

You must provide your name on the answer sheet. In addition, you are asked to voluntarily "provide" your social security number in order to verify your identity and avoid confusion between "two students" with the same name. Your social security number is sought for identification purposes pursuant to Public Law 93-579 and is being used as part of a system of "student" records that has been in effect prior to 1970. If you do not want to provide your social security number, you must write on the answer form an alternative identifier. [An alternative choice "would be" the University 'Net-ID' (network identification name) or the 9-digit blue number on each person's I-card].

ECE 340 – Spring 2003

Name Sol'n

Net ID# \_\_\_\_\_

Section \_\_\_\_\_

**ECE 340 - EXAM No. 2**  
 Thursday April 10, 2003  
 7:00-9:00 p.m.

**ROOM ASSIGNMENTS**

**Room 151 Everitt Lab**

G. Timp, Section A

C. Liu, Section E

**Room 100 Materials Science and Engineering Building**

K. Kim, Section C

K.C. Hsieh, Section X

J. Tucker, Section G

**Conflict Exam #2 – Thursday, April 10, 2003, 4:00-6:00 p.m.**  
**Room 365 Everitt Lab.**

NOTE: This is a closed book and closed notes exam. Unless stated otherwise, do your work on the page of the problem and if necessary on the preceding blank page. It is mandatory that proper units be included explicitly along with the numerical value for each term in a quantitative calculation, showing how the units of the final answers are derived. Failure to do this will result in no credit. Circle your answer. Be neat!

For each problem, you must show complete work and indicate your reasoning. No credit will be given if you do not show the complete work and describe your procedure, even if the answer is correct. Write your name, ID#, and section on this page and sign below.

	1-4	5	6	7	8	9
	20 pts.	20 pts.	20 pts.	12 pts.	8 pts.	20 pts.
a	---	5	4	4	4	5
b	---	5	6	4	4	4
c	---	5	5	4	---	3
d	---	5	5	---	---	3
e	---	---	---	---	---	5
T						

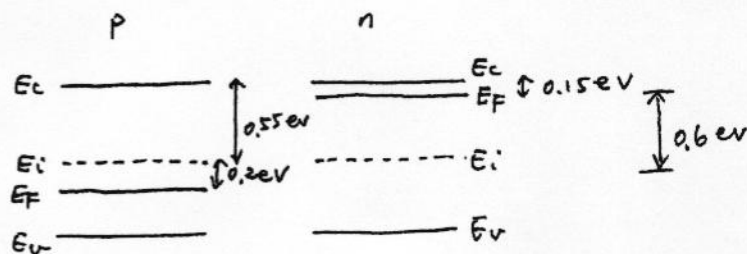
Signature: \_\_\_\_\_

Your Exam Score: \_\_\_\_\_

You must give a reason along with your answer to receive ANY credit.

1. (5 points) A silicon p-n junction diode at 300K is made of a p type material with  $E_F - E_i = -0.2$  eV and an n type material with  $E_F - E_c = -0.15$  eV. The bandgap of Si is 1.1 eV. What is the value of the contact potential of this p-n junction under thermal equilibrium? Assume that the intrinsic Fermi level coincides with the mid-gap of the energy band.

- (a) 0.35 eV  
 (b) 0.60 eV  
 (c) 0.05 eV  
 (d) 0.75 eV  
 (e) None of the above



The difference between two Fermi levels  
 is  $0.6 \text{ eV} = 0.4 \text{ eV} + 0.2 \text{ eV}$

2. A p-n junction under thermal equilibrium has a contact potential of 0.8 V. The p and n side have identical doping concentrations. The width of the space charge region on the n side is  $1 \mu\text{m}$ . What is the maximum magnitude of electric field in the p-n junction?

- (a) 0 V/cm  
 (b) 8000 V/cm  
 (c) 16000 V/cm  
 (d) Not enough information given.

Equal doping in p and n sides  
 indicate the depletion distances  
 are the same too.

