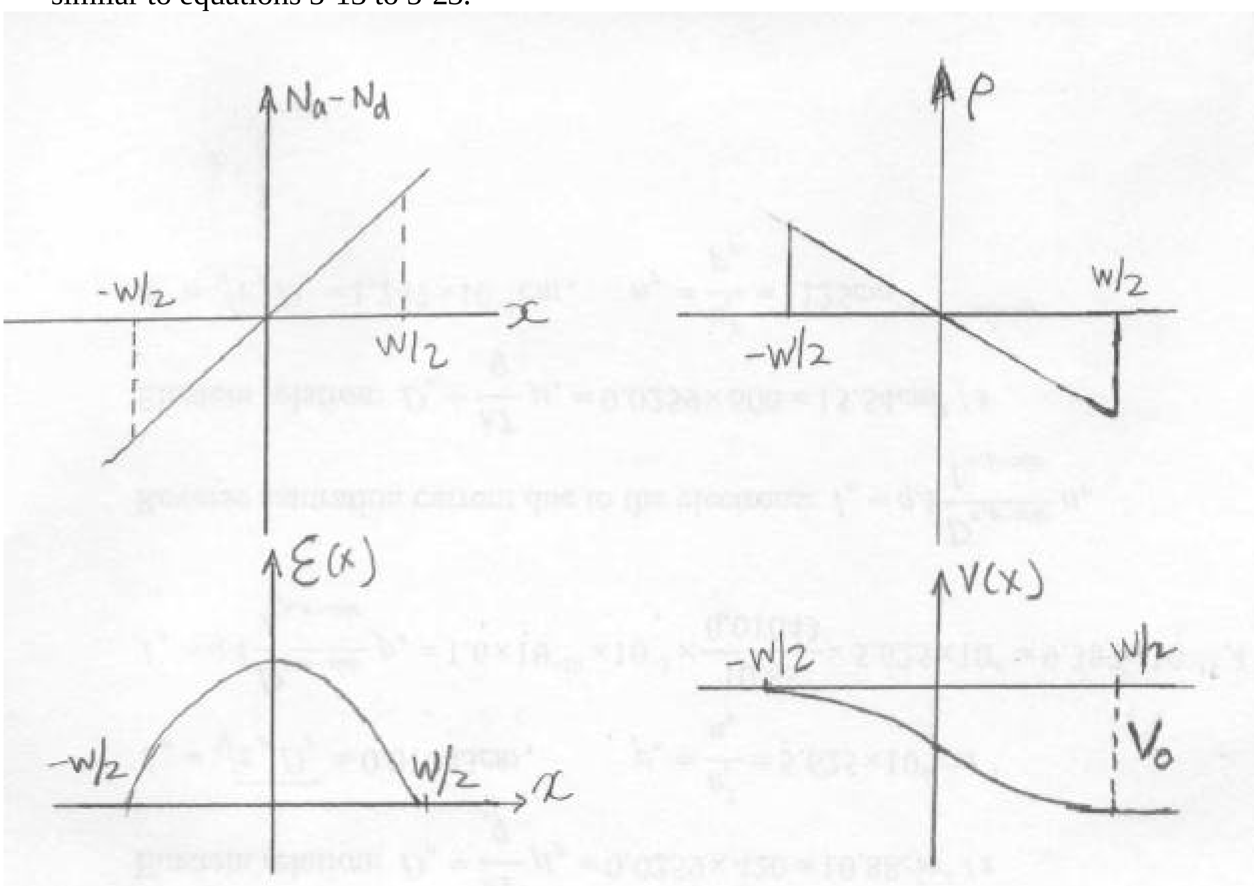
ECE 340 Homework IX Spring 2003

Due: Monday, March 17, 2003

1. When a prolonged diffusion or a high-energy implantation is conducted to form a p/n junction. The doping profile near the junction is usually graded, and the step-junction approach is no longer suitable to find the relationship between the width of the depletion region and the contact potential. However, the underlying principle used to establish equations 5-13 to 5-23 remains intact, and they can still be used to determine similar equations for the graded junction. Assume that the doping profile varies as N_a-N_d=Gx where G is $10^{20}/cm^4$ in a linear junction.
   (a) Find and plot the electric field, $e(x)$, for $-\infty < x < +\infty$.
   (b) Determine the relationship between the width of the depletion region and contact potential for the junction at equilibrium.

