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Shop floor lot-sizing and scheduling with a two-stage stochastic programming model considering uncertain demand and workforce efficiency

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ABSTRACT

Efficient and flexible production planning is necessary for the manufacturing industry to stay competitive in today's global market. Shop floor lot-sizing and scheduling is one of the most challenging and rewarding subjects for the management. In this study, a two-stage stochastic programming model is proposed to solve a single-machine, multi-product shop floor lot-sizing and scheduling problem. Two sources of uncertainties – product demand from the market, and workforce efficiency – are considered simultaneously, which is the major contribution of this study. The workforce efficiency affects the system productivity, and we propose different distributions to model its uncertainty given insufficient information. The model aims to determine optimal lot sizes and the production sequence that minimizes expected total system costs over the planning horizon, including setup, inventory, and production costs. A case study is performed on a supply chain producing brake equipment in the automotive industry. The numerical results illustrate the usefulness of the stochastic model under volatile environment, and the solution quality is analyzed.

1. Introduction

Production planning plays an essential role in the effective and economic operation of a manufacturing unit. In general, production planning aims to achieve effective utilization of resources; ensure steady flow of production and optimal inventory; improve labor productivity and product quality; enhance consumer satisfaction; reduce production costs; and thus capture the market when facing competition. Production planning is commonly adopted in scheduling, dispatch, capacity planning, quality management, inventory management, supply management, and equipment management (Chan & Prakash, 2012; Dal-Mas, Giarola, Zamboni, & Bezzo, 2011; Kanyalkar & Adil, 2008; Phan, Abdallah, & Matsui, 2011). Manufacturing firms consider three time ranges for decision-making: long-term (e.g., facility design and process choices), medium-term (e.g., capacity planning and material requirements planning), and short-term planning (e.g., day-to-day operations and job control) (Karimi, Ghomi, & Wilson, 2003). Lot-sizing and scheduling problem is applicable in medium- to short-term planning.

Lot-sizing and scheduling is one of the most challenging subjects in production planning (Almada-Lobo & James, 2010). The lot-sizing problem determines how much to produce of each product in each planning period. The scheduling problem determines the order of production lots to mitigate influences of setup time and costs in the manufacturing system. The decisions of lot-sizing and scheduling are made to meet the final product demand requirements and to minimize system costs, including setup, production, and holding costs. These operational strategies directly affect system performance, such as the utilization rate and the productivity of the shop floor, and thus are essential to enhancing a company's competitiveness in the market. The idea of incorporating uncertainty in mathematical models applied in lot-sizing and scheduling could significantly improve the decision-making, and provide more robust and stable scheduling decisions.

This paper focuses on the lot-sizing and scheduling decision-making in production planning considering uncertain demand and workforce efficiency data simultaneously, which could arise in many manufacturing companies. The uncertainty in demand is common and production plans usually rely on demand forecast, based on historical demand data, as well as the insight into market prospect. The uncertainty in workforce efficiency could be caused by different operational issues (e.g., proficiency of workers, parts availability). Both uncertain factors might cause a company not

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having sufficient production capability to meet the demand. To cope with volatile demand and production efficiency, we make it possible to deliver products later than their demand periods, and backorder costs will be incurred. A two-stage stochastic programming model is proposed to assist in the decision making to minimize total system costs.

A case study for a manufacturing company producing braking equipment in an automotive industry is conducted. Two sources of uncertainty are considered simultaneously. While demand uncertainty has fitted distributional representations based on historical data, uncertainty in workforce efficiency is modeled with insufficient information from experts' experiences. Given the circumstance, the impact of workforce efficiency is further discussed considering different choices of distributional models, with sensitivity analysis on distribution parameters. Computational experiments are implemented to demonstrate the effectiveness of the model compare to the deterministic model, and the solution qualities are analyzed. In particular, suggestions on scenario set cardinality is discussed, which is an important factor that balances the computational effort and solution quality.

The remainder of this paper is organized as follows. We review the related literature to lot-sizing and scheduling problem in Section 2. Section 3 formulates the scheduling and lot-sizing problem as a two-stage stochastic programming problem. The numerical results and discussions are included in Section 4. Section 5 concludes the paper and proposes directions of further research.

2. Related literature

Lot-sizing and scheduling problem has been studied extensively in the literature (Drexl & Kimms, 1997; Jans & Degraeve, 2008), and has been applied in many real-world industries, including a soft drink plant (Ferreira, Morabito, & Rangel, 2009), a diary company (Amorim, Antunes, & Almada-Lobo, 2011), a pharmaceutical company (Stadtler, 2011), and a metal foundry (Hans & Velde, 2011), to name a few.

According to Karimi et al. (2003), there are many variants of lot-sizing and scheduling problems. A simple economic order quantity (EOQ) model deals with single-level production without capacity constraints (Erlenkotter, 1990). Developed upon the EOQ model, the economic lot scheduling problem (ELSP) considers optimal sharing of scarce resources in a capacitated single-level, multi-item problem, while keeping the model with continuous time infinite planning horizon (Rogers, 1958); Wagner-Whitin (WW) problem assumes a discrete time finite planning horizon, while keeping the model without capacity limits (Wagner & Whitin, 1958). Extending the WW problem by including capacity constraints, we have the capacitated lot-sizing problem (CLSP or a large bucket problem), where multiple items may be produced per period. The following variants integrated lot-sizing with scheduling decision-making. Divide the macro-periods into several micro-periods, we have a discrete lot-sizing and scheduling problem (DLSP, or a small bucket problem) (Fleischmann, 1990). DLSP is developed with the so-called 'all-or-nothing' assumption, meaning only one item may be produced per micro-period, and, if so, full capacity will be applied. The continuous setup lot-sizing problem (CSLP) relaxed the 'all-or-nothing' assumption in DLSP, while still restricting that only one item could be produced in each period, i.e., the use of partial capacity is allowed, which is a shortcoming of CSLP. To fulfill the utilization of remaining capacity, the proportional lot-sizing and scheduling problem (PLSP) allows a second item being produced in one micro-period, i.e., the setup of a machine could change at most once in a micro-period (Kaczmarczyk, 2011). A further generalization is called the general lot-sizing and scheduling problem (GLSP), which allows multiple lots per period, where the maximum number of lots is user-defined (Fleischmann & Meyr, 1997).

