Independence

STAT 330 - Iowa State University

Outline

In this lecture students will learn about independence of two events and the intuition behind it. We will

1. Use the definition of independence to deduce if two events are independent.
2. Simplify intersection probabilities is we have two independent events.

Independence

Independence of Events

$\mathbb{P}(m) = .4$, $\mathbb{P}(m|I) = .46$

In Example 1, knowing an event occurred changed the probability of another event occurring.

However, sometimes knowing an event occurs *doesn't change* the probability of the other event.
In this case, we say the events are *independent*.

Definition

Events A and B are *independent* iff...

dependent

1. $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$

   or equivalently

2. $\mathbb{P}(A|B) = \mathbb{P}(A)$ if $\mathbb{P}(B) \neq 0$

Example 2: Check if events are independent

Is owning an Iphone and owning MacBook independent?
Recall that $\mathbb{P}(I) = 0.65$, $\mathbb{P}(M) = 0.4$, $\mathbb{P}(I \cap M) = 0.35$

$$\mathbb{P}(I \cap m) = .35$$

$$\mathbb{P}(I)\mathbb{P}(m) = (.65)(.4) \neq .35 = \mathbb{P}(I \cap m)$$

$$\Rightarrow \boxed{\text{dependent}}$$

$$\mathbb{P}(m | I) \stackrel{?}{=} \mathbb{P}(m)$$

$$.46 \neq .4$$

$$\Rightarrow \boxed{\text{dependent}}$$

Independence of Events Cont.

Example 3: Using independence to simplify calculations

If A, B independent $\rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(B)\mathbb{P}(A|B) = \mathbb{P}(B)\mathbb{P}(A)$

Roll a die 4 times. Assuming that rolls are independent, what is the probability of obtaining at least one '6'?

$\mathbb{P}(\text{at least 1 '6'}) = 1 - \mathbb{P}(\text{No '6's})$

$= 1 - \mathbb{P}(\text{no '6' on roll 1} \cap \text{no '6' on roll 2} \cap \cdots \cap \text{no '6' on roll 4})$

$= 1 - [\mathbb{P}(\text{No 6 on Roll 1}) \cdot \ldots \cdot \mathbb{P}(\text{No 6 on roll 4})]$

$= 1 - [(5/6)(5/6)(5/6)(5/6)]$

$= 1 - (5/6)^4$

$= .517$

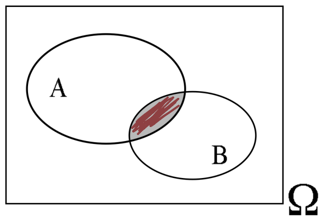
Independent vs. Disjoint

Independent ≠ Disjoint!!!

Completely different concepts!

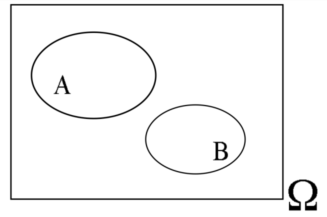
Always dependent

Independent:



$$\boxed{\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)}$$

Disjoint:



$$\mathbb{P}(A \cap B) = \mathbb{P}(\emptyset) = 0$$

$$\mathbb{P}(A) = q \qquad q \in [0,1]$$

$$\mathbb{P}(A|B) = 0 \neq q$$

Recap

Students should now know the definition of independence for two events. They should be able to determine if events are independent, and calculate probabilities with independent events.