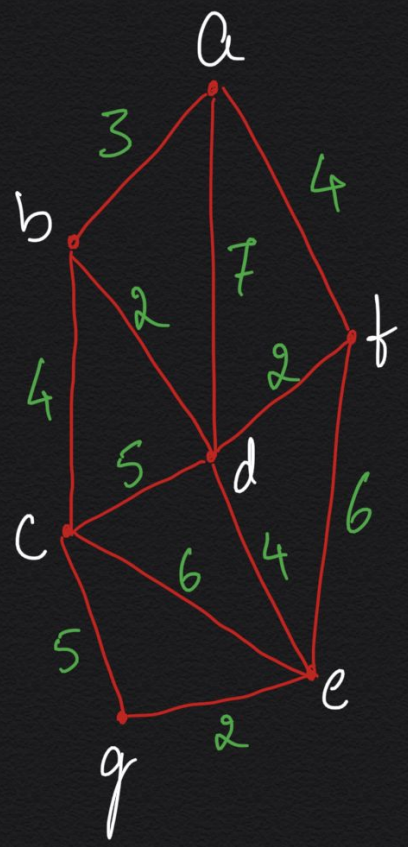
Design and Analysis of Algorithms

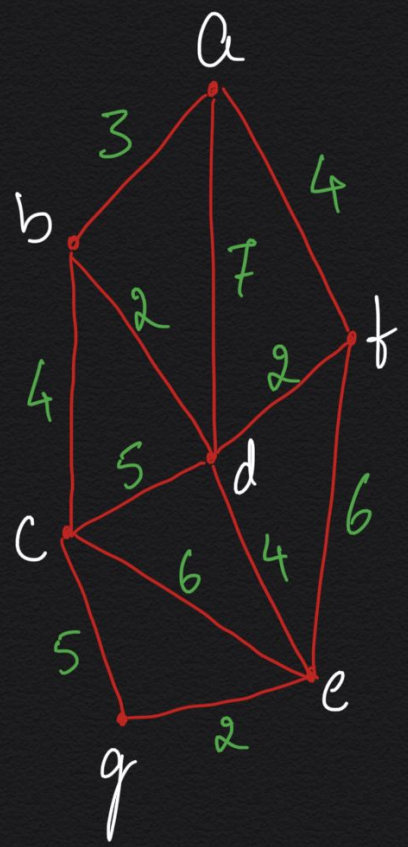
Greedy Technique

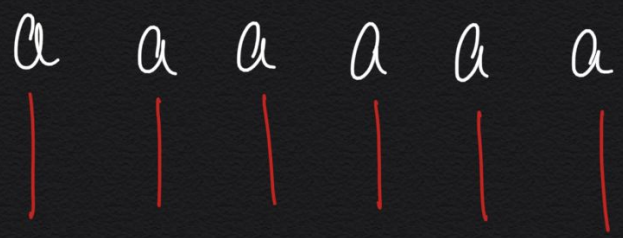
Mr. Channa Bankapur

Single-source Shortest-paths Problem

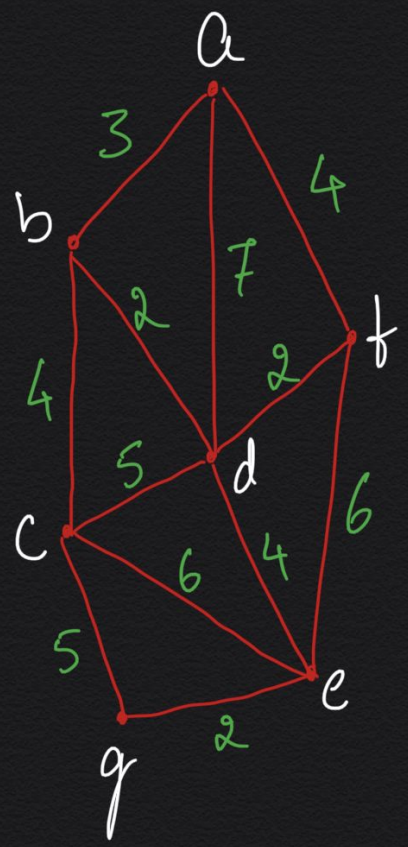


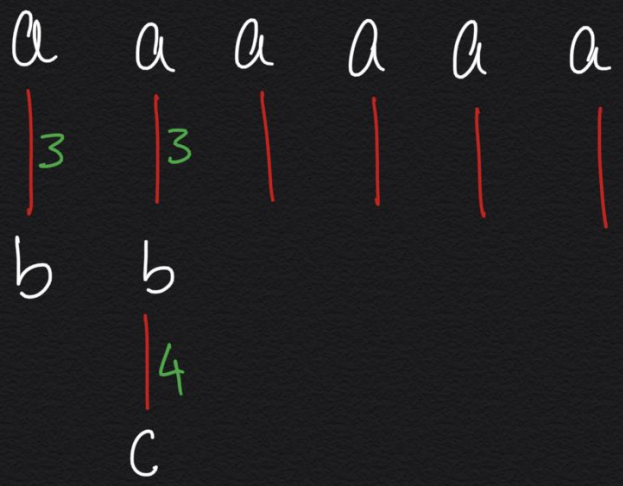
Single-source Shortest-paths Problem



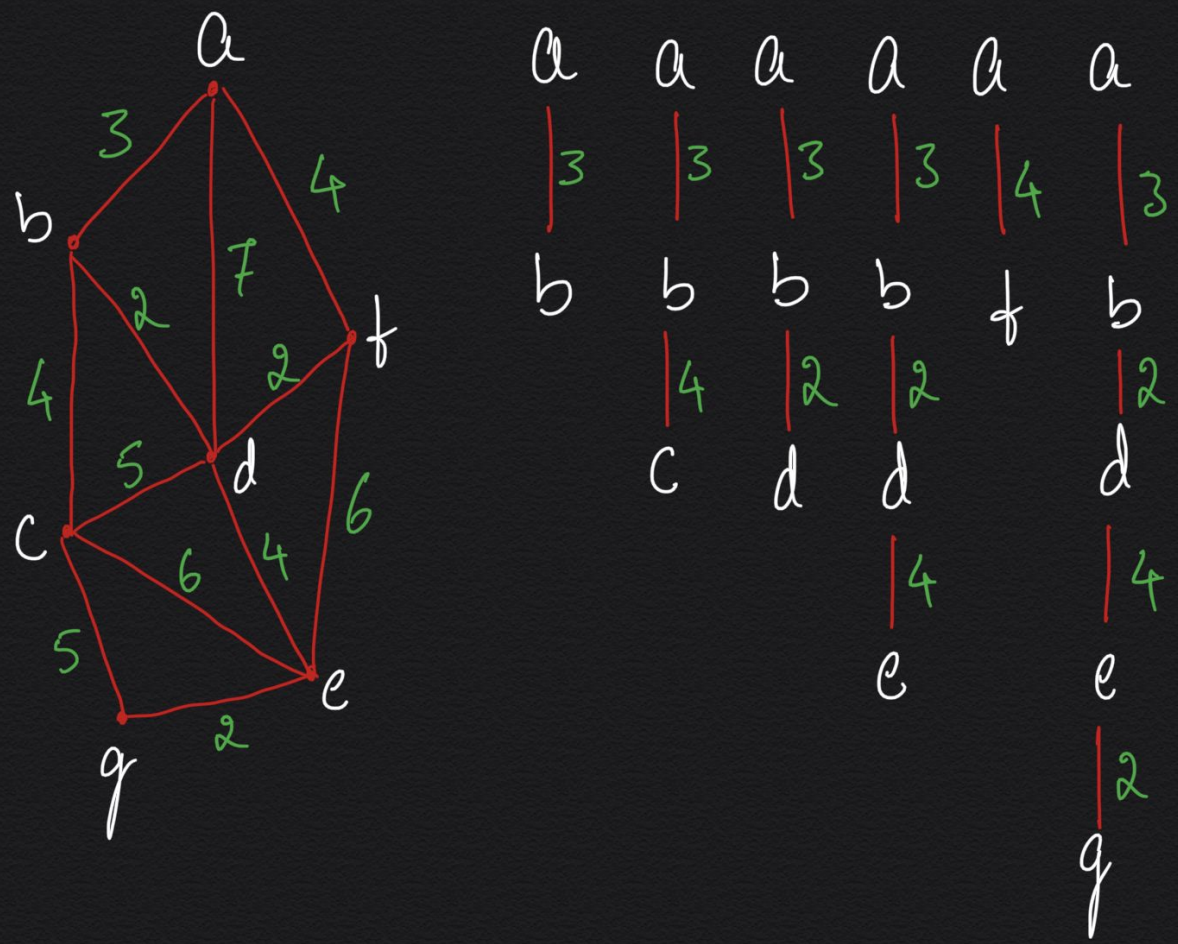


Single-source Shortest-paths Problem

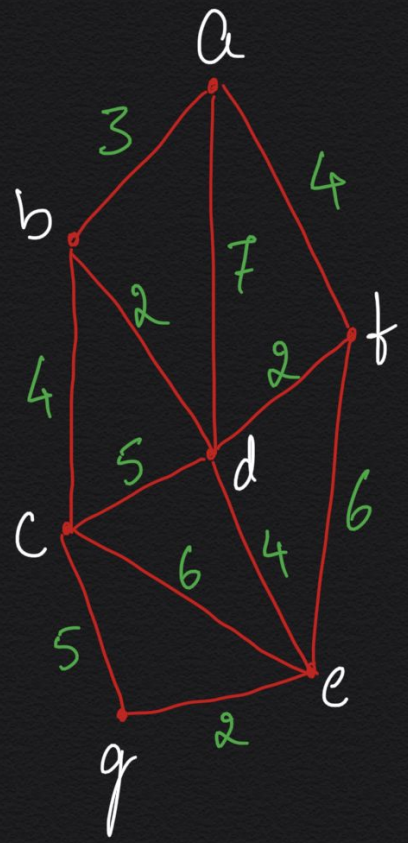


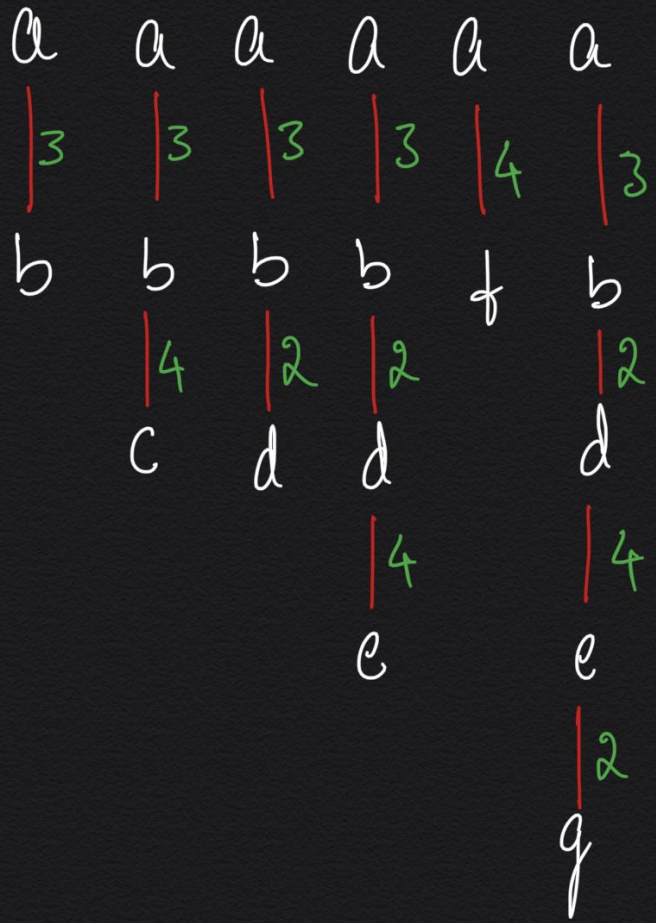


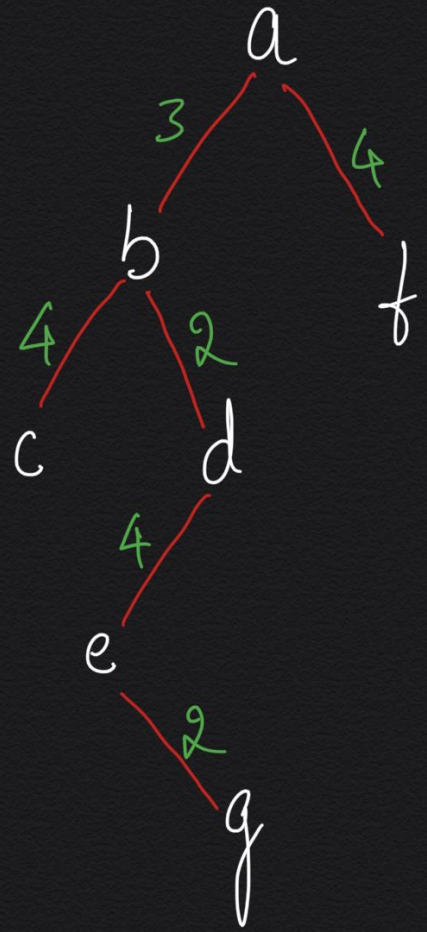
Single-source Shortest-paths Problem

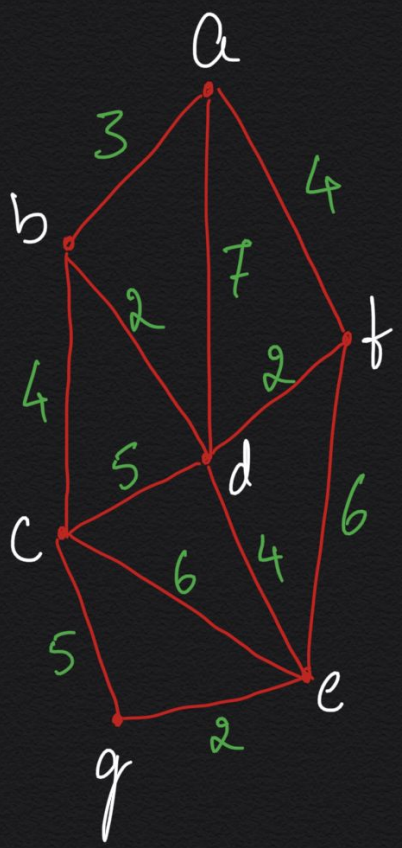


Single-source Shortest-paths Problem





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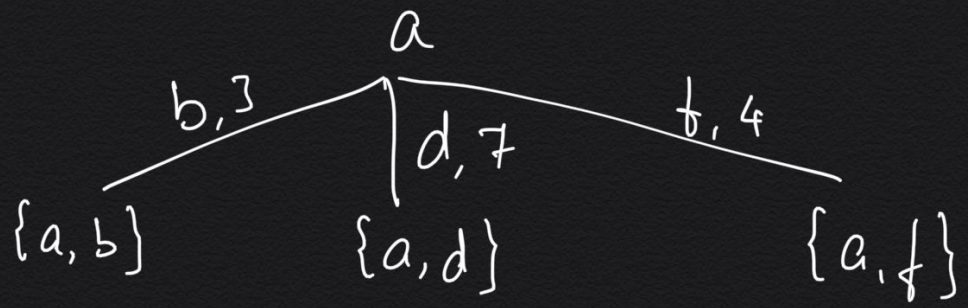
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f, 4
