Reparameterization of common distributions

We will work with Torch throughout this notebook.

In [ ]:
```
import torch
from torch.distributions import Exponential, Categorical #, ...  import the distributions you need here
from torch.nn import functional as F
```

A helper function to visualize the generated samples:

In [ ]:
```
import matplotlib.pyplot as plt
def compare_samples (samples_1, samples_2, bins=100, range=None):
  fig = plt.figure()
  if range is not None:
    plt.hist(samples_1, bins=bins, range=range, alpha=0.5)
    plt.hist(samples_2, bins=bins, range=range, alpha=0.5)
  else:
    plt.hist(samples_1, bins=bins, alpha=0.5)
    plt.hist(samples_2, bins=bins, alpha=0.5)
  plt.xlabel('value')
  plt.ylabel('number of samples')
  plt.legend(['direct','via reparameterization'])
  plt.show()
```

Q1. Exponential Distribution

Below write a function that generates N samples from $Exp(\lambda)$.

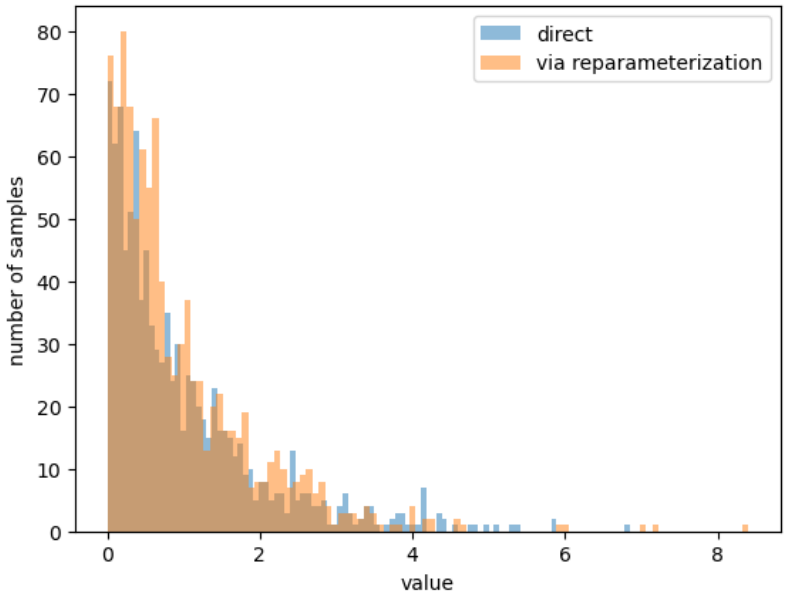
In [ ]:
```
def exp_sampler(l, N):
  # insert your code
  samples = Exponential(l).sample((N,))
  return samples # should be N-by-1
```

Now, implement the reparameterization trick:

In [ ]:
```
def exp_reparametrize(l,N):
  # this function should return N samples via reparametrization,
  # insert your code
  samples = -1/l *torch.log(1-torch.rand(N,))
  return samples
```

Generate samples for $\lambda = 1$ and compare:

In [ ]:
```
l = 1    #Lambda
N = 1000
direct_samples = exp_sampler(l, N)
reparametrized_samples = exp_reparametrize(l, N)
direct_samples = direct_samples.detach().numpy()
reparametrized_samples = reparametrized_samples.detach().numpy()
compare_samples(direct_samples, reparametrized_samples)
```



Q2. Categorical Distribution

Below write a function that generates N samples from Categorical (**a**), where **a** = $[a_0, a_1, a_2, a_3]$.

In [ ]:
```
def categorical_sampler(a, N):
  # insert your code
  samples = Categorical(a).sample((N,))
  return samples  # should be N-by-1
```

Now write a function that generates samples from Categorical (**a**) via reparameterization:

In [ ]:
```
# Hint: approximate the Categorical distribution with the Gumbel-Softmax distribution
def categorical_reparametrize(a, N, temp=0.1, eps=1e-20):  # temp and eps are hyperparameters for Gumbel-Softmax
  # insert your code
  samples = torch.zeros(N, a.shape[0])
  for i in range(N):
    samples[i] = F.gumbel_softmax(a, tau=temp, hard=True, eps=eps)

  return samples # make sure that your implementation allows the gradient to backpropagate
```

Generate samples when $a = [0.1, 0.2, 0.5, 0.2]$ and visualize them:

In [ ]:
```
a = torch.tensor([0.1,0.2,0.5,0.2])
N = 1000
direct_samples = categorical_sampler(a, N)
reparametrized_samples = categorical_reparametrize(a, N, temp=0.1, eps=1e-20)
direct_samples = direct_samples.detach().numpy()
reparametrized_samples = reparametrized_samples.detach().numpy()
compare_samples(direct_samples, reparametrized_samples)
```