

ARIZONA STATE UNIVERSITY

HONORS THESIS

DIY Supercube

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*A thesis submitted in fulfillment of the requirements
for the degree of Honors Thesis in Software Engineering
in the*

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Declaration of Authorship

I, Joseph HALE, declare that this thesis titled, "DIY Supercube" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed:

Date:

“Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism.”

Dave Barry

ARIZONA STATE UNIVERSITY

Abstract

Dr. Robert Heinrichs
Barrett, The Honors College

Honors Thesis in Software Engineering

DIY Supercube

by Joseph HALE

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor...

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List of Abbreviations

TPS Turns Per Second

Physical Constants

Speed of Light $c_0 = 2.997\,924\,58 \times 10^8 \text{ m s}^{-1}$ (exact)

List of Symbols

a	distance	m
P	power	$\text{W} (\text{J s}^{-1})$
ω	angular frequency	rad

For/Dedicated to/To my...

Chapter 1

Introduction

1.1 The Advent of Smart Cubes

Speedsolving, the sport of solving twisty puzzles like the Rubik's Cube as fast as possible, has seen a resurgence of popularity since the early 2000s. [TODO] Over the past two decades many advances in cube technology have produced ever higher performing puzzles.

Recently, the speedcubing community has seen the entrance of smart cubes, special versions of a Rubik's Cube built around hardware that can connect to a mobile device over Bluetooth. These smart cubes have sparked a wave of excitement with the vast opportunities they offer for automatic turn tracking, performance analysis, personalized improvement feedback, and networked competition.

1.2 Obstacles to Adoption

While a revolutionary idea, smart cubes still face several obstacles to widespread adoption.

- *Cost:* Smart cubes can cost up to eight times as much as a comparable non-smart speedcube.¹
- *Performance:* Existing smart cubes turn slower than comparable non-smart cubes. [TODO]
- *Reliability:* Many smart cube owners report inability to connect the smart cube to a mobile device and missed/inaccurate turn tracking. [TODO]
- *Regulation:* Current competition rules ban the use of electronics during timed solves, thus banning the use of smart cubes. There is no foreseeable change to this rule. [TODO]

As a result of these obstacles, many speedcubers refrain from purchasing a smart cube, despite expressing significant interest in the opportunities smart cubes offer.

¹For example, one popular budget speedcube, the Moyu Weilong, costs only \$5, while the cheapest smartcube, the Giiker Cube, starts at \$40. [TODO]. On the higher end, a premium speedcube, like the Gans 356 XS, retails for just over \$60 while a premium smartcube, like the GoCube, retails for over \$100. [TODO]

Furthermore, all current smartcubes have been specifically built for the primary purpose of providing move-tracking functionality. There is no existing way to automatically track the moves of a standard, "non-smart" speedcube.

1.3 Purpose of this Thesis

The primary goal of this thesis is to create a proof-of-concept for a smart cube design that can enable a speedcuber to use his/her personal favorite cube, while still having all the benefits of a smart cube.

In other words, this thesis seeks to answer the following question:

Is it possible to track the face turns of a standard, "non-smart" speedcube in a non-destructive, competition-legal way?

1.4 Thesis Overview

TODO give an overview of the rest of the Thesis document.

Chapter 2

Background

TODO Each chapter starts with a paragraph that briefly outlines the purpose of each of the sections.

2.1 A Brief History of the Rubik's Cube

In 1974, Erno Rubik, a Hungarian professor of architecture, sought to help his students visualize space in three dimensions. To that end, he created a special cube whose faces could independently rotate around all three physical axes [1]. When he added colored stickers to further aid in visualizing the movements, Mr. Rubik realized he had created a new puzzle. He patented his cube in 1975, [2] and since then over 450 million units have been sold [3], allowing an estimated 1 in 7 humans on earth to try their hand at solving it [4].

Since then, the cube has been the subject of academic research, competition, leisure, and cultural iconography.

2.2 The Anatomy of a Rubik's Cube

The Rubik's Cube, like a standard geometric cube, has six faces, all of which are squares and positioned at right angles to each other. When solved, each of these faces has a single, unique color.

The Rubik's Cube is further subdivided into a 3x3x3 arrangement of smaller "cubies" such that each face consists of nine individual colored stickers/tiles. There are three different types of cubies: centers, edges, and corners. Each type of cubie is distinguished by the number of unique colors it binds together into a single physical unit.

Type of Cubie	Number of Colors	Count per Rubik's Cube
Center	1	6^a
Edge	2	12
Corner	3	8

^aAll six center pieces are attached a single core

TABLE 2.1: The number of each type of Rubik's Cube cubie

Each of the six center cubies are also attached to a common core which allows them to rotate freely, but fixes their position relative to each other. As such, the single color of each center cubie is also the color shared by the corresponding face when the entire cube is solved.

2.2.1 Algorithm Notation

TODO

2.2.2 The Laws of the Cube

TODO Describe the basic concepts of group theory that stipulate what positions are and aren't legal. It might also be fun to discuss the derivation of the 43 quintillion possible positions on the cube.

2.3 Speedsolving

TODO

2.3.1 The World Cube Association

TODO

2.3.2 Competition Regulations

TODO

2.4 The Rise of Smart Cubes

TODO

Chapter 3

State of the Art

3.1 Introduction

This chapter seeks to provide a comprehensive summary of the existing approaches to tracking the face turns of a Rubik's Cube. The most widely used solutions to date are found in Commercial Smartcubes (3.2), but significant research has also been carried out into Computer Vision based solutions (3.3.1). Other researchers have also explored the use of magnetic resonance and a muscle-tracking armband (3.4)

This chapter will also explore a selection of wireless communication techniques that at the time of writing have not been applied to the challenge of tracking the moves of a Rubik's Cube. Specifically, this chapter will review the potential usage of sound (3.4.1), Radio Frequency Identification (RFID) (3.4.2), and Off-Axis Magnetic Angle Sensors (3.4.3).

Finally, this chapter will close by detailing the specific research questions this thesis will seek to answer (3.5).

3.2 Commercial Smartcubes

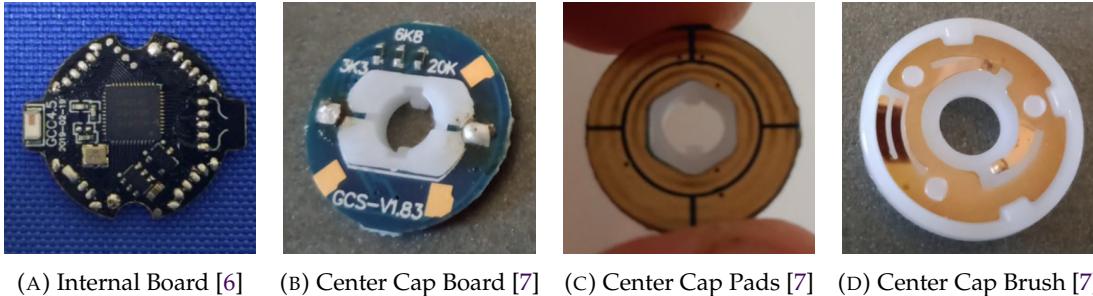
Commercial Smartcubes are special Rubik's Cubes built around sensors that can detect face turns and transmit that information over Bluetooth. Some models can also measure and transmit data about the cube's orientation.

