ECE 340 Homework III Spring 2003

Due: Friday, February 07, 2003

1. *In practice we assume that the intrinsic Fermi level, E_i, coincides with the center of the band gap. In reality it is not true. Derive an expression relating the intrinsic level E_i to the center of the band gap. Find the displacement of E_i from the center of the band gap for both silicon and germanium at room temperature. The effective mass values are $m_n^* = 0.55\ m_o$ for Ge and $1.1\ m_o$ for Si, $m_p^* = 0.37\ m_o$ for Ge and $0.56 m_o$ for Si where m_o is the free electron mass.*

   Solutions:

   Equation (3-15): $n_0 = N_c e^{-(E_c - E_F)/kT}$, for intrinsic material, $n_0 = n_i$, $E_F = E_i$, $n_i = N_c e^{-(E_c - E_i)/kT}$

   Equation (3-23): $n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$

   Therefore, $\sqrt{N_c N_v}\, e^{-E_g/2kT} = N_c e^{-(E_c - E_i)/kT}$, after simplification, we have:

   $$E_i = E_c - \frac{E_g}{2} + kT \ln \sqrt{\frac{N_v}{N_c}}$$

   The energy level at the center of band gap $E_{center} = E_c - \frac{E_g}{2}$

   The displacement $E_i - E_{center} = kT \ln \sqrt{\frac{N_v}{N_c}}$

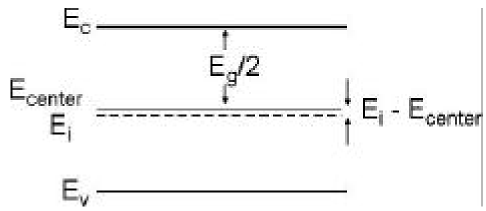
   Equation (3-16), (3-20), $N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$, $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$

   So we have $E_i - E_{center} = \frac{3}{4} kT \ln \frac{m_p^*}{m_n^*}$ (**)

   At T = 300K, $kT = 1.38 \times 10^{-23} \times 300 = 4.14 \times 10^{-21} J = 0.0259 eV$

   For silicon, $m_n^* = 1.1\ m_0$, $m_p^* = 0.56\ m_0$

   $$E_i - E_{center} = \frac{3}{4} kT \ln \frac{m_p^*}{m_n^*} = -0.013 eV$$

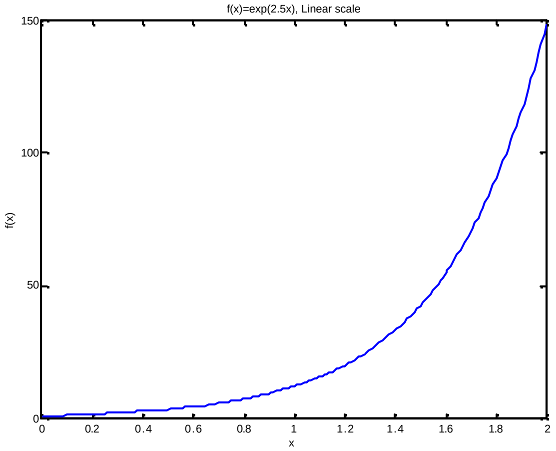
   

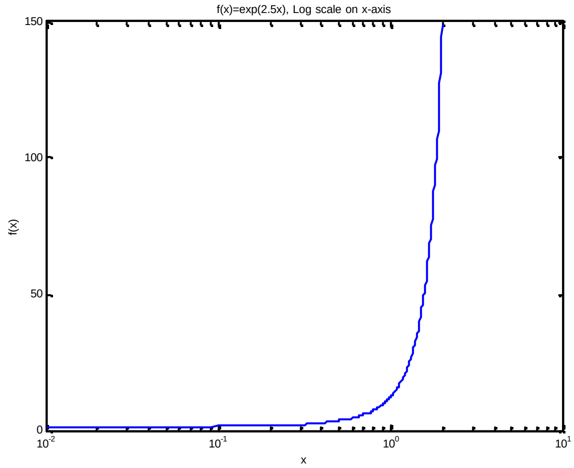
   For Ge, $m_n^* = 0.55\ m_0$, $m_p^* = 0.37\ m_0$

   $$E_i - E_{center} = \frac{3}{4} kT \ln \frac{m_p^*}{m_n^*} = -0.0077 eV$$

2. *Math exercise: plot the function f(x)=exp(2.5x) from x=0.01 to 2.0 in two graphs. One is on a linear scale and the other on semi-logarithmic scale.*

Solution:





The graphs plotted here has log scale in x-axis. It's also correct if having log scale on y-axis (which is a straight line). But with log scale on both axis is not correct.

3. *A silicon sample is doped with 1.5x10^{14} Ga atoms/cm^3. (a) What are the electron and hole concentrations, respectively, at 450 K under equilibrium conditions? (b) Where is E_F positioned relative to E_i ? (c) Draw the energy band diagram for the material assuming that E_i coincides with the center of the band gap.*

   Refer to Fig. 3-17 to obtain the intrinsic carrier concentration at 450 K. Also, the band gap energy is reduced to 1.08 eV at 450 K.

Solutions:

(a), Doped with Ga→p-type.

Look up the Fig 3-17, at 450K, $n_i \approx 8\times10^{13}cm^{-3}$. Since $n_i \sim N_a$, can't assume $p = N_a$, or $p = N_a + n_i$

Charge neutrality equation: $N_d^+ + p = N_a^- + n$, $N_a = 1.5\times10^{14}cm^{-3}$, $N_d = 0$

At thermal equilibrium, $n = n_i^2 / p$,

We have: $p = N_a^- - N_d^+ + \dfrac{n_i^2}{p}$, or $p^2 - \left(N_a^- - N_d^+\right)p - n_i^2 = 0$, solving this quadratic equation gives:

$$p = \frac{1}{2}\left(N_a \pm \sqrt{N_a^2 + 4n_i^2}\right) = 1.85\times10^{14}\,cm^{-3},$$

$$n = \frac{n_i^2}{p} = 3.47\times10^{13}cm^{-3}$$

(b), $E_F - E_i = kT\ln\dfrac{n_0}{n_i} = 0.0259\ln\dfrac{3.47\times10^{13}}{8\times10^{13}} = -0.022eV$

E_F lies 0.022eV below E_i,