



AN APPLICATION OF MULTI-OBJECTIVE PORT OF CALL OPTIMIZATION

DETERMINISTIC OPERATIONS
RESEARCH MODELS
NUS FINAL PROJECT

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1. Background

Containerised maritime transport is a key pillar of our global trade system, accounting for more than 50% of all global trade by volume. Container trade has the following key features:

- 1) It is highly standardised and easy to measure in terms of 20-foot-equivalent units (TEUs)
- 2) It encompasses a very wide network with a large number of supply and demand nodes
- 3) It is a highly competitive market driven by cost as very limited product differentiation
- 4) It is relatively time insensitive, therefore allowing a diverse selection of route options

These features mean that container traffic is highly flexible in order to find the lowest cost routes between ports. This frequently means that the units are transferred between vessels at intermediate ports rather than shipped directly between the source and destination. This process is termed transshipment.

The Port of Singapore Authority (PSA) is currently one of the largest and busiest container ports in the world. The PSA currently operates a total of 67 berths with a total handling capacity of 50 million TEUs per annum. In 2017, PSA handled 33 million TEUs, almost 4% of global container throughput. This is dominated by container transshipment, representing more than 90% of the total traffic. Transshipment is driven by two key factors, Singapore's strategic location on the global shipping lanes between Europe and Asia, and the rapid economic development of the local South East Asian markets.

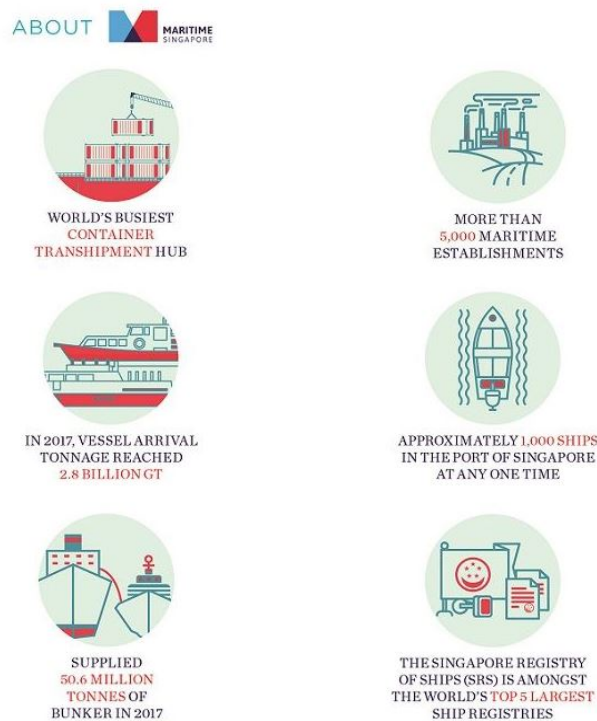


Figure 1: A snapshot of Singapore as a global hub port in 2017

The PSA generates revenue primarily through the application of Port Fees (the price leveled to berth and load/unload a vessel). Port fees are driven by the fixed cost of maintaining the port infrastructure, and the variable costs of personnel and berthing/unberthing costs (i.e. tugs, cranes and stevedores). Therefore, to maximise the port profitability, it is necessary to maximise both the throughput of container traffic and port fees. However, container ship operations are a highly competitive and flexible market with extremely tight margins. Fleet operators are therefore highly cost driven and will be heavily influenced by variation in the port fees.

Competition to the PSA comes from direct trade between supply and destination ports (i.e. avoiding transshipment in shipments), and from alternative regional transshipment ports. In 2016, Malaysian ports leveraged a cheaper currency and lower port costs to compete with Singapore for regional market share. This resulted in the PSA reducing port fees by 10 percent in order to counter the increasing transshipment volumes ceded to Malaysia.

Port capacity in both Singapore and Malaysia is large but limited by the capacity of berths and the speed of vessel turnaround. The capacity of the global container fleet to manage direct shipments is also limited due to constraints on berth capacity in most local ports (i.e. only smaller vessels can enter).

As PSA has limited influence to control global trade, the aim of this study is to calculate the optimal port fees that should be set by PSA in order to maximise the annual revenue resulting from container transshipment.

This study will model the minimum cost network flow of container traffic between South East Asia and the global market, considering direct trade as well as possible transshipment via Singapore or Malaysia. This model will then be utilised to test the sensitivity of container shipping to the port fees applied.

