Energy barrier

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Protocol

The protocol is as follows.

1. An element is selected for triggering (in the example below chosen in the center of the system). Its location is denoted by $\vec{r}'$.

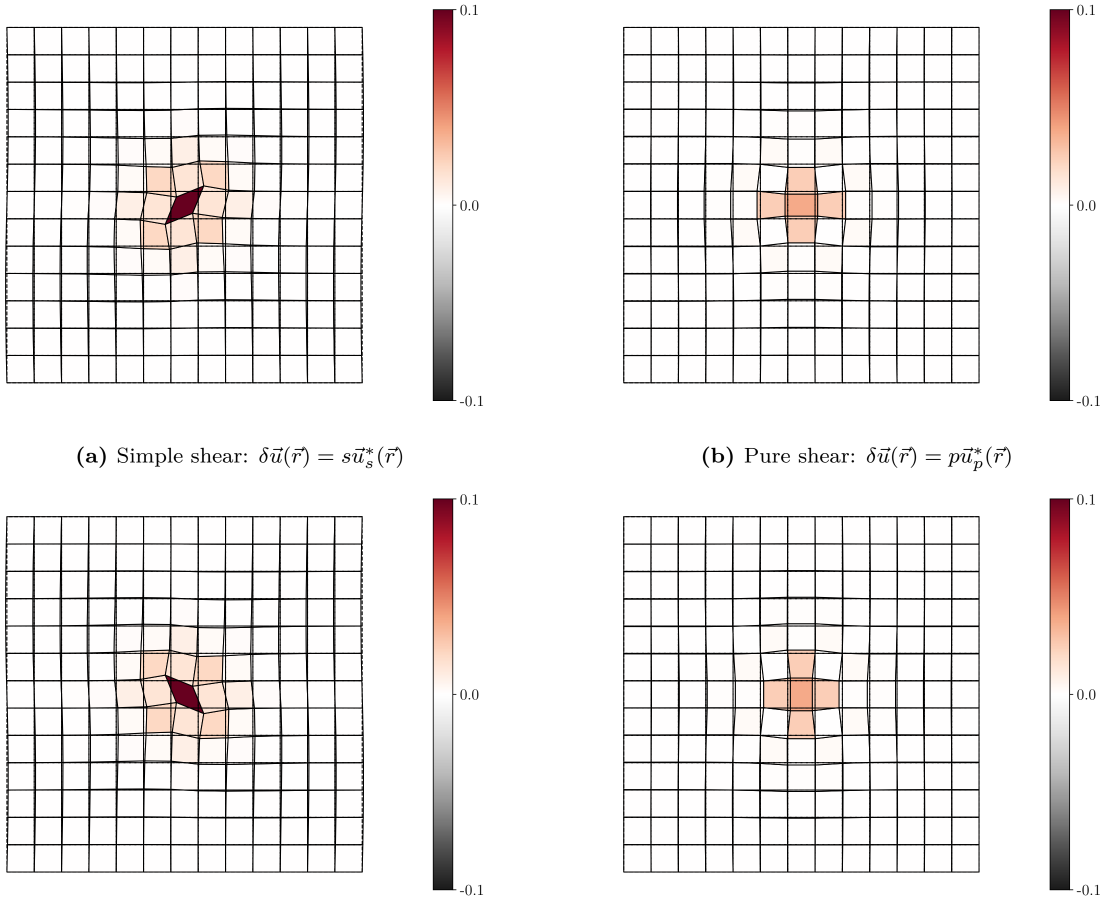
2. A perturbation around a stress and strain free configuration is considered. To this end, the selected element (only) is subjected to an eigen stress $\boldsymbol{\sigma}'$. The corresponding equilibrium configuration then constitutes to the perturbation that will be used. It is characterised by the stress field $\vec{u}^*(\vec{r})$, and corresponding stress $\boldsymbol{\sigma}^*(\vec{r})$ and strain $\boldsymbol{\varepsilon}^*(\vec{r})$ fields.

3. Two types of perturbations are considered:

   - Simple shear: $\boldsymbol{\sigma}' = \boldsymbol{\sigma}'_s = \vec{e}_x\vec{e}_y + \vec{e}_x\vec{e}_y$. Gives: $\vec{u}^*_s(\vec{r})$, $\boldsymbol{\sigma}^*_s(\vec{r})$, and $\boldsymbol{\varepsilon}^*_s(\vec{r})$.
   - Pure shear: $\boldsymbol{\sigma}' = \boldsymbol{\sigma}'_p = \vec{e}_x\vec{e}_x - \vec{e}_y\vec{e}_y$. Gives: $\vec{u}^*_p(\vec{r})$, $\boldsymbol{\sigma}^*_p(\vec{r})$, and $\boldsymbol{\varepsilon}^*_p(\vec{r})$.

