Discrete Distributions: Geometric Distribution

STAT 330 - Iowa State University

Outline

In this lecture, students will learn about the Geometric distribution. We will see how this distribution is derived and how to calculate probabilities for a Geometric random variable.

Geometric Distribution

Geometric Distribution

Set up:

- Experiment where each trial is Bernoulli (only 2 outcomes) with P(success) $= p$
- Repeat the trials until you obtain the *first success*.

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$$X = \text{“ \# of trials until first success”}$$

This random variable X follows a *Geometric Distribution*

$$X \sim Geo(p)$$

where p is the probability of success for each trial.

Geometric Random Variable

- Probability Mass Function (pmf)
  1. $Im(X) = \{1, 2, 3, 4, \ldots,\} = \mathbb{N}$
  2. $\mathbb{P}(X = x) = (1-p)^{x-1}p$

$$p_X(x) = (1-p)^{x-1}p \quad \text{for } x = 1, 2, 3, \ldots$$

- Cumulative distribution function (cdf)

$$F_X(t) = \mathbb{P}(X \leq t) = 1 - (1-p)^{\lfloor t \rfloor}$$

Why?

- $\mathbb{P}(X > t) = (1-p)^{\lfloor t \rfloor}$ because this is the probability that the first $\lfloor t \rfloor$ trials are failures
- $\mathbb{P}(X \leq t) = 1 - \mathbb{P}(X > t) = 1 - (1-p)^{\lfloor t \rfloor}$

Geometric Random Variable Cont.

- Expected Value

$$\mathbb{E}(X) = \sum_{x=1}^{\infty} x(1-p)^{x-1}p = \cdots = \frac{p}{(1-[1-p])^2} = \frac{1}{p}$$

- Variance

$$Var(X) = \sum_{x=1}^{\infty} \left(x - \frac{1}{p}\right)^2 (1-p)^{x-1}p = \cdots = \frac{1-p}{p^2}$$

- "Memoryless" Property of Geometric Dist.

$$\mathbb{P}(X \geq i + j | X \geq i) = \mathbb{P}(X \geq j) \text{ for } i, j = 0, 1, 2, \ldots$$

Geometric Distribution Examples

Geometric Distribution Examples

Example 1: Suppose we have an unfair 2−sided coin with $\mathbb{P}(Head) = 0.3$. We flip a coin until we get our first head, and stop flipping once we obtain the head.

What is the probability that ...

1. the first head occurs on the third flip?
2. we get the first head before the third flip?
3. we have to flip the coin at least 3 times, but at most 7 times to get the first head?
4. What is the expected number of flips until we obtain the first head?
5. What is the variance?

Geometric Distribution Examples

Start by defining the R.V and stating it's distribution.

1. What is the probability that the first head occurs on the third flip?

Geometric Distribution Examples

2. What is the probability that we get the first head before the third flip?

Geometric Distribution Examples

3. What is the probability that we have to flip the coin at least 3 times, but at most 7 times?

4. What is the expected value?

5. What is the variance?

Recap

Students should now be familiar with the Geometric distribution. They should know the scenario where the Geometric distribution is used, and how to calculate probabilities for a Geometric random variable.