Most scheduling problems in practice involve setup times and costs. In general, setup implies the activities that are required to prepare a machine to produce an item of a given type, including setting jigs and fixtures, adjusting tools, and acquiring materials. Shim, Kim, Doh, & Lee (2011) proposed an two-stage heuristic for the CLSP with sequence-dependent setup costs. Their heuristic suggests that after an initial solution is obtained, it is improved with a backward and forward improvement method with various priority rules to select the items to be moved among the periods.

Parallel machine setup is another type of extension that could complicate the model. Marinelli, Nenni, & Sforza (2007) illustrated a capacitated lot-sizing and scheduling problem with unrelated parallel machines with shared and capacitated buffers. The model was formulated as a hybrid continuous setup and capacitated lot-sizing problem, and solved with a two-stage heuristic approach. The model was applied in a yogurt packaging company. Quadt & Kuhn (2009) addressed a capacitated lot-sizing and scheduling problem with setup times, setup carry-over, and back-orders on identical parallel machines in a semiconductor assembly facility. The authors presented a MIP model and a solution procedure based on a novel "aggregate model." Afzalirad & Rezaeian (2016) addressed an unrelated parallel machine scheduling problem with resource constraints, sequence-dependent setup times, different release dates, machine eligibility, and precedence constraints. Two new meta-heuristic algorithms including genetic algorithm (GA) and artificial immune system (AIS) are developed to find optimal or near optimal solutions for this pure integer model.

Special modeling requirements should be taken into account given product characteristics. For example, Amorim et al. (2011) applied the lot-sizing and scheduling problem to perishable products (yogurt). To consider the trade-off between freshness of delivered product and total costs, the problem was extended to solving multi-objective models.

While the deterministic lot-sizing and scheduling assumes all the information that defines a problem instance is known with certainty in advance, in the real world, a production process can be affected by many forms of uncertainty. Ho (1989) categorized the uncertain factors into two groups: (1) environmental uncertainty, such as demand and supply uncertainty, and (2) system uncertainty, such as operation yield, quality, and system failure uncertainty. Therefore, a straightforward extension assumes that some of the problem data are subject to random fluctuations.

Brandimarte (2006) proposed a multi-stage stochastic programming approach for multi-item capacitated lot-sizing with uncertain demand; a time-sweep-based heuristic solution strategy is applied to solve the large-scale mixed integer linear programming model. Helber, Sahling, & Schimmelpfeng (2013) dealt with a multi-item stochastic capacitated lot-sizing problem under δ-service-level measure. The nonlinear functions of backlog and inventory are approximated with two different linear models, and the piecewise linear model is solved with a MIP-based heuristic. Lu, Cui, & Han (2015) addressed a problem of finding a robust and stable schedule for a single machine with availability constraints. A proactive approach generating a long-term initial schedule under failure uncertainty, which jointly determines the production planning and preventive maintenance (PM) is proposed.

To summarize the literature review, most of the previous researches deal with deterministic lot-sizing and scheduling problems, those consider stochasticity typically consider one uncertain factor. In our paper, two sources of uncertainties are taken into consideration simultaneously. The modeling of uncertainty with limited information is represented when modeling workforce efficiency uncertainty, and the impact from insufficient data is also discussed.

3. Model formulation

This paper studies a single-machine multi-product scheduling and lot-sizing problem with demand and workforce efficiency uncertainties. The goal is to find a robust production schedule and optimal lot-sizing over a planning horizon that minimize the expected total system costs. The model in this paper is based on Goplakrishnan (2000), with modifications in incorporating uncertain factors and subtour elimination constraints.

3.1. Problem description

A two-stage stochastic programming model is adopted to formulate this single-machine multi-product production planning problem under uncertain demand and workforce efficiency. In this two-stage stochastic programming model, the production schedule is the first stage decision, which is determined under all scenarios for the planning horizon, while the lot-sizing, inventory, and backorder amounts are the second stage decisions that are planned given observed demand and employees' proficiency. Main assumptions used in the model are listed as follows:

- Inventory, overtime production, and backorder are allowed to hedge against volatility in demand and workforce efficiency.
- The production capacity is embodied in regular manufacturing time capacity.
- Minimum safety stock is not required.
- The changeover towards each type of product could at most occur once in each period; the setup state could carry over to the subsequent period.

3.2. Mathematical notation

The notation used in the model formulation is listed below.

Sets

$\mathcal{N}$	Product type, $\mathcal{N} = \{1, 2, \ldots, N\}$
$\mathcal{T}$	Planning horizon, $\mathcal{T} = \{1, 2, \ldots, T\}$
$\mathcal{S}$	Scenario set, $\mathcal{S} = \{1, 2, \ldots, S\}$

Parameters

p_s	Probability associated with scenario s
d_{its}	Demand of product type i in period t under scenario s
c_i^r	Cost of producing one product i in regular time ($)
c_i^o	Cost of producing one product i in overtime ($)
c_i^b	Backorder cost for one product i ($)
c_i^h	Cost of holding one product i for one period ($)
c_{ij}^c	Changeover cost for switching from product i to product j ($)
τ_{its}	Processing time for producing one unit of product i in period t under scenario s
τ_{ij}^c	Changeover time for switching from product i to product j
q_t	Production time capacity in period t
α	Maximum overtime ratio, $\alpha = 0.2$
β	Minimum percentage of capacity that must be maintained, $\beta = 0.6$
I_{i0}	Initial inventory for product i
Q_{i0}^b	Initial backorder amount for product i
Z_{i0}	Initial setup status, indicate which product the machine is initially setup for