At the time of writing, there are four major smartcubes on the market: the Giiker Cube, the Go Cube, the Rubik's Connected (which is powered by GoCube technology) and the Gans 356i. This section will discuss the internal components of each of these cubes that provide this move-tracking functionality.

3.2.1 Giiker Cube

The Xiaomi Giiker Cube was released in September 2018 making it the first commercial smartcube on the market [5]. This "Supercube" as it was branded, used a relatively simple system for tracking the cube's movements. The core of the cube is built around a small circuit board with a microcontroller that measures the cube's movements and

a bluetooth antenna that transmits those moves wirelessly (Figure 3.1a). The microcontroller detects each face turn by reading the voltage drop across a small circuit embedded within each center cap (Figure 3.1b). The center cap circuit controls its output voltage by using a copper brush to change between four separate electric paths, three different resistors and ground, as each face rotates (Figure 3.1d). [6]



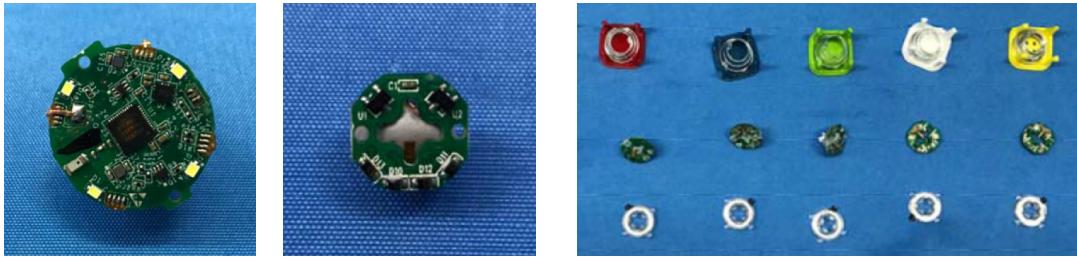
(A) Internal Board [6] (B) Center Cap Board [7] (C) Center Cap Pads [7] (D) Center Cap Brush [7]

FIGURE 3.1: The internal components of the Giiker Cube

3.2.2 Go Cube

Announced on Kickstarter in June 2018, Patricula's GoCube was the first smartcube to include a gyroscope that would track a Rubik's Cube's orientation in addition to the face turns applied to it. [8] Like the Giiker Cube before it, the GoCube's core contains a small circuit board with the main electronics including a microcontroller, bluetooth antenna, and the added gyroscope (Figure 3.2a). Though the teardown pictures from the Go Cube's FCC filing aren't particularly clear, it appears that the cube registers face turns similarly to the Giiker Cube: by producing a voltage drop via changing which one of the four resistors shown across the bottom of the center cap board in Figure 3.2b is in series with the circuit.

GoCube also serves as the underlying technology for the Rubik's Connected, the official smartcube from The Rubik's Company. [9]



(A) Internal Board

(B) Center Cap Board

(C) Full Center Cap Teardown

FIGURE 3.2: The internal components of the Go Cube [10]

3.2.3 Gans 356i

Released in July 2019, the Gans 356i was the first commercial smartcube produced by a traditional speedcube manufacturer. [11] While the Gans 356i also uses a microcontroller to process the face turns and Bluetooth to transmit the move data, it tracks moves

not through changing resistors in and out of a circuit, but via six plastic rods that connect the outer center caps to internal rotary encoders (Figure 3.3).

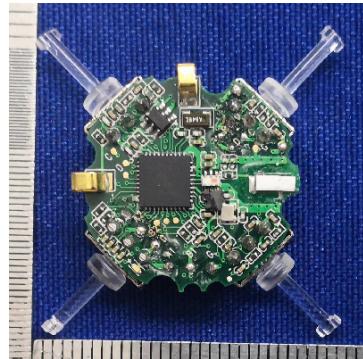


FIGURE 3.3: The Gans 356i Cube's internal components. [12]

3.3 Academia

In addition to the various commercial smartcubes, many academic research projects have involved some element of tracking the state/face turns of a Rubik's Cube.

This section summarizes the current state of academic research into using computer vision, magnetic resonance, and a muscle-tracking armband to track the state of a Rubik's Cube.

3.3.1 Computer Vision

Computer Vision refers to the "field of Artificial Intelligence (AI) that enables computers and systems to derive meaningful information from digital images, videos, and other visual inputs." [13] Since human manipulation of a Rubik's Cube is a physical, observable process, Computer Vision algorithms could be developed to extract face turn information from videos of Rubik's Cube solutions.

This section summarizes some of the relevant research in this area, including computer vision algorithms capable of extracting individual sticker colors from video, measuring the angle of rotation of a specific face, and detection of entire face turns and face turn sequences.

Sticker Color Classification

In 2015, Jay Hack, a graduate student studying Computer Science at Stanford developed a neural network capable of recognizing the colors of a Rubik's Cube face from video in various lighting conditions. His algorithm could classify frames within 7 milliseconds with 92% accuracy. [14]

Measuring a Face's Angle of Rotation

In 2019, OpenAI et al. published a viral video of a robot hand that had taught itself to solve a Rubik's Cube. While the final, most successful version of the robot hand's software used a Giiker Cube to obtain the current rotational state of the cube, OpenAI et al. also researched the viability of tracking a Rubik's Cube's position using only computer vision. Their most successful vision-only algorithm measured only the rotation angle of the top-most face on the Rubik's Cube and assumed significant hardware requirements: a modified sticker set for the Rubik's Cube, a well-lit environment, three strategically positioned RBG Basler cameras, and a neural network trained on "a pool of optimizer nodes, each of which uses 8 NVIDIA V100 GPUs and 64 CPU cores". At peak performance, their vision-only algorithm's average error (the difference between the predicted face angle and the actual face angle) was 15.92° , nearly three times the 5.90° average error of the hardware-based face angle measurement. [15]

Classification of Single Moves and Entire Move Sequences

In 2020, Junshen Kevin Chen, Wanze Xie, and Zhouheng Sun, graduate Computer Science students at Stanford created the DeepCube dataset consisting of over 20,000 videos of Rubik's Cube face turns with consistent lighting and backgrounds. They also built a neural network to classify the videos with the face turn they contained. Their best performing model only made "one mistake every 15 moves" which corresponds to a 93.3% accuracy. [16]

3.3.2 Magnetic Resonance

In 2018, Maria Mannone et al. used the IM3D magnetic 3D motion tracking technology introduced by Huang et al. [17] to track the state of a Rubik's Cube across various movements for the purpose of generating a sequence of musical chords. This approach to turn tracking requires a special array of magnetic coils as shown in Figure 3.4a and the installation of "multiple small, light-weight, wireless markers (LC coils) with unique IDs" (a process that requires permanent modifications to the cube as evidenced by the damaged plastic in Figure 3.4b). Mannone et al. reported no issues with mistakes in this move tracking technology. [18]

3.3.3 Muscle-Tracking Armband

In 2017, Richard Polfreman and Benjamin Oliver researched ways to use the face turns of a Rubik's Cube as controls for a music synthesizer. They explored the use of a muscle-tracking armband (specifically the Myo Armband) to track the human solver's finger movements while manipulating the cube. However, since "the Myo moved a little when 'cubing'", they ultimately found greater success with a computer vision based tracking solution similar to those discussed in 3.3.1.