2. Problem Statement

Figure 2 presents a snapshot of regional container shipping. This figure indicates the sheer volume of shipping in the region, but also demonstrates the limited number of major routes adopted by container traffic.

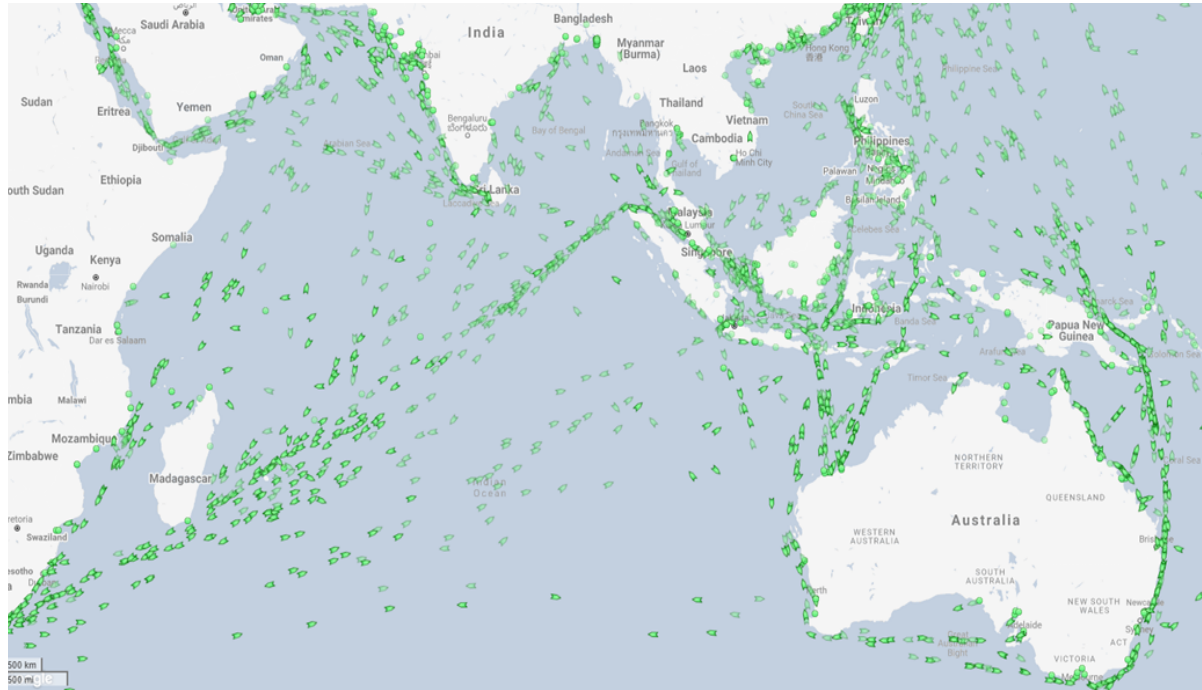


Figure 2: A snapshot of regional container vessels

Due to the large and highly complex nature of global trade, a number of key assumptions have been made to limit the complexity of the problem.

2.1 Container Volumes

The study considers containerised trade on an annual basis in terms of Twenty-foot Equivalent Units (TEUs). Containers are available in both twenty and forty foot equivalent units, however, all trade is measured in terms of TEU (a forty foot unit is equivalent to two TEUs). Other forms of trade import and export are excluded. Container import/export volumes are considered fixed for a year (i.e. price takers) and the final destination of each individual container is not considered. This means that the container trade is modelled as a series of balanced source and destination nodes.

Only two ports are sufficiently large in the region to act as major transshipment ports, Singapore and Malaysia. The total annual throughput capacity (imports and exports) of Singapore is 50 MTEU whereas the total capacity Malaysia is 30 MTEU. However, as these values assume maximum continuous throughput throughout the year, this is not considered achievable in reality. Therefore, a maximum utilisation factor of 80% has been applied to represent shipping

inefficiencies, blocked berths, etc. Therefore, maximum realistic annual capacity for Singapore and Malaysia are taken as 40 MTEU and 24 MTEU, respectively.

As the purpose of the study is to focus on transshipment via Singapore, the source and destination nodes have been simplified to consist of only the major regional import/export markets in South East Asia (Thailand, Vietnam, Indonesia and Australia). Singapore and Malaysia also import/export containers, however, the volume is considered low (around 10%) compared to transshipment and has been ignored for simplicity.

