Unit 4 Homework: Random Variables

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9/28/2017

1. Best Game in the Casino

(a) How much do you get pai if the coin comes up heads 3 times?

$$X = \begin{cases} \$0 & \text{0 heads} \\ \$2 & \text{1 head} \\ \$4 & \text{2 heads} \\ \$x & \text{3 heads} \end{cases}$$

$$P(X) = \begin{cases} 1/6 & \text{0 heads} \\ 1/3 & \text{1 head} \\ 1/3 & \text{2 heads} \\ 1/6 & \text{3 heads} \end{cases}$$

$$E(X) = 0 * \frac{1}{6} + 2 * \frac{1}{3} + 4 * \frac{1}{3} + x * \frac{1}{6} = 6$$

$$\frac{x}{6} = 6 - \frac{2}{3} - \frac{4}{3} = 4$$

$$x = 24$$

You would get paid \$24 if the coin comes up heads 3 times.

(b) Cumulative probability function for winning from the game

$$F(X) = P(X <= x) = \begin{cases} 1/6 & \text{x = 0} \\ 1/2 & \text{x = 1} \\ 5/6 & \text{x = 2} \\ 1 & \text{x = 3} \end{cases}$$

2. Processing Pasta

(a) Cumulative probability function of L

$$F(L) = P(L <= l) = \int_{-\infty}^{l} f(y)dy = \int_{0}^{l} \frac{y}{2}dy = \frac{y^2}{4} = \frac{l^2}{4}$$

(b) Expected length of the pasta E(L)

$$E(L) = \int_{-\infty}^{\infty} l * f(l)dl = \int_{0}^{2} l * \frac{l}{2} = \int_{0}^{2} \frac{l^2}{2} = \frac{l^3}{2 * 3} = \frac{l^3}{6}|_0^2 = \frac{2^3}{6} - 0 = \frac{4}{3}$$

3. The Warranty is Worth It

(a) Expected payout from the contract, E(X) = E(g(T)) using the expression for the expectation of a function of a random variable

$$f(x) = \begin{cases} 1 \text{ 0<=x<=1} \\ 0 \text{ Otherwise} \end{cases}$$

$$E(X) = E(g(T)) = \int_{-\infty}^{\infty} g(t) * f(x)dx = \int_{0}^{1} 100(1-t)^{\frac{1}{2}} * 1dx = 100\int_{0}^{1}(1-t)^{\frac{1}{2}} * dx$$

$$= -100 * \frac{2}{3}(1-x)^{\frac{3}{2}} = -\frac{200}{3}(1-x)^{\frac{3}{2}}|_0^1 = \frac{200}{3}$$

$66.67

(b) E(X) another way, by first characterizing the random variable X.

i. Value for T that results in the payoff X = x

$$X = x$$

$$F(X) = P(X <= x) = \begin{cases} 0 & \text{x<0} \\ \text{100(1-t)\^{}(1/2)} & \text{0<= x <= 100} \end{cases}$$

$$\begin{aligned} 100(1-t)^{\frac{1}{2}} &= x \\ (1-t)^{\frac{1}{2}} &= \frac{x}{100} \\ \sqrt{(}1-t) &= \frac{x}{100} \\ 1-t &= \frac{x^2}{10000} \\ t &= 1-\frac{x^2}{10000} \end{aligned}$$

ii.

$$\begin{aligned} X &<= x \\ t &<= 1-\frac{x^2}{10000} \end{aligned}$$

iii.

$$F(X) = P(X <= x) = \int_0^x f(y)dy = \int_0^x 1-\frac{x^2}{10000}dy = x-\frac{x^3}{30000}$$

iv.

$$F'(X) = 1 - \frac{x^2}{10000}$$

v.

$$E(X) = \int_{-\infty}^{\infty} x * f(x)dx = \int_{0}^{100} x * (1 - \frac{x^2}{10000})dx$$
$$= 2x^2 - \frac{x^4}{40000}|_0^{100} = 100 - \frac{1000000}{10000}$$

4. The Baseline for Measuring Deviations

(a) Expression for E(Y) and use properties of expectation

(b) Taking a partial derivative with respect to t, compute the value of t that minimizes E(Y)

(c) What is the value of E(Y) for this choice of t?