

Unit 4 Homework: Random Variables

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1. Best Game in the Casino

(a) How much do you get paid if the coin comes up heads 3 times?

$$X = \begin{cases} \$0 & 0 \text{ heads} \\ \$2 & 1 \text{ head} \\ \$4 & 2 \text{ heads} \\ \$x & 3 \text{ heads} \end{cases}$$

$$P(X) = \begin{cases} 1/6 & 0 \text{ heads} \\ 1/3 & 1 \text{ head} \\ 1/3 & 2 \text{ heads} \\ 1/6 & 3 \text{ heads} \end{cases}$$

$$E(X) = 0 * \frac{1}{6} + 2 * \frac{1}{3} + 4 * \frac{1}{3} + x * \frac{1}{6} = 6$$

$$\frac{x}{6} = 6 - \frac{2}{3} - \frac{4}{3} = 4$$

$$x = 24$$

You would get paid \$24 if the coin comes up heads 3 times.

(b) Cumulative probability function for winning from the game

$$F(X) = P(X \leq x) = \begin{cases} 1/6 & x = 0 \\ 1/2 & x = 1 \\ 5/6 & x = 2 \\ 1 & x = 3 \end{cases}$$

2. Processing Pasta

(a) Cumulative probability function of L

$$F(L) = P(L \leq l) = \int_{-\infty}^l f(y)dy = \int_0^l \frac{y}{2}dy = \frac{y^2}{4} = \frac{l^2}{4}$$

(b) Expected length of the pasta E(L)

$$E(L) = \int_{-\infty}^{\infty} l * f(l)dl = \int_0^2 l * \frac{l}{2} = \int_0^2 \frac{l^2}{2} = \frac{l^3}{2 * 3} = \frac{l^3}{6} \Big|_0^2 = \frac{2^3}{6} - 0 = \frac{4}{3}$$

3. The Warranty is Worth It

(a) Expected payout from the contract, $E(X) = E(g(T))$ using the expression for the expectation of a function of a random variable

$$f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{Otherwise} \end{cases}$$
$$E(X) = E(g(T)) = \int_{-\infty}^{\infty} g(t) * f(x) dx$$

(b) $E(X)$ another way, by first characterizing the random variable X .

i.

ii.

iii.

iv.

v.

4. The Baseline for Measuring Deviations

(a) Expression for $E(Y)$ and use properties of expectation

(b) Taking a partial derivative with respect to t , compute the value of t that minimizes $E(Y)$

(c) What is the value of $E(Y)$ for this choice of t ?