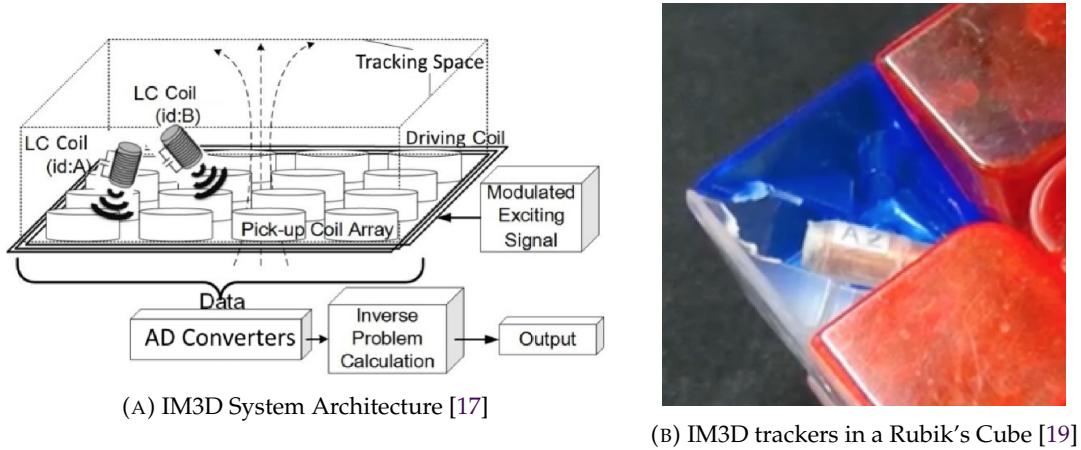


FIGURE 3.4: The IM3D technology as used in the Cube Harmonic

3.4 Other Relevant Research

Finally, there are a number of research papers/commercial products that seek to transmit data in highly-constrained environments that are potentially relevant to the challenge of tracking the turns of a Rubik's Cube.

This section discusses some of these potential alternate move tracking mediums, specifically sound, RFID, and off-angle magnetic rotation sensors.

3.4.1 Sound

Sound is another communication medium that could be leveraged to track the moves of a Rubik's Cube. In 2015, Jonas Michel, a researcher at The University of Texas at Austin documented his exploration of the viability of creating an "acoustic modem" to transmit an arbitrary sequence of bits using sound. He observed that "as commercial off-the-shelf (COTS) smartphones become more powerful, it is worthwhile to revisit the use of sound as a medium for aerial digital device-to-device communications." [20]

Indeed, since many speedcubers practice by timing solves on a microphone-equipped smartphone or laptop¹, sound is a promising alternative communication medium to existing Bluetooth-based smartcubes.

3.4.2 Radio Frequency Identification (RFID)

Radio Frequency Identification (RFID) is a wireless technology often used in supply-chain systems [23] based on individual tags that can transmit a fixed set of information to a nearby reader. [24]. In 2018, Genovesi et al. proposed a rotary encoder based on RFID that produced angle measurements accurate to within 3° after a calibration that "must only be done once (when the sensor is put in place) if the distance between the reader and the tag does not change." [23]

¹One of the highest rated cubing timers on Android, Twisty Timer, has over 100,000 downloads on the Google Play Store [21]. By comparison, SpeedSolving.com, the central forum for speedcubing related discussion, has about 43,000 members [22]

While this approach may not be particularly useful for tracking the moves of human speedcubers since their movement of the cube would require constant re-calibration, it could be used in robotics based applications desiring to track the ongoing state of the cube.

3.4.3 Off-Axis Magnetic Angle Sensors

Another unique type of rotary encoder is an off-angle magnetic rotational sensor like the ones produced by NVE Corporation. [25] In the context of a Rubik's Cube, a properly sized diametric ring magnet could be fastened to the inner screw below the center cap of each face of the cube and the resulting optimal position for the magnetic sensor would stay within the walls of the center cap as shown by the calculations in Figure 3.5.

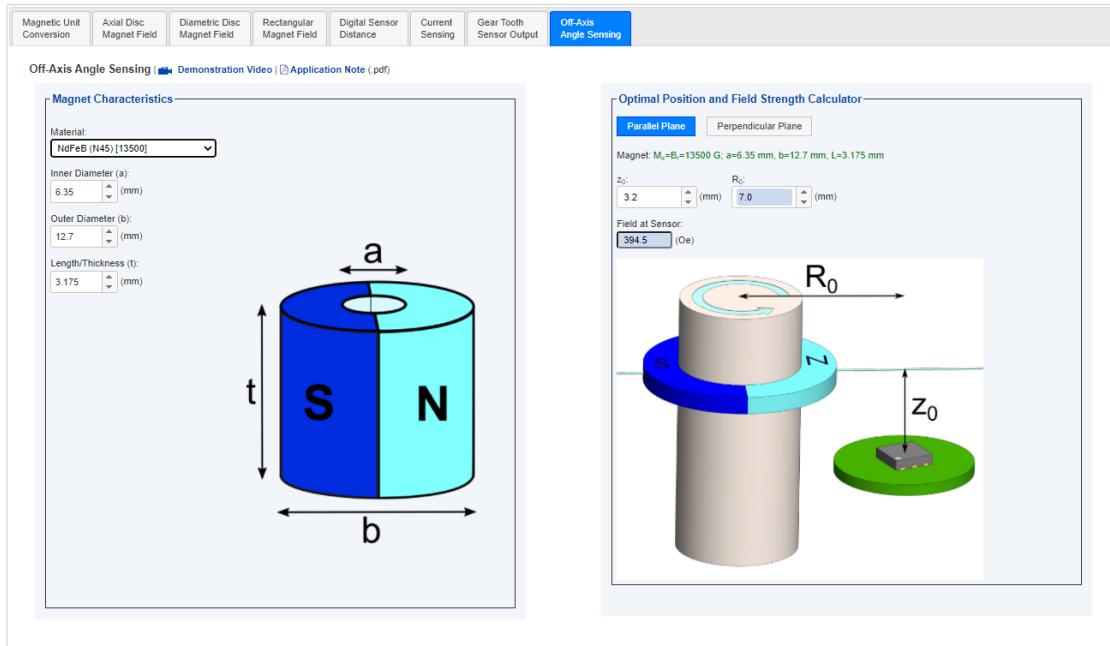


FIGURE 3.5: Off-Angle Magnetic Sensor Calculations. Notice how the z_0 and R_0 values are smaller than the 8 mm distance between the center screw and the inner wall of the center cap in a standard-sized cube like the Gans 356. [26]

3.5 Research Questions

While there are many effective solutions for tracking the moves of a Rubik's Cube if the cube is built for that purpose, there does not yet exist a comparably effective technique for tracking the moves of a standard, "non-smart" Rubik's Cube. As shown above, significant effort has been spent researching and developing solutions that can leverage the Bluetooth transmitters and camera sensors of consumer grade smartphones and laptops with varying degrees of success. However, little to no research has been carried out exploring the viability of using the microphones readily available on the same devices for this purpose.

