Intros + Lambda Calculus

CS 130 sp 21
4/2/21

Your TAs

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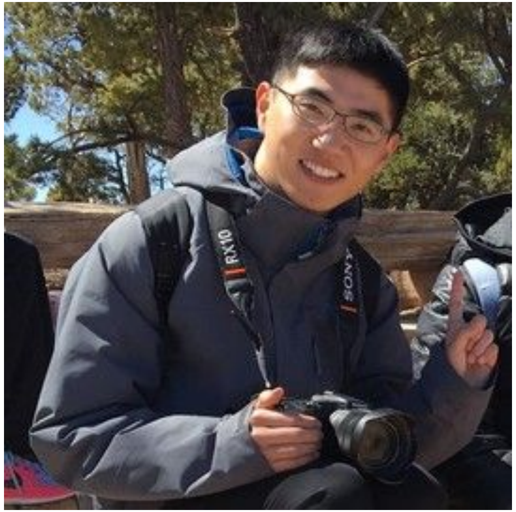

Zheng Guo

Agenda

Setup

What is the lambda calculus

Syntax

Beta reductions

PA0 tips

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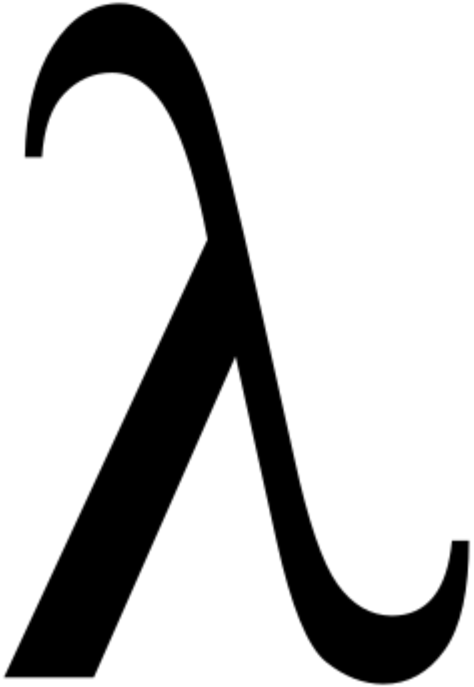


