DD2434 - Machine Learning, Advanced Course
Assignment 1B

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Contents

1 CAVI for Earth quakes

1.1 Question 1.1

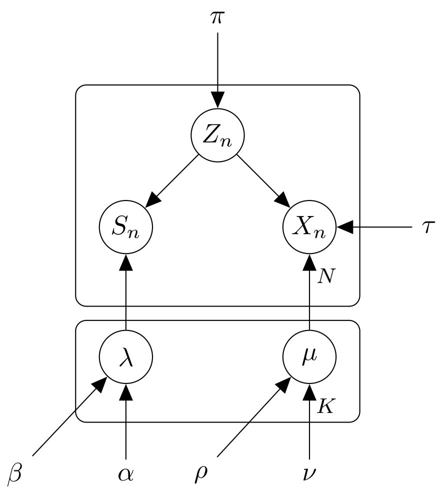

Figure 1: Directed Graphical Model for the Earthquake problem

1.2 Question 1.2

Here, we know these distributions:

- $p(Z_n|\pi) = Categorical(\pi)$
- $p(S_n|Z_n = k, \lambda_k) = Poisson(\lambda_k)$
- $p(X_n|Z_n = k, \mu_k, \tau) = Normal(\mu_k, \tau \cdot I)$
- $p(\mu_k|\nu, \rho) = Normal(\nu, \rho \cdot I)$
- $p(\lambda_k|\alpha, \beta) = Gamma(\alpha, \beta)$

Where, ρ and τ define precision and not standard variation. Then we have:

$$\begin{aligned}
\log p(X, S, Z, \lambda, \mu|\pi, \tau, \alpha, \beta, \nu, \rho) &= \log p(X|S, Z, \lambda, \mu, \pi, \tau, \alpha, \beta, \nu, \rho) \\
&\quad + \log p(S, Z, \lambda, \mu|\pi, \alpha, \beta, \nu, \rho) \\
&= \log p(X|Z, \mu, \tau) + \log p(S|Z, \lambda, \mu, \pi, \alpha, \beta, \nu, \rho) \\
&\quad + \log p(Z, \lambda, \mu|\pi\alpha, \beta, \nu, \rho) \\
&= \log p(X|Z, \mu, \tau) + \log p(S|Z, \lambda) + \log p(Z|\pi) \\
&\quad + \log p(\lambda, \mu|\alpha, \beta, \nu, \rho) \\
\log p(X, S, Z, \lambda, \mu|\pi, \tau, \alpha, \beta, \nu, \rho) &= \log p(X|Z, \mu, \tau) + \log p(S|Z, \lambda) + \log p(Z|\pi) \\
&\quad + \log p(\mu|\nu, \rho) + \log p(\lambda|\alpha, \beta)
\end{aligned} \tag{1}$$

Where:

$$\begin{aligned}
\log p(X|Z,\mu,\tau) &= \sum_{n=1}^{N}\sum_{k=1}^{K}\log p(X_n|Z_n=k,\mu_k,\tau) \\
\log p(S|Z,\lambda) &= \sum_{n=1}^{N}\sum_{k=1}^{K}\log p(S_n|Z_n=k,\lambda_k) \\
\log p(Z|\pi) &= \sum_{n=1}^{N}\log p(Z_n|\pi) \\
\log p(\mu|\nu,\rho) &= \sum_{k=1}^{K}\log p(\mu_k|\nu,\rho) \\
\log p(\lambda|\alpha,\beta) &= \sum_{k=1}^{K}\log p(\lambda_k|\alpha,\beta)
\end{aligned} \tag{2}$$

1.3 Question 1.3

Here, the mean field approximation is not an approximation but an equality because Z, μ, λ are independent. Therefore we have:

$$\begin{aligned}
\log q^*(Z_n) &\stackrel{\pm}{=} \mathbb{E}_{\mu,\lambda}[\log p(X_n, S_n, Z_n, \lambda, \mu|\pi, \tau, \alpha, \beta, \nu, \rho)] \\
&\stackrel{\pm}{=} \mathbb{E}_{\mu,\lambda}[\log p(X_n|Z_n,\mu,\tau) + \log p(S_n|Z_n,\lambda) + \log p(Z_n|\pi)] \\
&= \mathbb{E}_{\mu}\left[\sum_{k=1}^{K}\mathbb{1}_{\{Z_n=k\}}\left(\log\left(\frac{\tau}{2\pi}\right) - \frac{\rho}{2}\left((x_n-\mu_k)^T(x_n-\mu_k)\right)\right)\right] \\
&\qquad + \mathbb{E}_{\lambda}\left[\sum_{k=1}^{K}\mathbb{1}_{\{Z_n=k\}}\mathbb{E}_{\lambda}\left(\log(\pi_k) + \sum_{j\in\mathbb{N}}\mathbb{1}_{\{S_n=j\}}\left[-\lambda_k + j\log(\lambda_k) - \log(j!)\right]\right)\right] \\
&\stackrel{\pm}{=} \sum_{k=1}^{K}\mathbb{1}_{\{Z_n=k\}}\Bigg(\log(\tau) - \frac{\rho}{2}\mathbb{E}_{\mu}\left[(x_n-\mu_k)^T(x_n-\mu_k)\right] \\
&\qquad + \log(\pi_k) + \sum_{j\in\mathbb{N}}\mathbb{1}_{\{S_n=j\}}\mathbb{E}_{\lambda}\left[-\lambda_k + j\log(\lambda_k)\right]\Bigg)
\end{aligned} \tag{3}$$

A Appendix

A.1 Question 1.2

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.stats import gamma, norm
from scipy.special import psi
np.random.seed(14)


def generate_data(mu, tau, N):
    # Insert your code here
    D = np.random.normal(mu, np.sqrt(1/tau), N)

    return D


MU = 1
TAU = 0.5

dataset_1 = generate_data(MU, TAU, 10)
dataset_2 = generate_data(MU, TAU, 100)
dataset_3 = generate_data(MU, TAU, 1000)

# Visulaize the datasets via histograms
# Insert your code here
fig, axs = plt.subplots(1, 3, figsize=(12, 4))
axs[0].hist(dataset_1, bins=20)
axs[1].hist(dataset_2, bins=20)
axs[2].hist(dataset_3, bins=20)
plt.tight_layout()
plt.savefig('12_data.png')
plt.show()
```