The bigintcalc package

Heiko Oberdiek <oberdiek@uni-freiburg.de>

$2007/11/11~{\rm v}1.1$

Abstract

This package provides expandable arithmetic operations with big integers that can exceed TeX's number limits.

Contents

1	Doc	umentation 2
	1.1	Introduction
	1.2	Conditions
		1.2.1 Preconditions
		1.2.2 Postconditions
	1.3	Error handling
	1.4	Operations
		1.4.1 Num
		1.4.2 Inv, Abs, Sgn
		1.4.3 Min, Max, Cmp
		1.4.4 Odd
		1.4.5 Inc, Dec, Add, Sub
		1.4.6 Shl, Shr
		1.4.7 Mul, Sqr, Fac, Pow
		1.4.8 Div, Mul
	1.5	Interface for programmers
2	Imp	lementation 8
	2.1	Reload check and package identification
	2.2	Catcodes
	2.3	ε-T _E X detection
	2.4	Help macros
	2.5	Expand number
	2.6	Normalize expanded number
	2.7	Num
	2.8	Inv, Abs, Sgn
	2.9	Cmp, Min, Max
	2.10	Odd
	2.11	Inc, Dec
	2.12	Add, Sub
	2.13	Shl, Shr
	2.14	\BIC@Tim
	2.15	Mul
	2.16	Sqr
	2.17	Fac
	2.18	Pow
		2.18.1 Help macros
		2.18.2 Recursive calculation

	2.19	Div							
	2.20	Mod							
3	Test	t.							
	3.1	Catcode checks for loading							
	3.2	Macro tests							
		3.2.1 Preamble with test macro definitions							
		3.2.2 Time							
		3.2.3 Test sets							
4	Installation								
	4.1	Download							
	4.2	Bundle installation							
	4.3	Package installation							
	4.4	Refresh file name databases							
	4.5	Some details for the interested							
5	Hist	tory							
	[200]	7/09/27 v1.0]							
		7/11/11 v1.1]							
6	Inde	av							

1 Documentation

1.1 Introduction

Package bigintcalc defines arithmetic operations that deal with big integers. Big integers can be given either as explicit integer number or as macro code that expands to an explicit number. Big means that there is no limit on the size of the number. Big integers may exceed TeX's range limitation of -2147483647 and 2147483647. Only memory issues will limit the usable range.

In opposite to package intcalc unexpandable command tokens are not supported, even if they are valid TeX numbers like count registers or commands created by \chardef. Nevertheless they may be used, if they are prefixed by \number.

Also ε -TeX's \numexpr expressions are not supported directly in the manner of package intcalc. However they can be given if \the\numexpr or \numexpr are used.

The operations have the form of macros that take one or two integers as parameter and return the integer result. The macro name is a three letter operation name prefixed by the package name, e.g. \bigintcalcAdd{10}{43} returns 53.

The macros are fully expandable, exactly two expansion steps generate the result. Therefore the operations may be used nearly everywhere in TEX, even inside \csname, file names, or other expandable contexts.

1.2 Conditions

1.2.1 Preconditions

- Arguments can be anything that expands to a number that consists of optional signs and digits.
- The arguments and return values must be sound. Zero as divisor or factorials of negative numbers will cause errors.

1.2.2 Postconditions

Additional properties of the macros apart from calculating a correct result (of course ©):

- The macros are fully expandable. Thus they can be used inside \edef, \csname, for example.
- Furthermore exactly two expansion steps calculate the result.
- The number consists of one optional minus sign and one or more digits. The first digit is larger than zero for numbers that consists of more than one digit.

In short, the number format is exactly the same as \number generates, but without its range limitation. And the tokens (minus sign, digits) have catcode 12 (other).

• Call by value is simulated. First the arguments are converted to numbers. Then these numbers are used in the calculations.

Remember that arguments may contain expensive macros or ε -TEX expressions. This strategy avoids multiple evaluations of such arguments.

1.3 Error handling

Some errors are detected by the macros, example: division by zero. In this cases an undefined control sequence is called and causes a TeX error message, example: \BigIntCalcError:DivisionByZero. The name of the control sequence contains the reason for the error. The TEX error may be ignored. Then the operation returns zero as result. Because the macros are supposed to work in expandible contexts. An traditional error message, however, is not expandable and would break these contexts.

1.4 Operations

Some definition equations below use the function Int that converts a real number to an integer. The number is truncated that means rounding to zero:

$$Int(x) := \begin{cases} \lfloor x \rfloor & \text{if } x \ge 0 \\ \lceil x \rceil & \text{otherwise} \end{cases}$$

1.4.1 Num

\bigintcalcNum $\{\langle x \rangle\}$

Macro \bigintcalcNum converts its argument to a normalized integer number without unnecessary leading zeros or signs. The result matches the regular expression:

3

1.4.2 Inv, Abs, Sgn

\bigintcalcInv $\{\langle x \rangle\}$

Macro \bigintcalcInv switches the sign.

$$Inv(x) := -x$$

\bigintcalcAbs $\{\langle x \rangle\}$

Macro \bigintcalcAbs returns the absolute value of integer $\langle x \rangle$.

$$Abs(x) := |x|$$

\bigintcalcSgn $\{\langle x \rangle\}$

Macro \bigintcalcSgn encodes the sign of $\langle x \rangle$ as number.

$$Sgn(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

These return values can easily be distinguished by \ifcase:

```
\ifcase\bigintcalcSgn{<x>}
    $x=0$
\or
    $x>0$
\else
    $x<0$
\fi</pre>
```

1.4.3 Min, Max, Cmp

\bigintcalcMin $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMin returns the smaller of the two integers.

$$Min(x, y) := \begin{cases} x & \text{if } x < y \\ y & \text{otherwise} \end{cases}$$

\bigintcalcMax $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMax returns the larger of the two integers.

$$Max(x,y) := \begin{cases} x & \text{if } x > y \\ y & \text{otherwise} \end{cases}$$

\bigintcalcCmp $\{\langle x angle\}$ $\{\langle y angle\}$

Macro \bigintcalcCmp encodes the comparison result as number:

$$Cmp(x,y) := \begin{cases} -1 & \text{if } x < y \\ 0 & \text{if } x = y \\ 1 & \text{if } x > y \end{cases}$$

These values can be distinguished by \ifcase:

1.4.4 Odd

\bigintcalcOdd $\{\langle x \rangle\}$

$$Odd(x) := \begin{cases} 1 & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even} \end{cases}$$

1.4.5 Inc, Dec, Add, Sub

\bigintcalcInc $\{\langle x \rangle\}$

Macro \bigintcalcInc increments $\langle x \rangle$ by one.

$$Inc(x) := x + 1$$

\bigintcalcDec $\{\langle x
angle\}$

Macro \bigintcalcDec decrements $\langle x \rangle$ by one.

$$Dec(x) := x - 1$$

\bigintcalcAdd $\{\langle x
angle\}$ $\{\langle y
angle\}$

Macro \bigintcalcAdd adds the two numbers.

$$Add(x,y) := x + y$$

\bigintcalcSub $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcSub calculates the difference.

$$Sub(x, y) := x - y$$

1.4.6 Shl, Shr

\bigintcalcShl $\{\langle x \rangle\}$

Macro \bigintcalcShl implements shifting to the left that means the number is multiplied by two. The sign is preserved.

$$Shl(x) := x * 2$$

\bigintcalcShr $\{\langle x \rangle\}$

Macro \bigintcalcShr implements shifting to the right. That is equivalent to an integer division by two. The sign is preserved.

5

$$Shr(x) := Int(x/2)$$

1.4.7 Mul, Sqr, Fac, Pow

\bigintcalcMul
$$\{\langle x \rangle\}$$
 $\{\langle y \rangle\}$

Macro \bigintcalcMul calculates the product of $\langle x \rangle$ and $\langle y \rangle$.

$$Mul(x, y) := x * y$$

\bigintcalcSqr $\{\langle x \rangle\}$

Macro \bigintcalcSqr returns the square product.

$$Sqr(x) := x^2$$

\bigintcalcFac $\{\langle x \rangle\}$

Macro \bigintcalcFac returns the factorial of $\langle x \rangle$. Negative numbers are not permitted.

$$Fac(x) := x!$$
 for $x \ge 0$

$$(0! = 1)$$

\bigintcalcPow Mx My

Macro \bigintcalcPow calculates the value of $\langle x \rangle$ to the power of $\langle y \rangle$. The error "division by zero" is thrown if $\langle x \rangle$ is zero and $\langle y \rangle$ is negative. permitted:

$$\operatorname{Pow}(x, y) := \operatorname{Int}(x^y)$$
 for $x \neq 0$ or $y \geq 0$

$$(0^0 = 1)$$

1.4.8 Div, Mul

\bigintcalcDiv $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcDiv performs an integer division. Argument $\langle y \rangle$ must not be zero.

$$\operatorname{Div}(x,y) := \operatorname{Int}(x/y)$$
 for $y \neq 0$

\bigintcalcMod $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMod gets the remainder of the integer division. The sign follows the divisor $\langle y \rangle$. Argument $\langle y \rangle$ must not be zero.

$$Mod(x, y) := x \% y$$
 for $y \neq 0$

The result ranges:

$$-|y| < \operatorname{Mod}(x, y) \le 0$$
 for $y < 0$

$$0 \le \operatorname{Mod}(x, y) < y \qquad \text{for } y \ge 0$$

1.5 Interface for programmers

If the programmer can ensure some more properties about the arguments of the operations, then the following macros are a little more efficient.

In general numbers must obey the following constraints:

- Plain number: digit tokens only, no command tokens.
- Non-negative. Signs are forbidden.
- Delimited by exclamation mark. Curly braces around the number are not allowed and will break the code.

```
\BigIntCalcOdd\ \langle number \rangle !
     1/0 is returned if \langle number \rangle is odd/even.
\BigIntCalcInc\ \langle number \rangle !
     Incrementation.
\BigIntCalcDec\ \langle number \rangle !
     Decrementation, positive number without zero.
\BigIntCalcAdd\ \langle number\ A \rangle \ ! \ \langle number\ B \rangle \ !
     Addition, A \geq B.
Subtraction, A \geq B.
\BigIntCalcShl\ \langle number \rangle!
    Left shift (multiplication with two).
\BigIntCalcShr\ \langle number \rangle !
     Right shift (integer division by two).
\BigIntCalcMul \langle number A \rangle ! \langle number B \rangle !
     Multiplication, A \geq B.
\BigIntCalcDiv\ \langle number\ A \rangle \ ! \ \langle number\ B \rangle \ !
```

Division operation.

Modulo operation.

 $\BigIntCalcMod \langle number A \rangle ! \langle number B \rangle !$

2 Implementation

1 (*package)

2.1 Reload check and package identification

Reload check, especially if the package is not used with LATEX.

```
2 \begingroup
      \catcode44 12 %,
     \catcode45 12 % -
     \catcode46 12 % .
  6 \catcode58 12 % :
  7 \catcode64 11 % @
     \catcode123 1 % {
  9
     \catcode125 2 % }
     \expandafter\let\expandafter\x\csname ver@bigintcalc.sty\endcsname
 10
      \ifx\x\relax % plain-TeX, first loading
 11
     \else
 12
        \def\encompty{}%
 13
        \ifx\x\empty % LaTeX, first loading,
 14
         % variable is initialized, but \ProvidesPackage not yet seen
 15
 16
       \else
 17
          \catcode35 6 % #
 18
          \expandafter\ifx\csname PackageInfo\endcsname\relax
 19
            \def\x#1#2{%}
              \immediate\write-1{Package #1 Info: #2.}%
 20
            }%
 21
          \else
 22
            23
 24
          \x{bigintcalc}{The package is already loaded}%
 25
 26
          \aftergroup\endinput
 27
        \fi
     \fi
 29 \endgroup
Package identification:
 30 \begingroup
      \catcode35 6 % #
 32
     \catcode40 12 % (
     \catcode41 12 % )
 33
     \catcode44 12 % ,
 34
      \catcode45 12 % -
 35
      \catcode46 12 % .
 36
 37
      \catcode47 12 % /
 38
      \catcode58 12 % :
 39
      \catcode64 11 % @
      \catcode91 12 % [
 41
      \catcode93 12 % ]
 42
      \catcode123 1 % {
 43
      \catcode125 2 % }
      \expandafter\ifx\csname ProvidesPackage\endcsname\relax
 44
        \def\x#1#2#3[#4]{\endgroup}
 45
 46
          \immediate\write-1{Package: #3 #4}%
 47
          \xdef#1{#4}%
       }%
 48
 49
      \else
 50
        \def \x#1#2[#3] {\endgroup}
 51
          #2[{#3}]%
 52
          \ifx#1\@undefined
            \xdef#1{#3}%
 53
          \fi
 54
          \frak{1}\operatorname{n}
 55
            \xdef#1{#3}%
 56
```

```
\fi
57
      }%
58
59
60 \expandafter\x\csname ver@bigintcalc.sty\endcsname
61 \ProvidesPackage{bigintcalc}%
    [2007/11/11 v1.1 Expandable big integer calculations (HO)]
```

2.2Catcodes

```
63 \begingroup
  64
             \catcode123 1 % {
  65
              \catcode125 2 % }
  66
             \def\x{\endgroup
                    \expandafter\edef\csname BIC@AtEnd\endcsname{%
  67
                         \catcode35 \the\catcode35\relax
  68
                         \catcode64 \the\catcode64\relax
  69
                         \catcode123 \the\catcode123\relax
  70
  71
                         \catcode125 \the\catcode125\relax
  72
                   }%
            }%
  73
  74 \x
  75 \catcode35 6 % #
  76\ \catcode64\ 11\ \%\ @
  77 \catcode123 1 % {
  78 \catcode125 2 % }
  79 \def\TMP@EnsureCode#1#2{%
             \edef\BIC@AtEnd{%
  80
                    \BIC@AtEnd
  81
  82
                    \catcode#1 \the\catcode#1\relax
  83
             \catcode#1 #2\relax
  85 }
  86 \TMP@EnsureCode{33}{12}%!
  87 \TMP@EnsureCode{36}{14}% $ (comment!)
  88 \TMP@EnsureCode{38}{14}\% & (comment!)
  89 \TMP@EnsureCode{40}{12}\% (
 90 \TMP@EnsureCode{41}{12}% )
 91 \TMP@EnsureCode\{42\}\{12\}\% *
 92 \TMP@EnsureCode{43}{12}% +
 93 \TMP@EnsureCode{45}{12}% -
 94 \TMP@EnsureCode{46}{12}% .
 95 \TMP@EnsureCode\{47\}\{12\}\% /
 96 \TMP@EnsureCode{58}{11}% : (letter!)
 97 \TMP@EnsureCode{60}{12}% <
 98 \TMP@EnsureCode{61}{12}% =
 99 \TMP@EnsureCode\{62\}\{12\}\% >
100 \TMP@EnsureCode\{63\}\{14\}\% ? (comment!)
101 \verb|\degingroup\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafte
102 \expandafter\ifx\csname BIC@TestMode\endcsname\relax
103 \ensuremath{\setminus} else
           \catcode63=9 % ? (ignore)
105 \fi
106 ? \let\BIC@@TestMode\BIC@TestMode
             \varepsilon-T<sub>E</sub>X detection
```

2.3

```
107 \begingroup\expandafter\expandafter\expandafter\endgroup
108 \expandafter\ifx\csname numexpr\endcsname\relax
109 \catcode36=9 % $ (ignore)
110 \else
111 \catcode38=9 % & (ignore)
112 \fi
```

2.4 Help macros

```
\BIC@Fi
                    113 \let\BIC@Fi\fi
    \BIC@AfterFi
                    114 \def\BIC@AfterFi#1#2\BIC@Fi{\fi#1}%
  \BIC@AfterFiFi
                    115 \def\BIC@AfterFiFi#1#2\BIC@Fi{\fi\fi#1}%
\BIC@AfterFiFiFi
                    116 \def\BIC@AfterFiFiFi#1#2\BIC@Fi{\fi\fi\fi#1}%
      \BIC@Space
                    117 \begingroup
                    118 \def\x#1{\endgroup
                    119
                           \let\BIC@Space= #1%
                    120 }%
                    121 \x{ }
                          Expand number
                   2.5
                    122 \begingroup\expandafter\expandafter\expandafter\endgroup
                    123 \expandafter\ifx\csname RequirePackage\endcsname\relax
                    124 \input pdftexcmds.sty\relax
                    125 \ensuremath{\setminus} else
                    126 \RequirePackage{pdftexcmds}[2007/11/11]%
                    127 \fi
                    128 \begingroup\expandafter\expandafter\expandafter\endgroup
                    129 \expandafter\ifx\csname pdf@escapehex\endcsname\relax
     \BIC@Expand
                         \def\BIC@Expand#1{%
                    130
                    131
                           \romannumeral0%
                    132
                           \BIC@@Expand#1!\@nil{}%
                    133
                         }%
    \BIC@@Expand
                         \def\BIC@@Expand#1#2\@nil#3{%
                    135
                            \expandafter\ifcat\noexpand#1\relax
                    136
                             \expandafter\@firstoftwo
                    137
                           \else
                             \expandafter\@secondoftwo
                    138
                           \fi
                    139
                    140
                             \verb|\expandafter| BIC@@Expand#1#2| @ni1{#3}% |
                    141
                           }{%
                    142
                    143
                             \ifx#1!%
                               \expandafter\@firstoftwo
                    144
                    145
                              \else
                    146
                               \expandafter\@secondoftwo
                    147
                             \fi
                             { #3}{%
                    148
                               \BIC@@Expand#2\@ni1{#3#1}%
                    149
                             }%
                    150
                           }%
                    151
```

