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Fuzzy decision support system for demand forecasting with a learning mechanism

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Abstract

In this paper, a new decision support system for demand forecasting DSS_DF is presented. A demand forecast is generated in DSS_DF by combining four forecasts values. Two of them are obtained independently, one by a customer and the other by a market expert. They represent subjective judgments on future demand, given as linguistic values, such as "demand is *around* a certain value" or "demand is *not lower* than a certain value", etc. Two additional forecasts are crisp values, obtained using conventional statistical methods, one using time-series analysis based on decomposition (TSAD), and the other using an auto regressive integrated moving average (ARMA) model. The combination of these four forecast values into one improved forecast is made by applying fuzzy IF-THEN rules. A modified Mamdani-style inference is used, which enables reasoning with fuzzy inputs. A new learning mechanism is developed and incorporated into the DSS_DF to adapt the rule bases that combine the individual forecasted values. The rule bases are adapted taking into consideration the performance of each of the forecast methods recorded in the past. The application of DSS_DF is demonstrated by an illustrative example. The forecasts obtained by DSS_DF are compared with results procured by applying the conventional TSAD and ARMA methods separately. The results obtained are encouraging and indicate that combining forecasts obtained by different methods may be beneficial.

Keywords: Fuzzy inference systems; Forecasting; Learning; Decision support systems

1. Introduction

A typical problem that most of manufacturers face is the scheduling of raw material purchases and available resources. Most often, the time required to acquire material and to manufacture a final product is longer than the acceptable time between an order and delivery of the product. Demand forecasting is a standard approach that manufacturers use to facilitate material purchase and allocation of resources. In addition to production decision making, demand forecasting is of great importance for marketing activities (e.g., sales-force allocation, new product introduction), finance (e.g., plant/equipment investment, budgetary planning), personnel management (e.g., workforce planning), etc. [4].

Various demand forecasting methods have been developed and used in practice. A large number of them are based on statistics, such as moving average, time series analysis based on decomposition (TSAD), auto regressive integrated

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moving average (ARMA), etc. [3,9]. These methods assume that historical data are recorded in the past and they are precisely known. Furthermore, the statistical forecast methods assume that a historical pattern of demand is a good indicator of future demand. They can be applied successfully when historical data are reliable and environments being forecasted are relatively stable. However, these methods are perceived as being slow to react to changes in dynamic environments.

Uncertain and imprecise data are present in many real-world decision-making processes including demand forecasting. Only a few forecasting methods that address this issue are reported in the literature. In [7], a fuzzy logic-based advisory tool (FLAT) is described, where demand forecasts for a telecommunication manufacturer are inferred based on eight crisp input values. They are obtained from a customer, a market expert, or by making use of standard forecasting methods, such as weighted average value and trend. Inference rules are represented in FLAT by linguistic relations and the reasoning applied is based on a linguistic equation framework technique. Song and Chisson [18,19] developed fuzzy time-invariant and time-variant time series methods that were applied to forecasting educational enrolments. The fuzzy time series methods developed can be applied when historical data are either crisp or linguistic values. They involve fuzzy arithmetic to calculate a fuzzy matrix that represents a relationship between the historical data. Subjective management judgment plays an important role in generating a forecast [1]. Goodwin [8] reviews two approaches to integrating subjective forecast judgment and statistical methods: (1) voluntary integration where a judicial forecaster is free to decide how to use/adjust a statistical forecast and (2) mechanical integration where a subjectively generated forecast and a statistical forecast are combined by applying a certain mathematical method.

In this paper, a new fuzzy logic based decision support system DSS_DF is presented. It combines subjective forecasts and statistical forecasts into one improved forecast using fuzzy IF-THEN rules. A standard Mamdani-style inference mechanism is applied which is appropriately modified to enable reasoning with fuzzy inputs. A standard approach to learning in fuzzy systems has been focused on the determination of an input/output space partition and modification of parameters that define associated membership functions [16]. This has usually involved integration of fuzzy systems with artificial neural networks that provide learning abilities [10,12], or, more recently, with genetic algorithms [17,13]. A new learning mechanism is developed and embedded into the DSS_DF, which dynamically changes the fuzzy relationships between the linguistic variables that are constituent parts of the fuzzy IF-THEN rules. The appropriateness of DSS_DF is evaluated using real data provided by an industrial collaborator and the results obtained are very encouraging.

The organization of the paper is as follows. Section 2 introduces the structure of DSS_DF including: (1) forecast values that DSS_DF operates with; (2) fuzzy rule bases; and (3) the inference mechanism. In Section 3, the new learning mechanism built into DSS_DF is described. An illustrative example and the results obtained are presented in Section 4. Conclusions are given in Section 5.

2. Structure of DSS_DF

The general structure of the proposed DSS_DF is presented in Fig. 1. All parts of the DSS_DF are explained in the following sections, including (1) the forecast values that are input into DSS_DF, (2) the fuzzy rule bases and (3) the inference mechanism.

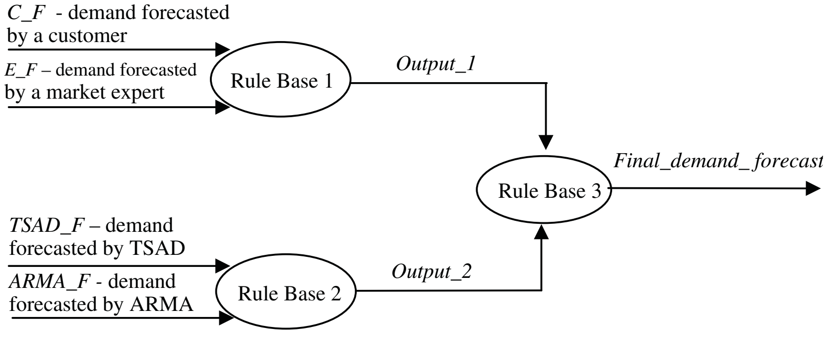

Fig. 1. Structure of DSS_DF.