Decision variables

X_{ijt}	First stage decision variable, binary, 1 if changeover from product i to product j occurs during period t, 0 otherwise
Y_{it}	First stage decision variable, integer, production sequence of product i in period t
Z_{it}	First stage decision variable, binary, 1 if the machine is set-up for product i at the end of period t, 0 otherwise
Q_{its}^r	Second stage decision variable, number of type i product produced in regular time in period t under scenario s
Q_{its}^o	Second stage decision variable, number of type i product produced overtime in time period t under scenario s
Q_{its}^b	Second stage decision variable, backorder quantity of type i product at the end of time period t under scenario s
I_{its}	Second stage decision variable, product inventory at the end of period t under scenario s

3.3. Objective function

The objective function is to minimize the total expected system costs over the planning horizon, including the regular time and overtime production costs, costs incurred from backorder, final product inventory cost, and machine changeover cost. The objective function is presented in Eq. (1):

$$\min z = \sum_{s\in\mathcal{S}} p_s \left[\sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} \left(C_i^r Q_{its}^r + C_i^o Q_{its}^o + C_i^b Q_{its}^b + C_i^h I_{its} \right) \right] + \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}, j\neq i} C_{ij}^c X_{ijt} \tag{1}$$

3.4. Constraints

In this lot-sizing and scheduling problem, the expected cost of the production system is minimized, as stated in the objective function, subject to the following constraints.

Product inventory balance constraints. Constraints (2) and (3) state that for each type of product, the beginning inventory plus regular time and overtime production amount during the current period equals the demand plus ending inventory. For the first decision period, the beginning inventory and backorder amount are taken from the actual initial parameter setup, as in Eq. (2).
Notice that under scenario s, at the end of period $t \in \mathcal{T}$, there is either excess or shortage in the production amount, but not both, i.e., at most one of I_{its} and Q_{its}^b could be strictly positive. Although this set of constraints does not restrain inventory and backorder amounts from both being strictly positive, the holding of this property is guaranteed due to the objective function (Eq. (1)).

$$I_{i0} - Q_{i0}^b + Q_{its}^r + Q_{its}^o - d_{its} = I_{its} - Q_{its}^b, \quad \forall i \in \mathcal{N},\ t = 1,\ s \in \mathcal{S} \tag{2}$$

$$I_{i,t-1,s} - Q_{i,t-1,s}^b + Q_{its}^r + Q_{its}^o - d_{its} = I_{its} - Q_{its}^b, \quad \forall i \in \mathcal{N},\ t \in \mathcal{T} \setminus \{1\},\ s \in \mathcal{S} \tag{3}$$

Production capacity constraints. Constraints (4) set a loose upper bound for regular time production quantity of each product type. With this set of constraints, it is guaranteed that each type of product can be produced during a period only if the machine is set up for this product type at the beginning of this period or a changeover to producing this product is performed during this period.

$$q_t \left(\sum_{j\in\mathcal{N}, j\neq i} X_{jit} + Z_{i,t-1} \right) \geqslant \tau_{its} Q_{its}^r, \quad \forall i \in \mathcal{N},\ t \in \mathcal{T},\ s \in \mathcal{S} \tag{4}$$

Production time capacity constraints. Constraints (5) ensure that the regular production time for all product types plus the amount of time necessary for changeover between different products does not exceed the time capacity in each period. Constraints (6) set an upper bound for overtime production, based on our assumption that overtime production must be limited within a certain fraction of regular time capacity. It is intuitive that regular time production should come before overtime production, since overtime cost exceeds regular time cost. This is not guaranteed with any set of constraints, but is caused by the objective function that minimizes the total system costs.

$$\sum_{i\in\mathcal{N}}\tau_{its}Q^r_{its}+\sum_{i=\in\mathcal{N}}\sum_{j\in\mathcal{N},j\neq i}\tau^c_{ij}X_{ijt}\leqslant q_t,\quad\forall t\in\mathcal{T},\ s\in\mathcal{S}\tag{5}$$

$$Q^o_{its}\leqslant\alpha Q^r_{its},\quad\forall i\in\mathcal{N},\ t\in\mathcal{T},\ s\in\mathcal{S}\tag{6}$$

Minimum production constraints. Typically in a manufacturing system, the system needs to reach a minimum percentage of capacity to maintain production. Constraints (7) define the minimum production for regular time during each period.

$$\sum_{i\in\mathcal{N}}\tau_{its}Q^r_{its}+\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N},j\neq i}\tau^c_{ij}X_{ijt}\geqslant\beta q_t,\quad\forall t\in\mathcal{T},\ s\in\mathcal{S}\tag{7}$$

Setup status balance constraints. Constraints (8) ensure that the machine is set up for exactly one type of product production at the end of each period. Constraints (9) and (10) are the setup flow condition constraints. Given initial setup at a period t and the changeover during t, it comes to the last product scheduled at t. On the left-hand side, the first term is whether setup to product i occurs at the beginning of a period (the setup status preserving from the previous period $t-1$), the second term is whether setup directed towards product i occurred in t, and the third term is whether setup directed away from product i occurred in t. These three terms determine whether it produces i at the end of t.

$$\sum_{i\in\mathcal{N}}Z_{it}=1,\quad\forall t\in\mathcal{T}\tag{8}$$

$$Z_{i0}+\sum_{j\in\mathcal{N},j\neq i}X_{jit}-\sum_{j\in\mathcal{N},j\neq i}X_{ijt}=Z_{it},\quad\forall i\in\mathcal{N},\ t=1\tag{9}$$

$$Z_{i,t-1}+\sum_{j\in\mathcal{N},j\neq i}X_{jit}-\sum_{j\in\mathcal{N},j\neq i}X_{ijt}=Z_{it},\quad\forall i\in\mathcal{N},\ t\in\mathcal{T}\setminus\{1\}\tag{10}$$

