Stamatios Morellas
COM S 342 - Homework 2
Due 2/13/20 @ 11:59pm

1. Compute the following (apply reductions till the expression cannot be reduced any further)

(a) $((\lambda x.\ (x\ \ x)\ \lambda y.\ y)\ \ \lambda y.\ y)$

$((\lambda y.\ y\ \ x)\ \ \lambda y.\ y)$
$((\lambda x.\ x)\ \lambda y.\ y)$

$\lambda y.\ y$ – **Answer**

(b) $((\lambda x.\ \lambda y.\ (x\ (y\ \ y))\ \lambda a.\ a)\ \ b)$

$((\lambda y.(\lambda a.a\ (y\ \ y)))\ \ b)$
$\lambda a.a\ (b\ \ b)$

$(b\ \ b)$ – **Answer**

(c) $((\lambda x.\ (x\ \ x)\ \ \lambda y.\ (y\ \ x))\ \ z)$

$(((\lambda y.\ (y\ \ n)\ \ m))\ \ z)$

$((m\ \ n)\ \ z)$ – **Answer**

(d) $(\lambda g.\ (g\ \lambda x.\ \lambda y.\ x)\ \ ((\lambda a\ .\lambda b.\ \lambda h.\ ((h\ \ a)\ \ b)\ \ z1)\ \ z2))$

$((\lambda a\ .\lambda b.\ \lambda h.\ ((h\ \ a)\ \ b)\ \ z_1)\ \ z_2)\quad \lambda x.\ \lambda y.\ x)$
$((\lambda b.\ \lambda h.\ ((h\ \ z_1)\ \ b)\ \ \ z_2)\quad \lambda x.\ \lambda y.\ x)$
$((\lambda h.\ ((h\ \ z_1)\ \ z_2)\quad \lambda x.\ \lambda y.\ x)$
$((\lambda x.\ \lambda y.\ x\ \ z_1)\ \ z_2)$
$((\lambda y.\ z_1\ y)\ \ z_2)$

$(z_1\ \ z_2)$ – **Answer**

(e) $((\lambda t.\, \lambda y.\, (t \;\; y) \;\; \lambda n.\, \lambda f.\, \lambda x.\, (f \;\; ((n \;\; f) \;\; x))) \;\; \lambda g.\, \lambda z.(g \;\; (g \;\; z)))$

$((\lambda t.\, \lambda y.\, (t \;\; y) \;\; \lambda n.\, \lambda f.\, \lambda x.\, (f \;\; ((n \;\; f) \;\; x))) \;\; \lambda g.\, \lambda z.(g \;\; (g \;\; z)))$
$(\lambda y.\, (\lambda n.\, \lambda f.\, \lambda x.\, (f \;\; ((n \;\; f) \;\; x)) \;\; y) \;\; \lambda g.\, \lambda z.(g \;\; (g \;\; z)))$
$(\lambda n.\, \lambda f.\, \lambda x.\, (f \;\; ((n \;\; f) \;\; x)) \;\; \lambda g.\, \lambda z.(g \;\; (g \;\; z)))$
$(\lambda f.\, \lambda x.\, (f \;\; (\lambda g.\, \lambda z.(g \;\; (g \;\; z)) \;\; f) \;\; x)))$
$(\lambda f.\, \lambda x.\, (f \;\; (\lambda z.(f \;\; (f \;\; z)) \;\; x))$