2.1. Forecast values

The forecast for product demand is generated in DSS_DF using four values, including: (1) *C_F*—demand forecasted by a customer; (2) *E_F*—demand forecasted by a market expert; (3) *TSAD_F*—demand forecasted by a traditional TSAD method; and (4) *ARMA_F*—demand forecasted by a traditional ARMA method.

A customer and an expert estimate demand for the next period independently using imprecise linguistic values, such as "demand is *about M* products". A linguistic value for the customer forecast is represented in DSS_DF by a fuzzy set $C\_F_{[M,D,d]}$ with a symmetrical trapezoidal membership function, where M is the mid-point of the lower base of the corresponding trapezium, D and d are the lengths of the lower and the upper trapezium bases, respectively.

In addition to these fuzzy forecasts, DSS_DF uses a crisp forecast, *TSAD_F*, generated by a TSAD method, which takes into consideration trend, seasonality, and cyclical components of a time series that represents historical demand [9]. Trend reflects the current rate of growth or decline in demand, seasonality indicates a pattern of changes in demand that recurs regularly over time, and the cyclical component corresponds to a series of irregular fluctuations of demand around the trend. A TSAD forecasting method that treats a time series as a product of these three components is implemented in DSS_DF.

The fourth forecast value *ARMA_F* is obtained using an ARMA model. An ARMA(p, d, q) model implemented in DSS_DF is based on the following equation [9]:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \omega_1 \varepsilon_{t-1} - \omega_2 \varepsilon_{t-2} - \cdots - \omega_q \varepsilon_{t-q}, \tag{1}$$

where y_t is demand forecast for period t, $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$ are actual demands recorded in periods $t-1, t-2, \ldots, t-p$, respectively, $\phi_0, \phi_1, \phi_2, \ldots, \phi_p$ are regression coefficients, ε_t is an estimation of the residual or the error of the forecast for period t, $\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}$ are previous values of the residuals for periods $t-1, t-2, \ldots, t-q$, respectively, $\omega_1, \omega_2, \ldots, \omega_q$ are weights, p is the order of the autoregressive term, d is the level of differencing, and q is the order of the moving average term.

Parameters p, d, q are determined by the user.

2.2. Rule bases

The demand forecast is generated by combining the four forecast values in two sequential phases using three rule bases (see Fig. 1). In the first phase, *C_F* is combined with *E_F*, and *TSAD_F* is combined with *ARMA_F*, using Rule Base 1 and Rule Base 2, respectively. In the second phase, the two combined demand forecasts, *Output_1* and *Output_2*, determined in the first phase are combined into a single combined *Final_demand_forecast*, using Rule Base 3.

The fuzzy rule bases involve the following linguistic variables:

Rule Base 1: *Customer_Forecast*, *Expert_Forecast* and *Combined_Forecast_1*,
Rule Base 2: *TSAD_Forecast*, *ARMA_Forecast*, and *Combined_Forecast_2*,
Rule Base 3: *Combined_Forecast_1*, *Combined_Forecast_2* and *Final_Forecast*.

Each of these linguistic variables can have 3 fuzzy values, namely *low*, *medium*, and *high*, and are represented by trapezoidal membership functions. The historical data recorded in the past are used to determine the domains of all the membership functions. Note that fuzzy historical data, such as forecasts given by the customer and the expert, are defuzzified first by making use of the mean-maximum defuzzification method. As an example, Fig. 2 illustrates the fuzzy values of the linguistic variable corresponding to the *TSAD_Forecast*.

Each rule base has nine IF-THEN rules that are defined for all the possible combinations of the fuzzy values of the linguistic variables involved. The rules reflect an initial strategy for combining the different forecast values that has been suggested by a user. For example, if the same level of trust is given to the customer forecast and expert forecast, the rules in Rule Base 1 can have the form as given in Table 1 (a). Rule Base 2 and Rule Base 3 are defined in a similar way (see Table 1 (a) and (b), respectively).

However, the proposed DSS_DF includes a learning mechanism that modifies and improves the initial rule bases over time, taking into consideration the performance of the rules, as described in Section 3.

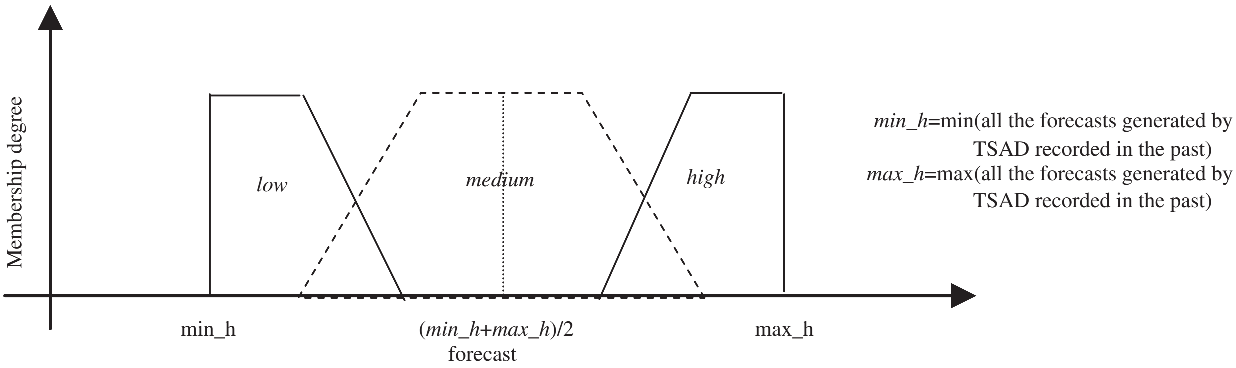

Fig. 2. Fuzzy values of linguistic variable *TSAD_Forecast*.

