5.3 CASCADE DESIGN REVISITED

In this chapter and in Chapter 4 we have developed a number of second-order sections $T_j(s)$ that permit us to realize arbitrary biquadratic functions. To realize more demanding specifications than are possible with second-order functions we need to be able to implement functions of higher order,

$$T(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{N(s)}{D(s)} = \frac{b_{2m}s^{2m} + b_{2m-1}s^{2m-1} + \cdots + b_1 s + b_0}{s^{2n} + a_{2n-1}s^{2n-1} + \cdots + a_1 s + a_0} \tag{5.52}$$

In connection with first-order circuits we discussed in Chapter 3, Section 3.6, a method that will allow us to address this problem with the knowledge gained so far is *cascade design*. The cascade method is used widely in industry for implementing high-order specifications. It is well understood, very easy to implement, has low sensitivity to component tolerances, and is efficient in its use of circuit elements. It uses a modular approach and results in filters that for the most part show good performance in practice. One of the main advantages of cascade filters is that they are very easy to adjust (or *tune*) because each biquad is responsible for the realization of only one pole pair and zero pair: in the realizations the individual critical frequencies of the filter are *decoupled* from each other. In addition, the design method is completely general in that arbitrary transfer functions of the form of Eq. (5.52) can be realized. Naturally, to keep the circuits stable and prevent them from oscillating, this means that the poles, the roots of $D(s)$, are restricted to the left half of the complex s-plane (the coefficients a_j are positive). However, the transmission zeros, the roots of $N(s)$, can be anywhere in the s-plane. As we illustrated in the first part of this chapter, their location depends only on the structure of the second-order sections. An additional advantage is that cascade design is easy and transparent because in the design we can focus on the low-order sections $T_j(s)$, which are generally simpler to implement than the high-order function $T(s)$.

To recapitulate, in the cascade procedure we need to factor the high-order circuit requirement of Eq. (5.52), with $n \geq m$ and $n > 1$, into a number of simpler specifications that result in practical circuits and that are readily implemented. These are then connected in a chain, that is, in a *cascade circuit*, as shown in Fig. 5.24, to realize the specified requirements. The degree of the specified function was assumed in Eq. (5.52) to be even, i.e., $2n$. If it is odd, one of the blocks T_j will be of first order and can be realized as discussed in Chapter 3, Sections 3.2 or 3.4. Thus, for our consideration, $T(s)$ in Eq. (5.52) is a ratio of two polynomials $N(s)$ and $D(s)$ of degrees $2m$ and $2n$, respectively. We selected, without loss of generality, the coefficient $a_{2n} = 1$, because we can always divide numerator and denominator by a_{2n}. The filter is then realized by connecting the functions $T_j(s) = V_{oj}/V_{ij}$ such that the total transfer behavior realized by that circuit is derived to be

$$\begin{aligned} T(s) = \frac{V_{\text{out}}}{V_{\text{in}}} &= \frac{V_{o1}}{V_{\text{in}}} \times \frac{V_{o2}}{V_{o1}} \times \frac{V_{o3}}{V_{o2}} \times \cdots \times \frac{V_{o(n-1)}}{V_{o(n-2)}} \times \frac{V_{\text{out}}}{V_{o(n-1)}} \\ &= T_1(s)T_2(s)\ldots T_{n-1}(s)T_n(s) = \prod_{j=1}^{n} T_j(s) \end{aligned} \tag{5.53}$$

i.e., the total transmission specification is obtained from the *product* of the low-order blocks. Equation (5.53) holds because the connection guarantees that the input voltage of circuit block

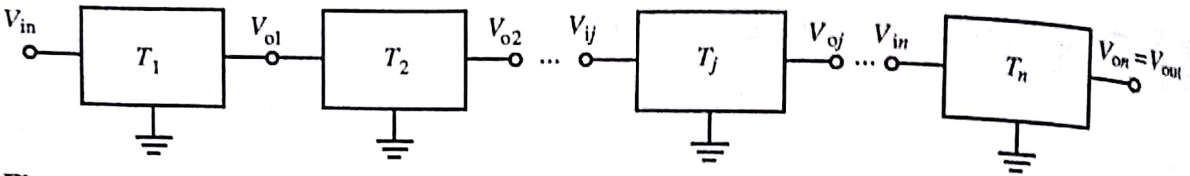


Figure 5.24 Cascade connection of second-order sections $T_j(s) = V_{oj}/V_{ij}$.

j is equal to the output voltage of block $(j-1)$, $V_{ij} = V_{o(j-1)}$. As we discussed in Section 3.6, Eq. (5.53) is valid under the assumption that the output impedance of block number j is much smaller than the input impedance of block $j+1$,

$$\left|Z_{\text{out},j}(j\omega)\right| << \left|Z_{\text{in},j+1}(j\omega)\right| \tag{5.54}$$

so that section T_{j+1} does not load T_j, i.e., two consecutive sections do not interact. Ideally $Z_{\text{out}} = 0$ and $Z_{\text{in}} = \infty$.

The goal is to realize the 2nth-order function $T(s)$ via simpler circuits in an efficient way with low sensitivities to component tolerances. Because both $N(s)$ and $D(s)$ are even, they can be factored into the product of second-order pole-zero pairs, as expressed in the following form:

$$T(s) = \frac{N(s)}{D(s)} = \frac{\prod_{i=1}^{m} k_i\left(\alpha_{2i}s^2 + \alpha_{1i}s + \alpha_{0i}\right)}{\prod_{j=1}^{n}\left(s^2 + s\omega_{0j}/Q_j + \omega_{0j}^2\right)}$$

$$= \prod_{j=1}^{n} k_j \frac{\alpha_{2j}s^2 + \alpha_{1j}s + \alpha_{0j}}{s^2 + s\omega_{0j}/Q_j + \omega_{0j}^2} = \prod_{j=1}^{n} T_j(s) \tag{5.55}$$

The notation assumes that both N and D are of degree $2n$; if $m < n$, the numerator will contain $2(n-m)$ factors of unity. If the degree of $T(s)$ is odd, the function can always be factored into the product of even terms as shown in Eq. (5.55) and a *first-order* factor. The first-order section, realized as discussed in Chapter 3, is then added to the higher order circuit as an additional block.

As shown in Eq. (5.55), we have again expressed the denominator in terms of the usual filter parameters, the quality factor Q and the pole frequency ω_0. Our problem is then reduced to realizing the second-order sections

$$T_j(s) = k_j \frac{\alpha_{2j}s^2 + \alpha_{1j}s + \alpha_{0j}}{s^2 + s\omega_{0j}/Q_j + \omega_{0j}^2} = k_j t_j(s) \tag{5.56}$$

We have also introduced the *gain constant* k_j. It is defined, e.g., such that the leading coefficient in the numerator of the *gain-scaled* transfer function $t_j(s)$ equals unity or such that $\left|t_j(j\omega_{0j})\right| = 1$.

EXAMPLE 5.13

The transfer function of a sixth-order filter, with a normalized frequency parameter $s = j\omega/\omega_0$, is

$$T(s) = \frac{s^5 + 2.5s^4 + 0.5625s}{s^6 + 0.390s^5 + 3.067s^4 + 0.785s^3 + 3.056s^2 + 0.387s + 0.989}$$

The function specifies a bandpass filter with two transmission zeros, one to the right and one to the left of the passband, which is centered around ω_0. We shall leave the discussion of how to find such a function from prescribed requirements until Chapter 8. The filter is to be realized as a cascade circuit. Find a possible set of second-order functions.

