ECE 340 | Homework XI | Spring 2003

Due: Friday, April 04, 2003

1. *For a p^+-n silicon junction, $N_a=10^{17}/cm^3$ in the p-side and $N_d=10^{15}/cm^3$ in the n-side. Determine the depletion capacitance per unit area of cm^2 at –4 V.*

Solutions:

$$V_0 = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2} = 0.0259\ln\frac{10^{17}\times10^{15}}{(1.5\times10^{10})^2} = 0.695V$$

$$W = \sqrt{\frac{2e(V_0+V_r)}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)} \xrightarrow{p^+n} W = \sqrt{\frac{2e(V_0+V_r)}{qN_d}}$$

$$\frac{C_j}{A} = \frac{e}{W} = \sqrt{\frac{qeN_d}{2(V_0+V_r)}} = \sqrt{\frac{1.6\times10^{-19}\times11.8\times8.85\times10^{-14}\times10^{15}}{2\times(0.695+4)}} = 4.22\times10^{-9}F/cm^2$$

2. A Schottky barrier is formed between a metal having a work function of 4.7 eV and n-type Si (electron affinity = 4 eV). The donor doping in the Si is $2\text{x}10^{17}/\text{cm}^3$. (a) Draw the equilibrium band diagram showing numerical values for qV_o and schottky barrier, $q\phi_n$. (b) Draw the band diagram with 0.5 V forward bias. Repeat for 2V reverse bias. For all three diagrams, determine and show the proper widths of the depletion region.

Solutions:

(a), At equilibrium:

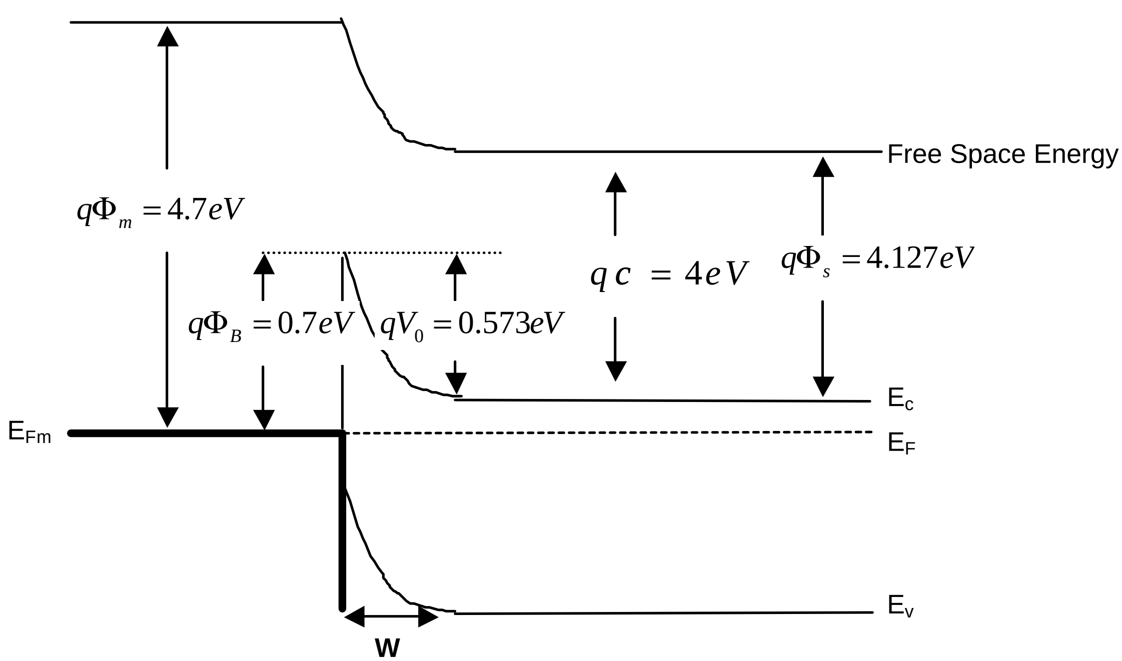
$$N_d = 2\times10^{17}cm^{-3},\ E_{Fs}-E_i = \frac{kT}{q}\ln\frac{N_d}{n_i} = 0.423eV$$