   For the triggered element the strain (and) stress are (by definition) of the following structure: $\boldsymbol{\varepsilon}^*_s(\vec{r}') = \chi_s(\vec{e}_x\vec{e}_y + \vec{e}_x\vec{e}_y)$ for the simple shear perturbation, and $\boldsymbol{\varepsilon}^*_p(\vec{r}') = \chi_p(\vec{e}_x\vec{e}_x - \vec{e}_y\vec{e}_y)$ for the pure shear perturbation.

4. A perturbation $\delta\vec{u}(\vec{r}) = s\vec{u}^*_s(\vec{r}) + p\vec{u}^*_p(\vec{r})$ is then applied such that it results in the minimal increase of potential energy in the landscape.



(a) Simple shear: $\delta\vec{u}(\vec{r}) = s\vec{u}_s^*(\vec{r})$

(b) Pure shear: $\delta\vec{u}(\vec{r}) = p\vec{u}_p^*(\vec{r})$

(c) Simple shear: $\delta\vec{u}(\vec{r}) = -s\vec{u}_s^*(\vec{r})$

(d) Pure shear: $\delta\vec{u}(\vec{r}) = -p\vec{u}_p^*(\vec{r})$

Figure 1. Perturbation modes. The shown colour is the energy change resulting from the perturbation.

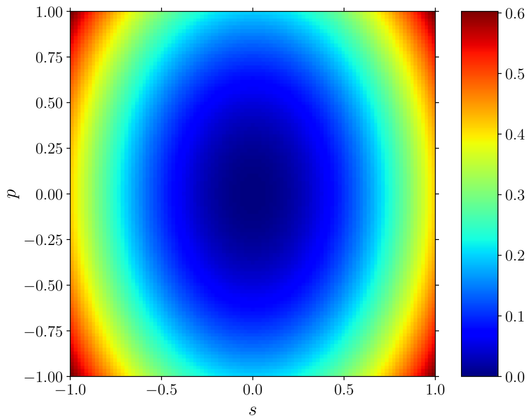
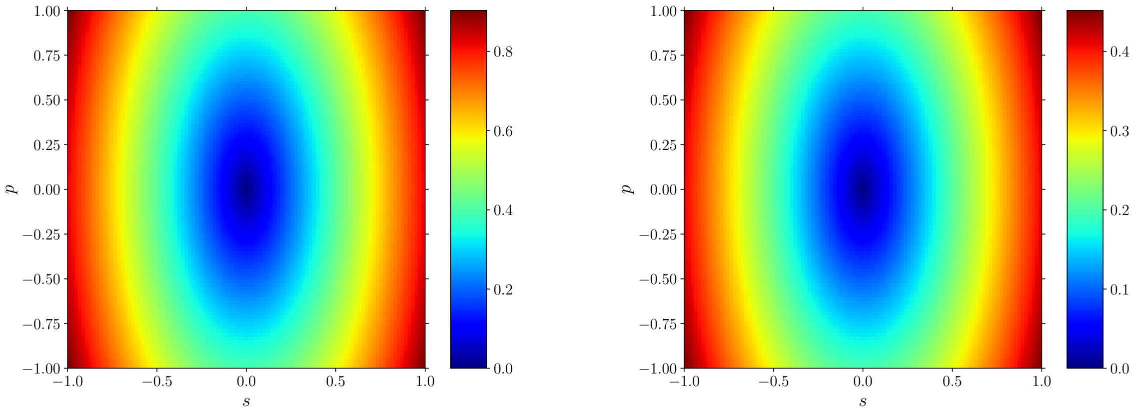


Figure 2. Energetic cost of a perturbation: $\delta\vec{u}(\vec{r}) = s\vec{u}_s^*(\vec{r}) + p\vec{u}_p^*(\vec{r})$.



