ECE 340 Homework VIII Spring 2003

Due: Monday, March 10, 2003

1. *An abrupt Si p-n junction is formed by alloying a uniformly doped n-type silicon bar where $N_d = 8x10^{16}/cm^3$ in the beginning. During the alloying process, a uniform counter doping of acceptors of $N_a = 1.4x10^{17}/cm^3$ is introduced in the region for x<0. Basically, x<0 is the p-side and x>0 is the n-side.*
   - *(a) Calculate the Fermi level positions at 300 K in the p and n regions.*
   - *(b) Draw an equilibrium band diagram for the junction and determine the contact potential V_o from the diagram.*
   - *(c) Compare the results of part (b) with V_o as calculated from Eq. (5-8).*
   - *(d) Using Eq. (5-8), calculate and plot V_O versus temperature ranging from 250 K to 500 K.*

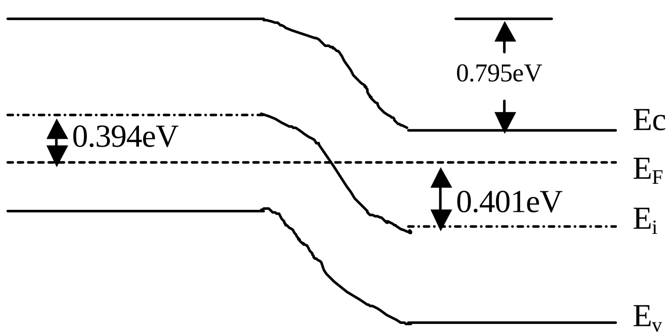
Solutions:

(a), At x<0, $p_p = N_a - N_d = 6\times10^{16} cm^{-3}$

$$\text{In p region, } E_{ip} - E_{Fp} = \frac{kT}{q}\ln\frac{p}{n_i} = 0.0259\times\ln\frac{6\times10^{16}}{1.5\times10^{10}} = 0.394eV$$

$$\text{In n region, } E_{Fn} - E_{in} = \frac{kT}{q}\ln\frac{n}{n_i} = 0.0259\times\ln\frac{8\times10^{16}}{1.5\times10^{10}} = 0.401eV$$

(b),



$V_0 = 0.795$ V

(c), Equations 5-8:

$$V_0 = 0.0259\ln\frac{N_a N_d}{n_i^2} = 0.0259\ln\frac{6\times10^{16}\times8\times10^{16}}{n_i^2} = 0.795V$$

Result is same as (b)

(d), Notice that the n_i is related to temperature through Eq. 3-26

Method 1:

Read n_i from Figure 3-17, plug into Eqn 5-8 to obtain the plot.

For example: at T=400K, $n_i = 8*10^{12}$ cm^{-3}.

$$V_0 = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2} = \frac{1.38\times10^{-23}\times400}{1.6\times10^{-19}}\ln\frac{6\times10^{16}\times8\times10^{16}}{(8\times10^{12})^2} = 0.626V$$

At least 5 points must be correctly read and calculated.

Method 2

$n_i(T) = 2\left(\frac{2p\,kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$, therefore, if we assume m_n^*, m_p^*, E_g is independent of T, we have:

$$\frac{n_i(T)}{n_i(300K)} = \left(\frac{T}{300}\right)^{3/2} e^{\left[-\frac{E_g}{2k}\left(\frac{1}{T}-\frac{1}{300}\right)\right]}, \qquad \text{i.e.,} \qquad n_i(T) = n_i(300K)\left(\frac{T}{300}\right)^{3/2} e^{\left[-\frac{E_g}{2k}\left(\frac{1}{T}-\frac{1}{300}\right)\right]}$$