Therefore the total depletion width  
 is  $2 \mu\text{m}$ .

$$V_0 = \frac{1}{2} \epsilon_m \cdot W$$

$$0.8 = \frac{1}{2} \cdot \epsilon_m \cdot (2 \times 10^{-4})$$

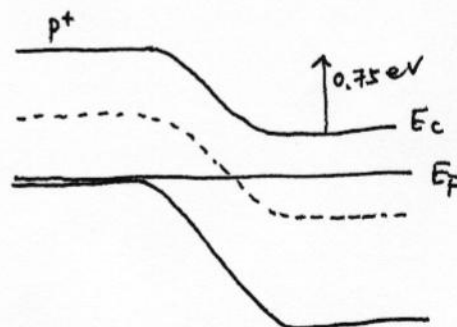
$$\Rightarrow \epsilon_m = 8000 \frac{\text{V}}{\text{cm}}$$

3. A silicon p-n junction under equilibrium has  $x_{n0} = 10 x_{p0}$ , where  $x_{n0}$  and  $x_{p0}$  refer to the widths of the depletion region extend into the n- and p-side, respectively. The dopant concentrations in the p and n side are  $N_a$  and  $N_d$ , respectively. Which one of the following statements is true for this diode under thermal equilibrium?

- (a)  $N_d = 10 N_a$  *false (  $N_a = 10 N_d$  )*  
 (b) The space charge region on the p side has more charge than the space charge region on the n side has. *false ( They are equal )*  
 (c) The minority injection current on the p side is greater than the minority injection current on the n side under forward biasing. *false ( the other way around )*  
 (d) The built-in electric field in the space charge region points from the p side to the n side.  
 (e) None of the above. *false ( just opposite )*

4. We have a  $p^+-n$  diode of an unknown material at an unknown temperature. If the Fermi level of the n side lies half way between the conduction band and the intrinsic Fermi level, and the contact potential is 0.75 eV. What is the best approximation of the band gap of the material?

- (a) 2 eV  
 (b) 0.75 eV  
 (c) 1 eV  
 (d) 1.5 eV  
 (e) None of the above.



$$E_F^n - E_F^p = 0.75 \text{ eV} = qV_0$$

$$E_F^i - E_F^p \approx \frac{1}{2} E_g \Rightarrow qV_0 = \frac{3}{4} E_g$$

$$E_F^n - E_F^i \approx \frac{1}{4} E_g \Rightarrow E_g = 1 \text{ eV}$$

5. A Si p-n junction with the following properties is under a forward bias of 0.5V at 300 K.

On the p-side:

$$N_A = 6 \times 10^{18} \text{ cm}^{-3}, D_n = 36 \text{ cm}^2/\text{s}, \tau_n = 10^{-8} \text{ s}$$

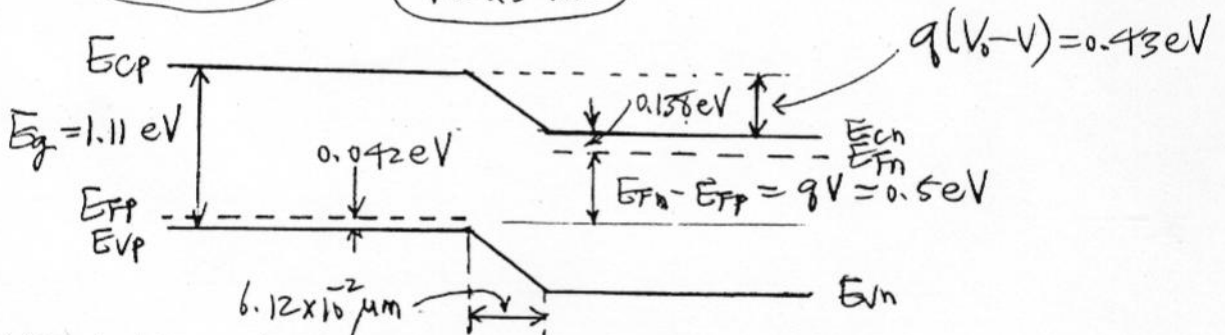
On the n-side:

$$N_D = 1.5 \times 10^{17} \text{ cm}^{-3}, D_p = 16 \text{ cm}^2/\text{s}, \tau_p = 10^{-8} \text{ s}$$