$$q\Phi_s = q\left[c + E_g/2 - \left(E_{Fs}-E_i\right)\right] = q(4+0.55-0.423) = 4.127eV$$

$$q\Phi_B = q(\Phi_m - c) = q(4.7-4) = 0.7eV$$

$$V_0 = \Phi_m - \Phi_s = 4.7 - 4.127 = 0.573V$$

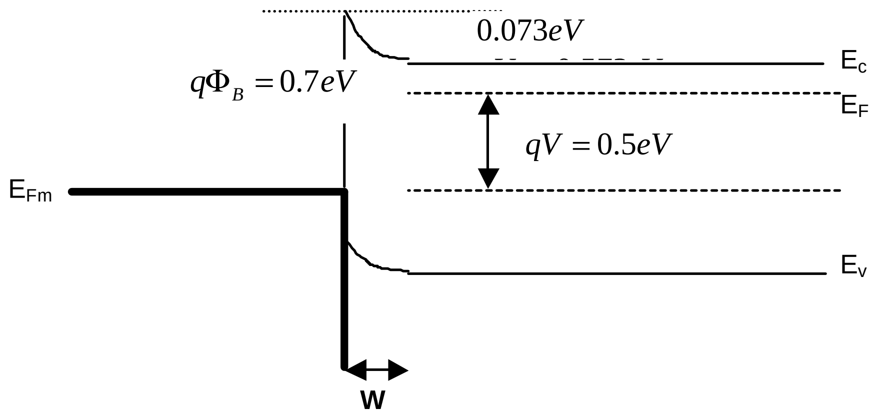
$$W = \sqrt{\frac{2eV_0}{qN_d}} = 6.11\times10^{-6}cm$$



(b), At forward bias:

$$V = V_0 - V_f = 0.573 - 0.5 = 0.073V$$

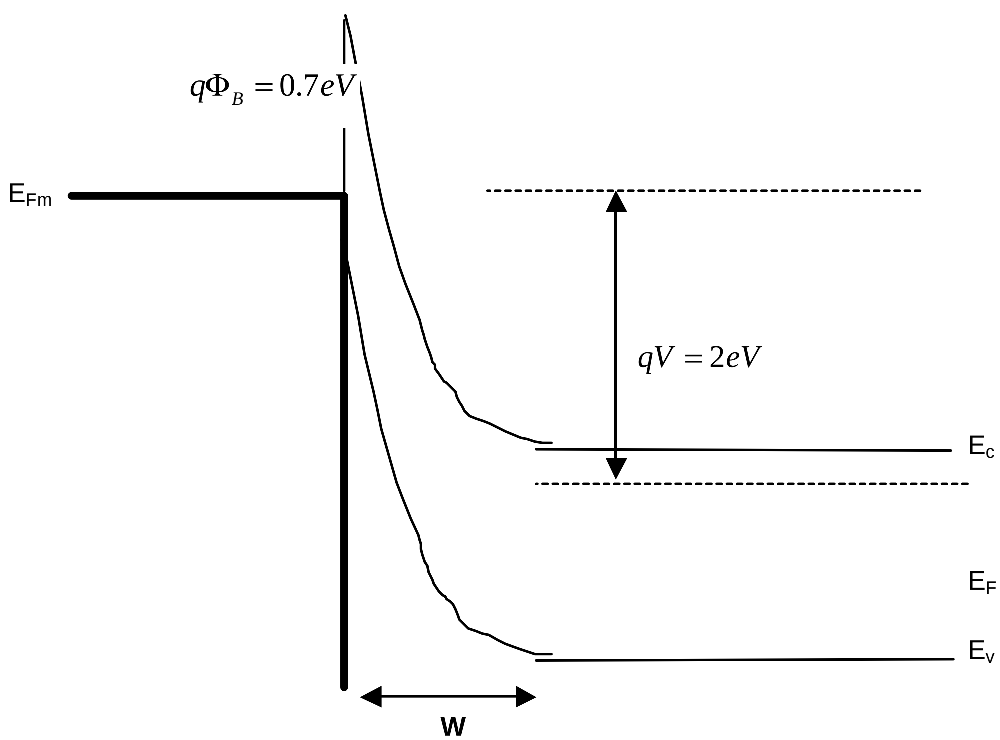
$$W = \sqrt{\frac{2e(V_0 - V_f)}{qN_d}} = 2.14\times 10^{-6} cm$$



