Conditional Probability

STAT 330 - Iowa State University

Outline

In this lecture students will learn about conditional probability. We introduce a contingency table when we have two events. Various probabilities, including conditional probability, can be calculated using the contingency table.

Contingency Table

Contingency Table

Definition

A *contingency table* gives the distribution of 2 variables.

Events

Example 1: Suppose in a small college of 1000 students, 650 students own Iphones, 400 students own MacBooks, and 300 students own both.

Define events: $I =$ "owns Iphone", and $M =$ "owns MacBook".

Phone \ Computer	M	$\overline{M}$	Total
I	300	?	650
$\overline{I}$	?	?	?
Total	400	?	1000

Contingency Table

Computer / Phone	M	$\overline{M}$	Total
I	300	350	650
$\overline{I}$	100	250	350
Total	400	600	1000

Marginal Probability

Marginal Probability

Definition

The *marginal probability* is the probability of a variable. It can be obtained from the *margins* of contingency table.

Phone \ Computer	M	$\overline{M}$	Total
I	300	350	650
$\overline{I}$	100	250	350
Total	400	600	1000

What is the probability of owning a Mac? (ie marginal probability of owning a Mac)

$\mathbb{P}(M) = \frac{400}{1000} = 0.40$

Conditional Probability

Conditional Probability

Does knowing someone owns an Iphone change the probability they own a Mac?

Informally, conditional probability is updating the probability of an event given information about another event.

If we *know* that someone owns an Iphone, then we can narrow our sample space to just the "owns Iphone" case (highlighted blue row) and ignore the rest!

Phone \ Computer	M	$\overline{M}$	Total
I	300	350	650
$\overline{I}$	~~100~~	~~250~~	~~350~~
Total	400	600	~~1000~~

Conditional Probability Cont.

What is the probability of owning a Mac *given* they own an Iphone?

Phone \ Computer	M	$\overline{M}$	Total
I	300	350	650
$\overline{I}$	100	250	350
Total	400	600	1000

$$\boxed{\mathbb{P}(M|I)} = \frac{300}{650} = 0.46$$

Conditional Probability Cont.

Definition

The *conditional probability* of event A given event B is

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

"given"

provided $\mathbb{P}(B) \neq 0$.

It can be obtained from the *rows/columns* of contingency table.

Back to Example 1 . . .

What is the probability of owning a Mac *given* they own an Iphone?

$$\mathbb{P}(M|I) = \frac{\mathbb{P}(I \cap M)}{\mathbb{P}(I)} = \frac{0.3}{0.65} = 0.46$$

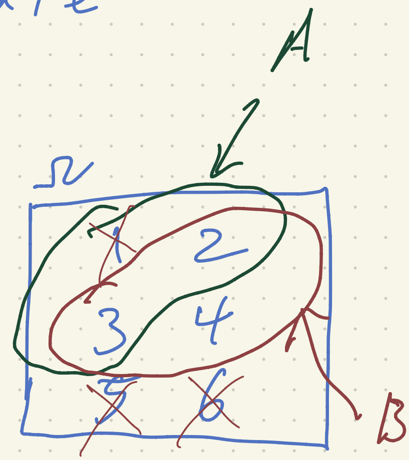
EX: Roll a fair die

$\Omega = \{1,2,3,4,5,6\}$

$A = \{1,2,3\}$

$B = \{2,3,4\}$

$A \cap B = \{2,3\}$



$\mathbb{P}(A) = 3/6 = 1/2$

$\mathbb{P}(B) = 3/6 = 1/2$

$\mathbb{P}(A \cap B) = 2/6 = 1/3$

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{1/3}{1/2} = \boxed{2/3}$$

Consequences of Conditional Probability

The definition of conditional probability gives useful results:

Multiplication Rule (Intersections)

1. $$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \to \mathbb{P}(A \cap B) = \underline{\mathbb{P}(B)\mathbb{P}(A|B)}$$

2. $$\mathbb{P}(B|A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)} \to \mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B|A)$$

This gives us two additional ways to calculate probability of intersections. Putting it together . . .

$$\underline{\mathbb{P}(A \cap B)} = \mathbb{P}(A|B)\mathbb{P}(B) = \mathbb{P}(B|A)\mathbb{P}(A)$$

Draw 2 cards from a deck. What is the probability you get an Ace and a King?

1.) <u>Counting</u>

A	K	other
4	4	44
1	1	

$$\mathbb{P}(\text{Ace and King}) = \frac{\binom{4}{1}\binom{4}{1}}{\binom{52}{2}} = .012$$

2.) mult. Rule: How do I get an ace and king? $(A_1 \cap K_2) \cup (K_1 \cap A_2)$

So, $\mathbb{P}(\text{Ace and King}) =$

$$\mathbb{P}(A_1 \cap K_2) + \mathbb{P}(K_1 \cap A_2)$$

$$= \mathbb{P}(A_1)\mathbb{P}(K_2 \mid A_1) + \mathbb{P}(K_1)\mathbb{P}(A_2 \mid K_1)$$

$$= \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) + \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) = .012$$

Probability Calculations

Probability Calculations

A contingency table can also be written with probabilities instead of counts. This is called a *probability table*.

Inner cells give "joint probabilities" $\rightarrow$ probability of intersections

- $\mathbb{P}(A \cap B), \mathbb{P}(\overline{A} \cap B)$, etc

Margins give "marginal probabilities" $\rightarrow$ probability of variables

- $\mathbb{P}(A), \mathbb{P}(B), \mathbb{P}(\overline{A})$, etc

Phone \ Computer	M	$\overline{M}$	Total
I	0.30	0.35	0.65
$\bar{I}$	0.10	0.25	0.35
Total	0.40	0.60	1

Probability Calculations Cont.

Phone \ Computer	M	$\overline{M}$	Total
I	0.30	0.35	0.65
$\bar{I}$	0.10	0.25	0.35
Total	0.40	0.60	1

$\mathbb{P}(\bar{I}) = .35$

$\mathbb{P}(M) = .40$

$\mathbb{P}(\bar{I} \cap M) = .10$

$\mathbb{P}(M|\bar{I}) = \frac{.10}{.35} = .286$

$\mathbb{P}(\bar{I}|M) = \frac{.10}{.40} = .25$

Recap

Students should understand conditional probability and how you are updating probabilities given another event has occurred. Students should now be able to calculate probabilities involving two events using a contingency table.