Solutions:

(a), Please refer to section 5.6.4 and Figure 5-39 (page 220 to obtain a general understanding of the device. We are going to use depletion approximation in the junction region and quasi-neutral approximation outside junction region. The example in section 5.6.4 has N_d-N_a=Gx, but we have N_a-N_d=Gx. Therefore, in our case, the r , E_x , and V will be the mirror image of those in figure 5-39 about the x-axis. After obtaining a general understanding of the device, we can solve the problem through steps similar to equations 5-13 to 5-23:



Since it's a symmetrical device, $x_{n0} = x_{p0} = W/2$. For x outside depletion region ($|x| > W/2$), electrical field is 0. For x within depletion region ($|x| < W/2$), start with Poisson equation,

$$\frac{d\mathrm{E}(x)}{dx} = \frac{q}{e}(p - n + N_d^+ - N_a^-) \xrightarrow[\text{approximation}]{\text{Depletion}} \frac{d\mathrm{E}(x)}{dx} = \frac{q}{e}(N_d^+ - N_a^-) = -\frac{q}{e}Gx$$

The general solution to above differential equation: $\mathrm{E}_x = -\frac{qG}{2e}x^2 + Const$

Applying boundary condition: $x = \frac{W}{2}$, $\mathrm{E}_x = 0$, we get $Const = \frac{qG}{8e}W^2$.

Therefore, $\mathrm{E}_x = -\frac{qG}{2e}x^2 + \frac{qG}{8e}W^2$, for $|x| < \frac{W}{2}$

But we still need to solve for W: following the method used in section 5.2.3 (page 166), we integrate E_x over depletion region to get V_0:

$$V_0 = -\int_{-W/2}^{W/2} \mathrm{E}_x dx = -\int_{-W/2}^{W/2}\left(-\frac{qG}{2e}x^2 + \frac{qG}{8e}W^2\right)dx$$

$$= -\left[-\frac{qG}{6e}x^3 + \frac{qG}{8e}W^2 x\right]_{-W/2}^{W/2} \qquad (*)$$

$$= -\frac{qGW^3}{12e} = -1.277\times10^{12}W^3(volts)$$

Noticing V_0 is also the Fermi-level difference across the depletion region:

$$V_0 = -0.0259\ln\frac{p_p n_n}{n_i^2} = -0.0259\ln\frac{(GW/2)^2}{n_i^2} = -0.0259\ln\left(1.111\times10^{19}W^2\right) \qquad (**)$$

Combining (*) and (**), $V_0 = -0.0259\ln\left(1.111\times10^{19}W^2\right) = -1.277\times10^{12}W^3$

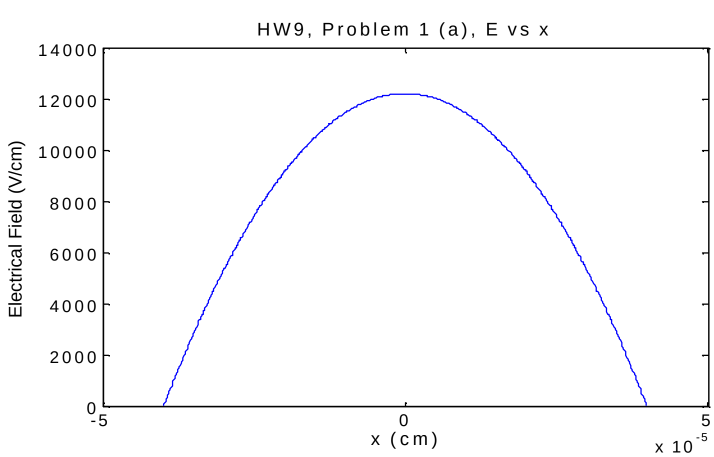
Use a graphic calculator or a computer program, the reasonable answer is:
$W = 7.97\times10^{-5}cm = 0.797mm$; $V_0 = 0.646V$.

Plug W into the E_x expression above, we get:

$$\mathrm{E}_x = -\frac{qG}{2e}x^2 + \frac{qG}{8e}W^2 = -7.66\times10^{12}x^2 + 1.22\times10^4(V/cm) \qquad \text{for } |x| < 3.99\times10^{-5}cm$$

$$\mathrm{E}_x = 0 \qquad \text{for} |x| > 3.99\times10^{-5}cm$$

The plot is shown below:



(b), As shown in part (a),

$$V_0 = -\frac{qGW^3}{12e} = -1.277\times10^{12}W^3(volts)$$

2. An abrupt Si p-n junction has the following properties at 300 K:

p-side	n-side	$A = 10^{-4}cm^2$
$N_a = 2x10^{17}/cm^3$	$N_d = 1x10^{16}/cm^3$	
$\tau_n = 0.1\ \mu s$	$\tau_p = 10\ \mu s$	
$\mu_p = 180\ cm^2/V\text{-}s$	$\mu_n = 1080\ cm^2/V\text{-}s$	
$\mu_n = 600\ cm^2/V\text{-}s$	$\mu_p = 400\ cm^2/V\text{-}s$	

(a) Determine the contact potential V_o of the junction, and the depletion widths at equilibrium, under a forward bias of $V_o/2$ and under a reverse bias of $4V_o$.