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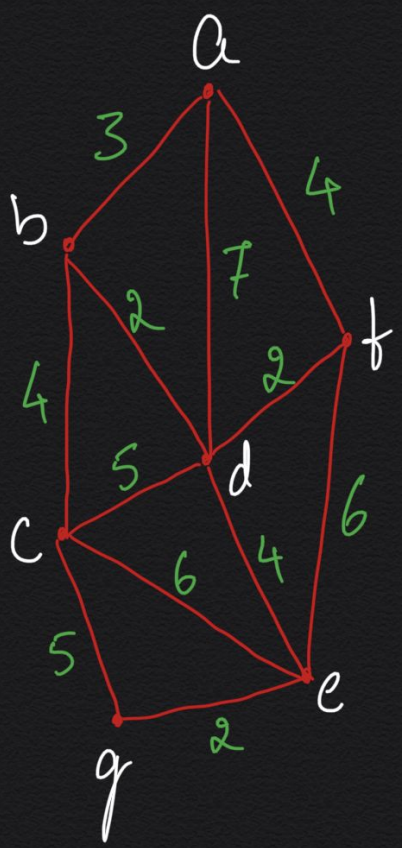


a

b,3 | d,7 | f,4

{a,b} {a,d} {a,f}

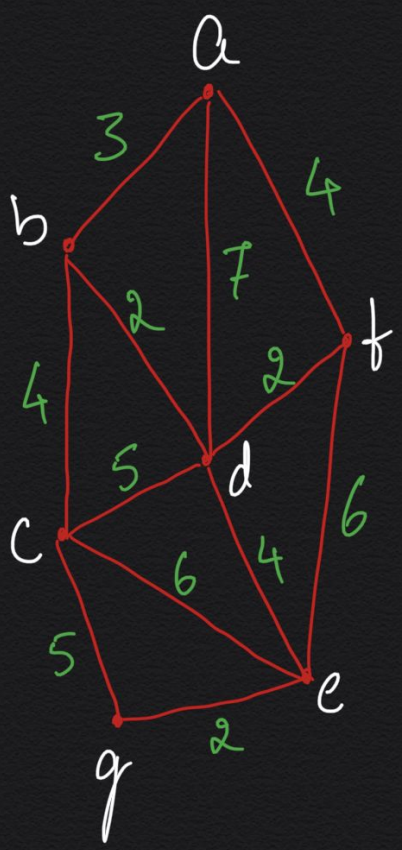
{a,b}: c,7 | d,5 | f,4

{a,b,c} {a,b,d} {a,b,f}

{a,d}: b | c | e

{a,f}: b,3 | d,6 | e,10

{a,b,f} {a,d,f} {a,e,f}



a

b,3 d,7 f,4

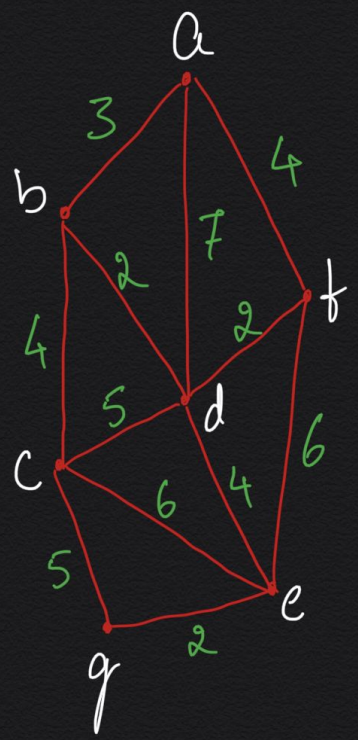
{a,b} {a,d} {a,f}

c,7 d,5 f,4 b c e f b,3 d,6 e,10

{a,b,c} {a,b,d} {a,b,f} {a,b,f} {a,d,f} {a,e,f}

c,7 d,5 e,10

abcf abdf abef



a

b,3 | d,7 | f,4

{a,b} | {a,d} | {a,f}

c,7 | d,5 | f,4 — b | c | e — b,3 | d,6 | e,10

{a,b,c} | {a,b,d} | {a,b,f} — {a,d,f} | {a,e,f}

c,7 | d,5 | e,10

abcf | abdf | abef

c,7 | e,9

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e,9 | g,12 — c | g

abcdef | abcdfg

g,11 | e

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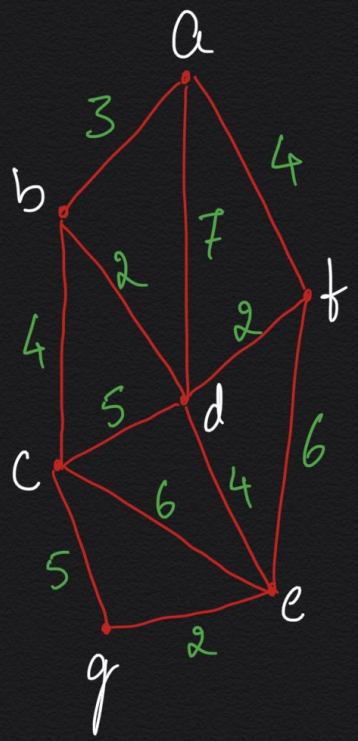