Thus, this thesis will seek to answer the overarching question from Chapter 1 "Is it possible to track the face turns of a standard, "non-smart" speedcube in a non-destructive, competition-legal way?" by detailing a proof-of-concept for a sound-based smartcube design created in response to the following questions:

1. **Feasibility on Consumer Hardware:** *What are the constraints for a sound-based smart cube design compatible with consumer-grade microphones like those found in common smartphones and laptops?*
2. **Compatibility with Standard Speedcubes:** *How could such a sound-based smart cube design be deployed within a standard, "non-smart" speedcube without requiring permanent modifications to the original cube?*
3. **Move Tracking Accuracy:** *How could such a sound-based smart cube design track the face turns of a Rubik's Cube with high accuracy?*
4. **Move Tracking Granularity:** *How could such a sound-based smart cube design record the time spent executing each individual face turn of a Rubik's Cube?*
5. **Competition Legality:** *How could such a sound-based smartcube design comply with competition regulations prohibiting the use of electronics while performing a competitive solve?*

Chapter 4

Protocol Design

4.1 Introduction

Designing a sound-based protocol for tracking the moves of a Rubik's Cube requires great care. Lots of things produce sound that could interfere with the protocol: people talking, machines operating, nature stirring, and so forth. Furthermore, the structure of the Rubik's Cube itself imposes stringent physical constraints on the size of any components used to produce the sounds used in the protocol.

This chapter first seeks to clearly detail the specific constraints that must be considered when designing a sound-based protocol for tracking the moves of a Rubik's Cube (4.2). From there, a specific protocol that meets these constraints will be proposed in Section 4.3. This proposed protocol will then be contrasted with an alternative option in Section 4.4. Finally, an overview of the plan to implement the proposed protocol in a proof-of-concept will be given in Section 4.5

4.2 Requirements

This section will detail the constraints required for designing a protocol for tracking the moves of a Rubik's Cube.

These constraints include a sufficiently strong signal-to-noise ratio (4.2.1), sufficient distinctiveness between tones (4.2.2), the frequency response range of consumer hardware (4.2.3), and the human auditory range (4.2.4).

4.2.1 Signal-to-Noise Ratio

Sound is an easily accessible, and therefore noisy, medium of communication. As a result, any data transmission protocol based on sound must be resilient to the presence of additional noise unrelated to the signal being transmitted. For a sound-based protocol to be effective, its tones must be easily distinguishable from background noise.

For the purpose of this thesis, "background noise" will be considered the ambient noise present in a quiet room when a speedcuber is actively solving a Rubik's Cube. To measure this background noise, three recordings were taken in a household bedroom (a typical place for a speedcuber to practice solving the cube) while solving a variety of

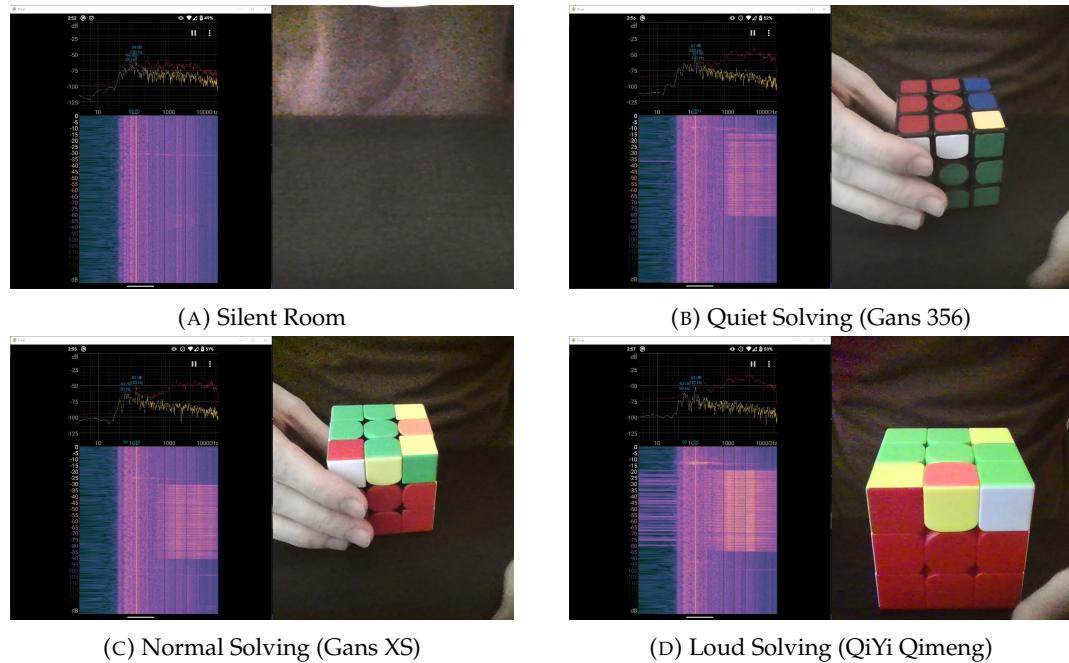


FIGURE 4.1: The background noise of a quiet room compared to solving several speedcubes

speedcubes selected based on their "noisiness" along with a fourth control recording taken of just the ambient noise in the bedroom (i.e. while no cubes were being solved).

Measurements were taken for the following specific background environments:

1. Background noise in a room without any use of a speedcube (Control).
2. Background noise in a room while solving a "quiet" speedcube (the Gans 356).
3. Background noise in a room while solving a "standard-volume" speedcube (the Gans XS).
4. Background noise in a room while solving a "noisy" speedcube (the QiYi Qimeng).

As shown in Figure 4.1, the predominant background noises in a "quiet" room are the tones between 80Hz - 500Hz which reach a max strength of -54dB. In contrast, the background noise of solving a speedcube spans all the way from 1000Hz - 20000Hz, with particular strength from the noisy QiYi cube in the 1000Hz - 10000Hz range reaching up to -30dB.

Thus, a sound-based move tracking protocol must account for the following items in order to achieve an adequate signal-to-noise ratio:

- Tones between 500Hz and 1000Hz are the easiest to detect while solving a speedcube.
- Tones between 1000Hz and 20000Hz must be significantly louder than -30dB to be properly detected.

4.2.2 Tone Distinctiveness

If more than one tone is required for the protocol, each tone must be unique enough to be easily distinguished from each other tone. Since a standard smartphone or laptop microphone will be used on the listening end of this protocol, the definition of "easily distinguishable" must be based on an assessment of how clearly smartphone and laptop-grade microphones can distinguish similar frequencies.

To measure the sensitivity of a standard smartphone microphone (in this case a Google Pixel 1), a recording was taken of two distinct tones that started 500 Hz apart and stepped closer together in 10 Hz increments every 0.5 seconds until their frequencies were identical.

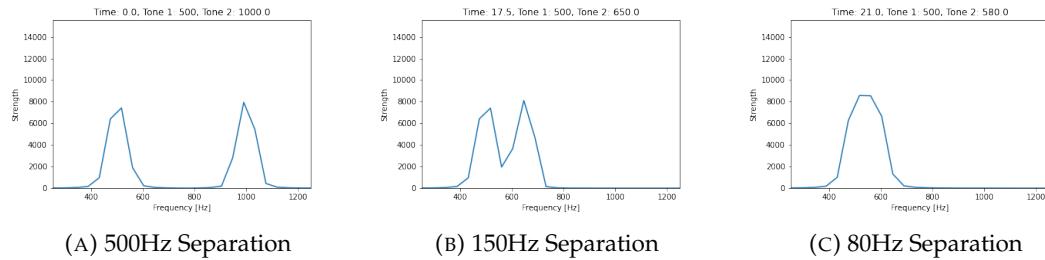


FIGURE 4.2: As two tones get more similar, they get harder to distinguish, particular when fewer than 100Hz apart.

