KAPITEL 2

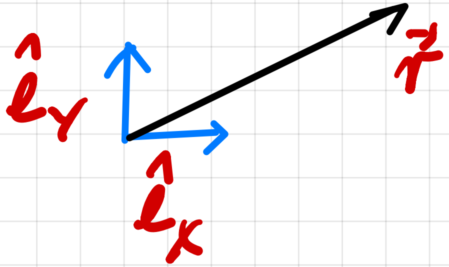
VEKTOREN $\neq$ $\underbrace{\text{KOORDINATENVEKTOREN}}_{\text{DARSTELLUNG VON VEKTOREN BZGL. EINER BASIS}}$

WÄHLEN IM RAUM EINEN REFERENZPUNKT

$\rightarrow$ ORT $\vec{r}$ EINES TEILCHENS RELATIV ZUM REF. PKT.

EIGENSCHAFTEN EINES VEKTORS:

- LÄNGE
- RICHTUNG



$$\left.\begin{aligned}\vec{r} &= x\,\hat{e}_x + y\,\hat{e}_y \\ &= r_x\,\hat{e}_x + r_y\,\hat{e}_y\end{aligned}\right\}\ \text{IN 2D}$$

3D:

$$\begin{aligned}\vec{r} &= x\,\hat{e}_x + y\,\hat{e}_y + z\,\hat{e}_z \\ &= r_x\,\hat{e}_x + r_y\,\hat{e}_y + r_z\,\hat{e}_z\end{aligned}$$

KOMPAKT:

$$\boxed{\begin{aligned}&\vec{r} = \sum_{i=1}^{3} r_i\,\hat{e}_i = \sum_i r_i\,\hat{e}_i \\ &\hat{e}_1 = \hat{e}_x,\quad \hat{e}_2 = \hat{e}_y,\quad \hat{e}_3 = \hat{e}_z\end{aligned}}$$

IM ALLG.: $\boxed{r_i = r_i(t)}$

ORTHONORMALITÄT:

$$\boxed{\hat{e}_i \cdot \hat{e}_j = \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}}$$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\varphi$$

$$\vec{a}=\vec{b} \Rightarrow \vec{a}\cdot\vec{a} = |\vec{a}|\,|\vec{a}|\cos 0 = |a|^2$$

$$\Rightarrow |\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}$$

$$\hat{e}_i\cdot\hat{e}_i = 1 \Rightarrow \sqrt{\hat{e}_i\cdot\hat{e}_i} = 1 \Rightarrow |\hat{e}_i| = 1$$

$$\vec{a}\perp\vec{b} \Rightarrow \varphi = 90^\circ \; \left(\frac{\pi}{2}\right) \Rightarrow \cos\varphi = \cos\frac{\pi}{2} = 0$$

$$\Rightarrow \vec{a}\cdot\vec{b} = 0$$

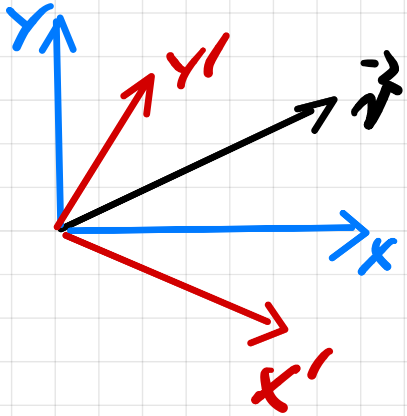
$$\vec{r} = \sum_i r_i\,\hat{e}_i \Rightarrow \boxed{r_i = \hat{e}_i\cdot\vec{r}}$$

BEACHTEN SIE: $\vec{r} = \sum_i r_i\,\hat{e}_i = \sum_j r_j\,\hat{e}_j = \sum_k r_k\,\hat{e}_k$

$$\vec{r} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

KOORDINATENVEKTOR

GLEICHSETZEN GEFÄHRLICH



$\vec{r} = \sum_i r_i \hat{e}_i \,, \quad r_i = \hat{e}_i \cdot \vec{r}$ "ALTE BASIS"

$\vec{r} = \sum_i r_i' \hat{e}_i' \,, \quad r_i' = \hat{e}_i' \cdot \vec{r}$ "NEUE BASIS"

$$\vec{r}\,' = \begin{pmatrix} r_x' \\ r_y' \\ r_z' \end{pmatrix} \neq \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

BEISPIEL: $\vec{r} = 2\,\hat{e}_x - 1\,\hat{e}_y + 3\,\hat{e}_z$

$\Rightarrow$ KOORD. VEK. $\vec{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$$\hat{e}_x' = -\frac{1}{\sqrt{2}}\,\hat{e}_x + 0\,\hat{e}_y + \frac{1}{\sqrt{2}}\,\hat{e}_z$$
$$\hat{e}_y' = 0\,\hat{e}_x + 1\,\hat{e}_y + 0\,\hat{e}_z$$
$$\hat{e}_z' = \frac{1}{\sqrt{2}}\,\hat{e}_x + 0\,\hat{e}_y + \frac{1}{\sqrt{2}}\,\hat{e}_z$$

$$\hat{e}_x' = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \hat{e}_y' = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{e}_z' = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

ES GILT: $\vec{r} = \sum_i r_i' \hat{e}_i'$, $\quad r_i' = \hat{e}_i' \cdot \vec{r}$

$$\Rightarrow r_x' = \hat{e}_x' \cdot \vec{r} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = -\frac{1}{\sqrt{2}} 2 + 0 \cdot (-1) + \frac{1}{\sqrt{2}} 3 = \frac{1}{\sqrt{2}}$$

$$r_y' = \hat{e}_y' \cdot \vec{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = -1$$

$$r_z' = \hat{e}_z' \cdot \vec{r} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \frac{5}{\sqrt{2}}$$

$$\vec{r}' = \begin{pmatrix} r_x' \\ r_y' \\ r_z' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{5}{\sqrt{2}} \end{pmatrix}$$