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c,7 d,5 e,10

abcf abdf abef

c,7 e,9

abcdf abdef

e,9 g,12 c g

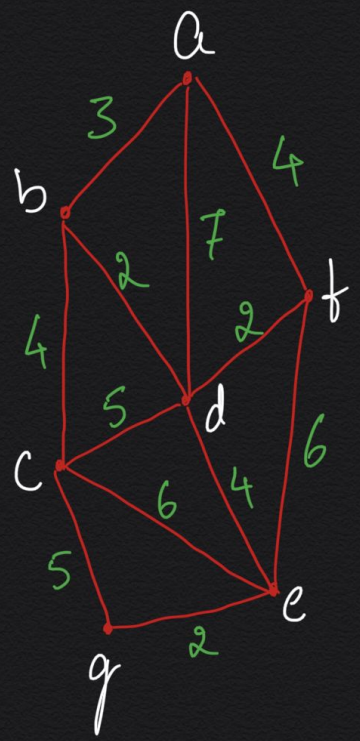
abcdef abcdfg

g,11 e

abcdefg

a

b,3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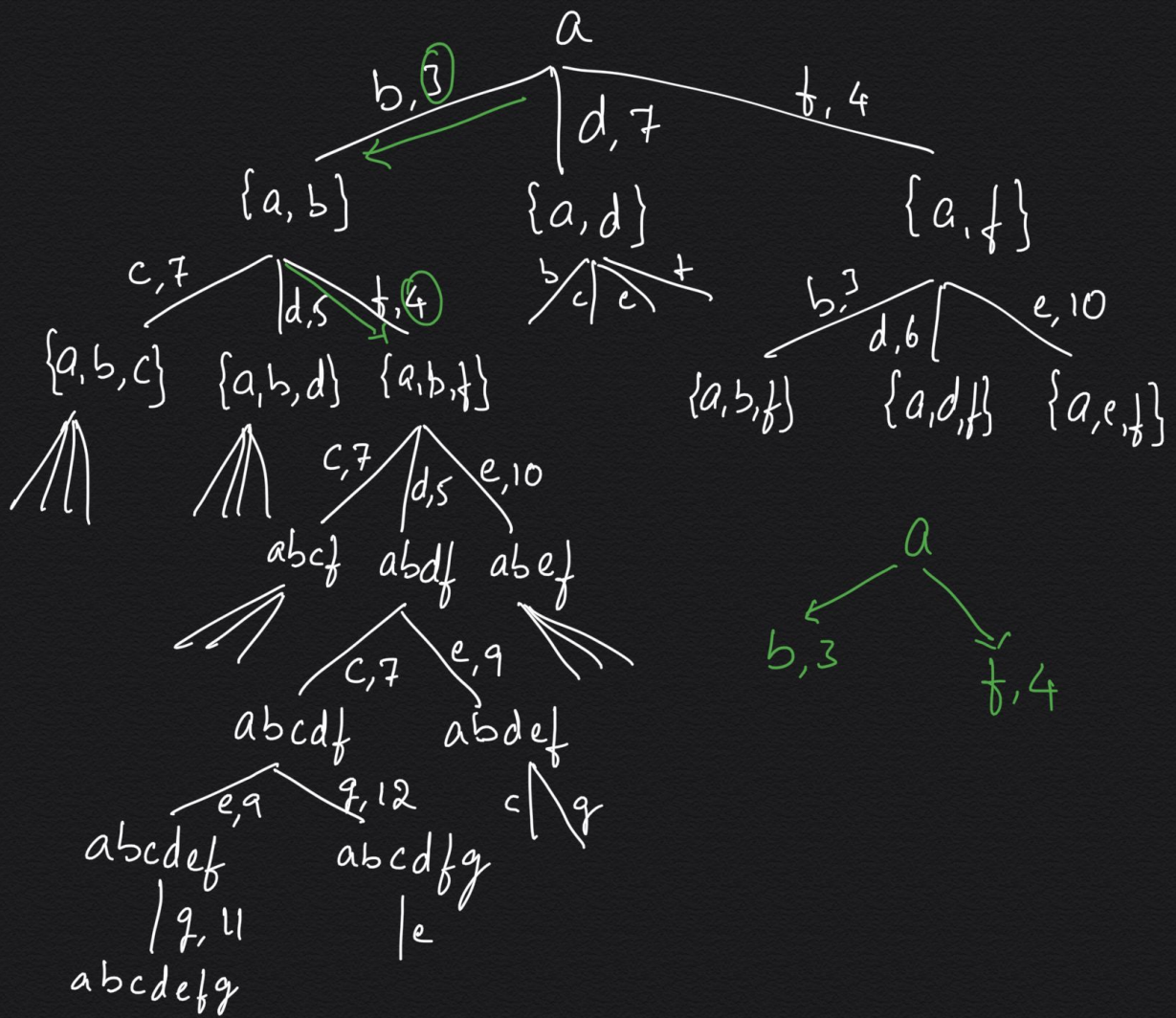
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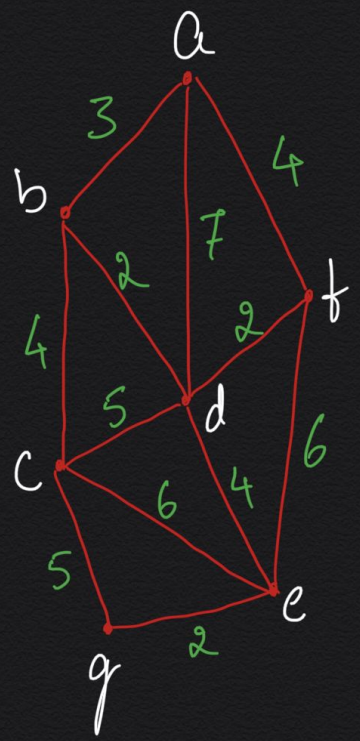