$(\lambda f.\, \lambda x.\, (f \;\; (f \;\; (f \;\; x))))$ – **Answer**

2. Given the following lambda expressions and corresponding interpretations:

- The interpretation of $\lambda f.\, \lambda x.x$ is natural number 0 (zero). The interpretation of $\lambda f.\, \lambda x.\, (f \;\; (f \;\; (\ldots \;\; x)))$, with n applications of f on x, is the natural number $n > 0$.
- The interpretation of $\lambda n.\, \lambda f.\, \lambda x.\, (f \;\; ((n \;\; f) \;\; x))$ is a successor function $succ$ for natural numbers, where n is the formal parameter corresponding to the number whose successor is computed.
- The interpretation of $\lambda m.\, \lambda n.\, ((m \;\; succ) \;\; n)$ is the addition function add for two natural numbers, where m and n are the formal parameters corresponding to the numbers whose sum is computed.
- The interpretation of $\lambda m.\, \lambda n.\, ((m \;\; (add \;\; n)) \;\; zero)$ is the multiplication function mul for two natural numbers, where m and n are the formal parameters corresponding to the numbers whose product is computed.
- The interpretation of $\lambda x.\, \lambda y.\, x$ is propositional constant $true$.
- The interpretation of $\lambda x.\, \lambda y.\, y$ is propositional constant $false$.
- The interpretation of $\lambda a.\, \lambda b.\, \lambda h.\, ((h \;\; a) \;\; b)$ is a pair of entities a and b on which some function h can be applied. We will refer to this function as $Pair$. The first or second element of the pair $((Pair \;\; z_1) \;\; z_2)$ can be obtained by applying on it the functions $\lambda g.(g \;\; (\lambda a.\, \lambda b.\, a))$ (referred to as fst) and $\lambda g.\, (g \;\; (\lambda a.\, \lambda b.\, b))$ (referred to as sec), respectively. That is. $(fst \;\; ((Pair \;\; z_1) \;\; z_2)) \;=\; z_1$ and $((sec \;\; ((Pair \;\; z_1) \;\; z_2) \;=\; z_2$.
- The interpretation of a pair $((Pair \;\; m) \;\; n)$ where m and n are natural numbers is a signed number whose valuation is difference between m and n (i.e., $m - n$). For instance, $((Pair \;\; \lambda f.\, \lambda x.\, x) \;\; \lambda f.\, \lambda x.\, (f \;\; x))$ represents a signed number -1.

Identify the mathematical/logical interpretation for the following expressions. Justify your answer. (In all these problems, apply the functions on some actual arguments and examine the results; does the result correspond to some interpretation that you already know about—basic arithmetic or logical

operations. We have done similar problems, when we identified the interpretation of functions representing
addition and multiplication of naturals, and negation, conjunction and disjunction of propositions.).

(a) $\lambda x. ((x \;\; false) \;\; true)$, where x is the formal parameter corresponding to **propositional constants**.

$\lambda x. ((x \;\; false) \;\; true)$
$\lambda x. ((x \;\; \lambda x.\lambda y.y) \;\; \lambda x.\lambda y.x)$
$(\lambda x.\lambda y.x \;\; \lambda x.\lambda y.y)$

$(\lambda x.\lambda y.x \;\; \lambda x.\lambda y.y)$ – **Simplest form**

When simplified, the above expression will evaluate to $(true \;\; false)$, also known as $(false)$. The logical interpretation for this is a logical NOT gate, or negation operator.

(b) $\lambda n. ((n \;\; \lambda p.((p \;\; false) \;\; true)) \;\; false)$, where n is the formal parameter corresponding to **natural numbers**.

$\lambda n. ((n \;\; \lambda p.((p \;\; false) \;\; true)) \;\; false)$
$\lambda n. ((n \;\; \lambda p.((p \;\; \lambda x.\lambda y.y) \;\; \lambda x.\lambda y.x)) \;\; \lambda x.\lambda y.y)$
$\lambda n. ((n \;\; \lambda p.((p \;\; \lambda x.\lambda y.y) \;\; \lambda x.\lambda y.x)) \;\; \lambda x.\lambda y.y)$
$(\lambda x.\lambda y.y \;\; \lambda p.((p \;\; \lambda x.\lambda y.y) \;\; \lambda x.\lambda y.x))$
$(\lambda x.\lambda y.y \;\; (\lambda x.\lambda y.x \;\; \lambda x.\lambda y.y))$

$(false \;\; (true \;\; false))$ – **Simplest form**

(c) $\lambda m. \lambda n. ((m \;\; (mul \;\; n)) \;\; (succ \;\; zero))$, where p is the formal parameter corresponding to some **signed number**.

(d) $\lambda p. ((Pair \;\; (sec \;\; p)) \;\; (fst \;\; p))$, where p is the formal parameter corresponding to some **signed number**.

(e) $\lambda p_1. \lambda p_2. ((Pair \;\; ((add \;\; (fst \;\; p_1)) \;\; (sec \;\; p_2))) \;\; ((add \;\; (sec \;\; p_1)) \;\; (fst \;\; p_2)))$, where p_1 and p_2 are formal parameters corresponding to some **signed numbers**.