Table 1
Fuzzy rule bases

(a) *Fuzzy IF-THEN rules of Rule Base 1 for determining Combined_Forecast_1*

Combined_Forecast_1		*Expert_Forecast*		
		Low	*Medium*	*High*
Customer_Forecast	*Low*	*Low*	*Low*	*Medium*
	Medium	*Low*	*Medium*	*High*
	High	*Medium*	*High*	*High*

(b) *Fuzzy IF-THEN rules of Rule Base 2 for determining Combined_Forecast_2*

Combined_Forecast_2		*ARMA_Forecast*		
		Low	*Medium*	*High*
TSAD_Forecast	*Low*	*Low*	*Low*	*Medium*
	Medium	*Low*	*Medium*	*High*
	High	*Medium*	*High*	*High*

(c) *Fuzzy IF-THEN rules of Rule Base 3 for determining Final_Forecast*

Final_Forecast		*Combined_Forecast_2*		
		Low	*Medium*	*High*
Combined_Forecast_1	*Low*	*Low*	*Low*	*Medium*
	Medium	*Low*	*Medium*	*High*
	High	*Medium*	*High*	*High*

2.3. Inference mechanism

Fuzzy rules have been widely accepted and used as a key tool for representing imprecise knowledge on relationships between variables [2]. Fuzzy rules have been interpreted in different ways, leading to different fuzzy inference mechanisms. Inference behaviour and possible applications of the different fuzzy rule models are discussed in [6]. Fuzzy rule models are mainly classified into two main groups, depending on the way the fuzzy rules are interpreted: (1) implication-based models, which include certainty rules, gradual rules, or impossibility rules; in these models, the rules are interpreted using a logical implication; and (2) conjunction-based models, which include possibility rules (e.g., Mamdani rules) or antigradual rules; in these models, the rules are interpreted using a Cartesian product. The main advantages of fuzzy rule models seem to be the possibility of implementing experience, intuition and heuristics, and the fact that they do not need a rigorous mathematical model of the process to be controlled.

The Mamdani style of inference [11] has been successfully applied in a wide range of applications in the field of fuzzy control. It is a rather simple inference mechanism which has found applicability in practice. For this reason, it is used in the DSS_DF, despite the lack of its rigor from logical point of view and its drawbacks that have been identified in literature [5]. It is worth noting that the learning mechanism built into the DSS_DF does not depend on the fuzzy inference mechanism applied. The analysis of appropriateness of other fuzzy inference mechanisms for combining different forecast values remains to be investigated in future research.

The standard Mamdani inference mechanism is used to combine both crisp forecasts *TSAD_F* and *ARMA_F* using Rule Base 2, and the crisp *Output_1* and crisp *Output_2* using Rule Base 3. The following steps are involved [14]: (1) fuzzification—to determine the degrees to which the crisp forecasts belong to each of the corresponding fuzzy sets *low*, *medium* and *high*; (2) rule evaluation—to evaluate the truth value of each appropriate IF part of the rule and to determine the consequent fuzzy set; the standard *Min* operator is used to model AND connective; (3) aggregation of the rule outputs; the standard *Max* operator is used; and (4) defuzzification; the standard moment method is used.

In order to enable reasoning with fuzzy inputs such as *C_F* and *E_F*, the Mamdani-style inference mechanism is modified in the following way [15]. Instead of the fuzzification step, where the input is a crisp value, compatibility between a fuzzy input and the appropriate fuzzy sets *low*, *medium* and *high* are calculated. Compatibility is represented by a number from on interval [0, 1], where 0 indicates complete non-compatibility and 1 indicates total compatibility between the two fuzzy sets. The following is used to calculate compatibility between two fuzzy sets F_1 and F_2:

$$\text{compatibility}(F_1, F_2) = \frac{\text{area of intersection of } F_1 \text{ and } F_2}{\text{area of union of } F_1 \text{ and } F_2}. \tag{2}$$

One should note that in the case when two fuzzy sets do not overlap, their compatibility is 0, whereas in the case when they are the same, their compatibility is 1.

3. Learning mechanism

The resulting performance of fuzzy reasoning within the DSS_DF is improved by introducing a new learning mechanism which dynamically modifies and improves the fuzzy IF-THEN rules. As explained in Section 2.2, the fuzzy rules are used to combine two demand forecasts obtained by different methods and express the confidence associated with each of the forecasts. In this learning mechanism, the confidence in the forecast is determined based on the performance of the rules recorded in the past.

Principles of the learning mechanism will be described using as an example Rule Base 1, which combines forecasts *C_F* and *E_F* into one improved combined forecast, using the linguistic variables *Customer_Forecast*, *Expert_Forecast* and *Combined_Forecast_1*. The learning mechanism involves three steps:

Step 1: The confidence in forecast *C_F* is determined as a real number $con(C\_F)$ on the interval [0,1], where 0 represents complete lack of confidence and 1 represents full confidence in the customer forecast. It is calculated in the following way:

$$con(C\_F) = e^{-|error(C\_F)| \cdot k}, \tag{3}$$

where $error(C\_F)$ is the arithmetic mean of the errors of the customer forecasts recorded in the past periods and $k \geqslant 1$ is a coefficient given by the user. The greater the k, the less confidence is attached to the method for the same generated error.