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c,7 d,5 e,10

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a

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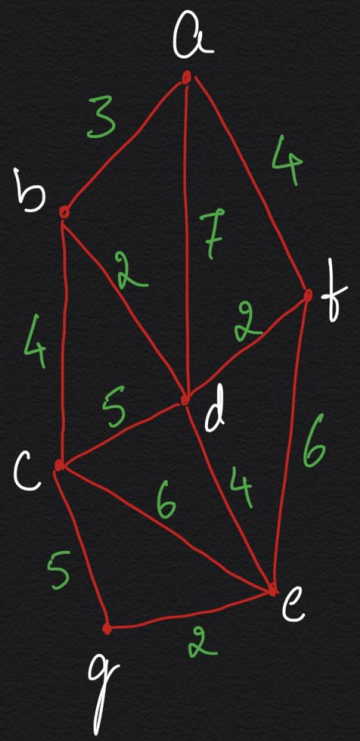
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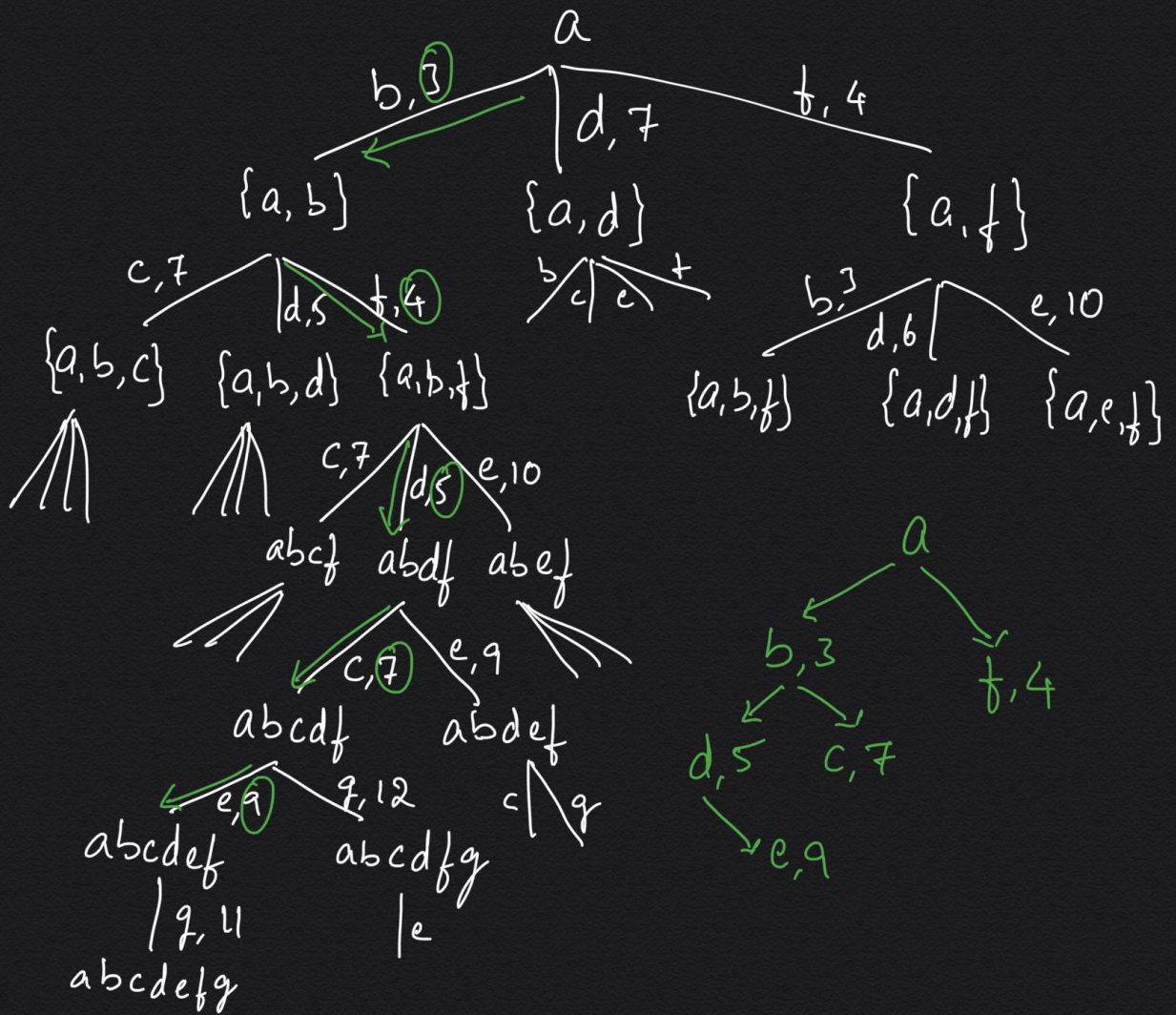
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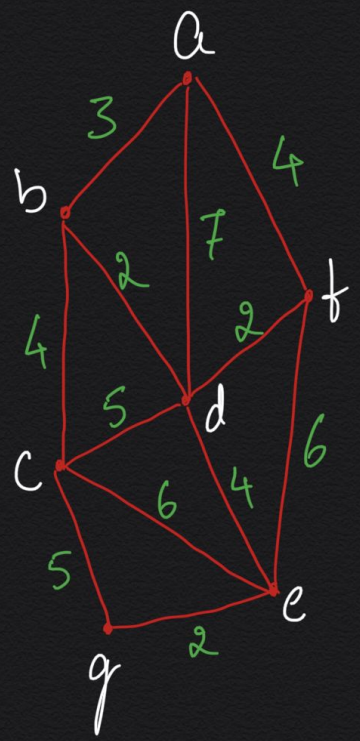
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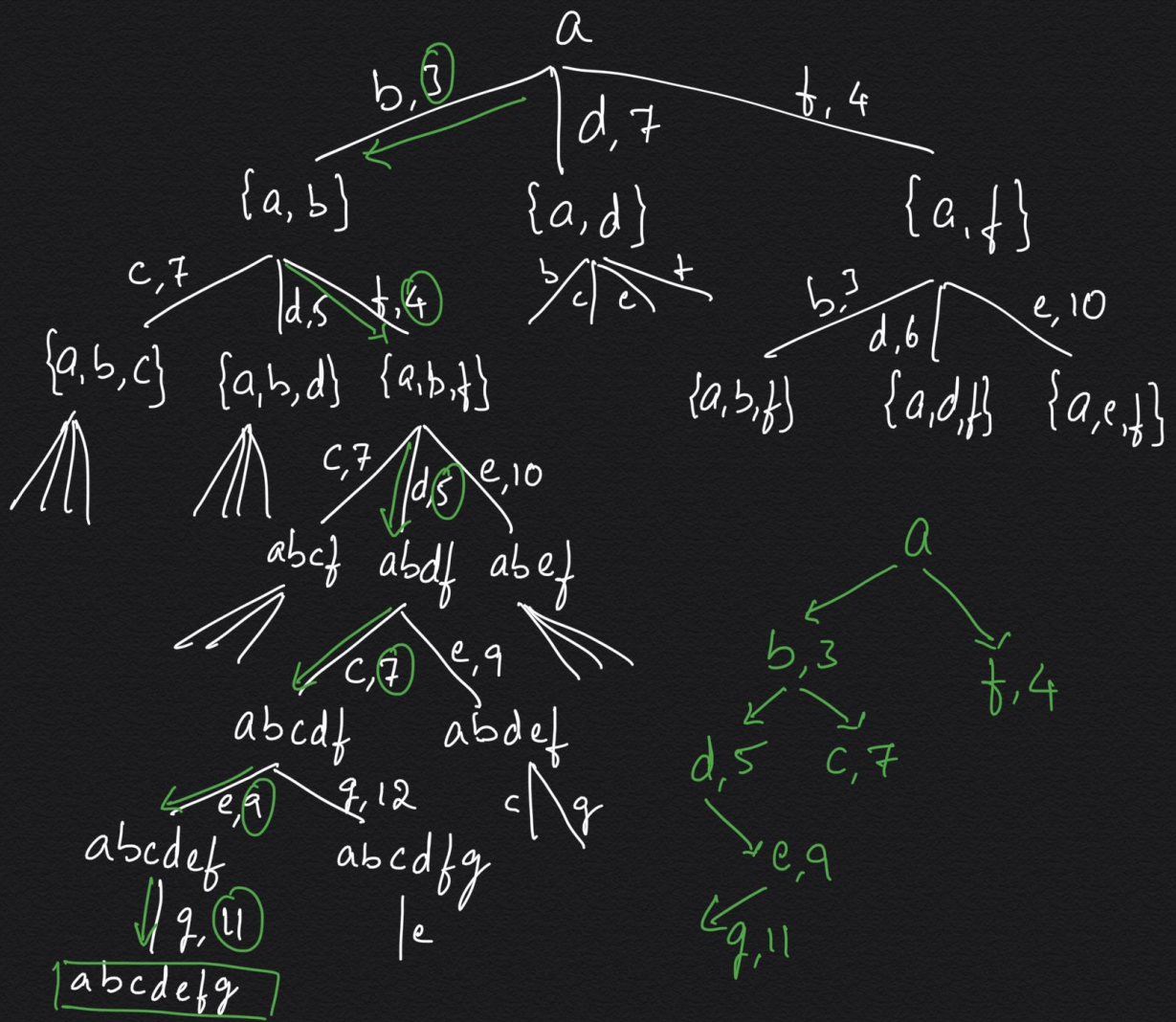
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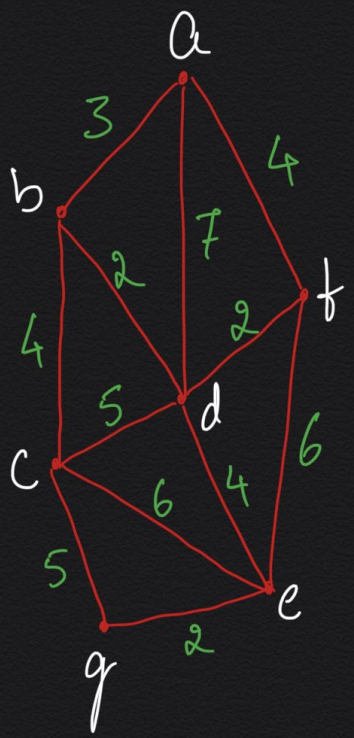
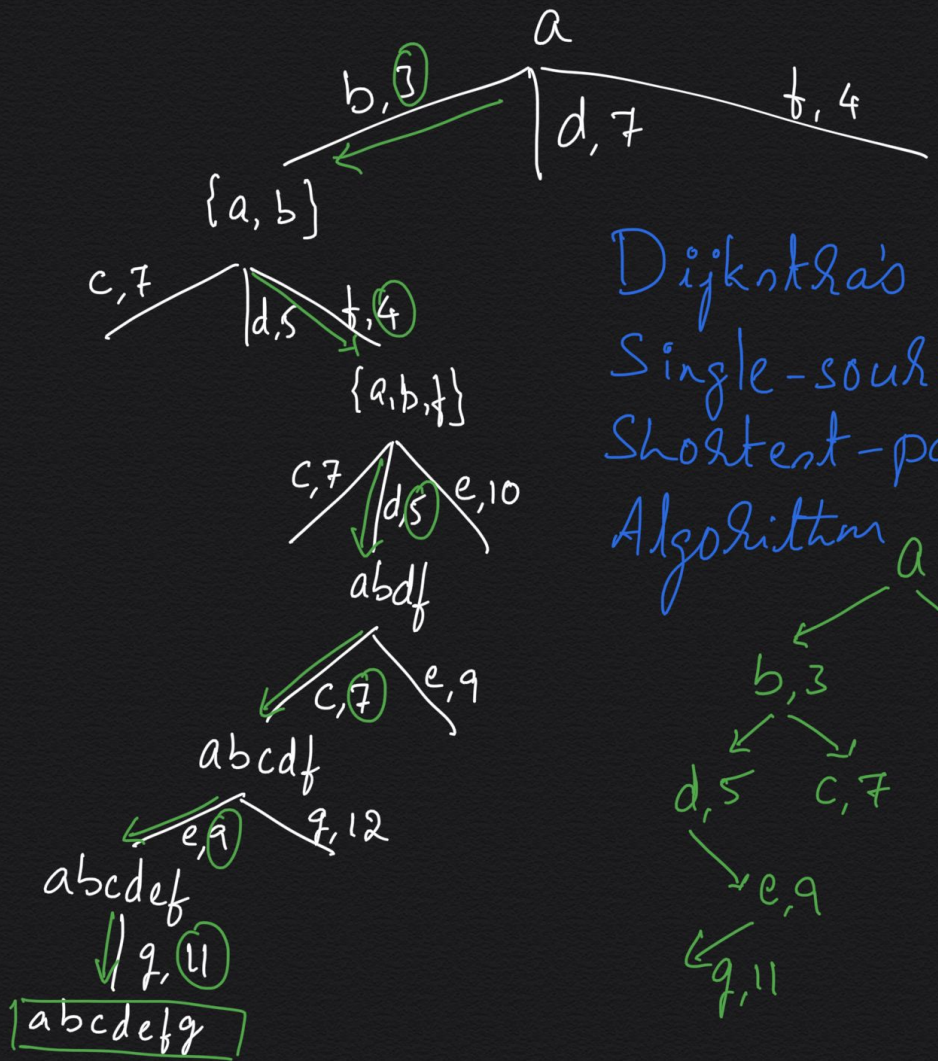
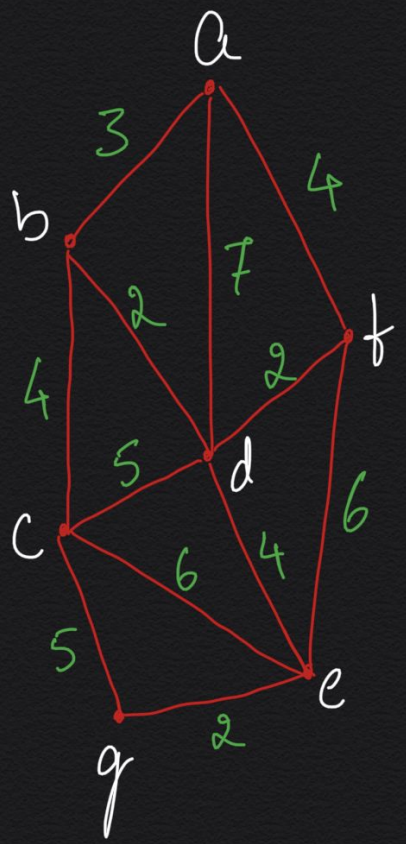




