Power System Dynamics

Most power system analysis is done in steady state, i.e. without consideration of the time-varying nature of electrical power.

However, the power flow in the system varies with time due to load changes, generator availability and faults.

Previously, it was shown that the electrical power in the system, P_E, is proportional to the voltage angle, δ, which is also known as the *load angle* for this reason.

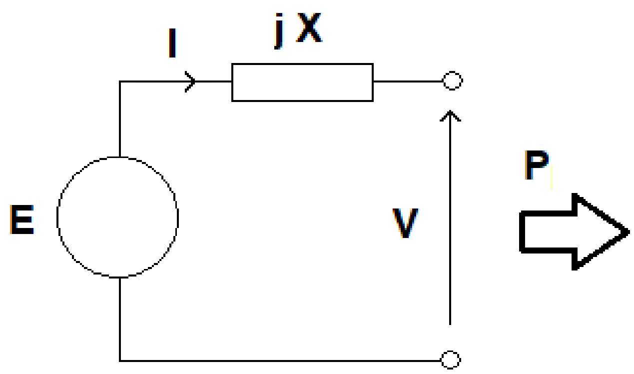
These power flow variations therefore affect this load angle, making it also a dynamic quantity.

Power system dynamics investigates the time-varying nature of δ and its effect on the stability of the system.

Power-Angle Curve

To understand the effects of varying δ, the power-angle relationship needs to be examined.

Consider a simple system (e.g. the Thevenin Equivalent of a power system):



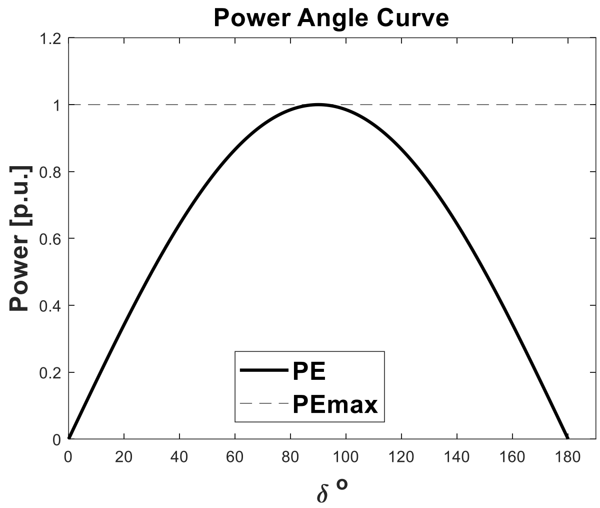
Here, the terminal voltage, V, is taken as the reference, so the supply voltage, E, is at angle δ relative to V, i.e. $\vec{E} = E\angle\delta$, $\vec{I} = I\angle\theta$ and $\vec{V} = V\angle 0°$

Through simple circuit analysis, the power-angle relationship is found to be $P_E = \frac{EV}{X} sin\ \delta$ or,

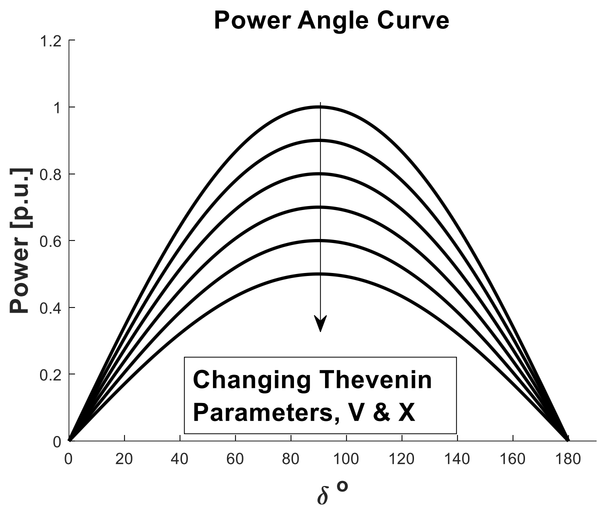
letting $P_{Emax} = \frac{EV}{X}$,

$$P_E = P_{Emax}\ sin\ \delta$$

So a sinusoidal relationship is observed:

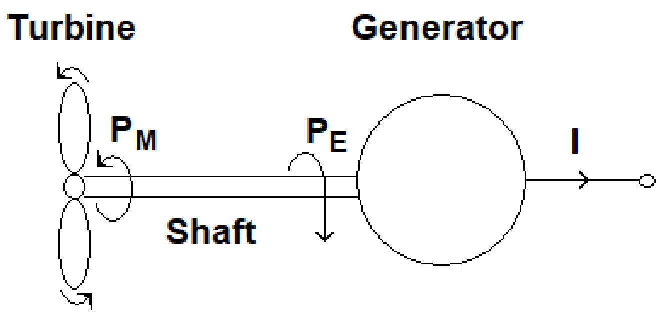


As electrical power varies due to load or generation changes, δ varies along this curve. However, if the configuration of the network changes, e.g. due to a fault, the Thevenin equivalent parameters, V and X change, and so this curve changes in amplitude. So a different curve exists for each electrical configuration of the system:



Generators

This angle, δ, is in fact caused by a relationship between the mechanical and electrical power on the shafts of the generators in the system and so is also called the *rotor angle*.



The mechanical power drives the shaft, trying to accelerate it, while the electrical power draws power out, causing deceleration.

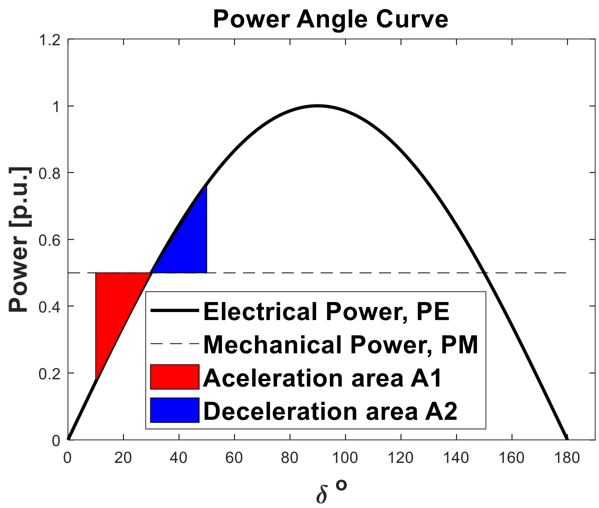
The net power variation is called the *accelerating power*:

$$P_A = P_M - P_E$$

Under normal conditions, the mechanical and electrical power are in equilibrium and $P_A = 0$ but if electrical power changes (i.e. if P_E varies), δ varies, which causes acceleration or deceleration of the shaft.

Acceleration and Deceleration Areas

The balancing act between the mechanical and electrical power can be seen on the power-angle curve by including the mechanical power line:



In area A1, $P_M > P_E$ causing acceleration.

In area A2, $P_E > P_M$ causing deceleration.

The extent to which δ changes, i.e. the range on the x-axis, will determine what the generator can withstand. If the range of δ is such that:

- $A1 > A2$ the machine will tend to accelerate, eventually becoming unstable.
- $A2 > A1$ the machine will tend to decelerate, eventually becoming stable.
- $A1 = A2$ the machine is at the critical point between stability and instability.

The Swing Equation

The dynamic relationship between the accelerating power, P_A, and the acceleration is determined by two things: the electrical frequency, *f*, and the inertia of the generator, *H*.

The electrical frequency describes the rate of change of the electrical power and so is proportional to the acceleration, whereas the inertia describes the heaviness of the generator so is inversely proportional, as a heavy generator will not change speeds readily.

This power-acceleration relationship is known as the *Swing Equation*:

$$a = \frac{\pi f}{H} P_A$$

Dynamic δ

From simple dynamics, acceleration is defined as the rate of change of speed. $a = \frac{d\omega}{dt}$

So the angular speed can be found through integration: $\omega = \int a \, dt$

Similarly, angular speed is the rate of change of angle, so this relates to δ as $\omega = \frac{d\delta}{dt}$

So δ can be found again through integration: $\delta = \int \omega \, dt$

While these relationships may seem straightforward, the integration is often tricky as the acceleration is proportional to the accelerating power, P_A, which is proportional to electrical power, P_E which is both proportional to δ and dynamic.

Numerical Solution

To solve these dynamics, the integrations are approximated numerically by summations of discrete values:

$$\omega = \sum_{\mathrm{n}=0}^{N} a.\Delta t \quad \text{and} \quad \delta = \sum_{n=0}^{N} \omega.\Delta t$$

where Δt is a small time increment, e.g. 1 millisecond, and *N* is the number of these increments from time zero.

This is solved iteratively, with the swing equation:

$$a_n = \frac{\pi f}{H} P_{A\,n}$$

$$\omega_{n+1} = \omega_n + \mathrm{a_n}\Delta t$$

$$\delta_{n+1} = \delta_n + \omega_n \Delta t$$

With initial conditions $P_{A\,0} = 0$, $\omega_0 = 0$ and $\delta = \delta_0$.