At reverese bias:

$$V = V_0 + V_r = 0.573 + 2 = 2.573V$$

$$W = \sqrt{\frac{2e(V_0 + V_r)}{qN_d}} = 1.30\times 10^{-5} cm$$



3. Assume that a p^+ -n diode with a uniform cross section area, A, is built with an n region width ℓ smaller than a hole diffusion length (ℓ < L_p). This is the so-called narrow base diode. Since for this case holes are injected into a short n region under forward bias, we cannot use the assumption $\delta p(x_n = ¥) = 0$ in Eq. 4-35. Instead, we must use as a boundary condition the fact that $\delta p = 0$ at $x_n = \ell$.

(a) Solve the diffusion equation to obtain

$$\delta p(x_n) = \frac{\Delta p_n\left[e^{(l-x_n)/L_p} - e^{(x_n-l)/L_p}\right]}{e^{l/L_p} - e^{-l/L_p}}$$

(b) Show that the current in the diode is

$$I = \frac{qAD_p p_n}{L_p} ctnh(l/L_p)\left(e^{qV/kT} - 1\right)$$

*(c) If the n-region is relatively short compared to the diffusion length, the excess hole $\delta p(x_n)$ can be approximated as a straight line, i.e. it varies linearly from Δp_n at x_n=0 to zero at the ohmic contact ($x_n = \ell$). Find the steady-state **total** excess charges Q_p in the n-region and determine the percentage of error comparing the **total** excess holes in the n-region obtained from par (a) with that from the straight-line approximation for ℓ/L_p = 0.05, 0.1, 0.5, 1 and 5.*

(d) Calculate the current due to recombination in the n region.

Solutions:

(a), Start from diffusion equation (Eq. 4-34b): $\dfrac{d^2\delta p(x_n)}{dx_n^2} = \dfrac{\delta p(x_n)}{L_p^2}$,

General solution has the form $\delta p = C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}$, apply boundary condition:

$\delta p(x_n = 0) = \Delta p_n$, $\delta p(x_n = l) = 0$, we have:

$C_1 + C_2 = \Delta p_n$, $C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p} = 0$, solve for C_1 and C_2,

$C_1 = \dfrac{\Delta p_n e^{-l/L_p}}{e^{l/L_p} - e^{-l/L_p}}$, $C_2 = \dfrac{\Delta p_n e^{l/L_p}}{e^{l/L_p} - e^{-l/L_p}}$, substitute into the general solution, we have:

$$\delta p(x_n) = \frac{-\Delta p_n e^{-l/L_p} e^{x_n/L_p} + \Delta p_n e^{l/L_p} e^{-x_n/L_p}}{e^{l/L_p} - e^{-l/L_p}} = \frac{\Delta p_n\left[e^{(l-x_n)/L_p} - e^{(x_n-l)/L_p}\right]}{e^{l/L_p} - e^{-l/L_p}}$$

(b), Since p+n diode, diffusion current is mostly hole current.

Since $\Delta p_n = p_n e^{qV/kT}$, $ctnh(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

$$I = -qAD_p \left.\frac{d\delta_p}{dx_n}\right|_{x_n=0} = \frac{qD_p A\Delta p_n\left[e^{l/L_p} + e^{-l/L_p}\right]}{L_p\left(e^{l/L_p} - e^{-l/L_p}\right)} = \frac{qAD_p p_n}{L_p} ctnh(l/L_p)\left(e^{qV/kT} - 1\right)$$

(c),

$$Q_p = qA\int_0^1 d\,p(x)dx = qA\Delta p_n \int_0^1 \frac{\left[e^{(l-x_n)/L_p} - e^{(x_n-l)/L_p}\right]}{e^{l/L_p} - e^{-l/L_p}} dx = \frac{qA\Delta p_n}{e^{l/L_p} - e^{-l/L_p}} \int_0^1 \left[e^{(l-x_n)/L_p} - e^{(x_n-l)/L_p}\right] dx$$

$$= \frac{qA\Delta p_n L_p}{e^{l/L_p} - e^{-l/L_p}} \left[-e^{(l-x_n)/L_p} - e^{(x_n-l)/L_p}\right]_0^l = qA\Delta p_n L_p \left[\frac{-2 + e^{l/L_p} + e^{-l/L_p}}{e^{l/L_p} - e^{-l/L_p}}\right]$$

