FAIR CAUSAL INFERENCE FOR FUNCTIONAL DATA

Computational Statistics Conference; Bologna, 2022

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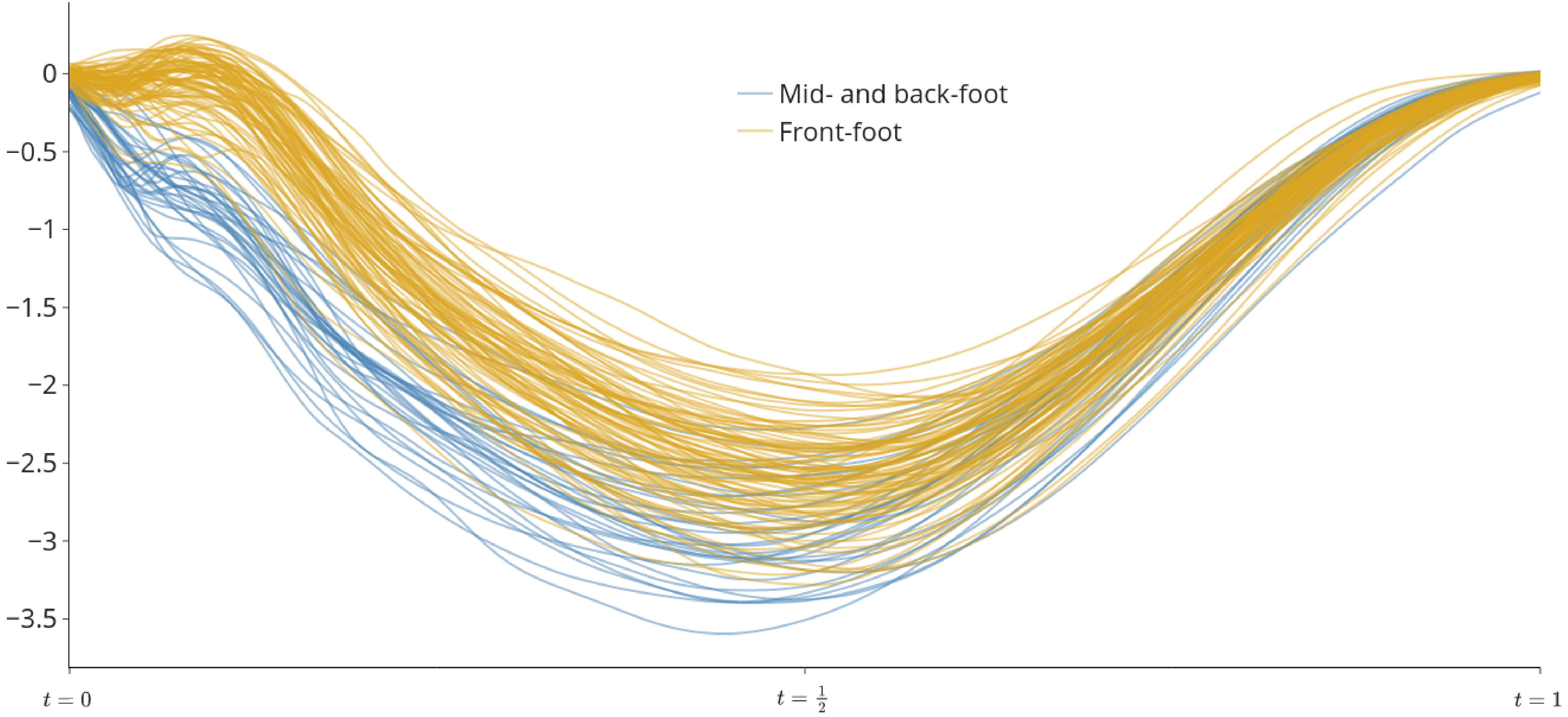
Who Am I?

- Tim Mensinger
- PhD in Economics, University of Bonn
- Focus:
  - Econometrics & Statistics
  - Causal Inference
  - Programming

Motivation

- Foot striking patterns:
  - fore foot or heel
- Is one of them *better*?
  - Consider one metric: Force on ankle joints
- ***What's the effect of fore-foot running on ankle joint loading?***

Data



Data Structure

Outcomes	Controls	Treatment
$Y_i \in L^2[0,1]$	$X_i \in \mathbb{R}^p$	$W_i \in \{0,1\}$

- Outcomes observed on a time-grid $\mathcal{T} \subset [0,1]$
- Potential Outcomes:
  - $Y_i(1), Y_i(0) \in L^2[0,1]$
  - $Y_i = Y_i(W_i) = W_i Y_i(1) + (1 - W_i) Y_i(0)$

Estimand

- Average treatment effect function:

$$\tau(t) = \mathbb{E}[Y_i(1)(t) - Y_i(0)(t)]$$

for $t \in [0, 1]$

Game Plan

1. **Choose a suitable estimator**
   - Utilize modern methods from *econometrics* literature
2. **Find relevant control variables**
   - Utilize causal graphs from *causal inference* literature
3. **Construct confidence bands**
   - Utilize novel results from *functional data* literature

