AGGREGATING FORECASTS TO OBTAIN FUZZY DEMANDS

ÖZGÜR KABAK
İstanbul Technical University, Management Faculty, Industrial Engineering Department, Macka 34357, Istanbul, Turkey – kabak@itu.edu.tr

FÜSUN ÜLENGİN
Dogus University, Engineering Faculty, Industrial Engineering Department, Zeamet Sokak, Acıbadem, 34722, Kadıköy / Istanbul, Turkey – fulengin@dogus.edu.tr

There are several studies in the literature that assumes fuzzy demands in supply chain or production planning models but most of them do not mention about how to derive the fuzzy demands from statistical and judgmental forecasts. In this study we propose a methodology to aggregate the forecasts coming based on different sources; namely statistical methods as well as the experts judgments, and to obtain an aggregated demand forecast that is represented by a possibilistic distribution. Results of the statistical and judgmental forecasts are represented by triangular possibilistic distributions. Subsequently, those results are combined by using weights of each input forecast. An illustrative example is also provided.

1. Introduction

Demand forecast is one of the most important inputs of production planning and supply chain planning models. In the manufacturing process, demands are forecasted to perform the basic planning activities such as capacity planning, resource planning, and raw material purchasing. Demand forecasts are of great importance for marketing activities and personnel management and its accuracy directly affects the profitability of the company. Underestimated demand causes raw-material and final product stockouts that surely leads to possible profit losses while overestimated demand results with increases in inventories that will cause an increase in the holding costs [1-3].

Due to the fact that it implies judgments about the future, demand forecasting is usually subject to uncertainties. Therefore in several studies in the literature (e.g. [4-7]) the demands are assumed to be fuzzy numbers or represented via possibilistic distributions for production and supply chain planning models. However there is no detailed procedure to generate fuzzy demands from the historical and/or judgmental data. For instance, in Petrovic [4] and Petrovic et al. [7] demands are specified based on managerial experience and subjective judgment by using linguistic expressions. Wang and Shu [5] represented the demands by six-point fuzzy number but did not mention the estimation procedure. Zhou and Liu [6] assumed the fuzzy demands are known.

In the literature, one of the important demand forecasting models taking into account both historical data and expert judgments is of Petrovic et al.'s [2]. It presented a new decision support system for demand forecasting where results of two linguistic forecasts and two conventional statistical methods are combined to generate a composite demand. The drawbacks of the model are crisp output and predetermined types of statistical and judgmental inputs.

In our previous work [8] a possibilistic linear programming model is proposed to make long-term resource allocation and outsourcing decisions and the demands of the products are considered to be triangular fuzzy numbers. Thus, in this study we propose a methodology to generate fuzzy demands by evaluating historical data and judgmental expressions for an input to our previous model as well as the other models that assumes fuzzy or possibilistic demands. In the second section, the details of the proposed methodology are given. Then, a numerical example is used to illustrate the working procedure of the model. Finally conclusions and further suggestions are provided.

2. Proposed Demand Forecast Methodology

A range of forecasting models and statistical techniques are available for demand forecasting. The source of the forecasts can be grouped into two categories. The first one is the historical data of the corresponding product, some other products, and the related industrial and macroeconomic variables. Statistical methods are used to analyze historical data. The second source is judgmental expressions of product and/or market experts. It is especially preferred when the historical data is either not available or not very relevant [1].

The proposed methodology combines the results of the statistical forecasts and judgmental forecasts using fuzzy logic-based aggregation procedure. For this purpose, initially, the forecasts generated by statistical methods are represented by triangular possibility distributions (TPDs). Then all the forecast results as well as the historical data are presented to experts to make a basis for their judgments. The experts indicate their judgments as optimistic, expected, and pessimistic values. If no historical data is available, the procedure starts with judgmental forecast based solely on opinions of experts. The judgmental forecasts are also represented by TPDs. Finally, all the forecast results obtained through multiple sources are aggregated using the weights that indicate the importance of the sources. Figure 1 shows the basic steps of the methodology.

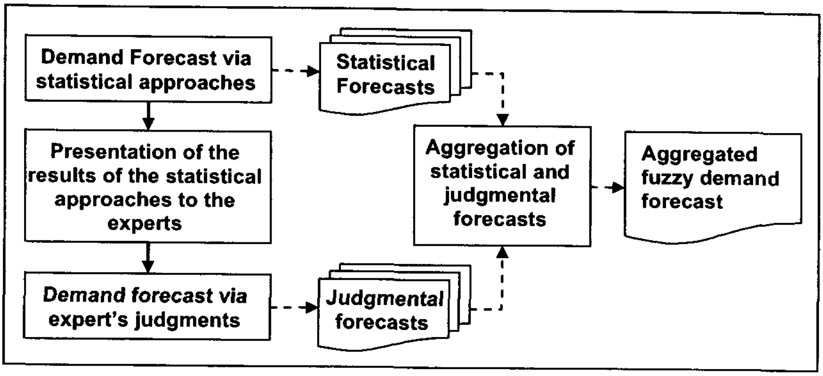

Figure 1. Proposed methodology

2.1. *Demand forecast via statistical approaches*

Statistical approaches for demand forecasts includes classical methods such as moving average, exponential smoothing, trend estimation as well as time series models such as Autoregressive Integrated Moving Average (ARIMA) etc., and causal models such as regression models etc.