Solution

Factoring the numerator and denominator polynomials by a suitable root-finding algorithm, we obtain

$$\begin{aligned} T(s) &= \frac{s\left(s^2+0.25\right)\left(s^2+2.25\right)}{\left(s^2+0.09s+0.83\right)\left(s^2+0.2s+1.01\right)\left(s^2+0.1s+1.18\right)} \\ &= T_1(s)T_2(s)T_3(s) \\ &= \frac{k_1 s}{s^2+0.09s+0.83} \times \frac{k_2\left(s^2+0.25\right)}{s^2+0.2s+1.01} \times \frac{k_3\left(s^2+2.25\right)}{s^2+0.1s+1.18} \end{aligned} \tag{5.57}$$

In this case, $n = 3$ and the numerator is odd, i.e., $N(s)$ is factored into a product of two second-order factors and a first-order term, s. For generality, we also have included three constants k_i to permit us to adjust the gain of the sections; the product of the three constants, of course, must equal unity, $k_1k_2k_3 = 1$, to meet the specified function. The denominator has three conjugate complex roots as shown in Eq. (5.57) and the function is broken into three terms according to Eq. (5.55).

Factoring a given transfer function is a simple numerical process that makes use of a suitable polynomial root finder available in many inexpensive software packages. The problem of designing the sixth-order filter is now simplified to realizing the three second-order functions $T_1(s)$, $T_2(s)$, and $T_3(s)$. We know how to handle this task: several implementation methods were discussed earlier in this chapter and in Chapter 4.

Although this process is simple and leads in a straightforward way to a possible cascade design, we still must resolve several difficulties:

1. We should determine which zero should be assigned to which pole in Eq. (5.55) when the biquadratic functions $T_j(s)$ are formed. For example, in Eq. (5.57), we could also form

$$T(s) = \frac{k_1\left(s^2+0.25\right)}{s^2+0.09s+0.83} \times \frac{k_2 s}{s^2+0.2s+1.01} \times \frac{k_3\left(s^2+2.25\right)}{s^2+0.1s+1.18}$$

or

$$T(s) = \frac{k_1\left(s^2+2.25\right)}{s^2+0.09s+0.83} \times \frac{k_2\left(s^2+0.25\right)}{s^2+0.2s+1.01} \times \frac{k_3 s}{s^2+0.1s+1.18}$$

There is a total of six possibilities. Since we have in general n pole pairs and n zero pairs (counting zeros at 0 and at ∞) we can select from n factorial, $n! = 1 \times 2 \times 3 \times \ldots \times n$, possible *pole-zero pairings*.

2. We might be concerned about the order in which the biquads in Eq. (5.55) should be cascaded. The question is whether the *cascading sequence* makes a difference. For n biquads, we have $n!$ possible sequences. In Example 5.13 we have the options of choosing

$$T_1T_2T_3 \quad T_1T_3T_2 \quad T_2T_1T_3 \quad T_2T_3T_1 \quad T_3T_1T_2 \quad T_3T_2T_1$$

3. Since only the product of the gain constants k_j is prescribed, we must ask whether there is an optimum *gain distribution*. The gain constants in Eq. (5.55), of course, do not affect the frequency response, but they determine the signal level for each biquad. In Eq. (5.57) we have $k_1k_2k_3 = 1$, but the value of each k_i is not specified.

Because the total transfer function $T(s)$ is simply the product of the biquad sections, we may be led to conclude that the answers to the three questions are quite arbitrary. This is certainly a correct conclusion for a "paper design," but not in practice. We will see shortly that the main consequence of our answers to the above three points is their effect on the signal level. That is, pole-zero pairing, section ordering, and gain distribution determine the magnitude of the voltages at the internal nodes in the cascade filter. The reason why this is important is that the second- or first-order sections $T_j(s)$ in the cascade circuit use active devices, in our case opamps. We saw in Chapter 2 that the maximum undistorted signal level that the opamps can handle is limited. The opamp output voltage V_o is, of course, always restricted to be less than the power supply voltage, but more importantly, it is limited by the slew rate SR. As we saw in Eq. (2.27), this limit is set by the amplifiers used and is inversely proportional to the operating frequency, $|V_{o,max}| < SR/\omega$. We will next develop a procedure for finding a solution that will result in practical cascade filters. We emphasize that good answers to these questions do not just amount to minor adjustments when designing a cascade filter, but that the cascade circuit will likely not perform to specifications in practice unless correct answers to the three points are available at the design stage. The concept is not difficult to understand but we have to write a number of equations to provide us with the tools to arrive at conclusive answers.

We have to make sure that the signal level at any section output, $|V_{oj}(j\omega)|$, satisfies

$$\max|V_{oj}(j\omega)| < V_{o,max}, \quad 0 \le \omega \le \infty, \quad j = 1, \ldots, n \tag{5.58}$$

that is, the maximum of the output voltage at all opamps stays below some level $V_{o,max}$ set by power supply or slew rate limits. Note that this condition must indeed be satisfied for all frequencies and not only in the passband, because large signals even outside the passband must not be allowed to overload and saturate the opamps. When opamps are overdriven, their operation becomes *nonlinear* and results in higher harmonics that are generated by the nonlinear opamp behavior. The problems that arise when saturating the opamps are harmonic and intermodulation distortion of the signal, changed operating points, and ultimately deviations in filter performance.

The restriction at the other extreme is the electrical noise. We must make certain that signals stay large enough to remain above the electrical noise created by random movements of electrons. They cause the currents to fluctuate ...

very small, it must be amplified back up to the prescribed output level, say 0 dB from input to output. If a signal is smaller than the random noise signals, it is very hard to recover; amplifying the signal to the desired output level will increase both the applied signals and the noise by the same amount—the *signal-to-noise ratio* cannot be improved by amplification. Consequently, the signal-to-noise ratio will be hurt if in the cascade filter the signal suffers excessive in-band attenuation, i.e., if it is permitted to become very small. As a consequence, the second condition we must fulfill in a successful filter design is that the minimum of the output voltages of any biquad must be made as large as possible: we must satisfy

$$\min \left|V_{oj}(j\omega)\right| \rightarrow \max \quad \text{for} \quad \omega_L \leq \omega \leq \omega_U, \quad j = 1, \ldots, n \tag{5.59}$$

ω_L and ω_U are, respectively, the lower and upper corners of the passband. In this case we are, of course, concerned only with signal frequencies in the passband, because in the stopband the signal-to-noise ratio is of no interest. Note, however, that the noise spectrum of a filter section usually has the same shape as the transfer function magnitude. This means that the highest noise peaks occur at the pole frequencies with the highest Q-values. Since these are mostly found just beyond the specified corners of the passband, they would not be included in the measurement defined in Eq. (5.59). Therefore, to avoid decreased dynamic range caused by possibly large noise peaks at the passband corners, it is advisable to extend the frequency range beyond the specified passband corners, ω_L, ω_U, into the transition band.

Let us proceed then to determine what can be done with the available design freedom to optimize the *dynamic range*, that is the range between the smallest and largest signal that the circuit can process without distortion. Let us start with pole-zero pairing.

5.3.1 Pole-Zero Pairing

Our strategy for choosing pole-zero assignments should be clear: in any biquad in the cascade circuit we must avoid undue attenuation in the passband as well as unneeded gain. In other words, we must strive to keep the signal level "as flat as possible," safely above the noise floor and below the maximum that the opamps can handle without distortion. If we suppose that the overall gain of the filter is specified, then any frequency components that become too small must be amplified to the prescribed level. But this will also amplify the noise! Similarly, if a frequency component of the signal becomes too large, it must be attenuated back to the prescribed level. But this also reduces the remaining smaller signal components whose magnitude comes closer to the noise level! We conclude that keeping the signal level in the passband as flat as possible optimizes the dynamic range.