Subtour elimination constraints. It is assumed that the setup towards each type of product occurs at most once during each period (initial setup at the beginning of a period accounts for one setup). Constraints (11) eliminate subtours. Without this set of constraints, it is possible to have a production cycle or multiple cycles during one period, for example, $1\to2\to1\to3$, or $1\to2\to3\to2\to1\to3$. With this set of constraints, the production of each type of product is assigned with a production sequence variable; therefore, the production of each product type would not reappear during one single period. With constraints (11), the changeover from product i to product j assigns j a production sequence that is 1 plus that of i.

$$Y_{jt}\geqslant Y_{it}+1-N(1-X_{ijt}),\quad\forall i\in\mathcal{N},\ j\in\mathcal{N}\setminus\{i\},\ t\in\mathcal{T}\tag{11}$$

Conditions of decision variables. Constraints (12) stipulate that all continuous decision variables, including regular time production, overtime production, backorder quantity, and product inventory quantity, are non-negative. Constraints (13) stipulate that indicator variables, including production changeover and end of period machine status indicators, are binary. Constraints (14) state that the production sequences in each period are integer number starting from 1 and not exceeding product type number N.

$$Q^r_{its},Q^o_{its},Q^b_{its},I_{its}\geqslant0,\quad\forall i\in\mathcal{N},\ t\in\mathcal{T},\ s\in\mathcal{S}\tag{12}$$

$$X_{ijt},Z_{it}\in\{0,1\},\quad\forall i\in\mathcal{N},\ j\in\mathcal{N},\ t\in\mathcal{T}\tag{13}$$

$$N\geqslant Y_{it}\geqslant1,\ \text{integer},\quad\forall i\in\mathcal{N},\ t\in\mathcal{T}\tag{14}$$

4. Case study

In this section we present a case study to demonstrate the performance of our proposed stochastic programming approach. The case refers to a supply chain producing braking equipment for the automotive industry, and the manufacturing supply chain consists of three sites (Gnoni, Iavagnilio, Mossa, Mummolo, & Di Leva, 2003). Constrained by data limitation, our model only focuses on Site 3 in the manufacturing system, where three types of servo-cylinders ($P1$, $P2$, $P3$) of hydraulic braking actuators are produced and provided to customers of the original equipment marketplace.

The mathematical models were implemented with the language of Python and Gurobi solver. All numerical experiments were performed on a PC equipped with an 3.50 GHz Intel Xeon CPU E3-1241 v3 processor with 16.0 GB of RAM running on a Windows 7 system.

4.1. Data sources

Average estimates of unit time setup cost, holding costs, and fixed regular time production cost are obtained from Gnoni et al. (2003). Overtime fixed production cost is set to be 1.5 times regular time cost, given extra payment to employees is required for overtime production (Zhang, Prajapati, & Peden, 2011). The occurrences of backorder result in extra costs related to urgent ordering and transporting; both raise the fixed cost of backorders. Here, the fixed backorder costs are established as doubled the fixed regular production cost (Rego & Mesquita, 2015). All these cost-related parameters are converted to dollars using 1 euro equals 1.07 US dollar (rate obtained on Jan. 22, 2017), and the calculated values are listed in Table 1.

The average operation times for each product type are listed in Table 1 (Gnoni et al., 2003). One source of uncertainty considered in our model is the workforce efficiency. The workforce efficiency of producing one unit of product is measured by the ratio of actual operation time to average operation time for the same product in a period. This ratio should always be positive and has a mean of 1. Any ratio above 1 means a longer than average operation time is used to finish a product, which indicates a less efficient performance; conversely, a ratio below 1 indicates a more efficient performance. In this study, we use a normal distribution $\mathcal{N}(\mu=1,\sigma^2)$ to represent this efficiency-related ratio. Based on input from manufacturing company experts, it is suggested that the actual operation times are within $\pm20\%$ of average, if not extreme cases. Following the experts' opinions, the normal distribution is truncated to interval $[0.8,1.2]$, and σ is chosen such that

Table 1
Product-based parameter values.

	$P1$	$P2$	$P3$
Setup cost ($/h)	16.585	16.585	16.585
Inventory cost ($/unit month)	0.149	0.148	0.358
Regular production cost ($/unit)	242.89	242.89	242.89
Overtime production cost ($/unit)	364.34	364.34	364.34
Backorder cost ($/unit)	485.78	485.78	485.78
Operation time (min/unit)	6	6.6	7.2
Pdf	Weibull	Weibull	Weibull
Scale	518	38	169
Shape	1.51	2.76	2.27
Mean	467.25	33.82	149.70
Variance	9.9422e+04	1.7542e+02	4.8777e+03

95% of the area under probability density function (pdf) falls within this interval, i.e.

$$\sigma = 20\%/\Phi(0.975) = 0.1020,$$

where $\Phi(\cdot)$ is the cumulative distribution function (cdf) for standard normal distribution.

According to Gnoni et al. (2003), the demands of three types of servo-cylinder equipment are uncertain, and are distributed according to Weibull distributions defined in Table 1. We assume the demands among different product types are independent of each other, and are also time-independent.

Changeover is the process of converting a machine from running one product to another. Changeovers require significant and sequence-dependent setup times; thus, an optimal production schedule is important for the manufacturing process. The changeover times are listed in Table 2 (Gnoni et al., 2003), and the corresponding costs can be calculated using changeover times multiplied by unit time setup costs (given in Table 1).

In this multi-period problem, we consider planning one year ahead due to the time capacity fluctuation over the year. Gnoni et al. (2003) provided the monthly nominal regular production time capacity. Breakdowns and repairs of critical resources could affect actual manufacturing capacity. Based on historical data, failure and repair times are stochastically distributed according to Weibull and Beta distributions. The actual production time capacity is calculated by removing expected failure and repair times proportionally from nominal production time capacity. Production time capacity before and after adjustment is shown in Table 3.