Dijkstra's Single-source Shortest-paths Algorithm



TREE VERTICES

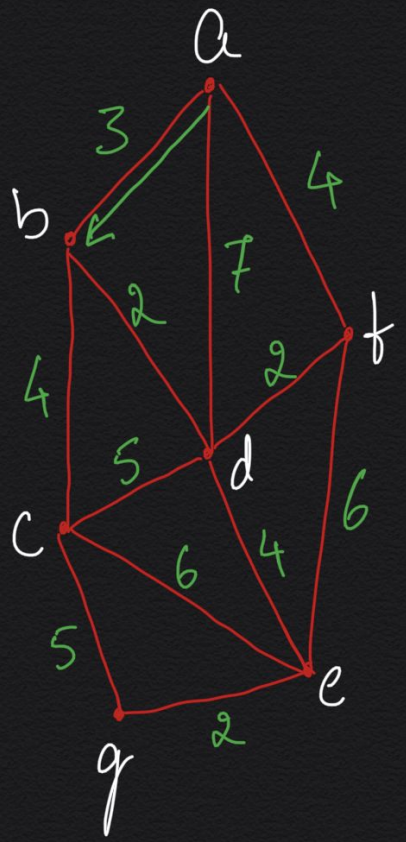
a

FRINGE VERTICES

b(a,3), d(a,7), f(a,4)

TREE

a



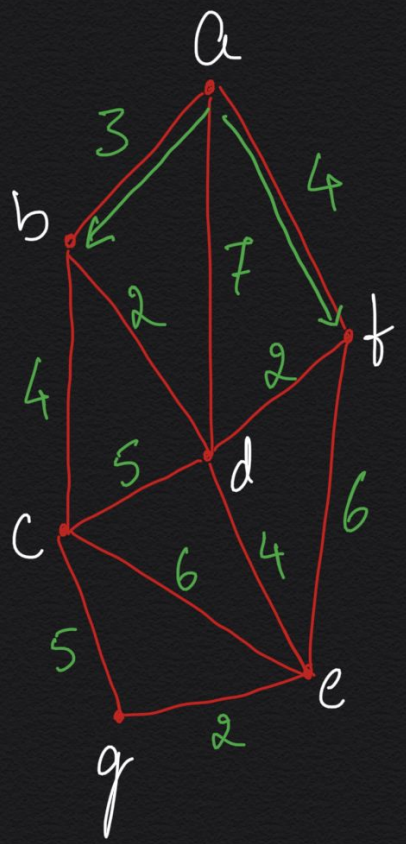
TREE VERTICES	FRINGE VERTICES
a(-,0)	b(a,3), d(a,7), f(a,4)
b(a,3)	c(b,7), d(b,5), f(a,4)

TREE

a

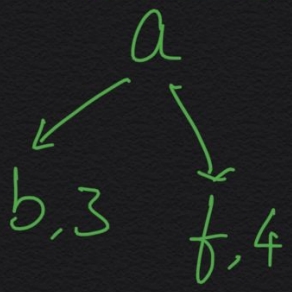
↙

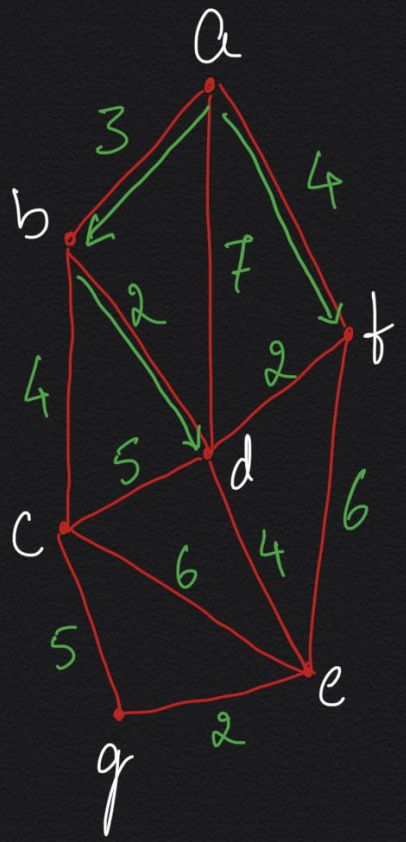
b,3



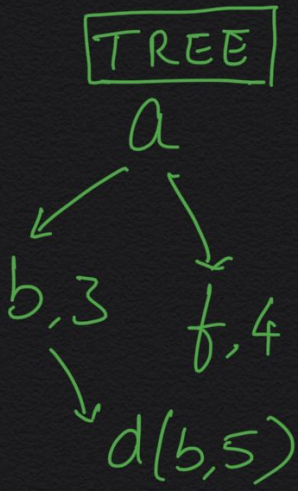
TREE VERTICES	FRINGE VERTICES
a(–,0)	b(a,3), d(a,7), f(a,4)
b(a,3)	c(b,7), d(b,5), f(a,4)
f(a,4)	c(b,7), d(b,5), e(f,10)

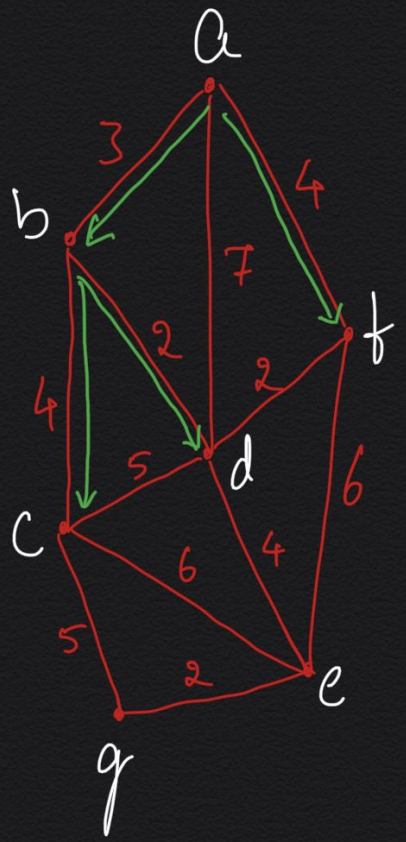
TREE



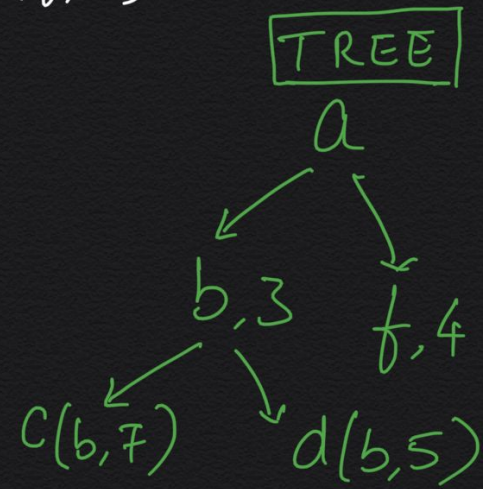


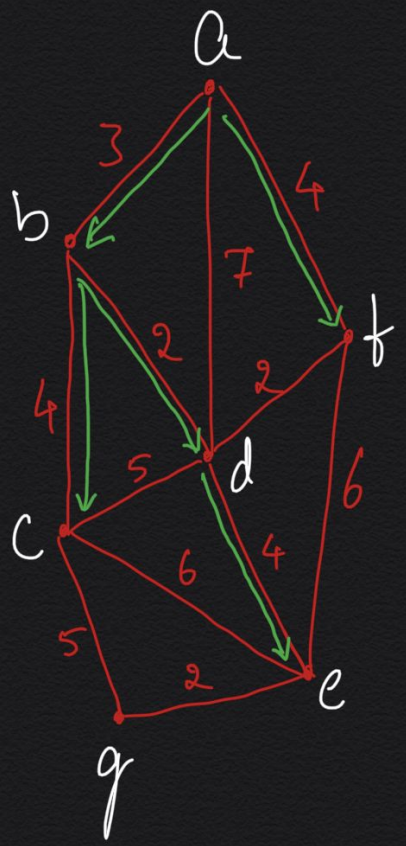
TREE VERTICES	FRINGE VERTICES
a(–,0)	b(a,3), d(a,7), f(a,4)
b(a,3)	c(b,7), d(b,5), f(a,4)
f(a,4)	c(b,7), d(b,5), e(f,10)
d(b,5)	c(b,7), e(d,9)