}%

152

```
\@firstoftwo
                           \expandafter\ifx\csname @firstoftwo\endcsname\relax
                     153
                            \long\def\@firstoftwo#1#2{#1}%
                     154
                          \fi
                     155
     \@secondoftwo
                          \expandafter\ifx\csname @secondoftwo\endcsname\relax
                     156
                             157
                     158
                     159 \else
       \BIC@Expand
                          \def\BIC@Expand#1{%
                     160
                             \romannumeralO\expandafter\expandafter\BIC@Space
                     161
                             \pdf@unescapehex{%
                     162
                     163
                               \expandafter\expandafter\expandafter
                     164
                               \BIC@StripHexSpace\pdf@escapehex{#1}20\@nil
                     165
                            }%
                          }%
                     166
\BIC@StripHexSpace
                          \verb|\def|BIC@StripHexSpace#120#2|@nil{||}|
                     167
                     168
                            #1%
                     169
                             \int x^{\#2}\
                             \else
                     170
                               \BIC@AfterFi{%
                     171
                                 \BIC@StripHexSpace#2\@nil
                     172
                              }%
                     173
                             \BIC@Fi
                     175
                          }%
                     176 \fi
                           Normalize expanded number
                    2.6
    \BIC@Normalize
                    #1: result sign
                    #2: first token of number
                     177 \def\BIC@Normalize#1#2{%
                     178
                          \ifx#2-%
                     179
                            \ifx\\#1\\%
                               \BIC@AfterFiFi{%
                     180
                                 \BIC@Normalize-%
                     181
                     182
                              }%
                             \else
                     183
                               \BIC@AfterFiFi{%
                     184
                     185
                                 \BIC@Normalize{}%
                              }%
                     186
                             \fi
                     187
                          \else
                     188
                            \ifx#2+%
                     189
                              \BIC@AfterFiFi{%
                     190
                                 \BIC@Normalize{#1}%
                     191
                              }%
                     192
                             \else
                     193
                               \ifx#20%
                     194
                     195
                                 \BIC@AfterFiFiFi{%
                     196
                                   \BIC@NormalizeZero{#1}%
                                }%
                     197
                              \else
                     198
                                \BIC@AfterFiFiFi{%
                     199
```

```
\BIC@NormalizeDigits#1#2%
                        200
                                   }%
                        201
                                 \fi
                        202
                        203
                               \fi
                        204
                             \BIC@Fi
                        205 }
  \BIC@NormalizeZero
                        206 \def\BIC@NormalizeZero#1#2{%
                        207 \ifx#2!%
                               \BIC@AfterFi{ 0}%
                        208
                        209
                            \else
                        210
                               \ifx#20%
                        211
                                 \BIC@AfterFiFi{%
                        212
                                   \BIC@NormalizeZero{#1}%
                        213
                                 }%
                        214
                               \else
                                 \BIC@AfterFiFi{%
                        215
                        216
                                   \BIC@NormalizeDigits#1#2%
                                 }%
                        217
                               \fi
                        218
                             \BIC@Fi
                        219
                        220 }
\BIC@NormalizeDigits
                        221 \def\BIC@NormalizeDigits#1!{ #1}
                       2.7
                              Num
      \bigintcalcNum
                        222 \def\bigintcalcNum#1{%
                             \romannumeral0%
                        223
                        224
                             \expandafter\expandafter\expandafter\BIC@Normalize
                        225
                             \expandafter\expandafter\expandafter{%
                             \expandafter\expandafter\expandafter}%
                        227
                             \BIC@Expand{#1}!%
                        228 }
                       2.8
                              Inv, Abs, Sgn
      \bigintcalcInv
                        229 \def\bigintcalcInv#1{\%
                        {\tt 230 } \verb|\romannumeral0\expandafter\expandafter\expandafter\BIC@Space|\\
                        231
                             \bigintcalcNum{-#1}%
                        232 }
      \bigintcalcAbs
                        233 \def\bigintcalcAbs#1{%
                        234 \romannumeral0%
                             \expandafter\expandafter\BIC@Abs
                             \bigintcalcNum{#1}%
                        237 }
            \BIC@Abs
                        238 \ensuremath{\mbox{MIC@Abs#1}}
                        239 \ifx#1-%
                               \expandafter\BIC@Space
                        240
                        241
                            \else
                        242
                               \expandafter\BIC@Space
                        243
                               \expandafter#1%
```

```
\fi
                 244
                 245 }
\bigintcalcSgn
                 246 \def\bigintcalcSgn#1{%
                 247 \number
                 248
                      \expandafter\expandafter\expandafter\BIC@Sgn
                      \bigintcalcNum{#1}! %
                 250 }
      \BIC@Sgn
                 251 \def\BIC@Sgn#1#2!{%
                 252 \ifx#1-%
                 253
                        -1%
                     \else
                 254
                        \ifx#10%
                 255
                          0%
                 256
                        \else
                 257
                 258
                          1%
                        \fi
                 259
                 260 \fi
                 261 }
                2.9
                       Cmp, Min, Max
\bigintcalcCmp
                 262 \def\bigintcalcCmp#1#2{%
                 263 \number
                     \expandafter\expandafter\expandafter\BIC@Cmp
                 264
                      \bigintcalcNum{#2}!{#1}%
                 265
                 266 }
      \BIC@Cmp
                 267 \ensuremath{$\backslash$} BIC@Cmp#1!#2{\%}
                 268 \expandafter\expandafter\expandafter\BIC@@Cmp
                 269 \bigintcalcNum{#2}!#1!%
                 270 }
     \BIC@@Cmp
                 271 \def\BIC@@Cmp#1#2!#3#4!{%
                 272 \ifx#1-%
                       \ifx#3-%
                 273
                 274
                          \BIC@AfterFiFi{%
                 275
                            \BIC@@Cmp#4!#2!%
                           }%
                 276
                 277
                         \else
                 278
                          \BIC@AfterFiFi{%
                 279
                            -1 %
                           }%
                 280
                        \fi
                 281
                      \else
                 282
                         \ifx#3-%
                 283
                           \BIC@AfterFiFi{%
                 284
                 285
                            1 %
                 286
                           }%
                 287
                         \else
                 288
                           \BIC@AfterFiFi{%
                 289
                             \BIC@CmpLength#1#2!#3#4!#1#2!#3#4!%
                           }%
                 290
                 291
                         \fi
                      \BIC@Fi
                 292
                 293 }
```

```
\BIC@PosCmp
                   294 \def\BIC@PosCmp#1!#2!{%
                   295 \BIC@CmpLength#1!#2!#1!#2!%
                   296 }
\BIC@CmpLength
                   297 \def\BIC@CmpLength#1#2!#3#4!{%
                        \ifx\\#2\\%
                   298
                           \ifx\\#4\\%
                   299
                   300
                             \BIC@AfterFiFi\BIC@CmpDiff
                   301
                           \else
                             \BIC@AfterFiFi{%
                   302
                   303
                               \BIC@CmpResult{-1}%
                   304
                   305
                           \fi
                   306
                       \else
                           \left| \frac{4}{\%} \right|
                   307
                             \BIC@AfterFiFi{%
                   308
                               \BIC@CmpResult1%
                   309
                             }%
                   310
                           \else
                   311
                   312
                             \BIC@AfterFiFi{%
                   313
                               \BIC@CmpLength#2!#4!%
                   314
                             }%
                   315
                        \BIC@Fi
                   316
                   317 }
\BIC@CmpResult
                   318 \def\BIC@CmpResult#1#2!#3!{#1 }
  \BIC@CmpDiff
                   319 \ensuremath{ \mbox{\sc def}\mbox{\sc BIC@CmpDiff#1#2!#3#4!} \ensuremath{ \%}
                   320 \ifnum#1<#3 %
                           \BIC@AfterFi{%
                   321
                   322
                             -1 %
                   323
                           }%
                   324
                        \else
                   325
                           \ifnum#1>#3 %
                             \BIC@AfterFiFi{%
                   326
                               1 %
                   327
                             }%
                   328
                           \else
                   329
                             \ifx\\#2\\%
                   330
                               \BIC@AfterFiFiFi{%
                   331
                   332
                                 0 %
                   333
                               }%
                   334
                             \else
                   335
                               \BIC@AfterFiFiFi{%
                   336
                                 \BIC@CmpDiff#2!#4!%
                               }%
                   337
                   338
                             \fi
                           \fi
                   339
                        \BIC@Fi
                  340
                   341 }
\bigintcalcMin
                   342 \def\bigintcalcMin#1{\%
                        \romannumeral0%
                        \verb|\expandafter| expandafter| BIC@MinMax|
                   344
                        \bigintcalcNum{#1}!-!%
                  345
                   346 }
```

```
347 \def\bigintcalcMax#1{%
                348 \romannumeral0%
                     \expandafter\expandafter\expandafter\BIC@MinMax
                349
                350 \bigintcalcNum{#1}!!%
                351 }
   \BIC@MinMax #1: x
                #2: sign for comparison
                #3: y
                 352 \def\BIC@MinMax#1!#2!#3{%
                     \expandafter\expandafter\expandafter\BIC@@MinMax
                     \bigintcalcNum{#3}!#1!#2!%
                355 }
 \BIC@@MinMax #1: y
                #2: x
                #3: sign for comparison
                356 \def\BIC@@MinMax#1!#2!#3!{%
                357 \ifnum\BIC@@Cmp#1!#2!=#31 %
                358
                       \BIC@AfterFi{ #1}%
                359
                     \else
                       \BIC@AfterFi{ #2}%
                360
                    \BIC@Fi
                361
                362 }
                2.10
                       Odd
\bigintcalcOdd
                 363 \def\bigintcalcOdd#1{%
                 364 \romannumeral0%
                     \expandafter\expandafter\expandafter\BIC@Odd
                     \bigintcalcAbs{#1}!%
                367 }
\BigIntCalcOdd
                 368 \def\BigIntCalcOdd#1!{%
                 369 \romannumeral0%
                370 \BIC@Odd#1!%
                371 }
     \BIC@Odd #1: x
                372 \def\BIC@Odd#1#2{%
                373 \ifx#2!%
                374
                       \ifodd#1 %
                         \BIC@AfterFiFi{ 1}%
                375
                376
                377
                         \BIC@AfterFiFi{ 0}%
                378
                       \fi
                379 \else
                       \expandafter\BIC@Odd\expandafter#2%
                380
                     \BIC@Fi
                381
                382 }
                        Inc, Dec
                2.11
\bigintcalcInc
                383 \def\bigintcalcInc#1{%
                     \romannumeral0%
                384
                 385
                     \expandafter\expandafter\BIC@IncSwitch
```

\bigintcalcMax

```
\bigintcalcNum{#1}!%
                 386
                 387 }
\BIC@IncSwitch
                 388 \def\BIC@IncSwitch#1#2!{%
                 389 \ifcase\BIC@@Cmp#1#2!-1!%
                 390
                        \BIC@AfterFi{ 0}%
                 391 \or
                      \BIC@AfterFi{%
                 392
                         \BIC@Inc#1#2!{}%
                 393
                        }%
                 394
                 395 \else
                 396
                        \BIC@AfterFi{%
                 397
                          \expandafter-\romannumeral0%
                 398
                          \BIC@Dec#2!{}\%
                 399
                        }%
                     \BIC@Fi
                 400
                 401 }
\bigintcalcDec
                 402 \def\bigintcalcDec#1{%
                 403 \romannumeral0%
                 404 \expandafter\expandafter\expandafter\BIC@DecSwitch
                 405 \bigintcalcNum{#1}!%
                 406 }
\BIC@DecSwitch
                 407 \def\BIC@DecSwitch#1#2!{%
                 408 \ifcase\BIC@Sgn#1#2! %
                        \BIC@AfterFi{ -1}%
                 409
                 410 \or
                 411
                        \BIC@AfterFi{%
                 412
                          \BIC@Dec#1#2!{}%
                        }%
                 413
                 414 \else
                        \BIC@AfterFi{%
                 415
                          \expandafter-\romannumeral0%
                 416
                          \BIC@Inc#2!{}%
                 417
                 418
                        }%
                 419
                      \BIC@Fi
                 420 }
\BigIntCalcInc
                 421 \def\BigIntCalcInc#1!{%
                 422 \romannumeral0\BIC@Inc#1!{}%
                 423 }
\BigIntCalcDec
                 424 \def\BigIntCalcDec#1!{%
                 425 \romannumeral0\BIC@Dec#1!{}%
                 426 }
      \BIC@Inc
                 427 \def\BIC@Inc#1#2!#3{%
                 428 \ifx\\#2\\%
                        \BIC@AfterFi{%
                 429
                          \BIC@@Inc1#1#3!{}%
                 430
                        }%
                 431
                 432
                      \else
                 433
                        \BIC@AfterFi{%
                 434
                          \BIC@Inc#2!{#1#3}%
```

```
435
                     }%
                   \BIC@Fi
              436
              437 }
 \BIC@@Inc
              438 \def\BIC@@Inc#1#2#3!#4{%
                  \ifcase#1 %
              440
                     \ifx\\#3\\%
                       \BIC@AfterFiFi{ #2#4}%
              441
              442
                     \else
                       \BIC@AfterFiFi{%
              443
                         \BIC@@IncO#3!{#2#4}%
              444
              445
                       }%
              446
                     \fi
              447
                   \else
              448
                     \ifnum#2<9 %
              449
                       \BIC@AfterFiFi{%
                         \verb|\expandafter\BIC@@Inc\the\numexpr#2+1\relax| \\
              450 &
                         \expandafter\expandafter\BIC@@@Inc
              451 $
              452 $
                         \ifcase#2 \expandafter1%
             453 $
                         \or\expandafter2%
              454 $
                         \or\expandafter3%
             455 $
                         \or\expandafter4%
             456 $
                         \or\expandafter5%
              457 $
                         \or\expandafter6%
              458 $
                         \or\expandafter7%
              459 $
                         \or\expandafter8%
              460 $
                         \or\expandafter9%
              461 $?
                         \else\BigIntCalcError:ThisCannotHappen%
                         \fi
              462 $
                         0#3!{#4}%
              463
                       }%
              464
              465
                     \else
              466
                       \BIC@AfterFiFi{%
                         \BIC@@@Inc01#3!{#4}%
              467
                       }%
              468
              469
              470
                   \BIC@Fi
              471 }
\BIC@@@Inc
              472 \def\BIC@@@Inc#1#2#3!#4{%
              473
                  \ifx\\#3\\%
                     \ifnum#2=1 %
              474
                       \BIC@AfterFiFi{ 1#1#4}%
              475
              476
                     \else
                       \BIC@AfterFiFi{ #1#4}%
              477
                     \fi
              478
              479
                   \else
                     \BIC@AfterFi{%
              480
              481
                       \BIC@@Inc#2#3!{#1#4}%
                     }%
              482
                   \BIC@Fi
              483
              484 }
  \BIC@Dec
              485 \def\BIC@Dec#1#2!#3{%
                   \ifx\\#2\\%
              486
                     \BIC@AfterFi{%
              487
                       \BIC@@Dec1#1#3!{}%
              488
              489
                     }%
                   \else
              490
```

```
\BIC@AfterFi{%
              491
              492
                       \BIC@Dec#2!{#1#3}%
              493
              494
                   \BIC@Fi
              495 }
 \BIC@@Dec
              496 \def\BIC@@Dec#1#2#3!#4{%
                  \ifcase#1 %
              497
                     \ifx\\#3\\%
              498
                       \BIC@AfterFiFi{ #2#4}%
             499
              500
                     \else
              501
                       \BIC@AfterFiFi{%
              502
                         \BIC@@Dec0#3!{#2#4}%
              503
                       }%
              504
                     \fi
              505
                   \else
                     \ifnum#2>0 %
              506
                       \BIC@AfterFiFi{%
              507
                          \verb|\expandafter\BIC@@@Dec\the\numexpr#2-1\relax|
              508 &
                          \verb|\expandafter| expandafter | BIC@@Dec| \\
             509 $
                          \ifcase#2
             510 $
             511 $?
                            \BigIntCalcError:ThisCannotHappen%
             512 $
                          \or\expandafter0%
                          \or\expandafter1%
             513 $
             514 $
                          \or\expandafter2%
              515 $
                         \or\expandafter3%
              516 $
                         \or\expandafter4%
             517 $
                         \or\expandafter5%
             518 $
                         \or\expandafter6%
             519 $
                         \or\expandafter7%
                         \or\expandafter8%
             520 $
             521 $?
                         \else\BigIntCalcError:ThisCannotHappen%
             522 $
                         \fi
                         0#3!{#4}%
              523
                       }%
              524
              525
                     \else
              526
                       \BIC@AfterFiFi{%
              527
                          \BIC@@@Dec91#3!{#4}%
                       }%
              528
                     \fi
              529
                   \BIC@Fi
             530
              531 }
\BIC@@@Dec
              532 \def\BIC@@@Dec#1#2#3!#4{%
                   \ifx\\#3\\%
              533
                     \ifcase#1 %
              534
                       \ifx\\#4\\%
              535
              536
                          \BIC@AfterFiFiFi{ 0}%
              537
                       \else
                          \BIC@AfterFiFiFi{ #4}%
              538
                       \fi
              539
                     \else
              540
                       \BIC@AfterFiFi{ #1#4}%
              541
                     \fi
             542
              543
                   \else
                     \BIC@AfterFi{%
              544
                       \BIC@@Dec#2#3!{#1#4}%
              545
                     }%
              546
             547
                   \BIC@Fi
             548 }
```

2.12 Add, Sub

```
\bigintcalcAdd
```

```
549 \ensuremath{\mbox{\sc Mdd}\#1}\
                 550 \romannumeral0%
                      \expandafter\expandafter\expandafter\BIC@Add
                      \bigintcalcNum{#1}!%
                 553 }
      \BIC@Add
                 554 \def\BIC@Add#1!#2{%
                 555 \expandafter\expandafter\expandafter
                      \BIC@AddSwitch\bigintcalcNum{#2}!#1!%
                 556
                 557 }
\bigintcalcSub
                 558 \ensuremath{\mbox{\sc Sub#1#2}\%}
                      \romannumeral0%
                      \expandafter\expandafter\BIC@Add
                      \bigintcalcNum{-#2}!{#1}%
                 561
                 562 }
```

\BIC@AddSwitch Decision table for \BIC@AddSwitch.

x < 0	y < 0	-x > -y	_	Add(-x, -y)
		else		Add(-y, -x)
	else	-x > y	_	Sub(-x,y)
		-x = y		0
		else	+	Sub(y, -x)
else	y < 0	x > -y	+	Sub(x, -y)
		x = -y		0
		else	_	Sub(-y,x)
	else	x > y	+	Add(x,y)
		else		Add(y,x)

```
563 \def\BIC@AddSwitch#1#2!#3#4!{%
564 \text{ } ifx#1-% x < 0
        \int x#3-% y < 0
565
           \expandafter-\romannumeral0%
566
           \label{localization} $$ \prod_{x \in \mathbb{Z}} BIC@PosCmp#2!#4!=1 % -x > -y $$
567
568
             \BIC@AfterFiFiFi{%
569
                \BIC@AddXY#2!#4!!!%
             }%
570
           \else % -x <= -y
571
             \BIC@AfterFiFiFi{%
572
                \BIC@AddXY#4!#2!!!%
573
             }%
574
           \fi
575
        \else % y >= 0
576
           \label{eq:local_problem} \label{eq:local_problem} $$ \left( \frac{3}{4} - x = y \right) $$
577
             \BIC@AfterFiFiFi{ 0}%
578
           579
580
             \expandafter-\romannumeral0%
581
             \BIC@AfterFiFiFi{%
                \BIC@SubXY#2!#3#4!!!%
582
             }%
583
           \else % -x <= y
584
             \BIC@AfterFiFiFi{%
585
                \BIC@SubXY#3#4!#2!!!%
586
             }%
587
           \fi
588
```

```
\fi
                589
                     \else % x >= 0
                590
                       \int x#3-% y < 0
                591
                          592
                593
                           \BIC@AfterFiFiFi{ 0}%
                594
                          595
                           \BIC@AfterFiFiFi{%
                596
                              \BIC@SubXY#1#2!#4!!!%
                           }%
                597
                         \else % x \le -y
                598
                           \expandafter-\romannumeral0%
                599
                           \BIC@AfterFiFiFi{%
                600
                              \BIC@SubXY#4!#1#2!!!%
                601
                           }%
                602
                603
                         \fi
                604
                        \else % y >= 0
                          605
                           \BIC@AfterFiFiFi{%
                606
                              \BIC@AddXY#1#2!#3#4!!!%
                607
                           }%
                608
                          \else % x <= y
                609
                           \BIC@AfterFiFiFi{%
                610
                              \BIC@AddXY#3#4!#1#2!!!%
                611
                           }%
                612
                613
                          \fi
                614
                        \fi
                      \BIC@Fi
                615
                616 }
\BigIntCalcAdd
                617 \def\BigIntCalcAdd#1!#2!{%
                     \romannumeral0\BIC@AddXY#1!#2!!!%
                618
                619 }
\BigIntCalcSub
                620 \def\BigIntCalcSub#1!#2!{%
                     \romannumeral0\BIC@SubXY#1!#2!!!%
                621
                622 }
   \BIC@AddXY
                623 \def\BIC@AddXY#1#2!#3#4!#5!#6!{%
                     \ifx\\#2\\%
                625
                        \ifx\\#3\\%
                         \BIC@AfterFiFi{%
                626
                           \BIC@DoAdd0!#1#5!#60!%
                627
                         }%
                628
                        \else
                629
                          \BIC@AfterFiFi{%
                630
                           \BIC@DoAdd0!#1#5!#3#6!%
                631
                632
                         }%
                       \fi
                633
                634
                     \else
                635
                        \ifx\\#4\\%
                636
                         \ifx\\#3\\%
                637
                           \BIC@AfterFiFiFi{%
                638
                              \BIC@AddXY#2!{}!#1#5!#60!%
                           }%
                639
                640
                          \else
                           \BIC@AfterFiFiFi{%
                641
                             \BIC@AddXY#2!{}!#1#5!#3#6!%
                642
                           }%
                643
                         \fi
                644
```

```
645
                                                                   \else
                                                                         \BIC@AfterFiFi{%
                                               646
                                                                              \BIC@AddXY#2!#4!#1#5!#3#6!%
                                               647
                                               648
                                               649
                                                                   \fi
                                               650
                                                             \BIC@Fi
                                               651 }
          \BIC@DoAdd #1: carry
                                             #2: reverted result
                                             #3#4: reverted x
                                             #5#6: reverted y
                                               652 \def\BIC@DoAdd#1#2!#3#4!#5#6!{%
                                               653 \ifx\\#4\\%
                                               654
                                                                   \BIC@AfterFi{%
                                               655 &
                                                                         \expandafter\BIC@Space
                                               656 &
                                                                         \theta = 1+#3+#5 relax#2%
                                               657 $
                                                                         \expandafter\expandafter\expandafter\BIC@AddResult
                                               658 $
                                                                         \BIC@AddDigit#1#3#5#2%
                                                                   }%
                                               659
                                                             \else
                                               660
                                                                   \BIC@AfterFi{%
                                               661
                                                                         \expandafter\expandafter\BIC@DoAdd
                                               662
                                                                         \BIC@AddDigit#1#3#5#2!#4!#6!%
                                               663
                                                                   }%
                                               664
                                                             \BIC@Fi
                                               665
                                               666 }
\BIC@AddResult
                                               667 $ \def\BIC@AddResult#1{%
                                                                  \ifx#10%
                                               668 $
                                                                        \expandafter\BIC@Space
                                               669 $
                                               670 $
                                                                   \else
                                               671 $
                                                                         \expandafter\BIC@Space\expandafter#1%
                                                                \fi
                                               672 $
                                               673 $ }%
  \BIC@AddDigit #1: carry
                                             #2: digit of x
                                             #3: digit of y
                                               674 \ensuremath{ \mbox{ GF}\BIC@AddDigit#1#2#3{\%} }
                                               675 \romannumeral0%
                                               676 & \expandafter\BIC@@AddDigit\the\numexpr#1+#2+#3!%
                                               677 $ \expandafter\BIC@@AddDigit\number%
                                               678 $ \csname
                                                                  BIC@AddCarry%
                                               679 $
                                               680 $
                                                                  \ifcase#1 %
                                               681 $
                                                                        #2%
                                               682 $
                                                                   \else
                                               683 $
                                                                        \footnote{Moreover the continuous of the conti
                                               684 $
                                                                   \fi
                                               685 $ \endcsname#3!%
                                               686 }
\BIC@@AddDigit
                                               687 \def\BIC@@AddDigit#1!{%
                                               688
                                                           \ifnum#1<10 %
                                               689
                                                                   \BIC@AfterFi{ 0#1}%
                                               690
                                                            \else
                                                                  \BIC@AfterFi{ #1}%
                                               691
                                                          \BIC@Fi
                                               692
                                               693 }
```

```
\BIC@AddCarry0
                     694 $\expandafter\def\csname BIC@AddCarryO\endcsname#1{#1}%
   \BIC@AddCarry10
                     695 $\expandafter\def\csname BIC@AddCarry10\endcsname#1{1#1}%
\BIC@AddCarry[1-9]
                     696 \ \def\BIC@Temp#1#2{%}
                             \expandafter\def\csname BIC@AddCarry#1\endcsname##1{%
                               \ifcase##1 #1\or
                     698 $
                     699 $
                     700 $?
                               \else\BigIntCalcError:ThisCannotHappen%
                     701 $
                     702 $
                             }%
                     703 $ }%
                     704 $ \BIC@Temp 0{1\or2\or3\or4\or5\or6\or7\or8\or9}%
                     705 $ \BIC@Temp 1{2\or3\or4\or5\or6\or7\or8\or9\or10}\%
                     706 $ \BIC@Temp 2{3\or4\or5\or6\or7\or8\or9\or10\or11}%
                     707 $ \BIC@Temp 3{4\or5\or6\or7\or8\or9\or10\or11\or12}%
                     708 $ \BIC@Temp 4{5\or6\or7\or8\or9\or10\or11\or12\or13}%
                     709 $ \BIC@Temp 5{6\or7\or8\or9\or10\or11\or12\or13\or14}%
                     710 $ \BIC@Temp 6{7\or8\or9\or10\or11\or12\or13\or14\or15}%
                     711 $ \BIC@Temp 7{8\or9\or10\or11\or12\or13\or14\or15\or16}%
                     712 $ \BIC@Temp 8{9\or10\or11\or12\or13\or14\or15\or16\or17}%
                     713 $ \BIC@Temp 9{10\or11\or12\or13\or14\or15\or16\or17\or18}%
        \BIC@SubXY Preconditions:
                        • x > y, x \ge 0, and y >= 0
                        • digits(x) = digits(y)
                     714 \def\BIC@SubXY#1#2!#3#4!#5!#6!{%
                     715 \ifx\\#2\\%
                     716
                             \ifx\\#3\\%
                               \BIC@AfterFiFi{%
                     717
                                 \BIC@DoSub0!#1#5!#60!%
                     718
                               }%
                     719
                     720
                             \else
                               \BIC@AfterFiFi{%
                     721
                                 \BIC@DoSub0!#1#5!#3#6!%
                     722
                               }%
                     723
                            \fi
                     724
                          \else
                     725
                             \ifx\\#4\\%
                     726
                               \ifx\\#3\\%
                     727
                                 \BIC@AfterFiFiFi{%
                     728
                                   \BIC@SubXY#2!{}!#1#5!#60!%
                     729
                                 }%
                     730
                     731
                               \else
                                 \BIC@AfterFiFiFi{%
                     732
                                   \BIC@SubXY#2!{}!#1#5!#3#6!%
                     733
                                 }%
                     734
                               \fi
                     735
                     736
                             \else
                     737
                               \BIC@AfterFiFi{%
                                 \BIC@SubXY#2!#4!#1#5!#3#6!%
                     738
                               }%
                     739
                     740
                             \fi
```