Step 2: The confidence in an expert forecast, $con(E\_F)$, is calculated in the same way:

$$con(E\_F) = e^{-|error(E\_F)| \cdot k}. \tag{4}$$

Step 3: The new relationship between the linguistic variables *Customer_Forecast*, *Expert_Forecast* and output variable *Combined_Forecast_1* is determined for each possible combination of the input linguistic values *low*, *medium* and *high*. It involves the following sub-steps, illustrated in Fig. 3.

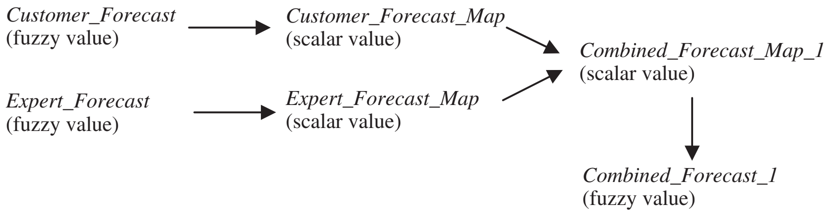

Fig. 3. Step 3 of the learning mechanism.

Sub-step 3.1: Map the input linguistic values, namely *low*, *medium* and *high* of the linguistic variable *Customer_ Forecast* into scalars −1, 0 and 1, respectively, as follows

$$Customer\_Forecast\_Map = \begin{cases} -1 & \text{if } Customer\_Forecast = low, \\ 0 & \text{if } Customer\_Forecast = medium, \\ +1 & \text{if } Customer\_Forecast = high. \end{cases} \tag{5}$$

The same mapping is used for the linguistic variable *Expert_Forecast*.

Sub-step 3.2: Combine the mapped values and the confidences in the forecasts into a scalar *Combined_Forecast_ Map_1*, using the following algebraic expression:

$$Combined\_Forecast\_Map\_1 = \frac{con(C\_F) \cdot Customer\_Forecast\_Map + con(E\_F) \cdot Expert\_Forecast\_Map}{con(C\_F) + con(E\_F)}. \tag{6}$$

It is worth noting that $Combined\_Forecast\_Map\_1 \in [-1, +1]$.

Sub-step 3.3: Map the scalar calculated in Sub-step 3.2 into a linguistic value of variable *Combined_Forecast_1*, as follows:

$$Combined\_Forecast\_1 = \begin{cases} low & \text{if } Combined\_Forecast\_Map\_1 \leqslant -0.5, \\ medium & \text{if } -0.5 < Combined\_Forecast\_Map\_1 < 0.5, \\ high & \text{if } Combined\_Forecast\_Map\_1 \geqslant 0.5. \end{cases} \tag{7}$$

It has been shown that (5)–(7) generate valid and reasonable rule base modifications (details are given in the Appendix). For example, if both forecasts are the same, i.e., they are *low*, *medium* or *high*, then the combined forecast determined using (5) to (7) remains *low*, *medium* or *high*, respectively. If the two forecasts are different, then they are combined into one, depending on the confidence determined for the two forecasts. For example, it has been shown that in the case when *Customer_Forecast* is *low* and *Expert_Forecast* is *medium* then *Combined_Forecast_1* is *medium* if the confidence in the forecasts determined by the customer in the past is lower than the confidence in the forecasts given by the market expert, i.e., $con(C\_F) < con(E\_F)$. Or for example, if *Customer_Forecast* is *low* and *Expert_Forecast* is *high* then their combination *Combined_Forecast_1* is *low* if $con(C\_F) \geqslant 3 \cdot con(E\_F)$; it is *medium* if $(1/3) \cdot con(C\_F) < con(E\_F) < 3 \cdot con(C\_F)$; it is *high* if $con(C\_F) \leqslant (1/3) \cdot con(E\_F)$; and so on.

It is worth noting that the proposed learning mechanism is general. In the DSS_DF it is applied to the rules with two input linguistic variables that represent the two forecasts obtained by different methods and one output linguistic variable that represents the combined forecast. The impact that each forecast has on the combined forecast is determined based on the confidence in each of the forecast methods recorded in the past. The learning mechanism can, in principle, be applied to any fuzzy IF-THEN rules that model a relationship between two inputs and one output linguistic variable and the impact of each input variable on the output can be determined quantitatively. In addition, (6) can be modified to take into consideration different weights that may be associated with the input variables, where the weights represent the user preference with respect to the impact that a given input variable has on the output variable.

4. The potential of using DSS_DF

The DSS_DF is implemented using Visual C++ programming language and it runs under the Microsoft NT Windows environment on a PC.

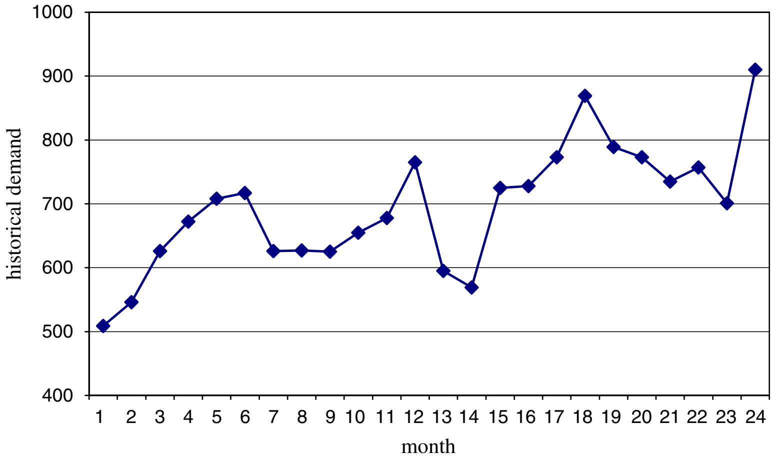

Fig. 4. Historical data that records monthly product demand.

