



Machine Intelligence Example of Particle Swarm Optimization



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MACHINE INTELLIGENCE

Example of Particle Swarm Optimization

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A Solved Example of PSO

Q:- Solve the constrained optimization problem:

$$\text{Min } f(x) = x \sin 10\pi x + 1.0$$

where $-1 \leq x \leq 2$.

Solⁿ: Let the position and velocity of i^{th} particle in the search space are represented as vectors:

$$x_i = (x_{i1}, x_{i2}, \dots, x_{iD})$$

$$v_i = (v_{i1}, v_{i2}, \dots, v_{iD}) \text{ resp.}$$

(D is the no. of variables in the function to be minimized.)

PSO Example contd....

Velocity & position of a particle is updated by

$$v_{ij}^{t+1} = w v_{ij}^t + c_1 \text{rand}_1(p_{ij}^t - x_{ij}^t) + c_2 \text{rand}_2(p_{gj}^t - x_{ij}^t)$$

$$w = \frac{(w_{\min} - w_{\max})(i_{\text{tu}} - 1)}{(i_{\text{temax}} - 1)} + w_{\max}$$

PSO Example contd....

$$\& \quad x_{ij}^{t+1} = x_{ij}^t + v_{ij}^{t+1}$$

where $i = \{1, 2, \dots, N\}$

$j = \{1, 2, \dots, D\}$

Step 1: Initialize PSO control parameters

Assumption:

particles = 5

Let $C_1 = 2, C_2 = 2$

$\omega_{max} = 0.9, \omega_{min} = 0.2$

PSO Example contd....

Given # variables = 1 , upper limit = 2,
lower limit = -1 , iter = 1, max iter = 100.

Step 2: Initial Generation of Population & Velocity

Generate random nos. for one variable
 $x_{ij}(x_{i1})$ with upper & lower limit, where i
is the no. of particles, j is the no. of
variables.
 $i=1,2,3,4,5$, $j=1$.

Velocity $v_{ij}(v_{i1})$ is also generated
randomly within limits.

PSO Example contd....

$$X_{i1} = \begin{cases} -1.00000 & \dots \dots x_{11} \\ 2.00000 & \dots \dots x_{21} \\ -0.2671582 & \dots \dots x_{31} \\ 1.28415387 & \dots \dots x_{41} \\ 0.188660 & \dots \dots x_{51} \end{cases}$$

$$V_{i1} = \begin{cases} 3.94634 & \dots \dots v_{11} \\ 4.00000 & \dots \dots v_{21} \\ 1.67851605 & \dots \dots v_{31} \\ -4.0000 & \dots \dots v_{41} \\ 2.7102747 & \dots \dots v_{51} \end{cases}$$

PSO Example contd....

Step 3: Evaluating fitness of the population.

Fitness value is computed by

$$\text{Fitness} = \begin{cases} \frac{1}{\text{Obj} + 1} & \text{when } \text{Obj} \geq 0 \\ 1 + |\text{Obj}| & \text{when } \text{Obj} < 0. \end{cases}$$

$$\text{Obj} = x \sin 10\pi x + 1.0$$

$$= \begin{cases} 1.00000 \\ 1.00000 \\ 1.2294 \\ 1.615081 \\ 0.934201 \end{cases}$$

PSO Example contd....

$$\text{Fitness} = \left\{ \begin{array}{l} 0.500025001 \\ 0.500025 \\ 0.44855 \\ 0.382397 \\ 0.51700 \end{array} \right.$$

Step 4:- Memorizes g_{best} value

g_{best} = Corresponding to $x_{51} = 0.1886$

g_{fit} = 0.51700

PSO Example contd....

Step 5 :- Set iteration = 1.

Step 6 :- Updating particle's velocity.

Weighting factor: for 1st iteration $w = [0.9]$

The corresponding velocity for the weighting factor

The inertial weighting factor w

$$w = \frac{(w_{\min} - w_{\max})(i_{\text{te}} - 1)}{(i_{\text{te}}_{\max} - 1)} + w_{\max}$$

$$= \frac{(0.2 - 0.9)(1 - 1)}{100 - 1} + 0.9 = 0.9$$

PSO Example contd....

Let $r_{\text{rand}_1} = 0.5498602$, $r_{\text{rand}_2} = 0.144954$.

$$\begin{aligned}\therefore V_{ii} &= (0.9 \times 3.9463) + 2(0.5498602) \times (1 - (-1)) \\ &\quad + 2(0.144954) + (0.1887 - (-1)) \\ &= 3.8962.\end{aligned}$$

$$\begin{aligned}V_{iy_1} \\ V_{21} &= (0.9 \times 4) + 2(0.5498602)(2 - 2) + \\ &\quad 2(0.144954)(0.1887 - 2) \\ &= 3.074\end{aligned}$$

PSO Example contd....

$$V_{31} = 1.6427$$

$$V_{41} = -3.9174$$

$$V_{51} = 2.43924$$

$$\therefore V = \begin{cases} 3.8962 \\ 3.074 \\ 1.6427 \\ -3.9174 \\ 2.43924 \end{cases}$$

In case, the velocity violates the limits,
set the velocity value = max. value.

PSO Example contd....

Step 7: Modification of Particle Position.

$$\therefore X_{11} = -1 + 3.8963 \\ = 2.8963$$

It violates the upper limit $\therefore \hat{x}_{11} = 2$

My, $X_{21} = 2 + 3.074 = 3.34$
 \therefore Set $X_{21} = 2$

My, $X_{31} = 1.711403, X_{41} = -1.000, X_{51} = 2.000$

PSO Example contd....

Step 8:- Evaluating of fitness of the modified position of particle.

$$Obj = \begin{cases} 1 \\ 0.39989 \\ 0.999 \\ 0.9999 \end{cases}$$

$$Fitness = \begin{cases} 0.5 \\ 0.5 \\ 0.714 \\ 0.5002 \\ 0.5002 \end{cases}$$

PSO Example contd....

Step 9:- Memorize g_{best} & p_{best} .

$$g_{best} = 1.7114$$

$$g_{fit} = 0.714$$

$$p_{best} = \begin{cases} 2.000 \\ 2.000 \\ 1.7114 \\ -1.000 \\ 2.000 \end{cases}$$

Step 10:- Memorize the best result so far
& increment $\underline{\underline{ite}} = \underline{\underline{ite}} + 1$.

PSO Example contd....

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PSO Example contd....

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THANK YOU

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