In general, when a forecast is made with a statistical approach, the result is given with expected value (f^e) and a confidence interval on the bases of α significance level ($\left[f_\alpha^l, f_\alpha^r\right]$). These results may be though as the most possible (the expected value), pessimistic and optimistic (left and right boundaries of the confidence interval) outcomes. Therefore, the demand forecast with s^{th} (s=1,...,S) statistical approach can be represented by a TPD, $\tilde{D}_s = \left(D_s^p, D_s^m, D_s^o\right)$, as follows:

$$D_s^o = f_{s\alpha}^r,\ D_s^p = f_{s\alpha}^l,\ D_s^m = f_{s\alpha}^e$$

where s is an index for statistical methods, p, m, and o indicate pessimistic, most possible, and optimistic demands, respectively and α is the significance level that is determined by the decision maker. In the proposed method the number and type of statistical approach to generate demand forecast is up the decision maker. All the results obtained through different approaches may be used in the model either individually, or by aggregating them in one result.

2.2. *Demand forecast via expert judgments*

The results of the statistical methods as well as the crude historical data are presented to experts to inform them about the data output. Experts are then requested to indicate their opinions about the future demands in pessimistic, optimistic and most possible perspectives. If the judgments are made in a group meeting, the experts may indicate their opinions individually or they may reach a consensus result. The resulting demand forecasts are represented by TPD as $\widetilde{D}_j = \left(D_j^p, D_j^m, D_j^o\right)$, where j is an index for different experts (j =1,...,J).

2.3. *Aggregation of statistical and judgmental forecasts*

All the forecast values coming from different sources are combined into an aggregated fuzzy forecast. One important feature added to aggregation process of the model is the weights of the sources that show the importance of the forecast coming from each different source. It is obvious that the importance of the experts as well as of the statistical methods may be different from each other. For instance if the uncertainty of the related the market is high, then the importance of the market expert will be higher. Besides, the standard errors of the statistical methods may be a reference to show the power of estimation, and thus the importance of the method. The problem owner decides on the weights.

In the proposed method, a five-point likert scale is suggested to evaluate the importance of the sources. The linguistic expressions and their numerical equivalent are very low (1), low (2), medium (3), high (4), and very high (5).

The aim of the aggregation procedure is to obtain a TPD by combining all the TPDs coming from different sources in the previous steps. In that perspective, the problem is converted to find an optimum TPD that has the minimum weighed distance from several TPDs. If $\widetilde{D}$ shows the aggregated demand, the problem can be formulated as follows

$$\text{Find } \widetilde{D} = (D^p, D^m, D^o)$$

$$\text{Minimizing } TotDist = \left(\sum_s w_s * Dist(\widetilde{D}_s, \widetilde{D}) + \sum_j w_j * Dist(\widetilde{D}_j, \widetilde{D}) \right) \tag{1}$$

where $Dist()$ is a function that indicates distance between two possibility distributions, w_s, is the weight of s^{th} statistical method (s=1,...S), w_j, is the weight of j^{th} expert (j=1,...,J).

Among the several distance measures in the literature, we select the following λ-cut based distance measure due to its applicability to the optimization of TPDs. For two arbitrary possibility distributions $\widetilde{A}$ and $\widetilde{B}$ with λ-cut sets, $\left[A^-(\lambda), A^+(\lambda)\right]$ and $\left[B^-(\lambda), B^+(\lambda)\right]$, [9]

$$Dist(\widetilde{A}, \widetilde{B}) = \int_0^1 \left[A^-(\lambda) - B^-(\lambda)\right]^2 d\lambda + \int_0^1 \left[A^+(\lambda) - B^+(\lambda)\right]^2 d\lambda \tag{2}$$

For a TPD, A = $\left(A^p, A^m, A^o\right)$ λ-cut levels can be found as follows:

$$A^{-}(\lambda) = A^{m} - (1-\lambda)(A^{m} - A^{p}) \qquad A^{+}(\lambda) = A^{m} + (1-\lambda)(A^{o} - A^{m}) \tag{3}$$

