Applied Spatiotemporal Data Mining

Statistical Analysis of Areal Data

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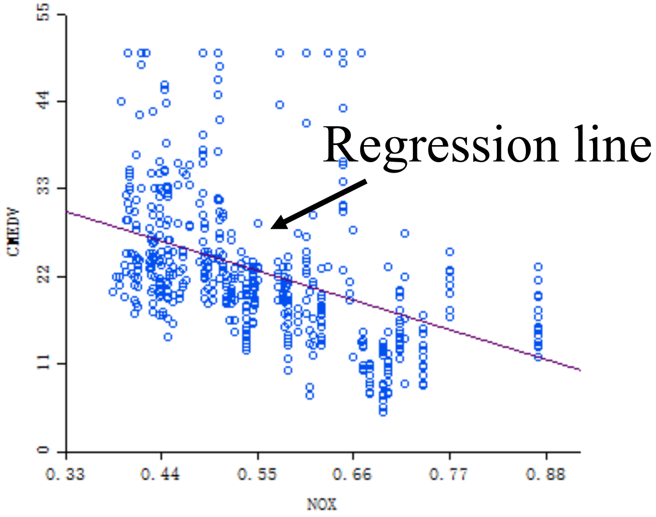
From Correlation to Regression

Correlation

- Co-variation
- Relationship or association
- No direction or causation is implied

Regression

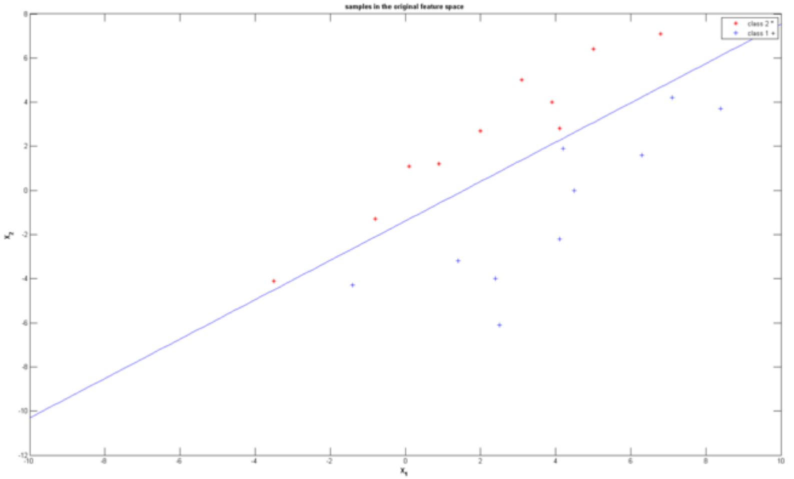
- Prediction of Y from X
- Implies, but does not prove, causation
- X (independent variable)
- Y (dependent variable)


Regression line

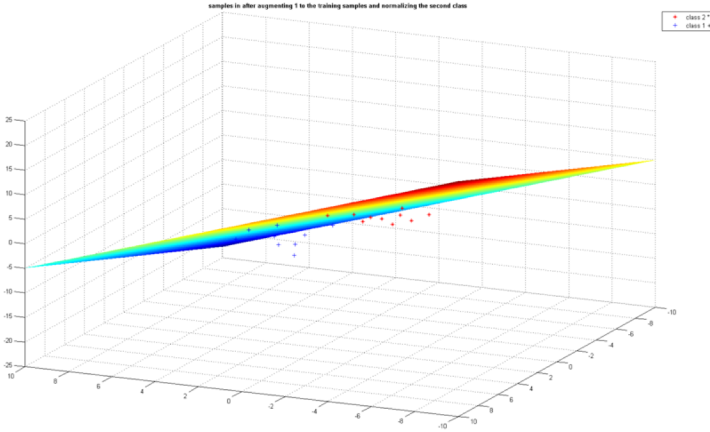
Regression

- Simple regression
  - Between two variables
    - One dependent variable (Y)
    - One independent variable (X)

$$Y = aX + b + \varepsilon$$



- Multiple Regression
  - Between three or more variables
    - One dependent variable (Y)
    - <u>Two</u> or independent variable (X_1 ,$X_{2\ldots}$)



$$Y = b_1 X_1 + b_2 X_2 + \cdots + b_0 + \varepsilon$$

Simple Linear Regression

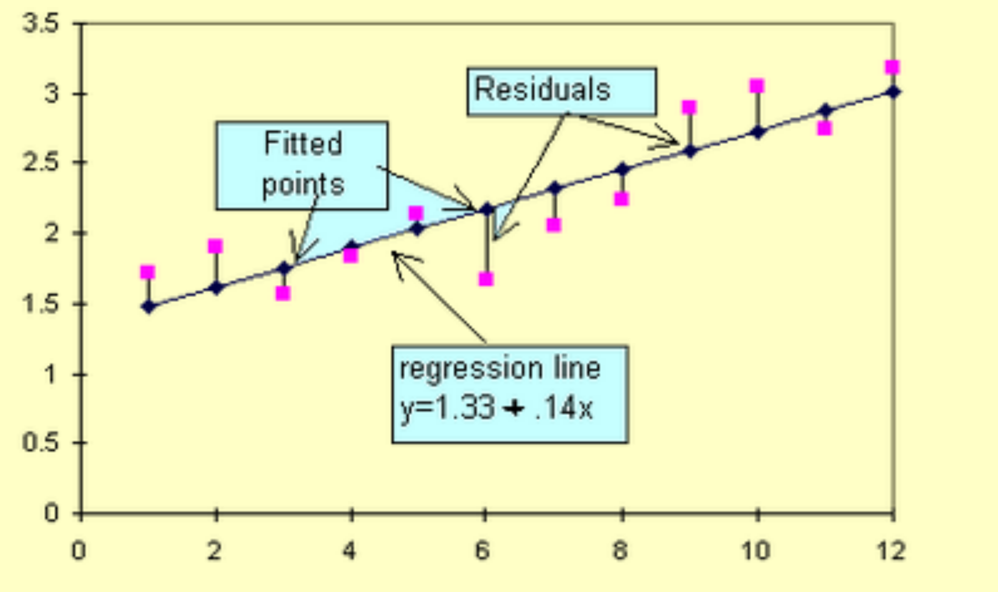
- Concerned with “predicting” one variable (Y - the dependent variable) from another variable (X - the independent variable)

$$Y = aX + b + \varepsilon$$

$$Y_i \sim \text{observations}$$

$$\hat{Y}_i \sim predictions$$

$$\varepsilon_i = Y_i - \hat{Y}_i$$



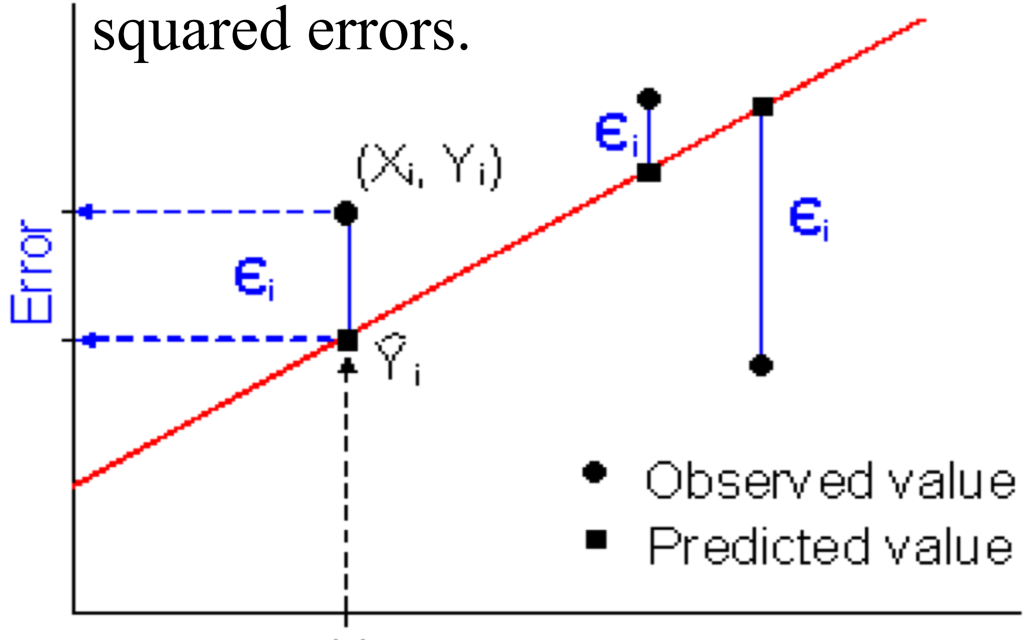
Residuals

Fitted points

regression line
y=1.33 + .14x

How to Find the line of the Best Fit

- **Ordinary Least Square** (OLS) is the mostly common used procedure
- This procedure evaluates the difference (or error) between each observed value and the corresponding value on the line of best fit.
- This procedure finds a line that minimizes the sum of the squared errors.