In terms of global trade, transshipment of goods for local markets is particularly common on the major shipping routes from East Asia to Europe and America. Rather than model a large number of origin/destination ports in these other regions, we will instead model only a single supply and demand node to represent the total import and export of each region (Europe, East Asia and Americas) to South East Asia. Our model does not consider direct trade between each of these regions (i.e. Europe to East Asian) as these cargos are sufficiently large that they would travel directly and not transship via East Asia.

Based upon the above assumptions, the shipment problem has been simplified to consist of just seven export and import nodes and two possible transshipment nodes. The model along with Annual volumes of containers in millions of TEU (MTEU) are presented in Figure 3.

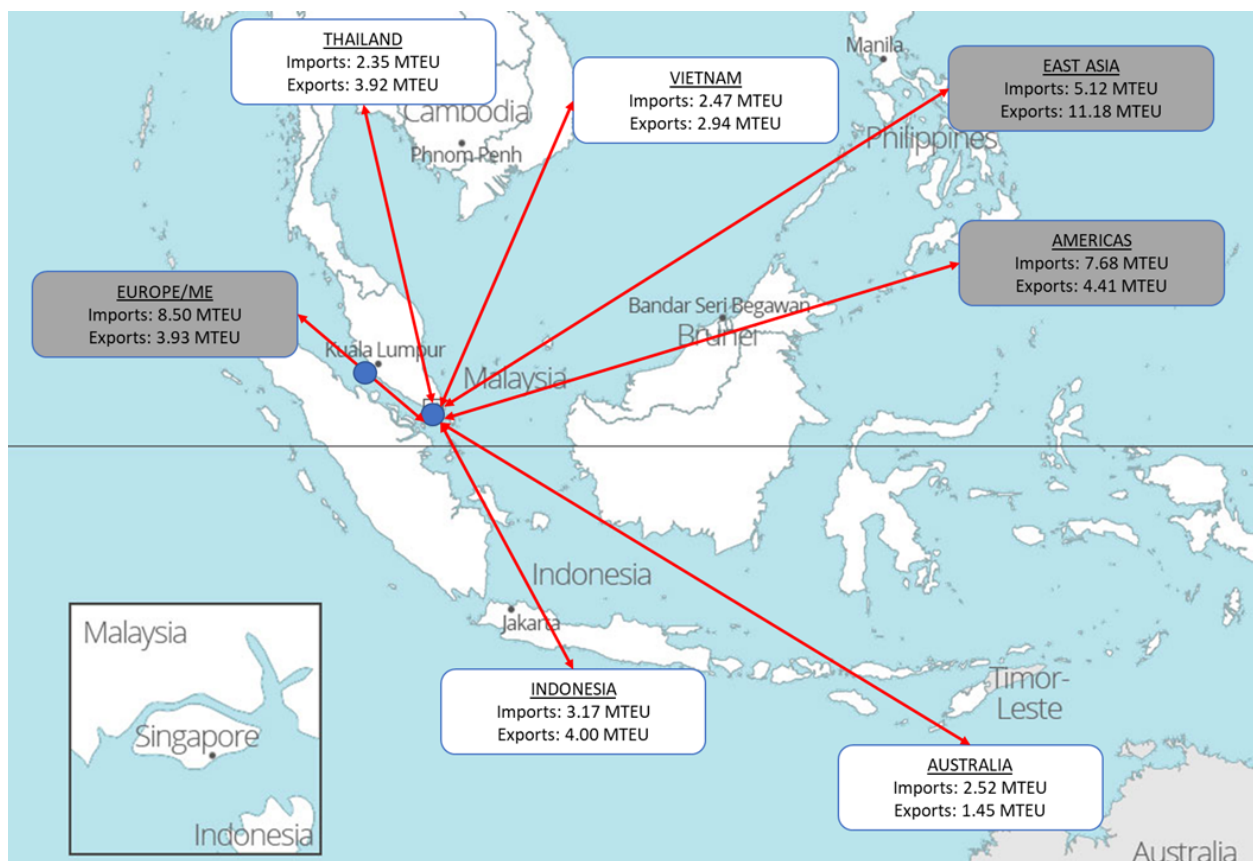


Figure 3: Simplified Import/Export Model (Millions TEU per annum)

2.2 Shipping Routes

Shipping routes between countries (nodes) have been modelled based upon actual sea route distance between major ports. For each country/region, only the single largest representative port has been selected to represent a realistic shipping route.