If using straight=line approximation,

$$Q_p^{'} = \frac{1}{2} qAl\Delta p_n$$

$$Error = \frac{Q_p^{'} - Q_p}{Q_p} = \frac{Q_p^{'}}{Q_p} - 1$$

$$= \left(\frac{l}{2L_p} \frac{e^{l/L_p} - e^{-l/L_p}}{e^{l/L_p} + e^{-l/L_p} - 2} - 1\right) \times 100\%$$

So, we have the following table:

l/L_p	0.05	0.1	0.5	1	5
Error	0.02%	0.08%	2.1%	8.2%	153%

(d). If we view the n-region as a node in KCL, there are three currents flowing to the node: the diffusion current flowing into the node at x=0, and diffusion current flowing out of the node at x=l, and the compensating recombination current.

Method 1:

By KCL, the recombination current is the difference between the diffusion current at x=0 and x=l.

$$I_{Diff,x_n=0} = -qAD_p \left.\frac{dd_p}{dx_n}\right|_{x_n=0}$$

$$\frac{dd\,p}{dx} = d\left(\frac{\Delta p_n\left[e^{(l-x_n)/L_p} - e^{(x_n-l)/L_p}\right]}{e^{l/L_p} - e^{-l/L_p}}\right) / dx = -\frac{\Delta p_n\left[e^{(l-x_n)/L_p} + e^{(x_n-l)/L_p}\right]}{L_p\left(e^{l/L_p} - e^{-l/L_p}\right)}$$

$$I_{Diff,x_n=0} = -qAD_p \left.\frac{dd_p}{dx_n}\right|_{x_n=0} = \frac{qD_p A\Delta p_n\left[e^{l/L_p} + e^{-l/L_p}\right]}{L_p\left(e^{l/L_p} - e^{-l/L_p}\right)}$$

Similarly,

$$I_{x_n=l} = -qAD_p \left.\frac{dd_p}{dx_n}\right|_{x_n=0} = \frac{qAD_p\Delta p_n 2}{L_p\left(e^{l/L_p} - e^{-l/L_p}\right)}$$

Therefore,

$$I_{rec} = I_{Diff,x_n=0} - I_{Diff,x_n=l} = \frac{qAD_p\Delta p_n\left[e^{l/L_p} + e^{-l/L_p} - 2\right]}{L_p\left(e^{l/L_p} - e^{-l/L_p}\right)}$$

Method 2:
The recombination current supplies the hole lost to recombination in the n-region:

$$I_{rec} = \frac{Q_{p,\,n\text{-}region}}{t_p} \xrightarrow{\text{Use partC}} = \frac{qA\Delta p_n L_p}{t_p}\left[\frac{-2+e^{l/L_p}+e^{-l/L_p}}{e^{l/L_p}-e^{-l/L_p}}\right]$$

$$\xrightarrow{t_p=\frac{L_p^2}{D_p}} = \frac{qA\Delta p_n D_p}{L_p}\left[\frac{e^{l/L_p}+e^{-l/L_p}-2}{e^{l/L_p}-e^{-l/L_p}}\right]$$

The two methods yield same results.

You may also further simplify the result by using relations below:

$$\left[\frac{e^{l/L_p}+e^{-l/L_p}-2}{e^{l/L_p}-e^{-l/L_p}}\right] = \frac{\left(e^{l/2L_p}-e^{-l/2L_p}\right)^2}{\left(e^{l/2L_p}+e^{-l/L_p}\right)\left(e^{l/2L_p}-e^{-l/2L_p}\right)} = \frac{\left(e^{l/2L_p}-e^{-l/2L_p}\right)}{\left(e^{l/2L_p}+e^{-l/L_p}\right)} = \tanh\left(e^{l/2L_p}\right)$$