(a) (5 points) Draw a band diagram pertaining to the p-n junction above. Make sure to fill in all the numerical details to make the diagram complete.

$$\left\{ \begin{array}{l} N_A = n_i e^{(E_i - E_F)_p / kT}, \quad (E_i - E_F)_p = kT \ln \left( \frac{N_A}{n_i} \right) = 0.513 \text{ eV} \\ N_D = n_i e^{(E_F - E_i)_n / kT}, \quad (E_F - E_i)_n = kT \ln \left( \frac{N_D}{n_i} \right) = 0.417 \text{ eV} \\ qV_0 = (E_i - E_F)_p + (E_F - E_i)_n = 0.930 \text{ eV} \\ W \approx \sqrt{\frac{2\epsilon(V_0 - V)}{qN_D}} = 6.12 \times 10^{-6} \text{ cm} = 6.12 \times 10^{-2} \mu\text{m} \end{array} \right.$$

$(N_A \gg N_D)$        $V = 0.5 V_0$



(b) (5 points) Determine the current density flowing through the junction.

$$N_A \gg N_D \Rightarrow p\text{-}n \text{ junction}$$

$$J \approx J_p(x_n = 0) = q \frac{D_p}{L_p} p_n (e^{qV/kT} - 1)$$

$$\begin{aligned} \left\{ \begin{array}{l} L_p = \sqrt{D_p \tau_p} = \sqrt{(16)(10^{-8})} = 4 \times 10^{-4} \text{ cm} \\ p_n = \frac{n_i^2}{N_D} = 1.5 \times 10^3 \text{ cm}^{-3} \end{array} \right. \\ = (1.6 \times 10^{-19}) \frac{(16)(1.5 \times 10^3)}{(4 \times 10^{-4})} (e^{0.5/0.0259} - 1) \\ = 2.32 \times 10^{-3} \text{ A/cm}^2 \end{aligned}$$

(c) (5 points) Determine the hole diffusion current density in the n-region at  $x_n = 2 L_p$ .

$$\begin{aligned}
 \boxed{J_p(x_n = 2 L_p)} &= -q D_p \frac{d}{dx_n} \delta p(x_n) \Big|_{x_n = 2 L_p}; \quad \delta p(x_n) = \delta p_n e^{-x_n/L_p} \\
 &= J_p(x_n = 0) e^{-x_n/L_p} \Big|_{x_n = 2 L_p} \\
 &= 2.32 \times 10^{-3} e^{-2} \\
 &= \boxed{-3.15 \times 10^{-4} \text{ A/cm}^2}
 \end{aligned}$$

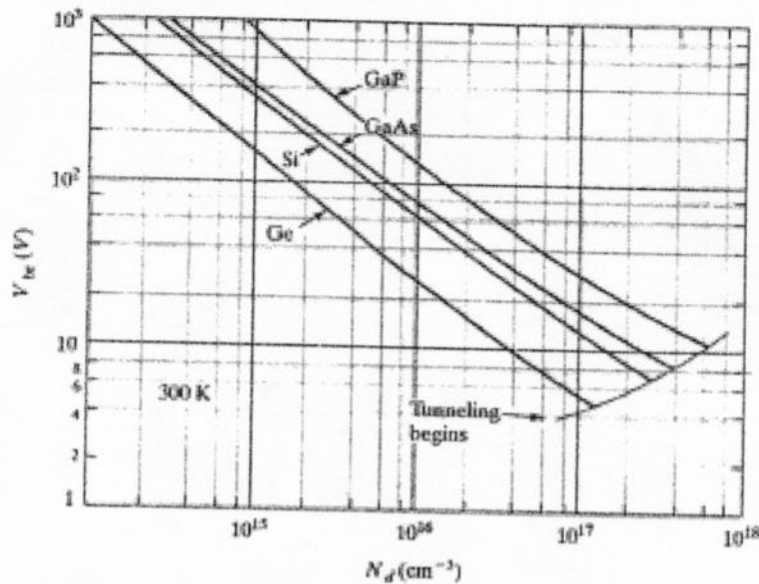
(d) (5 points) Determine the electric field in the n-region at  $x_n = 10 L_p$ .

