Probability Axioms

STAT 330 - Iowa State University

Outline

In this lecture students will:

1. Learn the three probability axioms that all probability models must follow.
2. See useful consequences of the axioms for finding probabilities

Kolmogorov's Axioms

Kolmogorov's Axioms

- Recall: $\mathbb{P}(A)$ is the probability that event A occurs
- Want to assign probabilities to events as a measure of their likelihood of occurring
- A *probability model* is an assignment of numbers $\mathbb{P}(A)$ to events $A \subseteq \Omega$ such that *Kolmogorov's axioms* are satisfied.

Kolmogorov's Axioms

1. $0 \leq \mathbb{P}(A) \leq 1$ for all A
2. $\mathbb{P}(\Omega) = 1$
3. If $A_1, A_2, A_3, \cdots$ are pairwise disjoint, then $\mathbb{P}(A_1 \cup A_2 \cup \cdots) = \mathbb{P}(A_1) + \mathbb{P}(A_2) + \cdots = \sum_i \mathbb{P}(A_i)$

Kolmogorov's Axioms Cont.

Kolmogorov's axioms...

- Give the logical framework that probability assignment must follow
- But don't tell us what probabilities to assign to events

Example 8: Draw a single card from a standard deck of playing cards: $\Omega = \{red, black\}$

Two different probability models are:

Model 1	Model 2
$\mathbb{P}(\Omega) = 1$	$\mathbb{P}(\Omega) = 1$
$\mathbb{P}(red) = 0.5$	$\mathbb{P}(red) = 0.3$
$\mathbb{P}(black) = 0.5$	$\mathbb{P}(black) = 0.7$

Both are *valid* probability models. However, real world experience tells us model 1 is more accurate for the scenario.

Consequences of Kolmogorov's Axioms

Let A, $B \subseteq \Omega$.

A. Probability of the Complementary Event:

$$\mathbb{P}(\overline{A}) = 1 - \mathbb{P}(A)$$

Corollary: $\mathbb{P}(\emptyset) = 0$

B. Addition Rule of Probability

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

C. If $A \subseteq B$, then $\mathbb{P}(A) \leq \mathbb{P}(B)$.

Corollary: For any A, $\mathbb{P}(A) \leq 1$.

Recap

Students should now be familiar with the three axioms that all probability models must follow. The should also know some useful consequences that will be used going forward.