To help tackle the problem, let us assume that the largest signal in biquad j is measured at its output V_{oj}. This assumption will always be correct in single-amplifier biquads, such as in Figs. 4.31, 4.37, and 5.22, where section output and opamp output are the same. In multiamplifier biquads, such in the two-integrator loop (Fig. 4.10) or the GIC circuit (Fig. 4.46), *each* opamp output has to be evaluated and the maximum opamp output voltage in the biquad section should be used.

According to our discussion, we should choose the pole-zero pairing such that in a given biquad the signal maximum, $M_j = \max\left|V_{oj}(j\omega)\right|$, is minimized for all frequencies, and the signal minimum, $m_j = \min\left|V_{oj}(j\omega)\right|$, is maximized in the passband. In other words, using Eq. (5.56), $\left|t_j(j\omega)\right|$ *should be as flat as possible in the frequency range of interest.* Figure 5.25 illustrates the situation. Plotted is a typical magnitude of V_{oj} as a function of

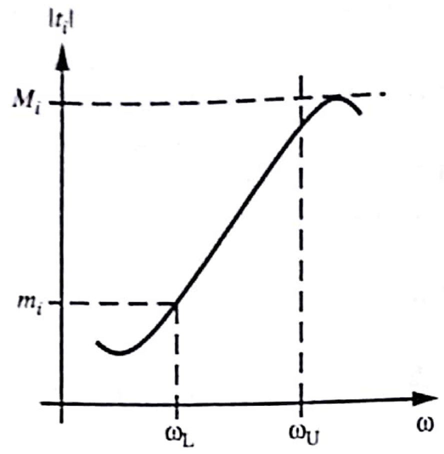


Figure 5.25 Typical biquad output voltage as a function of frequency. Notice that the maximum M_j lies just outside the upper edge ω_U of the passband and the minimum m_j lies at the lower passband corner ω_L.

frequency over the passband $\omega_L \leq \omega \leq \omega_U$, and maximum M_j and minimum m_j are indicated. Notice that M_j may lie outside and m_j at the edge of the passband; the actual minimum of the magnitude $|V_{oj}(j\omega)|$ or of $|t_j(j\omega)|$ lies, of course, in the stopband and is of no concern. Because the values of M_j and m_j change when the relative positions of the poles and the zeros of $t_j(s)$ are altered, the pole-zero assignment must be chosen such that the ratio M_j/m_j is as close to unity as possible. This means that for *each* biquad the "measure of flatness"

$$d_j = \left(\frac{M_j}{m_j} - 1\right), \quad j = 1, 2, \ldots, n \tag{5.60}$$

should be minimized. The optimal pole-zero assignment for the total $2n$th-order cascade filter is then the one that minimizes the maximum value of d_j:

$$d_{\max} = \max\{d_j\} \rightarrow \min, \quad j = 1, 2, \ldots, n \tag{5.61}$$

Algorithms that accomplish this task are available in the literature.[1] Because we have to explore $n!$ choices, the problem of pole-zero assignment is very computation intensive even in fairly simple low-order cases; it requires substantial software and computer resources.

Fortunately, a simple solution can be used that provides good suboptimal results if the appropriate computing facilities are not available. It is based on the intuitive insight that if the pole and zero pair were identical, their ratio t_j would equal unity. The circuit would, of course, provide no filtering at all, but the function would be optimally flat. Following this clue, we simply *assign each pole or pole pair to the closest zero*. On occasion, depending on system requirements, we may also preassign some pole-zero pair(s) and leave them out of the remaining pairing process. For instance, if the numerator contains a term s^2, we may prefer to factor it into $s \times s$ instead of $s^2 \times 1$, i.e., we may prefer to realize two second-order bandpass sections instead of a highpass and a lowpass because excellent bandpass circuits are available.

EXAMPLE 5.14

Determine the optimal pole-zero pairing for the transfer function of Eq. (5.57) of Example 5.13. Use the simple pole-zero assignment suggested above.

[1]See References 1–5.

Solution

The transfer function was

$$T(s) = \frac{k_1 s}{s^2 + 0.09s + 0.83} \times \frac{k_2 (s^2 + 0.25)}{s^2 + 0.2s + 1.01} \times \frac{k_3 (s^2 + 2.25)}{s^2 + 0.1s + 1.18}$$

The zeros are located at $z_1 = 0$ and $z_2 = \infty$, $z_{3,4} = \pm j0.05$, and at $z_{5,6} = \pm j1.5$, and the poles are at $p_{1,2} = -0.045 \pm j0.9099$, $p_{3,4} = -0.1 \pm j1.0$, and $p_{5,6} = -0.05 \pm j1.085$. According to the approximate assignment rule just stated, we should pair ($z_{1,2}$, $p_{3,4}$), ($z_{3,4}$, $p_{1,2}$), and ($z_{5,6}$, $p_{5,6}$). This is the choice indicated in Fig. 5.26 and in Eq. (5.62):

$$T(s) = T_1(s)T_2(s)T_3(s) = \frac{k_1 (s^2 + 0.25)}{s^2 + 0.09s + 0.83} \times \frac{k_2 s}{s^2 + 0.2s + 1.01} \times \frac{k_3 (s^2 + 2.25)}{s^2 + 0.1s + 1.18} \quad (5.62)$$

Notice that we shifted only the zeros but kept the gain constants associated with their poles.

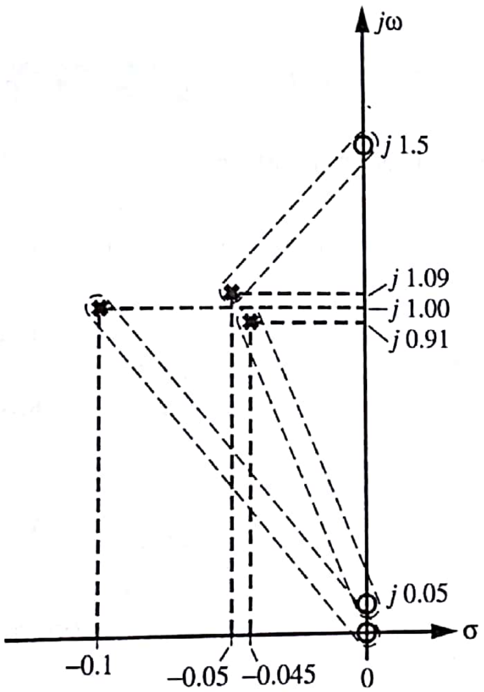


Figure 5.26 Pole-zero diagram for Example 5.14. Only the first quadrant is shown.

Having solved (at least approximately) the pole-zero assignment, we next attend to the question of section ordering

5.3.2 Section Ordering

Since we have n sections, there are $n!$ possibilities in which the biquads can be connected to form the cascade network. For example, for the sixth-order network with three sections in Example 5.13, we saw that there existed six possible ways to cascade the biquads. The best ordering sequence is the one that maximizes the dynamic range. Our task is to find that optimal sequence.