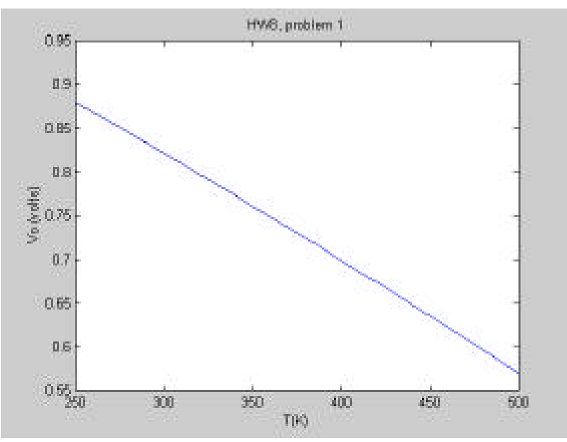
$$V_0 = \frac{kT}{q}\ln\frac{N_aN_d}{n_i^2} = \frac{kT}{q}\ln\frac{N_aN_d}{\left(n_i(300K)\left(\frac{T}{300}\right)^{3/2} e^{\left[-\frac{E_g}{2k}\left(\frac{1}{T}-\frac{1}{300}\right)\right]}\right)^2}$$

$$= \frac{kT}{q}\left[\ln\frac{N_aN_d}{n_i^2(300K)} - 3(\ln T - \ln 300) + \frac{E_g}{k}\left(\frac{1}{T}-\frac{1}{300}\right)\right]$$

$$= \frac{1.38\times10^{-23}\times T}{1.6\times10^{-19}}\left[30.69 - 3\ln T + 3\times\ln 300 + \frac{1.1\times1.6\times10^{-19}}{1.38\times10^{-23}}\left(\frac{1}{T}-\frac{1}{300}\right)\right]$$

$$= 8.625\times10^{-5}\times T\times\left(47.8 - 3\ln T + 1.275\times10^{4}\times\left(\frac{1}{T}-\frac{1}{300}\right)\right)(volts)$$

Plot:



2. Refer to problem 1, the silicon bar has a cross section with diameter 20 µm. Assume that the depletion approximation holds. (a) Calculate W, X_{no} and X_{po} at 300 K. (b) Determine the total positive ion charge in the depletion region. (c) Sketch to scale the charge density ρ(x), electrical field **E** (x), and electrostatic potential V(x) in the depletion region. Assume that the electrostatic potential is zero at x= 0. (d) Draw the energy band diagram for the device.

(a): $W = \left[\frac{2eV_0}{q}\left(\frac{1}{N_a}+\frac{1}{N_a}\right)\right]^{1/2} = \left[\frac{2(11.8\times8.85\times10^{-14})(0.795)}{1.6\times10^{-19}}\left(\frac{1}{6\times10^{16}}+\frac{1}{8\times10^{16}}\right)\right]^{1/2} = 0.174mm$

$$x_{n0} = \frac{W}{1 + N_d / N_a} = 0.0746mm$$

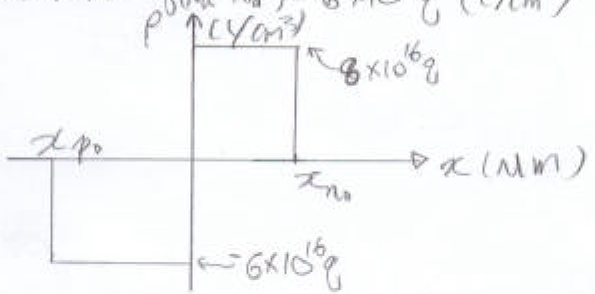
$$x_{p0} = \frac{W}{1 + N_a / N_d} = 0.0994mm$$

(b): $Q_+ = qAx_{n0}N_d = (1.6\times10^{-19})[p\times(10\times10^{-4})^2](7.46\times10^{-6})(8\times10^{16}) = 3.00\times10^{-13}C$

(c):

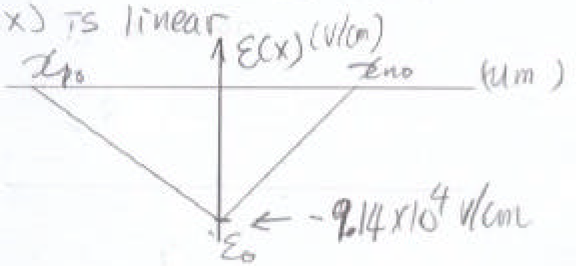
(c) $x>0$, $\rho(x) = q N_d = 8\times10^{16} q \; (C/cm^{-3})$

$x<0$, $\rho(x) = q(N_a^- - N_d) = -6\times10^{16} q \; (C/cm^{-3})$



$\mathcal{E}_0 = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{1.6\times10^{-19}}{11.8\times8.85\times10^{-14}}\times8\times10^{16}\times0.0746\times10^{-4} = -9.14\times10^{4}\ V/cm$