$$\begin{aligned}
 J(x_n = 10 L_p) &\approx J_{n, \text{drift}}(x_n = 10 L_p) \\
 &= q n_n \mu_n E(x_n = 10 L_p)
 \end{aligned}$$

$$\begin{cases}
 J(x_n = 10 L_p) \approx J_p(x_n = 0) \\
 n_n = N_D \\
 \mu_n \text{ for } N_D = 1.5 \times 10^{17} \approx 650 \text{ cm}^2/\text{V}\cdot\text{s} \text{ from curve}
 \end{cases}$$

$$\begin{aligned}
 \Rightarrow \boxed{E(x_n = 10 L_p)} &= \frac{2.32 \times 10^{-3}}{(1.6 \times 10^{-19})(1.5 \times 10^{17})(650)} \\
 &= \boxed{1.49 \times 10^{-4} \text{ V/cm}}
 \end{aligned}$$

6. Given the relationships between the avalanche breakdown voltage and donor concentration in the n-side of  $p^+$ -n junctions for different materials, determine the breakdown electric field and other physical properties of the following diodes.

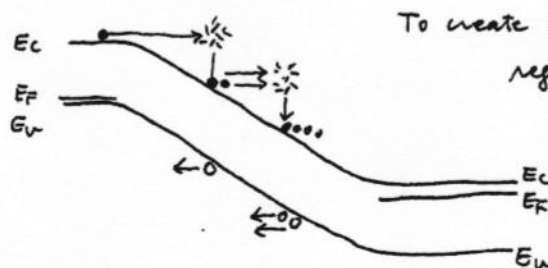


- (a) (4 points) Circle the **correct** statement(s) about the reverse breakdown of a  $p^+$ -n junction.

- (1) For a chosen semiconductor such as Si, the breakdown voltage decreases with the increasing donor concentration because it is easier to make ohmic contact to the n-side when the doping is higher. As a result, less energy is wasted for carriers to move across the metal/n-semiconductor contact leading to a lower breakdown voltage.
- (2) For a chosen semiconductor such as Si, the breakdown voltage decreases with the increasing donor concentration because the resistance decreases significantly in the n-side and current can increase easily leading to breakdown.
- (3) For a given donor concentration, a GaAs or GaP  $p^+$ -n junction has a larger breakdown voltage than those made of Si or Ge because the former is of zinc-blende structure while the latter is of diamond structure.
- (4) For a given donor concentration, a GaAs or GaP  $p^+$ -n junction has a larger breakdown voltage than those made of Si or Ge because the former is a III-V compound while the latter is made of only group IV element. The slightly ionic nature in the zinc-blende bonding is stronger than the purely covalent bonding in the diamond structure.

(5) All of the above.

(6) None of the above.

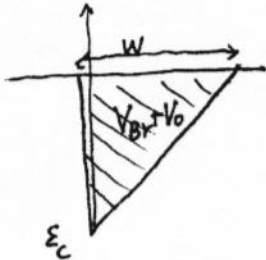


To create EHP, an electron in the depletion region accelerates and impinges on matrix atoms. However, it takes an energy of  $E_g$  to create one EHP. The higher the bandgap energy is, the ~~more~~ larger breakdown field and hence breakdown voltage it needs.



- (b) (6 points) For a  $p^+n$  silicon diode, the donor concentration and the physical dimension of the n-side are  $2 \times 10^{16}/\text{cm}^3$  and  $250 \mu\text{m}$ , respectively. Estimate the critical field required to induce avalanche.