As shown in Figure 4.2, the two tones are clearly distinguishable from 500Hz apart to 150Hz apart. However, once the tones came within 80Hz of each other, they became entirely indistinguishable.

Thus, a sound-based move tracking protocol must account for the following items in order to achieve adequate tone distinctiveness:

- Tones must be separated by at least 100Hz in order to be clearly distinguished from other tones in the protocol.

4.2.3 Frequency Response Range of Consumer Hardware

Since a standard smartphone or laptop microphone will be used on the listening end of this protocol, any tones used in the protocol must be within the range of tones that a smartphone or laptop-grade microphones can pick up. This range is formally known as the "frequency response range" of the microphone.

According to a Stanford research paper that analyzed over 10,000 mobile devices, the typical smartphone microphone has a frequency response range of 20Hz to 20kHz as shown in Figure 4.3

Thus, a sound-based move tracking protocol must account for the following restrictions on which tones can be used for the protocol:

- Tones used in the protocol must fall on the range of 20Hz-20kHz in order to be detectable by a typical smartphone or laptop microphone.

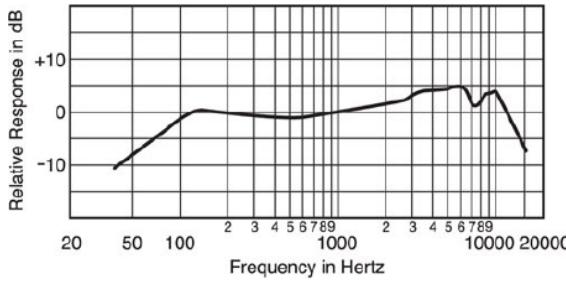


FIGURE 4.3: "A typical frequency response curve for a microphone." [27]

4.2.4 Human Auditory Range

An audible protocol could be distracting to a speedcuber. The human ear can detect audible frequencies from 20Hz to 20kHz, though this usually degrades with age with many people unable to notice sounds above 16kHz. [28]. However, not all tones are pleasant to listen to, particularly higher frequency tones. Tones within the frequency range of a piano (27.5Hz - 4kHz) are generally considered acceptable, while tones above the piano's upper range are often irritating. [29]

Thus, a sound-based move tracking protocol should be considerate of the human ear's sensitivity to various frequencies:

- The most acceptable tones for human speedsolvers are in the musical range of up to 4kHz or above the standard audible range of 16kHz

4.3 Specification

Given the above constraints, this section will detail a sound-based protocol for tracking the moves of a Rubik's Cube by continuously transmitting the current state of the cube. In this protocol, changes to the cube's state cause a change in the transmitted tones which can be recorded and analyzed to determine the face turn applied.

4.3.1 Representing the Cube's Current State

As mentioned in Section 2.2, a Rubik's Cube has six centerpieces that are fixed relative to each other, but can each rotate freely through four possible rotational positions as shown in Figure 4.4.

The state of a centerpiece is defined as its current rotational position. Implicit in this definition is the fact that a centerpiece is guaranteed to occupy one and only one of its four possible states at any given time.

Each of the six centerpieces has an independent set of four possible states, yielding a total of 24 different centerpiece states for the Rubik's Cube, of which there will always be exactly six active at any given time.

It's important to note that a knowledge of the current state of each centerpiece does not imply knowledge of the exact state of each of edge and corner cubie. For example, after

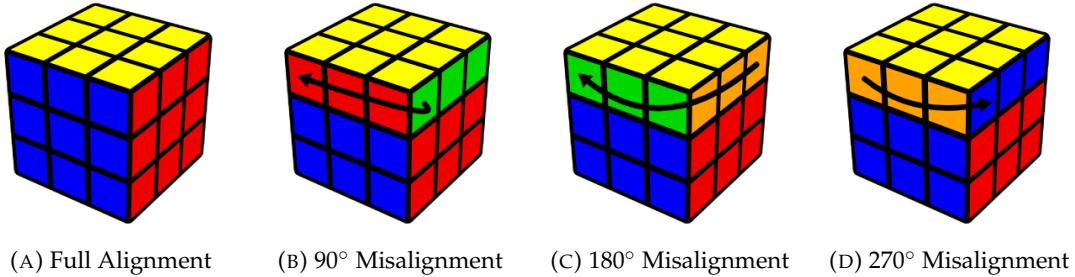


FIGURE 4.4: Each face of the Rubik’s Cube can occupy one of four rotational positions at any given time (Pictures from [30])

applying the algorithm $R U R' U'$ all centerpieces have the same state they occupied prior to the algorithm’s execution while, most of the edges and corners in the R and U layers will have been moved or rotated. However, in the reverse case, knowledge of the applied move sequence is sufficient information to determine the exact state of all cubies. Section 4.3.2 will explain how extract full move sequences using only the state information of the centers.

4.3.2 Tracking Face Turns

The only way to change a centerpiece’s state is by applying a face turn. As such, when a centerpiece is rotated to a new position, its state changes to that of the new position. From there, a simple comparison of the new state to the previous state reveals the exact face turn applied to the cube.

Thus, in order to determine the sequences of moves applied to the Rubik’s Cube, one simply needs to track how the state of its centerpieces changes over time.

4.3.3 Conveying State Through Sound

Conveying the current state of the cube’s centerpieces through sound can be done by simply associating each of the 24 possible centerpiece states with a specific tone as shown in Table 4.1 and broadcasting a signal composed of the six tones corresponding to the active states of the cube’s centerpieces. Whenever a face turn is applied, the tone associated with the rotated centerpiece would change to the tone representing the piece’s new state. From there, a microphone equipped device can record the broadcast frequencies, measure the changes in the frequencies over time, convert the frequency changes to state changes, and finally extract any move sequence applied to the cube.

Alignment	Yellow (U)	White (D)	Red (R)	Orange (L)	Blue (F)	Green (B)
Full	800 Hz	1300 Hz	1800 Hz	2300 Hz	2800 Hz	3300 Hz
90°	900 Hz	1400 Hz	1900 Hz	2400 Hz	2900 Hz	3400 Hz
180°	1000 Hz	1500 Hz	2000 Hz	2500 Hz	3000 Hz	3500 Hz
270°	1100 Hz	1600 Hz	2100 Hz	2600 Hz	3100 Hz	3600 Hz

TABLE 4.1: Example frequencies for representing a Rubik’s Cube’s centerpiece state in a sound-based move tracking protocol

4.4 Alternative Protocol

Instead of transmitting the current state of the cube's centerpieces, an alternative protocol design could seek to directly transmit the face turns applied to the cube.

This perspective focuses on the fact that all move sequences can be broken down into a series of 90° face turns. Since the cube consists of 6 faces, and each face can be turned either clockwise or counterclockwise, one could design a two-tone protocol using only 8 discrete audio frequencies to build the smart cube. The first tone would come from one of six predefined audio bands, one for each face of the cube. The second tone would come from one of two separately predefined audio bands, one for each possible direction of rotation. From this, an audio processing model could be designed to process a sequence of these two-tone pairs and reconstruct the sequence of face rotations by recording the rotated face followed by its direction of rotation.

