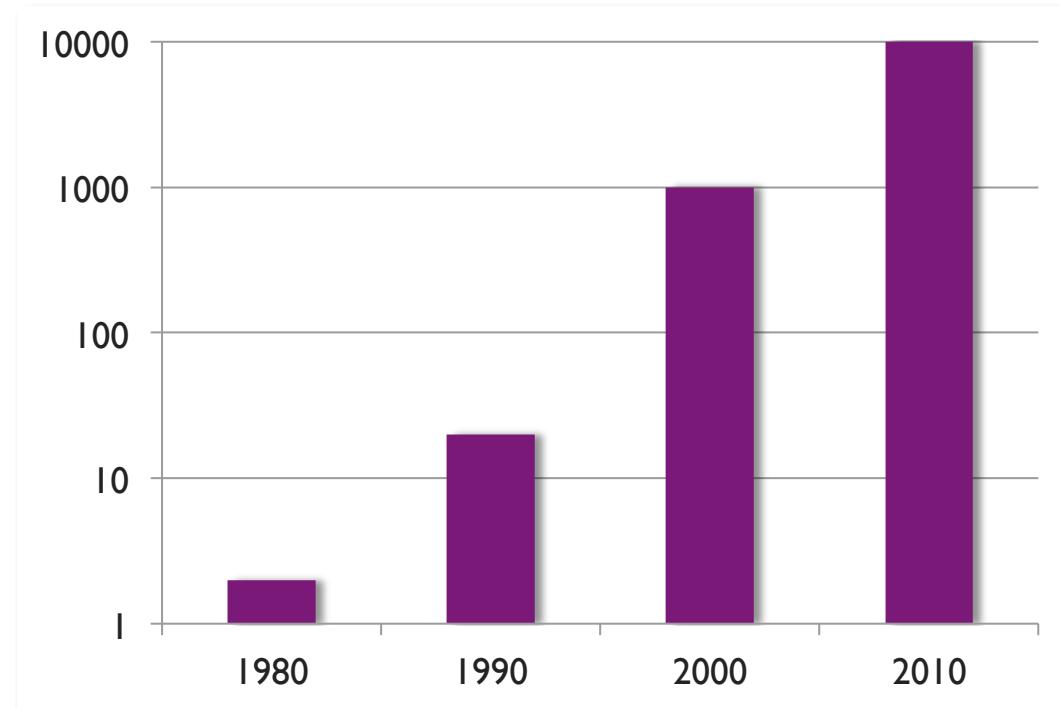


Aerial Robotics



Unmanned Aerial Vehicles in 2010

*Number of UAVs
worldwide*

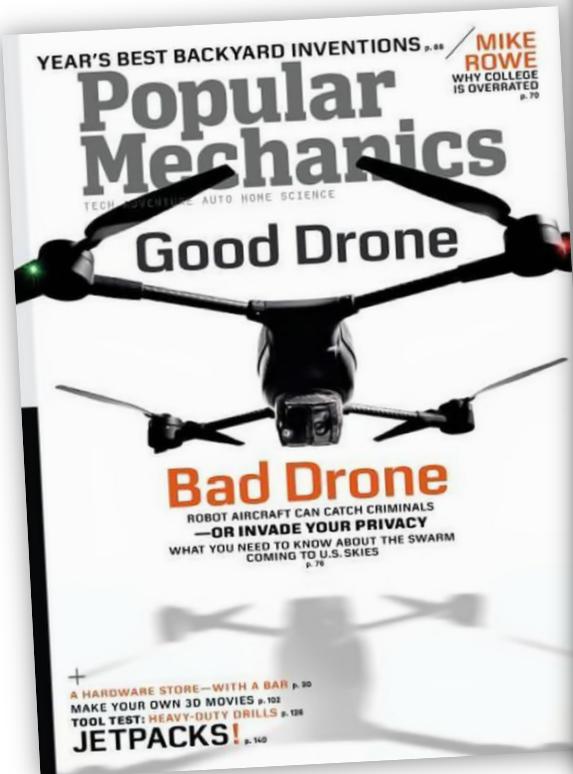


Predictions of a \$10B industry

- Military: Surveillance, force protection, warfare
- Civilian commercial: Transport, environment
- Civilian private: DIY Drones

