Uniform Distribution

STAT 330 - Iowa State University

Outline

In this lecture, students will introduced to some known continuous distributions that are commonly used in practice. We begin here with the Uniform distribution.

Continuous Distributions

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Common distributions for continuous random variables

- Uniform distribution

$$X \sim \mathit{Unif}(a, b)$$

- Exponential distribution

$$X \sim \mathit{Exp}(\lambda)$$

- Gamma distribution

$$X \sim \mathit{Gamma}(\alpha, \lambda)$$

- Normal distribution

$$X \sim \mathit{Normal}(\mu, \sigma^2)$$

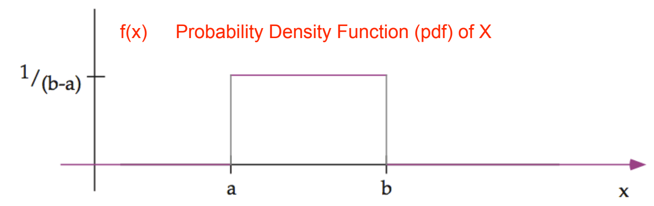
Uniform Distribution

Uniform Distribution

If a random variable follows a *uniform distribution*, then the R.V has constant probability between values a and b.

$$X \sim Unif(a, b)$$

- Probability Density Function (pdf)
  - Im(X) = (a,b)
  - $f_X(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$



Uniform Distribution Cont.

- Cumulative Distribution Function (cdf)

$$F_X(t) = \begin{cases} 0 & \text{for } t < a \\ \frac{t-a}{b-a} & \text{for } a \leq t \leq b \\ 1 & \text{for } t > b \end{cases}$$

- Expected Value: $\mathbb{E}(X) = \frac{a+b}{2}$

$$\mathbb{E}(X) = \int_a^b \frac{x}{b-a} dx = \frac{1}{b-a} \left(\frac{x^2}{2} \right) \Big|_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{a+b}{2}$$

- Variance: $Var(X) = \frac{(b-a)^2}{12}$

$$Var(X) = \int_a^b \left(x - \frac{a+b}{2} \right)^2 \frac{1}{b-a} dx = \ldots = \frac{(b-a)^2}{12}$$

Can also get variance by $Var(X) = \mathbb{E}(X^2) - [\mathbb{E}(X)]^2$

Example

Uniform Distribution Example

Example 1: A basic (pseudo) random number generator creates realizations of $\text{Unif}(0, 1)$ random variables.
X = number obtained from the random number generator.

1. What is $\text{Im}(X)$?
2. Give the pdf and cdf of X

Uniform Distribution Example

3. What is the probability that it generates a number greater than 0.85?

$$\mathbb{P}(X > .85) \;=\; \int_{.85}^{1} 1 dx = (x)\Big|_{.85}^{1} = .15$$

Uniform Distribution Example

3. What is the probability that it generates a number between 0.1 and 0.85?

$$\mathbb{P}(.1 < X < .85) \;\; = \;\; F_X(.85) - F_X(.1) = .85 - .1 = .75$$

4. What is the expected value?
   $\frac{a+b}{2} = \frac{0+1}{2} = \frac{1}{2}$
5. What is the variance?
   $\frac{(b-a)^2}{12} = \frac{(1-0)^2}{12} = \frac{1}{12}$

Uniform Distribution Example

Example 2: Suppose X has a uniform distribution between 5 and 10. Calculate

1. $P(X < 7) =$
2. $P(6 < X < 7) =$

Recap

Students have been introduced to some commonly used known continuous distributions. Students should be familiar with the Uniform distribution, and be able to use it to answer questions.