The procedure is completely analogous to the previous discussion where pole-zero pairs were chosen to keep the transfer functions of the individual sections as flat as possible. Now the *section ordering* is chosen such that the transfer functions,

$$H_i(s) = \frac{V_{oi}}{V_{in}} = \prod_{j=1}^{i} T_j(s), \qquad i = 1, \ldots n \tag{5.63}$$

from filter input to the output of the ith intermediate biquad are as flat as possible. H_n is, of course, equal to the total transfer function $T(s)$. This choice will help ensure that the maximum signal voltages do not overdrive the opamps and that, over the passband, the smallest signal stays well above the noise floor. Consequently, relationships similar to Eqs. (5.51) and (5.52) must be satisfied: we need to make certain that for all i the circuit satisfies

$$\max|V_{oi}(j\omega)| < V_{o,\max}, \quad 0 \le \omega \le \infty \tag{5.64}$$

$$\min|V_{oi}(j\omega)| \to \max \quad \text{for} \quad \omega_L \le \omega \le \omega_U \tag{5.65}$$

$V_{oi}(s)$ is the output voltage *of the cascade of the first i sections* when driven by an input signal V_{in}. With $H_i(s)$ given in Eq. (5.63), we define the maximum M_i of the transfer function from the input to the output of Section i for all ω,

$$M_i = \frac{\max|V_{oi}(j\omega)|}{|V_{in}|} = \max\left|\frac{V_{oi}(j\omega)}{V_{in}}\right| = \max|H_i(j\omega)| \quad \text{for} \quad 0 \le \omega \le \infty \tag{5.66}$$

and the minimum m_i of that function over the passband,

$$m_i = \frac{\min|V_{oi}(j\omega)|}{|V_{in}|} = \min\left|\frac{V_{oi}(j\omega)}{V_{in}}\right| = \min|H_i(j\omega)| \quad \text{for} \quad \omega_L \le \omega \le \omega_U \tag{5.67}$$

where we assumed that the input signal spectrum is constant for all frequencies. We then require again that the flatness criterion of Eq. (5.60) is minimized, now, however, *by choice of the cascading sequence*. The optimal sequence is the one that minimizes the maximum number d_i as prescribed in Eq. (5.61),

$$\max\{d_i\} = \max\left(\frac{M_i}{m_i} - 1\right) \to \min \tag{5.68}$$

Note that we do not have to consider d_n because, with all sections connected in the cascade filter, d_n is nothing but a measure of the *prescribed* passband variations.

With the problem identified, the optimum cascading sequence can be found in principle by calculating d_i for all $n!$ sequences and selecting the one that satisfies Eq. (5.68). As in the pole-zero assignment problem, a *brute-force* optimization approach involves a considerable amount of computation, and more efficient methods have been developed which use linear programming techniques, such as the "branch and bound" method, or "back track programming." The necessary computer algorithms are described in the literature.[2]

If the required software routines are not available to us, we can for most designs use a cascading sequence that, based on experience or intuition, appears to be a good choice. A selection that is often very close to the optimum is the one that chooses the section sequence in the order of increasing values of Q_i, i.e.,

[2]See References 6 and 7.

$$Q_1 < Q_2 < \cdots < Q_n \tag{5.69}$$

so that the section with the flattest transfer function magnitude (the lowest Q) comes first, the next flattest one second, and so on. The possible choices are frequently further limited by other considerations. For example, it is often desirable to have as the first section in the cascade a lowpass or a bandpass biquad so that high-frequency signal components are kept from the amplifiers in the filter in order to minimize slew-rate problems. Similarly, the designer may wish to employ a highpass or a bandpass biquad as the last section to eliminate low-frequency noise, dc offset, or power supply ripple from the filter output. If such practical constraints are important, we choose the placement of these sections first and perform the optimum sequencing only on the remaining sections. The following example will illustrate some of the steps discussed.

EXAMPLE 5.15

Continue Example 5.14 to find the optimal cascading sequence for the three second-order sections.

Solution

Since the coefficient of s in the denominators of the second-order sections of Eq. (5.62) equals ω_{0i}/Q_i, and the constant coefficient equals ω_{0i}^2, we have

$$Q_1 = \frac{\omega_{01}}{0.09} = \frac{\sqrt{0.83}}{0.09} \approx 10.12, \quad Q_2 = \frac{\sqrt{1.01}}{0.2} \approx 5.03, \quad Q_3 = \frac{\sqrt{1.18}}{0.1} \approx 10.86$$

Using the suggestion of Eq. (5.69) and continuing with the section numbering in Eq. (5.62), the optimal ordering is, therefore, $T_2T_1T_3$. If we were instead to emphasize the elimination of high-frequency signals from the filter and low-frequency noise from the output, the ordering $T_2T_3T_1$,

$$T(s) = \frac{k_2 s}{s^2 + 0.2s + 1.01} \times \frac{k_3\left(s^2 + 2.25\right)}{s^2 + 0.1s + 1.18} \times \frac{k_1\left(s^2 + 0.25\right)}{s^2 + 0.09s + 0.83} \tag{5.70}$$

would be preferable because the bandpass section, T_2, has the best high-frequency attenuation, and section T_1 provides reasonable attenuation at low frequencies ($0.25/0.83 = 0.30 \approx -10$ dB), whereas T_3/k_3 has a high-frequency gain of unity and amplifies low-frequency noise by more than $2.25/1.18 = 1.91 \approx 5.6$ dB. In our case, this suboptimal ordering gives almost identical results to the optimal one, $T_2T_1T_3$, because $Q_1 \approx Q_3$.

5.3.3 Gain Assignment

The last step we have to attend to in the realization of a cascade filter is the assignment of the gain constants k_i. Generally, the selection is again based on dynamic range concerns with the goal of keeping the signals below amplifier saturation limits. To see how we might proceed, we note that filters are linear circuits and all voltages rise in proportion to V_{in}. It is clear then that if we raise the input signal voltage we will overdrive that opamp first, which sees the largest output voltage after the previous two steps (pole-zero assignment and section ordering) were completed. It follows that we place equal stress on all opamps and can process the maximum undistorted input signal if we choose the gain constants such that *all internal output voltages* V_{oi}, $i = 1, \ldots, n-1$, *are equal in magnitude* to the prescribed magnitude of the output voltage, V_{on}:

$$\max |V_{oi}(j\omega)| = \max |V_{on}(j\omega)| = \max |V_{\text{out}}(j\omega)|, \quad i = 1, \ldots, n-1 \tag{5.71}$$

Assuming as before that the *output* voltage of the biquads (rather than an internal opamp) reaches the critical magnitude, this choice assures that for a given signal level none of the opamps in the blocks of Fig. 5.24 is overdriven sooner than any other one. Note, however, the earlier comments about precautions necessary in multiamplifier biquads.

For the analysis it is convenient to use the notation of Eqs. (5.55) and (5.56), i.e.,

$$T(s) = \prod_{j=1}^{n} T_j(s) = \prod_{j=1}^{n} k_j \prod_{j=1}^{n} t_j(s) \tag{5.72}$$

and, for the intermediate transfer functions of Eq. (5.63),

$$H_i(s) = \prod_{j=1}^{i} T_j(s) = \prod_{j=1}^{i} k_j \prod_{j=1}^{i} t_j(s) \tag{5.73}$$

Further, for convenience of notation we introduce the label K for the product of all section gain constants,

$$K = \prod_{j=1}^{n} k_j \tag{5.74}$$

Then the *prescribed* gain of the total cascade circuit is

$$\max |T(j\omega)| = \max |H_n(j\omega)| = K \max \prod_{j=1}^{n} |t_j(j\omega)| = K M_n \tag{5.75}$$

where we defined

$$M_n = \max \prod_{j=1}^{n} |t_j(j\omega)| \tag{5.76}$$

Similar to the definition of M_n, let us denote the maxima of the intermediate $n-1$ gain-scaled transfer functions by M_i, i.e.,

$$\max \prod_{j=1}^{i} \left| t_j(j\omega) \right| = M_i, \quad i = 1, \ldots, n-1 \tag{5.77}$$

We now assume that the input signal magnitude $|V_{\text{in}}|$ is constant, independent of frequency. This means that the requirement of equal voltage maxima throughout the cascade circuit,

$\max|V_{oi}(j\omega)| = \max|V_{out}(j\omega)|$, corresponds to $\max|H_i(j\omega)| = \max|H_n(j\omega)| = \max|T(j\omega)|$. Thus, with Eqs. (5.73) to (5.77) we have $\max|V_{o1}(j\omega)| = \max|V_{out}(j\omega)|$, i.e., $k_1 M_1 = K M_n$ or

$$k_1 = K\frac{M_n}{M_1} \tag{5.78}$$

Similarly, $\max|V_{o2}(j\omega)| = \max|V_{out}(j\omega)|$ results in $k_1 k_2 M_2 = K M_n$, i.e., with Eq. (5.78),

$$k_2 = \frac{M_1}{M_2} \tag{5.79}$$

Proceeding in the same manner with $\max|V_{oi}(j\omega)| = \max|V_{out}(j\omega)|$, $i = 3, \ldots, n$, yields

$$k_j = \frac{M_{j-1}}{M_j}, \quad j = 2, \ldots, n \tag{5.80}$$

Choosing the gain constants as in Eqs. (5.78) and (5.80) guarantees that all opamps see the same maximum voltage to ensure that the largest possible signal can be processed without distortion. Note that changing the total gain of the n-section cascade filter affects only K, i.e., the gain constant k_1 of the first section. The voltages in all other sections $T_i(s)$, $i = 2, \ldots, n$ increase or decrease proportionally, but their *relative* magnitudes stay the same, as determined by k_i in Eq. (5.80).