(b) Draw the band diagrams qualitatively at equilibrium and under forward and reverse bias showing the varying depletion widths, Fermi level and the quasi-Fermi levels.

(c) Calculate the reverse saturation current due to holes, due to electrons and the total reverse saturation current.

(d) Calculate the total injected minority carrier current for $V = V_o/3$ and $V_o/2$.

(e) Poor heat dissipation often leads to a rise of the device temperature. Assuming μ's and τ's do not change with temperature, repeat part (c) for a temperature of T = 350 K at which the intrinsic carrier concentration $n_i = 4x10^{11}/cm^3$.

Solutions:

(a), $V_0 = 0.0259\ln\frac{N_aN_d}{n_i^2} = 0.772V$

$$W_0 = \sqrt{\frac{2eV_0}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)} = \sqrt{\frac{2\times11.8\times8.85\times10^{-14}\times0.772}{1.6\times10^{-19}}\left(\frac{1}{2\times10^{17}}+\frac{1}{1\times10^{16}}\right)} = 0.33mm$$

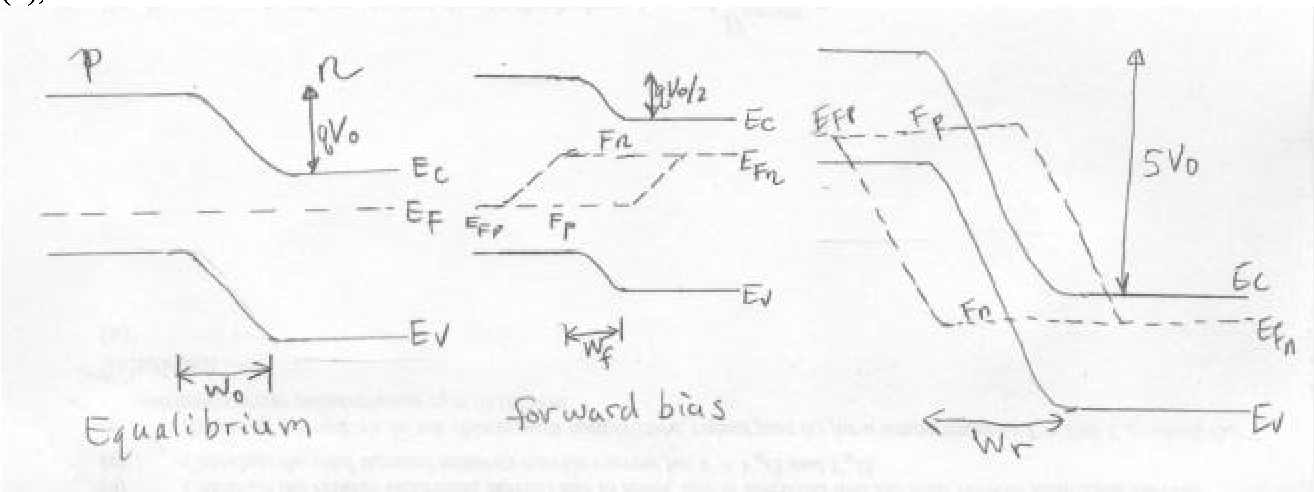
Under forward bias:

$$W_f = \sqrt{\frac{2e(V_0-V_f)}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)} = \sqrt{\frac{2\times11.8\times8.85\times10^{-14}\times0.386}{1.6\times10^{-19}}\left(\frac{1}{2\times10^{17}}+\frac{1}{1\times10^{16}}\right)} = 0.23mm$$

Under reverse bias:

$$W_r = \sqrt{\frac{2e(V_0+V_r)}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)} = \sqrt{\frac{2\times11.8\times8.85\times10^{-14}\times5\times0.772}{1.6\times10^{-19}}\left(\frac{1}{2\times10^{17}}+\frac{1}{1\times10^{16}}\right)} = 0.73mm$$