$\mathcal{E}(x)$ is linear



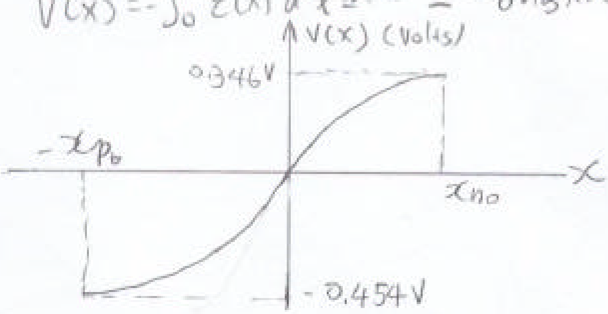
When $x<0$,

$V(x) = -\int_0^x \mathcal{E}(x)\,dx = -\int_0^x \frac{x+x_p}{x_{p0}}\mathcal{E}_0\,dx = \int_0^x -\frac{x}{x_{p0}}\mathcal{E}_0\,dx - \mathcal{E}_0 x$

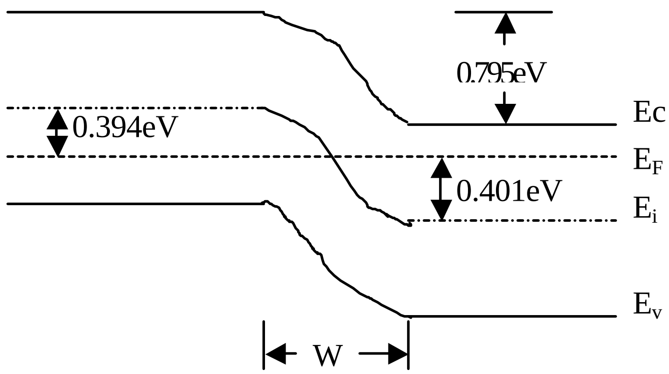
$= -\frac{\mathcal{E}_0}{2x_{p0}}x^2 - x\mathcal{E}_0 \simeq 4.6\times10^{9}x^2 + 9.14\times10^{4}x$ (x in cm)

when $x>0$

$V(x) = -\int_0^x \mathcal{E}(x)\,dx = \cdots = -6.13\times10^{9}x^2 + 9.14\times10^{4}x$



(d):



3. Refer to problem 1 again. In reality, the alloying process will introduce a much higher concentration of acceptor. Assume that the uniform counter doping is $N_a = 3 \times 10^{19}/cm^3$ instead. Determine and plot the contact potential V_o and depletion widths W, X_{no} and X_{po} versus temperature ranging from 250 K to 500 K.

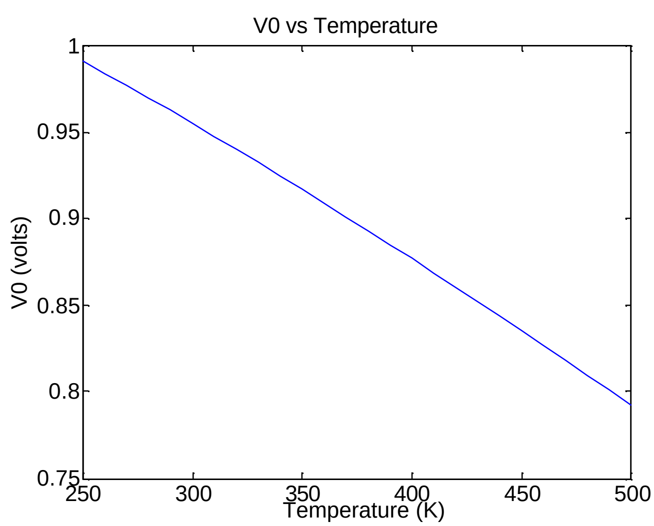
$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}, \qquad |E_0| = \frac{qN_d x_{n0}}{e} = \sqrt{\frac{2qV_0}{e}\left(\frac{N_a N_d}{N_a + N_d}\right)}, \quad W = \sqrt{\frac{2eV_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)},$$