\BIC@Fi

741 742 }

```
\BIC@DoSub #1: carry
                                            #2: reverted result
                                            #3#4: reverted x
                                            #5#6: reverted y
                                              743 \def\BIC@DoSub#1#2!#3#4!#5#6!{%
                                              744
                                                          \ifx\\#4\\%
                                              745
                                                                 \BIC@AfterFi{%
                                                                      \expandafter\expandafter\BIC@SubResult
                                               746
                                                                      \BIC@SubDigit#1#3#5#2%
                                              747
                                                                 }%
                                               748
                                               749
                                                         \else
                                               750
                                                                 \BIC@AfterFi{%
                                                                      \expandafter\expandafter\BIC@DoSub
                                               751
                                                                      \BIC@SubDigit#1#3#5#2!#4!#6!%
                                               752
                                               753
                                                                 }%
                                               754 \BIC@Fi
                                              755 }
  \BIC@SubResult
                                               756 \def\BIC@SubResult#1{%
                                              757 \ifx#10%
                                                                 \expandafter\BIC@SubResult
                                               758
                                               759
                                                          \else
                                               760
                                                                 \expandafter\BIC@Space\expandafter#1%
                                               761
                                                            \fi
                                               762 }
     \BIC@SubDigit #1: carry
                                            #2: digit of x
                                            #3: digit of y
                                               763 \def\BIC@SubDigit#1#2#3{\%
                                               764 \romannumeral0%
                                              765 & \expandafter\BIC@GSubDigit\the\numexpr#2-#3-#1!\%
                                              766 \ \expandafter\BIC@@AddDigit\number
                                              767 $
                                                             \csname
                                                                   BIC@SubCarry%
                                               768 $
                                              769 $
                                                                     \ifcase#1 %
                                              770 $
                                                                          #3%
                                              771 $
                                              772 $
                                                                           \footnote{Missing the continuous of the contin
                                              773 $
                                                                      \fi
                                              774 $
                                                              \endcsname#2!%
                                              775 }
  \BIC@@SubDigit
                                              776 & \def\BIC@@SubDigit#1!{%
                                              777 & \ifnum#1<0 %
                                              778 &
                                                                     \BIC@AfterFi{%
                                                                           \expandafter\BIC@Space
                                              779 &
                                              780 &
                                                                           \expandafter1\the\numexpr#1+10\relax
                                              781 &
                                                                  }%
                                              782 & \else
                                                                    \BIC@AfterFi{ 0#1}%
                                               783 &
                                              784 & \BIC@Fi
                                              785 & }%
  \BIC@SubCarry0
                                               786 $\expandafter\def\csname BIC@SubCarryO\endcsname#1{#1}%
\BIC@SubCarry10
                                               787 $\expandafter\def\csname BIC@SubCarry10\endcsname#1{1#1}%
```

```
\BIC@SubCarry[1-9]
                     788 $ \def\BIC@Temp#1#2{%
                            \expandafter\def\csname BIC@SubCarry#1\endcsname##1{%
                     789 $
                     790 $
                               \ifcase##1 #2%
                     791 $?
                               \else\BigIntCalcError:ThisCannotHappen%
                     792 $
                               \fi
                     793 $
                            }%
                     794 $ }%
                     795 $ \BIC@Temp 1{19\or0\or1\or2\or3\or4\or5\or6\or7\or8}%
                     796 $ \BIC@Temp 2{18\or19\or0\or1\or2\or3\or4\or5\or6\or7}%
                     797 $ \BIC@Temp 3{17\or18\or19\or0\or1\or2\or3\or4\or5\or6}%
                     798 $ \BIC@Temp 4{16\or17\or18\or19\or0\or1\or2\or3\or4\or5}%
                     799 $ \BIC@Temp 5{15\or16\or17\or18\or19\or0\or1\or2\or3\or4}%
                     800 $ \BIC@Temp 6{14\or15\or16\or17\or18\or19\or0\or1\or2\or3}%
                     801 \ BIC@Temp 7{13\or14\or15\or16\or17\or18\or19\or0\or1\or2}%
                     802 $ \BIC@Temp 8{12\or13\or14\or15\or16\or17\or18\or19\or0\or1}%
                     803 $ \BIC@Temp 9{11\or12\or13\or14\or15\or16\or17\or18\or19\or0}%
                    2.13
                            ShI, Shr
    \bigintcalcShl
                     804 \def\bigintcalcShl#1{%
                     805 \romannumeral0%
                     806
                          \expandafter\expandafter\BIC@Shl
                     807
                          \bigintcalcNum{#1}!%
                     808 }
          \BIC@Shl
                     809 \def\BIC@Shl#1#2!{%
                     810 \ifx#1-%
                     811
                            \BIC@AfterFi{%
                     812
                              \expandafter-\romannumeral0%
                              \BIC@@Sh1#2!!%
                     813 &
                     814 $
                              \BIC@AddXY#2!#2!!!%
                            }%
                     815
                     816 \else
                            \BIC@AfterFi{%
                     817
                              \BIC@@Shl#1#2!!%
                     818 &
                     819 $
                               \BIC@AddXY#1#2!#1#2!!!%
                     820
                            }%
                     821
                          \BIC@Fi
                     822 }
    \BigIntCalcShl
                     823 \def\BigIntCalcShl#1!{%}
                     824 \romannumeral0%
                     825 & \BIC@@Shl#1!!%
                     826 $ \BIC@AddXY#1!#1!!!%
                     827 }
         \BIC@@Shl
                     828 & \def\BIC@@Shl#1#2!{%
                     829 &
                            \ifx\\#2\\%
                     830 &
                              \BIC@AfterFi{%
                     831 &
                                \BIC@@@Sh10!#1%
                     832 &
                              }%
```

833 &

834 &

835 & 836 &

837 & \\
838 & \}%

\else

}% \BIC@Fi

\BIC@AfterFi{% \BIC@@Shl#2!#1%

```
\BIC@@@Shl #1: carry
               #2: result
               #3#4: reverted number
                839 & \def\BIC@@@Shl#1#2!#3#4!{%
                      \ifx\\#4\\%
                840 &
                         \BIC@AfterFi{%
                841 &
                           \expandafter\BIC@Space
                842 &
                           \theta = 12\%
                843 &
                        }%
                844 &
                845 & \else
                846 &
                        \BIC@AfterFi{%
                          \expandafter\BIC@@@GShl\the\numexpr#3*2+#1!#2!#4!%
                847 &
                         }%
                848 &
                      \BIC@Fi
                849 &
                850 & }%
   \BIC@@@@Shl
                851 & \def\BIC@@@@Shl#1!{%
                      \ifnum#1<10 %
                852 &
                853 &
                         \BIC@AfterFi{%
                           \BIC@@@Sh10#1%
                854 &
                         }%
                855 &
                856 & \else
                857 &
                         \BIC@AfterFi{%
                858 &
                           \BIC@@@Shl#1%
                         }%
                859 &
                      \BIC@Fi
                860 &
                861 & }%
\bigintcalcShr
                862 \def\bigintcalcShr#1{\%
                863 \romannumeral0%
                864
                     \expandafter\expandafter\BIC@Shr
                     \bigintcalcNum{#1}!%
                865
                866 }
      \BIC@Shr
                867 \def\BIC@Shr#1#2!{%
                868 \ifx#1-%
                869
                       \expandafter-\romannumeral0%
                       \BIC@AfterFi{%
                870
                         \BIC@@Shr#2!%
                871
                       }%
                872
                873 \else
                       \BIC@AfterFi{%
                874
                         \BIC@@Shr#1#2!%
                875
                       }%
                876
                     \BIC@Fi
                877
                878 }
\BigIntCalcShr
                879 \def\BigIntCalcShr#1!{%
                880 \romannumeral0%
                     \BIC@@Shr#1!%
                881
                882 }
    \BIC@@Shr
                883 \def\BIC@@Shr#1#2!{%
                884 \ifcase#1 %
                       \BIC@AfterFi{ 0}%
                885
                886 \or
```

```
\ifx\\#2\\%
                 887
                           \BIC@AfterFiFi{ 0}%
                 888
                 889
                         \else
                           \BIC@AfterFiFi{%
                 890
                 891
                             \BIC@@@Shr#1#2!!%
                 892
                           }%
                 893
                         \fi
                 894
                       \else
                         \BIC@AfterFi{%
                 895
                           \BIC@@@Shr0#1#2!!%
                 896
                         }%
                 897
                       \BIC@Fi
                 898
                 899 }
    \BIC@@@Shr #1: carry
                #2#3: number
                #4: result
                 900 \def\BIC@@@Shr#1#2#3!#4!{%
                 901
                      \ifx\\#3\\%
                         \ifodd#1#2 %
                 902
                           \BIC@AfterFiFi{%
                 903
                             \verb|\expandafter\BIC@ShrResult\the\numexpr(#1#2-1)/2\relax|
                 904 &
                             \expandafter\expandafter\expandafter\BIC@ShrResult
                 905 $
                 906 $
                             \csname BIC@ShrDigit#1#2\endcsname
                 907
                             #4!%
                 908
                           }%
                 909
                         \else
                 910
                           \BIC@AfterFiFi{%
                 911 &
                             \expandafter\BIC@ShrResult\the\numexpr#1#2/2\relax
                 912 $
                             \verb|\expandafter| expandafter| BICQShrResult|
                 913 $
                             \csname BIC@ShrDigit#1#2\endcsname
                             #4!%
                 914
                           }%
                 915
                         \fi
                 916
                       \else
                 917
                         \ifodd#1#2 %
                 918
                 919
                 920 &
                             \expandafter\BIC@@@@Shr\the\numexpr(#1#2-1)/2\relax1%
                 921 $
                             \expandafter\expandafter\expandafter\BIC@@@@Shr
                 922 $
                             \csname BIC@ShrDigit#1#2\endcsname
                             #3!#4!%
                 923
                           }%
                 924
                 925
                         \else
                           \BIC@AfterFiFi{%
                 926
                 927 &
                             \expandafter\BIC@@@Shr\the\numexpr#1#2/2\relax0%
                 928 $
                             \expandafter\expandafter\BIC@@@@Shr
                 929 $
                             \csname BIC@ShrDigit#1#2\endcsname
                             #3!#4!%
                 930
                 931
                           }%
                 932
                         \fi
                       \BIC@Fi
                 933
                 934 }
\BIC@ShrResult
                 935 & \def\BIC@ShrResult#1#2!{ #2#1}%
                 936 $ \def\BIC@ShrResult#1#2#3!{ #3#1}%
   \BIC@@@@Shr #1: new digit
                #2: carry
                #3: remaining number
                #4: result
                 937 \def\BIC@@@@Shr#1#2#3!#4!{%
```

```
\BIC@@@Shr#2#3!#4#1!%
                       938
                       939 }
\BIC@ShrDigit[00-19]
                       940 $ \def\BIC@Temp#1#2#3#4{%
                              \expandafter\def\csname BIC@ShrDigit#1#2\endcsname{#3#4}%
                       942 $ }%
                       943 $ \BIC@Temp 0000%
                       944 $ \BIC@Temp 0101%
                       945 $ \BIC@Temp 0210%
                       946 $ \BIC@Temp 0311%
                       947 $ \BIC@Temp 0420%
                       948 $ \BIC@Temp 0521%
                       949 $ \BIC@Temp 0630%
                       950 $ \BIC@Temp 0731%
                       951 $ \BIC@Temp 0840%
                       952 $ \BIC@Temp 0941%
                       953 $ \BIC@Temp 1050%
                       954 $ \BIC@Temp 1151%
                       955 $ \BIC@Temp 1260%
                       956 $ \BIC@Temp 1361%
                       957 $ \BIC@Temp 1470%
                       958 $ \BIC@Temp 1571%
                       959 $ \BIC@Temp 1680%
                       960 $ \BIC@Temp 1781%
                       961 $ \BIC@Temp 1890%
                       962 $ \BIC@Temp 1991%
                       2.14
                               \BIC@Tim
            \BIC@Tim Macro \BIC@Tim implements "Number times digit".
                       #1: plain number without sign
                       #2: digit
           \BIC@@Tim #1#2: number
                       #3: reverted number
                       963 \def\BIC@@Tim#1#2!{%
                       964 \ifx\\#2\\%
                       965
                               \BIC@AfterFi{%
                                 \BIC@ProcessTim0!#1%
                       966
                              }%
                       967
                       968 \else
                               \BIC@AfterFi{%
                       969
                                 \BIC@@Tim#2!#1%
                       970
                       971
                              }%
                       972
                            \BIC@Fi
                       973 }
     \BIC@ProcessTim #1: carry
                       #2: result
                       #3#4: reverted number
                       #5: digit
                       974 \def\BIC@ProcessTim#1#2!#3#4!#5{%
                       975
                            \ifx\\#4\\%
                       976
                               \BIC@AfterFi{%
                                 \expandafter\BIC@Space
                       977
                                 \the\numexpr#3*#5+#1\relax
                       978 &
                       979 $
                                 \romannumeral0\BIC@TimDigit#3#5#1%
                                 #2%
                       980
                              }%
                       981
                       982 \else
```

```
\BIC@AfterFi{%
                      983
                                \expandafter\BIC@@ProcessTim
                      984
                                \the\numexpr#3*#5+#1%
                      985 &
                                \romannumeral0\BIC@TimDigit#3#5#1%
                      986 $
                      987
                                !#2!#4!#5%
                      988
                              }%
                      989
                            \BIC@Fi
                      990 }
  \BIC@@ProcessTim #1#2: carry?, new digit
                     #3: new number
                     #4: old number
                     #5: digit
                      991 \def\BIC@@ProcessTim#1#2!{%
                      992 \ifx\\#2\\%
                              \BIC@AfterFi{%
                      993
                                \BIC@ProcessTim0#1%
                      994
                      995
                              }%
                      996
                            \else
                      997
                              \BIC@AfterFi{%
                      998
                                \BIC@ProcessTim#1#2%
                      999
                              }%
                            \BIC@Fi
                     1000
                     1001 }
     \BIC@TimDigit #1: digit 0-9
                     #2: digit 3-9
                     #3: carry 0-9
                     1002 $ \def\BIC@TimDigit#1#2#3{%
                     1003 $
                             \ifcase#1 % 0
                     1004 $
                                \BIC@AfterFi{ #3}%
                     1005 $
                              \or % 1
                     1006 $
                                \BIC@AfterFi{%
                     1007 $
                                  \expandafter\BIC@Space
                     1008 $
                                  \number\csname BIC@AddCarry#2\endcsname#3 %
                                }%
                     1009 $
                     1010 $
                              \else
                                \ifcase#3 %
                     1011 $
                     1012 $
                                  \BIC@AfterFiFi{%
                     1013 $
                                    \expandafter\BIC@Space
                     1014 $
                                    \number\csname BIC@MulDigit#2\endcsname#1 %
                                  }%
                     1015 $
                                \else
                     1016 $
                     1017 $
                                  \BIC@AfterFiFi{%
                     1018 $
                                    \expandafter\BIC@Space
                     1019 $
                                    \romannumeral0%
                     1020 $
                                    \expandafter\BIC@AddXY
                     1021 $
                                    \number\csname BIC@MulDigit#2\endcsname#1!%
                     1022 $
                                    #3!!!%
                     1023 $
                                  }%
                     1024 $
                                \fi
                     1025 $
                             \BIC@Fi
                     1026 $ }%
\BIC@MulDigit[3-9]
                     1027 $ \def\BIC@Temp#1#2{%
                     1028 $
                              \expandafter\def\csname BIC@MulDigit#1\endcsname##1{%
                     1029 $
                                \ifcase##1 0%
                                \or ##1%
                     1030 $
                                \or #2%
                     1031 $
                                \else\BigIntCalcError:ThisCannotHappen%
                     1032 $?
                     1033 $
```

```
1034 $ }%
1035 $ }%
1036 $ \BIC@Temp 3{6\or9\or12\or15\or18\or21\or24\or27}%
1037 $ \BIC@Temp 4{8\or12\or16\or20\or24\or28\or32\or36}%
1038 $ \BIC@Temp 5{10\or15\or20\or25\or30\or35\or40\or45}%
1039 $ \BIC@Temp 6{12\or18\or24\or30\or36\or42\or48\or54}%
1040 $ \BIC@Temp 7{14\or21\or28\or35\or42\or49\or56\or63}%
1041 $ \BIC@Temp 8{16\or24\or32\or40\or48\or56\or64\or72}%
1042 $ \BIC@Temp 9{18\or27\or36\or45\or54\or63\or72\or81}%
```

2.15 Mul

\bigintcalcMul

```
1043 \end{substitute} $1044 \end{substitute} $$ 1044 \end{substitute} $$ \end{substi
```

\BIC@Mul

\BIC@MulSwitch Decision table for \BIC@MulSwitch.