The effectiveness and efficiency of the proposed DSS_DF have been tested using real data provided by a textile manufacturing company Cash's (UK) Ltd. The company operates in a dynamic environment in the presence of uncertainty in demands for products. The DSS_DF has been successfully used to forecast demand for two types of products where demand for one product has a dominant seasonal characteristic and demand for the other has been rather more erratic over the two-year period considered. In both cases, DSS_DF has generated forecasts combining two statistical forecasts obtained using the TSAD and ARMA methods and fuzzy market expert/customer forecasts. The DSS_DF provided satisfactory results which have been welcomed by the company. However, due to the need to respect the confidential nature of the data used whilst recognising the need to demonstrate the potential of DSS_DF in dealing with both types of forecasts, i.e., crisp and fuzzy, a hypothetical illustrative example of a time series and results obtained are presented.

4.1. Illustrative example

Assume a time series that records monthly demand for a final product for a period of two years is available as represented in Fig. 4.

Mamdani-style inference used to combine demand forecasts. It is instructive to demonstrate how DSS_DF generates the forecast for the next period, i.e., month 25. Let imprecise customer and expert forecasts for month 25 be expressed by the linguistic expressions "demand will be *about* 750 products" and "demand will be *about* 730 products", respectively. They are represented by fuzzy sets with trapezoidal membership functions, $C\_F_{[750,100,40]}$ and $E\_F_{[730,100,40]}$, respectively. The forecast generated by the TSAD method is $TSAD\_F = 667$ products, where the trend, seasonality and cyclical components, determined using all historical data available, are 817.3, 0.859 and 0.96, respectively. An ARMA(1,1,0) model is used to generate *ARMA_F* forecast and it indicates 849 products. Initially, the same trust is given to all forecasts generated by the customer, market expert, TSAD and ARMA methods. The corresponding rule bases include the rules outlined in Section 2.2.

Initially, the fuzzy sets that represent linguistic values *low*, *medium* and *high* for the four linguistic variables, namely *Customer_Forecast*, *Expert_Forecast*, *TSAD_Forecast*, and *ARMA_Forecast*, are determined using the corresponding historical data recorded over the last 6 months, i.e., from month 20 to 25 (with the new forecasts obtained for month 25). The corresponding fuzzy sets for the linguistic variables *Combined_Forecast_1*, *Combined_Forecast_2* and *Final_Forecast* are determined using the actual demands recorded in the past periods, i.e., from months 20 to 24. The lower and the upper bases of the trapezoidal membership functions are set empirically to 100 and 40, respectively.

The DSS_DF combined the four forecasts as illustrated in Fig. 5.

The two fuzzy forecasts, $C\_F_{[750,100,40]}$ and $E\_F_{[730,100,40]}$ are combined as follows. The compatibilities between $C\_F_{[750,100,40]}$ and the corresponding linguistic values *low*, *medium* and *high* customer forecasts are obtained using (2) and they are found to be 1, 0.01 and 0, respectively. In this case, $C\_F_{[750,100,40]}$ and *low* customer forecast coincide, and, therefore, their compatibility is 1 (see Fig. 6).

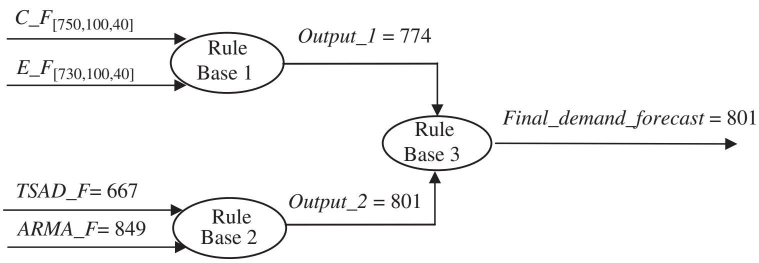

Fig. 5. The inference process in DSS_DF.

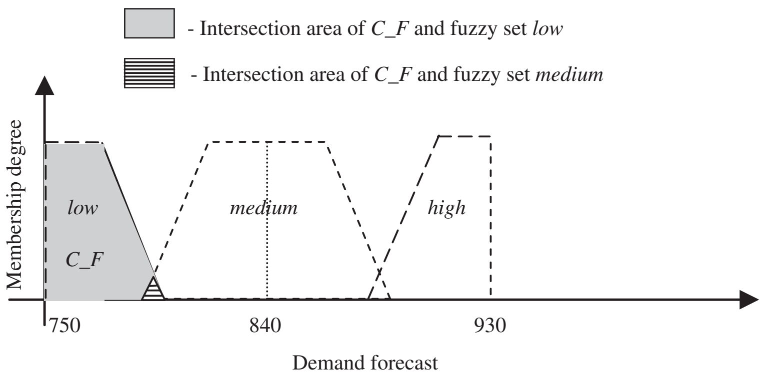

Fig. 6. Compatibility between *C_F* and fuzzy sets *low*, *medium* and *high* customer forecasts.