Evaluating the Goodness of Fit: Coefficient of Determination (r^2)

- The coefficient of determination (r^2) measures the proportion of the variance in Y (the dependent variable) which can be predicted or “explained by” X (the independent variable). Varies from 1 to 0.
- It equals the correlation coefficient (r) squared.

$$r^2 = \frac{\sum (\hat{Y}_i - \bar{Y})^2 \longleftarrow}{\sum (Y_i - \bar{Y})^2 \longleftarrow}$$

SS Regression or Explained Sum of Squares

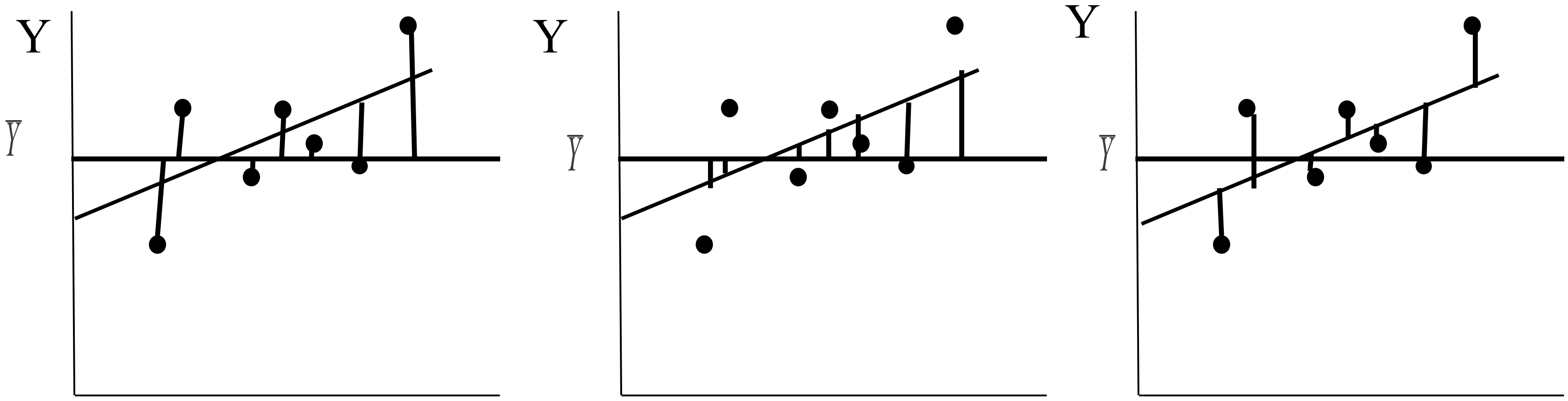
SS Total or Total Sum of Squares

Note:

$$\sum \underset{\uparrow}{(Y_i - \bar{Y})^2} = \sum \underset{\uparrow}{(\hat{Y}_i - \bar{Y})^2} + \sum \underset{\uparrow}{(Y_i - \hat{Y}_i)^2}$$

SS Total or Total Sum of Squares = SS Regression or Explained Sum of Squares + SS Residual or Error Sum of Squares

Partitioning the Variance on Y



$$\sum (Y_i - \bar{Y})^2 = \sum (\hat{Y}_i - \bar{Y})^2 + \sum (Y_i - \hat{Y}_i)^2$$

SS Total
or Total Sum of
Squares

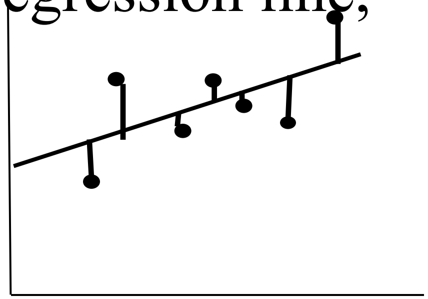
SS Regression
or Explained Sum of
Squares

SS Residual
or Error Sum of Squares

$$r^2 = \frac{\sum (\hat{Y}_i - \bar{Y}_i)^2}{\sum (Y_i - \bar{Y}_i)^2}$$

Standard Error of the Estimate

- Measures *predictive accuracy*: the bigger the standard error, the greater the spread of the observations about the regression line, thus the predictions are less accurate



- $\sigma =$ error mean square, or average squared residual
  = variance of the estimate, variance about regression

$$\sigma = \sqrt{\frac{\sum (Y_i - \hat{Y}_i)^2}{n-k}}$$

Sum of squared residuals

Number of observations minus *degrees of freedom*
(for simple regression, degrees of freedom = 2)

Sample Statistics, Population Parameters and Statistical Significance tests

$$Y = aX + b + \varepsilon \qquad Y = \alpha + \beta X + \varepsilon$$

- a and b are *sample statistics* which are estimates of *population parameters* α and β
- β (and b) measure the change in Y for a one unit change in X. If β = 0 then X has no effect on Y
- **Significant test**
  - Test whether X has a statistically significant affect on Y
  - **Null Hypothesis** (H_0)**:** in the population $\beta = 0$
  - **Alternative Hypothesis** (H_1)**:** in the population $\beta \neq 0$

Test Statistics in Simple Regression

- Student's t test, similar to normal, but with heavier tails

$$t = \frac{b}{\mathrm{SE(b)}} = \frac{b}{\sqrt{\dfrac{\sigma_e^2}{\sum_i (X - \overline{X})^2}}}$$

where σ_{e}^2 is the variance of the estimate, with degrees of freedom = n – 2

- F-test, A test can also be conducted on the *coefficient of determination* (r^2) to test if it is significantly greater than zero, using the *F* frequency distribution.

$$F = \frac{\text{Regression S.S./d.f.}}{\text{Residual S.S./d.f.}} = \frac{\sum (\hat{\mathrm{Y}}_{\mathrm{i}} - \bar{\mathrm{Y}})^2 / 1}{\sum (\mathrm{Y}_{\mathrm{i}} - \hat{\mathrm{Y}}_{\mathrm{i}})^2 / n - 2}$$

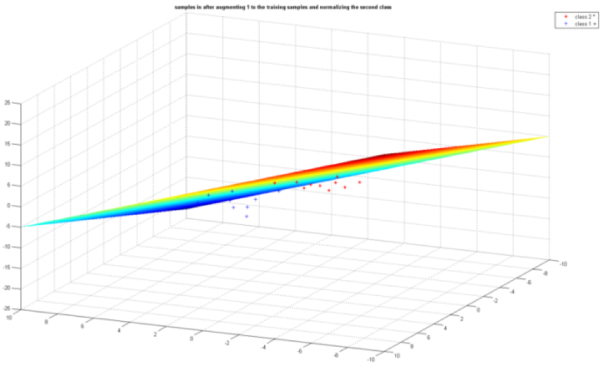
- Mathematically identical to each other

Multiple regression

- Multiple regression: Y is predicted from 2 or more independent variables

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_m X_m + \varepsilon$$

- β_0 is the *intercept* —the value of Y when values of all $X_j = 0$
- $\beta_1 \ldots \beta_m$ are *partial regression coefficients* which give the change in Y for a one unit change in X_j, all other X variables held constant
- *m* is the number of independent variables



How to Decide the Best Multiple Regression Hyperplane?

- Least square - Same as in the simple regression case

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_m X_m + \varepsilon$$

$$\text{or } Y_i = \sum_{j=0}^{m} X_{ij}\beta_j + \varepsilon_i \text{ (actual } \mathrm{Y}_{\mathrm{i}}\text{).}$$

$\hat{Y}_i = \sum_{j=0}^{m} X_{ij} b_j$ predicted values for Y (regression hyperplane)

)= residuals ¿

$e_i = Y_i - \sum_{j=0}^{m} X_{ij} b_j = (Y_i - \hat{Y}_i) = ($Actual Y_i - Predicted $\{\hat{Y}$ $_i$

$$\text{Min } \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

Evaluating the Goodness of Fit: Coefficient of Multiple Determination (R^2)

- Similar to simple regression, the coefficient of multiple determination (R^2) measures the proportion of the variance in Y (the dependent variable) which can be predicted or "explained by" all of X variables in combination.

Varies from 0 to 1.

$$R^2 = \frac{\sum(\hat{Y}_i - \bar{Y})^2 \longleftarrow}{\sum(Y_i - \bar{Y})^2 \longleftarrow}$$

SS Regression or Explained Sum of Squares

Formulae identical to simple regression

SS Total or Total Sum of Squares

As with simple regression

$$\underset{\substack{\uparrow \\ \text{SS Total or Total Sum of Squares}}}{\sum(Y_i - \bar{Y})^2} = \underset{\substack{\uparrow \\ \text{SS Regression or Explained Sum of Squares}}}{\sum(\hat{Y}_i - \bar{Y})^2} + \underset{\substack{\uparrow \\ \text{SS Residual or Error Sum of Squares}}}{\sum(Y_i - \hat{Y}_i)^2}$$

SS Total or Total Sum of Squares = SS Regression or Explained Sum of Squares + SS Residual or Error Sum of Squares

Reduced or Adjusted $\overline{R}^2$

- R^2 will <u>always</u> increase each time another independent variable is included
  - an additional dimension is available for fitting the regression *hyperplane* (the multiple regression equivalent of the regression line)
- Adjusted $\overline{R}^2$ is normally used instead of R^2 in multiple regression

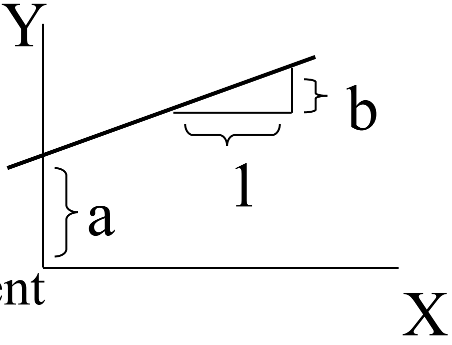
$$\overline{R}^2 = 1 - (1 - R^2)\left(\frac{n-1}{n-k}\right)$$

k is the number of coefficients in the regression equation, normally equal to the number of independent variables plus 1 for the intercept.