x = 0				0
x > 0	y = 0			0
	y > 0	x > y	+	Mul(x, y)
		else		Mul(y,x)
	y < 0	x > -y	_	Mul(x, -y)
		else		Mul(-y,x)
x < 0	y = 0			0
	y > 0	-x > y	_	Mul(-x,y)
		else		Mul(y, -x)
	y < 0	-x > -y	+	Mul(-x, -y)
		else		Mul(-y, -x)

```
1052 \def\BIC@MulSwitch#1#2!#3#4!{%
1053 \ifcase\BIC@Sgn#1#2! % x = 0
1054
       \BIC@AfterFi{ 0}%
1055
     1056
       \ightharpoonup \ \ifcase\BIC@Sgn#3#4! % y = 0
1057
        \BIC@AfterFiFi{ 0}%
1058
       1059
1060
          \BIC@AfterFiFiFi{%
            \BIC@ProcessMul0!#1#2!#3#4!%
1061
          }%
1062
1063
         \else % x <= y
          \BIC@AfterFiFiFi{%
1064
            \BIC@ProcessMul0!#3#4!#1#2!%
1065
1066
          }%
1067
        \fi
1068
       \else % y < 0
1069
        \expandafter-\romannumeral0%
         1070
          \BIC@AfterFiFiFi{%
1071
            \BIC@ProcessMul0!#1#2!#4!%
1072
1073
          }%
```

```
1074
                             \else % x <= -y
                  1075
                               \BIC@AfterFiFiFi{%
                  1076
                                 \BIC@ProcessMul0!#4!#1#2!%
                  1077
                  1078
                            \fi
                  1079
                          \fi
                  1080
                        \else % x < 0
                          \ightharpoonup \ \ifcase\BIC@Sgn#3#4! % y = 0
                  1081
                            \BIC@AfterFiFi{ 0}%
                  1082
                           \or % y > 0
                  1083
                             \expandafter-\romannumeral0%
                  1084
                             \ifnum\BIC@PosCmp#2!#3#4!=1 % -x > y
                  1085
                               \BIC@AfterFiFiFi{%
                  1086
                  1087
                                 \BIC@ProcessMul0!#2!#3#4!%
                               }%
                  1088
                  1089
                             \else % -x <= y
                               \BIC@AfterFiFiFi{%
                  1090
                                 \BIC@ProcessMul0!#3#4!#2!%
                  1091
                               }%
                  1092
                  1093
                             \fi
                           \else % y < 0
                  1094
                             \infty BIC@PosCmp#2!#4!=1 % -x > -y
                  1095
                               \BIC@AfterFiFiFi{%
                  1096
                                 \BIC@ProcessMul0!#2!#4!%
                  1097
                               }%
                  1098
                             \else % -x <= -y
                  1099
                               \BIC@AfterFiFiFi{%
                  1100
                                 \BIC@ProcessMul0!#4!#2!%
                  1101
                               }%
                  1102
                            \fi
                  1103
                           \fi
                  1104
                  1105
                        \BIC@Fi
                  1106 }
 \BigIntCalcMul
                  1107 \def\BigIntCalcMul#1!#2!{%
                        \romannumeral0%
                  1108
                        \BIC@ProcessMul0!#1!#2!%
                  1109
                  1110 }
\BIC@ProcessMul #1: result
                  #2: number x
                  #3#4: number y
                  1111 \def\BIC@ProcessMul#1!#2!#3#4!{%
                        \ifx\\#4\\%
                  1112
                           \BIC@AfterFi{%
                  1113
                             \expandafter\expandafter\BIC@Space
                  1114
                             \bigintcalcAdd{\BIC@Tim#2!#3}{#10}%
                  1115
                          }%
                  1116
                  1117
                        \else
                           \BIC@AfterFi{%
                  1118
                  1119
                             \expandafter\expandafter\expandafter\BIC@ProcessMul
                  1120
                             \bigintcalcAdd{\BIC@Tim#2!#3}{#10}!#2!#4!%
                  1121
                          }%
                        \BIC@Fi
                  1122
                  1123 }
                  2.16
                          Sqr
 \bigintcalcSqr
                  1124 \def\bigintcalcSqr#1{%
                  1125
                        \romannumeral0%
```

```
\expandafter\expandafter\BIC@Sqr
                 1126
                       \bigintcalcNum{#1}!%
                 1127
                 1128 }
       \BIC@Sqr
                 1129 \def\BIC@Sqr#1{%
                       \ifx#1-%
                          \expandafter\BIC@@Sqr
                 1131
                 1132
                       \else
                          \expandafter\BIC@@Sqr\expandafter#1%
                 1133
                 1134
                        \fi
                 1135 }
      \BIC@@Sqr
                 1136 \def\BIC@@Sqr#1!{%
                 1137
                       \BIC@ProcessMul0!#1!#1!%
                 1138 }
                 2.17
                         Fac
 \bigintcalcFac
                 1139 \def\bigintcalcFac#1{%
                      \romannumeral0%
                       \expandafter\expandafter\BIC@Fac
                 1142
                      \bigintcalcNum{#1}!%
                 1143 }
       \BIC@Fac
                 1144 \def\BIC@Fac#1#2!{%
                 1145 \ifx#1-%
                 1146
                         \BIC@AfterFi{ 0\BigIntCalcError:FacNegative}%
                 1147
                      \else
                         \ifnum\BIC@PosCmp#1#2!13!<0 %
                 1148
                           \ifcase#1#2 %
                 1149
                              \BIC@AfterFiFiFi{ 1}% 0!
                 1150
                           \or\BIC@AfterFiFiFi{ 1}% 1!
                 1151
                 1152
                           \or\BIC@AfterFiFiFi{ 2}% 2!
                 1153
                           \or\BIC@AfterFiFiFi{ 6}% 3!
                 1154
                           \or\BIC@AfterFiFiFi{ 24}% 4!
                 1155
                           \or\BIC@AfterFiFiFi{ 120}% 5!
                           \or\BIC@AfterFiFiFi{ 720}% 6!
                 1156
                           \or\BIC@AfterFiFiFi{ 5040}% 7!
                 1157
                           \or\BIC@AfterFiFiFi{ 40320}% 8!
                 1158
                           \or\BIC@AfterFiFiFi{ 362880}% 9!
                 1159
                           \or\BIC@AfterFiFiFi{ 3628800}% 10!
                 1160
                           \or\BIC@AfterFiFiFi{ 39916800}% 11!
                 1161
                 1162
                           \or\BIC@AfterFiFiFi{ 479001600}% 12!
                 1163 ?
                           \else\BigIntCalcError:ThisCannotHappen%
                 1164
                           \fi
                         \else
                 1165
                           \BIC@AfterFiFi{%
                 1166
                             \BIC@ProcessFac#1#2!479001600!%
                 1167
                           }%
                 1168
                 1169
                         \fi
                       \BIC@Fi
                 1170
                 1171 }
\BIC@ProcessFac #1: n
                 #2: result
                 1172 \def\BIC@ProcessFac#1!#2!{%
                 1173 \ifnum\BIC@PosCmp#1!12!=0 %
```

```
\BIC@AfterFi{ #2}%
                  1174
                  1175
                        \else
                          \BIC@AfterFi{%
                  1176
                  1177
                            \expandafter\BIC@@ProcessFac
                  1178
                            \romannumeral0\BIC@ProcessMul0!#2!#1!%
                  1179
                            !#1!%
                  1180
                          }%
                        \BIC@Fi
                  1181
                  1182 }
\BIC@@ProcessFac #1: result
                  #2: n
                  1183 \def\BIC@@ProcessFac#1!#2!{%
                  1184 \expandafter\BIC@ProcessFac
                       \romannumeral0\BIC@Dec#2!{}%
                  1186 !#1!%
                  1187 }
                  2.18
                          Pow
  \bigintcalcPow
                 #1: basis
                  #2: power
                  1188 \def\bigintcalcPow#1{%
                       \romannumeral0%
                  1189
                       \expandafter\expandafter\expandafter\BIC@Pow
                        \bigintcalcNum{#1}!%
                  1192 }
        \BIC@Pow #1: basis
                  #2: power
                  1193 \def\BIC@Pow#1!#2{%
                       \expandafter\expandafter\expandafter\BIC@PowSwitch
                       \bigintcalcNum{#2}!#1!%
                  1195
                  1196 }
  \BIC@PowSwitch #1#2: power y
                  #3#4: basis x
```

Decision table for \BIC@PowSwitch.

y = 0			1
y=1			x
y=2	x < 0		Mul(-x, -x)
	else		Mul(x, x)
y < 0	x = 0		DivisionByZero
	x = 1		1
	x = -1	ifodd(y)	-1
		else	1
	else $(x > 1)$		0
y > 2	x = 0		0
	x = 1		1
	x = -1	ifodd(y)	-1
		else	1
	$x < -1 \ (x < 0)$	ifodd(y)	$-\operatorname{Pow}(-x,y)$
		else	Pow(-x,y)
	else $(x > 1)$		Pow(x, y)

```
1197 \def\BIC@PowSwitch#1#2!#3#4!{%
1198 \ifcase\ifx\\#2\\%
1199 \ifx#100 % y = 0
1200 \else\ifx#111 % y = 1
```

```
\left| \frac{y}{2} \right|
1201
               \else4 % y > 2
1202
               \fi\fi\fi
1203
             \else
1204
1205
               \int 1-3 \% y < 0
1206
               \else4 % y > 2
1207
               \fi
             \fi
1208
        \BIC@AfterFi{ 1}% y = 0
1209
      \ \ y = 1
1210
        \BIC@AfterFi{ #3#4}%
1211
      \ \ y = 2
1212
        \int x#3-% x < 0
1213
1214
          \BIC@AfterFiFi{%
1215
            \BIC@ProcessMul0!#4!#4!%
          }%
1216
        \else % x >= 0
1217
          \BIC@AfterFiFi{%
1218
            \BIC@ProcessMul0!#3#4!#3#4!%
1219
          }%
1220
        \fi
1221
      \or % y < 0
1222
        \ifcase ifx \#4\%
1223
                  \int x#300 % x = 0
1224
1225
                  \left( x = 1 \right)
1226
                  \else3 % x > 1
                  \fi\fi
1227
1228
               \else
                  \ifcase\BIC@MinusOne#3#4! %
1229
                   3 \% |x| > 1
1230
                  \or
1231
1232
                    2 \% x = -1
1233 ?
                  \else\BigIntCalcError:ThisCannotHappen%
                  \fi
1234
1235
               \fi
          \BIC@AfterFiFi{ 0\BigIntCalcError:DivisionByZero}% x = 0
1236
1237
        BIC@AfterFiFi{ 1}% x = 1
1238
1239
        \or % x = -1
          \ifcase\BIC@ModTwo#2! % even(y)
1240
1241
            \BIC@AfterFiFiFi{ 1}%
          \or % odd(y)
1242
            \BIC@AfterFiFiFi{ -1}%
1243
1244 ?
          \else\BigIntCalcError:ThisCannotHappen%
1245
1246
        1247
          \BIC@AfterFiFi{ 0}%
1248 ?
        \else\BigIntCalcError:ThisCannotHappen%
1249
        \fi
1250
      \left( \frac{1}{x}\right) 
1251
                  \int x#300 % x = 0
1252
                  \left( x = 1 \right)
1253
1254
                  \left( x > 1 \right)
                  \fi\fi
1255
1256
               \else
1257
                  \ifx#3-%
1258
                    \ifcase\BIC@MinusOne#3#4! %
1259
                      3 \% x < -1
                    \else
1260
                      2 \% x = -1
1261
1262
                    \fi
```

```
1263
                 \else
                   4 % x > 1
1264
                 \fi
1265
               \fi
1266
          \BIC@AfterFiFi{ 0}\% x = 0
1267
1268
        BIC@AfterFiFi{ 1}% x = 1
1269
1270
        \ifcase\BIC@ModTwo#1#2! % even(y)
1271
            \BIC@AfterFiFiFi{ 1}%
1272
          \or % odd(y)
1273
            \BIC@AfterFiFiFi{ -1}%
1274
          \else\BigIntCalcError:ThisCannotHappen%
1275 ?
1276
          \fi
1277
        1278
          \ifcase\BIC@ModTwo#1#2! % even(y)
1279
            \BIC@AfterFiFiFi{%
              \BIC@PowRec#4!#1#2!1!%
1280
            }%
1281
1282
          \or % odd(y)
            \expandafter-\romannumeral0%
1283
            \BIC@AfterFiFiFi{%
1284
              \BIC@PowRec#4!#1#2!1!%
1285
1286
1287 ?
          \else\BigIntCalcError:ThisCannotHappen%
1288
          \fi
1289
        \or % x > 1
1290
          \BIC@AfterFiFi{%
1291
            \BIC@PowRec#3#4!#1#2!1!%
         }%
1292
        \else\BigIntCalcError:ThisCannotHappen%
1293 ?
1294
1295 ? \else\BigIntCalcError:ThisCannotHappen%
1296
      \BIC@Fi
1297 }
```

2.18.1 Help macros

\BIC@ModTwo \BIC@ModTwo expects a number without sign and returns digit 1 or 0 if the number is odd or even.

```
1298 \def\BIC@ModTwo#1#2!{%
1299
      \ifx\\#2\\%
1300
        \ifodd#1 %
           \BIC@AfterFiFi1%
1301
         \else
1302
           \BIC@AfterFiFi0%
1303
        \fi
1304
1305
      \else
         \BIC@AfterFi{%
1306
1307
           \BIC@ModTwo#2!%
        }%
1308
1309
      \BIC@Fi
1310 }
```

\BIC@MinusOne

Macro \BIC@MinusOne expects a number and returns digit 1 if the number equals minus one and returns 0 otherwise.

```
1311 \def\BIC@MinusOne#1#2!{%

1312 \ifx#1-%

1313 \BIC@@MinusOne#2!%

1314 \else

1315 0%

1316 \fi
```

```
1317 }
```

```
\BIC@@MinusOne
```

```
1318 \def\BIC@@MinusOne#1#2!{%
1319
      \ifx#11%
1320
        \ifx\\#2\\%
1321
          1%
1322
        \else
          0%
1323
        \fi
1324
1325
      \else
1326
        0%
1327
      \fi
1328 }
```

2.18.2 Recursive calculation

```
\BIC@PowRec
```

```
Pow(x, y) {
       PowRec(x, y, 1)
     PowRec(x, y, r) {
       if y == 1 then
         return r
       else
         ifodd y then
           return PowRec(x*x, y div 2, r*x) % y div 2 = (y-1)/2
         else
           return PowRec(x*x, y div 2, r)
         fi
       fi
     }
   #1: x (basis)
#2#3: y (power)
#4: r (result)
1329 \def\BIC@PowRec#1!#2#3!#4!{%
1330
     1331
        \infnum\BIC@PosCmp#1!#4!=1 % x > r
1332
         \BIC@AfterFiFi{%
1333
            \BIC@ProcessMul0!#1!#4!%
         }%
1334
        \else
1335
          \BIC@AfterFiFi{%
1336
           \BIC@ProcessMul0!#4!#1!%
1337
         }%
1338
       \fi
1339
1340
     \or
        \ifcase\BIC@ModTwo#2#3! % even(y)
1341
1342
         \BIC@AfterFiFi{%
1343
            \expandafter\BIC@@PowRec\romannumeral0%
1344
            \BIC@@Shr#2#3!%
1345
            !#1!#4!%
         }%
1346
        \or % odd(y)
1347
         \mbox{ifnum}BIC@PosCmp#1!#4!=1 % x > r
1348
            \BIC@AfterFiFiFi{%
1349
              \expandafter\BIC@@@PowRec\romannumeral0%
1350
              \BIC@ProcessMul0!#1!#4!%
1351
1352
              !#1!#2#3!%
           }%
1353
1354
         \else
            \BIC@AfterFiFiFi{%
1355
              \expandafter\BIC@@@PowRec\romannumeral0%
1356
```

```
\BIC@ProcessMul0!#1!#4!%
                1357
                              !#1!#2#3!%
                1358
                            }%
                1359
                1360
                          \fi
                1361 ?
                        \else\BigIntCalcError:ThisCannotHappen%
                1362
                1363 ? \else\BigIntCalcError:ThisCannotHappen%
                1364
                      \BIC@Fi
                1365 }
 \BIC@@PowRec #1: y/2
                #2: x
                #3: new r (r \text{ or } r * x)
                1366 \def\BIC@@PowRec#1!#2!#3!{%
                1367 \expandafter\BIC@PowRec\romannumeral0%
                1368
                     \BIC@ProcessMul0!#2!#2!%
                1369
                      !#1!#3!%
                1370 }
\BIC@@@PowRec #1: r*x #2: x #3: y
                1371 \def\BIC@@@PowRec#1!#2!#3!{%
                     \expandafter\BIC@@PowRec\romannumeral0%
                     \BIC@@Shr#3!%
                1373
                     !#2!#1!%
                1374
                1375 }
                2.19
                        Div
\bigintcalcDiv #1: x
                #2: y (divisor)
                1376 \def\bigintcalcDiv#1{%
                1377 \romannumeral0%
                      \expandafter\expandafter\BIC@Div
                1378
                1379
                      \bigintcalcNum{#1}!%
                1380 }
      \BIC@Div #1: x
                #2: y
                1381 \def\BIC@Div#1!#2{%
                1382 \expandafter\expandafter\expandafter\BIC@DivSwitchSign
                1383 \bigintcalcNum{#2}!#1!%
                1384 }
\BigIntCalcDiv
                1385 \def\BigIntCalcDiv#1!#2!{%
                1386
                     \romannumeral0%
                     \BIC@DivSwitchSign#2!#1!%
                1388 }
```

 $\verb|\BIC@DivSwitchSign| Decision table for \verb|\BIC@DivSwitchSign|.$

y = 0		DivisionByZero
y > 0	x = 0	0
	x > 0	DivSwitch(+, x, y)
	x < 0	DivSwitch(-, -x, y)
y < 0	x = 0	0
	x > 0	DivSwitch(-, x, -y)
	x < 0	DivSwitch(+, -x, -y)

```
#1: y (divisor)
#2: x
1389 \def\BIC@DivSwitchSign#1#2!#3#4!{%
      \ifcase\BIC@Sgn#1#2! % y = 0
1391
        \BIC@AfterFi{ 0\BigIntCalcError:DivisionByZero}%
1392
      \or % y > 0
        \left| \frac{8100 \text{ Sgn} #3#4! \% x = 0}{100 \text{ Sgn} #3#4! \% x} \right|
1393
           \BIC@AfterFiFi{ 0}%
1394
1395
        1396
           \BIC@AfterFiFi{%
             \BIC@DivSwitch{}#3#4!#1#2!%
1397
           }%
1398
1399
        \else % x < 0
          \BIC@AfterFiFi{%
1400
1401
             \BIC@DivSwitch-#4!#1#2!%
1402
           }%
1403
        \fi
1404
      \else % y < 0
        \left| \frac{8100 \text{Sgn} #3#4! \% x = 0}{100 \text{Sgn} #3#4! \% x} \right|
1405
1406
           \BIC@AfterFiFi{ 0}%
1407
         \BIC@AfterFiFi{%
1408
             \BIC@DivSwitch-#3#4!#2!%
1409
           }%
1410
         \else % x < 0
1411
           \BIC@AfterFiFi{%
1412
             \BIC@DivSwitch{}#4!#2!%
1413
1414
1415
         \fi
1416
      \BIC@Fi
1417 }
```

\BIC@DivSwitch Decision table for \BIC@DivSwitch.

y = x	sign 1	
y > x	0	
y < x	y=1	sign x
	y=2	sign Shr(x)
	y=4	sign Shr(Shr(x))
	else	sign ProcessDiv (x, y)

```
#1: sign
#2: x
#3#4: y (y \neq 0)
1418 \def\BIC@DivSwitch#1#2!#3#4!{%
1419 \ifcase\BIC@PosCmp#3#4!#2!\% y = x
1420
        \BIC@AfterFi{ #11}%
1421
      \or % y > x
        \BIC@AfterFi{ 0}%
1422
1423
     \else % y < x
        \ifx\\#1\\%
1424
1425
        \else
          \expandafter-\romannumeral0%
1426
        \fi
1427
1428
        \left( \frac{1}{x}\right) 
1429
                  \int x#310 \% y = 1
1430
                  \left( x \right) = 10^{10}
1431
                  \left( x \right) = 1
                  \else3 % y > 2
1432
                  \fi\fi\fi
1433
                \else
1434
                  3 \% y > 2
1435
```

```
1436
                                   \fi
                   1437
                             BIC@AfterFiFi{ #2}% y = 1
                           1438
                   1439
                             \BIC@AfterFiFi{%
                   1440
                               \BIC@@Shr#2!%
                             }%
                   1441
                   1442
                           1443
                             \BIC@AfterFiFi{%
                               \expandafter\BIC@@Shr\romannumeral0%
                   1444
                                  \BIC@@Shr#2!!%
                   1445
                             }%
                   1446
                           \or % y > 2
                   1447
                             \BIC@AfterFiFi{%
                   1448
                   1449
                               \BIC@DivStartX#2!#3#4!!!%
                   1450
                   1451 ?
                           \else\BigIntCalcError:ThisCannotHappen%
                   1452
                           \fi
                         \BIC@Fi
                   1453
                   1454 }
 \BIC@ProcessDiv #1#2: x
                   #3#4: y
                   #5: collect first digits of x
                   #6: corresponding digits of y
                   1455 \def\BIC@DivStartX#1#2!#3#4!#5!#6!{%
                         \ifx\\#4\\%
                   1456
                           \BIC@AfterFi{%
                   1457
                   1458
                             \BIC@DivStartYii#6#3#4!{#5#1}#2=!%
                   1459
                           }%
                   1460
                        \else
                   1461
                           \BIC@AfterFi{%
                             \BIC@DivStartX#2!#4!#5#1!#6#3!%
                   1462
                           }%
                   1463
                   1464 \BIC@Fi
                   1465 }
\BIC@DivStartYii #1: y
                   #2: x, =
                   1466 \def\BIC@DivStartYii#1!{%
                   1467 \expandafter\BIC@DivStartYiv\romannumeral0%
                         \BIC@Shl#1!%
                   1468
                   1469
                         !#1!%
                   1470 }
\BIC@DivStartYiv #1: 2y
                   #2: y
                   #3: x, =
                   1471 \def\BIC@DivStartYiv#1!{%
                   1472 \expandafter\BIC@DivStartYvi\romannumeral0%
                   1473
                         \BIC@Shl#1!%
                   1474
                         !#1!%
                   1475 }
\BIC@DivStartYvi #1: 4y
                   #2: 2y
                   #3: y
                   #4: x, =
                   1476 \ensuremath{ \mbox{\sc MIC@DivStartYvi#1!#2!} \label{lem:mass} } \\
                   1477 \expandafter\BIC@DivStartYviii\romannumeral0%
                         \BIC@AddXY#1!#2!!!%
                   1478
                        !#1!#2!%
                   1479
                   1480 }
```

```
\BIC@DivStartYviii #1: 6y
                    #2: 4y
                    #3: 2y
                    #4: y
                    #5: x, =
                    1481 \def\BIC@DivStartYviii#1!#2!{%
                          \expandafter\BIC@DivStart\romannumeral0%
                          \BIC@Sh1#2!%
                    1484
                          !#1!#2!%
                    1485 }
     \BIC@DivStart #1: 8y
                    #2: 6y
                    #3: 4y
                    #4: 2y
                    #5: y
                    #6: x, =
                    1486 \def\BIC@DivStart#1!#2!#3!#4!#5!#6!{%
                    1487 \BIC@ProcessDiv#6!!#5!#4!#3!#2!#1!=%
                    1488 }
  \BIC@ProcessDiv #1#2#3: x, =
                    #4: result
                    #5: y
                    #6: 2y
                    #7: 4y
                    #8: 6y
                    #9: 8y
                    1489 \def\BIC@ProcessDiv#1#2#3!#4!#5!{%
                    1490 \ifcase\BIC@PosCmp#5!#1!% y = #1
                            \ifx#2=%
                    1491
                              \BIC@AfterFiFi{\BIC@DivCleanup{#41}}%
                    1492
                            \else
                    1493
                    1494
                              \BIC@AfterFiFi{%
                    1495
                                \BIC@ProcessDiv#2#3!#41!#5!%
                    1496
                              }%
                    1497
                            \fi
                    1498
                          \ifx#2=%
                    1499
                              \BIC@AfterFiFi{\BIC@DivCleanup{#40}}%
                    1500
                             \else
                    1501
                               \ifx\\#4\\%
                    1502
                                \BIC@AfterFiFiFi{%
                    1503
                                   \BIC@ProcessDiv{#1#2}#3!!#5!%
                    1504
                                }%
                    1505
                               \else
                    1506
                    1507
                                 \BIC@AfterFiFiFi{%
                    1508
                                   \BIC@ProcessDiv{#1#2}#3!#40!#5!%
                    1509
                                }%
                              \fi
                    1510
                    1511
                             \fi
                          \else % y < #1
                    1512
                            \BIC@AfterFi{%
                    1513
                               \BIC@@ProcessDiv{#1}#2#3!#4!#5!%
                    1514
                            }%
                    1515
                          \BIC@Fi
                    1516
                    1517 }
  \BIC@DivCleanup
                   #1: result
                    #2: garbage
                    1518 \def\BIC@DivCleanup#1#2={ #1}%
```