Table 2
Evaluation of the rules of Rule Base 1; truth values of input and the output linguistic variables are given in brackets

Combined_Forecast_1		*Expert_Forecast*		
		Low (1)	*Medium* (0.36)	*High* (0)
Customer_Forecast	*Low* (1)	*Low* (1)	*Low* (0.36)	*Medium* (0)
	Medium (0.01)	*Low* (0.01)	*Medium* (0.01)	*High* (0)
	High (0)	*Medium* (0)	*High* (0)	*High* (0)

Similarly, the compatibilities of $E\_F_{[730,100,40]}$ and the corresponding linguistic values *low*, *medium* and *high* expert forecasts are found to be 1, 0.36 and 0, respectively. The truth values of the IF parts of the fuzzy rules are determined using the Min operator, as represented in Table 2.

The rule outputs are then aggregated using the Max operator. This means that *Combined_Forecast_1* is *low* with degree equal to $\text{Max}\{1, 0.36, 0.01\} = 1$, it is *medium* with degree equal to $\text{Max}\{0, 0.01\} = 0.01$ and it is *high* with degree equal to $\text{Max}\{0\}$. Finally, the output fuzzy set, given in Fig. 7, is defuzzified, obtaining crisp $Output\_1 = 774$.

The additional two crisp forecasts, $TSAD\_F = 667$ and $ARMA\_F = 849$ are combined using a standard Mamdani-style of inference applied to Rule Base 2, as follows. First, *TSAD_F* is fuzzified to determine the degrees to which the crisp forecast belongs to the fuzzy sets *low*, *medium* and *high*, and they are found to be 0.95, 0.04 and 0, respectively (see Fig. 8). Similarly, the degrees to which *ARMA_F* belongs to the fuzzy sets *low*, *medium* and *high* forecasts are determined. The remaining steps of the Mamdani-style inference are executed, including rule evaluation, aggregation and defuzzification, and $Output\_2 = 801$ is obtained. Once crisp combined forecasts *Output_1* and *Output_2* are generated, they are combined into one *Final_demand_forecast* using a standard Mamdani-style of inference applied to Rule Base 3.

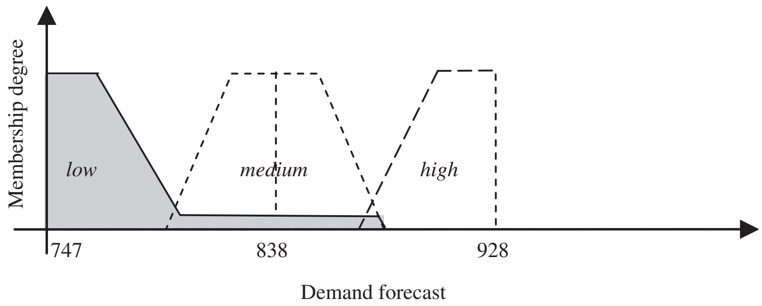

Fig. 7. Aggregation of the rule outputs.

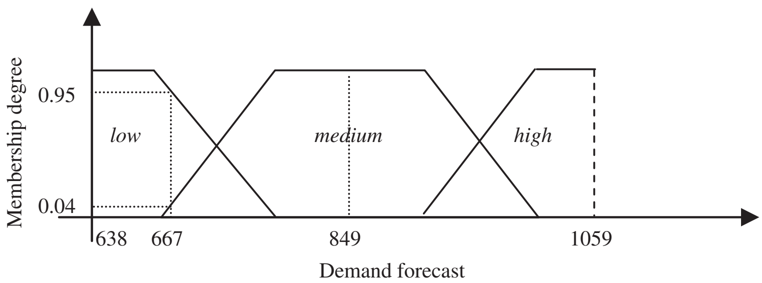

Fig. 8. Fuzzification of *TSAD_F*.

Table 3
C_F, *E_F*, *Output_1* and actual demand recorded over 6 month period

Time period (month)	*C_F*	*E_F*	*Output_1*	Actual demand
25	$C\_F_{[750,100,40]}$	$E\_F_{[730,100,40]}$	774	747
26	$C\_F_{[770,100,40]}$	$E\_F_{[750,100,40]}$	811	782
27	$C\_F_{[780,100,40]}$	$E\_F_{[760,100,40]}$	819	835
28	$C\_F_{[900,100,40]}$	$E\_F_{[770,100,40]}$	847	837
29	$C\_F_{[920,100,40]}$	$E\_F_{[780,100,40]}$	859	886
30	$C\_F_{[930,100,40]}$	$E\_F_{[820,100,40]}$	882	928

Learning mechanism. It is now illustrated how the learning mechanism, incorporated within the DSS_DF, modifies the rule bases by considering the fuzzy IF-THEN rules in Rule Base 1, that are used to combine the fuzzy customer forecast, *C_F*, and the fuzzy market expert forecast, *E_F*. Table 3 shows data generated for the period of 6 months, i.e., months 25–30, including fuzzy *C_F* and fuzzy *E_F*, their combined forecast *Output_1* generated using Rule Base 1 and actual demand recorded in the same period.

The learning mechanism uses the historical data recorded for the 6 month period to modify Rule Base 1, following the steps given in Section 3 as follows.

Step 1: The mean error generated by the customer in the six month period is 3.4%. Note that fuzzy customer forecasts are defuzzified in order to calculate the mean errors, where mean values of the support of the corresponding trapezoidal membership functions are used.

The confidence in Customer Forecast is determined using (3) and selecting $k = 15$ empirically: $con(C\_F) = 0.60$.

Step 2: The mean error generated by the expert in the 6 month period is 7.8% and the corresponding confidence is obtained as $con(E\_F) = 0.31$ with $k = 15$.

