Probability Intro: Sample Space and Events

STAT 330 - Iowa State University

Outline

In this lecture students will:

1. Be introduced to Probability and Statistics
2. Begin Probability Basics
    - 2.1 Random Experiments
    - 2.2 Outcomes and Sample Space
    - 2.3 Events

Introduction

Probability and Statistics

Definitions

- *Probability* is a mathematical theory for modeling processes where outcomes occur randomly
- *Statistics* is learning about the real world from data under the assumption that the data was generated by a random process

Goals

1. Probability: model and assign probabilities to outcomes
2. Statistics: use probability models to draw conclusions/learn

Random Experiment

Random Experiment

Definition

A *random experiment* is an experiment (or process) for which outcome cannot be predicted with certainty

Example 1: Various random experiments

- A message can take two network routers to reach a recipient computer. We record the status of router 1, the status of router 2, and the status of the recipient computer, where the status is either up (U) or down (D).
- Record the time for a web page to respond.
- Roll a die and record the face up.
- Flip a coin until you get a head. Record all the faces that your obtain.

Components of random experiment

Outcome

Definition

The *outcome* (ω) is the result of an experiment

Example 2: Outcomes

- **Network Routers:** $\omega =$ (router 1 down, router 2 down, recipient computer up) $=$ DDU
- **Access web page:** $\omega =$
- **Roll a die:** $\omega =$
- **Toss coin until head:** $\omega =$

Sample space

Definition

The *sample space* (Ω) is the set of ALL possible outcomes

Example 3: Sample spaces

- **Network Routers:** $\Omega = \{$DDD, DDU, DUD, UDD, UUD, UDU, DUU, UUU$\}$
- **Access web page:** $\Omega =$
- **Roll a die:** $\Omega =$
- **Toss coin until head:** $\Omega =$

$|\Omega| \equiv \#$ of outcomes in sample space.

For the network routers example, $|\Omega| = 8$

Types of sample space

Sample space can be ...

- finite $\rightarrow$ discrete
- countable infinite $\rightarrow$ discrete
- uncountable infinite $\rightarrow$ not discrete

Example 4: Discrete/not discrete sample spaces

- **Network Routers:**
  $\Omega = \{\text{DDD, DDU, DUD, UDD, UUD, UDU, DUU, UUU}\}$
  $\rightarrow$ discrete
- **Access web page:** $\Omega = (0, \infty) \rightarrow$ not discrete
- **Roll a die:** $\Omega = \{1, 2, 3, 4, 5, 6\} \rightarrow$ discrete
- **Toss coin until head:** $\Omega = \{H, TH, TTH, TTTH, \cdots\}$
  $\rightarrow$ discrete

Event

Definition

An *event* (A, B, C, etc) is a collections of outcomes from the sample space that we are interested in. $A \subseteq \Omega$

Example 5: Events

- **Network Routers:** A message is transmitted successfully if at least one router is up and recipient computer is up.
  $A =$ successful transmission $= \{\text{DUU, UDU, UUU}\}$
- **Access web page:** B = More than 10 seconds $= (10, \infty)$
- **Roll a die:** C =
- **Toss coin until head:** D =

Probability

Probability

- Let A be an event (set of outcomes from the sample space)
- Then, probability of event A is written as $\mathbb{P}(A)$

Example 6:

- Consider event C as successful transmission in the Network Router example
- Suppose the chance that a message is successfully transmitted is 90%
- $\mathbb{P}(C) =$

To calculate probability of events, start with understanding set theory

Recap

Students should now be familiar with basic concepts and terms that serve as the starting point for beginning probability.

1. Random Experiments
2. Outcomes and Sample Space
3. Events