The initial state parameters, including initial inventory (I_{i0}), backorder amount (Q^b_{i0}), and machine setup status (Z_{i0}), are supposed to be based on real-time values of the supply chain. In the case study, we assign 0 to initial inventory and backorder amount, and assume the machine is set up for type 1 production ($P1$).

4.2. Scenario generation

In Section 4.1, we provided the continuous distribution for the two uncertain factors. However, in the model formulation (Section 3), the uncertainty is represented with a number of realizations based on the underlying distributions. This gap is filled with a discretization approach called scenario generation.

Table 2
Sequence dependent changeover times (min/setup).

From\To	$P1$	$P2$	$P3$
$P1$	0	270	90
$P2$	180	0	270
$P3$	90	180	0

Table 3
Production time capacity over a one year planning horizon.

Month	Nominal time capacity (min)	Actual time capacity (min)
1	8400	5500
2	7680	5029
3	8400	5500
4	8400	5500
5	6720	4400
6	6720	4400
7	6720	4400
8	2400	1572
9	6720	4400
10	6720	4400
11	6720	4400
12	4800	3143

To be rigorous, a convex stochastic program can be written in the following form:

$$z^* = \min_{\mathbf{x}\in X} \mathbb{E}_P f(\mathbf{x};\xi) = \int_\Omega f(\mathbf{x};\xi)P(d\xi) \tag{15}$$

where $X \subset \mathbb{R}^n$ is a non-empty convex closed set, Ω is a closed subset in $\mathbb{R}^s$, and ξ is a random vector following probability measure P. In a two-stage stochastic programming problem, X is the set of feasible first-stage decisions and the function value $f(\mathbf{x};\xi)$ evaluates the best outcome of decision $\mathbf{x} \in X$ given observed ξ. z^* is the true optimal value of (15).

Except for some trivial cases, (15) cannot be solved with continuous distributions. Hence, in practical applications, we solve only an approximation of (15), and the stochastic parameters are assigned to discrete distributions with a limited number of realizations.

In our case study, we use the most common sampling method to generate scenarios. A scenario set is generated with sampling according to the probability distributions for demand and workforce efficiency, due to the assumption that the uncertain factors are not correlated with each other and are also time-independent. The discretized problem, or sample average approximation (SAA) problem is then

$$\min_{\mathbf{x}\in X} \mathbb{E}_P f(\mathbf{x};\xi) \approx \min_{\mathbf{x}\in X} \frac{1}{n}\sum_{i=1}^{n} f(\mathbf{x};\xi^i) = z_n^*. \tag{16}$$

The cardinality of the support of the discrete distribution is limited by the available computing power, considering the complexity of the decision-making model. In Section 4.3, optimal solutions and objective values with different numbers of realizations are presented.

4.3. Results and discussion

In this section, scheduling and lot-sizing decisions under different numbers of realizations are presented. A comparison of decision-making between stochastic model and deterministic model is performed, where the deterministic model, or expected value problem, assumes the uncertain parameters in the stochastic programming model to be known with certainty, and are replaced by the expected values of the random parameters, i.e.,

$$EV = \min_{\mathbf{x}\in X} f(\mathbf{x};\bar{\xi}), \quad \bar{\xi} = \mathbb{E}[\xi]$$

The quality of the stochastic solution is also evaluated.

4.3.1. Solution of deterministic and stochastic model

Although sampling can provide a discrete distribution that is arbitrarily close to the underlying distribution, the true optimal value cannot be well approximated with a small sampled scenario set. Yet, the computational effort of solving scenario-based models depends on the number of scenarios. Choosing the size of the scenario set is critical to achieve certain computational precision within a feasible computing capability. Here, we solve the stochastic model with different numbers of scenarios ($|\mathcal{S}| = \{10, 20, 50, 100, 200, 500, 1000\}$) and the deterministic model.

The model complexity, including number of decision variables and constraints, as well as the model solving CPU time for both the deterministic model and stochastic model (with different number of scenarios) are listed in Table 4. A summary of machine utilization rates and cost components for both deterministic and stochastic ($|\mathcal{S}| = 1000$) models are presented in Table 5, and the scheduling results under different numbers of scenarios are listed in Table 6.

Table 4
Model complexity and CPU time.

Model	Decision variables	Constraints	CPU time (s)
Deterministic	324	432	0.21
Stochastic $\lvert\mathcal{S}\rvert = 10$	1620	1620	0.95
Stochastic $\lvert\mathcal{S}\rvert = 20$	3060	2940	2.42
Stochastic $\lvert\mathcal{S}\rvert = 50$	7380	6900	12.76
Stochastic $\lvert\mathcal{S}\rvert = 100$	14,580	13,500	45.97
Stochastic $\lvert\mathcal{S}\rvert = 200$	28,980	26,700	170.47
Stochastic $\lvert\mathcal{S}\rvert = 500$	72,180	66,300	1308.59
Stochastic $\lvert\mathcal{S}\rvert = 1000$	144,180	132,300	6248.18

Table 5
Machine utilization rates and cost components for deterministic model and stochastic model ($\lvert\mathcal{S}\rvert = 1000$).

	Deterministic	Stochastic ($\lvert\mathcal{S}\rvert = 1000$)
Utilization rate	93.36%	88.20%
Changeover cost ($)	429 (0.02%)	554 (0.03%)
Inventory cost ($)	866 (0.05%)	870 (0.04%)
Regular production cost ($)	1,894,220 (99.73%)	1,810,919 (89.67%)
Overtime production cost ($)	3846 (0.20%)	121,775 (6.03%)
Backorder cost ($)	0 (0.00%)	85,349 (4.23%)

It is seen in Table 5 that in the ideal case, assuming all information is known with certainty, the machine is well-utilized to meet the demand with just a small requirement of overtime production in the deterministic model. However, after considering uncertain factors, the system productivity is affected by the uncertain efficiency and results in a lower regular time production amount. Due to the uncertainty in demand, overtime production and backorder become more inevitable, and these two parts of cost form over 10% of total cost.