Interpreting *partial regression coefficients*

- The regression coefficients (b_j) tell us the change in Y for a 1 unit change in X_j, all other X variables "held constant"
- Can we compare these b_j values to tell us the relative importance of the independent variables in affecting the dependent variable?
  - If $b_1 = 2$ and $b_2 = 4$, is the affect of X_2 twice as big as the affect of X_1 ?
  - NO!
- The size of b_j depends on the measurement scale used for each independent variable
  - if X_1 is income, then a 1 unit change is \$1
  - but if X_2 is rmb or Euro(€) or even cents (₵)

    1 unit is not the same!
  - And if X_2 is *% population urban,* 1 unit is very different



- Regression coefficients are only directly comparable if the units are all the same: all \$ for example

Standardized partial regression coefficients

- How do we compare the relative importance of independent variables?
- We know we cannot use partial regression coefficients to directly compare independent variables unless they are all measured on the same scale
- However, we can use *standardized partial regression coefficients* (also called *beta weights*, *beta coefficients*, or *path coefficients*).
- They tell us the number of standard deviation (SD) unit changes in Y for a one SD change in X)
- They are the partial regression coefficients if we had measured every variable in *standardized form*

$$\beta_{YX_j}^{std} = b_j \left(\frac{s_{X_j}}{s_Y} \right)$$

Test Statistics in Multiple Regression:

- Similar as in the simple regression case, but for each independent variable
- The student's test can be conducted for each partial regression coefficient b_j to test if the associated independent variable influences the dependent variable.
  - Null Hypothesis H_o : $b_j = 0$

$$t = \frac{b_j}{\mathrm{SE}(b_j)}$$

with degrees of freedom = n – k, where ***k*** is the number of coefficients in the regression equation, normally equal to the number of independent variables plus 1 for the intercept (m+1).

- The formula for calculating the standard error (SE) of b_j is more complex than for simple regression , so it is not shown here.

Test Statistics in Mutiple Regression

testing the overall model

- We test the *coefficient of multiple determination* (R^2) to see if it is significantly greater than zero, using the *F* frequency distribution.
- It is an overall test to see if at least one independent variable, or two or more in combination, affect the dependent variable.
- Does not test if each and every independent variable has an effect

$$F = \frac{\text{Regression S.S./d.f.}}{\text{Residual S.S./d.f.}} = \frac{\sum (\hat{Y}_i - \bar{Y})^2 / k-1}{\sum (Y_i - \hat{Y}_i)^2 / n-k}$$

Again, k is the number of coefficients in the regression equation, normally equal to the number of variables (m) plus 1.

- Similar to the F test in simple regression.
  - But unlike simple regression, it is not identical to the t tests.
- It is possible (but unusual) for the F test to be significant but all t tests *not significant.*

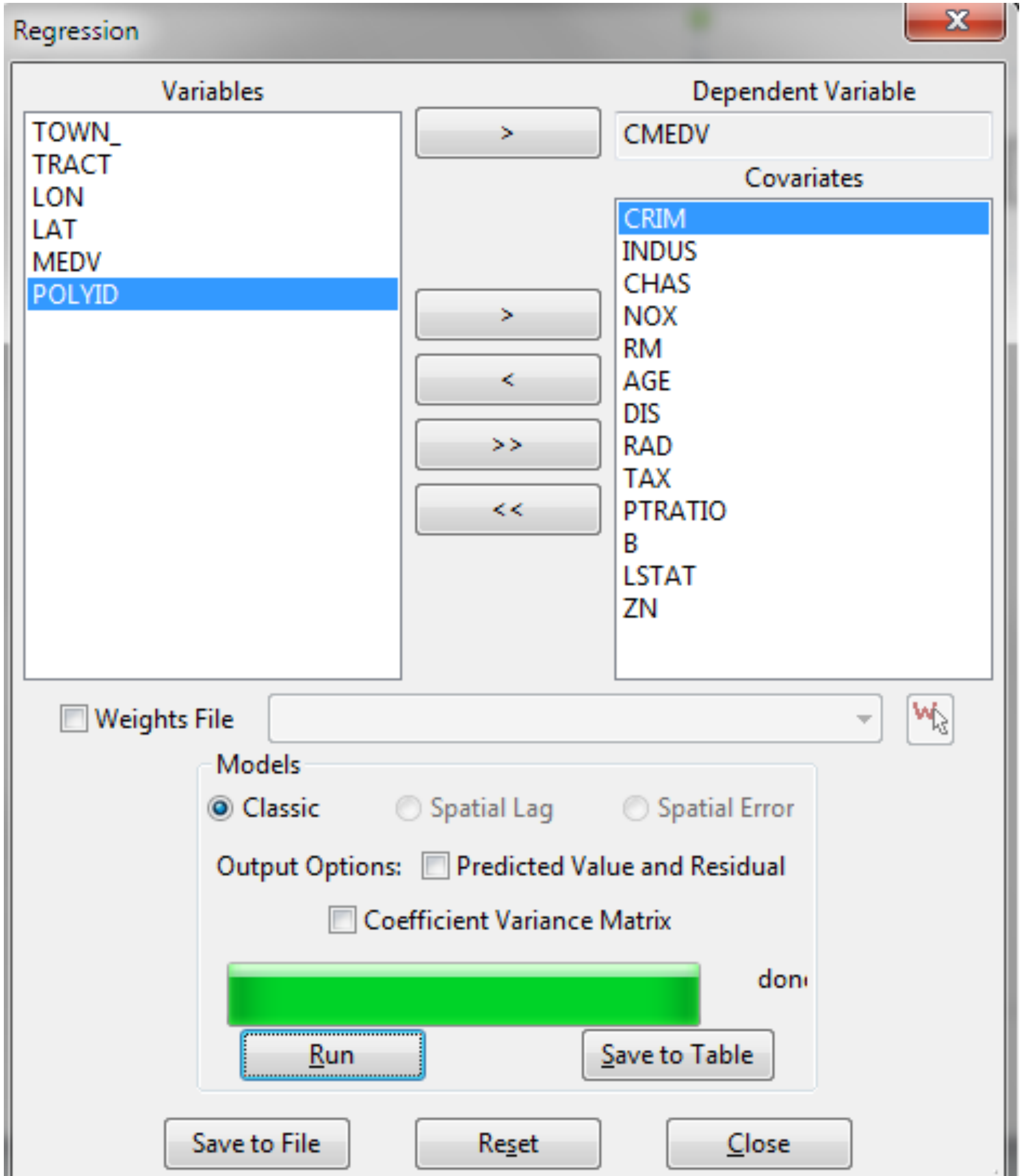
Model/Variable Selection

- Model selection is usually an iterative process
- R^2 nor Adjusted $\overline{R}^2$
- P-value of coefficient
- Maximum likelihood
- *Akaike Information Criteria* (AIC)
  - the <u>smaller</u> the AIC value the <u>better</u> the model

$$AIC = 2\mathrm{k} + n[\ln(\text{Residual Sum of Squares})]$$

k is the number of coefficients in the regression equation, normally equal to the number of independent variables plus 1 for the intercept term.

