

Energy barrier

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Protocol

The protocol is as follows.

1. An element is selected for triggering (in the example below chosen in the center of the system). Its location is denoted by \vec{r}' .
2. A perturbation around a stress and strain free configuration is considered. To this end, the selected element (only) is subjected to an eigen stress $\boldsymbol{\sigma}'$. The corresponding equilibrium configuration then constitutes to the perturbation that will be used. It is characterised by the stress field $\vec{u}^*(\vec{r})$, and corresponding stress $\boldsymbol{\sigma}^*(\vec{r})$ and strain $\boldsymbol{\varepsilon}^*(\vec{r})$ fields.
3. Two types of perturbations are considered:
 - Simple shear: $\boldsymbol{\sigma}' = \boldsymbol{\sigma}'_s = \vec{e}_x \vec{e}_y + \vec{e}_y \vec{e}_x$. Gives: $\vec{u}_s^*(\vec{r})$, $\boldsymbol{\sigma}_s^*(\vec{r})$, and $\boldsymbol{\varepsilon}_s^*(\vec{r})$.
 - Pure shear: $\boldsymbol{\sigma}' = \boldsymbol{\sigma}'_p = \vec{e}_x \vec{e}_x - \vec{e}_y \vec{e}_y$. Gives: $\vec{u}_p^*(\vec{r})$, $\boldsymbol{\sigma}_p^*(\vec{r})$, and $\boldsymbol{\varepsilon}_p^*(\vec{r})$.

For the triggered element the strain (and) stress are (by definition) of the following structure: $\boldsymbol{\varepsilon}_s^*(\vec{r}') = \chi_s(\vec{e}_x \vec{e}_y + \vec{e}_y \vec{e}_x)$ for the simple shear perturbation, and $\boldsymbol{\varepsilon}_p^*(\vec{r}') = \chi_p(\vec{e}_x \vec{e}_x - \vec{e}_y \vec{e}_y)$ for the pure shear perturbation.

4. A perturbation $\delta \vec{u}(\vec{r}) = s \vec{u}_s^*(\vec{r}) + p \vec{u}_p^*(\vec{r})$ is then applied such that it results in the minimal increase of potential energy in the landscape.