Let us summarize this somewhat involved procedure: To maximize the dynamic range in a cascade filter, we need to maximize in each filter section the minimum signal level in the passband and minimize the maximum signal level. In addition, the maxima of the output voltages of all sections should be made equal. The way to accomplish this is first choose the pole-zero pairing, next perform section ordering, and last determine the gain assignment. Specifically, if the necessary software for an optimal solution is not available, we

1. assign the zeros to the closest poles,
2. order the sections in the cascade in the order of increasing values of Q, and
3. compute the gain constants according to Eqs. (5.78) and (5.80). The values M_i needed to evaluate these equations are readily computed from the known functions $t_i(s)$ or with the help of a circuit simulator, such as SPICE or Electronics Workbench (the program used throughout this text), from an initial trial design.

EXAMPLE 5.16

Continue Example 5.15 to find the optimal gain constants for the three second-order sections so that their maximum output levels are equalized. The specified midband filter gain is 10 dB. Use the section ordering in Eq. (5.70). Realize the filter with three suitable Åckerberg–Mossberg biquads based on LM741 opamps. The center frequency is $f_0 = 15.7$ kHz.

Solution

The transfer function is

$$T(s) = \frac{k_1 s}{s^2 + 0.2s + 1.01} \times \frac{k_2 (s^2 + 2.25)}{s^2 + 0.1s + 1.18} \times \frac{k_3 (s^2 + 0.25)}{s^2 + 0.09s + 0.83} \tag{5.81}$$

where we have renamed the gain constants k_1, k_2, and k_3 such that their subscripts indicate the order of the sections. For the prescribed 10-dB gain we find from Eq. (5.75),

$$KM_3 = 3.16 \tag{5.82}$$

The maximum of Section 1, M_1, follows directly from the maximum Q of a bandpass function: $M_1 \approx 1/0.2 = 5$. The remaining maxima M_2 and M_3 can be computed by evaluating the functions in Eq. (5.76) with Eq. (5.81). We find

$$M_1 \approx \frac{1}{0.2} = 5, \quad M_2 \approx 46, \quad M_3 \approx 125$$

With M_i known, and using Eq. (5.82), Equations (5.78) and (5.80) give

$$k_1 = \frac{KM_3}{M_1} = \frac{3.16}{5} = 0.63, \quad k_2 = \frac{M_1}{M_2} \approx \frac{5}{46} = 0.11, \quad k_3 = \frac{M_2}{M_3} \approx \frac{46}{125} = 0.38$$

that is

$$T(s) = T_2 T_3 T_1 = \frac{0.63s}{s^2 + 0.2s + 1.01} \times \frac{0.11 (s^2 + 2.25)}{s^2 + 0.1s + 1.18} \times \frac{0.38 (s^2 + 0.25)}{s^2 + 0.09s + 0.83} \tag{5.83}$$

These values result in all section outputs being equal to $KM_3 = 3.16$ times the input voltage level for a uniform gain of 10 dB. If the designer were to find out later that system performance would improve for a different filter gain, say, 25 dB ($KM_3 = 17.8$) rather than 10 dB, the redesign is easy: it is necessary only to alter the first section in the cascade from $k_1 = 0.63$ to

$$k_1 = KM_3/M_1 = 17.8/5 = 3.56$$

to achieve the new circuit for which dynamic range is still optimized.

This completes the steps required to optimize the dynamic range. We next have to select the appropriate Åckerberg–Mossberg sections. The first section, T_2, is the bandpass filter of Fig. 4.29. We select $C_1 = C_2 = C = 0.01\ \mu\text{F}$ so that with $f_0 = 15.7$ kHz we obtain $R_1 = 1/(\omega_0 C) = 1.013\ \text{k}\Omega$. Because $Q_2 = 5.03$ from Example 5.15, we have for the resistor QR the value $5.13\ \text{k}\Omega$. Finally, the numerator coefficient of the first section in Eq. (5.83) equals $k_1 = 0.63$ which results in $R/k_1 = 1.61\ \text{k}\Omega$.

The circuit for the two notch sections is given in Fig. 5.13 with $k = b = 0$ from Eq. (5.32). We choose again $C_1 = C_2 = C = 0.01\ \mu\text{F}$ so that for the second section, T_3, we have

$$R_1 = 1/\left(\sqrt{1.18} \times 2\pi \times 15700\ \text{Hz} \times 0.01\ \mu\text{F}\right) = 933\ \Omega$$

and $QR = 10.13\ \text{k}\Omega$. The feed-in components are

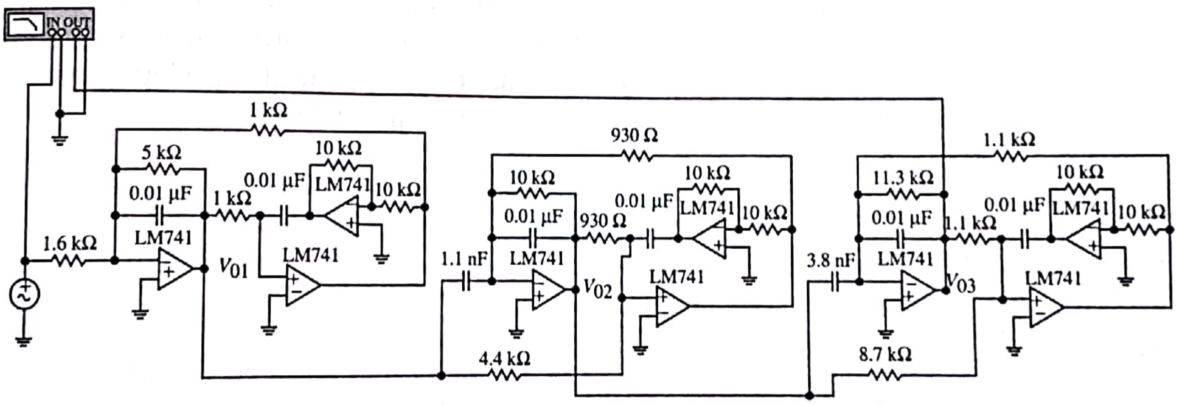
$$aC = 0.11 \times 0.01\ \mu\text{F} = 1.1\ \text{nF} \quad \text{and} \quad \frac{R}{c} = \frac{933\ \Omega}{0.11 \times 2.25/1.18} = 4.4\ \text{k}\Omega$$

For the last section, T_1, we obtain similarly $R = 1.1\ \text{k}\Omega$ and $QR = 11.3\ \text{k}\Omega$. The feed-in elements are

