DD2434 - Machine Learning, Advanced Course
Assignment 1A

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1 Exponential Family

1.1 Question 1.1

$$\begin{aligned}
p(x|\theta) &= h(x)\exp(\eta(\theta)\cdot T(x) - A(\eta)) \\
&= h(x)\exp(\eta(\lambda)\cdot T(x) - A(\eta(\lambda))) \\
&= h(x)\exp(\log\lambda\cdot x - A(\log\lambda)) \\
&= h(x)\exp(\log\lambda\cdot x - \lambda) \\
&= h(x)\exp(\log\lambda\cdot x)\exp(-\lambda) \\
&= e^{-\lambda}\frac{\lambda^x}{x!}
\end{aligned} \tag{1}$$

We can see that the distribution correspond to a Poisson distribution of parameter λ.

1.2 Question 1.2

$$\begin{aligned}
p(x|\theta) &= h(x)\exp(\eta(\theta)\cdot T(x) - A(\eta)) \\
&= \exp(\eta([\alpha,\beta])\cdot[\log x, x] - A(\alpha-1,-\beta)) \\
&= \exp([\alpha-1,-\beta]\cdot[\log x, x] - \log\Gamma(\alpha) + \alpha\log(\beta)) \\
&= \exp((\alpha-1)\log x - \beta x - \log\Gamma(\alpha) + \alpha\log(\beta)) \\
&= \frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}
\end{aligned} \tag{2}$$

We can see that the distribution correspond to a Gamma distribution of parameters α and β.

1.3 Question 1.3

$$\begin{aligned}
p(x|\theta) &= h(x)\exp(\eta(\theta)\cdot T(x) - A(\eta)) \\
&= \frac{\exp(\eta([\mu,\sigma^2])\cdot[x,x^2] - A(\eta([\mu,\sigma^2])))}{\sqrt{2\pi}} \\
&= \frac{\exp([\frac{\mu}{\sigma^2},-\frac{1}{2\sigma^2}]\cdot[x,x^2] - A([\frac{\mu}{\sigma^2},-\frac{1}{2\sigma^2}]))}{\sqrt{2\pi}} \\
&= \frac{\exp(\frac{\mu x}{\sigma^2} - \frac{x^2}{2\sigma^2} - \frac{\mu^2}{2\sigma^2} - \log\sigma)}{\sqrt{2\pi}} \\
&= \frac{\exp(-\frac{(x-\mu)^2}{2\sigma^2})}{\sigma\sqrt{2\pi}}
\end{aligned} \tag{3}$$

We can see that the distribution correspond to a Normal distribution of parameters μ and σ^2.

1.4 Question 1.4

$$
\begin{aligned}
p(x|\theta) &= h(x)\exp(\eta(\theta)\cdot T(x) - A(\eta)) \\
&= 2\exp(\eta(\lambda)\cdot x - A(\eta(\lambda))) \\
&= 2\exp(-\lambda x - A(-\lambda)) \\
&= 2\exp\left(-\lambda x + \log\left(\frac{\lambda}{2}\right)\right) \\
&= \lambda e^{-\lambda x}
\end{aligned} \tag{4}
$$

We can see that the distribution correspond to a Exponential distribution of parameter λ.

1.5 Question 1.5

$$
\begin{aligned}
p(x|\theta) &= h(x)\exp(\eta(\theta)\cdot T(x) - A(\eta)) \\
&= \exp(\eta([\psi_1,\psi_2])\cdot[\log x, \log(1-x)] - A(\eta([\psi_1,\psi_2]))) \\
&= \exp([\psi_1 - 1, \psi_2 - 1]\cdot[\log x, \log(1-x)] - A([\psi_1 - 1, \psi_2 - 1])) \\
&= \exp((\psi_1 - 1)\log x + (\psi_2 - 1)\log(1-x) - \log\Gamma(\psi_1) - \log\Gamma(\psi_2) + \log\Gamma(\psi_1+\psi_2)) \\
&= \frac{\Gamma(\psi_1+\psi_2)}{\Gamma(\psi_1)\Gamma(\psi_2)} x^{\psi_1 - 1}(1-x)^{\psi_2 - 1}
\end{aligned} \tag{5}
$$

We can see that the distribution correspond to a Beta distribution of parameters ψ_1 and ψ_2.

2 Dependencies in a Directed Graphical Model

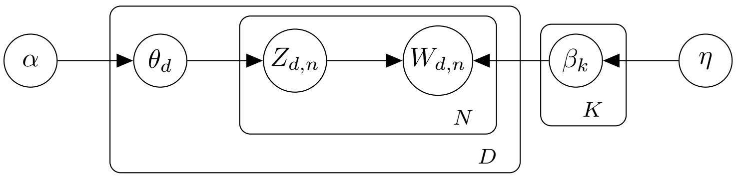

Figure 1: Graphical model of smooth LDA.

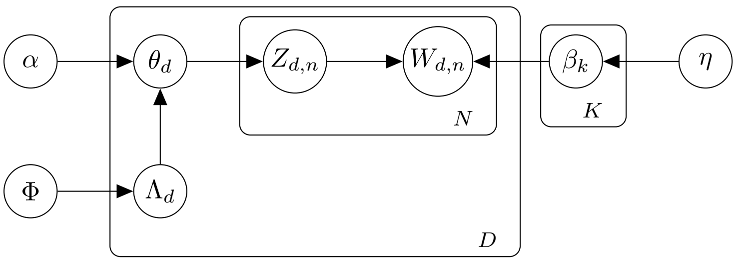

Figure 2: Graphical model of Labeled LDA.

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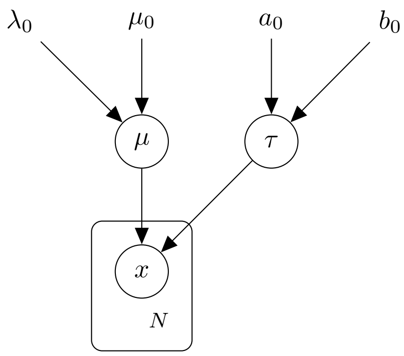

Figure 3: DGM