Week 4 Pre-Class Warm-up

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The 'Pyramid' Distribution

a. Find the cumulative density function of X, Fx, and plot it

$$F(x) = \int_{-\infty}^{x} f(y)dy = \int_{0}^{x} ydy = \frac{y^2}{2}\Big|_{y=0}^{y=x} = \frac{x^2}{2} - 0 = \frac{x^2}{2}$$

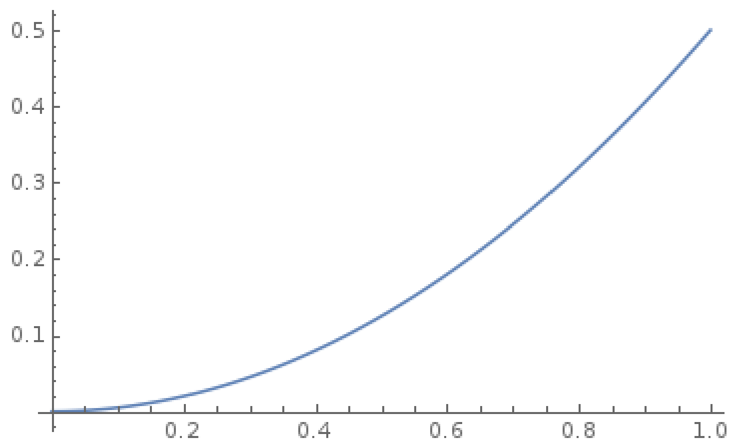

Figure 1: F(x) - x^2/2

$$F(x) = \int_{-\infty}^{x} f(y)dy = \int_{1}^{x} (2-y)dy = 2y - \frac{y^2}{2}\Big|_{y=1}^{y=x} = 2x - \frac{x^2}{2} - (2 - \frac{1}{2}) = 2x - \frac{x^2}{2} - \frac{3}{2}$$

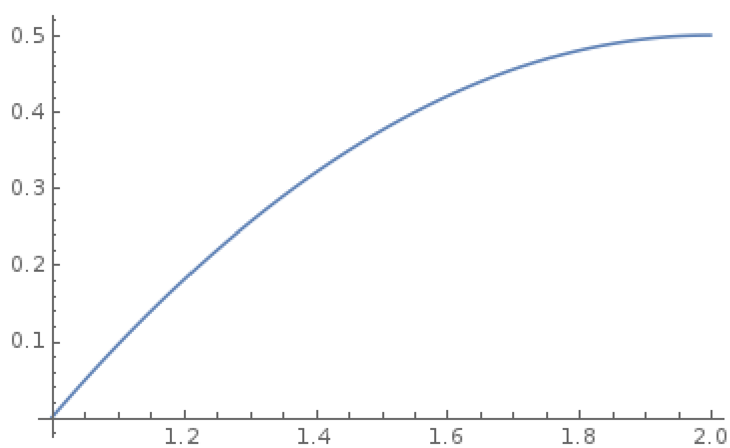

Figure 2: F(x) - 2x - x^2/2 - 3/2

b. Compute E(X)

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x * xdx = \int_0^1 x^2 dx = \frac{x^3}{3}|_{x=0}^{x=1} = \frac{1}{3}$$

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_1^2 (2-x) * xdx = \int_1^2 2x - x^2 dx = x^2 - \frac{x^3}{3}|_{x=1}^{x=2} = 4 - \frac{8}{3} - (1 - \frac{1}{3}) = \frac{2}{3}$$

c. Compute V(X)

$$V(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^1 x^2 * xdx = \int_0^1 x^3 dx = \frac{x^4}{4}|_{x=0}^{x=1} = \frac{1}{4}$$

$$V(X) = \frac{1}{4} - (\frac{1}{3})^2 = \frac{5}{36}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_1^2 x^2(2-x)xdx = \int_1^2 (2x^2 - x^3)dx = \frac{2x^3}{3} - \frac{x^4}{4}|_{x=1}^{x=2} = \frac{16}{3} - \frac{16}{4} - (\frac{2}{3} - \frac{1}{4}) = \frac{16}{12} - \frac{5}{12} = \frac{11}{12}$$

$$V(X) = \frac{11}{12} - (\frac{2}{3})^2 = \frac{33}{36} - \frac{16}{36} = \frac{17}{36}$$

d. Suppose Y(X) = X^2. Explain why Y is also a random variable

Y(X) is a function whose parameter is random variable X. This means the outcome of Y(X) depends on the value of the random variable X and therefore makes Y also a random variable.

e. Compute E(Y)

$$E(Y) = \int_{-\infty}^{\infty} h(x)f(x)dx = \int_0^1 x^2 * xdx = \int_0^1 x^3 dx = \frac{x^4}{4}|_{x=0}^{x=1} = \frac{1}{4}$$

$$E(Y) = \int_{-\infty}^{\infty} h(x)f(x)dx = \int_1^2 x^2(2-x)dx = \int_1^2 2x^2 - x^3 dx = (\frac{2x^3}{3} - \frac{x^4}{4})|_{x=1}^{x=2} = \frac{16}{3} - \frac{16}{4} - (\frac{2}{3} - \frac{1}{4}) = \frac{11}{12}$$