Table 1 presents the shipping distances between each of the ports in the region as well as distances to representative ports in each of the major regions. All distances are given in km. Note that the major representative port in each region is specified beneath the country. All shipping route to Europe are assumed to pass through the Suez Canal as this is the shortest route.

	Singapore	Malaysia	Vietnam	Thailand	Indonesia	Australia
Singapore (Brani)	-					
Malaysia (Port Dickson)	291	-				
Vietnam (Ho Chi Min)	1,196	1,487	-			
Thailand (Laem Chabang)	1,452	1,743	1,174	-		
Indonesia (Jakarta)	972	1,198	1,911	2,304	-	
Australia (Sydney)	7,914	8,180	8,477	9,143	7,119	-
East Asia (Shanghai)	4,143	4,434	3,124	4,082	4,673	8,578
Americas (Los Angeles)	14,203	14,494	13,316	14,359	14,629	12,057
Europe/ME (Rotterdam)	15,349	15,060	16,546	16,801	15,831	21,428

Table 1: Arc lengths between key ports (km)

2.3 Shipping Costs

Container ships come in a range of capacities ranging from local coastal vessels (<1000 TEU) up to the latest Ultra Large Container Ships (ULCSs) with capacities >18,000 TEU. For the purposes of this study, we have grouped vessels into several standard categories used in international trade, namely Feedermax (1,000 to 5,000 TEU) used for local shipping; Panamax (5,000 to 10,000 TEU) used for global shipping on less dense routes; and Suezmax (10,000 to 18,000 TEU) used only on major international shipping routes. We have then selected a representative average size for each of these categories, Feedermax (3,000 TEU), Panamax (8,000 TEU), and Suezmax (14,000 TEU).

In general, the costs of shipping per TEU will reduce with increasing vessel size due to economies of scale. Daily operating costs (fuel, crew, fees, etc) do not increase linearly in proportion with container capacity. In addition, larger ships typically travel at higher average speeds meaning short voyage durations. Table 2 provides an overview of the average daily vessel operating cost per TEU and, accounting for design speed, the average operating cost per km per TEU. This demonstrates that on average, the cost of shipping a container on a Suezmax carrier is roughly a third of the cost of shipping that same container on a Feedermax carrier. Table 2 also includes the approximate cost of transiting the Suez Canal which will apply for all shipments to and from Europe.

Ship Capacity (TEU)	Daily Operating Cost (\$/TEU)	Design Speed (knots)	Operating Cost (\$/km/kTEU)	Suez Transit Cost (\$/kTEU)
Feedermax (3k)	3.00	20.0	3.37	53,000
Panamax (8k)	1.70	22.5	1.70	36,000
Suezmax (14k)	1.15	24.0	1.08	29,000

Table 2: Vessel Operating Costs

Although larger vessels are clearly optimal from a simple route cost perspective, there are limitations on the maximum size of vessels that can enter most ports and berths. Typically, only the largest global ports have sufficient berth strength, crane size and water depth (>17m) to accommodate Suezmax vessels. Most regional ports are limited to Panamax or even Feedermax vessels. Table 3 provides an overview of the maximum vessel capacity permitted in the various countries under consideration.

	Singapore	Malaysia	Vietnam	Thailand	Indonesia	Australia
Singapore	-					
Malaysia	14k	-				
Vietnam	8k	8k	-			
Thailand	8k	8k	8k	-		
Indonesia	3k	3k	3k	3k	-	
Australia	8k	8k	8k	8k	3k	-
East Asia	14k	14k	8k	8k	3k	8k
Americas	14k	14k	8k	8k	3k	8k
Europe/ME	14k	14k	8k	8k	3k	8k

Table 3: Maximum vessel size per route

A further limitation on the size of containerships is the necessity to operate at high utilisation rates (container capacity >80%) in order to maintain profitability. Ships will not sail empty and therefore the same number of ships of a certain size must import and export containers from Singapore each year. This limitation means that annual container shipments entering Singapore on vessels of each size category must approximately balance the shipments leaving Singapore each year (plus/minus 20%)

2.4 Cost Model

Given the large market size, homogenous project and intense competition, we assume that the selection of a route is highly cost sensitive. Container shipments are also typically non-perishable meaning that routes are non-time critical. This means that when selecting a route, shipping companies will select the route based solely upon the cost. As shippers are typically price takers, these costs are assumed to remain fixed for all shipping routes.