(b),



(c), $D_p(n-side) = 0.0259 \times 400 = 10.36 cm^2/s \quad D_n(p-side) = 0.0259 \times 600 = 15.54 cm^2/s$

$$L_p(n-side) = \sqrt{D_p t_p} = \sqrt{10.36 \times 10 \times 10^{-6}} = 1.018 \times 10^{-4} cm$$

$$L_n(p-side) = \sqrt{D_n t_n} = \sqrt{15.54 \times 0.1 \times 10^{-6}} = 1.25 \times 10^{-3} cm$$

$$I_p = qA\frac{D_p}{L_p} p_n = qA\frac{D_p}{L_p}\frac{n_i^2}{n_n} = 3.66 \times 10^{-16} A$$

$$I_n = qA\frac{D_n}{L_n} n_p = qA\frac{D_n}{L_n}\frac{n_i^2}{p_p} = 2.24 \times 10^{-16} A$$

$$I_{total} = I_n + I_p = 5.90 \times 10^{-16} A$$

(d), For V = $V_o/3$, $I = I_0(e^{qV/kT} - 1) = I_0(e^{qV_0/3kT} - 1) = 1.22 \times 10^{-11} A$

For V = $V_o/2$, $I = I_0(e^{qV/kT} - 1) = I_0(e^{qV_0/2kT} - 1) = 1.75 \times 10^{-9} A$

(e), $D_p(n-side) = 0.0302 \times 400 = 12.07 cm^2/s \quad D_n(p-side) = 0.0302 \times 600 = 18.1 cm^2/s$

$$L_p(n-side) = \sqrt{D_p t_p} = \sqrt{12.07 \times 10 \times 10^{-6}} = 1.098 \times 10^{-4} cm$$

$$L_n(p-side) = \sqrt{D_n t_n} = \sqrt{18.1 \times 0.1 \times 10^{-6}} = 1.345 \times 10^{-3} cm$$

$$I_p = qA\frac{D_p}{L_p} p_n = qA\frac{D_p}{L_p}\frac{n_i^2(350K)}{n_n} = 2.81 \times 10^{-13} A$$

$$I_n = qA\frac{D_n}{L_n} n_p = qA\frac{D_n}{L_n}\frac{n_i^2}{p_p} = 1.72 \times 10^{-13} A$$

$$I_{total} = I_n + I_p = 4.53 \times 10^{-13} A$$

3. Assume that an abrupt Si p-n junction with area 10^{-4} cm^2 has $N_a = 5 \times 10^{16}/cm^3$ on the p-side and $N_d = 2 \times 10^{17}/cm^3$ on the n side. The diode has a forward bias 0.5 volts.

(a) Using mobility values from Fig. 3-23 (or better yet from the information sheet used for HR Exam) and assuming that $\tau_n = \tau_p$ =1 μs, plot I_p and I_n versus distance on a diagram such as Fig. 5-17, including both sides of the junction. Neglect recombination within the space charge region, W.

(b) Plot $\delta n(x_p)$ and $\delta p(x_n)$.

(c) Determine the separation of quasi-Fermi levels at the position 5 μ m into the quasi-neutral region in both sides of the junction.

(a), Using Eqn (5-21) to (5-23), we obtain:

$V_0 = 0.814V$

From graph, $m_{p,n-side} = 180cm^2/Vs$, therefore, $D_p = 0.0259m_p = 4.662cm^2/s$

$m_{n,\,p-side} = 810cm^2/Vs$, therefore, $D_n = 0.0259m_n = 20.98cm^2/s$

$$L_n = \sqrt{t_n D_n} = 4.58\times10^{-3}cm, \qquad n_p = \frac{n_i^2}{p_p} = 4500cm^{-3}$$

$$L_p = \sqrt{t_p D_p} = 2.16\times10^{-3}cm, \qquad p_n = \frac{n_i^2}{p_p} = 1125cm^{-3}$$

$$\Delta n_p = n_p\left(e^{qV/kT}-1\right) = 4500\times\left(e^{0.5/0.0259}-1\right) = 1.09\times10^{12}cm^{-3}$$

$$\Delta p_n = p_n\left(e^{qV/kT}-1\right) = 1125\times\left(e^{0.5/0.0259}-1\right) = 2.72\times10^{11}cm^{-3}$$

In p-region,

$$I_n(x_p) = qA\frac{D_n}{L_n}\Delta n_p e^{-x_p/L_p} = 1.6\times10^{-19}\times10^{-4}\frac{20.98}{4.58\times10^{-3}}1.09\times10^{12}\times e^{-x_p(mm)/45.8}$$

$$= 7.98\times10^{-8}\times e^{-x_n(mm)/45.8}A$$

In n-region:

