

SMT-RAT 25.06

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SMT-RAT [3] is an open-source C++ toolbox for strategic and parallel SMT solving consisting of a collection of SMT compliant implementations of methods for solving quantifier-free first-order formulas with a focus on non-linear real and integer arithmetic. Further supported theories include linear real and integer arithmetic, difference logic, bit-vectors and pseudo-Boolean constraints. A more detailed description of SMT-RAT can be found at <https://smtrat.github.io/>.

For *quantifier-free non-linear real arithmetic (QF-NRA)*, SMT-RAT uses our implementation of the MCSAT framework [4] inspired by [8]. We employ incomplete methods to handle simpler problem classes more efficiently. Thus, our implementation is equipped with multiple explanation backends based on Fourier-Motzkin variable elimination, interval constraint propagation, virtual substitution as in [17], and a novel level-wise variant of the one-cell CAD [2, 11], which are called in this order. The level-wise one-cell CAD uses linear approximations of some cell boundaries which would otherwise be defined by polynomials with high degree, as described in [15]. The general MCSAT framework is integrated in our adapted `minisat` [5] solver. Depending on the number of Boolean variables in the input, we either use a fully dynamic variable ordering as suggested in [7, 13], or a static feature-based ordering as in [14]. Furthermore, we supplement our solver with an incomplete check for subtropical satisfiability [6] before the main MCSAT solver is called. For algebraic operations, we use `libpoly` [1].

For *non-linear real arithmetic (NRA)*, SMT-RAT uses the cylindrical algebraic covering (CAIC) method [16] extended for quantifiers [9, 12].

Both the one-cell CAD used in MCSAT and the CAIC method use a novel technique for merging adjacent cells during the construction, as presented in [10].

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