For the purposes of this study, costs are broken down into only two principal components, shipping cost and port fees at transshipment ports. In reality, a range of other costs will apply such as handling fees, storage costs, source and destination port fees, distribution costs, insurances, duties and tariffs. However, as the majority of these costs would apply regardless of whether or not transshipment occurred, these origin and destination costs will have no impact on the volume of container traffic travelling through transshipment ports.

The PSA is assumed to only make revenue from port fees which will be the same for all vessel sizes and operators. In reality, container fleet operators arrange alliance agreements (similar to Airline alliances) in order to take advantage of economies of scale and negotiate better port fee prices. The intention of this model is to maximise the revenue of the PSA by varying the port fee price. Revenue is taken as simply the product of the port fee and the container transshipment volume.

3. Literature Review

The transshipment problem is a case of the minimum cost flow problem, in which commodities from supply nodes can be transported to demand nodes not just directly, but also via transshipment nodes. The objective is to determine how many units should be shipped over each node so that all the demand requirements are met with the minimum transportation cost.

Though our model makes unique assumptions and considerations, there are some relationships between this study and existing literature: Orden (1956) first introduced the transshipment problem as an extension to the basic transportation problem. Rhody (1963) solved the transshipment problem using the reduced matrix technique. Judge et al. (1965) formulated the transshipment problem as a general linear programming model.

Finding the cost-minimizing transportation route is more common, but Garg and Prakash (1985), studied a time-minimizing transshipment problem. Herer and Tzur (2001) studied the transshipment problem in a deterministic setting. Khurana and Arora (2011) solved the transshipment problem with mixed constraints by converting it to an equivalent transportation problem.

In the maritime context, S. Krille (2004), devised a heuristic algorithm for solving the minimum cost multicommodity flow problem (MCMCF) - i.e. transporting multiple types of cargo from several sources to several sinks. Each ship carries different types of containers for the different kinds of commodities. So, an optimal loading/unloading sequence for each port, for each cargo type, must also be found to minimize overall voyage costs. The algorithm minimizes loading, discharging, and transshipment costs, while meeting cargo demands of multiple destination ports.

In more recent work, Kumari and Kumar (2017) proposed a “max-min method” to solve a transshipment problem with mixed constraints. The mixed constraints arise from introducing the flexibility that some nodes may supply a finite amount, whereas others may supply at least/at most a given amount ‘a’. Similarly, while some destinations may demand a fixed amount, others may demand at least/most some amount ‘b’. This is to account for supply and demand fluctuations. In our case, we consider fixed supplies and demands which must be exactly met.

They first convert the transshipment problem into an equivalent linear transportation problem. Their objective is to find the shipping schedule that minimizes total shipping cost, while satisfying demand and supply constraints. Similar to our case, they make the assumption of a uniform commodity being shipped. They consider both balanced and unbalanced transshipment problems, while ours is balanced (total exports are equal to total imports).

All their nodes can act as transshipment nodes, while source nodes have only supply and destination nodes have only demand. Though we too consider our source nodes to be supply-only, and final destinations to be demand-only (though a supply and demand node could both be referring to the same real-life port), the transshipment nodes have neither supply nor demand

(just finite capacities). So, the definition of transshipment node in the reviewed paper differs: it could be an intermediate point for a given source-destination pair but doesn't have zero supply and zero demand. That is, the supply/demand nodes themselves can be intermediate points. Additionally, they allow the case of having multiple transshipment stops between source and destination nodes.

Closer to our problem, is work on export routes for the Brazilian soybean by Lopes, H.C., et al. (2016). They considered seven origin ports in Brazil, two destinations (Shanghai and Hamburg), with nine transshipment points (within Brazil), leading to 126 different flows across different modes of transport (not just sea). As in our case, the transshipment points are neither centers of production or consumption. They proposed three different scenarios (based on varied capacity of transshipment ports), and optimized these scenarios using Excel Solver. Each scenario was modeled as a linear transshipment problem. A key difference from our modeling, is that they do not consider direct flows from origins to destinations. The results from two of their scenarios showed reductions in operating costs of US \$236 million and US \$ 926 million.

The consideration of multiple arcs between any pair of nodes (whether supply-demand, supply-transshipment, or transshipment-demand) to represent shipment by different types of ships (i.e. with different cargo capacities, measured in TEUs, or twenty-foot equivalent unit) appears to be unique to our work.