From the given graph, one finds that the breakdown voltage for a  $p^+n$  junction is about 39 volts for  $N_d = 2 \times 10^{16}/\text{cm}^3$ .



$$V_0 + V_{br} = \frac{1}{2} E_c \cdot W$$

$$(V_0 + V_{br})^2 = \frac{1}{4} E_c^2 \cdot W^2 = \frac{1}{4} E_c^2 \cdot \left[ \frac{2\epsilon (V_0 + V_{br})}{q} \cdot \frac{1}{N_d} \right]$$

$$\Rightarrow E_c^2 = \left[ \frac{2qN_d}{\epsilon} (V_0 + V_{br}) \right]$$

$$\therefore E_c \approx \left( \frac{2qN_d}{\epsilon} V_{br} \right)^{\frac{1}{2}} \quad V_0 \ll V_{br}$$

$$= \left( \frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^{16}}{11.8 \times 8.854 \times 10^{-14}} \cdot 39 \right)^{\frac{1}{2}}$$

$$= 4.89 \times 10^5 \left( \frac{\text{V}}{\text{cm}} \right)$$

Self check:  $W \approx \frac{2V_{br}}{E_c}$

$$= 1.6 \times 10^{-4} \text{ cm}$$

$$= 1.6 \mu\text{m} \ll 250 \mu\text{m} \text{ so avalanche takes place.}$$

- (c) (5 points) Assume that the donor concentration is reduced to  $1 \times 10^{15}/\text{cm}^3$  and the dimension of the n-region decreases to  $25 \mu\text{m}$ . The contact potential of the junction is 0.84 volts. Does the junction break down by avalanche or punchthrough? Show your work and reasoning.

From the graph, one can find the breakdown voltage to be 380 volts for  $N_d = 10^{15}/\text{cm}^3$ .

$$\therefore W_{br} = \left[ \frac{2\epsilon (V_0 + V_{br})}{q} \cdot \frac{1}{N_d} \right]^{\frac{1}{2}}$$

$$= \left[ \frac{2 \times 11.8 \times 8.854 \times 10^{-14} \cdot (0.84 + 380)}{1.6 \times 10^{-19} \cdot 1 \times 10^{15}} \right]^{\frac{1}{2}}$$

$$= 2.23 \times 10^{-3} \text{ cm}$$

$$= 22.3 \mu\text{m}.$$

- ①  $W_{br} < 25 \mu\text{m}$  so avalanche does take place before the n-region is completely consumed (depleted).

- (d) (5 points) Referring to part (c), estimate the depletion capacitance per  $\text{cm}^2$  for the junction when it is reverse-biased at  $-10\text{V}$ .

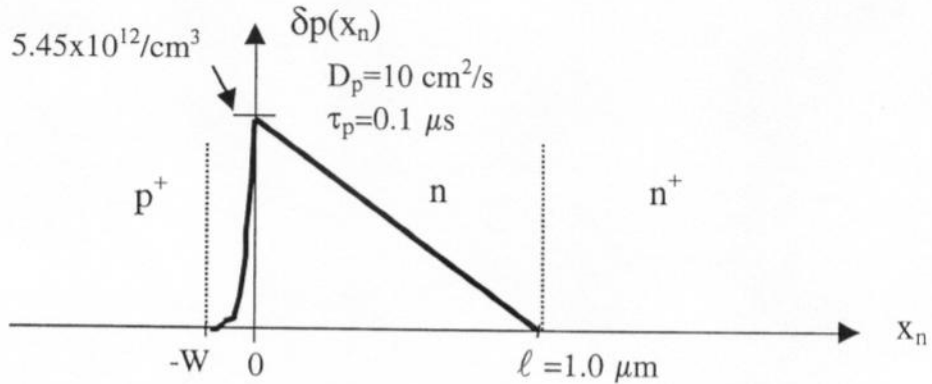
at  $V_r = -10 \text{ volts}$

$$\begin{aligned}
 W &= \left[ \frac{2\epsilon(V_0 - V_r)}{q} \cdot \frac{1}{N_d} \right]^{\frac{1}{2}} \\
 &= \left[ \frac{2 \times 11.8 \times 8.854 \times 10^{-14} \times (10.8)}{1.6 \times 10^{-19}} \cdot \frac{1}{1 \times 10^{15}} \right]^{\frac{1}{2}} \\
 &= 3.76 \times 10^{-4} (\text{cm})
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{C_j}{A} &= \frac{\epsilon}{W} = \frac{8.854 \times 10^{-14} \times 11.8}{3.76 \times 10^{-4}} \\
 &= 2.78 \times 10^{-9} \text{ F/cm}^2 \\
 &= 2.78 \text{ nF/cm}^2
 \end{aligned}$$