Using this approach, δ can be calculated at each time interval and observed graphically. This graph of δ vs time is called the *Swing Curve.*

Swing Curve

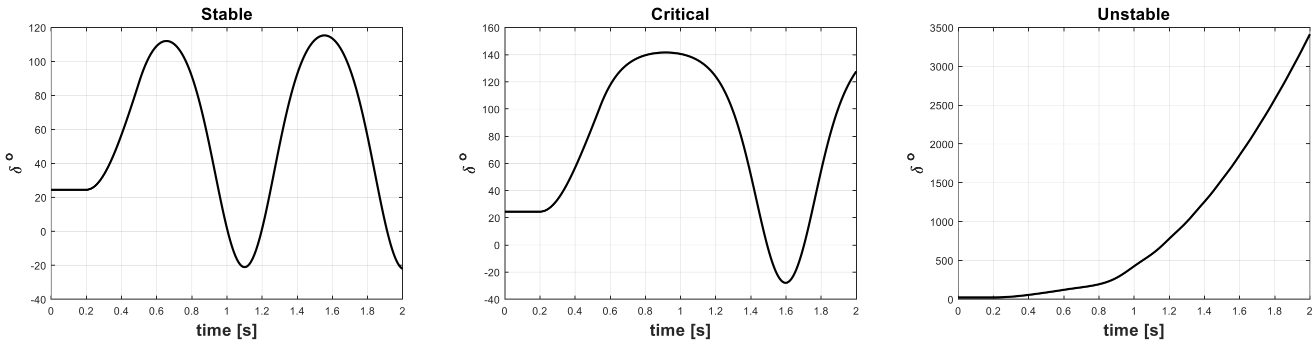
The swing curve is very useful for determining whether the system is stable.

Under stable conditions, δ will eventually stop increasing, whereas in an unstable system, δ continues to increase indefinitely as the generator spins out of control, forcing it to desynchronise.

Somewhere between stable and unstable is the *critical point*, where the system is on the verge of instability.

The swing curve allows the angle δ to be converted to a time value which is useful in selecting appropriate protection settings and measuring stability.

Sample swing curves for each state can be see here:



Transient Stability

It is important in a power system to know the limits of its stability.

This is defined by a maximum δ that the system can withstand before instability occurs.

The most dramatic changes in δ are observed during transient faults.

A transient fault is one which lasts for a short time, either by its nature (e.g. lightning or switching surges) or because of protection systems isolating the fault.

To determine whether a system can withstand a transient and remain stable, the critical point must be identified.

This is defined by the *critical clearing angle*, δ_{CC}, the maximum δ that the system can withstand before the fault *must* be cleared.

The time at which this angle occurs is the *critical clearing time*, t_{CC}, the longest tolerable duration of the transient.

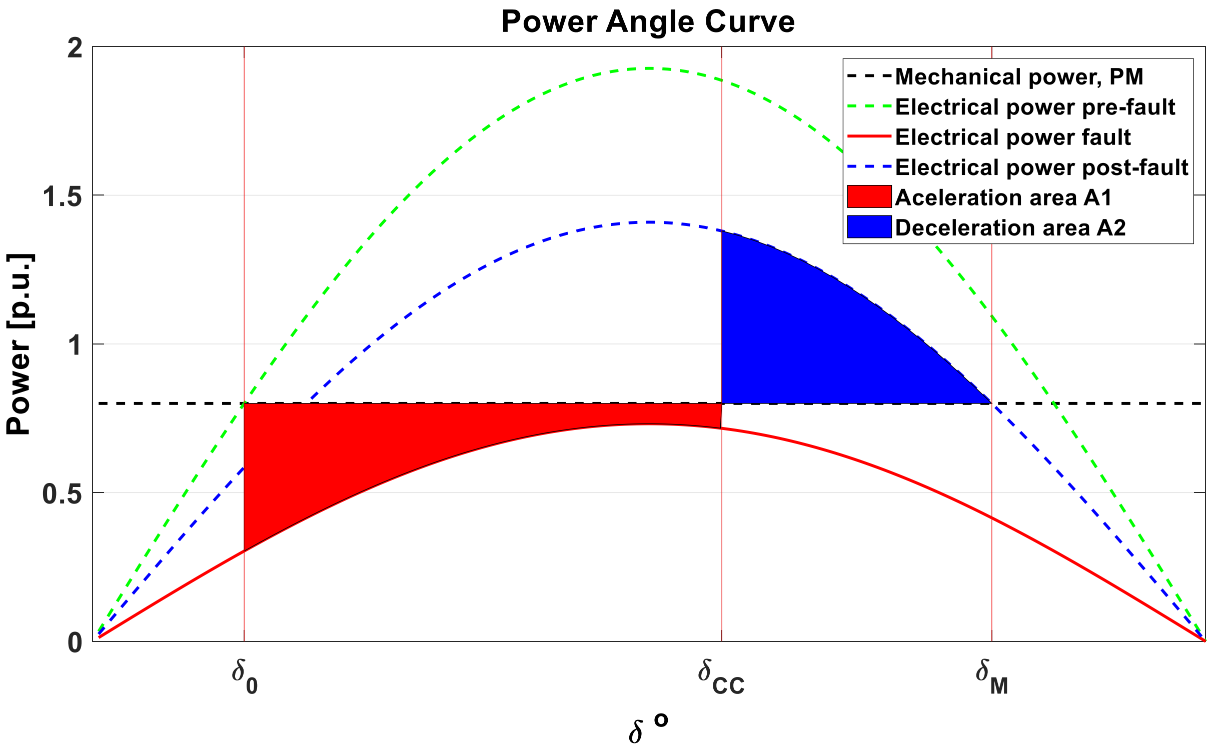
As such, the critical clearing time, t_{CC}, is a measure of stability.