4. Model Formulation

4.1 Objective Function

$$\text{Minimize: } \sum \sum \sum [c(v, w, k) + p(w, k)] * f(v, w, k)$$

for

$\forall v$ belonging to S or T, and
 $\forall w$ belonging to D or T, and
 $\forall k$ belonging to K

where

S is a supply node
T is a transshipment node
D is a demand node
K is ship type (3k TEUs, 8k TEUs, 14k TEUs)
c is arc cost,
f is amount shipped via the arc (in kTEUs)
(v, w, k) is a directed arc $e \in E$ from v to w via ship type k,
b(v) is Supply/Demand on each node
a(w) is port capacity where $w \in T$
p(w, k) is the fixed transshipment port fee per vessel where $w \in T, k \in K$

s.t

$$\begin{aligned} \sum f(v, w, k) - \sum f(w, v, k) &= b(v) & \forall v \in S, D, T \\ \sum f(v, w, k) &\leq a(w) & \forall w \in T \\ 0.8 * \sum f(v, w, k) &\leq \sum f(w, v, k) \leq 1.2 * \sum f(v, w, k) & \forall w \in T, \forall k \in K \\ f(v, w, k) &\geq 0 & \forall (v, w, k) \in E \\ f(w, v, k) &\geq 0 & \forall (w, v, k) \in E \end{aligned}$$

4.2 Nodes

All Network Nodes:

$$N = \{Americas, Europe, East Asia, Singapore, \\ Malaysia, Vietnam, Thailand, Indonesia, Australia\}$$

Supply Nodes:

$$S = \{Americas, Europe, East Asia, Thailand, Vietnam, Indonesia, Australia\}$$

Demand Nodes:

$$D = \{Americas, Europe, East Asia, Thailand, Vietnam, Indonesia, Australia\}$$

Transshipment Nodes:

$$T = \{Singapore, Malaysia\}$$

4.3 Assumptions

1. Total Supply equals Total Demand
2. We assume that there are no capacity constraints on arcs, or, in other words, that each arc has infinite capacity
3. The nodes that produce flow are supply nodes. Nodes that consume flow are demand nodes. If $b(v) = 0$, node v does not consume nor produce flow, i.e., it is a transshipment node
4. We assume this network cost minimization to be done on a yearly basis.
5. We consider port fees applicable only for routes going via transshipment. Port fees at the supply and demand destinations are not considered assuming they are mandatory fees.

4.4 Model Constraints

1. Conservation of flow
 - 1.1. For each transit node, total incoming flow equals total outgoing flow
 - 1.2. For each demand node, the total incoming flow equals node demand plus total outgoing flow
 - 1.3. For each supply node, the total outgoing flow equals node supply plus total incoming flow
2. Capacity for transshipment hubs
 - 2.1. Total incoming flow should be less than or equal to the maximum capacity of the transshipment hub
3. Utilization of transshipment hubs
 - 3.1. The incoming containers to transshipment hub should have high utilisation rates ($>80\%$) and ($<120\%$) to maintain profitability. Ships will not sail empty and therefore the same number of ships of a certain size must enter and leave transshipment hub each year
4. All arc flows are non-negative

5. Results and Discussions

5.1. Model Results and Impacts on Reality

The network model has been run based upon the various modelling assumptions described above and assuming an initial port fee of zero. Figure 4 presents the minimal network cost model in such a case showing that transshipment hubs are the most cost-effective solutions if transshipment costs are kept low.