Regression in GeoDa



Regression

Variables

TOWN_
TRACT
LON
LAT
MEDV
POLYID

Dependent Variable

CMEDV

Covariates

CRIM
INDUS
CHAS
NOX
RM
AGE
DIS
RAD
TAX
PTRATIO
B
LSTAT
ZN

□ Weights File

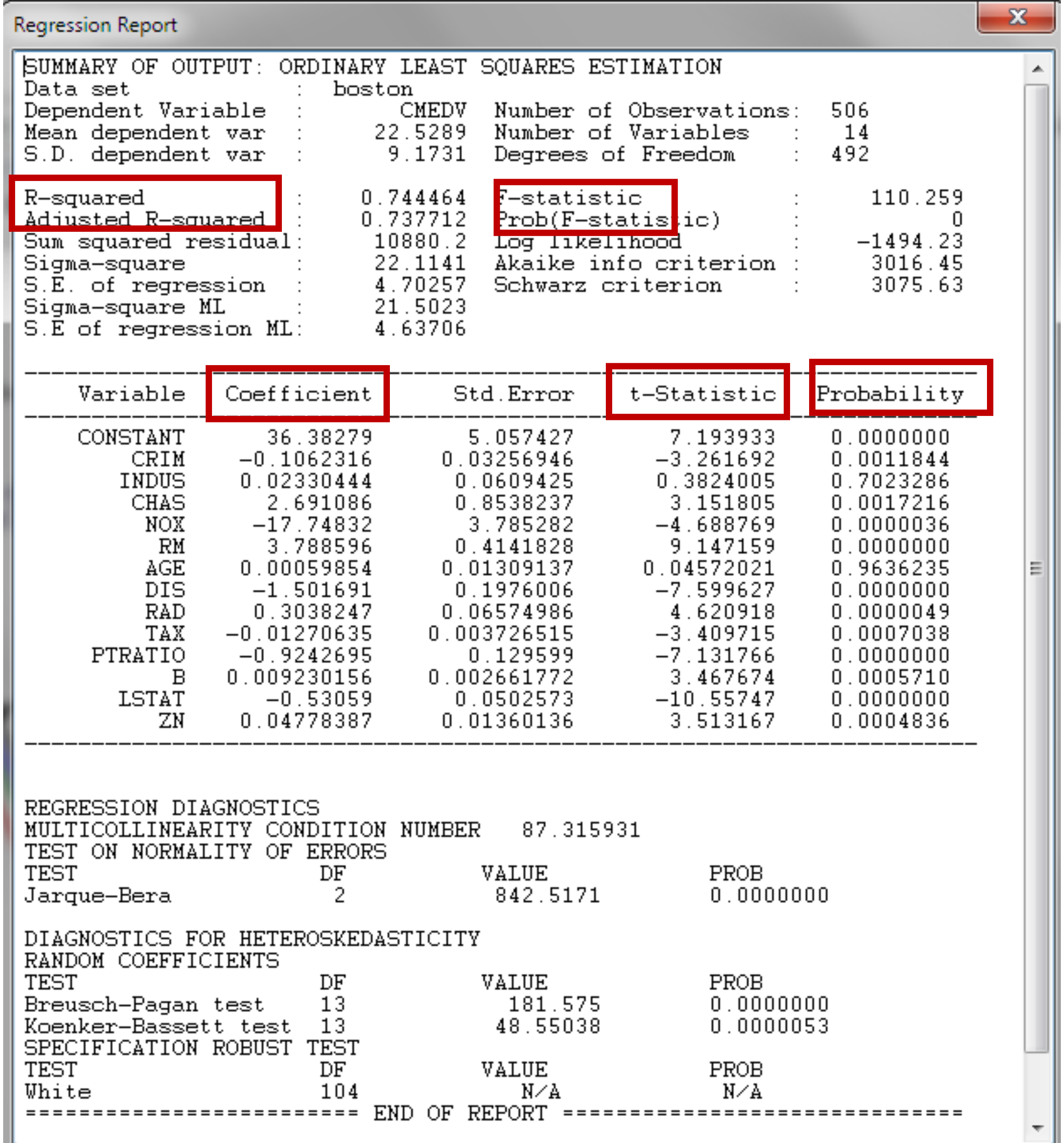
Models

● Classic ○ Spatial Lag ○ Spatial Error

Output Options: □ Predicted Value and Residual

□ Coefficient Variance Matrix

don



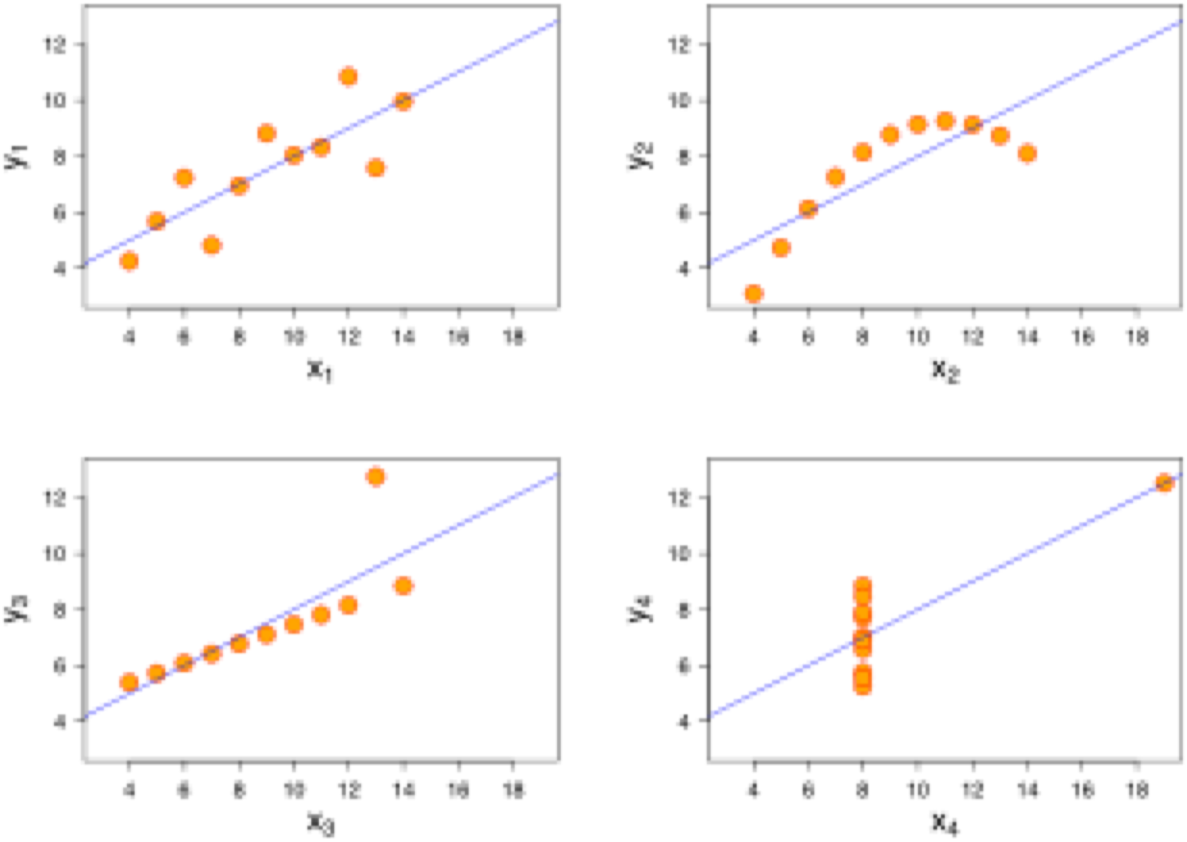
```
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES ESTIMATION
Data set            :  boston
Dependent Variable  :       CMEDV  Number of Observations:  506
Mean dependent var  :     22.5289  Number of Variables   :   14
S.D. dependent var  :      9.1731  Degrees of Freedom    :  492

R-squared           :    0.744464  F-statistic           :     110.259
Adjusted R-squared  :    0.737712  Prob(F-statistic)     :           0
Sum squared residual:     10880.2  Log likelihood        :    -1494.23
Sigma-square        :     22.1141  Akaike info criterion :     3016.45
S.E. of regression  :     4.70257  Schwarz criterion     :     3075.63
Sigma-square ML     :     21.5023
S.E of regression ML:     4.63706

-----------------------------------------------------------------------
    Variable   Coefficient      Std.Error    t-Statistic   Probability
-----------------------------------------------------------------------
    CONSTANT      36.38279       5.057427       7.193933     0.0000000
        CRIM    -0.1062316     0.03256946      -3.261692     0.0011844
       INDUS    0.02330444      0.0609425      0.3824005     0.7023286
        CHAS      2.691086      0.8538237       3.151805     0.0017216
         NOX     -17.74832       3.785282      -4.688769     0.0000036
          RM      3.788596      0.4141828       9.147159     0.0000000
         AGE    0.00059854     0.01309137     0.04572021     0.9636235
         DIS     -1.501691      0.1976006      -7.599627     0.0000000
         RAD     0.3038247     0.06574986       4.620918     0.0000049
         TAX   -0.01270635    0.003726515      -3.409715     0.0007038
     PTRATIO    -0.9242695       0.129599      -7.131766     0.0000000
           B   0.009230156    0.002661772       3.467674     0.0005710
       LSTAT      -0.53059      0.0502573      -10.55747     0.0000000
          ZN    0.04778387     0.01360136       3.513167     0.0004836
-----------------------------------------------------------------------


REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER   87.315931
TEST ON NORMALITY OF ERRORS
TEST                  DF          VALUE            PROB
Jarque-Bera            2           842.5171        0.0000000

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                  DF          VALUE            PROB
Breusch-Pagan test    13            181.575        0.0000000
Koenker-Bassett test  13           48.55038        0.0000053
SPECIFICATION ROBUST TEST
TEST                  DF          VALUE            PROB
White                104             N/A             N/A
========================= END OF REPORT ==============================
```

Procedures for Regression

- Diagnostic
  - Outlier
  - Constant variance
  - Normality
- Transformation
  - Transforming the response
  - Transforming the predictors
- Scale Change, principal component and collinearity, and auto/cross-correlation
- Variable selection
  - Step-wise procedures
- Model fit and analysis

Always look at your data

Statistics might lie

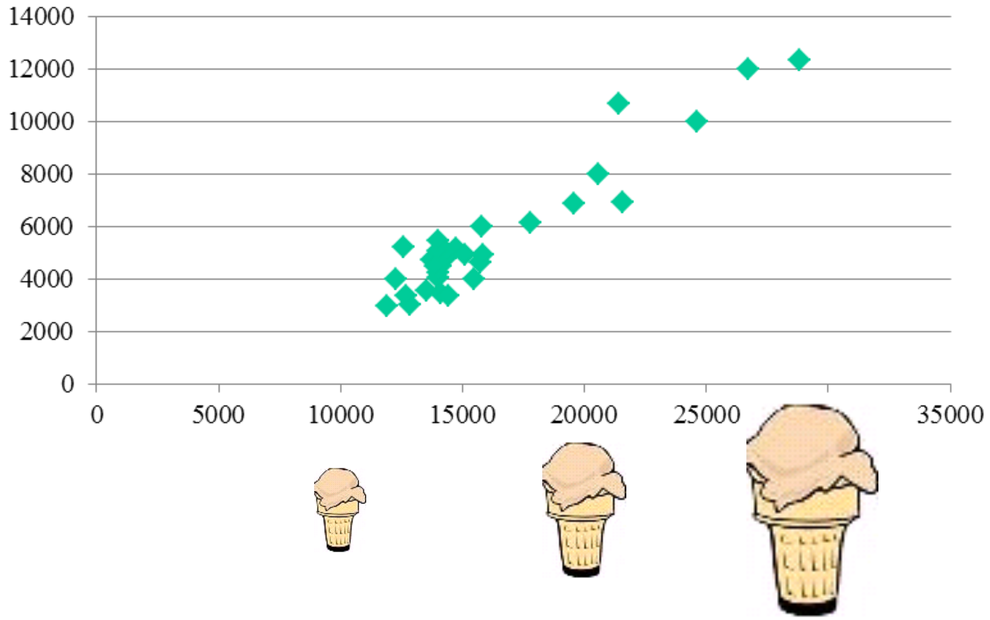


Anscombe, Francis J. (1973). "Graphs in statistical analysis". *The American Statistician* **27**: 17–21.