Step 3: The new relationship between linguistic variables *Customer_Forecast*, *Expert_Forecast* and *Combined_Forecast_1* determined using equations (5)–(7) are given in Table 4.

Table 4
Relationship between *Customer_Forecast*, *Expert_ Forecast* and *Combined_ Forecast_1*

Customer_Forecast	*Expert_Forecast*	*Combined_Forecast_Map_1*	*Combined_Forecast_1*
Low	*Low*	−1	*Low*
Low	*Medium*	−0.66	*Low*
Low	*High*	−0.32	*Medium*
Medium	*Low*	−0.34	*Medium*
Medium	*Medium*	0	*Medium*
Medium	*High*	0.34	*Medium*
High	*Low*	0.32	*Medium*
High	*Medium*	0.66	*High*
High	*High*	+1	*High*

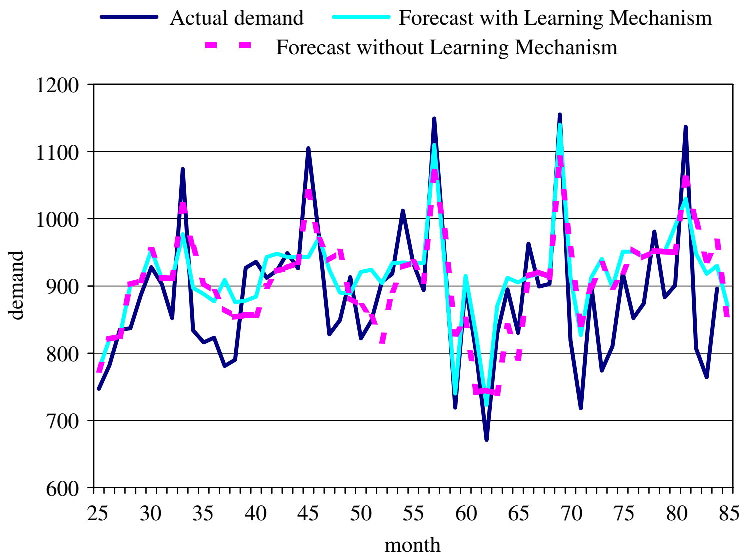

Fig. 9. Actual demand and demand forecasted by DSS_DF with and without learning.

The relationship between linguistic variables *Customer_Forecast* and *Expert_Forecast* is now changed. Note, for example, that *medium Customer_Forecast* and *low Expert_Forecast* now lead to *medium Combined_Forecast_1*, rather than *low*, as a consequence of the customer forecasts performing better to a certain degree than the expert forecasts in the 6 month period considered.

4.2. Comparison between DSS_DF forecasts generated with and without the learning mechanism

In order to test the effectiveness of the learning mechanism built into DSS_DF, the following experiment is performed. For comparison purposes, the DSS_DF is first used to generate monthly forecasts for a period of five years, using the historical data given in Section 4.1, in the absence of the learning mechanism. The process is then repeated with the learning mechanism implemented. In the latter case, the rule bases are modified every month based on the performance of each of the forecast methods recorded in the previous six months. The parameters of the statistical methods, namely the trend, seasonality and cyclical components for the TSAD method, and p, d, and q for the ARMA method are modified each month, taking into consideration new data, i.e., actual demand that becomes available. Results presented in Fig. 9 clearly show the benefit of the adaptive rule base which arises when applying the learning mechanism. The mean forecasting error is reduced from 7.1% to 5.8% per month by making use of the learning mechanism.

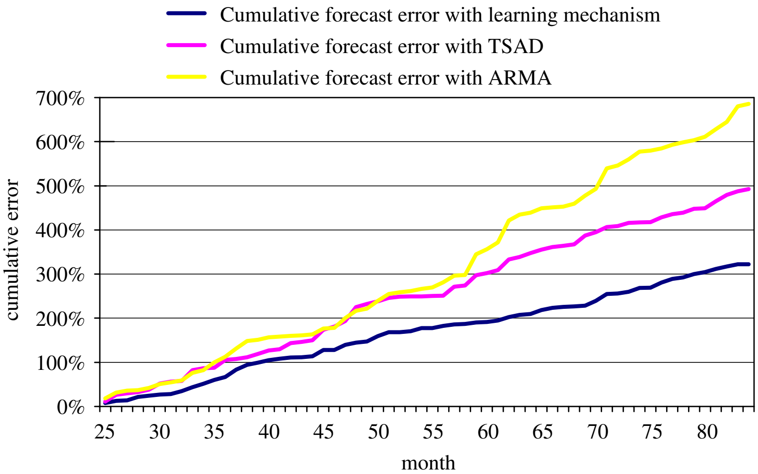

Fig. 10. Cumulative forecast errors generated by DSS_DF, TSAD and ARMA.

4.3. Comparison between DSS_DF forecasts and forecasts obtained by individual forecast methods

The forecasts generated for the same time series over the period of five years using DSS_DF with the learning mechanism is compared with the forecasts generated by the TSAD and ARMA methods, separately. The mean forecasting errors generated by TSAD and ARMA methods are 7.4% and 10.9% per month, respectively. Fig. 10 presents the cumulative forecast errors generated by each of these forecasting methods. It should be noted that, generally, standard forecasting methods, such as TSAD and ARMA, require careful selection and tuning of their parameters, and, therefore, the experience of the user is of great importance in achieving high forecasting accuracy. This example demonstrates that combining the forecast values may lead to a better forecast than by applying each of the methods separately. It is a form of adaptive data fusion.