FAA predicts 15,000 civilian drones by 2020

Unmanned Aerial Vehicles in 2015



- Over 15,000 drones sold in the US every month
- \$15B industry, projected to grow to \$25B by 2020
- Expectations for leading industry applications
 - ▼ Agriculture
 - ▼ Infrastructure inspection
 - ▼ Border patrols
 - ▼ Photography
 - ▼ Construction
 - ▼ Film production

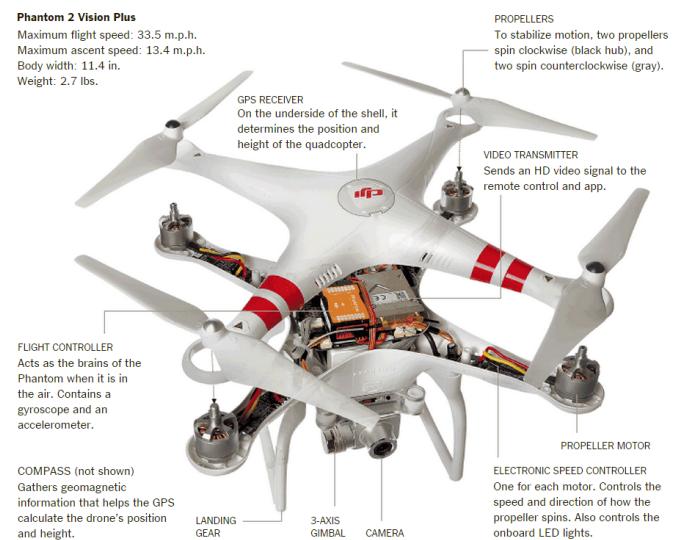
Unmanned Aerial Vehicles



Aerial Robots



Remotely
Piloted Vehicles
(RPVs)



Drones

Remote Control
Includes a mount for a smartphone and a range extender that allows the phone to communicate with the drone up to 2,300 feet away.



Drones mischaracterize what these things are. They're not dumb. Nor are they unmanned, actually. They're remotely piloted aircraft. - Gen. Norton Schwarz, August 10, 2012



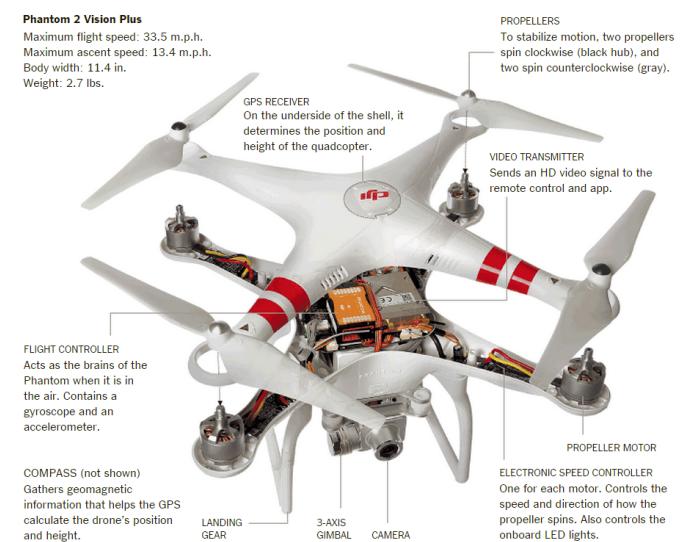
UAVs = RPVs = Aerial Robots = Drones



Aerial Robots



Remotely Piloted Vehicles (RPVs)