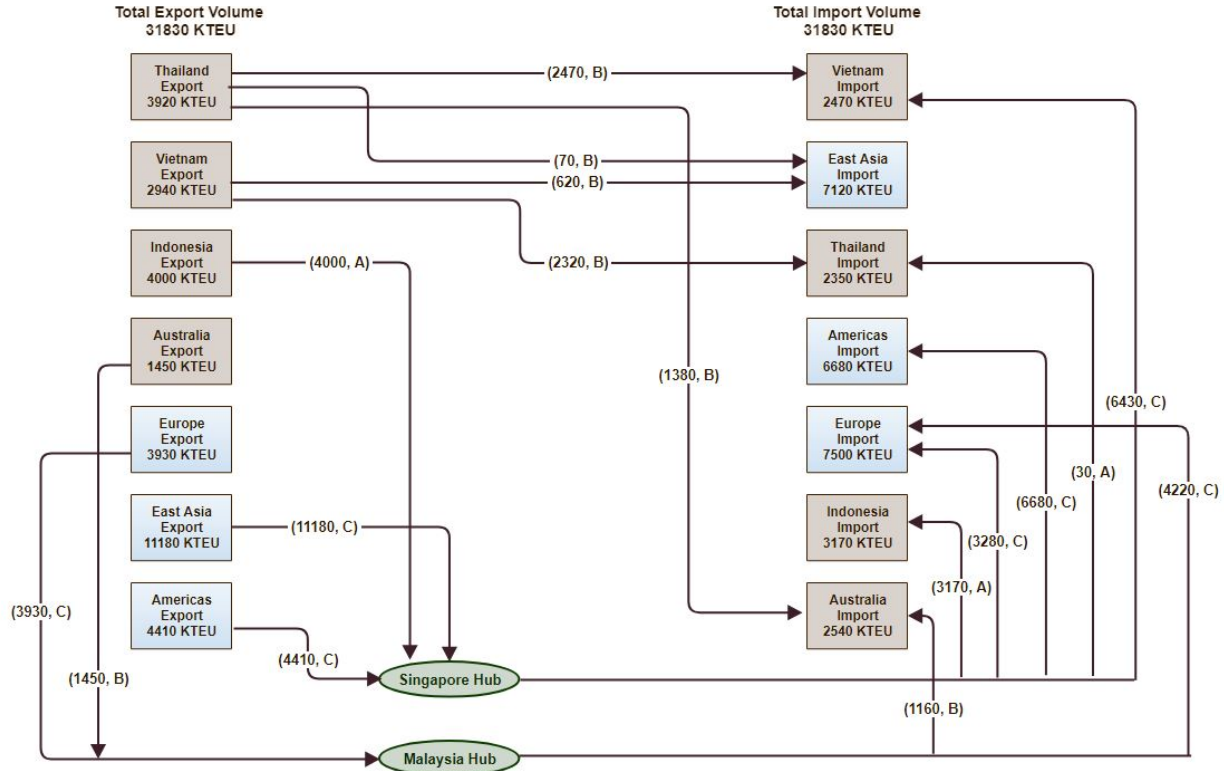


Figure 4: Base Case Minimal Network Cost model (Singapore Port Fee set to zero)

Based upon the zero port fee model, transshipment volumes via Singapore are around 39.2 million TEU per annum, almost equal to the maximum capacity of 40 million TEU. The transshipment via Malaysia is approximately 10.8 million TEU.

The port fee has been introduced for all container traffic shipping via Singapore. The fee is based upon a fixed fee per 1000 TEU added to all arcs connected to Singapore. By varying the port fee, it is possible to see the impact that this has on the network balance. By multiplying the port fee by the container throughput in Singapore, it is possible to gain an assessment of the PSA revenue. Figure 5 presents the results of this assessment, varying the Singapore port fee from 0 to 8,800 USD per 1000 TEU.

Singapore Port fee against Revenue and Volume



Singapore Port Fee vs. Singapore Revenue and Singapore Volume.

Figure 5: Singapore Port Revenue (USD) and Transshipment Volume (kTEU) against Port Fee (USD)

As would be expected, the transshipment volume decreases from peak throughput of nearly 40 million TEU, the maximum port capacity, at zero port fee. Down to zero for port fee exceeding 8,800 USD per 1,000 TEU. This decrease occurs in steps as the various cost points cause shipments to transition from a transshipment model via Singapore, to either transship via Malaysia instead or ship directly thereby avoiding transshipment costs entirely.

PSA revenue is calculated by multiplying the port fee by the volume of throughput. This allows us to identify the point of maximum revenue which occurs at a port fee of 4,400 USD per 1,000 TEU. This port fee value results in a total throughput of 26.4 million TEU per annum. This results in a maximum revenue of 116.2 million USD per annum from port fees in Singapore.

It should be noted that the transshipment values presented exclude the local demand and supply for Singapore of 3.4 million TEU. Combining the transshipment and local container demand gives a total container volume of 29.8 million TEU per annum. This is close to the actual volume of container traffic handled by the port in 2017 (33 million TEU) and provides credibility to the model.

The current model demonstrates unrealistically sharp transition points. In reality, different shipping companies may consider shifting operating models at different price points to avoid the sharp changes in throughput volume. However, historical behaviour does back this effect up to some extent as typically a shipping company or alliance will shift all of its business to a new transshipment port rather than piecemeal transition in order to take full advantage of the economies of scale.

The current model has focused on the variation in Singapore port fees to maximise revenue. Malaysian port fees have been set to zero for all cases. In reality, Malaysia will set its own port fees which will tend to be lower than those of Singapore due to the lower costs of operation. The effect of increasing Malaysian port fees primarily serves to drive up the allowable port fees and revenue for Singapore with limited impact on throughput.