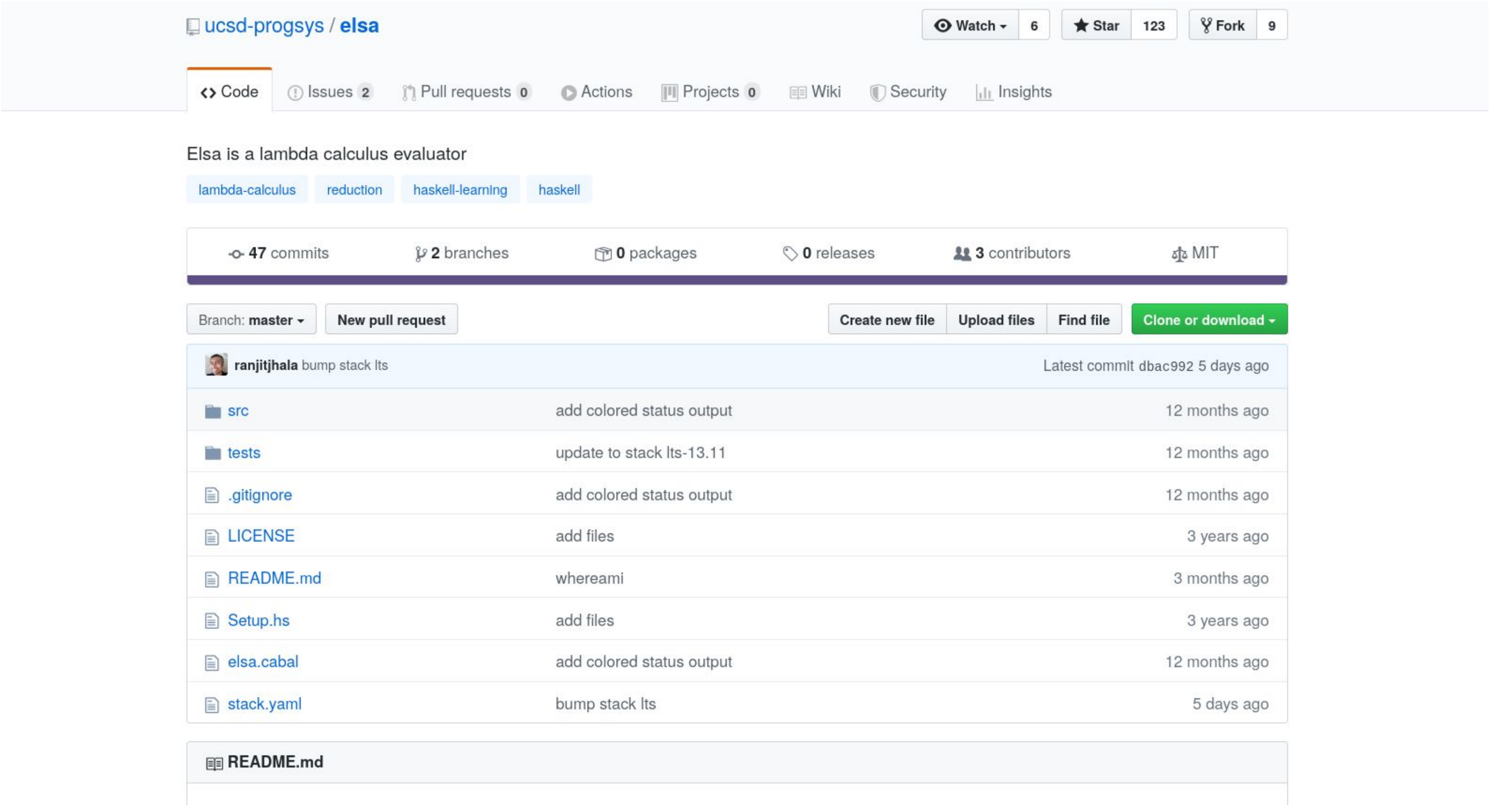
Setup

HW0 will have you *manually* evaluate lambda calculus terms

Elsa checks that **reductions** are valid

Elsa - what is it and how do I use it?

Elsa is implemented as a Haskell package



How do I run elsa and do the HW?

Options:

1. SSH into ieng6
2. Install stack locally
3. Use online demo

SSH into ieng6

Pros:

- Should have everything installed already
- Standardized and easy for us to help us with

Cons:

- Requires internet connection
- Watch out for quota!

Install stack locally

Pros:

- Everything can be done offline
- We will use Haskell throughout the class, you might want it locally

Cons:

- Installing Haskell's stack tool might be annoying
- Unix: should be easy
- Mac: should also be easy with brew
- Windows: Installer from Stack website
- WSL: ??
- `$ stack install elsa`

Online demo

Pros:

- Will “just work”

Cons:

- Very clumsy for doing the homework

Doing the homework

`make test` will check your work

Make sure to commit your work to GH!

Submit a .zip of your git repo to gradescope (use GH's "download zip" feature)

Confused? [The submission instructions are quite detailed]

Do not use =*> or =~> operators!

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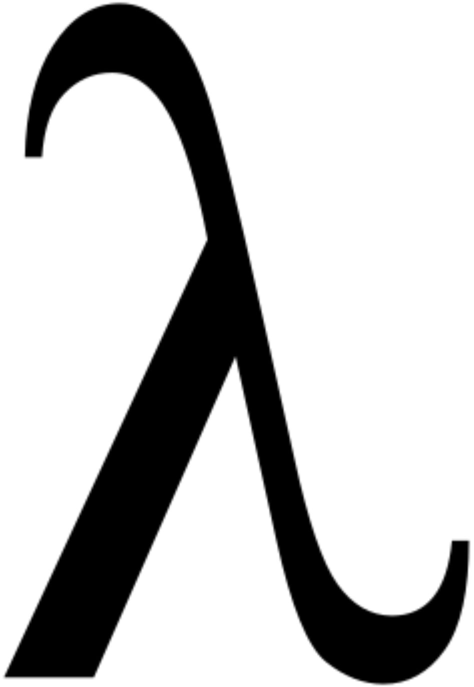

What is the lambda calculus

Very simple programming language

Still Turing complete!

What is the lambda calculus

It might look silly but...

- Simple **formal model** of programming
- Provides a minimal framework for exploring and reasoning about various PL concepts
- Fundamental to lots of PL research (especially functional programming)
- **Definitely on the exam**

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λ

Syntax

`x` : Variable

`(\x -> M)` : Function abstraction (M is a lambda term)

`(M N)` : Function application (M, N are lambda terms)

All we can do is declare functions and apply functions!