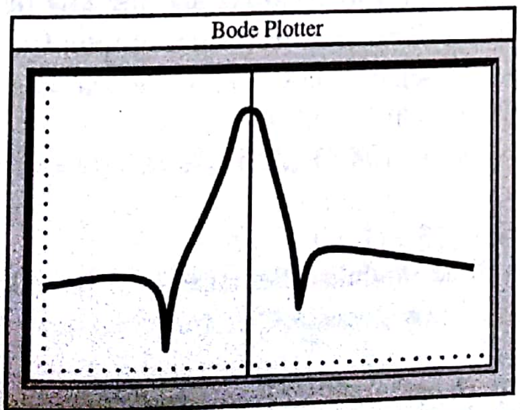
$$aC = 3.8\ \text{nF} \quad \text{and} \quad \frac{R}{c} = \frac{933\ \Omega}{0.38 \times 0.25/0.83} = 8.16\ \text{k}\Omega$$

Figure 5.27a shows the circuit and Fig. 5.27b the test results. The filter has a flat passband at 10.6 dB gain with $f_0 = 15.6$ kHz as required, with a 1-dB gain variation (ripple) in $14.8\ \text{kHz} \le f \le 16.8$ kHz. The transmission zeros are at 8.3 and 23.7 kHz, close to the specified values. Looking at all opamps, we find signal level variations over the passband to be less than 9 dB (a factor 2.8) with the maximum gain at 10.6 dB. To verify that our design indeed equalizes the maximum signal levels at the three section outputs, we show in Fig. 5.27c the simulated output voltages. Evidently, their maxima are at approximately 10 dB as specified.

To demonstrate that gain equalization is very important in cascade realizations, consider the case where equalization is *not* performed. Had the designer chosen all $k_i = 1$ in Eq. (5.81), the maximum output levels would have been 13.9 dB at V_{o1}, 31.9 dB at V_{o2}, and 41.3 dB at V_{o3}. Reducing the overall gain from 41.3 dB to the prescribed level of 10 dB by increasing the input resistor would result in a maximum level of −17 dB at V_{o1} and −7.9 dB at V_{o2}. The difference of (10 dB + 17 dB) = 27 dB, a factor of approximately 23 in opamp output voltages, would force the input voltage to be 23 times smaller than in the optimized design.

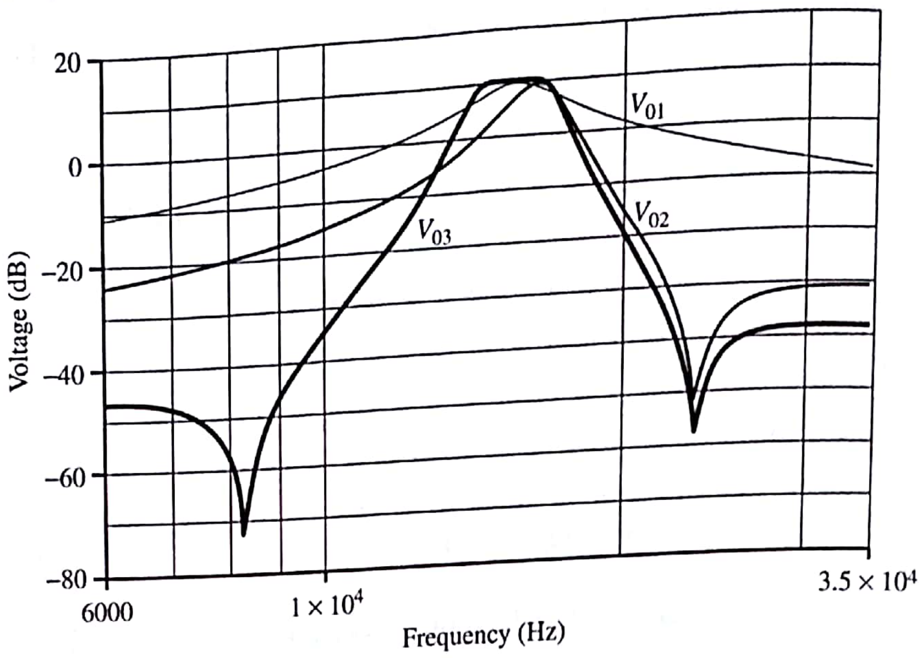


(a)



(b)

Figure 5.27 (a) The cascade circuit of Example 5.16; (b) test results; (c) simulated voltage levels. (Bode Plotter scales: 3 to 100 kHz; −80 to 20 dB; cursor at 15.61 kHz, 10.59 dB.)



(c)

Figure 5.27 Continued

Notice that the optimal cascading routine we discussed, pole-zero pairing, section ordering, and gain assignment, is entirely general and does not depend on the transfer function or its type. It is independent of the actual implementation of the second-order building blocks. The designer may choose any convenient technology, and the circuit architecture that seems preferable from the point of view of sensitivity, element numbers, types, values and element value spreads, power consumption, or other practical considerations. As a further example consider the realization of a fifth-order lowpass filter.

EXAMPLE 5.17

Realize the filter function

$$T(s) = T_1(s)T_2(s)T_3(s) = \frac{k\left(s^2 + 29.2^2\right)\left(s^2 + 43.2^2\right)}{(s + 16.8)\left(s^2 + 19.4s + 20.01^2\right)\left(s^2 + 4.72s + 22.52^2\right)}$$

The frequency is normalized with respect to 1000 Hz and the low-frequency gain is to be 13.5 dB. Use single-amplifier biquads. This is a fifth-order elliptic lowpass filter used in telephone channel bank applications. We shall discuss the method to arrive at this function later in this book.

Solution

We have already factored $T(s)$ into three modules. Because $T(s)$ is odd, there will be one first-order term. We have

$$T(s) = T_1(s)T_2(s)T_3(s) = \frac{h_1}{s + 16.8}\,\frac{h_2\left(s^2 + 29.2^2\right)}{s^2 + 19.4s + 20.01^2}\,\frac{h_3\left(s^2 + 43.2^2\right)}{s^2 + 4.72s + 22.52^2} \qquad (5.84)$$

We have labeled the three gain constants h_1, h_2, and h_3; they must be determined such that the low-frequency gain is 13.5 dB. We have from $T(0)$

$$h_1 h_2 h_3 \frac{1}{16.8} \frac{29.2^2}{20.01^2} \frac{43.2^2}{22.52^2} = 0.4664 h_1 h_2 h_3 = 10^{13.5/20} = 4.732 \tag{5.85}$$

i.e., $h_1 h_2 h_3 = 10.145$. The pole-zero pairing assigns zeros to the closest poles; this is the assignment in Eq. (5.84). To select the cascading sequence we need to find the quality factors. From Eq. (5.84) we have

$$Q_2 = 20.01/19.4 = 1.03 \quad \text{and} \quad Q_3 = 22.52/4.72 = 4.77 \tag{5.86}$$

We also try to reduce high-frequency signals from entering the filter, which suggests taking the first-order function T_1 as first section. Cascading the sections in the order of increasing values of Q results in the sequence in Eq. (5.84), which we now proceed to realize.

