Energy barrier

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Protocol

The protocol is as follows.

1. An element is selected for triggering (in the example below chosen in the center of the system). Its location is denoted by $\vec{r}'$.

2. A perturbation around a stress and strain free configuration is considered. To this end, the selected element (only) is subjected to an eigen stress $\boldsymbol{\sigma}'$. The corresponding equilibrium configuration then constitutes to the perturbation that will be used. It is characterised by the stress field $\vec{u}^*(\vec{r})$, and corresponding stress $\boldsymbol{\sigma}^*(\vec{r})$ and strain $\boldsymbol{\varepsilon}^*(\vec{r})$ fields.

3. Two types of perturbations are considered:

   - Simple shear: $\boldsymbol{\sigma}' = \boldsymbol{\sigma}'_s = \vec{e}_x\vec{e}_y + \vec{e}_x\vec{e}_y$. Gives: $\vec{u}^*_s(\vec{r})$, $\boldsymbol{\sigma}^*_s(\vec{r})$, and $\boldsymbol{\varepsilon}^*_s(\vec{r})$.
   - Pure shear: $\boldsymbol{\sigma}' = \boldsymbol{\sigma}'_p = \vec{e}_x\vec{e}_x - \vec{e}_y\vec{e}_y$. Gives: $\vec{u}^*_p(\vec{r})$, $\boldsymbol{\sigma}^*_p(\vec{r})$, and $\boldsymbol{\varepsilon}^*_p(\vec{r})$.

   For the triggered element the strain (and) stress are (by definition) of the following structure: $\boldsymbol{\varepsilon}^*_s(\vec{r}') = \chi_s(\vec{e}_x\vec{e}_y + \vec{e}_x\vec{e}_y)$ for the simple shear perturbation, and $\boldsymbol{\varepsilon}^*_p(\vec{r}') = \chi_p(\vec{e}_x\vec{e}_x - \vec{e}_y\vec{e}_y)$ for the pure shear perturbation.

4. The two perturbative modes are scaled such that the energy increase to a perturbation with $\delta\vec{u}(\vec{r}) = s\vec{u}_s(\vec{r}) + p\vec{u}_p(\vec{r})$ is isotropic in the (s, p)-space. In particular, prefactors ψ_s and ψ_p are used to obtain

   $$\vec{u}_i = \psi_i \vec{u}^*_i, \qquad \boldsymbol{\sigma}_i = \psi_i \boldsymbol{\sigma}^*_i, \qquad \boldsymbol{\varepsilon}_i = \psi_i \boldsymbol{\varepsilon}^*_i, \qquad \text{for } i \in (s, p) \tag{1}$$

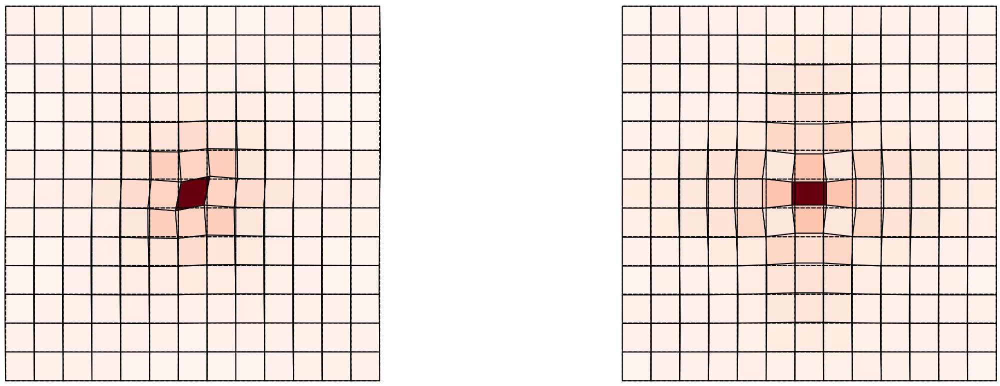
   whereby

   - Simple shear: $1/\psi_s = (\sigma_{xy})^*_s(\varepsilon_{xy})^*_s$
   - Simple shear: $1/\psi_p = (\sigma_{yy})^*_p(\varepsilon_{yy})^*_p$

   taking the stress and strain for the triggered element.

   The resulting perturbative displacement modes are shown in Fig. 1, whereby the corresponding equivalent deviatoric stress is shown in colour.

5. A perturbation with $\delta\vec{u}(\vec{r}) = s\vec{u}_s(\vec{r}) + p\vec{u}_p(\vec{r})$ now results in an energy increase in the system that is isotropic in the (s, p) domain, see Fig. 2. In contrast the increases in equivalent stress and strain span an ellipse in the (s, p) domain, see Fig. 3.



(a) Simple shear **(b)** Pure shear

Figure 1. Perturbation modes. The shown colour is the equivalent deviatoric stress resulting from the perturbation.