Functions are *first-class*: We can apply functions to other functions, and a function can return another function

Syntax

```
\a -> (\b -> b)  -- Function that takes a parameter “a” and
                 -- returns a function that takes a param “b”
```

```
\a -> \b -> b    -- Syntactic sugar for above
```

```
\a b -> b        -- More syntactic sugar
```

Syntax Quiz

Which is equivalent to:

```
(\foo -> (\bar -> (\baz -> baz bar foo)))
```

No reductions! Just syntactic sugar!

```
a.  (\foo bar baz -> baz bar foo)
b.  (\foo -> (\bar -> (\baz -> (baz (bar foo)))))
c.  (\foo -> \bar) -> (\baz -> baz bar foo)
d.  A & B
```

Syntax Quiz

Which is equivalent to:

```
(\foo -> (\bar -> (\baz -> baz bar foo)))
```

No reductions! Just syntactic sugar!

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a.  (\foo bar baz -> baz bar foo)
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c.  (\foo -> \bar) -> (\baz -> baz bar foo)
d.  A & B
```

Syntax Quiz

Which is equivalent to:

```
(\foo -> (\bar -> (\baz -> baz bar foo)))
```

No reductions! Just syntactic sugar!

Not the same!

```
a.  (\foo bar baz -> baz bar foo)
b.  (\foo -> (\bar -> (\baz -> (baz (bar foo)))))
c.  (\foo -> \bar) -> (\baz -> bar bar foo)
d.  A & B
```

Syntactic Sugar

- `\x -> (\y -> (\z -> E))` we can rewrite: `\x -> \y -> \z -> E`
- `\x -> \y -> \z -> E` we can rewrite: `\x y z -> E`
- `(((E1 E2) E3) E4)` we can rewrite: `E1 E2 E3 E4`

Associativity

Application is **left** associative!

```
E1 E2 E3 =

((E1 E2) E3)
```

Abstraction is **right** associative!

```
\x -> \y -> \z -> E =

\x -> (\y -> (\z -> E)
```

Quiz

Fully parenthesize: **`\b1 b2 -> ITE b1 b2 FALSE`**

```
a.  ((((\b1 b2 -> ITE) b1) b2) FALSE)

b.  (\b1 b2 -> (ITE (b1 (b2 FALSE))))

c.  (\b1 b2 -> ((ITE b1) b2) FALSE)

d.  (\b1 b2 -> (ITE b1) (b2 FALSE))
```

Quiz

Fully parenthesize: `\b1 b2 -> ITE b1 b2 FALSE`

```
a.  ((((\b1 b2 -> ITE) b1) b2) FALSE)

b.  (\b1 b2 -> (ITE (b1 (b2 FALSE))))
```

c. `(\b1 b2 -> ((ITE b1) b2) FALSE)`

```
d.  (\b1 b2 -> (ITE b1) (b2 FALSE))
```

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Alpha/Beta reductions

Beta step: Calling a function

Alpha step: Renaming a variable inside a function

Beta step

What do we do with `(\x -> x) y` ?

We can **substitute** y for x inside the body of the function: we just get `y`

More examples:

`(\a b c -> b) d` becomes `(\b c -> b)`

`(\b c -> b) e` becomes `(\c -> e)`

`(\a b c -> b) d e` becomes `(\c -> e)`

$\boxed{a\ (\backslash b \to b)\ c}$

~~-> b a c~~

No more reduction

$\underbrace{(a\ (\backslash b \to b))}\ c$

In Elsa

```

eval beta :
  (\f x -> f (f x)) g

```

In Elsa

```
eval beta :
  (\f x -> f (f x)) g
  =b> \x -> g (x x)
```

g

$(\lambda x \to g\,(g\,x))$

In Elsa