We shall use the first-order section of Fig. 3.24 with $C_1 = 0$. The second-order sections will be Sallen–Key circuits with additional feed-in components, Fig. 5.22. The filter calls for two lowpass notch sections described by Eq. (5.49):

$$T(s) = K \frac{c_k}{2} \frac{s^2 + 2\frac{a - c_k}{c_k}\omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2} = k_2 \frac{s^2 + \omega_N^2}{s^2 + s\omega_0/Q + \omega_0^2}$$

With k_2 and f_N specified we have to satisfy from Eqs. (5.50) and (5.51)

$$a = \left(2 + \frac{f_N^2}{f_0^2}\right) \frac{k_2}{K} \le 1, \quad b = \frac{3k_2}{K} \le 1, \quad c = \frac{2k_2}{K - 1} \le 1 \tag{5.87}$$

At this point let us recall that the Sallen–Key filter experiences a decrease in f_0 and an increase in Q when real opamps with $A \neq \infty$ are used. Let us estimate the size of these errors and see how they can be minimized in the design. Referring to Eqs. (4.113) to (4.116), we have

$$f_{0R} \approx f_0 \left(1 - \frac{\varepsilon}{2}\right) \quad \text{and} \quad Q_R \approx Q \left(1 + \frac{\varepsilon}{2}\right) \tag{5.88}$$

where the error was

$$\varepsilon = \frac{\omega_0}{\omega_t} K^2 = \frac{K^2}{|A(j\omega_0)|}$$

The parameter K for each biquad is determined through Q by Eq. (4.105), $K = 3 - 1/Q$. With Q_2 and Q_3 in Eq. (5.86), we have for the two sections

$$K_2 = 3 - 1/1.03 = 2.03 \quad \text{and} \quad K_3 = 3 - 1/4.77 = 2.79$$

so that the errors in Sections 2 and 3 are

$$\frac{\varepsilon_2}{2} = \frac{2.03^2}{1500/20.01} \approx 0.0275 \quad \text{and} \quad \frac{\varepsilon_3}{2} = \frac{2.79^2}{1500/22.52} \approx 0.0584$$

By Eq. (5.88) the opamps cause f_0 and Q to decrease and increase, respectively, by $\varepsilon/2$. Let us anticipate these nominal errors and *predistort* the specified values. Predistortion means we design for a nominally larger f_0 and smaller Q so that the opamp effect restores the parameters to the required values. Thus, we design the filter section T_2 with

$$\begin{aligned} \omega_{02} &= 20.01\left(1+\frac{\varepsilon_2}{2}\right) = 20.01 \times 1.0275 = 20.57 \\ Q_2 &= 1.03\left(1-\frac{\varepsilon_2}{2}\right) = 1.03 \times 0.9725 = 1.00 \end{aligned} \tag{5.89a}$$

and T_3 with

$$\begin{aligned} \omega_{03} &= 22.52\left(1+\frac{\varepsilon_3}{2}\right) = 22.52 \times 1.0584 = 23.84 \\ Q_3 &= 4.77\left(1-\frac{\varepsilon_3}{2}\right) = 4.77 \times 0.9416 = 4.49 \end{aligned} \tag{5.89b}$$

Let us choose $C = 0.01\ \mu\text{F}$ for all capacitors. Then we find for $T_2(s)$ with the predistorted parameters in Eq. (5.89a)

$$R = \frac{1}{2\pi \times 1000\text{s}^{-1} \times 20.57 \times 0.01\ \mu\text{F}} = 773.72\ \Omega \quad \text{and} \quad K = 3 - 1/Q = 2.00$$

Also, from Eq. (5.87),

$$a = \left(2 + \frac{29.2^2}{20.57^2}\right)\frac{h_2}{2.00} = 2.008h_2 \le 1, \quad b = \frac{3h_2}{2} = 1.500h_2 \le 1,$$

$$c = \frac{2h_2}{1.00} = 2.00h_2 \le 1$$

This means that we must choose $h_2 \le 1/2.008 = 0.498$. Let us pick this value of h_2 since Eq. (5.85) requires a large gain. We get

$$a = 1, \quad b = 0.747, \quad \text{and} \quad c = 0.996 \approx 1$$

Similarly, we find for $T_3(s)$ with Eq. (5.89b),

$$R = \frac{1}{2\pi \times 1000\text{s}^{-1} \times 23.84 \times 0.01\ \mu\text{F}} = 667.6\ \Omega \quad \text{and} \quad K = 2.778$$

and

$$a = \left(2 + \frac{43.2^2}{23.84^2}\right)\frac{h_3}{2.778} = 1.902h_3 \le 1, \quad b = \frac{3h_3}{2.778} = 1.080h_3 \le 1,$$

$$c = \frac{2h_3}{1.778} = 1.125h_3 \le 1$$

We must choose $h_3 \le 1/1.902 = 0.526$. Let us pick this value of h_3 to get

$$a = 1, \quad b = 0.568, \quad \text{and} \quad c = 0.592$$

With h_2 and h_3 determined, we find from Eq. (5.85) $h_1 = 38.73$. Because of the constraints on the values of h_2 and h_3 we could not equalize the signal levels at the three opamps. We have at the output of T_1 a gain of 7.27 dB, at the output of T_2 a gain of 7.78 dB, and at the filter output, of course, the required 13.5 dB gain. The third opamp, therefore, determines the maximum signal amplitude.

The first-order section T_1 is the lossy integrator from Fig. 3.24 described by Eq. (3.70),

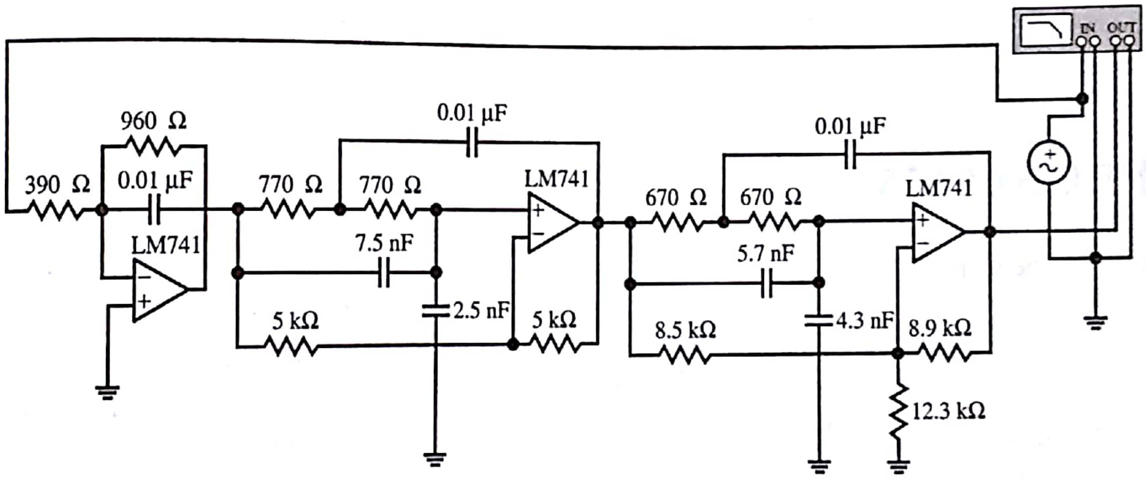
$$T_1(s) = \frac{G_1}{sC + G_2} = \frac{G_1/C}{s + G_2/C} \rightarrow \frac{h_1}{s + 16.8}$$

Consequently, we obtain

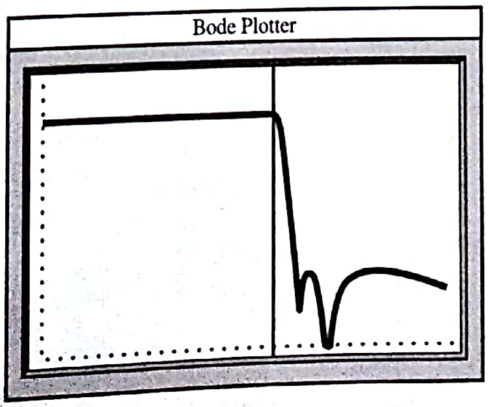
$$R_2 = \frac{1}{2\pi \times 1000\text{s}^{-1} \times 16.8 \times 0.01\ \mu\text{F}} = 947.35\ \Omega$$

$$R_1 = \frac{1}{h_1 \times 2000\pi\text{s}^{-1} \times 0.01\ \mu\text{F}} = 411\ \Omega$$

Finally, choosing $R_0 = 5\ \text{k}\Omega$ in Fig. 5.22 we have the circuit in Fig. 5.28. We notice that the filter performs as specified. In this example we had no more degrees of freedom to adjust the gain constants to equalize the signal level according to Eqs. (5.75) to (5.79). Adjusting h_2 and h_3 to their maxima is the best choice available. Of course, there is no limit on h_1.