Choose a suitable estimator

Estimator

- Linear model:

$$Y_i(t) = \tau_i(t)W_i + \beta(t)^\top X_i + e_i(t)$$

  - Estimate $\tau(t) = \mathbb{E}[\tau_i(t)]$ for all $t \in \mathcal{T}$ via OLS

- **Doubly robust**:
  - Estimate (mean) potential outcome functions
  - Correct bias using inverse propensity score weighting

Doubly Robust

$$\hat{A}(t) = \frac{1}{n}\sum_{i=1}^{n} \hat{\mathbb{E}}[Y_i(1)(t)|X_i] - \hat{\mathbb{E}}[Y_i(0)(t)|X_i]$$

$$\hat{B}(t) = \frac{1}{n}\sum_{i=1}^{n} W_i \frac{Y_i(t) - \hat{\mathbb{E}}[Y_i(1)(t)|X_i]}{\hat{\mathbb{P}}[W_i = 1|X_i]} - (1 - W_i)\frac{Y_i(t) - \hat{\mathbb{E}}[Y_i(0)(t)|X_i]}{\hat{\mathbb{P}}[W_i = 0|X_i]}$$

$$\hat{\tau}(t) = \hat{A}(t) - \hat{B}(t)$$

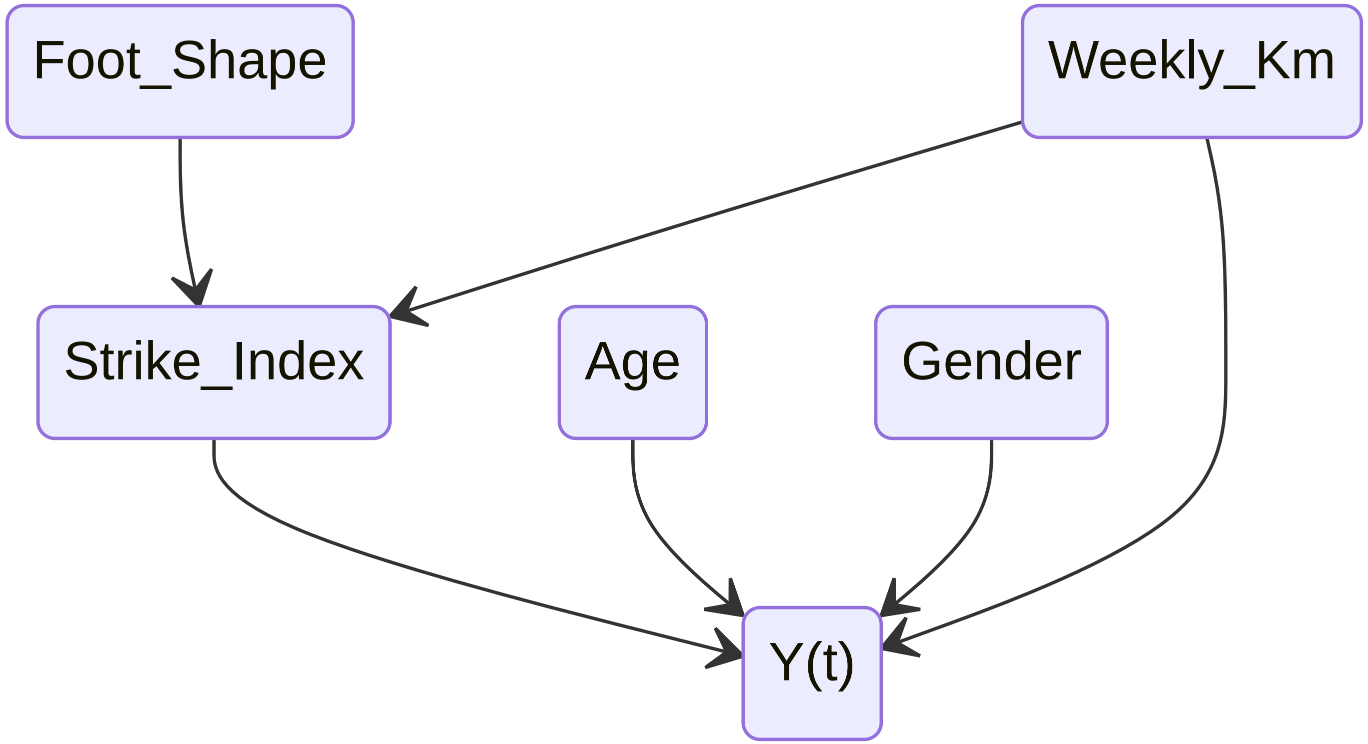
- Propensity score estimator: $\hat{\mathbb{P}}[W_i = w|X_i]$
- Conditional mean estimators: $\hat{\mathbb{E}}[Y_i(w)(t)|X_i]$

Find relevant control variables

Application

- Observations: $n = 112$
- Features: $p = 9$
  - Examples: `gender`, `age`, `weekly_km`, `...`
- Develop graphical model to
  - Better communicate model
  - Gain efficiency
  - Filter bad controls