\BIC@@ProcessDiv

```
1519 \def\BIC@@ProcessDiv#1#2#3!#4!#5!#6!#7!{%
                   1520 \ifcase\BIC@PosCmp#7!#1!% 4y = #1
                   1521
                            \ifx#2=%
                              \BIC@AfterFiFi{\BIC@DivCleanup{#44}}%
                   1522
                   1523
                             \BIC@AfterFiFi{%
                   1524
                               \BIC@ProcessDiv#2#3!#44!#5!#6!#7!%
                   1525
                   1526
                            }%
                   1527
                            \fi
                         \or % 4y > #1
                   1528
                            \ifcase\BIC@PosCmp#6!#1!% 2y = #1
                   1529
                              \ifx#2=%
                   1530
                                \BIC@AfterFiFiFi{\BIC@DivCleanup{#42}}%
                   1531
                   1532
                              \else
                   1533
                                \BIC@AfterFiFiFi{%
                   1534
                                  \BIC@ProcessDiv#2#3!#42!#5!#6!#7!%
                   1535
                              \fi
                   1536
                            \or % 2y > #1
                   1537
                   1538
                              \ifx#2=%
                                \BIC@AfterFiFiFi{\BIC@DivCleanup{#41}}%
                   1539
                              \else
                   1540
                                \BIC@AfterFiFiFi{%
                   1541
                   1542
                                  \BIC@DivSub#1!#5!#2#3!#41!#5!#6!#7!%
                   1543
                                }%
                              \fi
                   1544
                            \else % 2y < #1
                   1545
                   1546
                              \BIC@AfterFiFi{%
                   1547
                                \expandafter\BIC@ProcessDivII\romannumeral0%
                   1548
                                \BIC@SubXY#1!#6!!!%
                   1549
                                !#2#3!#4!#5!23%
                                #6!#7!%
                   1550
                              }%
                   1551
                            \fi
                   1552
                          \else % 4y < #1
                   1553
                   1554
                            \BIC@AfterFi{%
                              \BIC@@@ProcessDiv{#1}#2#3!#4!#5!#6!#7!%
                   1555
                   1556
                   1557
                          \BIC@Fi
                   1558 }
                  Next token group: #1-#2 and next digit #3.
      \BIC@DivSub
                    1559 \def\BIC@DivSub#1!#2!#3{%
                         \expandafter\BIC@ProcessDiv\expandafter{%
                   1560
                   1561
                            \romannumeral0%
                   1562
                            \BIC@SubXY#1!#2!!!%
                   1563
                            #3%
                   1564 }%
                   1565 }
\BIC@ProcessDivII #1: x'-2y
                   #2#3: remaining x, =
                   #4: result
                   #5: y
                   #6: first possible result digit
                   #7: second possible result digit
                   1566 \def\BIC@ProcessDivII#1!#2#3!#4!#5!#6#7{%
                         \infty SIC@PosCmp#5!#1!% y = #1
                   1567
                            \ifx#2=%
                   1568
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#7}}%
                   1569
                   1570
                            \else
```

```
1571
                              \BIC@AfterFiFi{%
                                \BIC@ProcessDiv#2#3!#4#7!#5!%
                    1572
                              }%
                    1573
                    1574
                            \fi
                    1575
                          1576
                            \ifx#2=%
                    1577
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#6}}%
                    1578
                            \else
                              \BIC@AfterFiFi{%
                    1579
                                \BIC@ProcessDiv{#1#2}#3!#4#6!#5!%
                    1580
                              }%
                    1581
                            \fi
                    1582
                          \else % y < #1
                    1583
                            \int x#2=%
                    1584
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#7}}%
                    1585
                    1586
                            \else
                    1587
                              \BIC@AfterFiFi{%
                                \BIC@DivSub#1!#5!#2#3!#4#7!#5!%
                    1588
                              }%
                    1589
                    1590
                            \fi
                          \BIC@Fi
                    1591
                    1592 }
\BIC@ProcessDivIV #1#2#3: x, =, x > 4y
                    #4: result
                    #5: y
                    #6: 2y
                    #7: 4y
                    #8: 6y
                    #9: 8y
                    1593 \def\BIC@@@ProcessDiv#1#2#3!#4!#5!#6!#7!#8!#9!{%
                    1594
                          \ifcase\BIC@PosCmp#8!#1!% 6y = #1
                    1595
                            \ifx#2=%
                    1596
                              \BIC@AfterFiFi{\BIC@DivCleanup{#46}}%
                    1597
                            \else
                    1598
                              \BIC@AfterFiFi{%
                                \BIC@ProcessDiv#2#3!#46!#5!#6!#7!#8!#9!%
                    1599
                              }%
                    1600
                            \fi
                    1601
                          \or % 6y > #1
                    1602
                            \BIC@AfterFi{%
                    1603
                              \expandafter\BIC@ProcessDivII\romannumeral0%
                    1604
                              \BIC@SubXY#1!#7!!!%
                    1605
                              !#2#3!#4!#5!45%
                    1606
                    1607
                              #6!#7!#8!#9!%
                    1608
                            }%
                    1609
                          \else % 6y < #1
                    1610
                            \ifcase\BIC@PosCmp#9!#1!% 8y = #1
                    1611
                              \ifx#2=%
                                \BIC@AfterFiFiFi{\BIC@DivCleanup{#48}}%
                    1612
                    1613
                              \else
                                \BIC@AfterFiFiFi{%
                    1614
                                   \BIC@ProcessDiv#2#3!#48!#5!#6!#7!#8!#9!%
                    1615
                                }%
                    1616
                    1617
                              \fi
                            \or % 8y > #1
                    1618
                    1619
                              \BIC@AfterFiFi{%
                                \expandafter\BIC@ProcessDivII\romannumeral0%
                    1620
                                \BIC@SubXY#1!#8!!!%
                    1621
                                !#2#3!#4!#5!67%
                    1622
                                #6!#7!#8!#9!%
                    1623
                              }%
                    1624
```

```
\else % 8y < #1
                1625
                         \BIC@AfterFiFi{%
                1626
                            \expandafter\BIC@ProcessDivII\romannumeral0%
                1627
                1628
                            \BIC@SubXY#1!#9!!!%
                1629
                            !#2#3!#4!#5!89%
                1630
                            #6!#7!#8!#9!%
                1631
                         }%
                1632
                        \fi
                     \BIC@Fi
                1633
                1634 }
                2.20
                        Mod
\bigintcalcMod #1: x
                #2: y
                1635 \def\bigintcalcMod#1{%
                1636 \romannumeral0%
                     \expandafter\expandafter\expandafter\BIC@Mod
                1638
                      \bigintcalcNum{#1}!%
                1639 }
      \BIC@Mod #1: x
                #2: y
                1641 \expandafter\expandafter\expandafter\BIC@ModSwitchSign
                1642 \quad \verb|\bigintcalcNum{#2}!#1!%|
                1643 }
\BigIntCalcMod
                1644 \def\BigIntCalcMod#1!#2!{%
                     \romannumeral0%
                1646
                     \BIC@ModSwitchSign#2!#1!%
                1647 }
```

\BIC@ModSwitchSign Decision table for \BIC@ModSwitchSign.

y = 0		DivisionByZero
y > 0	x = 0	0
	else	ModSwitch(+, x, y)
y < 0		ModSwitch(-, -x, -y)

```
#1#2: y
#3#4: x
1648 \def\BIC@ModSwitchSign#1#2!#3#4!{%
1649 \ifcase\ifx\\#2\\%
             \int x#100 \% y = 0
1650
              \else1 % y > 0
1651
1652
             \fi
1653
             \else
              \int 1^2 x^4 - 2  % y < 0
1654
              \else1 % y > 0
1655
1656
              \fi
1657
             \fi
1658
       \BIC@AfterFi{ 0\BigIntCalcError:DivisionByZero}%
1659
       1660
         \BIC@AfterFiFi{ 0}%
1661
       \else
1662
         \BIC@AfterFiFi{%
1663
1664
          \BIC@ModSwitch{}#3#4!#1#2!%
1665
         }%
```

```
1666
        \fi
      \else % y < 0
1667
        \left( \frac{1}{2} \right)^{4}
1668
                  \int x#300 % x = 0
1669
1670
                   \else1 % x > 0
1671
                   \fi
1672
                \else
                   \int x#3-2 % x < 0
1673
                   \else1 % x > 0
1674
                  \fi
1675
                \fi
1676
          \BIC@AfterFiFi{ 0}%
1677
         1678
1679
           \BIC@AfterFiFi{%
             \BIC@ModSwitch--#3#4!#2!%
1680
          }%
1681
         \else % x < 0
1682
           \BIC@AfterFiFi{%
1683
             \BIC@ModSwitch-#4!#2!%
1684
1685
          }%
1686
         \fi
      \BIC@Fi
1687
1688 }
```

\BIC@ModSwitch Decision table for \BIC@ModSwitch.

y = 1		0
y=2	ifodd(x)	sign 1
	else	0
y > 2	x < 0	$z \leftarrow x - (x/y) * y; (z < 0) ? z + y : z$
	x > 0	x - (x/y) * y

```
#1: sign
#2#3: x
#4#5: y
1689 \def\BIC@ModSwitch#1#2#3!#4#5!{%
      \ifcase\ifx\\#5\\%
1691
                \int x#410 \% y = 1
1692
                \left( x + 421 \% y = 2 \right)
1693
                \ensuremath{\mbox{lelse2}} % y > 2
                fi\fi
1694
              \else2 % y > 2
1695
1696
             \fi
        \BIC@AfterFi{ 0}\% y = 1
1697
      \ \ y = 2
1698
        \ifcase\BIC@ModTwo#2#3! % even(x)
1699
1700
           \BIC@AfterFiFi{ 0}%
1701
        \BIC@AfterFiFi{ #11}%
1702
        \else\BigIntCalcError:ThisCannotHappen%
1703?
        \fi
1704
1705
      \or % y > 2
        \ifx\\#1\\%
1706
1707
1708
           \expandafter\BIC@Space\romannumeral0%
           \expandafter\BIC@ModMinus\romannumeral0%
1709
1710
1711
        \int x^2-x^2 x < 0
1712
          \BIC@AfterFiFi{%
             \expandafter\expandafter\expandafter\BIC@ModX
1713
             \bigintcalcSub{#2#3}{%
1714
               \bigintcalcMul{#4#5}{\bigintcalcDiv{#2#3}{#4#5}}%
1715
```

```
}!#4#5!%
               1716
                         }%
               1717
                       \else % x > 0
               1718
                         \BIC@AfterFiFi{%
               1719
               1720
                           \expandafter\expandafter\expandafter\BIC@Space
               1721
                           \bigintcalcSub{#2#3}{%
               1722
                             \label{limits} $$  \left( \frac{4445}{\sigma} \right) = CDiv{#2#3}{#4#5}} $$
                           }%
               1723
                         }%
               1724
                       \fi
               1725
               \BIC@Fi
               1728 }
\BIC@ModMinus
               1729 \def\BIC@ModMinus#1{\%
               1730 \ifx#10%
                      \BIC@AfterFi{ 0}%
               1731
               1732 \else
                       \BIC@AfterFi{ -#1}%
               1733
               1734 \BIC@Fi
               1735 }
   \BIC@ModX #1#2: z
               #3: x
               1736 \def\BIC@ModX#1#2!#3!{%
               1737 \ifx#1-% z < 0
                       \BIC@AfterFi{%
               1738
                         \expandafter\BIC@Space\romannumeral0%
               1739
                         \BIC@SubXY#3!#2!!!%
               1740
                       }%
               1741
                     \leq % z >= 0
               1742
                       \BIC@AfterFi{ #1#2}%
               1743
                    \BIC@Fi
               1744
               1745 }
               1746 \BIC@AtEnd
               1747 (/package)
```

3 Test

3.1 Catcode checks for loading

```
1748 (*test1)
1749 \catcode'\{=1 %
1750 \catcode'\}=2 %
1751 \catcode'\#=6 %
1752 \catcode \@=11 %
1753 \expandafter\ifx\csname count@\endcsname\relax
1754 \countdef\count@=255 %
1755 \fi
1756 \expandafter\ifx\csname @gobble\endcsname\relax
1757 \long\def\@gobble#1{}%
1759 \expandafter\ifx\csname @firstofone\endcsname\relax
     \long\def\@firstofone#1{#1}%
1761 \fi
1762 \expandafter\ifx\csname loop\endcsname\relax
1763 \expandafter\@firstofone
1764 \else
1765 \expandafter\@gobble
```

```
1766 \fi
1767 {%
1768
      \def\loop#1\repeat{%
1769
        \left( \frac{1}{x} \right)
1770
        \iterate
1771
      }%
1772
      \def\iterate{%
1773
        \body
          \let\next\iterate
1774
        \else
1775
          \let\next\relax
1776
        \fi
1777
1778
        \next
1779
      }%
      \let\repeat=\fi
1780
1781 }%
1782 \def\RestoreCatcodes{}
1783 \count@=0 %
1784 \loop
1785
      \edef\RestoreCatcodes{%
        \RestoreCatcodes
1786
        \catcode\the\count@=\the\catcode\count@\relax
1787
1788
1789 \ifnum\count@<255 %
      \advance\count@ 1 %
1790
1791 \repeat
1792
1793 \def\RangeCatcodeInvalid#1#2{%
1794
      \count@=#1\relax
1795
      \loop
1796
        \catcode\count@=15 %
      \ifnum\count@<#2\relax
1797
1798
        \advance\count@ 1 %
1799
      \repeat
1800 }
1801 \expandafter\ifx\csname LoadCommand\endcsname\relax
      \def\LoadCommand{\input bigintcalc.sty\relax}%
1803 \fi
1804 \left\lceil \text{Test} \right\rceil
      \verb|\RangeCatcodeInvalid{0}{47}||%
1805
      1806
      \RangeCatcodeInvalid{91}{96}%
1807
      \RangeCatcodeInvalid{123}{255}%
1808
1809
      \catcode'\@=12 %
1810
      \catcode'\\=0 %
      \ccite{1 %}
1811
      \ccite{1}=2 %
1812
      \catcode'\#=6 %
1813
      \cite{12 }%
1814
      \cite{12 }%
1815
      \c \catcode '\%=14 %
1816
      \catcode'\ =10 %
1817
      \catcode13=5 %
1818
1819
      \LoadCommand
      \RestoreCatcodes
1820
1821 }
1822 \Test
1823 \csname @@end\endcsname
1824 \end
1825 (/test1)
```