However, while this model minimizes the number of discrete frequencies required to communicate changes in the cube's state, it carries many challenges. Consider the example of a speedcuber averaging 5 turns per second (common for a 12-15 second solver) with bursts up to 10 TPS. The burst TPS would require the successful transmission of 20 sequential tones within a single second - only 50ms per tone, all in the midst of additional noise from the cube's pieces hitting each other harder at the higher turn speed. Adding to the difficulty, since each tone is only ever transmitted once, the audio detection model must achieve 100% tone recognition to be able to accurately reconstruct the originating move sequence. As a result, this model fails to support any sort of error correction that would make it resistant to the common challenges to data transmission through sound.

4.5 Implementation

Implementing the protocol proposed in Section 4.3 requires the creation of a transmitter and a receiver that support the protocol. The details of implementing such a receiver are documented in Chapter 5, while the details of the transmitter are documented in Chapter 6.

Chapter 5

The Receiver

5.1 Introduction

The receiver for the sound-based move tracking protocol is in charge of decoding a sequence of face turns from the transmitted audio.

To do this, the receiver must go through the following steps:

1. Record the transmitted audio.
2. Measure the audible frequencies at each time step.
3. Convert each time step's audible frequencies to the cube's state at that moment.
4. Decode the applied face turns from the sequence of cube states.

This chapter will describe the development of a software algorithm in Python that can serve as a receiver for this sound-based move tracking protocol. This development began with the creation of synthetic audio recordings representing the tones that would be emitted by a Rubik's Cube equipped with an ideal transmitter (Section 5.2) followed by the implementation of an algorithm capable of decoding that ideal synthetic audio (Section 5.3). The algorithm was then made more robust by adding realistic noise to the synthetic audio to better simulate a real speedcubing environment (Section 5.4) followed by enhancing the previously designed algorithm to continue to decode the applied move sequence in the midst of the added noise (Section: 5.5).¹

5.2 Synthetic Audio Generation

The first step of designing a receiver is to synthesize an audio signal representative of the output of the ideal receiver.

This is done by encoding the frequency corresponding to each centerpiece state (Section 5.2.1), creating a virtual Rubik's Cube (Section 5.2.2), and finally generating the audio signal from the virtual Rubik's Cube's state (Section 5.2.3).

¹The contents of this Chapter have been specially written so the reader can copy them into a Jupyter Notebook running a Python 3.9 kernel and see the same results for himself/herself.

5.2.1 Representing the Audio Protocol

For the synthetic audio generator to produce a realistic signal, it needs to know which frequencies to transmit for each centerpiece state. For this design the frequencies listed in Table 4.1 are converted to a dictionary.

```
CENTERPIECE_ROTATION_TO_FREQUENCY_MAPPINGS = {
    "U": {
        0: 800,
        1: 900,
        2: 1000,
        3: 1100,
    },
    "D": {
        0: 1300,
        1: 1400,
        2: 1500,
        3: 1600,
    },
    "R": {
        0: 1800,
        1: 1900,
        2: 2000,
        3: 2100,
    },
    "L": {
        0: 2300,
        1: 2400,
        2: 2500,
        3: 2600,
    },
    "F": {
        0: 2800,
        1: 2900,
        2: 3000,
        3: 3100,
    },
    "B": {
        0: 3300,
        1: 3400,
        2: 3500,
        3: 3600,
    }
}
```

With this dictionary in place, determining the frequency to transmit for any particular centerpiece's current rotation is reduced to a simple lookup.

```
def frequency_of(centerpiece: str, rotation: int) -> float:
    return CENTERPIECE_ROTATION_TO_FREQUENCY_MAPPINGS[centerpiece][rotation]
```

5.2.2 Representing the Rubik's Cube

Next, the synthetic audio generator needs a representation of a Rubik's Cube on which face turns can be virtually applied, and the resulting cube state can be read out.

```
class RubiksCube:

    CLOCKWISE = 1
    COUNTERCLOCKWISE = 3

    def __init__(self):
        self.state = { "U": 0, "D": 0, "R": 0, "L": 0, "F": 0, "B": 0 }

    def apply_move(self, move: str):
        # Extract the face and direction from the move string.
        face = move[0]
        if len(move) == 1:                                # e.g. U
            direction = RubiksCube.CLOCKWISE
        else:                                              # e.g. U'
            direction = RubiksCube.COUNTERCLOCKWISE
        # Update the state to apply the move.
        self.state[face] = (self.state[face] + direction) % 4
```

5.2.3 Generating Synthetic Audio for an Arbitrary Algorithm

Now the synthetic audio can be generated for any valid algorithm. This is done using the "tones" library [31] created by Erik Nyquist. First, a separate audio track is created for each centerpiece on the cube and the frequencies representing the initial state of each centerpiece are added to its corresponding audio track. From there, the rest of the synthetic audio can be created by iterating through each step of the given algorithm, applying it to the virtual Rubik's Cube, and adding the frequency for each centerpiece's resulting state to its corresponding audio track.

```
from tones.mixer import Mixer # https://pypi.org/project/tones/
from tones import SINE_WAVE

def _create_mixer(rubiks_cube: RubiksCube) -> Mixer:
    mixer: Mixer = Mixer(sample_rate=44100, amplitude=1)
    # Add a separate track for each centerpiece.
    for face, _ in rubiks_cube.state.items():
        mixer.create_track(face, SINE_WAVE, attack=0, decay=0)
    return mixer
```

```

def _render_cube_state(mixer: Mixer, rubiks_cube: RubiksCube, tps: float):
    for face, rotation in rubiks_cube.state.items():
        mixer.add_tone(face, frequency_of(face, rotation), duration=1 / tps)

def render_audible_alg(alg: str, wav_path: str=None, tps: float=4):
    # Create the virtual Rubik's Cube.
    rubiks_cube: RubiksCube = RubiksCube()

    # Create the audio mixer used to create the synthesized audio.
    mixer = _create_mixer(rubiks_cube)

    # Add the initial cube state to the mixer.
    _render_cube_state(mixer, rubiks_cube, tps)

    # Iterate over the moves in the algorithm, adding
    # the cube state to the mixer after each move.
    moves = alg.split(" ")
    for move in moves:
        rubiks_cube.apply_move(move)
        _render_cube_state(mixer, rubiks_cube, tps)

    # Save the final audio to a .wav file.
    mixer.write_wav(wav_path if wav_path else f"{alg}.wav")

```

With these functions in place, synthetic audio can be easily created for any valid Rubik's Cube algorithm. For example, generating synthetic audio that sweeps through every possible centerpiece state can be done with the following snippet of code:

```

demo_alg = "U U U U D D D D R R R R L L L L F F F F B B B B"
demo_wav_path = "demo_all_states.wav"
render_audible_alg(demo_alg, demo_wav_path)

```

5.3 Decoding Move Sequences from Synthetic Audio

With synthetic audio now available for any valid Rubik's Cube algorithm, the next step is to create an initial software algorithm that can decode that audio back into the original move sequence.