7. The excess minority hole density in a  $p^+-n-n^+$  narrow-base diode is sketched below, where  $x_n=0$  is the starting point of the quasi-neutral region.



- (a) (4 points) Estimate the diode current per unit area ( $\text{A}/\text{cm}^2$ ).

$$\begin{aligned}
 J &= -q D_p \left. \frac{d\delta p(x_n)}{dx} \right|_{x_n=0} \\
 &\approx q D_p \frac{\Delta p_n}{\ell} \quad (\text{straight-line approximation}) \\
 &= 1.6 \times 10^{-19} \text{ C} \times 10 \frac{\text{cm}^2}{\text{sec}} \times \frac{5.45 \times 10^{12} \text{ cm}^{-3}}{10^{-4} \text{ cm}} \\
 &= 8.7 \times 10^{-2} \frac{\text{A}}{\text{cm}^2}
 \end{aligned}$$

- (b) (4 points) Estimate the total number of excess minority holes per unit area ( $\text{cm}^2$ ) inside the n-region.

$$\int_0^l \delta p(x_n) dx_n \approx \frac{1}{2} \Delta p_n \cdot l$$

$$= \frac{1}{2} \times 5.45 \times 10^{12} \text{ cm}^{-3} \times 10^{-4} \text{ cm}$$

$$= 2.7 \times 10^8 \text{ cm}^{-2}$$

$$\therefore \frac{Q_p}{q} = 2.7 \times 10^8 \text{ holes / cm}^2$$

- (c) (4 points) Where do most of the holes recombine? Circle your answer and state your reasoning.

(1) in the n-region,  $0 < x_n < l$

(2) at  $x_n = l$  in the  $n^+$ -region.

(3) several diffusion lengths inside the  $n^+$ -region.

(4) inside the depletion region, W.

(5) None of the above.

The recombination current

$$I_{\text{rec}} = \frac{q Q_p}{\tau_p}$$

$$= \frac{2.7 \times 10^8 \times 1.6 \times 10^{-19}}{0.1 \times 10^{-6}}$$

$$= 4.32 \times 10^{-4} \frac{\text{Amp}}{\text{cm}^2}$$

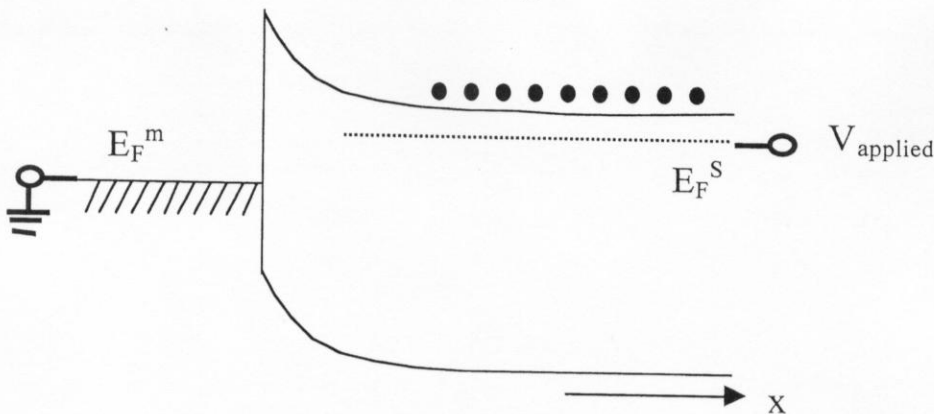
Recombination Current ~~at~~ at  $n^+$ -region

$$= 8.7 \times 10^{-2} - 4.32 \times 10^{-4}$$

$$\approx 8.66 \times 10^{-2} \frac{\text{A}}{\text{cm}^2}$$

$\therefore$  most holes recombine at  $x_n = l$

8. The band diagram of a forward-biased Schottky barrier diode is sketched below, and a current of  $I = 2.0 \text{ mA}$  flows with an unknown bias of  $V_{\text{applied}}$ .