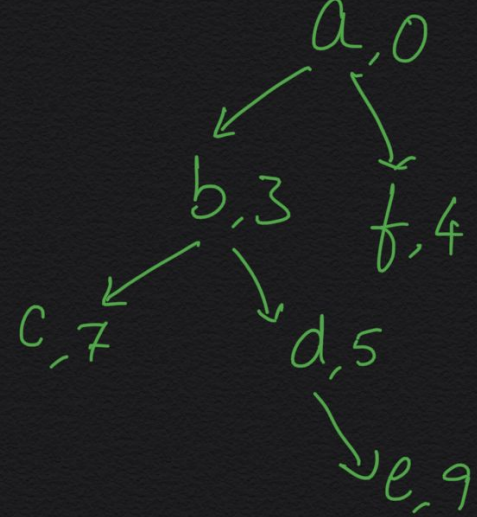
TREE VERTICES	FRINGE VERTICES
a(–,0)	b(a,3), d(a,7), f(a,4)
b(a,3)	c(b,7), d(b,5), f(a,4)
f(a,4)	c(b,7), d(b,5), e(f,10)
d(b,5)	c(b,7), e(d,9)
c(b,7)	e(d,9), g(c,12)

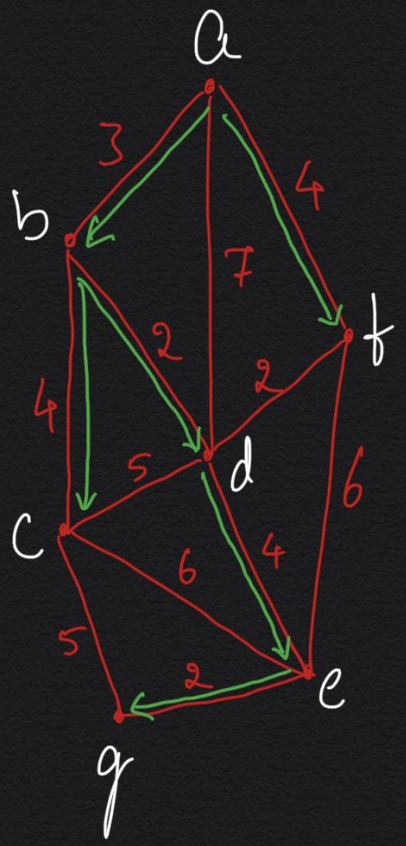




TREE VERTICES	FRINGE VERTICES
a(-,0)	b(a,3), d(a,7), f(a,4)
b(a,3)	c(b,7), d(b,5), f(a,4)
f(a,4)	c(b,7), d(b,5), e(f,10)
d(b,5)	c(b,7), e(d,9)
c(b,7)	e(d,9), g(c,12)
e(d,9)	g(e,11)

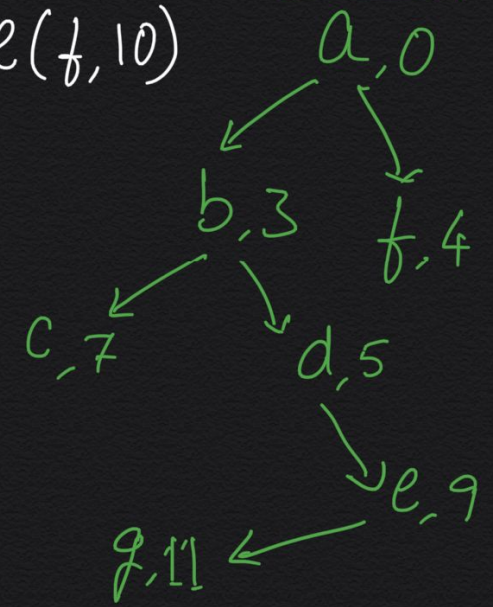
TREE

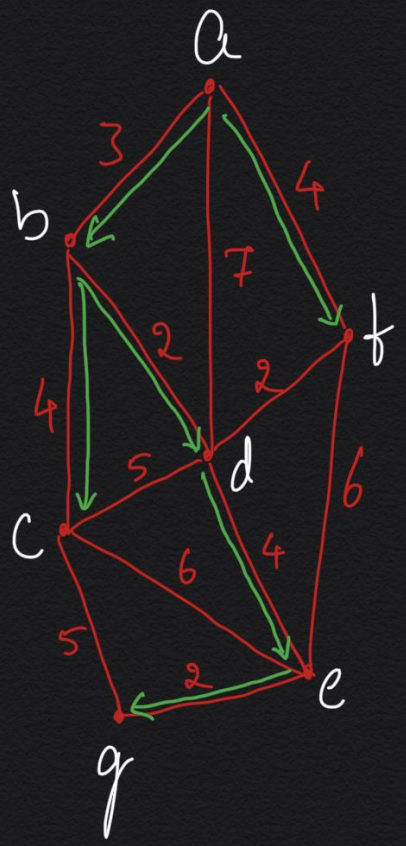




TREE VERTICES	FRINGE VERTICES
a(–,0)	b(a,3), d(a,7), f(a,4)
b(a,3)	c(b,7), d(b,5), f(a,4)
f(a,4)	c(b,7), d(b,5), e(f,10)
d(b,5)	c(b,7), e(d,9)
c(b,7)	e(d,9), g(c,12)
e(d,9)	g(e,11)
g(e,11)	—

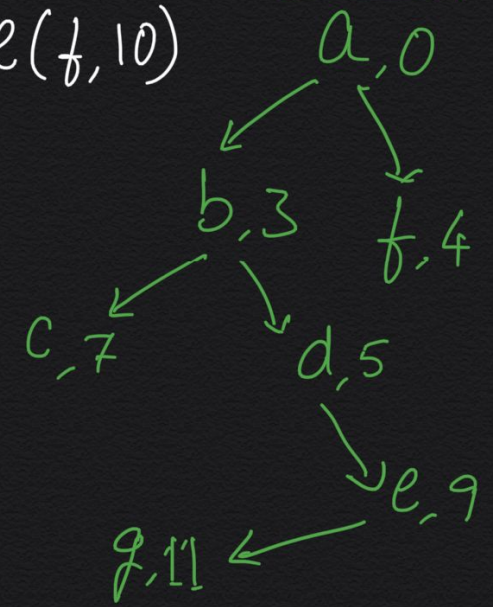
TREE





TREE VERTICES	FRINGE VERTICES
a(–,0)	b(a,3), d(a,7), f(a,4)
b(a,3)	c(b,7), d(b,5), f(a,4)
f(a,4)	c(b,7), d(b,5), e(f,10)
d(b,5)	c(b,7), e(d,9)
c(b,7)	e(d,9), g(c,12)
e(d,9)	g(e,11)
g(e,11)	—

TREE



ALGORITHM *Dijkstra*(G, s)

```
//Dijkstra's algorithm for single-source shortest paths
//Input: A weighted connected graph G = ⟨V, E⟩ with nonnegative
//       weights and its vertex s
//Output: The length d_v of a shortest path from s to v
//       and its penultimate vertex p_v for every vertex v in V
Initialize(Q)  //initialize priority queue to empty
for every vertex v in V
    d_v ← ∞;  p_v ← null
    Insert(Q, v, d_v)  //initialize vertex priority in the priority queue
d_s ← 0;  Decrease(Q, s, d_s)  //update priority of s with d_s
V_T ← ∅
for i ← 0 to |V| − 1 do
    u* ← DeleteMin(Q)  //delete the minimum priority element
    V_T ← V_T ∪ {u*}
    for every vertex u in V − V_T that is adjacent to u* do
        if d_u* + w(u*, u) < d_u
            d_u ← d_u* + w(u*, u);  p_u ← u*
            Decrease(Q, u, d_u)
```

Time Complexity of the Dijkstra's Algorithm

Let n = |V| and m = |E|

Graph is represented as **adjacency lists**
Priority Queue is implemented as a **min-heap**
T(n) = O(?)

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Graph is represented as adjacency lists
Priority Queue is implemented as a min-heap
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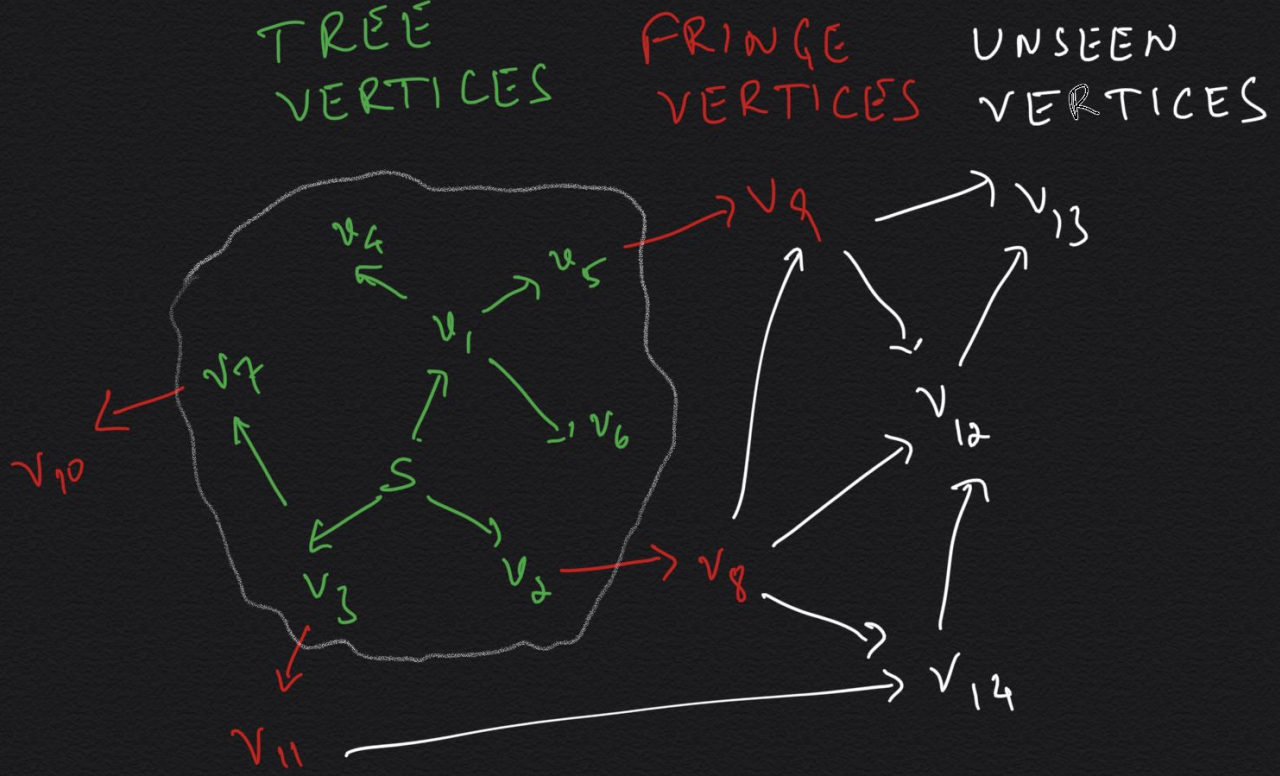
Graph is represented as weight matrix
Priority Queue is implemented as an unordered array
$T(n) = O(n^2)$

Dijkstra's Algorithm:

- First, it finds the shortest path from the source to a vertex nearest to it, then to the second nearest, and so on.
- The algorithm finds the shortest paths to the graph's vertices in order of their shortest distance from the source vertex.
- All the vertices including the source, and the edges of the shortest paths leading to them from the source form a tree, which is a subgraph of the given graph.
- The algorithm is applicable to undirected and directed graphs with **nonnegative weights** only.