We see in the scheduling results listed in Table 6 that product 2 is often over-produced during the first few months and the demand is satisfied with inventory amount in some other periods. This situation is also seen in product 3, but less often. This production pattern is due to the highly imbalanced total production time distribution over the year. Due to the trade-off between inventory cost and changeover time/cost, products with comparatively lower demands, are more likely to be chosen to over-produce during periods with higher time capacity, and holding additional products for subsequent periods. To an extent, this shows the necessity of planning the production for an entire year-cycle when time capacity varies dramatically during the production cycle.

With the machine setup carryover being allowed, products, especially high-volume products (in our case, product 1) could be produced continuously over two or more adjacent planning periods to avoid the machine from unnecessary setups.

It is also seen in Table 6, as the number of scenarios goes above 200, the first stage solution stabilizes. This, in a way, gives us a guidance of discrete distribution cardinality selection. Therefore, for our case study, a sample size greater than 200 would be adequate to achieve a good enough solution.

4.3.2. Performance of stochastic programming solutions

To evaluate the importance of uncertainties in mathematical models, the following notions are introduced (Birge & Louveaux, 2011).

A perfect information solution chooses optimal first-stage decisions for each realization of ξ, i.e. assuming the decision-maker has perfect foresight of future uncertainty. The expected value of this solution is the *wait-and-see* value,

$$WS = \mathbb{E}_P\left[\min_{\mathbf{x}\in X} f(\mathbf{x};\xi)\right].$$

The *here-and-now* value is the expected value of the recourse problem,

$$RP = \min_{\mathbf{x}\in X} \mathbb{E}_P f(\mathbf{x};\xi).$$

Analyses of the effect of uncertainty in the stochastic program is generally concentrated to be the expected value of perfect information

$$EVPI = RP - WS.$$

This quantity represents the maximum amount a decision-maker is willing to pay in return for perfect information about the future.

Let $\bar{\mathbf{x}}(\bar{\xi}) = \arg\min_{\mathbf{x}\in X} f(\mathbf{x};\bar{\xi})$ be the expected value solution. The expected result of using the EV solution measures the performance of $\bar{\mathbf{x}}(\bar{\xi})$, where optimal second-stage solution given $\bar{\mathbf{x}}(\bar{\xi})$ is performed after ξ is realized,

$$EEV = \mathbb{E}_P[f(\bar{\mathbf{x}}(\bar{\xi});\xi)].$$

The value of the stochastic solution (VSS) is defined as

$$VSS = EEV - RP.$$

This value reveals the potential benefit from solving the stochastic program over solving a deterministic one.

In producing the values of basic summary quantities, different numbers of scenarios are used. The stochastic program results summary is shown in Table 7. It is also observed that applying deterministic model scheduling decision to stochastic scenarios, the production line encountered larger backorder and overtime production amounts, and also higher inventory levels, than that with first-stage decisions from stochastic model. This observation and VSS in Table 7 both indicate that the stochastic solution outperforms the deterministic solution. The EVPI column in Table 7 implies the potential worth of obtaining more accurate forecasts.

4.3.3. Solution quality evaluation

We use the method in Mak, Morton, & Wood (1999) to evaluate the quality of a candidate solution $\hat{\mathbf{x}}$. In this section, the batch-means approach to develop confidence intervals on the optimality gap with respect to any candidate solution $\hat{\mathbf{x}}$ is briefly summarized.

The expected cost of operating under a suboptimal candidate solution is estimated as an upper bound $\overline{U}(n)$,

$$\overline{U}(n) = \frac{1}{n}\sum_{i=1}^{n} f(\hat{\mathbf{x}};\xi^i).$$

A lower bound $\overline{L}(n)$ could be stated as

$$\overline{L}(n) = \mathbb{E}z_n^* = \mathbb{E}\left[\min_{\mathbf{x}\in X}\frac{1}{n}\sum_{i=1}^{n} f(\mathbf{x};\xi^i)\right] \leqslant \min_{\mathbf{x}\in X}\mathbb{E}\frac{1}{n}\sum_{i=1}^{n} f(\mathbf{x};\xi^i)$$
$$= \min_{\mathbf{x}\in X}\mathbb{E}f(\mathbf{x};\xi) = z^*.$$

In this paper, we use the variance-reduction technique of common random numbers (CRN) to develop a confidence interval for the optimality gap with respect to $\hat{\mathbf{x}}$, i.e. $\mathbb{E}f(\hat{\mathbf{x}};\xi) - z^*$.

$$\mathbb{E}G_n = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} f(\hat{\mathbf{x}};\xi^i) - \min_{\mathbf{x}\in X}\frac{1}{n}\sum_{i=1}^{n} f(\mathbf{x};\xi^i)\right] \geqslant \mathbb{E}f(\hat{\mathbf{x}};\xi) - z^*.$$

Let $\xi^{i1},\ldots,\xi^{in}, i = 1,\ldots,n_g$ be i.i.d. batches of realizations from the distribution of ξ. Calculate an observation of the optimality gap in each batch G_n^i. Let $\overline{G}(n_g) = n_g^{-1}\sum_{i=1}^{n_g} G_n^i$. With the central limit theorem (CLT),

$$\sqrt{n_g}\left[\overline{G}(n_g) - \mathbb{E}G_n\right] \Rightarrow N(0,\sigma_g^2) \text{ as } n_g \to \infty.$$

Table 6
Production sequence for deterministic model and stochastic model with $|\mathcal{S}| = \{10, 20, 50, 100, 200, 500, 1000\}$ scenarios.