- (a) (4 points) If the voltage applied to the semiconductor is changed to  $V_{\text{applied}} + 0.06 \text{ V}$ , what will be the new current (mA)?

positive voltage at the semiconductor contact reduces the forward bias

$$\begin{aligned} \therefore I_{\text{new}} &= I_{\text{old}} \times e^{-q\Delta V/kT} \\ &= I_{\text{old}} \times e^{-0.06/0.0259} = 2 \times 9.86 \times 10^{-2} \text{ (mA)} \\ &= 0.2 \text{ (mA)} \end{aligned}$$

- (b) (4 points) If the metal work-function was reduced by  $0.12 \text{ eV}$ , what would be the current with  $V_{\text{applied}}$  as shown?

Reducing the metal work function reduces  $q\phi_{B,n}$  ( $\because \phi_{B,n} = \phi_m - \chi$ )

and increases  $I_0$

$$I = I_0 (e^{qV/kT} - 1) \quad \text{and} \quad I_0 \propto e^{-\phi_{B,n}/kT}$$

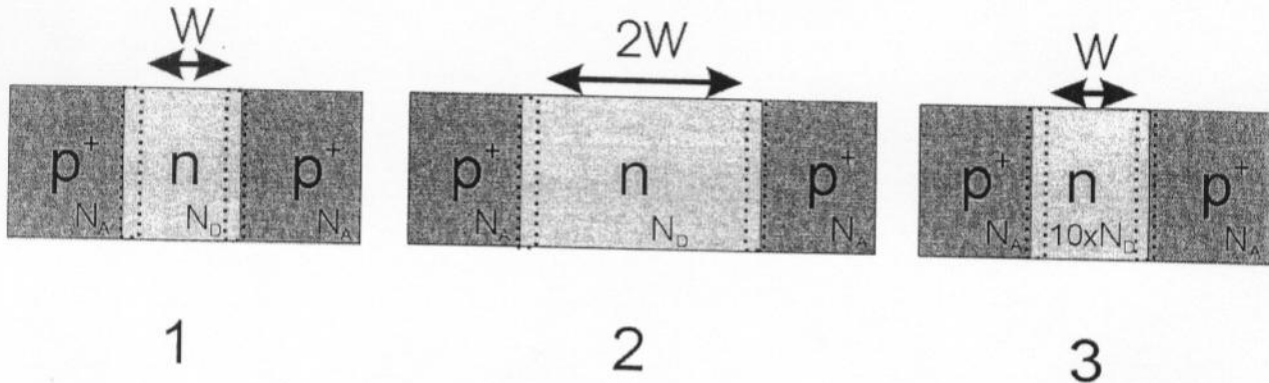
$$\begin{aligned} \therefore I_{\text{new}} &= I_{\text{old}} \times e^{q\phi_{B,n}/kT} \\ &= 2 \text{ mA} \times e^{0.12/0.0259} \\ &= 205.6 \text{ mA} \end{aligned}$$

9. Assume that the silicon  $p^+ - n - p^+$  transistors below are identical EXCEPT:

(i) the base width of transistor 2 is twice that of transistor 1 and 3;

AND

(ii) the doping in the base of transistor 3 is 10x that in transistor 1 and 2.



List the transistors in order from the largest to smallest value of the parameter listed below AND give a concise mathematical reason for your answer (or you will received NO CREDIT). Assume that all three transistors are biased in the normal active mode with identical voltages in the emitter-base junctions and with identical voltages in the base-collector junctions. Also assume that the base widths are much smaller than the minority carrier diffusion lengths.