(a)



(b)

Figure 5.28 The fifth-order lowpass circuit for Example 5.17 and test results. (Bode Plotter scales: 1 to 200 kHz; −40 to 20 dB; cursor at 21.0 kHz, 12.0 dB.)

For building active filters we have now available in our arsenal a variety of first- and second-order building blocks with which we can realize poles anywhere in the left half s-plane and zeros anywhere in the s-plane. Further, with the cascade method we can connect these building blocks to implement sophisticated filters of high order. In Examples 5.16 and 5.17 we have illustrated the process of implementing a high-order transfer function. How to generate these transfer functions to satisfy some required design goal we shall explore in the following chapters, beginning with general lowpass magnitude specifications.

REFERENCES

[1] S. Halfin, "An Optimization Method for Cascaded Filters," *Bell Syst. Tech. J.*, Vol. 49, pp. 185–190, 1970.

[2] S. Halfin, "Simultaneous Determination of Ordering and Amplifications of Cascaded Subsystems," *J. Optimiz. Theory Applic.*, Vol. 6, pp. 356–360, 1970.

[3] E. Lüder, "Optimization of the Dynamic Range and the Noise Distance of RC-Active Filters by Dynamic Programming," *Int. J. Circuit Theory Applic.*, Vol. 3, pp. 365–370, 1975.

[4] G. S. Moschytz, *Linear Integrated Networks—Design*. van Nostrand Reinhold, New York, 1975.

[5] R. Schaumann, M. S. Ghausi and K. R. Laker, *Design of Analog Filters: Passive, Active RC and Switched Capacitor*. Prentice-Hall, Englewood Cliffs, NJ, 1990.

[6] A. S. Sedra and P. O. Brackett, *Filter Theory and Design: Active and Passive*. Matrix, Portland, OR, 1978.

[7] W. M. Snelgrove and A. S. Sedra, "Optimization of Dynamic Range in Cascade Active Filters," *Proc. IEEE Int. Symp. Circ. Syst.*, pp. 151–155, 1978.

PROBLEMS

5.1 In the transfer function

$$T(s) = \frac{N(s)}{D(s)} = \frac{k_2 s^2 + k_1 s + k_0}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

determine the conditions the coefficients k_0, k_1, and k_2 must satisfy so that $T(s)$ is a (a) lowpass, (b) highpass, (c) bandpass, (d) allpass; (e) band-rejection function; (f) a magnitude equalizer that provides (f1) a gain dip; (f2) a gain boost.

5.2 Use the general Åckerberg–Mossberg biquad in Fig. 5.1 to realize the filters in Parts (a) through (m) below. The following abbreviations or symbols are used: HP = highpass; LP = lowpass; BP = bandpass; BE = band elimination; AP = allpass; N = notch; ME/B = magnitude equalizer with gain boost; ME/D = magnitude equalizer with gain dip; ω_0 or f_0 = pole frequency; ω_z or f_z = zero frequency; Q = pole quality factor; BW = bandwidth; H_0 = dc gain; H_M = midband gain; H_∞ = high-frequency gain. Use practical component values and LM741 or HA 2542-2 opamps as appropriate. In all cases test your designs with Electronics Workbench (EWB) and comment on any inaccuracies observed.

(a) HP with $\omega_0 = 9$ krad/s; $Q = 4.5$; $H_\infty = 0$ dB

(b) HP with $\omega_0 = 600$ krad/s; $Q = 4.5$; $H_\infty = 0$ dB

(c) BP with $\omega_0 = 6.3$ krad/s; $Q = 35$; $H_M = 24$

(d) BP with $\omega_0 = 64$ krad/s; bandwidth = 2.2 kHz; $H_M = 1$

(e) BE with $f_z = 24$ kHz; signal level −15 dB below dc gain in a band of 3.2 kHz; $H_\infty = H_0 = 0$ dB

(f) BE with $\omega_z = 1.2$ Mrad/s; signal level −36 dB below H_0 in a band of 60 kHz; $H_\infty = H_0 = +4$ dB

(g) LPN with $\omega_0 = 17$ krad/s; $\omega_z = 47$ krad/s; $H_\infty = 0$ dB; band edge gain peaking of 5 dB above H_0

(h) LPN with $f_0 = 2$ MHz; $H_0 = 0$ dB; $H_\infty <$ −30 dB band edge gain peaking of 2 dB above H_0

(counting zeros at 0 and at ∞) we can select from n factorial, $n! = 1 \times 2 \times 3 \times \ldots \times n$, possible *pole-zero pairings*.

2. We might be concerned about the order in which the biquads in Eq. (5.55) should be cascaded. The question is whether the *cascading sequence* makes a difference. For n biquads, we have $n!$ possible sequences. In Example 5.13 we have the options of choosing

$$T_1T_2T_3 \quad T_1T_3T_2 \quad T_2T_1T_3 \quad T_2T_3T_1 \quad T_3T_1T_2 \quad T_3T_2T_1$$

3. Since only the product of the gain constants k_j is prescribed, we must ask whether there is an optimum *gain distribution*. The gain constants in Eq. (5.55), of course, do not affect the frequency response, but they determine the signal level for each biquad. In Eq. (5.57) we have $k_1k_2k_3 = 1$, but the value of each k_i is not specified.

Because the total transfer function $T(s)$ is simply the product of the biquad sections, we may be led to conclude that the answers to the three questions are quite arbitrary. This is certainly a correct conclusion for a "paper design," but not in practice. We will see shortly that the main consequence of our answers to the above three points is their effect on the signal level. That is, pole-zero pairing, section ordering, and gain distribution determine the magnitude of the voltages at the internal nodes in the cascade filter. The reason why this is important is that the second- or first-order sections $T_j(s)$ in the cascade circuit use active devices, in our case opamps. We saw in Chapter 2 that the maximum undistorted signal level that the opamps can handle is limited. The opamp output voltage V_o is, of course, always restricted to be less than the power supply voltage, but more importantly, it is limited by the slew rate SR. As we saw in Eq. (2.27), this limit is set by the amplifiers used and is inversely proportional to the operating frequency, $|V_{o,\max}| < SR/\omega$. We will next develop a procedure for finding a solution that will result in practical cascade filters. We emphasize that good answers to these questions do *not* just amount to minor adjustments when designing a cascade filter, but that the cascade circuit will likely not perform to specifications in practice unless correct answers to the three points are available at the design stage. The concept is not difficult to understand but we have to write a number of equations to provide us with the tools to arrive at conclusive answers.

We have to make sure that the signal level at any section output, $|V_{oj}(j\omega)|$, satisfies

$$\max |V_{oj}(j\omega)| < V_{o,\max}, \quad 0 \le \omega \le \infty, \quad j = 1, \ldots, n \tag{5.58}$$

that is, the maximum of the output voltage at all opamps stays below some level $V_{o,\max}$ set by power supply or slew rate limits. Note that this condition must indeed be satisfied for *all* frequencies and not only in the passband, because large signals even outside the passband must not be allowed to overload and saturate the opamps. When opamps are overdriven, their operation becomes *nonlinear* and results in higher harmonics that are generated by the nonlinear opamp behavior. The problems that arise when saturating the opamps are harmonic and intermodulation distortion of the signal, changed operating points, and ultimately deviations in filter performance.

The restriction at the other extreme is the electrical noise. We must make certain that signals stay large enough to remain above the electrical noise created by random movements of electrons. They cause the currents to fluctuate, which results in small random signals in the circuit (in the microvolt to millivolt range). Once the applied signal *in the passband* becomes