$\lvert\mathcal{S}\rvert$	Product	1	2	3	4	5	6	7	8	9	10	11	12
Deterministic	$P1$	1	2	3	1	2	1	1	–	2	1	1	1
	$P2$	3	1	2	–	–	–	–	–	–	–	–	3
	$P3$	2	3	1	2	1	–	2	1	1	–	–	2
10	$P1$	1	2	2	1	2	1	1	1	2	1	1	2
	$P2$	3	1	–	–	–	–	–	–	–	–	–	3
	$P3$	2	3	1	2	1	–	–	2	1	–	2	1
20	$P1$	1	2	2	2	2	1	2	1	1	2	1	3
	$P2$	3	1	3	1	–	–	–	–	–	–	3	1
	$P3$	2	3	1	3	1	2	1	–	2	1	2	2
50	$P1$	1	2	2	1	2	2	1	1	2	1	2	2
	$P2$	3	1	–	3	1	–	–	–	–	3	1	–
	$P3$	2	3	1	2	3	1	–	2	1	2	3	1
100	$P1$	1	2	2	1	3	1	2	1	1	2	2	1
	$P2$	3	1	–	–	2	–	–	–	3	1	–	2
	$P3$	2	3	1	2	1	2	1	–	2	3	1	3
$\geqslant$ 200	$P1$	1	2	2	1	3	1	2	1	1	2	1	2
	$P2$	3	1	–	–	2	–	–	–	–	–	3	1
	$P3$	2	3	1	2	1	2	1	–	2	1	2	3

Table 7
Basic summary of stochastic programming results ($|\mathcal{S}| = \{10, 20, 50, 100, 200, 500, 1000\}$).

$\lvert\mathcal{S}\rvert$	EV	WS	RP	EEV	EVPI	VSS
10	1,899,361	1,867,336	1,879,427	1,891,932	12,091	12,505
20	1,899,361	1,925,629	1,943,944	1,987,110	18,315	43,166
50	1,899,361	1,950,564	1,973,192	2,019,564	22,628	46,372
100	1,899,361	1,947,307	1,972,957	2,020,853	25,650	47,896
200	1,899,361	1,986,128	2,013,843	2,055,270	27,715	41,427
500	1,899,361	1,994,214	2,025,098	2,068,405	30,884	43,307
1000	1,899,361	1,993,306	2,019,468	2,062,314	26,162	42,846

Define $\tilde{\varepsilon}_g = \frac{t_{n_g-1,\alpha} s_g(n_g)}{\sqrt{n_g}}$, where $s_g^2(n_g)$ is the sample variance estimator of σ_g^2. Note that $G_n \geqslant 0$, an approximate $(1-\alpha)$-level confidence interval for the optimality gap at $\hat{\mathbf{x}}$ is $[0, \overline{G}(n_g) + \tilde{\varepsilon}_g]$.

Table 8 displays the computational results for sampling based on CRN. We assess the quality of previous stochastic solutions generated with $|\mathcal{S}| = \{10, 20, 50, 100, 200, 500, 1000\}$ scenarios. We see a tighter confidence interval on the optimality gap is obtained as the scenario number of generating $\hat{\mathbf{x}}$ increases. Together with a stabilizing first-stage solution as scenario number exceeds 200, it further suggests that a sample size of 200 is adequate for our case.

4.3.4. Impact of uncertain workforce efficiency

4.3.4.1. Choice of distribution. In Section 4.1, we used a truncated normal distribution to represent the workforce efficiency uncertainty. Yet, this distribution is based on limited information that in general the actual operation times are within $\pm 20\%$. In Table 9, we display results together with if applying two other commonly adopted distributions to model workforce efficiency, uniform distribution (unif(0.8, 1.2)) and symmetric triangular distribution (triangular(0.8, 1.0, 1.2)). It could be seen that the model is rather robust on the shape of distribution, since the results are not deviating much.

4.3.4.2. Sensitivity analysis on distribution parameters. In Table 10, the expected system cost and the system utilization rate with a distribution shift in mean level is presented. When the distribution is shifted rightward, say $+5\%$ or $+10\%$, it means that on average the workers are using 5 or 10% additional time to produce one unit of product. On the contrary, a leftward shift of the distribution, say -5% or -10%, means a higher average productivity of workers. It is seen that more efficient workers lowers the costs and system utilization.

Table 11 listed the expected system cost and the system utilization rate when the assumed support of distribution of workforce efficiency changes. In the baseline, we assumed the support of workforce efficiency is $[0.8, 1.2]$. If the support interval gets wider, it means there exists higher volatility in worker's production proficiency, and vise versa. From the results, it could be seen that the more volatile the workforce efficiency, the lower the system cost and utilization.

4.3.5. Sensitivity analysis in failure time

From Table 3, we see that the amount of failure and repair times consume 35% of the time capacity, which affects the production flexibility. Therefore, a good system quality control could be essential to guarantee production performance and profitability. Here,

Table 8
Optimality gap with CRN.

Sample size used to generate $\hat{\mathbf{x}}$	10	20	50	100	$\geqslant$ 200
Batch size n	100	100	100	100	100
No. of batches n_g	100	100	100	100	100
Point estimate $\overline{G}(n_g)$	39056	6581	2927	1970	1099
Error estimate ($\alpha = 0.05$) $\tilde{\varepsilon}_g$	1488	422	290	214	167
Confidence interval (95%)	[0, 40,544]	[0, 7003]	[0, 3217]	[0, 2184]	[0, 1266]

Table 9
Summary of stochastic programming results with different underlying workforce efficiency distributions ($|\mathcal{S}| = 200$).

Distribution	EV	WS	RP	EEV	EVPI	VSS
Truncated normal	1,899,361	1,986,128	2,013,843	2,055,270	27,715	41,427
Uniform	1,899,361	2,006,186	2,034,254	2,076,044	28,068	41,790
Triangular	1,899,361	2,010,343	2,037,334	2,075,911	26,991	38,577

Table 10
Expected system cost and utilization rate with shifting mean level of workforce efficiency ($|\mathcal{S}| = 200$).