Source: Brigs UT Dallas

Spurious relationships

Ice Cream sales related to Drownings



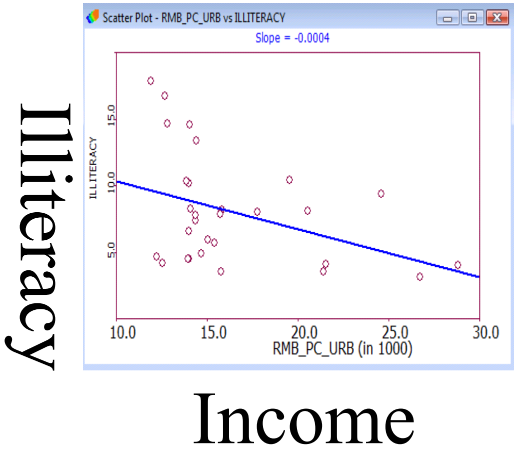
- *Omitted variable* problem

  --both are related to a third variable not included in the analysis

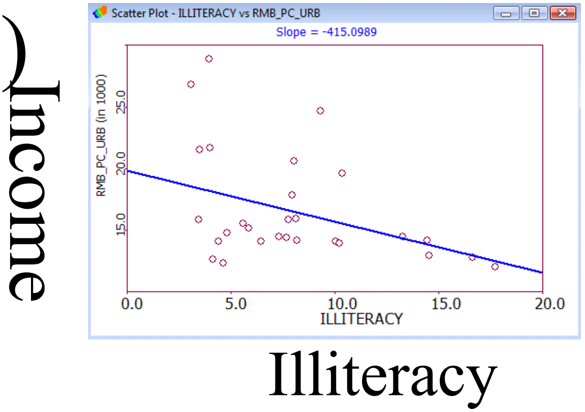
Source: Briggs UT Dallas

Linear Regression does not prove causal effects!

- States with higher incomes can afford to spend more on education, so illiteracy is lower
  - Higher Income -> Less Illiteracy

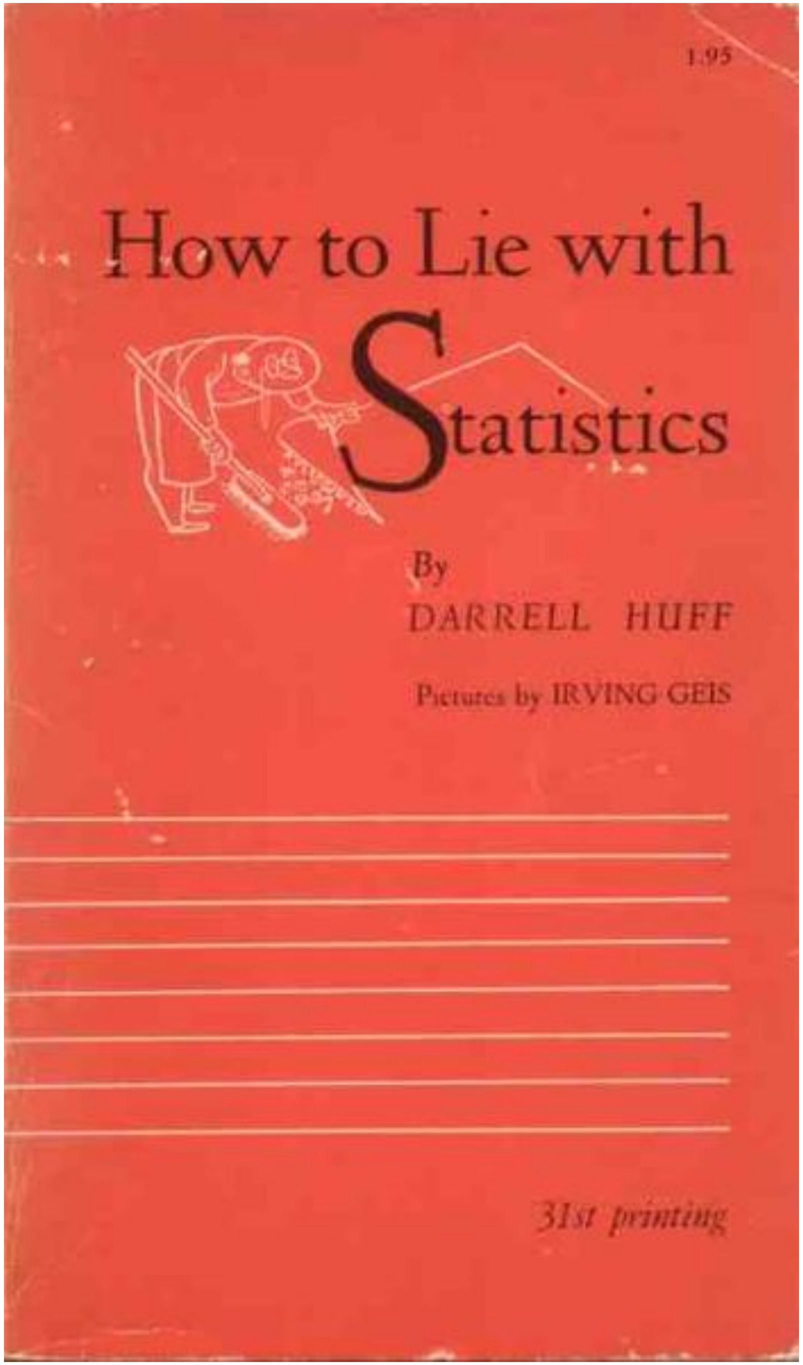


- The higher the level of literacy (and thus the lower the level of illiteracy) the more high income jobs.
  - Less Illiteracy -> Higher Income



- Regression will not decide!