3.2 Macro tests

3.2.1 Preamble with test macro definitions

```
1827 \NeedsTeXFormat{LaTeX2e}
1828 \setminus nofiles
1829 \documentclass{article}
1830 \langle noetex \rangle \setminus Iet \setminus SavedNumexpr \setminus numexpr
1831 \langle noetex \rangle \setminus let \setminus numexpr \setminus UNDEFINED
1832 \text{ } \text{\colored}
1833 \chardef\BIC@TestMode=1 %
1834 \makeatother
1835 \usepackage{bigintcalc} [2007/11/11]
1836 \langle noetex \rangle \setminus let \setminus numexpr \setminus Saved Numexpr
1837 \usepackage{qstest}
1838 \IncludeTests{*}
1839 \setminus LogTests\{log\}\{*\}\{*\}
1840 \newcommand*{\TestSpaceAtEnd}[1]{%
1841 (noetex) \let\SavedNumexpr\numexpr
1843 \edef\resultA{\#1}%
1844 \edef\resultB{#1 }%
1845 (noetex) \let\numexpr\SavedNumexpr
1846 \Expect*{\resultA\space}*{\resultB}%
1848 \newcommand*{\TestResult}[2]{%
1850 (noetex) \let\numexpr\UNDEFINED
1851 \edef\result{#1}%
1853 \Expect*{\result}{#2}%
1854 }
1855 \newcommand*{\TestResultTwoExpansions}[2]{%
1856 (*noetex)
1857
                           \begingroup
                                    \label{let_numexpr} $$ \end{area} $$ \end{
1858
1859
                                    \verb|\expandafter| expandafter| expandafter|
1860
                           \endgroup
1861 (/noetex)
                          \expandafter\expandafter\Expect
1862
1863
                           \expandafter\expandafter\expandafter{#1}{#2}%
1864 }
1865 \newcount\TestCount
1866 \langle \text{etex} \rangle = 1866 \langle \text{et
1867 (noetex) \newcommand*{\TestArg}[1]{#1}
1868 \newcommand*{\TestTeXDivide}[2]{%
                         \TestCount=\TestArg{#1}\relax
1870
                         \divide\TestCount by \TestArg{#2}\relax
                          1871
1872 }
1873 \newcommand*{\Test}[2]{\%
1874 \TestResult{#1}{#2}%
1875
                         \TestResultTwoExpansions{#1}{#2}%
1876
                         \TestSpaceAtEnd{#1}%
1877 }
1878 \mbox{newcommand}*{\mbox{TestExch}[2]{\mbox{#2}{#1}}}
1879 \newcommand*{\TestInv}[2]{%
1880
                       \Test{\bigintcalcInv{#1}}{#2}%
1881 }
1882 \newcommand*{\TestAbs}[2]{%
1883 \Test{\bigintcalcAbs{#1}}{#2}%
1884 }
```

```
1885 \newcommand*{\TestSgn}[2]{%
      \Test{\bigintcalcSgn{#1}}{#2}%
1886
1887 }
1888 \newcommand*{\TestMin}[3]{%
1889
      Test{\bigintcalcMin{#1}{#2}}{#3}%
1890 }
1891 \newcommand*{\TestMax}[3]{%
1892
      \texttt{Test{\bigintcalcMax{#1}{#2}}{#3}%
1893 }
1894 \newcommand*{\TestCmp}[3]{%
      \verb|\Test{\bigintcalcCmp{#1}{#2}}{#3}||
1895
1896 }
1897 \newcommand*{\TestOdd}[2]{%
      \Test{\bigintcalcOdd{#1}}{#2}%
1898
      \left( x_{x}\right) 
1899
1900
         \noexpand\Test{%
1901
           \noexpand\BigIntCalcOdd
           \bigintcalcAbs{#1}!%
1902
        }{#2}%
1903
1904
      }%
1905
      \x
1906 }
1907 \newcommand*{\TestInc}[2]{%
      \Test{\bigintcalcInc{#1}}{#2}%
1908
       \ifnum\bigintcalcSgn{#1}>-1 %
1909
1910
         \left( x_{x}\right) 
1911
           \noexpand\Test{%}
             \noexpand\BigIntCalcInc\bigintcalcNum{#1}!%
1912
1913
           }{#2}%
        }%
1914
         \x
1915
1916
      \fi
1917 }
1918 \newcommand*{\TestDec}[2]{%
      \Test{\bigintcalcDec{#1}}{#2}%
1919
1920
      \ifnum\bigintcalcSgn{#1}>0 %
1921
         \left( x_{x}\right) 
1922
           \noexpand\Test{%}
1923
             \noexpand\BigIntCalcDec\bigintcalcNum{#1}!%
1924
           }{#2}%
        }%
1925
1926
         ١x
1927
      \fi
1928 }
1929 \newcommand*{\TestAdd}[3]{%
1930
      \Test{\bigintcalcAdd{#1}{#2}}{#3}%
1931
       \ifnum\bigintcalcSgn{#1}>0 %
1932
         \ifnum\bigintcalcSgn{#2}> 0 %
1933
           \ifnum\bigintcalcCmp{#1}{#2}>0 %
1934
             \left( x_{x}\right) 
               \verb|\noexpand\Test{%|}
1935
1936
                  \noexpand\BigIntCalcAdd
                  \bigintcalcNum{#1}!\bigintcalcNum{#2}!%
1937
1938
               }{#3}%
             }%
1939
1940
             \x
1941
           \else
1942
             \left( x_{x}\right) 
1943
               \noexpand\Test{%}
                  \noexpand\BigIntCalcAdd
1944
                  \bigintcalcNum{#2}!\bigintcalcNum{#1}!%
1945
               }{#3}%
1946
```

```
1947
            }%
1948
             \x
          \fi
1949
        \fi
1950
1951
1952 }
1953 \newcommand*{\TestSub}[3]{%
1954
      \Test{\bigintcalcSub{#1}{#2}}{#3}%
      \ifnum\bigintcalcSgn{#1}>0 %
1955
        1956
          \ifnum\bigintcalcCmp{#1}{#2}>0 %
1957
             \left( x_{x}\right) 
1958
               \noexpand\Test{%
1959
                 \noexpand\BigIntCalcSub
1960
                 \bigintcalcNum{#1}!\bigintcalcNum{#2}!%
1961
1962
1963
            }%
1964
             \x
          \fi
1965
        \fi
1966
1967
      \fi
1968 }
1969 \newcommand*{\TestSh1}[2]{%
      \Test{\bigintcalcShl{#1}}{#2}%
1970
      \left( x_{x}\right) 
1971
1972
        \noexpand\Test{%
          \noexpand\BigIntCalcShl\bigintcalcAbs{#1}!%
1973
1974
        }{\bigintcalcAbs{#2}}%
1975
      }%
1976
      \backslash x
1977 }
1978 \newcommand*{\TestShr}[2]{%
1979
      \Test{\bigintcalcShr{#1}}{#2}%
1980
      \left( x_{x}\right) 
        \noexpand\Test{%
1981
1982
          \noexpand\BigIntCalcShr\bigintcalcAbs{#1}!%
1983
        }{\bigintcalcAbs{#2}}%
1984
      }%
1985
      /x
1986 }
1987 \newcommand*{\TestMul}[3]{%
      1988
      \left( x_{x}\right) 
1989
1990
        \noexpand\Test{%
1991
          \noexpand\BigIntCalcMul
1992
          \bigintcalcAbs{#1}!\bigintcalcAbs{#2}!%
1993
        }{\bigintcalcAbs{#3}}%
1994
      }%
1995
1996 }
1997 \newcommand*{\TestSqr}[2]{%
1998
      \Test{\bigintcalcSqr{#1}}{#2}%
1999 }
2000 \newcommand*{\TestFac}[2]{%
      \expandafter\TestExch\expandafter{%
2001
2002
        \the\numexpr#2%
2003
      }{\bigintcalcFac{#1}}%
2004 }
2005 \newcommand*{\TestFacBig}[2]{%
     \Test{\bigintcalcFac{#1}}{#2}%
2006
2007 }
2008 \newcommand*{\TestPow}[3]{%
```

```
2009
                      Test{\big(\propty \propty \prop
2010 }
2011 \newcommand*{\TestDiv}[3]{%
                       \Test{\bigintcalcDiv{#1}{#2}}{#3}%
                       \TestTeXDivide{#1}{#2}%
2014 }
2015 \newcommand*{\TestDivBig}[3]{%
2016
                      \Test{\bigintcalcDiv{#1}{#2}}{#3}%
2017
                       \edef\x{%
                               \noexpand\Test{%
2018
                                       \noexpand\BigIntCalcDiv\bigintcalcAbs{#1}!\bigintcalcAbs{#2}!%
2019
2020
                              }{\bigintcalcAbs{#3}}%
2021
                      }%
2022 }
2023 \newcommand*{\TestMod}[3]{%
                       \Test{\bigintcalcMod{#1}{#2}}{#3}%
2025
                       \ifcase\ifcase\bigintcalcSgn{#1} 0%
2026
                                                 \or
                                                          \ifcase\bigintcalcSgn{#2} 1%
2027
                                                         \or 0%
2028
                                                         \else 1%
2029
                                                         \fi
2030
2031
                                                  \else
                                                          \ifcase\bigintcalcSgn{#2} 1%
2032
2033
2034
                                                          \else 0%
2035
                                                         \fi
                                                 \fi\relax
2036
2037
                               \left( x_{x}\right) 
                                      \noexpand\Test{%
2038
                                              \noexpand\BigIntCalcMod
2039
2040
                                              \bigintcalcAbs{#1}!\bigintcalcAbs{#2}!%
2041
                                      }{\bigintcalcAbs{#3}}%
                              }%
2042
2043
                              \x
2044
                      \fi
2045 }
3.2.2
                            Time
2046 \verb|\begingroup\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandaft
2047 \verb|\expandafter\ifx\csname| pdfresettimer\endcsname\relax|
2048 \else
2049
                       \makeatletter
2050
                       \newcount\SummaryTime
2051
                       \newcount\TestTime
2052
                       \SummaryTime=\z@
2053
                       \newcommand*{\PrintTime}[2]{%
2054
                               \typeout{%
2055
                                       [Time #1: \strip@pt\dimexpr\number#2sp\relax\space s]%
2056
                              }%
                      }%
2057
                       \newcommand*{\StartTime}[1]{%
2058
2059
                               \renewcommand*{\TimeDescription}{#1}%
2060
                               \pdfresettimer
                      }%
2061
2062
                       \newcommand*{\TimeDescription}{}%
2063
                       \newcommand*{\StopTime}{%
2064
                               \TestTime=\pdfelapsedtime
2065
                               \global\advance\SummaryTime\TestTime
2066
                               \PrintTime\TimeDescription\TestTime
                      }%
2067
                       \let\saved@qstest\qstest
2068
                       \let\saved@endqstest\endqstest
2069
```

```
\def\qstest#1#2{%
2070
        \saved@qstest{#1}{#2}%
2071
        \StartTime{#1}%
2072
2073
2074
      \def\endqstest{%
2075
        \StopTime
2076
        \saved@endqstest
2077
      ጉ%
      \AtEndDocument{%
2078
        \PrintTime{summary}\SummaryTime
2079
     }%
2080
2081
      \makeatother
2082 \fi
3.2.3
      Test sets
2083 \makeatletter
2084
2085 \begin{qstest}{inv}{inv}%
2086
     TestInv{0}{0}%
      \TestInv{1}{-1}%
2087
      TestInv{-1}{1}%
2088
2089
      \TestInv{10}{-10}%
2090
      \TestInv{-10}{10}%
      \TestInv{2147483647}{-2147483647}%
      \TestInv{-2147483647}{2147483647}%
2093
      \TestInv{12345678901234567890}{-12345678901234567890}%
2094
      \TestInv{-12345678901234567890}{12345678901234567890}%
      \TestInv{ 0 }{0}%
2095
2096
      \TestInv{ 1 }{-1}%
      \TestInv{--1}{-1}%
2097
      \TestInv{\number\z0}{0}%
2098
2099
      \TestInv{\ifx\relax\relax1\fi}{-1}%
2100
      \TestInv{\ifx\relax-fi\ifx234\else1\fi}{1}%
2101 \end{qstest}
2102
2103 \begin{qstest}{abs}{abs}%
2104 \TestAbs{0}{0}%
2105
     \texttt{TestAbs}\{1\}\{1\}\%
2106
     TestAbs{-1}{1}%
      \texttt{TestAbs}\{10\}\{10\}\%
2107
2108
      \TestAbs{-10}{10}%
      \TestAbs{2147483647}{2147483647}%
2109
2110
      \TestAbs{-2147483647}{2147483647}%
2111
      \TestAbs{12345678901234567890}{12345678901234567890}%
2112
      \TestAbs{-12345678901234567890}{12345678901234567890}%
2113
      \TestAbs{ 0 }{0}%
2114
      \TestAbs{ 1 }{1}%
2115
      TestAbs{--1}{1}%
2116
      \TestAbs{-+-+1}{1}%
2117
      \TestAbs{00000000000}{0}%
      \TestAbs{00000001000}{1000}%
2118
      2119
2120 \end{qstest}
2121
2122 \begin{qstest}{sign}{sign}%
2123
     TestSgn{0}{0}%
2124
      TestSgn{1}{1}%
2125
      \TestSgn{-1}{-1}%
2126
      TestSgn{10}{1}%
      \texttt{\TestSgn}\{-10\}\{-1\}\%
2127
      \TestSgn{2147483647}{1}%
2128
2129
      \TestSgn{-2147483647}{-1}%
      \TestSgn{12345678901234567890}{1}%
2130
```

```
\TestSgn{-12345678901234567890}{-1}%
2131
      \TestSgn{ 0 }{0}%
2132
      \TestSgn{ 2 }{1}%
2133
      TestSgn{ -2 }{-1}%
2134
2135
      \TestSgn{--2}{1}%
2136
      \TestSgn{
\quad \number\z0}{0}%
2137
      \TestSgn{
\underline{0ne}{1}}
2138
      \TestSgn{\number\m@ne}{-1}%
2139
      \TestSgn{%
        -+-+\number\z@\number\z@
2140
        \iftrue1\fi\iftrue2\fi\iftrue3\fi
2141
     }{1}%
2142
2143 \end{qstest}
2144
2145 \begin{qstest}{min}{min}%
     \TestMin{0}{1}{0}%
2147
      \TestMin{1}{0}{0}%
     \TestMin{-10}{-20}{-20}%
2148
      \TestMin{ 1 }{ 2 }{1}%
2149
     \TestMin{ 2 }{ 1 }{1}%
2150
2151
      \TestMin{1}{1}{1}%
      \TestMin{\number\z0}{\number\@ne}{0}%
2152
2153
      \TestMin{\number\@ne}{\number\m@ne}{-1}%
2154 \end{qstest}
2155
2156 \begin{qstest}{max}{max}%
2157
     \text{TestMax}\{0\}\{1\}\{1\}\%
2158
      \text{TestMax}\{1\}\{0\}\{1\}\%
2159
      TestMax{-10}{-20}{-10}%
     \TestMax{ 1 }{ 2 }{2}%
2160
      \TestMax{ 2 }{ 1 }{2}%
2161
2162
     \TestMax{1}{1}{1}}%
2163
     \TestMax{\number\z0}{\number\@ne}{1}%
2164
     \TestMax{\number\@ne}{\number\m@ne}{1}%
2165 \end{qstest}
2166
2167 \begin{qstest}{cmp}{cmp}%
2168
     \TestCmp{0}{0}{0}%
2169
      \TestCmp{-21}{17}{-1}%
2170
     \TestCmp{3}{4}{-1}%
      \texttt{\TestCmp}\{-10\}\{-10\}\{0\}\%
2171
      \texttt{\TestCmp}\{-10\}\{-11\}\{1\}\%
2172
      \TestCmp{100}{5}{1}%
2173
2174
      \TestCmp{9}{10}{-1}%
2175
      TestCmp{10}{9}{1}%
2176
      TestCmp{ 3 }{ 3 }{ 0}
2177
      TestCmp{-9}{-10}{1}%
2178
      TestCmp{-10}{-9}{-1}%
2179
      TestCmp{-3}{-3}{0}%
2180
      TestCmp{0}{-2}{1}%
      TestCmp{0}{2}{-1}%
2181
      \texttt{\TestCmp}\{2\}\{0\}\{1\}\%
2182
      TestCmp{-2}{0}{-1}%
2183
      \TestCmp{12}{11}{1}%
2184
2185
      \TestCmp{11}{12}{-1}%
2186
      \TestCmp{2147483647}{-2147483647}{1}%
2187
      \TestCmp{-2147483647}{2147483647}{-1}%
2188
      \TestCmp{2147483647}{2147483647}{0}%
2189
      2190
      \TestCmp{\number\@ne}{\number\m@ne}{1}%
      TestCmp{ 4 }{ 5 }{-1}%
2191
2192
      TestCmp{ -3 }{ -7 }{1}%
```

```
2193 \end{qstest}
2194
2195 \begin{qstest}{odd}{odd}
2196 \tracingmacros=1
2197
     \TestOdd{0}{0}%
2198
     \TestOdd{1}{1}%
2199
     \TestOdd{2}{0}%
2200
     \TestOdd{3}{1}%
     \TestOdd{14}{0}%
2201
     \Test0dd{15}{1}%
2202
     \TestOdd{12345678901234567896}{0}%
2203
     \TestOdd{12345678901234567897}{1}%
2204
2205 \end{qstest}
2206
2207 \begin{qstest}{inc}{inc}%
     TestInc{0}{1}%
2208
2209
     TestInc{1}{2}%
2210
     \TestInc{-1}{0}%
     \TestInc{10}{11}%
2211
2212
     TestInc{-10}{-9}%
2213
     \TestInc{ 3 }{4}%
     \TestInc{999}{1000}%
2214
2215
     \TestInc{-1000}{-999}%
2216
     \TestInc{129}{130}%
     \TestInc{2147483646}{2147483647}%
2217
     \TestInc{-2147483647}{-2147483646}%
2218
2219
     \TestInc{12345678901234567890}{12345678901234567891}%
2220
     2221
     TestInc{-12345678901234567891}{-12345678901234567890}%
     2222
2223 \end{qstest}
2224
2225 \begin{qstest}{dec}{dec}%
     TestDec{0}{-1}%
2226
     \TestDec{1}{0}%
2227
2228
     TestDec{-1}{-2}%
2229
     \TestDec{10}{9}%
2230
     \TestDec{-10}{-11}%
2231
     \TestDec{1000}{999}%
2232
     \TestDec{-999}{-1000}%
2233
     \TestDec{130}{129}%
     \TestDec{2147483647}{2147483646}%
2234
2235
     \TestDec{-2147483646}{-2147483647}%
2236
     \TestDec{12345678901234567891}{12345678901234567890}%
2237
     \TestDec{-12345678901234567890}{-12345678901234567891}%
2239
     2240 \end{qstest}
2241
2242 \begin{qstest}{add}{add}%
     \TestAdd{0}{0}{0}%
2243
2244
     \TestAdd{1}{0}{1}%
     \TestAdd{0}{1}{1}%
2245
2246
     \TestAdd{1}{2}{3}%
2247
     TestAdd{-1}{-1}{-2}%
2248
     \TestAdd{2147483646}{1}{2147483647}%
2249
     \TestAdd{-2147483647}{2147483647}{0}%
2250
     \TestAdd{20}{-5}{15}%
2251
     TestAdd{-4}{-1}{-5}%
2252
     TestAdd{-1}{-4}{-5}%
     TestAdd{-4}{1}{-3}%
2253
2254 \ \text{TestAdd}{-1}{4}{3}%
```

```
TestAdd{4}{-1}{3}%
2255
      TestAdd{1}{-4}{-3}%
2256
      TestAdd{-4}{-1}{-5}%
2257
      TestAdd{-1}{-4}{-5}%
2259
      TestAdd{ -4 }{ -1 }{-5}%
2260
      TestAdd{ -1 }{ -4 }{-5}%
2261
      TestAdd{ -4 }{ 1 }{-3}%
      TestAdd{ -1 }{ 4 }{3}%
2262
      TestAdd{ 4 }{ -1 }{3}%
2263
      TestAdd{ 1 }{ -4 }{-3}%
2264
      TestAdd{ -4 }{ -1 }{-5}%
2265
      TestAdd{ -1 }{ -4 }{-5}%
2266
      \texttt{\TestAdd} \{876543210\} \{111111111\} \{987654321\} \%
2267
      \TestAdd{999999999}{2}{1000000001}%
2269 \end{qstest}
2270
2271 \begin{qstest}{sub}{sub}
      \TestSub{0}{0}{0}}%
2272
      \texttt{\TestSub}\{1\}\{0\}\{1\}\%
2273
      \texttt{\TestSub}\{1\}\{2\}\{-1\}\%
2274
2275
      TestSub{-1}{-1}{0}%
      \TestSub{2147483646}{-1}{2147483647}%
2276
2277
      \TestSub{-2147483647}{-2147483647}{0}%
2278
      TestSub{-4}{-1}{-3}%
      TestSub{-1}{-4}{3}%
2279
2280
      TestSub{-4}{1}{-5}%
2281
      TestSub{-1}{4}{-5}%
2282
      TestSub{4}{-1}{5}%
2283
      TestSub{1}{-4}{5}%
2284
      TestSub{-4}{-1}{-3}%
      TestSub{-1}{-4}{3}%
2285
2286
      TestSub{ -4 }{ -1 }{-3}%
      TestSub{ -1 }{ -4 }{3}%
2287
      TestSub{ -4 }{ 1 }{-5}%
2288
      TestSub{ -1 }{ 4 }{-5}%
2289
2290
      TestSub{ 4 }{ -1 }{5}
2291
      TestSub{ 1 }{ -4 }{5}%
2292
      TestSub{ -4 }{ -1 }{-3}%
2293
      TestSub{ -1 }{ -4 }{3}%
      \TestSub{1000000000}{2}{999999998}%
2294
2295
      \TestSub{987654321}{111111111}{876543210}\%
2296 \end{qstest}
2297
2298 \begin{qstest}{shl}{shl}
2299
      TestShl{0}{0}%
2300
      TestShl{1}{2}%
2301
      TestSh1{2}{4}%
2302
      \TestSh1{5621}{11242}%
2303
      \TestSh1{1073741823}{2147483646}%
2304 \end{qstest}
2305
2306 \left\{ shr}{shr} \right\}
      TestShr{0}{0}%
2307
2308
      \TestShr{1}{0}%
2309
     TestShr{2}{1}%
2310
     TestShr{3}{1}%
2311
      TestShr{4}{2}%
2312
      TestShr{5}{2}%
2313
      \TestShr{6}{3}%
2314
      \TestShr{7}{3}%
      \TestShr{8}{4}%
2315
2316 \TestShr{9}{4}%
```

```
\TestShr{10}{5}%
2317
      \TestShr{11}{5}%
2318
      \TestShr{12}{6}%
2319
      \TestShr{13}{6}%
2320
2321
      \TestShr{14}{7}%
2322
      \TestShr{15}{7}%
2323
      \TestShr{16}{8}%
2324
      \TestShr{17}{8}%
      \TestShr{18}{9}%
2325
      \TestShr{19}{9}%
2326
      \TestShr{20}{10}%
2327
      \TestShr{21}{10}%
2328
2329
      \TestShr{22}{11}%
      \TestShr{11241}{5620}%
2330
      \TestShr{73054202}{36527101}%
2331
2332
      \TestShr{2147483646}{1073741823}%
2333 \end{qstest}
2334
2335 \left[ \text{mul} \right] 
2336
      \TestMul{0}{0}{0}%
      \TestMul{1}{0}{0}%
2337
      \TestMul{0}{1}{0}%
2338
2339
      \TestMul{1}{1}{1}%
      \TestMul{3}{1}{3}%
2340
      TestMul{1}{-3}{-3}%
2341
2342
      TestMul{-4}{-5}{20}%
2343
      \TestMul{3}{7}{21}%
2344
      \TestMul{7}{3}{21}%
2345
      TestMul{3}{-7}{-21}%
2346
      TestMul{7}{-3}{-21}%
      TestMul{-3}{7}{-21}%
2347
2348
      TestMul{-7}{3}{-21}%
2349
      \TestMul{-3}{-7}{21}%
2350
      \TestMul{-7}{-3}{21}%
      \TestMul{12}{11}{132}%
2351
2352
      \TestMul{999}{333}{332667}%
2353
      \TestMul{1000}{4321}{4321000}%
2354
      \TestMul{12345}{173955}{2147474475}%
2355
      \TestMul{1073741823}{2}{2147483646}%
2356
      \TestMul{2}{1073741823}{2147483646}%
2357
      \TestMul{-1073741823}{2}{-2147483646}%
      \TestMul{2}{-1073741823}{-2147483646}%
2358
2359
      \TestMul{6706022400}{13}{87178291200}%
2360 \end{qstest}
2361
2362 \begin{qstest}{sqr}{sqr}
2363
      \TestSqr{0}{0}%
2364
      \TestSqr{1}{1}%
2365
      \TestSqr{2}{4}%
2366
      \TestSqr{3}{9}%
      \TestSqr{4}{16}%
2367
2368
      \TestSqr{9}{81}%
      \TestSqr{10}{100}%
2369
2370
      \TestSqr{46340}{2147395600}%
2371
      \TestSqr{-1}{1}%
2372
      \TestSqr{-2}{4}%
2373
      \TestSqr{-46340}{2147395600}%
2374 \end{qstest}
2375
2376 \left[ \frac{qstest}{fac} \right]
2377
      \TestFac{0}{1}%
2378
      \TestFac{1}{1}%
```

```
\TestFac{2}{2}%
2379
      \TestFac{3}{2*3}%
2380
      \TestFac{4}{2*3*4}%
2381
      \TestFac{5}{2*3*4*5}%
2382
2383
      \TestFac{6}{2*3*4*5*6}%
2384
      \TestFac{7}{2*3*4*5*6*7}%
2385
      \TestFac{8}{2*3*4*5*6*7*8}%
2386
      \TestFac{9}{2*3*4*5*6*7*8*9}%
      \TestFac{10}{2*3*4*5*6*7*8*9*10}%
2387
      \TestFac{11}{2*3*4*5*6*7*8*9*10*11}%
2388
      \TestFac{12}{2*3*4*5*6*7*8*9*10*11*12}%
2389
2390
      \TestFacBig{13}{6227020800}%
2391
      \TestFacBig{14}{87178291200}%
      \TestFacBig{15}{1307674368000}%
2392
      \TestFacBig{16}{20922789888000}%
2393
2394
      \TestFacBig{17}{355687428096000}%
2395
      \TestFacBig{18}{6402373705728000}%
      \TestFacBig{19}{121645100408832000}%
2396
      \TestFacBig{20}{2432902008176640000}%
2397
2398
      \TestFacBig{21}{51090942171709440000}%
2399
      \TestFacBig{22}{1124000727777607680000}%
2400 \end{qstest}
2401
2402 \geq 2402 \leq (qstest) \leq pow 
      TestPow{-2}{0}{1}%
2403
2404
      TestPow{-1}{0}{1}%
2405
      \TestPow{0}{0}{1}%
2406
      \TestPow{1}{0}{1}%
2407
      \TestPow{2}{0}{1}%
2408
      \TestPow{3}{0}{1}%
      TestPow{-2}{1}{-2}%
2409
      TestPow{-1}{1}{-1}%
2410
2411
      \TestPow{1}{1}{1}}%
2412
      \TestPow{2}{1}{2}%
      \TestPow{3}{1}{3}%
2413
2414
      \TestPow{-2}{2}{4}%
2415
      TestPow{-1}{2}{1}%
2416
      TestPow{0}{2}{0}%
2417
      \TestPow{1}{2}{1}%
2418
      \TestPow{2}{2}{4}%
2419
      \TestPow{3}{2}{9}%
      \TestPow{0}{1}{0}%
2420
      \TestPow{1}{-2}{1}%
2421
2422
      \TestPow{1}{-1}{1}%
2423
      TestPow{-1}{-2}{1}%
      TestPow{-1}{-1}{-1}%
2425
      TestPow{-1}{3}{-1}%
2426
      \TestPow{-1}{4}{1}%
2427
      \TestPow{-2}{-1}{0}%
      TestPow{-2}{-2}{0}%
2428
      TestPow{2}{3}{8}%
2429
2430
      \TestPow{2}{4}{16}%
      \TestPow{2}{5}{32}%
2431
2432
      \TestPow{2}{6}{64}%
2433
      \TestPow{2}{7}{128}%
2434
      \TestPow{2}{8}{256}%
2435
      \TestPow{2}{9}{512}%
2436
      \TestPow{2}{10}{1024}%
2437
      TestPow{-2}{3}{-8}%
2438
      TestPow{-2}{4}{16}%
      TestPow{-2}{5}{-32}%
2439
2440
      TestPow{-2}{6}{64}%
```

```
\TestPow{-2}{7}{-128}%
2441
      \TestPow{-2}{8}{256}%
2442
      \TestPow{-2}{9}{-512}%
2443
      \TestPow{-2}{10}{1024}%
2445
      \TestPow{3}{3}{27}%
2446
      \TestPow{3}{4}{81}%
2447
      \TestPow{3}{5}{243}%
      TestPow{-3}{3}{-27}%
2448
2449
      TestPow{-3}{4}{81}%
      TestPow{-3}{5}{-243}%
2450
      \TestPow{2}{30}{1073741824}%
2451
      \TestPow{-3}{19}{-1162261467}%
2452
      \TestPow{5}{13}{1220703125}%
2453
      \TestPow{-7}{11}{-1977326743}%
2455 \end{qstest}
2456
2457 \begin{qstest}{div}{div}
      \TestDiv{1}{1}{1}%
2458
      TestDiv{2}{1}{2}%
2459
      TestDiv{-2}{1}{-2}%
2460
2461
      TestDiv{2}{-1}{-2}%
      \TestDiv{-2}{-1}{2}%
2462
2463
      \TestDiv{15}{2}{7}%
2464
      \TestDiv{-16}{2}{-8}%
      \TestDiv{1}{2}{0}%
2465
2466
      \TestDiv{1}{3}{0}%
2467
      \TestDiv{2}{3}{0}%
2468
      TestDiv{-2}{3}{0}%
2469
      TestDiv{2}{-3}{0}%
2470
      TestDiv{-2}{-3}{0}%
      \TestDiv{13}{3}{4}%
2471
2472
      \TestDiv{-13}{-3}{4}%
2473
      \TestDiv{-13}{3}{-4}%
2474
      TestDiv{-6}{5}{-1}%
      TestDiv{-5}{5}{-1}%
2475
2476
      TestDiv{-4}{5}{0}%
2477
      TestDiv{-3}{5}{0}%
2478
      TestDiv{-2}{5}{0}%
2479
      TestDiv{-1}{5}{0}%
2480
      \TestDiv{0}{5}{0}%
      \TestDiv{1}{5}{0}%
2481
2482
      \TestDiv{2}{5}{0}%
2483
      \TestDiv{3}{5}{0}%
2484
      \TestDiv{4}{5}{0}%
2485
      \TestDiv{5}{5}{1}%
      \TestDiv{6}{5}{1}%
2487
      TestDiv{-5}{4}{-1}%
2488
      TestDiv{-4}{4}{-1}%
2489
      TestDiv{-3}{4}{0}%
2490
      TestDiv{-2}{4}{0}%
      \TestDiv{-1}{4}{0}%
2491
2492
      TestDiv{0}{4}{0}%
      \TestDiv{1}{4}{0}%
2493
2494
      \TestDiv{2}{4}{0}%
2495
      \TestDiv{3}{4}{0}%
2496
      \TestDiv{4}{4}{1}%
2497
      \TestDiv{5}{4}{1}%
2498
      \TestDiv{12345}{678}{18}%
2499
      \TestDiv{32372}{5952}{5}%
      \TestDiv{284271294}{18162}{15651}%
2500
      \TestDiv{217652429}{12561}{17327}%
2501
```