Distribution	Shift of mean level				
	−10%	−5%	0%	+5%	+10%
Truncated normal	1,933,712 81.71%	1,976,901 84.97%	2,013,843 87.63%	2,113,545 90.04%	2,271,472 91.72%
Uniform	1,936,615 81.27%	1,981,080 84.65%	2,034,254 87.38%	2,118,119 89.63%	2,228,178 91.42%
Triangular	1,947,374 81.78%	1,983,022 85.19%	2,037,334 87.56%	2,121,821 89.93%	2,246,033 91.64%

Table 11
Expected system cost and utilization rate with changing volatility in workforce efficiency ($|\mathcal{S}| = 200$).

Distribution	Support of distribution			
	[0.9, 1.1]	[0.8, 1.2]	[0.7, 1.3]	[0.6, 1.4]
Truncated normal	2,019,544 87.95%	2,013,843 87.63%	1,982,594 87.42%	1,965,193 86.33%
Uniform	2,045,774 87.87%	2,034,254 87.38%	2,002,434 86.91%	1,998,789 85.53%
Triangular	2,046,782 87.92%	2,037,334 87.56%	2,022,590 87.43%	2,031,113 86.62%

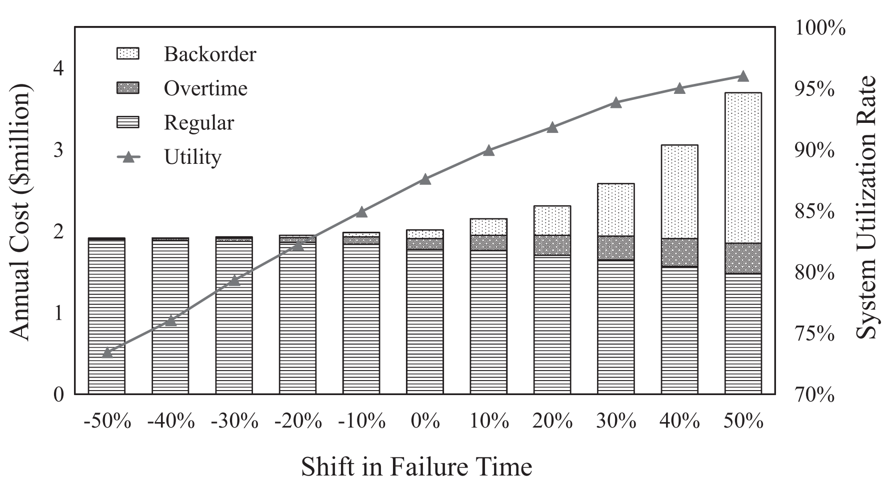

Fig. 1. Expected value of cost components ($million) and system utilization rate (%) vs. shift in failure time.

we perform a sensitivity analysis with respect to failure time, where this parameter is shifted from −50% to +50%, and the effect of changing failure time on the production performance could be seen in Fig. 1.

In Fig. 1, system cost is partitioned, and a bar chart is included to show different production cost components. Note that inventory costs and changeover costs are not illustrated in the figure, because they form a small proportion ($\ll 1\%$) of total system cost. The utilization rate is presented in Fig. 1 using a line with triangle markers.

Comparing the cost components, we see that as failure time increases, the regular production cost decreases, due to the negative effect on the company's productivity. The requirement of overtime production increases to cover the gap between regular time productivity and demand. However, the overtime production amount is capped given the constraint that overtime cannot exceed 20% of regular time capacity, and as a consequence, the backorder amount increases dramatically when the system failure time exceeds our benchmark setting (0%). Intuitively, the utilization rate increases as the proportion of failure time enlarges.

At the baseline demand level, remain the current level of system maintenance should be adequate, since the marginal benefit from decreasing failure time is indistinctive (halve the failure time only causes 5% decrease in system cost). However, if the demand level is to increase in the future, enhance the strength of maintenance to reduce the occurrence of system failure could be a valid approach to improve productivity and reduce system cost.

5. Conclusion

Production planning decisions are vital for manufacturing companies, and this study solves a shop floor lot-sizing and scheduling problem. Two uncertain factors considered are demand and workforce efficiency. This paper proposes a two-stage stochastic programming model that explicitly addresses both uncertain factors. This single-machine multi-product model is sufficient to address multi-machine case with limited interactions among different machines, which is typically the case when machines produce different sets of products. Such machines could be modeled separately. When multiple machines working on the same set of products in parallel, an extension to multiple identical machines or production facilities can be performed by adding another index dimension for machines.

A case study for an automotive brake equipment manufacturer is performed. The numerical results and the solution evaluation for this problem in the auto-part industry case study provide the optimal scheduling decisions, and illustrate the usefulness of our stochastic programming approach when planning under an uncertain environment. The optimal value of the optimization model can be well-approximated when using a large enough number of scenarios (in our case 200 scenarios), considering the balance of solution quality and computational power. In addition, the sensitivity analysis suggests enhancing system maintenance (i.e., reduce failure and repair time) as an approach to increase productivity and decrease cost, and this approach becomes more powerful when the usable time capacity is not adequate to meet the demand.

Although using this model, a production scheduling decision for the entire planning horizon (one year in the case study) could be optimized, it is more practical that the model is done on a rolling horizon basis and only the immediate decision would actually be applied in the manufacturing system. To make decisions further in the future, we could run the model in a rolling horizon manner with system status at the moment and necessary updates in parameter settings.

The findings of this study illustrate the importance of considering uncertain factors when making lot-sizing and scheduling decisions on a shop floor. One future research topic might be developing a more realistic and thorough representation of the system stochasticity. For example, besides uncertainty considered in this paper, many other sources of randomness affect the manufacturing systems, such as quality uncertainty, failure of a production system, and supply uncertainty. Also, the correlation between different uncertain factors and time dependency would also influence system performance. These could be accounted for with proper scenario generation and modeling procedure given data availability.

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