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COM S 474 - Homework 5 Written Solutions

11/11/20

Question 1:

The result of the Hadamard product $A \circ B$ is shown below:

$$A \circ B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 0.5 & 0.1 & 0.3 \\ -1 & -20 & 1.5 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.2 & 0.9 \\ -3 & -40 & 1.5 \end{pmatrix}$$

Question 2:

The result of AB^T:

$$AB^T = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0.5 & -1 \\ 0.1 & -20 \\ 0.3 & 1.5 \end{pmatrix} = \begin{pmatrix} 1.6 & -36.5 \\ 2 & -41.5 \end{pmatrix}$$

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Question 3:

There is **not** a product AB because the dimensions of the two matrices are different. Matrix A has dimensions (2 x 3) and matrix B also has dimensions (2 x 3), so they **cannot** be multiplied. To multiply matrices A and B, they need to have dimensions such that A has m rows and n columns, while matrix B has n rows and l columns in order to produce a matrix with m rows and l columns.

Question 4:

Given $f(x) = x + 1$, the value of $f(AB^T) = \mathbf{AB^T + 1}$.

Question 6

Given that $d = 3$, $\mathbf{x} = [x_0, x_1, x_2, x_3] = [1, 0, 1, 0]^T$, and $\mathbf{w} = [w_0, w_1, w_2, w_3] = [5, 4, 6, 1]$, and $\phi(x) = x^2$, we can find $\hat{y}$ by following the steps below.

First, we must find $w^T x$:

$$w^T x = \begin{pmatrix} 5 & 4 & 6 & 1 \end{pmatrix} * \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = 11$$

Then, we use $\phi(x) = x^2$ to find $\phi(w^T x)$:

$$\phi(w^T x) = \phi(11) = 11^2 = 121$$

So from this, we get:

$$\hat{y} = \phi(w^T x) = (w^T x)^2 = 121$$

Question 7:

Given the value of the loss function $E = \hat{y} - y$, we can find the respective values of $\partial E/\partial x_1$ and $\partial E/\partial w_1$ by following the process below:

First, we will find the value of $\partial E/\partial w_1$:

$$\frac{\partial E}{\partial w_1} = (\frac{\partial E}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial w_1}) = (\frac{\partial E}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial (w^T x)} * \frac{\partial (w^T x)}{\partial w_1}) = 1 * 22 * 0 = \mathbf{0}$$

Next, we will find the value of $\partial E/\partial x_1$:

$$\frac{\partial E}{\partial x_1} = (\frac{\partial E}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial (w^T x)} * \frac{\partial (w^T x)}{x_1}) = 1 * 22 * 4 = \mathbf{88}$$

Question 8 (Bonus):

We saw from question 7 that $\frac{\partial E}{\partial x_1} = 1 * 22 * x_1 = \mathbf{22x_1}$ and $\frac{\partial E}{\partial w_1} = 1 * 22 * w_1 = \mathbf{22w_1}$.

Now, to find the values of $\frac{\partial E}{\partial \mathbf{x}}$ and $\frac{\partial E}{\partial \mathbf{w}}$, we will use the following equations to find the results:

For $\frac{\partial E}{\partial \mathbf{x}}$:

$$\frac{\partial E}{\partial \mathbf{x}} = 22 * \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 22 \\ 0 \\ 22 \\ 0 \end{pmatrix}$$

For $\frac{\partial E}{\partial \mathbf{w}}$:

$$\frac{\partial E}{\partial \mathbf{w}} = 22 * \begin{pmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 110 \\ 88 \\ 132 \\ 22 \end{pmatrix}$$