```
eval beta :
  (\f x -> f (f x)) g
  =b> \x ->  g (g x)
```

Quiz

What does: **`(\f x -> f (f x)) incr one`** beta-reduce to (in one step)?

a. `(\x -> incr (f x)) one`

b. `(\x -> one (one x)) incr`

c. `(\x -> incr (incr x)) one`

d. `(\f -> f (f one)) incr`

Quiz

What does `(\f x -> f (f x)) incr one` beta-reduce to (in one step)?

1st 2nd

a. `(\x -> incr (f x)) one`

b. `(\x -> one (one x)) incr`

c. `(\x -> incr (incr x)) one`

d. `(\f -> f (f one)) incr`

(\f x -> (f (f x))) incr one

=b> (\x -> incr (incr x)) one

=b> incr (incr one)

Quiz

What does: **`(\f x -> f (\f -> f x) x) incr one`** beta-reduce to?

a. `(\x -> incr (\f -> incr x) x) one`
b. `(\x -> incr (\incr -> incr x) x) one`
c. `(\f -> incr (incr x) x) one`
d. `(\x -> incr (\f -> f x) x) one`

Quiz

x is free

free fs

bound

What does: `(\f x -> f (\f -> f x) x) incr one` beta-reduce to?

a. `(\x -> incr (\f -> incr x) x) one`

b. `(\x -> incr (\incr -> incr x) x) one`

c. `(\f -> incr (incr x) x) one`

d. **`(\x -> incr (\f -> f x) x) one`** ← start

(incr (\f -> f one)) one

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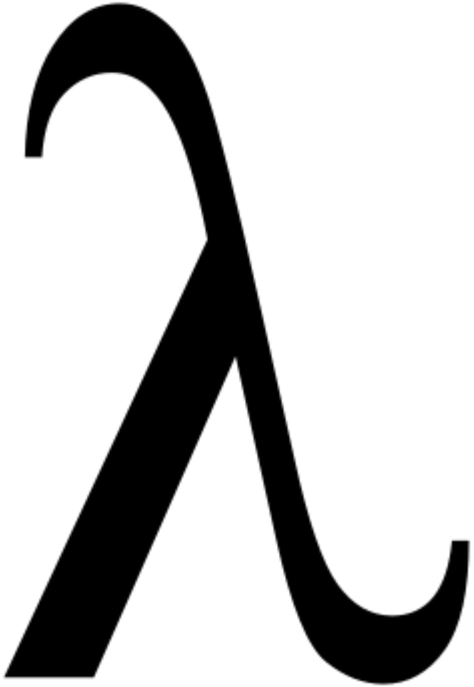

PA0 Overview

Goal: Simplify lambda calculus expressions via alpha/beta steps

You will need to understand:

- How to apply alpha/beta steps
- The definitions in each source file

Be aware: the lambda calculus is weird! This might take time

PA0 Overview

Each problem will define higher-level concepts with lambda terms:

```
--------------------------------------------------------------------------------
-- DO NOT MODIFY THIS SEGMENT
--------------------------------------------------------------------------------

let TRUE  = \x y -> x
let FALSE = \x y -> y
let ITE   = \b x y -> b x y
let NOT   = \b x y -> b y x
let AND   = \b1 b2 -> ITE b1 b2 FALSE
let OR    = \b1 b2 -> ITE b1 TRUE b2
```

Most of these definitions will not make sense on their own!

TRUE and FALSE make no sense without the definition of ITE -- you need to read all the definitions and try to figure out how they work together

PA0 overview

(\b xy-> b xy) Not

apply Not

Expand (\b xy-> y x)

Elsa also offers a =d> operator

This allows you to replace symbols with their definition -- this is key! Use it early

```
--------------------------------------------------------------------------------
-- DO NOT MODIFY THIS SEGMENT
--------------------------------------------------------------------------------

let TRUE  = \x y -> x
let FALSE = \x y -> y
let ITE   = \b x y -> b x y
let NOT   = \b x y -> b y x
let AND   = \b1 b2 -> ITE b1 b2 FALSE
let OR    = \b1 b2 -> ITE b1 TRUE b2

--------------------------------------------------------------------------------
-- YOU SHOULD ONLY MODIFY THE TEXT BELOW, JUST THE PARTS MARKED AS COMMENTS
--------------------------------------------------------------------------------

eval not_true :
  NOT TRUE
  -- (a) fill in your reductions here
  =d> FALSE
```

let APPLE = \x -> x

However, you can also make the problems too complicated...

If we replace **all** definitions, we might end up with too much complexity!

Which of these is easier to work with? Why?

```
eval not_true :
  NOT TRUE
  =d> (\b x y -> b y x) TRUE
```

```
eval not_true :
  NOT TRUE
  =d> (\b x y -> b y x) (\x y -> x)
```

Q & A / Live Examples