By using these values distance measure can be updated as follows:

$$Dist(\widetilde{A},\widetilde{B}) = \begin{array}{l} \int_0^1 \left[A^{m} - (1-\lambda)(A^{m} - A^{p}) - B^{m} + (1-\lambda)(B^{m} - B^{p})\right]^2 d\lambda + \\ \int_0^1 \left[A^{m} + (1-\lambda)(A^{o} - A^{m}) - B^{m} - (1-\lambda)(B^{o} - B^{m})\right]^2 d\lambda \end{array} \tag{4}$$

To solve the problem given in Eq. (1) with the given distance function (Eq. 4), partial derivatives of the function with respect to three parameters of the aggregated demand ($\widetilde{D} = (D^{p}, D^{m}, D^{o})$) is get and it is equaled to zero [9].

$$\frac{\partial TotDist}{\partial D^{i}} = \left(\sum_{s} w_{s} * \frac{\partial Dist(\widetilde{D}_{s},\widetilde{D})}{\partial D^{i}} + \sum_{j} w_{j} * \frac{\partial Dist(\widetilde{D}_{j},\widetilde{D})}{\partial D^{i}} \right) = 0 \quad (i=p,m,o) \tag{5}$$

The resulting values of these partial derivatives are given below:

$$D^{i} = \left(\sum_{s} D_{s}^{i} * w_{s} + \sum_{j} D_{j}^{i} * w_{j} \right) \Big/ \left(\sum_{s} w_{s} + \sum_{j} w_{j} \right) \quad (i=p,m,o) \tag{6}$$

The resulting aggregated demand function is the weighted average of the input demand forecasts.

3. Numerical Example

To illustrate the proposed methodology an example referred in Petrovic et al. [2] is used. A time series that records monthly demands for a product for a period of two years is given: {506; 545; 626; 674; 707; 716; 625; 625; 625; 655; 679; 765; 595; 570; 725; 727; 772; 870; 790; 773; 735; 756; 701; 911}.

Using STATGRAPHICS Centurion software package, three different time series analysis techniques, namely ARIMA(1,1,0), linear trend, and simple exponential smoothing, are applied to given data. These methods are chosen because the first one is refereed in Petrovic et al., and other two gives best results w.r.t. root mean square error (RMSE). The results are shown in Table 1.

According to the results (Table 1), the corresponding TPDs are determined:

$$\widetilde{D}_1 = (720,889,1057);\ \widetilde{D}_2 = (662,818,973);\ \widetilde{D}_3 = (692,844,997) \tag{7}$$

To illustrate the judgmental forecasts, the expert's opinions given in Petrovic et al. [2] are used. As a customer forecast, the expected demand is 750 while the pessimistic and optimistic forecasts are 700 and 800 respectively. The same values for the expert forecast are 730, 680, and 780, respectively. Therefore the related TPDs are as follows:

$$\tilde{D}_1 = (700,750,800);\ \tilde{D}_2 = (680,730,780) \tag{8}$$

Table 1. Results of the statistical methods

s	Method	Forecast for the 25^{th} month	%95 confidence interval for forecast	RMSE
1	ARIMA (1,1,0)	889	[720;1057]	81,12
2	Linear trend	818	[662, 973]	67,67
3	Simple Exponential smoothing	844	[692, 997]	79,32

Finally, the weights are assumed to be medium, very high, medium for the statistical results, respectively, and high, for both of the judgmental results. When Eq. (6) is employed, the aggregated demand is found to be $\tilde{D}(688,800,913)$. When compared to the crisp result of Petrovic et al [2], 774, and the actual demand, 747, the result of the proposed model is satisfactory.

4. Conclusions

In this study a methodology to aggregate forecasts of different sources is proposed in order to reach a combined forecast that is represented by TPDs. The results of the model can be used in production and supply chain planning models. It is also beneficial to have range estimation rather than point estimation in such uncertain phenomena.

Related future studies may be about getting more accurate information from the experts and determining the weights of the different forecasts.

References

1. J.F. Shapiro, Modeling the Supply Chain. *Duxbury Thomson*, USA. (2001)
2. D. Petrovic, Y. Xie, K. Burnham, *Fuzzy Sets&Sys.* **157**, 1713 – 1725 (2006)
3. H.-M. Hsu, W.-P. Wang, *Fuzzy Sets and Sys.* **119**, 59-70 (2001)
4. D. Petrovic, *Int. J. of Production Economics* **71**, 429-438 (2001)
5. J. Wang, Y.-F. Shu, *Fuzzy Sets and Systems* **150,** 107–127 (2005)
6. J. Zhou, B. Liu, *Computers & Industrial Engineering* **53**, 454–468 (2007)
7. D. Petrovic, Y. Xie, K. Burnham, R. Petrovic, *European Journal of Operational Research* **185**, 146–158 (2008)
8. Ö. Kabak, F. Ülengin, *IFAC Sym. Cefis, 9-11 Oct., Istanbul*, 38-43 (2007)
9. W. Zeng, H. Li, *Int. J. of Approximate Reasoning* **46**, 137–150 (2007)