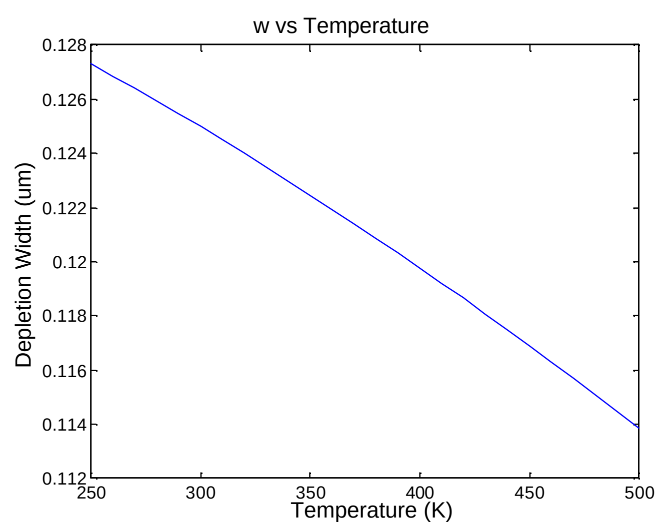
$$x_{n0} = \frac{W}{1 + N_d / N_a}, \qquad x_{p0} = \frac{W}{1 + N_a / N_d}$$

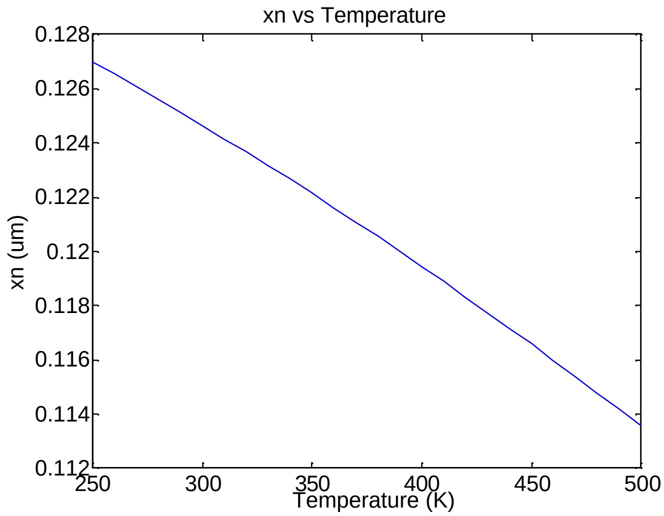
Notice that n_i is a function of temperature. You can either use equation 3-26 or figure 3-17
Both are ok. If you use figure 3-17, at least 5 points need to be correctly read. Below I use eqn 3-26.

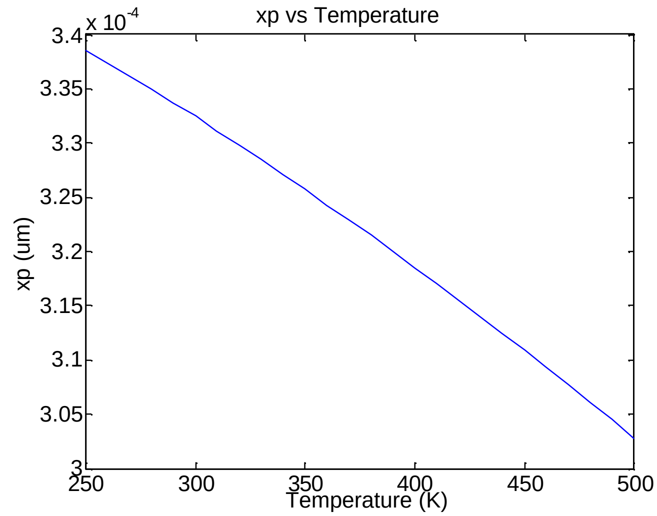
$$V_0 = \frac{kT}{q}\left[\ln \frac{N_a N_d}{n_i^2(300K)} - 3(\ln T - \ln 300) + \frac{E_g}{k}\left(\frac{1}{T} - \frac{1}{300}\right)\right]$$

Using a computer program can greatly speed up calculation.






xn vs Temperature
0.128
0.126
0.124
0.122
0.12
0.118
0.116
0.114
0.112
xn (um)
250
300
350
400
450
500
Temperature (K)


xp vs Temperature
x 10^-4
3.4
3.35
3.3
3.25
3.2
3.15
3.1
3.05
3
xp (um)
250
300
350
400
450
500
Temperature (K)