Dijkstra's Algorithm:

- The set of vertices not in ***Ti***, which are adjacent to the vertices in ***Ti*** can be referred to as "fringe vertices"; they are the candidates from which Dijkstra's algorithm selects the next vertex nearest to the source.
- To identify the **i^{th}** nearest vertex, the algorithm computes, for every fringe vertex u, the shortest distance from the source through an adjacent vertex in ***Ti*** and then selects the vertex with the smallest such distances. (This selection is the **greedy** step!)
- Why is this **locally optimal** choice is part of a **globally optimal** solution?

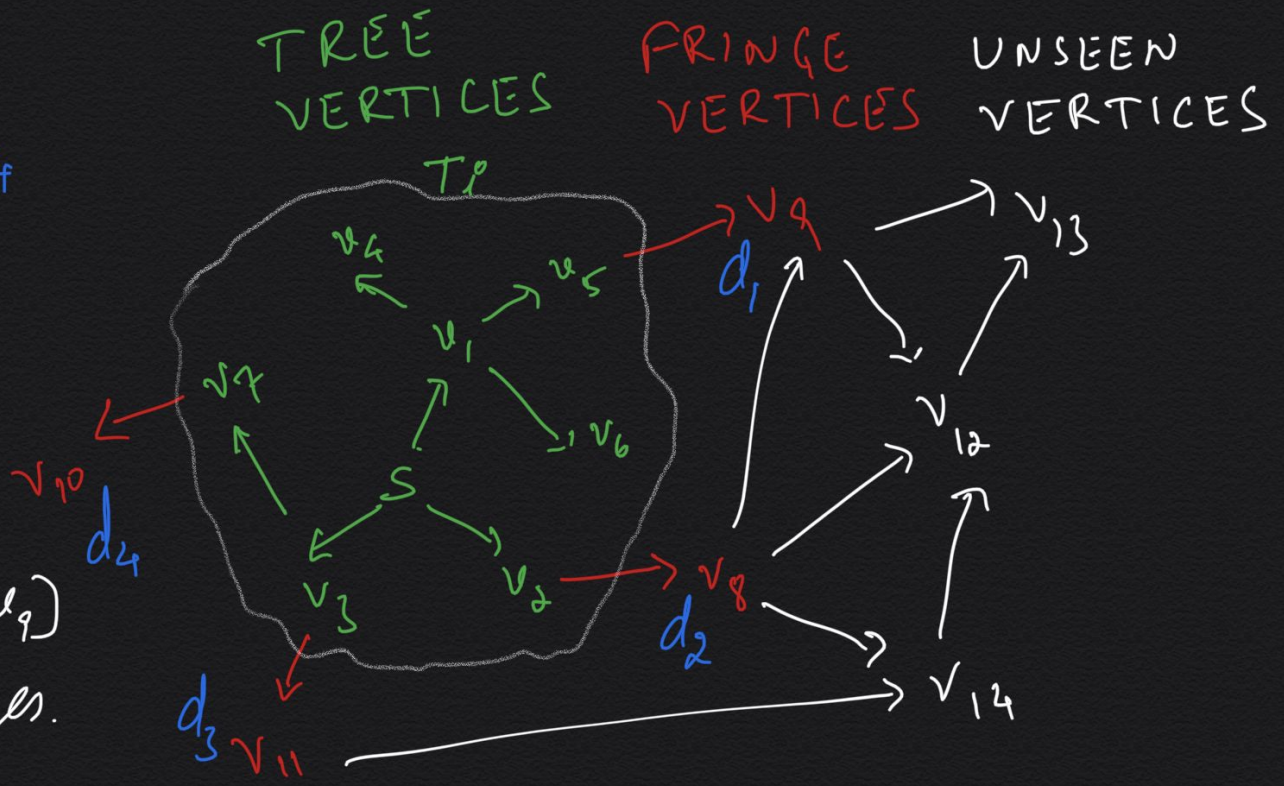

TREE VERTICES
FRINGE VERTICES
UNSEEN VERTICES
v_4
v_5
v_1
v_7
v_6
v_{10}
s
v_3
v_2
v_{11}
v_9
v_{13}
v_{12}
v_8
v_{14}

Let $d_1 < d_2 < d_3 < d_4$

Greedy choice:

$v_9(v_5, d_1)$

i.e., add edge $[v_5, v_9]$ into tree edges.

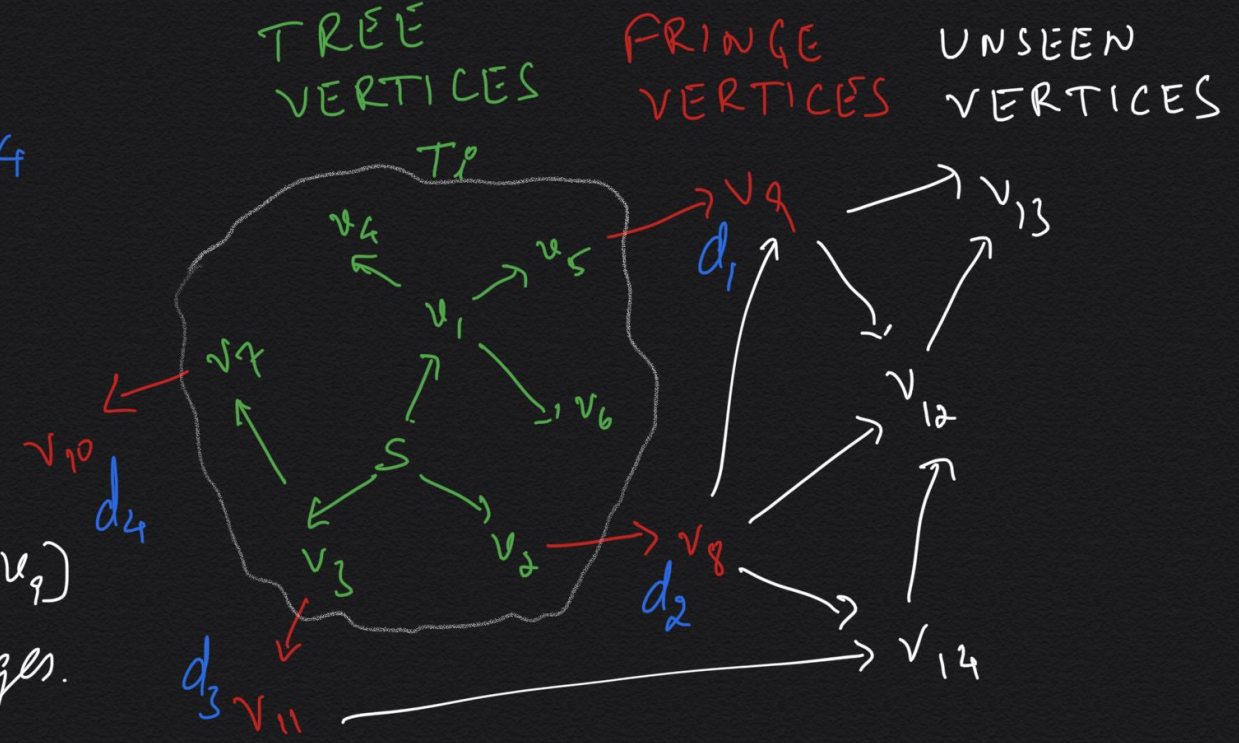


Let $d_1 < d_2 < d_3 < d_4$

Greedy choice:

$v_9(v_5, d_1)$

i.e., add edge $[v_5, v_9]$ into three edges.



BASIS STEP: SMALLEST OUTGOING EDGE OF s IS IN THE OPTIMAL TREE.

INDUCTION STEP: $T_i \rightarrow T_{i+1}$. PROVE BY CONTRADICTION.

Huffman Trees and Codes:

- **Codeword**: Encoding a text that comprises of **n** characters from some alphabet by assigning to each of the text's characters some sequence of bits. This bit-sequence is called codeword.
- **Fixed length encoding**: Assigns to each character a bit string of the same length.
- **Variable length encoding**: Assigns codewords of different lengths to different characters.
- **Prefix free code**: In Prefix free code, no codeword is a prefix of a codeword of another character.

Huffman Trees and Codes:

Binary prefix code :

- The characters are associated with the leaves of a binary tree.
- All left edges are labeled as 0, and right edges as 1.
- Codeword of a character is obtained by recording the labels on the simple path from the root to the character’s leaf.
- Since, there is no simple path to a leaf that continues to another leaf, no codeword can be a prefix of another codeword.

