Discrete Distributions: Bernoulli Distribution

STAT 330 - Iowa State University

Outline

In this lecture, we will overview the common named discrete distributions used in practice. We first look at the Bernoulli distribution.

Discrete Distributions

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Common distributions for discrete random variables

- Bernoulli distribution

$$X \sim Bern(p)$$

- Binomial distribution

$$X \sim Bin(n, p)$$

- Geometric distribution

$$X \sim Geo(p)$$

- Poisson distribution

$$X \sim Pois(\lambda)$$

We will also discuss *joint distributions* for 2 or more discrete random variables

Bernoulli Distribution

Bernoulli Distribution

Bernoulli Experiment: Random experiment with only 2 outcomes:

- Success (S)
- Failure (F)

where $\mathbb{P}(\text{Success}) = \mathbb{P}(\text{S}) = p$ for $p \in [0, 1]$

Example 1: (Bernoulli experiments):

1. Flip a coin: S = heads, F = tails
2. Watch stock prices: S = increase, F = decrease
3. Cancer screening: S = cancer, F = no cancer

Working with Bernoulli Random Variable

Suppose we have a situation that matches a Bernoulli experiment (only 2 outcomes: "success" and "failure").

We obtain the outcome "success" with probability p

When random variable X follows a *Bernoulli Distribution*, we write

$$X \sim Bern(p)$$

- Define a random variable X

$$X = \begin{cases} 1 & \text{Success (S)} \\ 0 & \text{Failure (F)} \end{cases}$$

Bernoulli Random Variable Cont.

- Probability Mass Function (pmf)
  1. $Im(X) = \{0, 1\}$
  2. $\mathbb{P}(X = 1) = \mathbb{P}(S) = p$
     $\mathbb{P}(X = 0) = \mathbb{P}(F) = 1 - p$

The pmf can be written in tabular form:

x	0	1
$p_X(x)$	$1-p$	p

The pmf can be written as a function:

$$p_X(x) = \begin{cases} p^x(1-p)^{1-x} & x \in \{0, 1\} \\ 0 & \text{otherwise} \end{cases}$$

Typically, we use the above functional form to describe the *probability mass function (pmf)* of Bernoulli random variable.

Bernoulli Random Variable Cont.

- Cumulative distribution function (cdf)

$$F_X(t) = \mathbb{P}(X \leq t) = \begin{cases} 0 & t < 0 \\ 1 - p & 0 \leq t < 1 \\ 1 & t \geq 1 \end{cases}$$

- Expected Value: $\mathbb{E}(X) = p$

$$\mathbb{E}(X) = \sum_{x \in \{0,1\}} x\mathbb{P}(X = x) = 0(1-p) + 1(p) = p$$

- Variance: $Var(X) = p(1-p)$

Recap

Students should now be aware that there are many named distributions that are used for various scenarios. Students should now be familiar with the Bernoulli distribution.