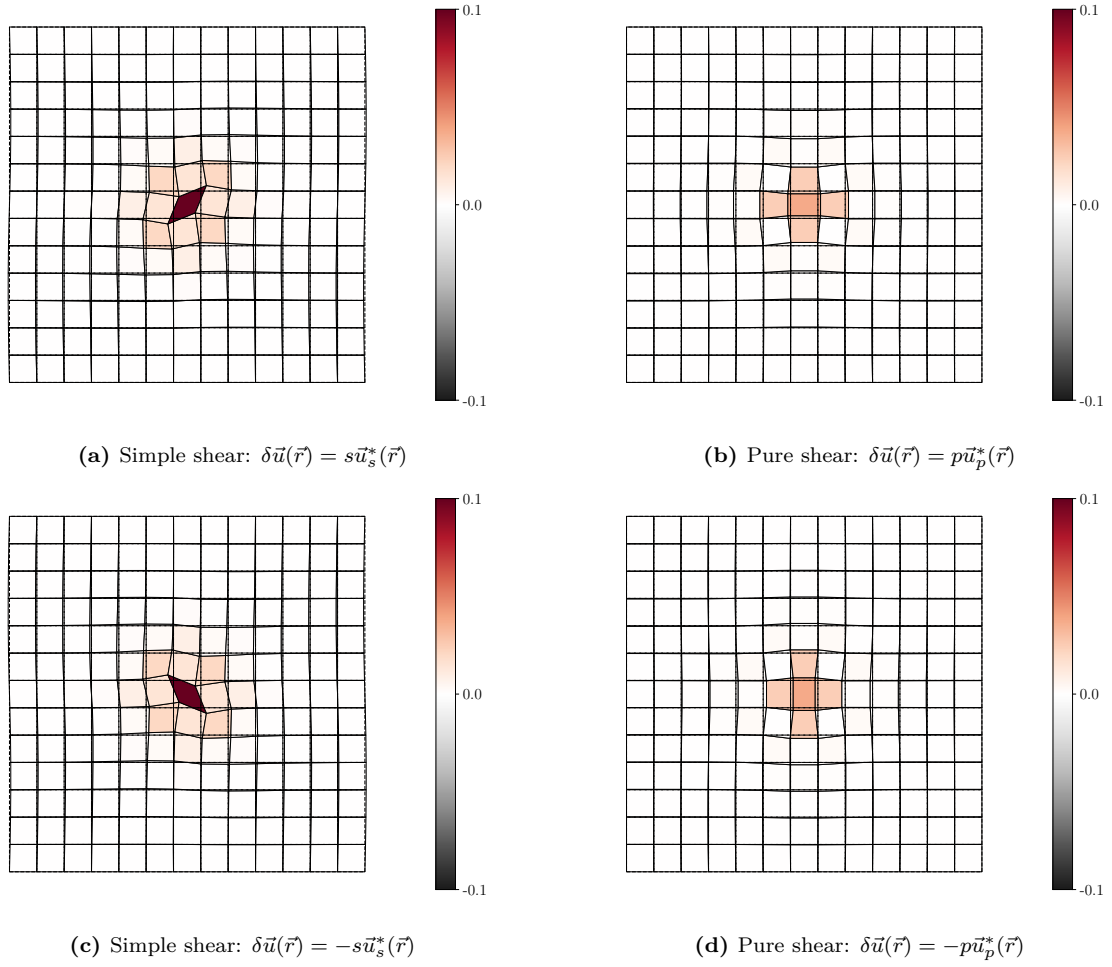


Figure 1. Perturbation modes. The shown colour is the energy change resulting from the perturbation.

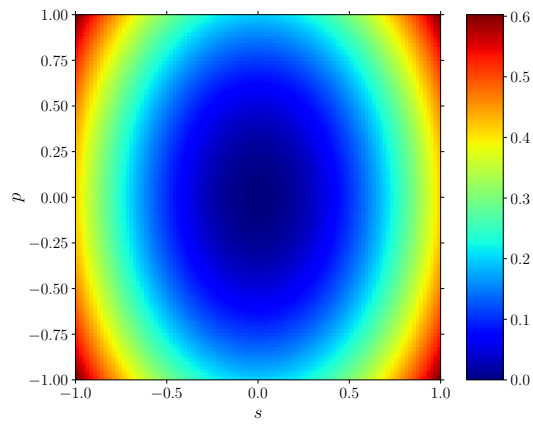
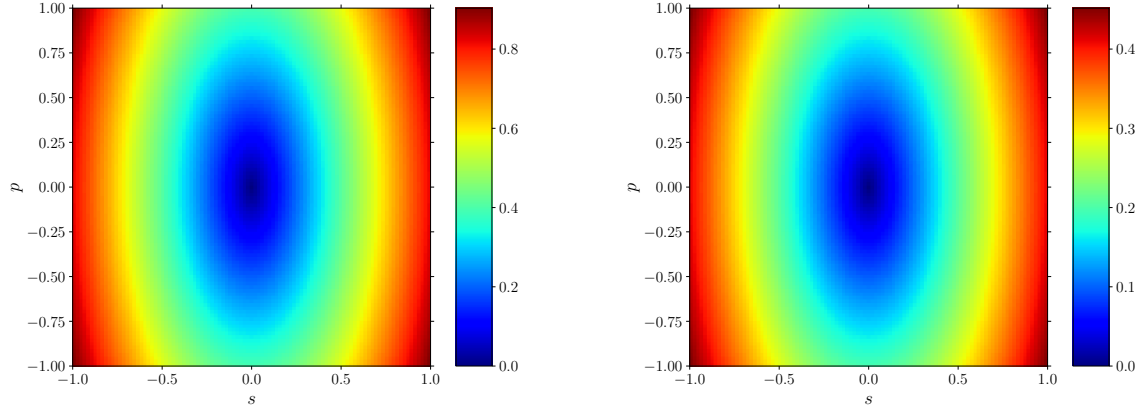


Figure 2. Energetic cost of a perturbation: $\delta\vec{u}(\vec{r}) = s\vec{u}_s^*(\vec{r}) + p\vec{u}_p^*(\vec{r})$.



(a) Equivalent stress.

(b) Equivalent strain.

Figure 3. Resulting (a) equivalent stress and (b) equivalent strain for a perturbation: $\delta \vec{u}(\vec{r}) = s \vec{u}_s^*(\vec{r}) + p \vec{u}_p^*(\vec{r})$.

Example

An example for two configurations in Fig. 4 is shown in Fig. 5.