Accomplishing this will require computing the synthetic audio's spectrogram (Section 5.3.1), followed by extracting the dominant frequencies present at each time step (Section 5.3.2), and converting those dominant frequencies into the corresponding Rubik's Cube centerpiece states (Section 5.3.3), all before finally recovering the originally applied move sequence (Section 5.3.4).

5.3.1 Compute the Spectrogram

The first step in decoding the synthetic audio is determining its component frequencies at any specific moment in time. These component frequencies can be easily visualized using a spectrogram, like the one in Figure 5.1 of the synthetic audio created in Section 5.2.3.

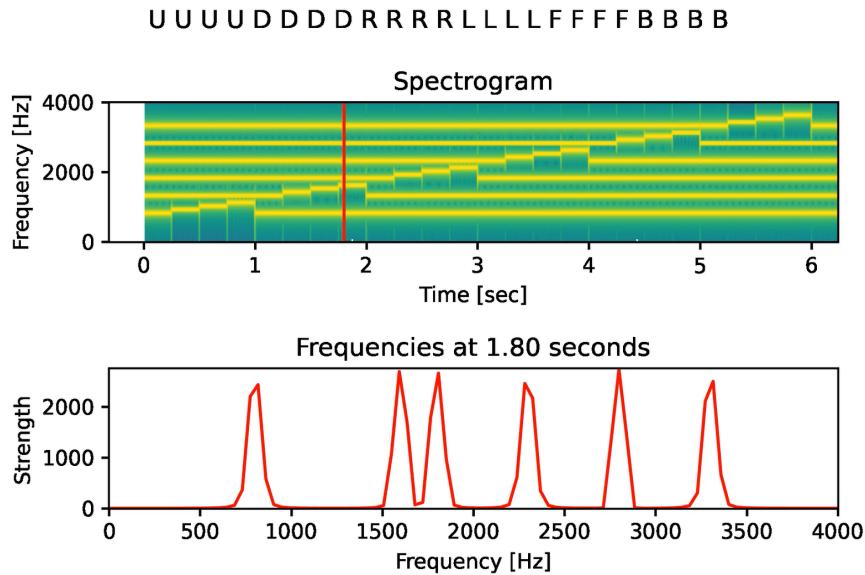


FIGURE 5.1: Spectrogram of the synthetic audio created in Section 5.2.3.
The source code for this diagram is available in Appendix A.1.

The bright, horizontal yellow bands in the spectrogram represent the component frequencies present in the audio signal at each point in time. The vertical red line on the spectrogram indicates the specific slice of the spectrogram represented by the graph of the component frequencies at that specific point in time.

While the Figure 5.1 was created using Matplotlib's specgram plot, the actual data of the spectrogram for a given .wav file can be easily obtained using the numpy and scipy packages

```
import numpy as np
from scipy import signal
from scipy.io import wavfile

def compute_spectrogram(wav_path: str):
    SAMPLES_PER_WINDOW = 1024 # Balances frequency/time precision
    sample_rate, audio_samples = wavfile.read(wav_path)
    freq, time, Zxx = signal.stft(audio_samples, fs=sample_rate,
        nperseg=SAMPLES_PER_WINDOW, noverlap=(SAMPLES_PER_WINDOW // 4) * 3)
    spectrogram = np.abs(Zxx).transpose()
    return freq, time, spectrogram
```

5.3.2 Extract the Dominant Component Frequencies

Notice how the component frequency graph in Figure 5.1 has six peaks: these are the transmitted frequencies representing the current state of the virtual Rubik's Cube's six centerpieces at that specific instant of time.

The exact frequencies of these peaks can be extracted by filtering out all frequencies not above a specific threshold. In this case, a simple threshold of 85% of the maximum strength of any component frequency (see the green line in Figure 5.2) isolates the peak of the six dominant component frequencies.

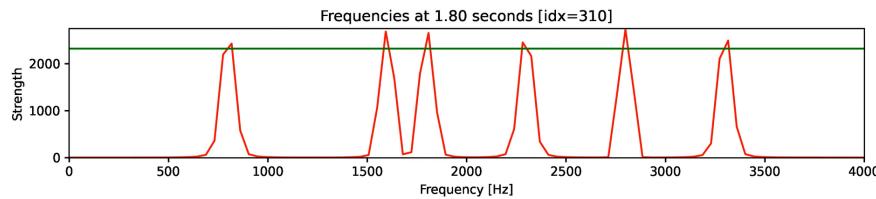


FIGURE 5.2: Spectrogram of the synthetic audio created in Section 5.2.3 with a threshold at 85% of the maximum strength of the component frequencies. The source code for this diagram is available in Appendix A.2.

Using the actual spectrogram data we can compute the exact values of these peaks.

```
def compute_threshold(values: list):
    return max(values) * 0.85

def extract_important_frequencies(freq, time, spectrogram, time_idx):
    important_freqs = []
    threshold = compute_threshold(spectrogram[time_idx])
    for freq_idx in range(len(freq)):
        if spectrogram[time_idx][freq_idx] > threshold:
            important_freqs.append(dict(
                hz=freq[freq_idx],
                power=spectrogram[time_idx][freq_idx]
            ))
    return important_freqs

freq, time, spectrogram = compute_spectrogram(demo_wav_path)
important_freqs = extract_important_frequencies(freq, time, spectrogram,
                                                time_idx=310) # 1.80 seconds

important_freqs_hz = [i["hz"] for i in important_freqs]
pretty_freqs = list(map(lambda x: f"{x:.0f} Hz", important_freqs_hz))
print(pretty_freqs)

>> ["818 Hz", "1593 Hz", "1809 Hz", "2283 Hz", "2799 Hz", "3316 Hz"]
```

5.3.3 Translating Component Frequencies to Centerpiece States

With the specific frequencies of each detected peak, the original state of the Rubik's Cube at that moment in time can be computed by finding the state whose corresponding frequency is closest to each detected peak frequency.

```
def _closest_state(detected_freq):
    closest_rotation = None
    closest_difference = None
    for face, rotations in CENTERPIECE_ROTATION_TO_FREQUENCY_MAPPINGS.items():
        for rotation, freq in rotations.items():
            difference = abs(detected_freq - freq)
            if closest_difference is None or difference < closest_difference:
                closest_difference = difference
                closest_rotation = {face: rotation}
    return closest_rotation

def get_state_from_freqs(important_freqs: list) -> dict:
    state = {}
    for freq in important_freqs:
        state.update(_closest_state(freq))
    return state

print(get_state_from_freqs(important_freqs_hz))

>> {"U": 0, "D": 3, "R": 0, "L": 0, "F": 0, "B": 0}

Repeating this process for each time step in the recorded audio will yield a sequence of the Rubik's Cube's centerpiece states over the course of the recorded audio sequence.

def get_state_over_time(freq, time, spectrogram):
    state_over_time = []
    for time_idx in range(len(time)):
        important_freqs = extract_important_frequencies(
            freq, time, spectrogram, time_idx)
        important_freqs_hz = [i["hz"] for i in important_freqs]
        state = get_state_from_freqs(important_freqs_hz)
        state_over_time.append(dict(
            time=time[time_idx],
            state=state
        ))
    return state_over_time

state_over_time = get_state_over_time(freq, time, spectrogram)
```