2502

\TestDiv{462028434}{5439}{84947}%

```
\TestDiv{2147483647}{1000}{2147483}%
2503
      \TestDiv{2147483647}{-1000}{-2147483}%
2504
      \TestDiv{-2147483647}{1000}{-2147483}%
2505
      TestDiv{-2147483647}{-1000}{2147483}%
2506
2507
      \TestDiv{0}{3}{0}%
2508
      \TestDiv{1}{3}{0}%
2509
      \TestDiv{2}{3}{0}%
2510
      \TestDiv{3}{3}{1}%
2511
      TestDiv{4}{3}{1}%
      \TestDiv{5}{3}{1}%
2512
      \TestDiv{6}{3}{2}%
2513
      \TestDiv{7}{3}{2}%
2514
      \TestDiv{8}{3}{2}%
2515
      \TestDiv{9}{3}{3}%
2516
      \TestDiv{10}{3}{3}%
2517
2518
      \TestDiv{11}{3}{3}%
2519
      \TestDiv{12}{3}{4}%
2520
      \TestDiv{13}{3}{4}%
      \TestDiv{14}{3}{4}%
2521
      \TestDiv{15}{3}{5}%
2522
2523
      \TestDiv{16}{3}{5}%
      \TestDiv{17}{3}{5}%
2524
2525
      \TestDiv{18}{3}{6}%
2526
      \TestDiv{19}{3}{6}%
      \TestDiv{20}{3}{6}%
2527
2528
      \TestDiv{21}{3}{7}%
2529
      \TestDiv{22}{3}{7}%
2530
      \TestDiv{23}{3}{7}%
2531
      \TestDiv{24}{3}{8}%
      \TestDiv{25}{3}{8}%
2532
      \TestDiv{26}{3}{8}%
2533
2534
      \TestDiv{27}{3}{9}%
2535
      \TestDiv{28}{3}{9}%
2536
      \TestDiv{29}{3}{9}%
      \TestDiv{30}{3}{10}%
2537
2538
      \TestDiv{31}{3}{10}%
2539
      \TestDivBig{17363436332507}{24702}{702916214}%
2540 \end{qstest}
2541
2542 \ \ \{mod} \ \ \
2543
      TestMod{-6}{5}{4}%
      TestMod{-5}{5}{0}%
2544
2545
      TestMod{-4}{5}{1}%
2546
      TestMod{-3}{5}{2}%
2547
      \TestMod{-2}{5}{3}%
      TestMod{-1}{5}{4}%
2549
      \TestMod{0}{5}{0}%
2550
      TestMod{1}{5}{1}%
2551
      \TestMod{2}{5}{2}%
2552
      \TestMod{3}{5}{3}%
      TestMod{4}{5}{4}%
2553
2554
      \TestMod{5}{5}{0}%
      \TestMod{6}{5}{1}%
2555
2556
      TestMod{-5}{4}{3}%
2557
      TestMod{-4}{4}{0}%
2558
      TestMod{-3}{4}{1}%
2559
      TestMod{-2}{4}{2}%
2560
      TestMod{-1}{4}{3}%
2561
      TestMod{0}{4}{0}%
2562
      \TestMod{1}{4}{1}%
      TestMod{2}{4}{2}%
2563
2564
      \TestMod{3}{4}{3}%
```

```
TestMod{4}{4}{0}%
2565
      TestMod{5}{4}{1}%
2566
      TestMod{-6}{-5}{-1}%
2567
      TestMod{-5}{-5}{0}%
2569
      TestMod{-4}{-5}{-4}%
2570
      TestMod{-3}{-5}{-3}%
2571
      TestMod{-2}{-5}{-2}%
2572
      TestMod{-1}{-5}{-1}%
      TestMod{0}{-5}{0}%
2573
      TestMod{1}{-5}{-4}%
2574
      TestMod{2}{-5}{-3}%
2575
     TestMod{3}{-5}{-2}%
2576
2577
      TestMod{4}{-5}{-1}%
     TestMod{5}{-5}{0}%
2578
     TestMod{6}{-5}{-4}%
2579
2580
     TestMod{-5}{-4}{-1}%
2581
      TestMod{-4}{-4}{0}%
      TestMod{-3}{-4}{-3}%
2582
      TestMod{-2}{-4}{-2}%
2583
2584
      TestMod{-1}{-4}{-1}%
      TestMod{0}{-4}{0}%
2585
2586
      TestMod{1}{-4}{-3}%
2587
      TestMod{2}{-4}{-2}%
      TestMod{3}{-4}{-1}%
2588
      TestMod{4}{-4}{0}%
2589
2590
      TestMod{5}{-4}{-3}%
2591
      \TestMod{2147483647}{1000}{647}%
2592
      \TestMod{2147483647}{-1000}{-353}%
2593
      \TestMod{-2147483647}{1000}{353}%
      TestMod{-2147483647}{-1000}{-647}%
2594
      TestMod{ 0 }{ 4 }{0}
2595
2596
      TestMod{1}{4}{1}%
2597
      TestMod{ -1 }{ 4 }{3}%
     TestMod{ 0 }{ -4 }{0}%
2598
     TestMod{1}{-4}{-3}%
2599
2600
     TestMod{ -1 }{ -4 }{-1}%
2601
      TestMod{18362}{25}{12}%
2602 \end{qstest}
2603
2604 \newcommand*{\TestError}[2]{%
2605
     \begingroup
        \expandafter\def\csname BigIntCalcError:#1\endcsname{}%
2606
        \Expect*{#2}{0}%
2607
2608
        \expandafter\def\csname BigIntCalcError:#1\endcsname{ERROR}%
2609
        \Expect*{#2}{OERROR}%
2610
      \endgroup
2611 }
2612 \begin{qstest}{error}{error}
      \TestError{FacNegative}{\bigintcalcFac{-1}}%
2614
      \TestError{FacNegative}{\bigintcalcFac{-2147483647}}%
      \TestError{DivisionByZero}{\bigintcalcPow{0}{-1}}%
2615
      \TestError{DivisionByZero}{\bigintcalcDiv{1}{0}}%
2616
      \TestError{DivisionByZero}{\bigintcalcMod{1}{0}}%
2617
2618 \end{qstest}
2619
2620 \begin{document}
2621 \end{document}
2622 (/test2)
```

4 Installation

4.1 Download

Package. This package is available on CTAN¹:

CTAN:macros/latex/contrib/oberdiek/bigintcalc.dtx The source file.

CTAN:macros/latex/contrib/oberdiek/bigintcalc.pdf Documentation.

Bundle. All the packages of the bundle 'oberdiek' are also available in a TDS compliant ZIP archive. There the packages are already unpacked and the documentation files are generated. The files and directories obey the TDS standard.

```
CTAN:install/macros/latex/contrib/oberdiek.tds.zip
```

TDS refers to the standard "A Directory Structure for TEX Files" (CTAN:tds/tds.pdf). Directories with texmf in their name are usually organized this way.

4.2 Bundle installation

Unpacking. Unpack the oberdiek.tds.zip in the TDS tree (also known as texmf tree) of your choice. Example (linux):

```
unzip oberdiek.tds.zip -d ~/texmf
```

Script installation. Check the directory TDS:scripts/oberdiek/ for scripts that need further installation steps. Package attachfile2 comes with the Perl script pdfatfi.pl that should be installed in such a way that it can be called as pdfatfi. Example (linux):

```
chmod +x scripts/oberdiek/pdfatfi.pl
cp scripts/oberdiek/pdfatfi.pl /usr/local/bin/
```

4.3 Package installation

Unpacking. The .dtx file is a self-extracting docstrip archive. The files are extracted by running the .dtx through plain- T_FX :

```
{\tt tex\ bigintcalc.dtx}
```

TDS. Now the different files must be moved into the different directories in your installation TDS tree (also known as texmf tree):

```
\begin{array}{lll} \mbox{bigintcalc.sty} & \rightarrow \mbox{tex/generic/oberdiek/bigintcalc.sty} \\ \mbox{bigintcalc.pdf} & \rightarrow \mbox{doc/latex/oberdiek/bigintcalc.pdf} \\ \mbox{test/bigintcalc-test1.tex} & \rightarrow \mbox{doc/latex/oberdiek/test/bigintcalc-test1.tex} \\ \mbox{test/bigintcalc-test2.tex} & \rightarrow \mbox{doc/latex/oberdiek/test/bigintcalc-test3.tex} \\ \mbox{bigintcalc-test3.tex} & \rightarrow \mbox{doc/latex/oberdiek/bigintcalc.dtx} \\ \mbox{} & \rightarrow \mbox{source/latex/oberdiek/bigintcalc.dtx} \\ \end{array}
```

If you have a docstrip.cfg that configures and enables docstrip's TDS installing feature, then some files can already be in the right place, see the documentation of docstrip.

4.4 Refresh file name databases

If your TEX distribution (teTEX, mikTEX, \dots) relies on file name databases, you must refresh these. For example, teTEX users run texhash or mktexlsr.

¹ftp://ftp.ctan.org/tex-archive/

4.5 Some details for the interested

Attached source. The PDF documentation on CTAN also includes the .dtx source file. It can be extracted by AcrobatReader 6 or higher. Another option is pdftk, e.g. unpack the file into the current directory:

```
pdftk bigintcalc.pdf unpack_files output .
```

Unpacking with LATEX. The .dtx chooses its action depending on the format:

plain-T_EX: Run docstrip and extract the files.

LATEX: Generate the documentation.

If you insist on using LATEX for docstrip (really, docstrip does not need LATEX), then inform the autodetect routine about your intention:

```
latex \let\install=y\input{bigintcalc.dtx}
```

Do not forget to quote the argument according to the demands of your shell.

Generating the documentation. You can use both the .dtx or the .drv to generate the documentation. The process can be configured by the configuration file ltxdoc.cfg. For instance, put this line into this file, if you want to have A4 as paper format:

```
\PassOptionsToClass{a4paper}{article}
```

An example follows how to generate the documentation with pdfIATEX:

```
pdflatex bigintcalc.dtx
makeindex -s gind.ist bigintcalc.idx
pdflatex bigintcalc.dtx
makeindex -s gind.ist bigintcalc.idx
pdflatex bigintcalc.dtx
```

5 History

```
[2007/09/27 \text{ v}1.0]
```

• First version.

[2007/11/11 v1.1]

• Use of package pdftexcmds for LuaTeX support.

6 Index

Numbers written in italic refer to the page where the corresponding entry is described; numbers underlined refer to the code line of the definition; numbers in roman refer to the code lines where the entry is used.

```
Symbols
                              \@nil . 132, 134, 141, 149, 164, 167, 172
\# ..... 1751, 1813
                              \@secondoftwo ..... 138, 146, 156
  \Qundefined ..... 52
\@ ..... 1752, 1809
                                \@firstofone ..... 1760, 1763
                                 169, 179, 298, 299, 307, 330, 428,
\@firstoftwo ...... 136, 144, <u>153</u>
                                   440, 473, 486, 498, 533, 535,
\@gobble ..... 1757, 1765
                                   624, 625, 635, 636, 653, 715,
\@ne ..... 2137,
                                   716, 726, 727, 744, 829, 840,
    2152,\ 2153,\ 2163,\ 2164,\ 2189,\ 2190
                                   887, 901, 964, 975, 992, 1112,
```

```
1198, 1223, 1251, 1299, 1320,
                                            895, 965, 969, 976, 983, 993,
                                            997, 1004, 1006, 1054, 1113,
      1330, 1424, 1428, 1456, 1502,
      1649, 1660, 1668, 1690, 1706, 1810
                                             1118, 1146, 1174, 1176, 1209,
  1211, 1306, 1391, 1420, 1422,
                                             1457, 1461, 1513, 1554, 1603,
  1658, 1697, 1731, 1733, 1738, 1743
                                      \BIC@AfterFiFi ......
                                             <u>115,</u> 180, 184, 190, 211, 215,
274, 278, 284, 288, 300, 302,
                                            308, 312, 326, 375, 377, 441,
                                            443, 449, 466, 475, 477, 499,
\advance ..... 1790, 1798, 2065
                                            501, 507, 526, 541, 626, 630,
\aftergroup ..... 26
                                            646, 717, 721, 737, 888, 890,
\AtEndDocument ..... 2078
                                            903, 910, 919, 926, 1012, 1017,
                                             1057, 1082, 1166, 1214, 1218,
                                             1236, 1238, 1247, 1267, 1269,
\begin 2085, 2103, 2122, 2145, 2156,
                                             1290, 1301, 1303, 1332, 1336,
      2167, 2195, 2207, 2225, 2242,
                                             1342, 1394, 1396, 1400, 1406,
      2271, 2298, 2306, 2335, 2362,
                                             1408, 1412, 1437, 1439, 1443,
      2376, 2402, 2457, 2542, 2612, 2620
                                             1448, 1492, 1494, 1500, 1522,
\BIC@@@@Shl ..... 847, <u>851</u>
                                             1524, 1546, 1569, 1571, 1577,
\BIC@@@Shr ... 920, 921, 927, 928, 937
                                             1579, 1585, 1587, 1596, 1598,
\BIC@@@Dec ..... 508, 509, 527, \underline{532}
                                             1619, 1626, 1661, 1663, 1677,
\BIC@@@Inc ..... 450, 451, 467, \underline{472}
                                             1679, 1683, 1700, 1702, 1712, 1719
\BIC@@@PowRec ..... 1350, 1356, 1371
                                      \BIC@AfterFiFiFi <u>116</u>, 195, 199, 331,
\BIC@@ProcessDiv \dots 1555, 1593
                                            335, 536, 538, 568, 572, 578,
\BIC@@@Shl ..... 831, 839, 854, 858
                                            581, 585, 593, 595, 600, 606,
\BIC@@@Shr ..... 891, 896, 900, 938
                                            610, 637, 641, 728, 732, 1060,
\BIC@@AddDigit .... 676, 677, 687, 766
                                            1064,\ 1071,\ 1075,\ 1086,\ 1090,
\BIC@@Cmp ..... 268, <u>271</u>, 357, 389
                                            1096, 1100, 1150, 1151, 1152,
\BIC@@Dec ..... 488, <u>496</u>, 545
                                            1153, 1154, 1155, 1156, 1157,
\BIC@@Expand ..... 132, <u>134</u>
                                            1158, 1159, 1160, 1161, 1162,
\BIC@@Inc ..... 430, <u>438</u>, 481
                                            1241, 1243, 1272, 1274, 1279,
\BIC@@MinMax ..... 353, <u>356</u>
                                            1284, 1349, 1355, 1503, 1507,
\BIC@@MinusOne ..... 1313, <u>1318</u>
                                            1531, 1533, 1539, 1541, 1612, 1614
\BIC@@PowRec ..... 1343, 1366, 1372
                                      \BIC@AtEnd ..... 80, 81, 1746
\BIC@@ProcessDiv \dots 1514, \underline{1519}
                                      \BIC@Cmp ..... 264, 267
\BIC@@ProcessFac ..... 1177, \underline{1183}
                                      \BIC@CmpDiff ..... 300, <u>319</u>
\BIC@@ProcessTim ..... 984, 991
                                      \BIC@CmpLength ..... 289, 295, <u>297</u>
\BIC@@Shl ..... 813, 818, 825, <u>828</u>
                                      \BIC@CmpResult ..... 303, 309, 318
\BIC@@Shr ..... 871, 875, 881,
                                      \BIC@Dec .... 398, 412, 425, 485, 1185
      883, 1344, 1373, 1440, 1444, 1445
                                      \BIC@DecSwitch ..... 404, 407
\BIC@@Sqr ..... 1131, 1133, <u>1136</u>
                                      \BIC@Div ..... 1378, <u>1381</u>
\BIC@@SubDigit ..... 765, 776
                                      \BIC@DivCleanup ......
\BIC@@TestMode ..... 106
                                             .\ \ 1492,\ 1500,\ \underline{1518},\ 1522,\ 1531,
1539, 1569, 1577, 1585, 1596, 1612
\BIC@Abs ..... 235, <u>238</u>
                                      \BIC@DivStart ..... 1482, <u>1486</u>
\BIC@Add ..... 551, <u>554</u>, 560
                                      \BIC@DivStartX .... 1449, 1455, 1462
\BIC@AddCarry0 ..... <u>694</u>
                                      \BIC@DivStartYii ..... 1458, 1466
\BIC@AddCarry10 ..... <u>695</u>
                                      \BIC@DivStartYiv ..... 1467, <u>1471</u>
\BIC@AddCarry[1-9] ..... <u>696</u>
                                      \BIC@DivStartYvi ..... 1472, 1476
\BIC@AddDigit ..... 658, 663, 674
                                      \BIC@DivStartYviii ..... 1477, 1481
\BIC@AddResult .... 657, 667
                                      \BIC@DivSub ..... 1542, 1559, 1588
\BIC@AddSwitch ..... 556, <u>563</u>
                                      \BIC@DivSwitch ......
\BIC@AddXY .... 569, 573, 607, 611,
                                             .... 1397, 1401, 1409, 1413, 1418
      618, <u>623</u>, 814, 819, 826, 1020, 1478
                                      \BIC@DivSwitchSign . . 1382, 1387, 1389
\BIC@AfterFi ..... <u>114</u>,
                                      \BIC@DoAdd ..... 627, 631, 652
      171, 208, 321, 358, 360, 390,
                                      \BIC@DoSub ..... 718, 722, <u>743</u>
      392, 396, 409, 411, 415, 429,
      433, 480, 487, 491, 544, 654,
                                      \BIC@Expand ..... <u>130</u>, <u>160</u>, <u>227</u>
                                      \BIC@Fac ..... 1141, <u>1144</u>
      661, 689, 691, 745, 750, 778,
      783, 811, 817, 830, 834, 841,
                                      \BIC@Fi ... <u>113</u>, 114, 115, 116, 174,
      846, 853, 857, 870, 874, 885,
                                            204, 219, 292, 316, 340, 361,
```