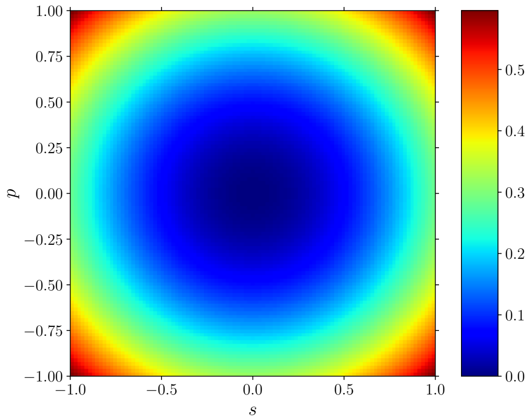
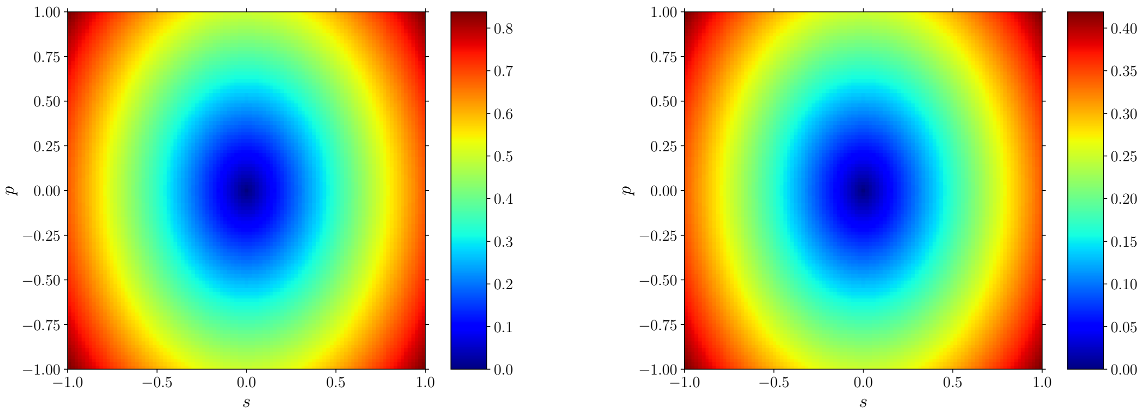


Figure 2. Energetic cost of a perturbation: $\delta\vec{u}(\vec{r}) = s\vec{u}_s(\vec{r}) + p\vec{u}_p(\vec{r})$.



(a) Equivalent stress. **(b)** Equivalent strain.

Figure 3. Resulting (a) equivalent stress and (b) equivalent strain for a perturbation: $\delta\vec{u}(\vec{r}) = s\vec{u}_s(\vec{r}) + p\vec{u}_p(\vec{r})$.

Example

As an example a configuration is considered in which an elastic medium is subjected to a shear deformation of the top boundary and a random prestress along the middle layer. The resulting equilibrium configuration is shown in Fig. 4(a).

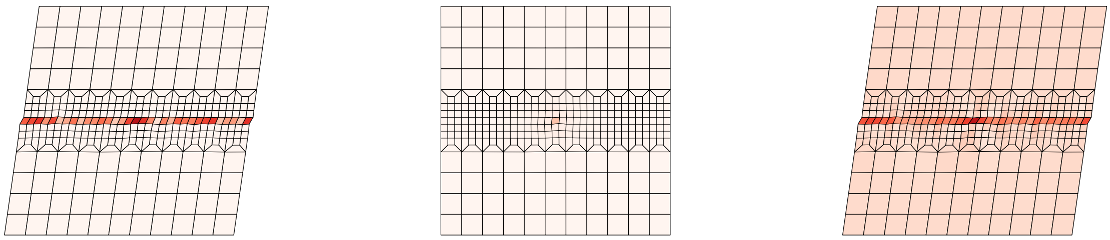
An element is then selected for triggering such that the equivalent strain in the element increases to some value ε_y. For the described protocol the perturbation of minimal energy, results in the following strain tensor in the triggered element

$$\boldsymbol{\varepsilon} = \begin{bmatrix} E_0 & \gamma_0 \\ \gamma_0 & -E_0 \end{bmatrix} + \eta \begin{bmatrix} \delta E & \delta\gamma \\ \delta\gamma & -\delta E \end{bmatrix} \tag{2}$$

where $\eta = s = p$ thus follows as the solution of

$$\varepsilon_y^2 = (E_0 + \eta\delta E)^2 + (\gamma_0 + \eta\delta\gamma)^2 \tag{3}$$

The perturbation is shown in Fig. 4(b) and the perturbed configuration where the equivalent strain in the triggered element is equal to ε_y is shown in Fig. 4(c).



(a) Starting configuration.

(b) Applied perturbation.

(c) Perturbed starting configuration.

Figure 4. Friction-like example.