About **Huffman Algorithm**:

- Invented by David A Huffman in 1951.
- Constructs binary prefix code tree.
- Huffman’s algorithm achieves data compression by finding the best variable length binary encoding scheme for the symbols that occur in the file to be compressed. Huffman coding uses frequencies of the symbols in the string to build a variable rate prefix code
    - Each symbol is mapped to a binary string
    - More frequent symbols have shorter codes
    - No code is a prefix of another code (prefix free code)
- Huffman Codes for data compression achieves 20-90% Compression.

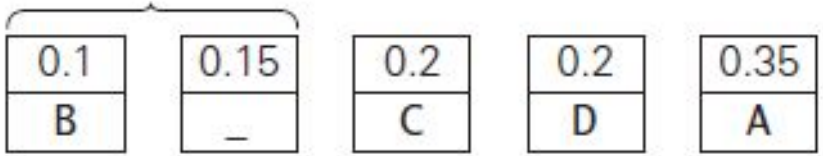
Huffman Algorithm:

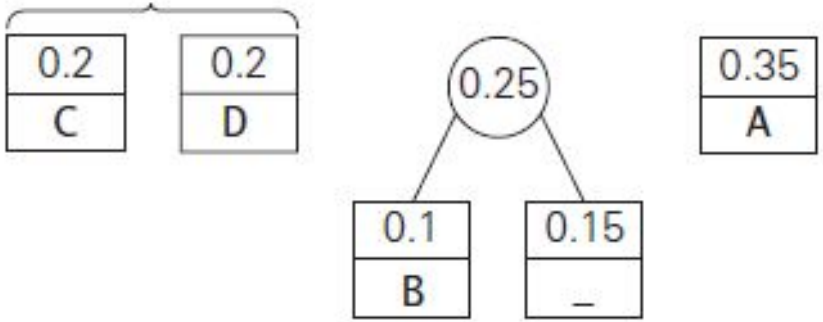
Input: Alphabet and frequency of each symbol in the text.

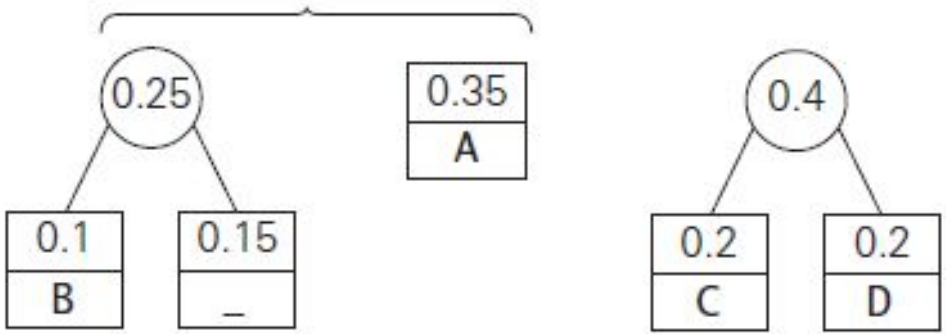
Step 1: Initialize **n** one-node trees (forest) and label them with the symbols of the alphabet given. Record the frequency of each symbol in its tree's root to indicate the tree's weight. (Generally, the weight of a tree will be equal to the sum of the frequencies in the tree's leaves.)

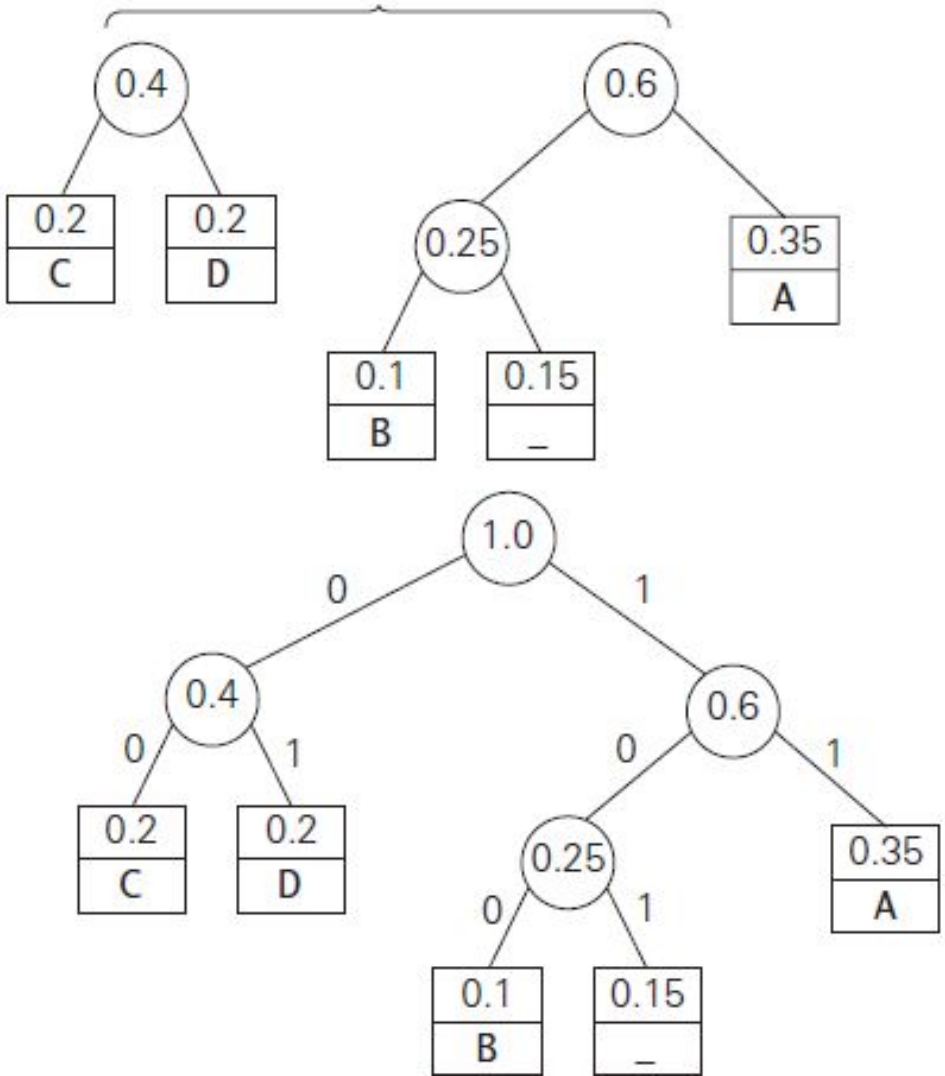
Step 2: Repeat the following operation until a single tree is obtained. Find two trees with the smallest weight. Make them the left and right subtree of a new tree and record the sum of their weights in the root of the new tree as its weight.

symbol	A	B	C	D	_
frequency	0.35	0.1	0.2	0.2	0.15









Huffman Algorithm:

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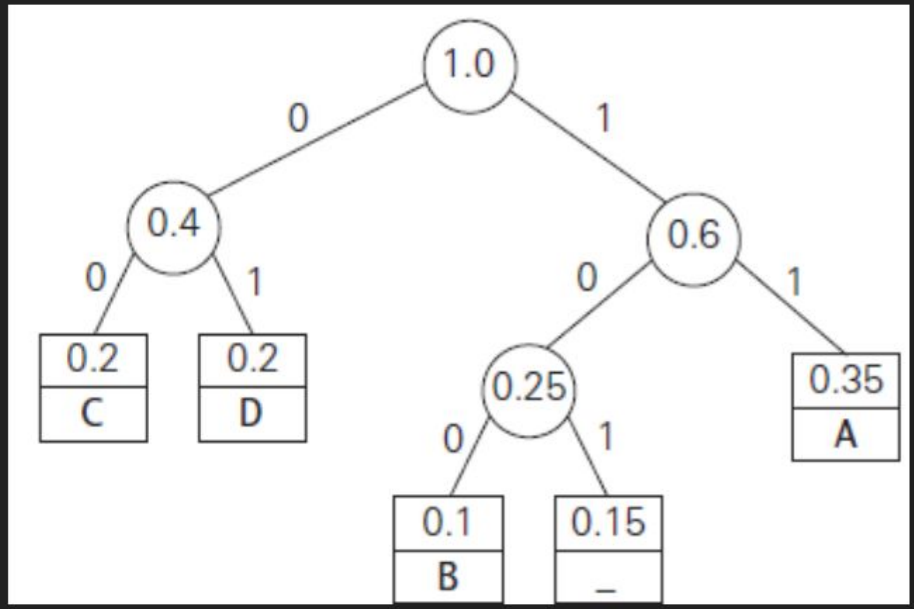

symbol	A	B	C	D	_
frequency	0.35	0.1	0.2	0.2	0.15
codeword	11	100	00	01	101

Huffman Algorithm:

Character	A	B	C	D	-
probability	0.4	0.1	0.2	0.15	0.15

Character	A	B	C	D	-
probability	0.4	0.1	0.2	0.15	0.15
Code word	0	100	111	101	110

Compute compression ratio:

Bits per character = Codeword length * Frequency

= (1 * 0.4) + (3 * 0.1) + (3 * 0.2) + (3 * 0.15) + (3 * 0.15)

= 2.20

Compression ratio is = (3 – 2.20)/ 3 . 100% = 26.6%

Huffman Coding:

Optimal Encoding: When the frequencies of symbol occurrences are independent and known in advance, the Huffman Algorithm yields an optimal, i.e., minimal-length, encoding.

Drawback: It is necessary to include the coding table into the encoded text to make its decoding possible.

Lempel-Ziv algorithm: assign codewords not to individual symbols but to strings of symbols, allowing them to achieve better and more robust compressions in many applications.

Q: For the given alphabet and their frequencies in a text, write

- Huffman tree
- Huffman codes
- Encode the text: A_CAB_AB_AD
- Decode the code string: 100010111110100100

Alphabet:	A	B	C	D	_
Frequencies:	45	17	2	1	35

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Alphabet:	A	B	C	D	_
Frequencies:	45	17	2	1	35
Codes:	0	100	1010	1011	11

A_CAB_AB_AD: 01110100100110100110101 1

10001011111010010 0: BAD_CAB

Thank You :-)

Mr. Channa Bankapur