Source: Briggs UT Dallas



1.95

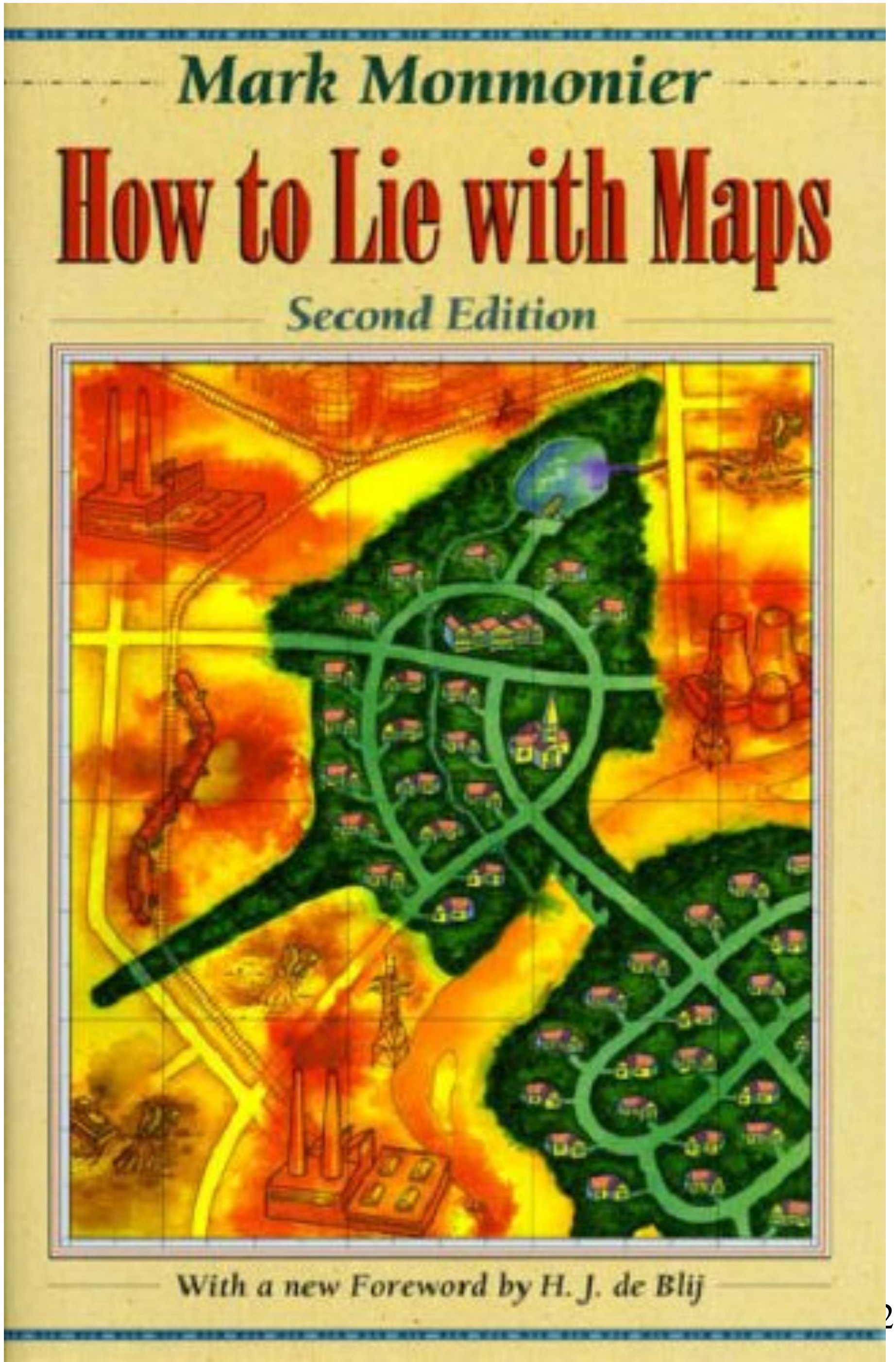
How to Lie with Statistics

By

DARRELL HUFF

Pictures by IRVING GEIS

31st printing



Mark Monmonier

How to Lie with Maps

Second Edition

With a new Foreword by H. J. de Blij

Spatial Regression

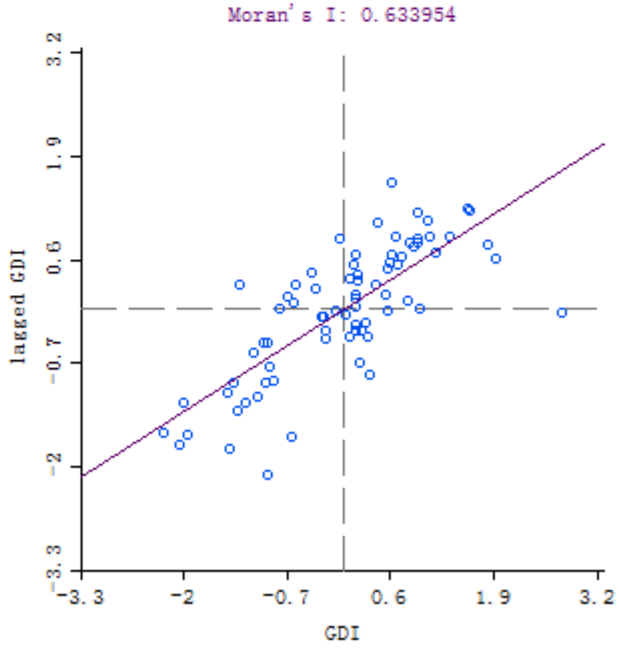
Spatial Autocorrelation vs Correlation

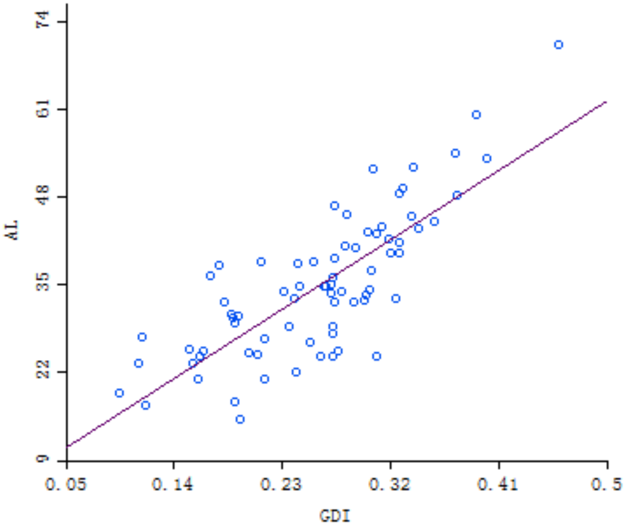
Spatial Autocorrelation:

shows the association or relationship between the same variable in "near-by" areas.

Standard Correlation

shows the association or relationship between two different variables





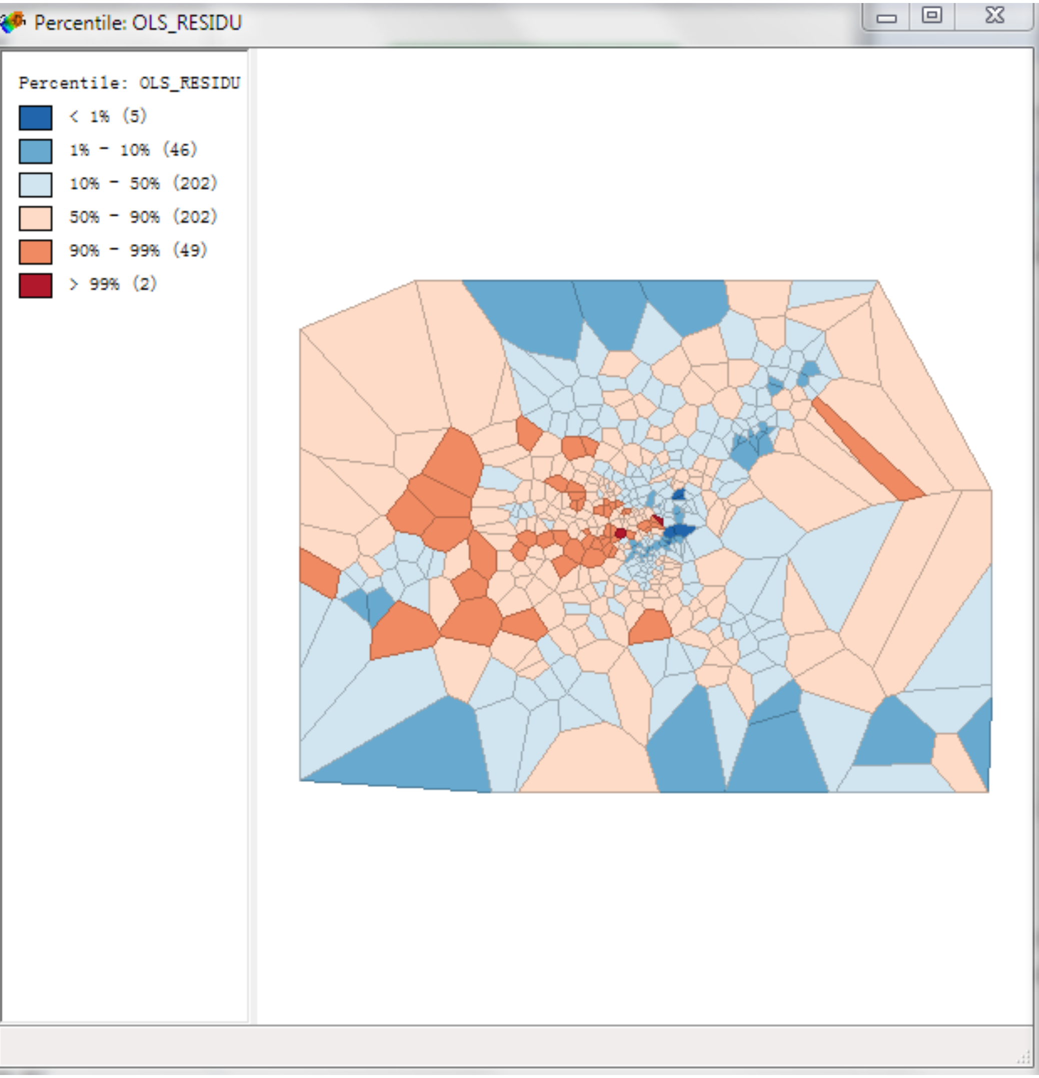
Consequences of Ignoring Spatial Autocorrelation

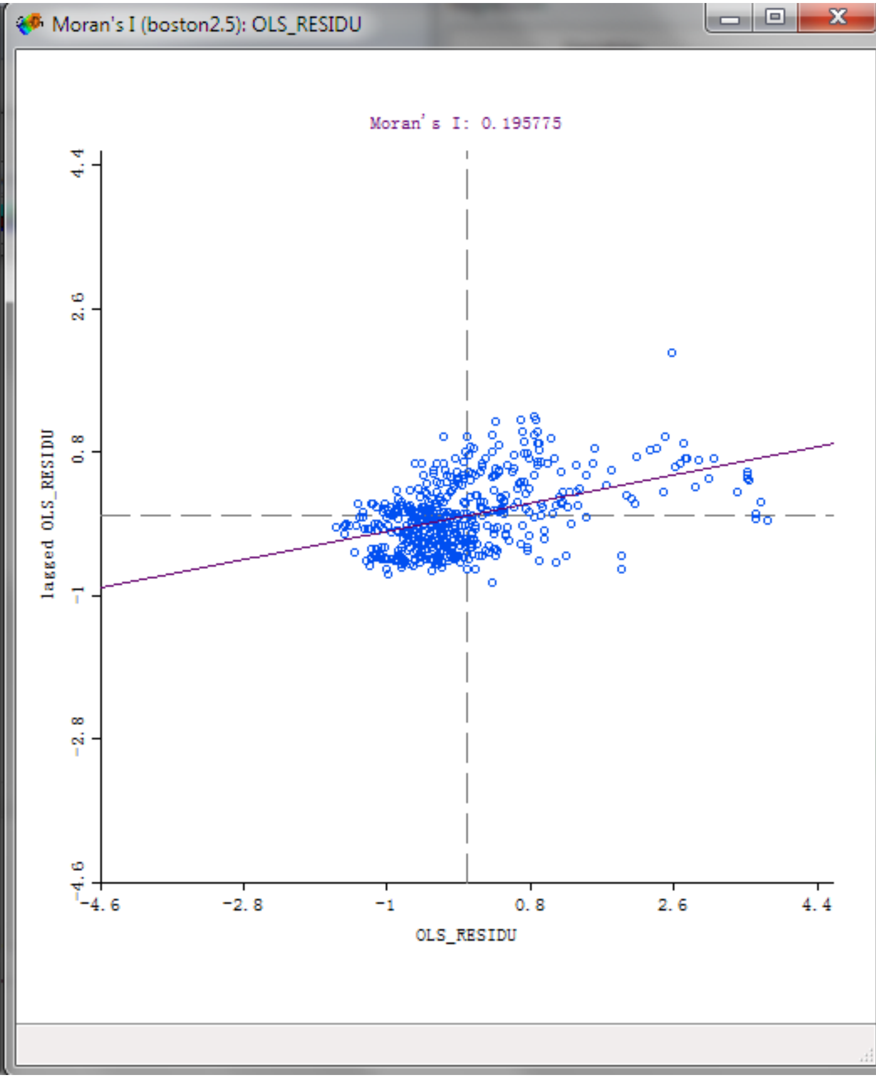
- correlation coefficients and coefficients of determination appear bigger than they really are
  - You think the relationship is stronger than it really is
  - the variables in nearby areas affect each other
- Standard errors appear smaller than they really are
  - *exaggerated precision*
  - You think your predictions are better than they really are since standard errors measure *predictive accuracy*
  - More likely to conclude relationship is *statistically significant*.