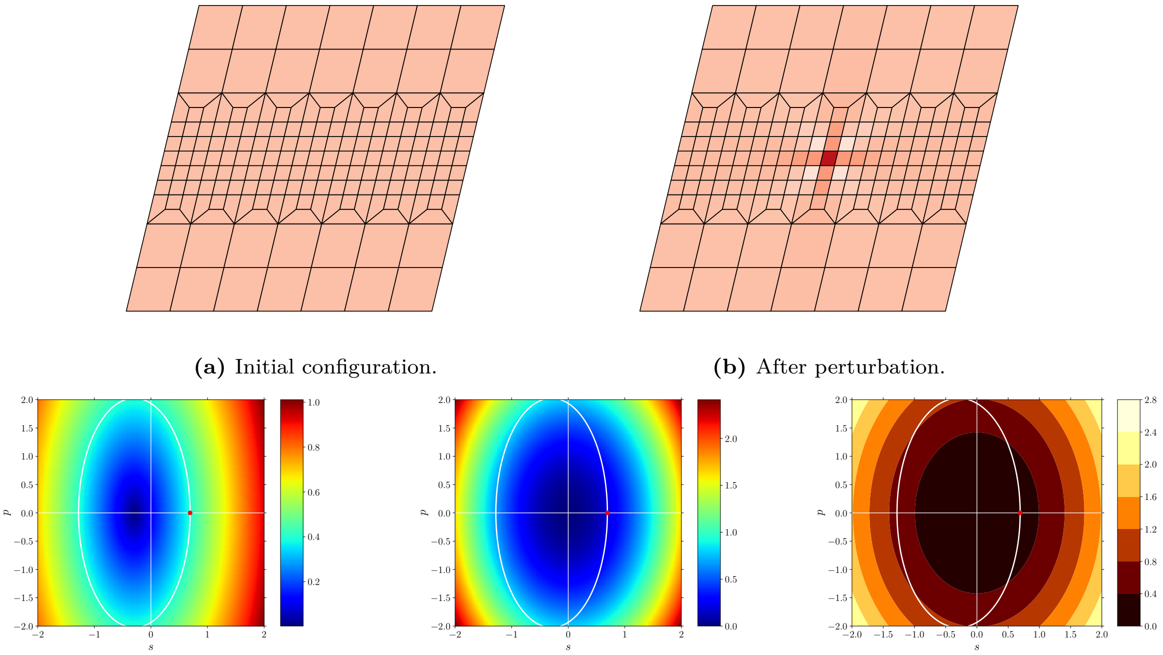
(a) Equivalent stress.

(b) Equivalent strain.

Figure 3. Resulting (a) equivalent stress and (b) equivalent strain for a perturbation: $\delta\vec{u}(\vec{r}) = s\vec{u}_s^*(\vec{r}) + p\vec{u}_p^*(\vec{r})$.

Example

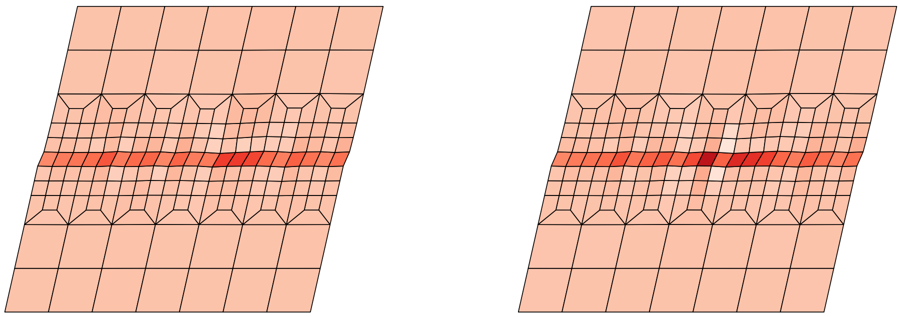
Two examples are included: A homogeneous medium that is subjected to shear in Fig. 4, and the sample problem additionally subject to a random stress perturbation in Fig. 5.



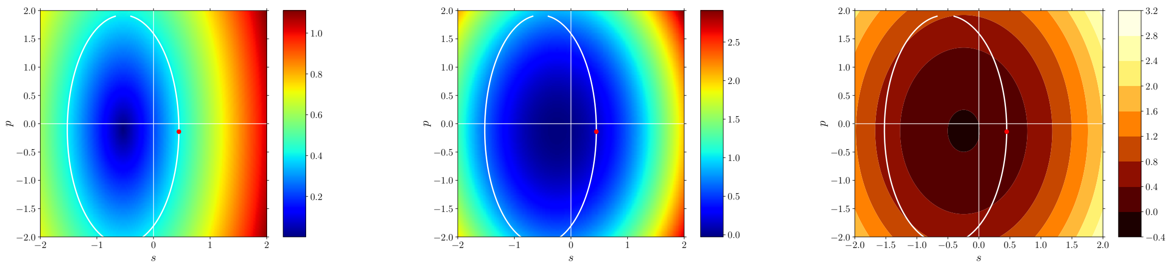
(a) Initial configuration. **(b)** After perturbation.

(c) ε **(d)** ΔE **(e)** ΔE

Figure 4. (a–b) Starting and perturbed configuration for a homogeneous sheared system. (c–e) Phase diagram of a perturbation $\delta\vec{u}(\vec{r}) = s\vec{u}_s^*(\vec{r}) + p\vec{u}_p^*(\vec{r})$ of the configuration in (a).



(a) Initial configuration. **(b)** After perturbation.



(c) ε **(d)** ΔE **(e)** ΔE

Figure 5. (a–b) Starting and perturbed configuration for a homogeneous sheared system. (c–e) Phase diagram of a perturbation $\delta\vec{u}(\vec{r}) = s\vec{u}^*_s(\vec{r}) + p\vec{u}^*_p(\vec{r})$ of the configuration in (a).