5. Conclusion

In this paper, a new fuzzy-logic based decision support system for product demand forecasting DSS_DF with an adaptive learning mechanism is presented. The DSS_DF is flexible and able to handle various types of information by applying different forecasting methods. Both fuzzy and crisp forecasts can be obtained. These forecasts are combined into one improved forecast value using fuzzy IF-THEN rules. The DSS_DF represented in this paper operates with four forecasts as inputs, however, it can combine any number of available forecasts into a single improved forecast. Consequently, the number of corresponding rule bases can be increased to accommodate combinations of additional forecasts. These features make the proposed DSS_DF widely applicable, which is considered highly relevant since companies usually keep different types of historical data, recorded in different formats. In addition, a new adaptive learning mechanism is embedded into the DSS_DF that dynamically modifies the fuzzy IF-THEN rules, based on the performance of each of the forecasting methods recorded in the past.

Some of the advantages of using DSS_DF over traditional forecasting methods are outlined as follows:

1. DSS_DF takes into consideration both historical crisp data and linguistic information such as customer forecasts and marketing and sales personnel expectations, while traditional forecasting methods rely on historical crisp data only.
2. DSS_DF combines forecasts generated by different forecasting methods in a way that may provide improved accuracy over each of the methods in isolation.
3. The learning mechanism built into DSS_DF enables dynamic adaptation of the fuzzy rule bases that combine the forecasts obtained by different methods, based on their performance recorded in the past.

Future work will focus on:

(a) implementing other fuzzy inference mechanisms in the DSS_DF and analysing their effects on combining different forecasts in fuzzy rule chains;
(b) analysing the effects of the parameters involved in the learning mechanism, such as the number of past periods that are considered in the learning process, and the parameter k to determine confidence in a forecasting method;
(c) analysing the impact of introducing weights to forecasts generated for different periods, used by the learning mechanism; and
(d) investigating effects of tuning rule bases by increasing the number of fuzzy values of the linguistic variables involved, e.g., from three (such as *low*, *medium*, *high*) to five (e.g., *very low*, *low*, *medium*, *high*, *very high*).

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Appendix

It has been shown that formulae (5)–(7) generate valid and reasonable rule modifications for all combinations of the fuzzy values of the corresponding linguistic variables. This will be shown using, as an example, rules in Rule Base 1, which involve three linguistic variables, *Customer_Forecast*, *Expert_Forecast* and *Combined_Forecast_1*.

In the case, when both *Customer_Forecast* and *Expert_Forecast* have the same fuzzy value (i.e., both are *low*, *medium* or *high*), then their combined value should be the same, i.e., the value of *Combined_Forecast_1* should also be *low*, *medium* or *high*, respectively. Indeed, if, for example, both *Customer_Forecast* and *Expert_Forecast* are *low*, then $Customer\_Forecast\_Map = -1$ and $Expert\_Forecast\_Map = -1$ and

$$Combined\_Forecast\_Map\_1 = \frac{-con(C\_F) - con(E\_F)}{+con(C\_F) + con(E\_F)} = -1$$

and, therefore,

$$Combined\_Forecast\_1 = low.$$

Similarly, it has been shown that combinations of *medium* or *high Customer_Forecast* and *medium* or *high Expert_Forecast* generate *medium* or *high Combined_Forecast_1*, respectively.

Further, if *Customer_Forecast* is *low* and *Expert_Forecast* is *medium*, then $Customer\_Forecast\_Map = -1$ and $Expert\_Forecast\_Map = 0$, and

$$Combined\_Forecast\_Map\_1 = \frac{-con(C\_F)}{+con(C\_F) + con(E\_F)}.$$

It is easy to show that $Combined\_Forecast\_Map\_1 < 0.5$. In addition, $Combined\_Forecast\_Map\_1 > -0.5$ if $con(C\_F) < con(E\_F)$, i.e., $|error(C\_F)| > |error(E\_F)|$. This means that *Combined_Forecast_1* is *medium* when $|error(C\_F)| > |error(E\_F)|$, otherwise it is *low*.

An interesting case is when *Customer_Forecast* is *low* and *Expert_Forecast* is *high*. Then, $Customer\_Forecast\_Map = -1$ and $Expert\_Forecast\_Map = +1$, and

$$Combined\_Forecast\_Map\_1 = \frac{-con(C\_F) + con(E\_F)}{+con(C\_F) + con(E\_F)}.$$

It has been shown that $Combined\_Forecast\_Map\_1 \leqslant -0.5$, and therefore, *Combined_Forecast_1* is *low*, if $con(C\_F) \geqslant 3 \cdot con(E\_F)$, i.e., $|error(C\_F)| - |error(E\_F)| \geqslant \ln 3/k$. Similarly, $-0.5 < Combined\_Forecast\_Map\_1 < 0.5$, and, therefore, *Combined_Forecast* is *medium*, if $con(C\_F) < 3 \cdot con(E\_F)$, and $con(C\_F) > (1/3) \cdot con(E\_F)$, i.e.,

$$|error(C\_F)| - |error(E\_F)| < \frac{\ln 3}{k} \quad \text{and} \quad |error(C\_F)| - |error(E\_F)| > \frac{\ln(1/3)}{k}.$$

Finally, $Combined\_Forecast\_Map\_1 \geqslant 0.5$, and therefore, *Combined_Forecast* is *high*, if $con(C\_F) \leqslant (1/3) \cdot con(E\_F)$, i.e., $|error(C\_F)| - |error(E\_F)| \leqslant \frac{\ln(1/3)}{k}$.

Similarly, it has been shown that the learning mechanism generates reasonable rule modifications for all other combinations of fuzzy values of the linguistic variables *Customer_Forecast* and *Expert_Forecast*.

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