Diagnostic of Spatial Dependence

- **For correlation**
  - calculate Moran's I for each variable and test its statistical significance
  - If Moran's I is significant, you may have a problem!
- **For regression**
  - calculate the residuals

    map the residuals: do you see any spatial patterns?
  - Calculate Moran's I for the residuals: is it statistically significant?


Percentile: OLS_RESIDU
Percentile: OLS_RESIDU
< 1% (5)
1% - 10% (46)
10% - 50% (202)
50% - 90% (202)
90% - 99% (49)
> 99% (2)


Moran's I (boston2.5): OLS_RESIDU
Moran's I: 0.195775
lagged OLS_RESIDU
4.4
2.6
0.8
-1
-2.8
-4.6
-4.6
-2.8
-1
0.8
2.6
4.4
OLS_RESIDU

When (spatial) correlation happens

- Try to think of omitted variables and include them in a multiple regression.
  - Missing (omitted) variables may cause spatial autocorrelation
- Regression assumes all relevant variables influencing the dependent variable are included
  - If relevant variables are missing, model is *misspecified*

Spatial Regression Methods

- Spatial Econometrics Approaches
  - Lag model
  - Error model
- Spatial Statistics Approaches
  - Simultaneous Autoregressive Models (SAR)
    - A more general case of Spatial Econometrics
  - Conditional Autoregressive Models (CAR)
- Other methods:
  - Generalized linear model with mixed effects
  - Generalized additive model
  - Generalized Estimating Equations

Source: Briggs UT Dallas

Spatial Econometrics Approaches

- **Spatial lag model**

$$Y = \beta_0 + \enclose{circle}{\lambda WY} + X\beta + \varepsilon$$

values of the dependent variable in neighboring locations (*WY)* are included as an extra explanatory variable

  - these are the "spatial lag" of Y

- **Spatial error model**

$$Y = \beta_0 + X\beta + \enclose{circle}{\rho W\varepsilon} + \xi$$

ξ is "white noise"

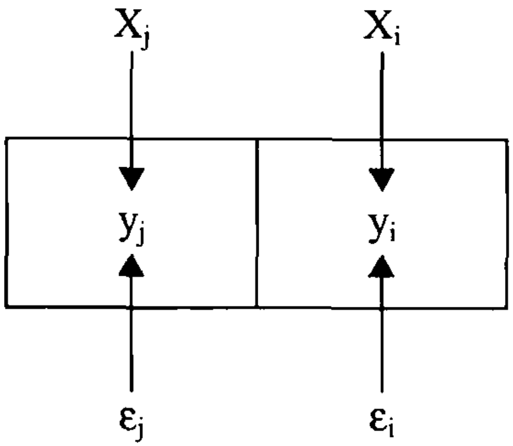
values of the residuals in neighboring locations (*W*ε) are included as an extra term in the equation;

  - these are "spatial error"

Spatial Lag and Spatial Error Models: *conceptual comparison*

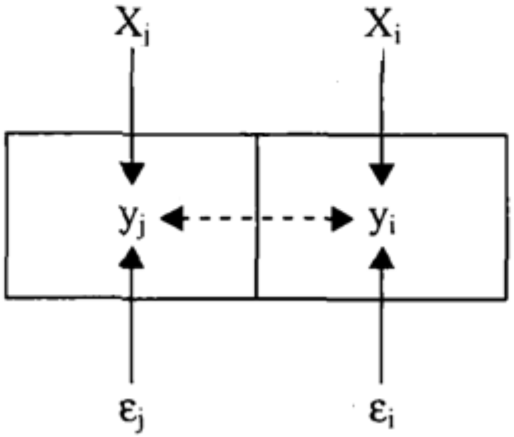
Ordinary Least Squares

OLS



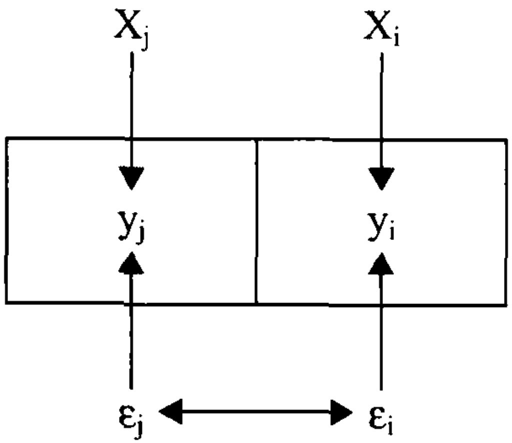
No influence from neighbors

SPATIAL LAG



Dependent variable influenced by neighbors

SPATIAL ERROR



Residuals influenced by neighbors

Baller, R., L. Anselin, S. Messner, G. Deane and D. Hawkins. 2001. *Structural covariates of US County homicide rates: incorporating spatial effects*,. Criminology , 39, 561-590

Source: Briggs UT Dallas

Spatial Lag Model

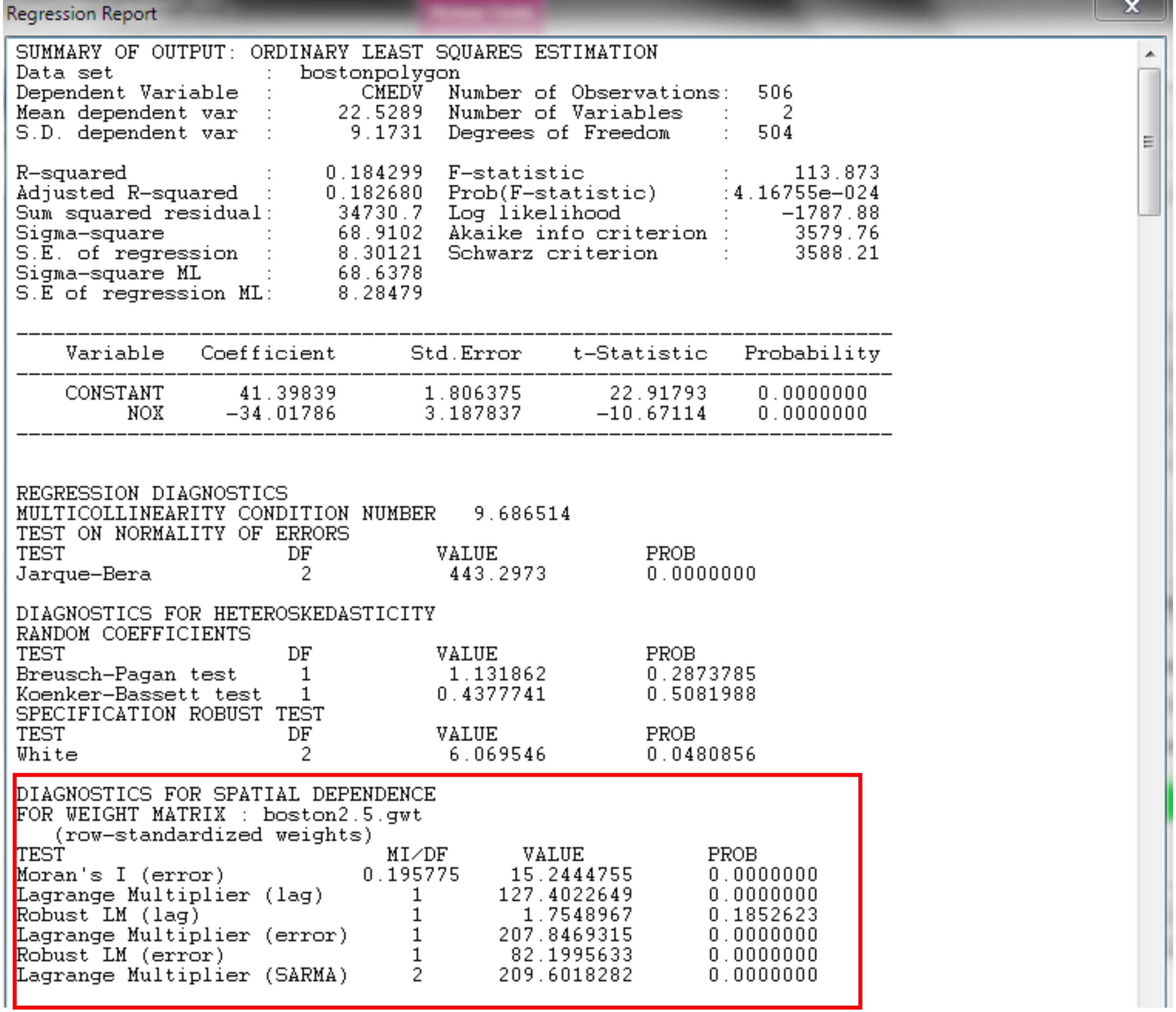
- Incorporates spatial effects by including a spatially lagged dependent variable as an additional predictor
- Outcome is dependent on the outcome for neighbors
- The ‘spatially lagged’ or ‘average neighbouring’ Wy is correlated with the unobserved error term, thus the model leads to biased and inefficient coefficients if using OLS

Spatial Error Model

- Incorporates spatial effects through error term
- Unobserved factors in neighboring locations are correlated
- With spatial error violate the assumption that error terms are uncorrelated and coefficients are inefficient if using OLS