5.2. Further Possible Model Developments

The team studies the concept of a minimum cost network flow (MCNF) model applied to the context of a transshipment problem, where we aim to minimize the cost based on the volume being shipped through the transshipment hubs of Singapore and Malaysia, subjected to the constraints of flow conservation, port capacity and ship sizes. The team proceeded to increase the port fees in order to obtain the highest possible revenue for PSA before shipping companies choose not to use Singapore as a transshipment stop.

With the non-commercial software resources (Python) available and to minimize the computational time, this report and case study is based on a few assumptions that helped to simplify our model but may seem rather lofty in terms of realism. For example, time is not an element to be considered or minimized (length of stay at the port, voyage time, etc), the distance travelled is also not a factor to be considered, container import/export volumes are considered fixed for a year, and total berth capacity in Singapore and other transshipment ports is fixed for the year, etc.

The current model is simplified to focus on minimum cost network flow without considering the distribution of containers between demand nodes. This has resulted in a simplification of the container traffic flow to fewer arcs. By improving the model to consider that all ports will likely consider some import and export from other regions, it is likely that the model will improve to consider a greater number of arcs.

The model is also oversimplified in terms of the sailing frequency and vessel size distribution. In reality, most ports require regular sailings (weekly) in order to ensure sufficient frequency of service. This frequency will typically result in greater use of smaller or intermediate sized vessels rather than concentration in fewer larger vessels on all but the busiest routes. Ports also maintain a range of berths with differing capacities which will tend to limit the number of larger vessels that can enter at any one time. These changes would again result in a greater distribution between the arc types.

There are many other advanced models to explore for possible future research, which will consider the factors listed above. The original transshipment problem is used to find the shortest route from one point in a network to another (*Orden 1956*). An extension of this model may include several additional linear constraints or mixed type constraints (*Gupta, Khanna, Puri 1993*). A variant for this problem also explores time minimization (*Khurana, Verma, Arora 2014*). For a combination of parameters such as time-cost trade off pairs, a non-linear model in the form of fixed charge bi-criterion indefinite quadratic transportation problem with various flows has also been studied (*Khurana A, Arora SR 2011*). Another angle on the transportation

problem involves an element of uncertainty in demand, by using scenario-based approach to formulate a two-stage stochastic mixed-integer linear program, where the decision, which is made under uncertainty, of the first-stage program, is followed by the second-stage decision that reacts to the observed demand (*Hrabec, Popela, Roupec, Mazel, Stodola 2015*). Lastly, a different but interesting take on the transshipment problem includes an analytic hierarchy process (AHP) on port selection decisions, based on service characteristics such as efficiency, frequency of ship visits and adequate infrastructure (*Ugboma, Ugboma, Ogwude 2006*).

For the purpose of this course in deterministic operations research models and in the interest of time, the team has utilized the basic minimum cost network flow operations research techniques considering different ship capacity sizes, to map the global shipping network as realistically as possible. However, other than minimizing costs/distance travelled and maximizing profits, there are factors such as geographical location, port operational efficiency, security and infrastructure for a shipping company to consider a transshipment hub. Hence there is always much room for improvement and experimentation from a real-life perspective in optimizing global transshipment networks.

6. Conclusion

This analysis is intended to model the flow of container traffic within the South East Asian region utilising the concept of minimum cost network flow. The novel contribution to existing research in solving problems of this type, is the study of a network with multiple arc types between any given pair of nodes. Though this complicates the model with additional constraints, it better captures the realities of container shipping networks.

The resulting transshipment model provides a reasonably accurate assessment of the volume of container traffic passing through the port of Singapore based upon the requirement to maximise port revenues. The optimal throughput calculated by the model is within 10% of the actual throughput in 2017, which demonstrates the power of the network flow modelling process to accurately model such a complex system.

In order to develop this model, it was necessary to make a number of major simplifications which limit the physical realism of the container flow in the region. A number of key model improvements have been identified and recommended to increase the accuracy in future. These include increased constraints on the frequency and size of vessels operating between ports. In addition, the distribution of container flow from a port needs to be considered since the current model tends to concentrate on only a reduced number of arcs.

This model may be further improved upon in future to assess the sensitivity of the local container market to various changes such as competitor pricing, vessel availability, container traffic increases, and changes in port capacity. This tool could be of considerable benefit to companies such as PSA who will depend on accurate forecasts of future capacity needs and revenue growth.

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