Equal Area Criterion

To determine δ_C, the *critical clearing angle*, the *Equal Area Criterion* is used.

As noted earlier, the critical point between stability and instability occurs when the accelerating area matches the decelerating area, i.e. when $A_1 = A_2$.

However, since the electrical conditions are different before, during and after the fault, three electrical power curves are needed to determine these areas:



Before the fault (pre-fault), the electrical and mechanical powers are in equilibrium and δ is at its initial value, δ_0.

When the fault occurs (fault), the electrical power drops, causing acceleration.

After the fault is cleared (post-fault), electrical power flows through alternative routes (as the faulted area is now isolated) and the shaft decelerates.

The angle which gives equal areas, $A_1$and A_2, is the critical clearing angle, δ_{CC}.

To find δ_C, these areas can be calculated by integration:

$$A_1 = \int_{\delta_0}^{\delta_{CC}} (P_M - P_{EF})\, d\delta$$

$$A_2 = \int_{\delta_{CC}}^{\delta_M} (P_{EPF} - P_M)\, d\delta$$

where P_{EF} is the electrical power flow under fault conditions, P_{EPF} is the electrical power flow post-fault and δ_M is the maximum possible value of δ post-fault.

The critical clearing time, t_{CC}, is the time at which this angle occurs and can be found from the swing curve.

Protection

Once t_{CC} is known for different fault scenarios, appropriate protection can be set within the stability limits of the system.

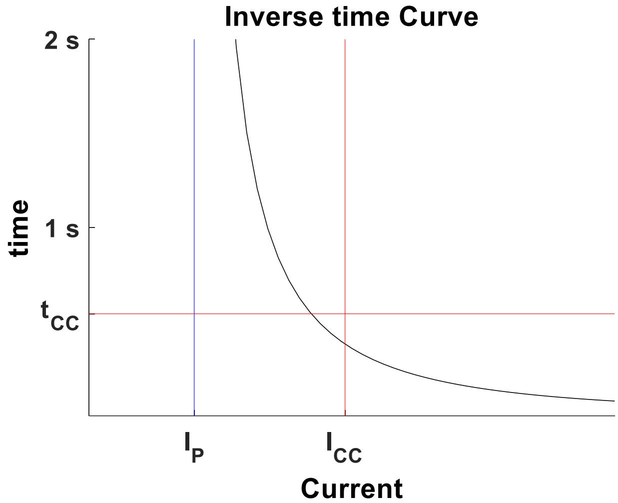
There are a wide range of protective devices such as directional, line differential, earth fault and overcurrent relays, but all types use time settings to set behaviour.

Any fault should be cleared quickly, but allowing some delay means that protection can be coordinated, e.g. allowing time for the closest relay to trip before an upstream one means cutting off less of the network.

Delays are also essential to prevent nuisance trips resulting from equipment inrush currents.

The severity of the fault and the stability of the system will determine how long the fault can be tolerated and the time setting must be within this limit.

This can be captured in a time-current curve and is a common design for overcurrent relays.



This curve has an inverse relationship between time and current, e.g. in the IEC Standard (common in Europe), which has the form:

$$t = \frac{K}{\left(\frac{I}{I_P}\right)^p - 1}$$

Where K and p are parameters of the relay, I_P is the relay pickup current.

At this critical point, the tripping time should be less than t_{CC}.