Lag or Error Model: *Which to use?*

- **Lag** model primarily controls spatial autocorrelation in the dependent variable
- **Error** model controls spatial autocorrelation in the residuals, thus it controls autocorrelation in both the dependent and the independent variables
- **Conclusion:** the error model is more robust and generally the better choice.
- **Statistical tests** called the *LM Robust* test can also be used to select
  - Will not discuss these



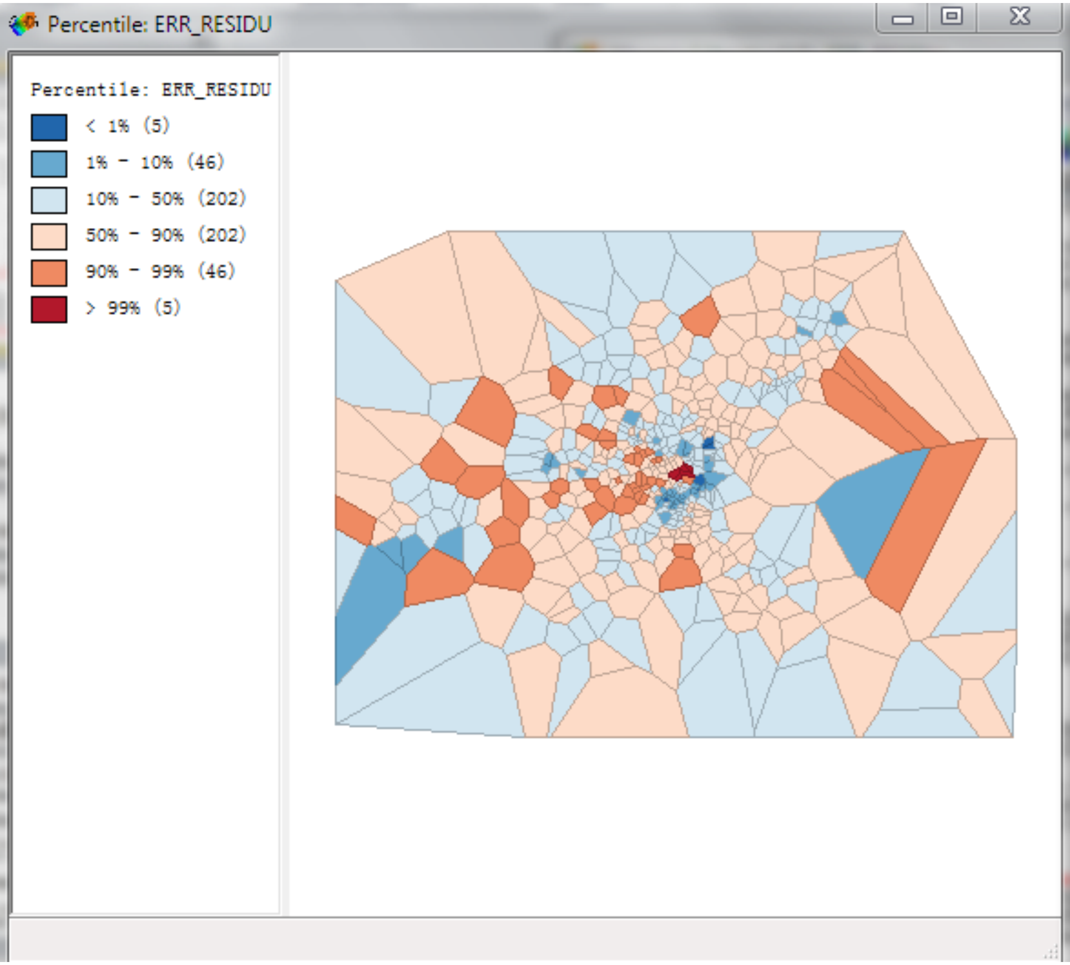
```
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES ESTIMATION
Data set            :  bostonpolygon
Dependent Variable  :       CMEDV  Number of Observations:  506
Mean dependent var  :     22.5289  Number of Variables   :    2
S.D. dependent var  :      9.1731  Degrees of Freedom    :  504

R-squared           :    0.184299  F-statistic           :      113.873
Adjusted R-squared  :    0.182680  Prob(F-statistic)     :4.16755e-024
Sum squared residual:     34730.7  Log likelihood        :     -1787.88
Sigma-square        :     68.9102  Akaike info criterion :      3579.76
S.E. of regression  :     8.30121  Schwarz criterion     :      3588.21
Sigma-square ML     :     68.6378
S.E of regression ML:     8.28479

-----------------------------------------------------------------------
    Variable   Coefficient      Std.Error    t-Statistic   Probability
-----------------------------------------------------------------------
    CONSTANT      41.39839       1.806375       22.91793     0.0000000
         NOX     -34.01786       3.187837      -10.67114     0.0000000
-----------------------------------------------------------------------


REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER   9.686514
TEST ON NORMALITY OF ERRORS
TEST                  DF           VALUE            PROB
Jarque-Bera            2            443.2973        0.0000000

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                  DF           VALUE            PROB
Breusch-Pagan test     1            1.131862        0.2873785
Koenker-Bassett test   1           0.4377741        0.5081988
SPECIFICATION ROBUST TEST
TEST                  DF           VALUE            PROB
White                  2            6.069546        0.0480856

DIAGNOSTICS FOR SPATIAL DEPENDENCE
FOR WEIGHT MATRIX : boston2.5.gwt
   (row-standardized weights)
TEST                          MI/DF       VALUE          PROB
Moran's I (error)           0.195775    15.2444755      0.0000000
Lagrange Multiplier (lag)       1      127.4022649      0.0000000
Robust LM (lag)                 1        1.7548967      0.1852623
Lagrange Multiplier (error)     1      207.8469315      0.0000000
Robust LM (error)               1       82.1995633      0.0000000
Lagrange Multiplier (SARMA)     2      209.6018282      0.0000000
```

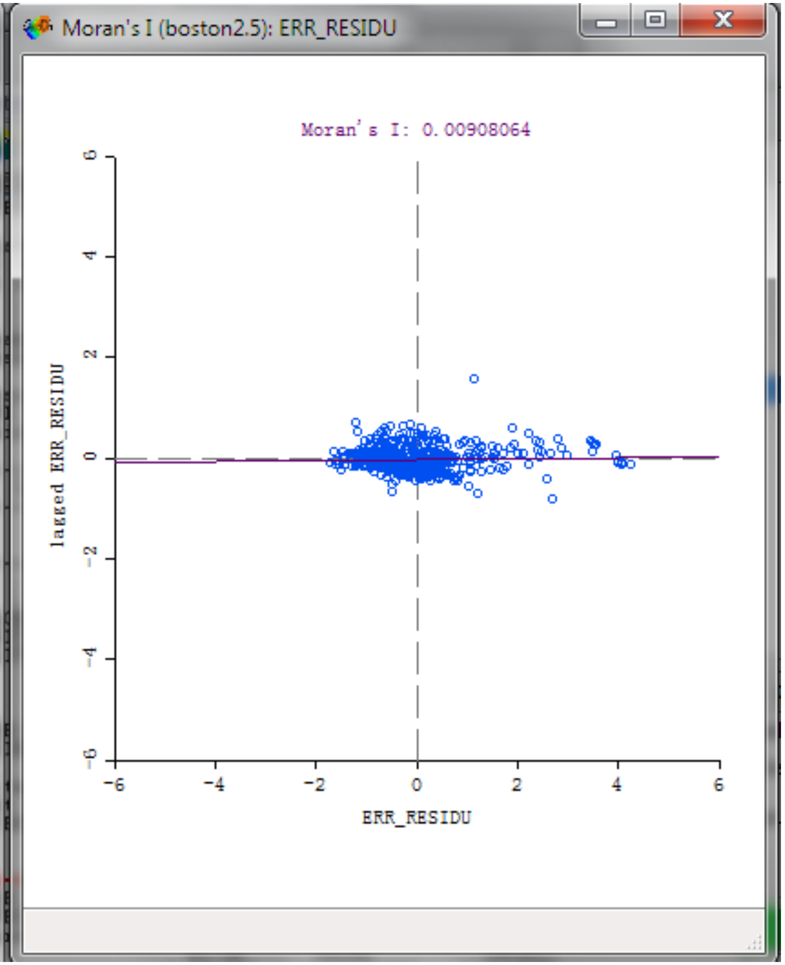
Model Fitting

- Maximum likelihood estimation

$$\varepsilon = Y-(\beta_0 + \lambda WY + X\beta)$$

- ε are assumed to be normally distributed
- Likelihood distribution of ε can be derived
- *$I-\lambda W$ must be invertible matrix (non-singular)*


Percentile: ERR_RESIDU
Percentile: ERR_RESIDU
< 1% (5)
1% - 10% (46)
10% - 50% (202)
50% - 90% (202)
90% - 99% (46)
> 99% (5)


Moran's I (boston2.5): ERR_RESIDU
Moran's I: 0.00908064
lagged ERR_RESIDU
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6
ERR_RESIDU

Model/Variable Selection

- Which model best predicts the dependent variable?
- Neither R^2 nor Adjusted $\bar{R}^2$ can be used to compare different spatial regression models
- We use *Akaike Information Criteria* (AIC)
  - the smaller the AIC value the better the model

$$AIC = 2\mathrm{k} + n\,[\,\ln(\,\text{Residual Sum of Squares}\,)\,]$$

k is the number of coefficients in the regression equation, normally equal to the number of independent variables plus 1 for the intercept term.

Note: can only be used to compare models with the same dependent variable

- End of this topic