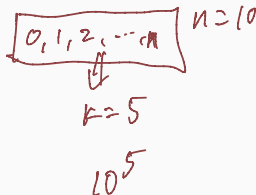
Sample Space: $\Omega = \{(x_1, \dots, x_k) : x_i \in \{1, \dots, n\}\}$

What is $|\Omega|$?

Break complex action into a series of k single draws.

1. n possibilities for x_1
2. n possibilities for x_2
- \vdots
- k . n possibilities for x_k

n n n . . . n



Multiplication principle: $|\Omega| = n \cdot n \cdot n \cdots n = n^k$

Permutation

Ordered Without Replacement

A box has n items numbered $1, \dots, n$. Select k items **without** replacement. This means once a number is chosen, it can't be selected again.

Sample Space: $\Omega = \{(\underline{x_1, \dots, x_k}) : x_i \in \{1, \dots, n\}, \underline{x_i \neq x_j}\}$

~~$(1, 2, \dots, 1)$~~

What is $|\Omega|$?

Break complex action into a series of k single draws.

1. n possibilities for x_1
2. $n - 1$ possibilities for x_2
3. $n - 2$ possibilities for x_2
- \vdots
- k. $n - (k - 1)$ possibilities for x_k

n $n-1$ $n-2$ \dots $n-(k-1)$

Multiplication principle: $|\Omega| = n \cdot (n - 1) \cdot (n - 2) \cdots (n - (k - 1))$

This is equivalent to $\frac{n!}{(n-k)!}$

Permutation

1, 2, 3 | 123, 132, ...

Definition

A **permutation** is an ordering of k distinct objects chosen from n objects. This is another name for the **ordered without replacement** scenario.

Theorem

"in the box"
we choose

$P(n, k)$, called the **permutation number**, is the number of permutations of k distinct objects out of n objects.

$$P(n, k) = \frac{n!}{(n - k)!}$$

Note (factorials): $n! = n \cdot (n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1$

$$0! = 1$$

$$\text{Ex. } 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

EX 4 books
How many ways
to order those
books?



Permutation Example

Example 5:

Out of a group of 10 students, I choose 3 distinct students to give prizes to. How many ways can I select 3 students?

$$\underline{n = 10}$$

$$\underline{k = 3}$$

$$P(n, k) = \frac{n!}{(n - k)!}$$

$$\underline{P(10, 3)} = \frac{10!}{(10 - 3)!}$$

$$= \frac{10!}{7!}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}}$$

$$= 10 \cdot 9 \cdot 8 = 720$$

Permutation Example

Example 6:

University phone exchange starts with 641 – _ _ _ _

What is the probability that a randomly selected phone number contains 7 distinct digits?

Sample space: (All possibilities for 4 chosen numbers)

$$|\Omega| = \underbrace{10} \underbrace{10} \underbrace{10} \underbrace{10} \Rightarrow 10^4$$

Event: (4 chosen numbers are distinct - no repeats!)

$$|A| = \boxed{n=7} \quad \downarrow \quad k=4 \quad P(7, 4) = \frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}} = 840$$

$$\mathbb{P}(A) = \frac{|A|}{|\Omega|} = \frac{840}{10^4} = \boxed{.084}$$

Recap

Students should be familiar with permutations and the permutation number. They should be able to apply it to probability questions under ordered without replacement samples.