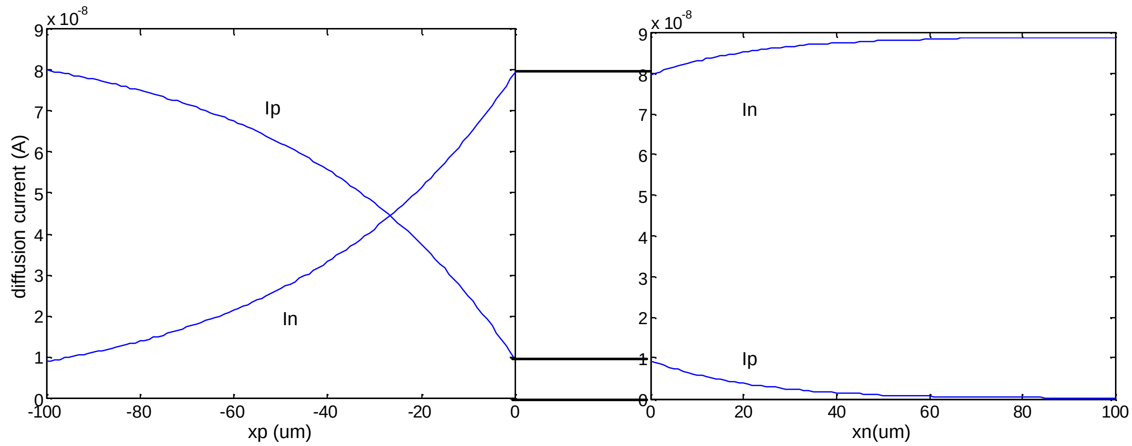
$$I_p(x_n) = qA\frac{D_p}{L_p}\Delta p_n e^{-x_n/L_p} = 9.37\times10^{-9}\times e^{-x_n(mm)/21.6}A$$

$$I_{total} = I_n + I_p = 8.89\times10^{-8}A$$

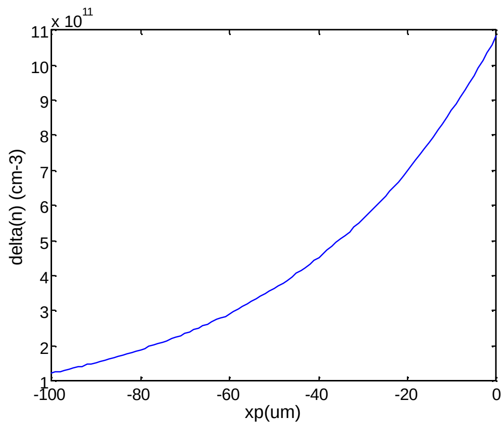
$$I_p(x_n) = I_{total} - I_{n,\,p-side}$$

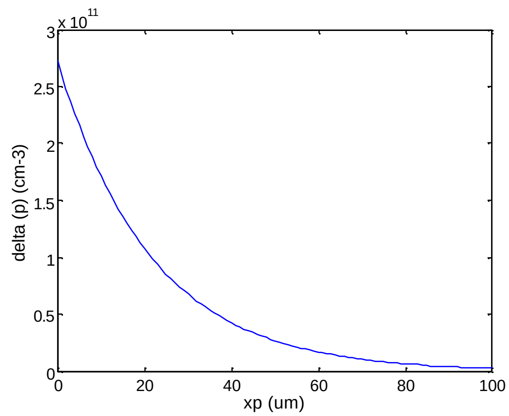
$$I_n(x_p) = I - I_p$$

Based on the above calculation, we can draw a graph similar to Figure 5-17



(b), $d_n(x_p) = 1.09\times10^{12} e^{-x_p(mm)/45.8} cm^{-3}$ $\quad$ $d_p(x_n) = 2.72\times10^{11} e^{-x_n(mm)/21} cm^{-3}$





(c),At 5μm into p-side, $d_n(x_p = 5mm) = 1.08\times10^{12} e^{-5/45.8} cm^{-3} = 9.68\times10^{11} cm^{-3}$

There is negligible change in majority carrier concentration.

$$F_n - F_p = 0.0259 \ln \frac{np}{n_i^2} = 0.497 eV$$

At 5μm into n-side, $d_p(x_n = 5mm) = 2.72\times10^{11} e^{-5/27.88} cm^{-3} = 2.15\times10^{11} cm^{-3}$

$$F_n - F_p = 0.0259 \ln \frac{np}{n_i^2} = 0.494 eV$$