Directed Acyclical Graph



- Not all variables relevant for estimation of $\hat{\mathbb{P}}$ and $\hat{\mathbb{E}}$
- Structure may change with t

Construct Confidence Bands

What do we have

- Can show that under regularity conditions

$$\sqrt{n}(\hat{\tau} - \tau) \xrightarrow{d} \mathcal{GP}(0, c)$$

- Can consistently estimate covariance kernel c

Liebl & Reimherr (2022)

- Need:
  - Asymptotically Gaussian estimator
  - Consistent estimate of its covariance kernel
- Get:
  - Simultaneous and fair confidence bands

Simultaneity & Fairness

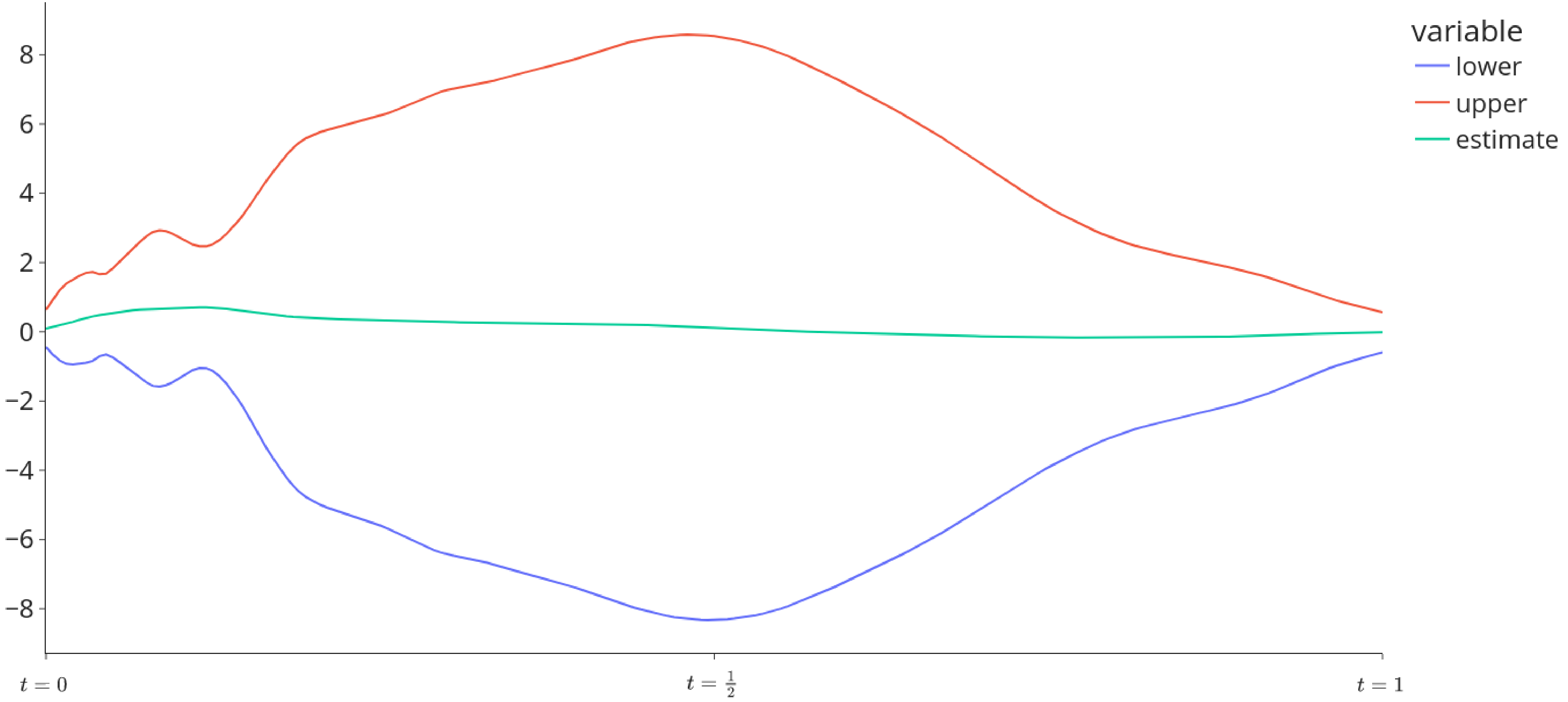
- This slide needs to be updated!!
- Simultaneity:

$$\mathbb{P}[\forall t \in [0,1] : \tau(t) \in \mathrm{SCB}(t)] \underset{n\to\infty}{\longrightarrow} 1-\alpha$$

- Fairness:

$$\lim_{n\to\infty} \mathbb{P}_{H_0}[\text{reject } H_0 \text{ over } [a_{j-1}, a_j]] \leq \alpha(a_j - a_{j-1})$$

Results



Final Remarks

Working on:

- Improving minimal assumption set
- Robustness checks for estimators $\hat{\mathbb{E}}[Y_i(w)|X_i]$ and $\hat{\mathbb{P}}[W_i = w|X_i]$
- Publishing a corresponding Python and R package
    - *(Code is online, but not in a package format)*

Contact

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