381, 400, 419, 436, 470, 483,	\BIC@SubXY
494, 530, 547, 615, 650, 665,	. 582, 586, 596, 601, 621, <u>714</u> ,
692, 741, 754, 784, 821, 837,	1548, 1562, 1605, 1621, 1628, 1740
849, 860, 877, 898, 933, 972,	\BIC@Temp 696, 704,
989, 1000, 1025, 1105, 1122,	705, 706, 707, 708, 709, 710,
1170, 1181, 1296, 1309, 1364,	711, 712, 713, 788, 795, 796,
1416, 1453, 1464, 1516, 1557,	797, 798, 799, 800, 801, 802,
1591, 1633, 1687, 1727, 1734, 1744	803, 940, 943, 944, 945, 946,
\BIC@Inc 393, 417, 422, <u>427</u>	947, 948, 949, 950, 951, 952,
\BIC@IncSwitch 385, 388	953, 954, 955, 956, 957, 958,
\BIC@MinMax 344, 349, <u>352</u>	959, 960, 961, 962, 1027, 1036,
\BIC@MinusOne 1229, 1258, 1311	1037, 1038, 1039, 1040, 1041, 1042
\BIC@Mod 1637, <u>1640</u>	\BIC@TestMode 106, 1833
\BIC@ModMinus 1709, <u>1729</u>	\BIC@Tim 963, 1115, 1120
\BIC@ModSwitch . 1664, 1680, 1684, 1689	\BIC@TimDigit 979, 986, 1002
\BIC@ModSwitchSign 1641, 1646, 1648	\bigintcalcAbs 4, <u>233</u> , 366, 1883,
\BIC@ModTwo	1902, 1973, 1974, 1982, 1983,
1240, 1271, 1278, <u>1298</u> , 1341, 1699	1992, 1993, 2019, 2020, 2040, 2041
\BIC@ModX 1713, <u>1736</u>	\BigIntCalcAdd 7, 617, 1936, 1944
\BIC@Mul 1045, <u>1048</u>	\bigintcalcAdd 5, <u>549</u> , 1115, 1120, 1930
\BIC@MulDigit[3-9] 1027	\bigintcalcCmp 4, <u>262</u> , 1895, 1933, 1957
\BIC@MulSwitch 1049, 1052	\BigIntCalcDec 7, <u>424</u> , 1923
\BIC@Normalize 177, 224	\bigintcalcDec 5, <u>402</u> , 1919
\BIC@NormalizeDigits 200, 216, 221	\BigIntCalcDiv 7, <u>1385</u> , 2019
\BIC@NormalizeZero 196, 206	\bigintcalcDiv 6, <u>1376</u> ,
\BIC@Odd 365, 370, <u>372</u>	1715, 1722, 1871, 2012, 2016, 2616
\BIC@PosCmp <u>294</u> , 567, 577, 592,	\BigIntCalcError 461, 511, 521, 700, 791, 1032, 1146,
605, 1059, 1070, 1085, 1095,	1163, 1233, 1236, 1244, 1248,
1148, 1173, 1331, 1348, 1419, 1490, 1520, 1529, 1567, 1594, 1610	1275, 1287, 1293, 1295, 1361,
\BIC@Pow 1190, 1193	1363, 1391, 1451, 1658, 1703, 1726
\BIC@PowRec 1280, 1285, 1291, 1329, 1367	\bigintcalcFac
\BIC@PowSwitch 1194, 1197	6, <u>1139</u> , 2003, 2006, 2613, 2614
\BIC@ProcessDiv	\BigIntCalcInc
<u>1455</u> , 1487, <u>1489</u> , 1525,	\bigintcalcInc 5, <u>383</u> , 1908
1534, 1560, 1572, 1580, 1599, 1615	\bigintcalcInv 3, <u>229</u> , 1880
\BIC@ProcessDivII	\bigintcalcMax 4, 347, 1892
1547, <u>1566</u> , 1604, 1620, 1627	\bigintcalcMin 4, 342, 1889
\BIC@ProcessDivIV <u>1593</u>	\BigIntCalcMod 7, <u>1644</u> , 2039
\BIC@ProcessFac 1167, 1172, 1184	\bigintcalcMod 6, <u>1635</u> , <u>2024</u> , <u>2617</u>
\BIC@ProcessMul . 1061, 1065, 1072,	\BigIntCalcMul
1076, 1087, 1091, 1097, 1101,	\bigintcalcMul 6, <u>1043</u> , 1715, <u>1722</u> , 1988
1109, <u>1111</u> , 1137, 1178, 1215,	\bigintcalcNum 3, <u>222</u> , <u>231</u> , <u>236</u> ,
1219, 1333, 1337, 1351, 1357, 1368	249, 265, 269, 345, 350, 354,
\BIC@ProcessTim 966, 974, 994, 998	386, 405, 552, 556, 561, 807,
\BIC@Sgn 248, <u>251</u> , 408,	865, 1046, 1050, 1127, 1142,
1053, 1056, 1081, 1390, 1393, 1405	1191, 1195, 1379, 1383, 1638,
\BIC@Shl 806 , 809 , 1468 , 1473 , 1483	1642, 1912, 1923, 1937, 1945, 1961
\BIC@Shr 864, <u>867</u>	\BigIntCalcOdd
\BIC@ShrDigit[00-19] <u>940</u>	\bigintcalcOdd
\BIC@ShrResult 904, 905, 911, 912, 935	\bigintcalcPow 6 , 1188 , 2009 , 2615
\BIC@Space $\dots \dots \underline{117}$,	\bigintcalcSgn
$161, \ 230, \ 240, \ 242, \ 655, \ 669,$	4, <u>246</u> , 1886, 1909, 1920, 1931,
671, 760, 779, 842, 977, 1007,	1932, 1955, 1956, 2025, 2027, 2032
1013, 1018, 1114, 1708, 1720, 1739	\BigIntCalcShl
\BIC@Sqr 1126, <u>1129</u>	\bigintcalcShl 5, <u>804</u> , <u>1970</u>
\BIC@StripHexSpace 164, <u>167</u>	\BigIntCalcShr
\BIC@SubCarry0	\bigintcalcShr 5, <u>862</u> , <u>1979</u>
\BIC@SubCarry10	\bigintcalcSqr 6, <u>1124</u> , 1998
\BIC@SubCarry[1-9]	\BigIntCalcSub 7, <u>620</u> , 1960
\BIC@SubDigit 747, 752, <u>763</u>	\bigintcalcSub 5, <u>558</u> , 1714, 1721, 1954
\BIC@SubResult 746, 756	\body 1769, 1773

\mathbf{C}	\:f 11 14 19 44 59 55 109 109
\catcode 3, 4, 5, 6, 7, 8, 9, 17, 31, 32,	\ifx . 11, 14, 18, 44, 52, 55, 102, 108, 123, 129, 143, 153, 156, 169,
33, 34, 35, 36, 37, 38, 39, 40, 41,	178, 179, 189, 194, 207, 210,
42, 43, 64, 65, 68, 69, 70, 71, 75,	239, 252, 255, 272, 273, 283,
76, 77, 78, 82, 84, 104, 109, 111,	298, 299, 307, 330, 373, 428,
$1749, \ 1750, \ 1751, \ 1752, \ 1787,$	440, 473, 486, 498, 533, 535,
1796, 1809, 1810, 1811, 1812,	564, 565, 591, 624, 625, 635,
1813, 1814, 1815, 1816, 1817, 1818	636, 653, 668, 715, 716, 726,
\chardef 1833	727, 744, 757, 810, 829, 840,
\count@ 1754, 1783, 1787,	868, 887, 901, 964, 975, 992,
1789, 1790, 1794, 1796, 1797, 1798	1112, 1130, 1145, 1198, 1199,
\countdef 1754	1200, 1201, 1205, 1213, 1223,
\csname 10,	1224, 1225, 1251, 1252, 1253,
18, 44, 60, 67, 102, 108, 123,	1257, 1299, 1312, 1319, 1320,
129, 153, 156, 678, 694, 695,	1330, 1424, 1428, 1429, 1430,
697, 767, 786, 787, 789, 906,	1431, 1456, 1491, 1499, 1502,
913, 922, 929, 941, 1008, 1014,	1521, 1530, 1538, 1568, 1576,
1021, 1028, 1753, 1756, 1759,	1584, 1595, 1611, 1649, 1650,
	1654, 1660, 1668, 1669, 1673,
1762, 1801, 1823, 2047, 2606, 2608	
D	1690, 1691, 1692, 1706, 1711,
_	1730, 1737, 1753, 1756, 1759,
\dimexpr 2055	1762, 1801, 2047, 2099, 2100, 2119
\divide 1870	\immediate 20, 46
\documentclass 1829	\IncludeTests 1838
	\input 124, 1802
E	\iterate 1770, 1772, 1774
\empty 13, 14	L
\end 1824,	
2101, 2120, 2143, 2154, 2165,	\LoadCommand 1802, 1819
2193, 2205, 2223, 2240, 2269,	\LogTests 1839
2296, 2304, 2333, 2360, 2374,	\loop 1768, 1784, 1795
2400, 2455, 2540, 2602, 2618, 2621	
$\ensuremath{\setminus} \text{endcsname} \dots 10,$	\mathbf{M}
10 44 en e7 100 100 100	
18, 44, 60, 67, 102, 108, 123,	\m@ne 2138, 2153, 2164, 2190
129, 153, 156, 685, 694, 695,	\makeatletter 1832, 2049, 2085
	\makeatletter 1832, 2049, 2085
129, 153, 156, 685, 694, 695,	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2083
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906,	\makeatletter 1832, 2049, 2085
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014,	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2083 N
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759,	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2083 N
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 \endinput	\makeatletter
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb+endinput $	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2081 \newcommand
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb+endinput $	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2083 \newcommand
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 \endinput	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2083 \n \newcommand
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 \endinput	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2083 \[\begin{align*} \textbf{N} \\ \textbf{NeedsTeXFormat} \tag{1.5} \\ \textbf{newcommand} \tag{1.5} \\ \textbf{1.866}, 1867, 1868, 1873, 1878, 1879, 1882, 1885, 1888, 1891, 1894, 1897, 1907, 1918, 1929, 1953, 1969, 1978, 1987, 1997, \end{align*}
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 \endinput	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2083 \[\begin{align*} \be
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 \endinput	\makeatletter 1832, 2049, 2083 \makeatother
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 \endinput	\makeatletter
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 \endinput	\makeatletter
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb+endinput $	\makeatletter
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb+endinput $	\makeatletter
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput \dots \dots$	\makeatletter
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput \dots \dots$	\makeatletter 1832, 2049, 2083 \makeatother
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter 1832, 2049, 2083 \makeatother
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 \endinput	\makeatletter 1832, 2049, 2083 \makeatother
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter 1832, 2049, 2083 \makeatother
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter 1832, 2049, 2083 \makeatother
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter 1832, 2049, 2083 \makeatother
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput 2669,\ 2074\\ \verb \Expect 1846,\ 1853,\ 1862,\ 1871,\ 2607,\ 2609\\ \hline \\ \hline \begin{array}{c} \textbf{I}\\ \verb \frace{1}\\ \ \ \ \ \ \ \ \ \ \ \ \ \$	\makeatletter 1832, 2049, 2083 \makeatother
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2083 \makeatother 1840, 1848, 1855,
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter 1832, 2049, 2083 \makeatother 1834, 2081 \textbf{N} \textbf{N} \textbf{Newcommand} 1840, 1848, 1855, 1866, 1867, 1868, 1873, 1878, 1879, 1882, 1885, 1888, 1891, 1894, 1897, 1907, 1918, 1929, 1953, 1969, 1978, 1987, 1997, 2000, 2005, 2008, 2011, 2015, 2023, 2053, 2058, 2062, 2063, 2604 \textbf{Newcount} 1865, 2050, 2051 \textbf{Newtount} 1774, 1776, 1778 \textbf{Nofiles} 1828 \textbf{Number} 247, 263, 677, 766, 1008, 1014, 1021, 2055, 2098, 2136, 2137, 2138, 2140, 2152, 2153, 2163, 2164, 2189, 2190 \textbf{Numexpr} 450, 508, 656, 676, 765, 780, 843, 847, 904, 911, 920, 927, 978, 985, 1830, 1831, 1836, 1841, 1842, 1845, 1849, 1850, 1852, 1858, 1866, 2002 \textbf{P}
$\begin{array}{c} 129,\ 153,\ 156,\ 685,\ 694,\ 695,\\ 697,\ 774,\ 786,\ 787,\ 789,\ 906,\\ 913,\ 922,\ 929,\ 941,\ 1008,\ 1014,\\ 1021,\ 1028,\ 1753,\ 1756,\ 1759,\\ 1762,\ 1801,\ 1823,\ 2047,\ 2606,\ 2608\\ \verb \endinput $	\makeatletter 1832, 2049, 208 \makeatother 1834, 208 \n \n \n \NeedsTeXFormat 1840, 1848, 1855,
129, 153, 156, 685, 694, 695, 697, 774, 786, 787, 789, 906, 913, 922, 929, 941, 1008, 1014, 1021, 1028, 1753, 1756, 1759, 1762, 1801, 1823, 2047, 2606, 2608 Andinput	\makeatletter 1832, 2049, 208 \makeatother 1834, 208 \textbf{N} \textbf{NeedsTeXFormat} 185 \textbf{1866}, 1867, 1868, 1873, 1878, 1879, 1882, 1885, 1888, 1891, 1894, 1897, 1907, 1918, 1929, 1953, 1969, 1978, 1987, 1997, 2000, 2005, 2008, 2011, 2015, 2023, 2053, 2058, 2062, 2063, 266 \textbf{Newcount} 1865, 2050, 208 \textbf{Newcount} 1865, 2050, 208 \textbf{Newtount} 1874, 1776, 177 \textbf{Nofiles} 1875 \textbf{Number} 247, 263, 677, 766, 1008, 1014, 1021, 2055, 2098, 2136, 2137, 2138, 2140, 2152, 2153, 2163, 2164, 2189, 2164, 2189, 2154, 2152, 2153, 2163, 2164, 2189, 2164, 911, 920, 927, 978, 985, 1830, 1831, 1836, 1841, 1842, 1845, 1849, 1850, 1852, 1858, 1866, 206 \textbf{P} \textbf{PackageInfo} 260 \textbf{P} \textbf{PackageInfo}

```
\pdfelapsedtime ..... 2064
                                             2177, 2178, 2179, 2180, 2181,
                                             2182, 2183, 2184, 2185, 2186,
\pdfresettimer ..... 2060
\PrintTime ..... 2053, 2066, 2079
                                             2187, 2188, 2189, 2190, 2191, 2192
\TestCount .... 1865, 1869, 1870, 1871
                                      \TestDec .. 1918, 2226, 2227, 2228,
                 \mathbf{Q}
                                             2229, 2230, 2231, 2232, 2233,
\qstest .....
                           2068, 2070
                                             2234, 2235, 2236, 2237, 2238, 2239
                                      \TestDiv ..... 2011,
                                             2458,\ 2459,\ 2460,\ 2461,\ 2462,
\RangeCatcodeInvalid .....
                                             2463, 2464, 2465, 2466, 2467,
      . . . . 1793, 1805, 1806, 1807, 1808
                                             2468, 2469, 2470, 2471, 2472,
\renewcommand ..... 2059
                                             2473, 2474, 2475, 2476, 2477,
\repeat ..... 1768, 1780, 1791, 1799
                                             2478, 2479, 2480, 2481, 2482,
\RequirePackage ..... 126
                                             2483, 2484, 2485, 2486, 2487,
\RestoreCatcodes 1782, 1785, 1786, 1820
                                             2488, 2489, 2490, 2491, 2492,
\result ..... 1851, 1853
                                             2493, 2494, 2495, 2496, 2497,
\resultA ..... 1843, 1846
                                             2498, 2499, 2500, 2501, 2502,
\resultB ..... 1844, 1846
                                             2503, 2504, 2505, 2506, 2507,
\romannumeral 131, 161, 223, 230, 234,
                                             2508, 2509, 2510, 2511, 2512,
      343, 348, 364, 369, 384, 397,
                                             2513, 2514, 2515, 2516, 2517,
      403, 416, 422, 425, 550, 559,
                                             2518, 2519, 2520, 2521, 2522,
      566, 580, 599, 618, 621, 675,
                                             2523, 2524, 2525, 2526, 2527,
      764, 805, 812, 824, 863, 869,
                                             2528, 2529, 2530, 2531, 2532,
      880, 979, 986, 1019, 1044, 1069,
                                             2533, 2534, 2535, 2536, 2537, 2538
      1084, 1108, 1125, 1140, 1178,
                                      \TestDivBig ..... 2015, 2539
      1185, 1189, 1283, 1343, 1350,
                                      \TestError ......
      1356, 1367, 1372, 1377, 1386,
                                             2604, 2613, 2614, 2615, 2616, 2617
      1426, 1444, 1467, 1472, 1477,
                                      \TestExch ..... 1878, 2001
      1482, 1547, 1561, 1604, 1620,
                                      \TestFac ..... 2000, 2377, 2378,
      1627, 1636, 1645, 1708, 1709, 1739
                                             2379. 2380. 2381. 2382. 2383.
                                             2384, 2385, 2386, 2387, 2388, 2389
                                      \TestFacBig .....
\saved@endqstest ..... 2069, 2076
                                             . 2005, 2390, 2391, 2392, 2393,
\saved@qstest ..... 2068, 2071
                                             2394, 2395, 2396, 2397, 2398, 2399
\SavedNumexpr .....
                                      \TestInc 1907, 2208, 2209, 2210, 2211.
      1830, 1836, 1841, 1845, 1849, 1852
                                             2212, 2213, 2214, 2215, 2216,
\space ..... 1846, 2055
                                             2217, 2218, 2219, 2220, 2221, 2222
\StartTime ..... 2058, 2072
                                      \TestInv 1879, 2086, 2087, 2088, 2089,
\StopTime ..... 2063, 2075
                                             2090, 2091, 2092, 2093, 2094,
\strip@pt .... 2055
                                             2095, 2096, 2097, 2098, 2099, 2100
\SummaryTime ... 2050, 2052, 2065, 2079
                                      \TestMax .... 1891, 2157, 2158,
                                             2159, 2160, 2161, 2162, 2163, 2164
                                      \TestMin ..... 1888, 2146, 2147,
\Test ..... 1804, 1822, 1873, 1878,
                                             2148, 2149, 2150, 2151, 2152, 2153
      1880, 1883, 1886, 1889, 1892,
                                      \TestMod .. 2023, 2543, 2544, 2545.
      1895, 1898, 1900, 1908, 1911,
                                             2546, 2547, 2548, 2549, 2550,
      1919, 1922, 1930, 1935, 1943,
      1954, 1959, 1970, 1972, 1979,
                                             2551, 2552, 2553, 2554, 2555,
                                             2556, 2557, 2558, 2559, 2560,
      1981, 1988, 1990, 1998, 2006,
                                             2561, 2562, 2563, 2564, 2565,
      2009, 2012, 2016, 2018, 2024, 2038
                                             2566, 2567, 2568, 2569, 2570,
\TestAbs ..... 1882,
                                             2571, 2572, 2573, 2574, 2575,
      2104, 2105, 2106, 2107, 2108,
      2109, 2110, 2111, 2112, 2113,
                                             2576, 2577, 2578, 2579, 2580,
                                             2581, 2582, 2583, 2584, 2585,
      2114, 2115, 2116, 2117, 2118, 2119
                                             2586,\ 2587,\ 2588,\ 2589,\ 2590,
\TestAdd ..... 1929,
                                             2591, 2592, 2593, 2594, 2595,
      2243, 2244, 2245, 2246, 2247,
                                             2596, 2597, 2598, 2599, 2600, 2601
      2248, 2249, 2250, 2251, 2252,
                                      \TestMul .. 1987, 2336, 2337, 2338,
      2253, 2254, 2255, 2256, 2257,
      2258, 2259, 2260, 2261, 2262,
                                             2339,\ 2340,\ 2341,\ 2342,\ 2343,
      2263,\ 2264,\ 2265,\ 2266,\ 2267,\ 2268
                                             2344,\ 2345,\ 2346,\ 2347,\ 2348,
\TestArg ..... 1866, 1867, 1869, 1870
                                             2349, 2350, 2351, 2352, 2353,
\TestCmp 1894, 2168, 2169, 2170, 2171,
                                             2354, 2355, 2356, 2357, 2358, 2359
      2172, 2173, 2174, 2175, 2176,
                                      \TestOdd ..... 1897, 2197, 2198,
```

2199, 2200, 2201, 2202, 2203, 2204 \text{Pow} \tag{2405}, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2439, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454	2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295 \TestTeXDivide
\TestResult 1848, 1874	\tracingmacros 2196
TestResultTwoExpansions . $1855, 1875$	\typeout 2054
\TestSgn 1885, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139	U \UNDEFINED 1831, 1842, 1850, 1858 \usepackage 1835, 1837
\TestShl	\mathbf{W}
1969, 2299, 2300, 2301, 2302, 2303 \TestShr 1978,	\write 20, 46
2307, 2308, 2309, 2310, 2311,	\mathbf{X}
2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332 \TestSpaceAtEnd 1840, 1876 \TestSqr 1997, 2363, 2364, 2365, 2366, 2367,	\x
2368, 2369, 2370, 2371, 2372, 2373 \TestSub 1953, 2272, 2273, 2274,	Z \z@2052,
2275, 2276, 2277, 2278, 2279,	2098, 2136, 2140, 2152, 2163, 2189