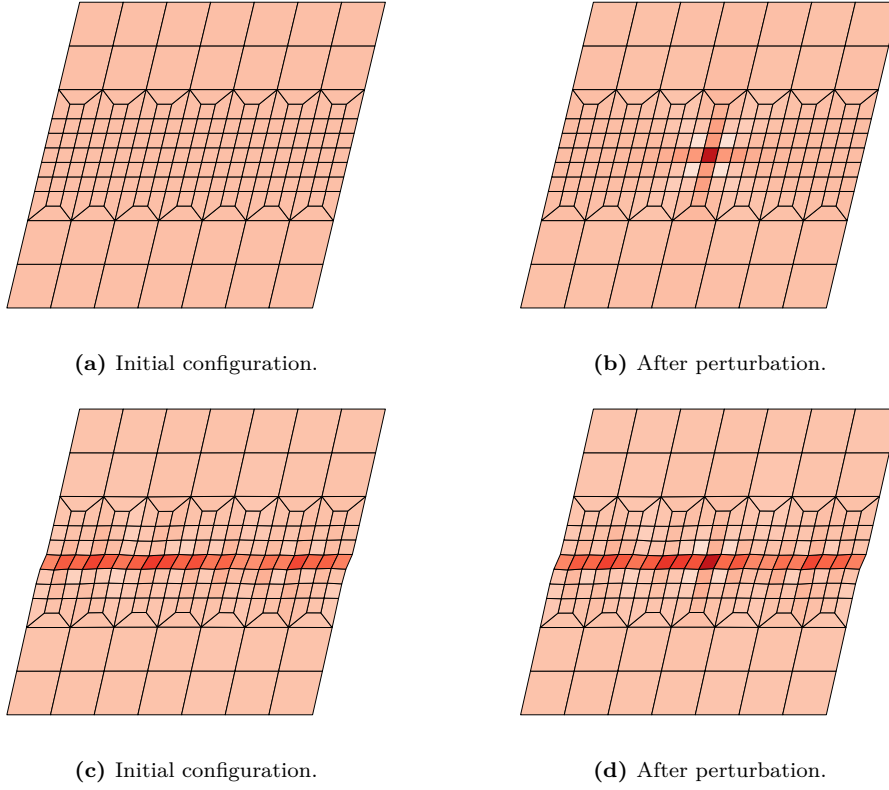


Figure 4. Starting and perturbed configuration, for (a-b) a homogeneous sheared system, and (c-d) a homogeneous sheared system subjected to a random pre-stress.

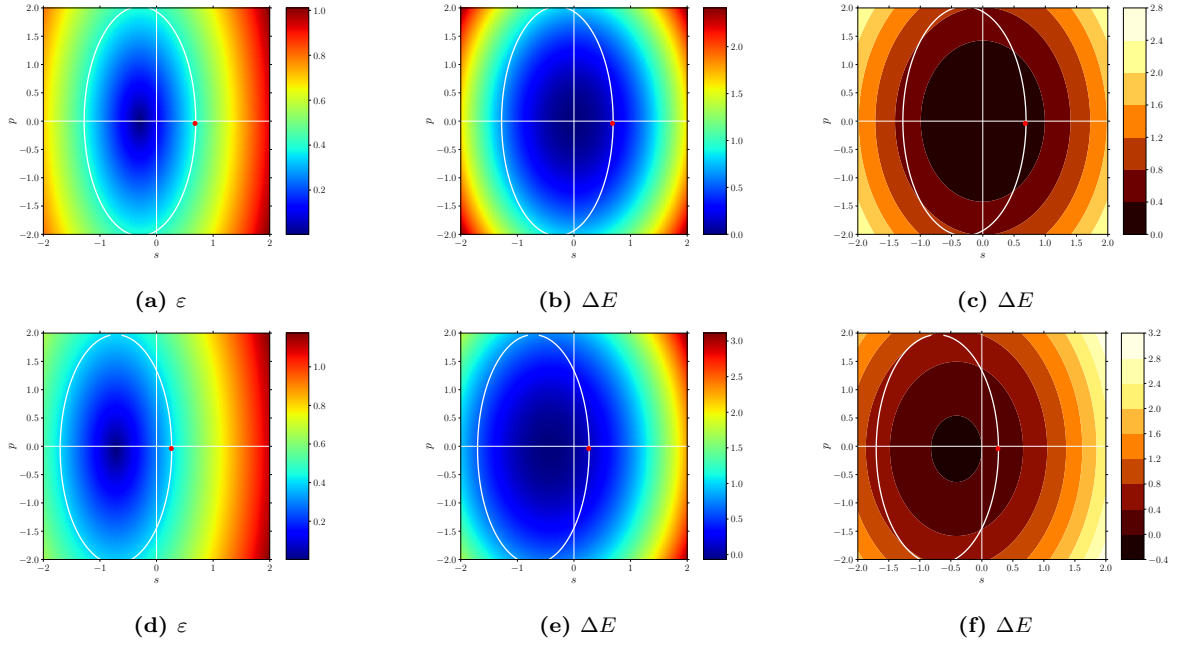


Figure 5. Phase diagram for the configurations in Fig. 4.