(a) (4 points)  $\gamma$ , emitter injection efficiency;

$$1 > 2 > 3 \quad \text{because} \quad \gamma = \frac{I_{Ep}}{I_{En} + I_{Ep}} \cong \left[ 1 + \frac{W n_n \mu_n^p}{L_n p_p \mu_p^n} \right]^{-1}$$

$1 > 2$  because the base width of 2 is larger;

$1 > 3$  because the base doping  $n_n$  is larger;

$2 > 3$  because the base doping of 3 is 10x larger, while the base width of 2 is only 2x.

(b) (4 points)  $B$ , base transport factor;

$$1 > 2 > 3 \quad \text{because} \quad B = \frac{I_C}{I_{Ep}} \cong 1 - \frac{1}{2} \left( \frac{W}{L_p} \right)^2$$

$$\text{where } L_p = \sqrt{D_p \tau_p} = \sqrt{\mu_p (N_D) kT \tau_p} \quad \text{and} \quad \tau_p \propto \frac{1}{\alpha_r n_n} = \frac{1}{\alpha_r N_D}$$

1 > 2 because the base width of 2 is larger, so more recombination;

1 > 3 because the base doping of 3 is larger therefore more recombination, i.e.  $\tau_p$  is smaller AND  $D_p$  is smaller since  $\mu_p$  is smaller so  $L_p$  is smaller;

2 > 3 because the base doping of 3 is 10x larger while the base width of 2 is only 2x. Therefore,  $L_p^2 \sim \tau_p$  is reduced by more than a factor of 10.

(c) (4 points) the recombination component of the base current;

$$\begin{aligned} 2 > 1 \geq 3 \quad \text{because} \quad I_r &= \frac{Q_p}{\tau_p} = \frac{1}{2} W \times q A p_{n0} (e^{qV_{EB}/kT} - 1) \times \frac{1}{\tau_p} \\ &\propto \frac{1}{2} W \times q A \frac{n_i^2}{n_n} (e^{qV_{EB}/kT} - 1) \times \alpha_r n_n \\ &\propto \frac{1}{2} W \times q A n_i^2 (e^{qV_{EB}/kT} - 1) \times \alpha_r \end{aligned}$$

2 > 1 because the base width of 2 is larger so more recombination;

1  $\geq$  3 because the lifetime in 3 is reduced according to  $\tau_p \sim 1/\alpha N_D$  which roughly cancels out the reduction in the minority carrier concentration in equilibrium  $p_{n0} = n_i^2/N_D$ .

(d) (5 points)  $\beta = I_C/I_B$ , the (common emitter) current gain;

$$1 > 2 > 3 \quad \text{because} \quad \beta = \frac{I_C}{I_B} \cong 2 \left( \frac{L_p}{W} \right)^2$$

$$\text{where } L_p = \sqrt{D_p \tau_p} = \sqrt{\mu_p kT \tau_p} \quad \text{and} \quad \tau_p \propto \frac{1}{\alpha_r n_n}$$

1 > 2 because the base width of 2 is larger;

1 > 3 because the base doping of 3 is larger therefore more recombination, i.e.  $\tau_p$  is smaller AND  $D_p$  is smaller since  $\mu_p$  is smaller so  $L_p$  is smaller;

2 > 3 because the base doping of 3 is 10x larger while the base width of 2 is only 2x. Therefore,  $L_p^2 \sim \tau_p$  is reduced by more than a factor of 10 in transistor 3.

(e) (3 points)  $\tau_t$ , the base transit time.

$$2 > 3 > 1 \quad \text{because} \quad \tau_t \cong \frac{1}{2} \frac{W^2}{D_p} \propto \frac{1}{2} \frac{W^2}{\mu_p}$$

2 > 1 because the base width is larger;

3 > 1 because the base doping is larger so  $\mu_p$  is smaller;

2 > 3 because the base doping in 3 is 10x larger while the base width of 2 is only 2x. The mobility drop in 3 is <50% (see graph.)