5.3.4 Extracting Move Sequences from Centerpiece State Sequences

Finally, the original move sequence can be recovered by iterating over the sequence of cube states, and registering any change to the cube state as a move applied to the cube.

```
def _move_from(face, current_rotation, new_rotation):
    direction = None
    if (current_rotation + RubiksCube.CLOCKWISE) % 4 == new_rotation:
        direction = "" # Clockwise
    elif (current_rotation + RubiksCube.COUNTERCLOCKWISE) % 4 == new_rotation:
        direction = '"' # Counterclockwise
    return f"{face}{direction}"

def detect_moves(state_over_time):
    detected_moves = []
    current_state = state_over_time[0]["state"]
    for timed_state in state_over_time:
        time, state = timed_state.values()
        for face, rotation in state.items():
            if rotation != current_state[face]:
                detected_moves.append(dict(
                    time=time,
                    move=_move_from(face, current_state[face], rotation)
                ))
                current_state[face] = rotation
    return detected_moves

detected_moves = detect_moves(state_over_time)

pretty_moves = [i["move"] for i in detected_moves]
pretty_moves_str = " ".join(pretty_moves)
print(pretty_moves_str)
print(f"Matches demo_alg? {demo_alg == pretty_moves_str}")

>> U U U U D D D R R R R L L L L F F F F B B B B
>> Matches demo_alg? True
```

Clearly, the detected move sequence matches the demo algorithm used to create the synthetic audio in Section 5.2.3, which means this algorithm is a functional receiver for an ideal transmitter.

5.4 Adding Realistic Noise to the Synthetic Audio

TODO pull from Jupyter Notebook

5.5 Decoding Move Sequences from Realistic Audio

TODO pull from Jupyter Notebook

5.5.1 Optimizing algorithm parameters

TODO - Share the strategy for finding the optimal parameters, and the end results, but defer the detailed analysis for the Evaluation.

Chapter 6

PCB Design

6.1 Requirements

TODO lay out the requirements for the PCB

6.1.1 Accuracy of Tone Generation

TODO Describe this requirement

6.1.2 Tone Response to Cube Rotations

TODO Describe this requirement

6.1.3 Signal-to-Noise Ratio

TODO Describe this requirement TODO Discuss the dampening effect of the plastic enclosure.

6.1.4 Prospects of Miniaturization

TODO Describe this requirement

6.2 Hardware Selection

TODO outline the process of choosing specific hardware to use for this project

6.2.1 Minimizing Sound Obstruction

TODO Discuss the "tupperware" tests -> design of various center caps.

6.3 Prototyping

TODO detail the process of building a prototype. Include pictures of the board and the generated spectrograms.

Chapter 7

Evaluation

TODO review the core goals outlined in the introduction, and methodically review how well my final prototypes stack up against those goals. This section serves to prove (with all the data) that the approach I've described in the previous chapters actually works.

7.1 Compatibility with Standard Speedcubes

TODO discuss how well my design meets this requirement from the Introduction

The design must be deployable within a standard (non-smart) speedcube.

1. The design must not require permanent modifications to the original speedcube.
2. The design must not significantly impact the turn-speed of the original speedcube.

7.2 Move Tracking Accuracy

TODO discuss how well my design meets this requirement from the Introduction

The design must be capable of tracking the face turns of a Rubik's Cube with over 99% accuracy.

7.3 Move Tracking Granularity

TODO discuss how well my design meets this requirement from the Introduction

The design must be capable of recording the time spent executing each individual face turn of a Rubik's Cube.

7.4 Competition Legality

TODO discuss how well my design meets this requirement from the Introduction

The design must be competition legal, meaning it results in a cube that either does not violate existing competition rules against the use of electronics or can be easily modified to regain compliance.

Chapter 8

Conclusion

8.1 Summary

TODO summarize the goals from the Introduction and broadly describe what techniques were most helpful in reaching them. Also detail a summary of exactly how effective they were.

8.2 Limitations

TODO detail the limits to which this research can be more broadly applied. Be clear about the effects various assumptions/decisions have on the reliability of the conclusions of this thesis.

8.3 Ideas for Future Research

TODO If I had more time/resources to work on it, what would I do next with this project?

- Live algorithm - build it small enough to fit in the cube.

Appendix A

Longer Code Snippets

A.1 Spectrogram Generation for Figure 5.1

This code snippet actually generates a full animation of how the component frequencies change over time. Figure 5.1 is only one frame of this animation.

Make sure to run this code after running the code in Section 5.3.1.

```
# Heavily aided by https://matplotlib.org/stable/gallery/animation/simple_anim.html
import matplotlib.pyplot as plt
import matplotlib.animation as animation

def compute_threshold(values: list):
    return -500 # Out of sight of the chart

def animate(wav_path: str, alg: str):
    # Parse the audio sample into its spectrogram
    freq, time, spectrogram = compute_spectrogram(wav_path)

    # Generate the plots
    fig, (ax_spectrogram, ax_fourier) = plt.subplots(2)

    # Set the spectrogram
    ax_spectrogram.specgram(audio_samples, Fs=sample_rate,
                            NFFT=SAMPLES_PER_WINDOW,
                            nooverlap=SAMPLES_PER_WINDOW // 4 * 3) # 3/4 of the window size
    line_spectrogram, = ax_spectrogram.plot([], [], '-',
                                             color='red')
    time_tracker_spectrogram = ax_spectrogram.axvline(0, color='red')
    ax_spectrogram.set_xlim(0, 4000)
    ax_spectrogram.set_title(f"Spectrogram")
    ax_spectrogram.set_ylabel('Frequency [Hz]')
    ax_spectrogram.set_xlabel('Time [sec]')

    # Set the Fourier transform
    STEP = 10
    indices = np.arange(0, len(time) + 1, step=STEP)
    line_fourier, = ax_fourier.plot(freq, spectrogram[0], color='red')
```

```

threshold_fourier = ax_fourier.axhline(-500, color='green')
ax_fourier.set_title("")
ax_fourier.set_xlim(0, 4000) # Frequency
ax_fourier.set_xlabel("Frequency [Hz]")
ax_fourier.set_ylim(0, npamax(spectrogram)) # Strength
ax_fourier.set_ylabel("Strength")

# Space figures nicely
fig.suptitle(alg)
fig.tight_layout(h_pad=3)

def draw_frame_at(i):
    time_tracker_spectrogram.set_xdata(time[i])
    line_fourier.set_ydata(spectrogram[i])
    threshold_fourier.set_ydata(compute_threshold(spectrogram[i]))
    ax_fourier.set_title(f" Frequencies at {time[i]:.2f} seconds [idx={i}]")
    return line_spectrogram, line_fourier

ani = animation.FuncAnimation(fig, draw_frame_at,
    frames=indices, interval=1, blit=True, save_count=len(time)/STEP)
ani.save(f"plt/animation/{wav_path[:-4]}.mp4",
    fps=len(time)/STEP/5, dpi=300)
plt.show()

animate(demo_wav_path, demo_alg)

```

A.2 Spectrogram Generation for Figure 5.2

This code snippet actually generates a full animation of how the component frequencies change over time. Figure 5.2 is only one frame of this animation.

Make sure to run this code after running the code in Section 5.3.2 as the new *compute_threshold* function there is what adds the threshold line to this animation.

```
animate(demo_wav_path, demo_alg)
```

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