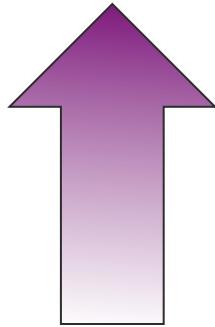
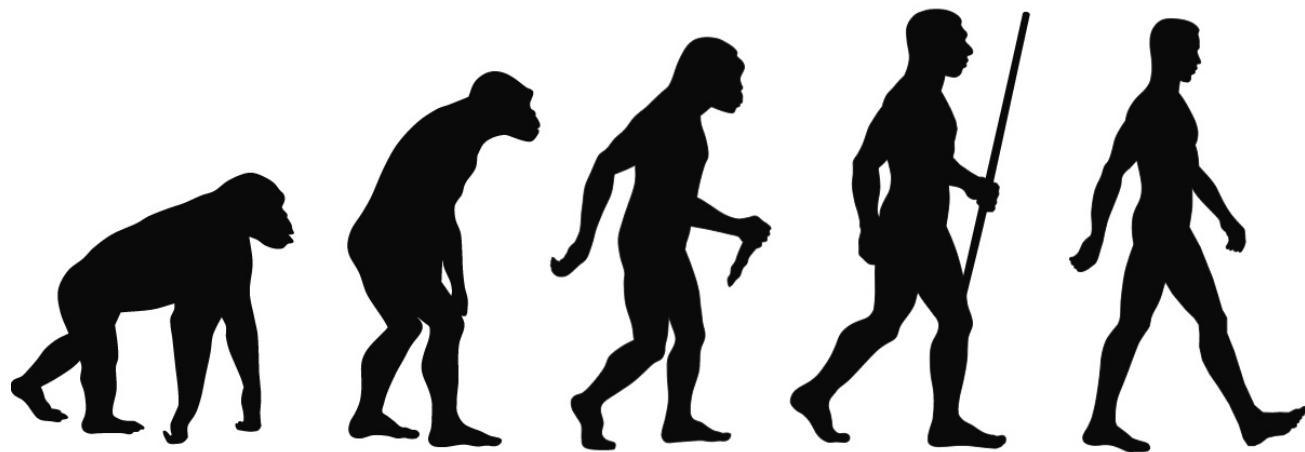


Drones

Remote Control
Includes a mount for a smartphone and a range extender that allows the phone to communicate with the drone up to 2,300 feet away.



Aerial Robotics



We are here!

THE WALL STREET JOURNAL.

U.S. NEWS

FAA Authorizes Commercial-Drone Testing

Six Operators Selected to Conduct Research, Setting Stage for Eventual Widespread Use

By ANDY PASZTOR

Updated Dec. 30, 2013 7:42 p.m. ET



Officials eventually want to allow widespread use of private drones. Above, an unarmed government Predator. Getty Images

Aviation officials on Monday selected a handful of universities and state agencies to operate sites for drone testing commercial unmanned aircraft into the U.S. aviation system.

Under the six operators chosen by the Federal Aviation Administration, research will be conducted by industry experts and academics on the safe operation of drones, or unmanned aerial vehicles, across a broad array of geographical areas, climates and types of airspace. The work is expected to target everything from federal certification of the safety of commercial drones to the reliability of air-to-ground communication links to verifying a generation of new, lower-cost sensors designed to avoid midair collisions.

The FAA, however, stopped short of committing itself to a specific timetable for permitting widespread use of unmanned commercial aircraft across U.S. skies.



Currently, law-enforcement agencies, universities and some environmental organizations are allowed to fly remotely piloted aircraft in clearly delineated corridors or swaths of U.S. airspace. The FAA has been facing escalating industry and congressional pressure to move more quickly to open up additional flight regions.

The winning applicants were the commerce department of North Dakota; the state of Nevada; a public airport some 250 miles north of New York City; the University of Alaska; Texas A&M University in Corpus Christi; and a partnership between Virginia Tech and Rutgers University. The first site is expected to begin operating within six months.

For Immediate Release

September 15, 2015

WASHINGTON – As the Papal visit approaches, the U.S. Department of Transportation's Federal Aviation Administration (FAA) is reminding residents of and visitors to Washington, DC, New York, and Philadelphia that these cities and the surrounding communities are No Drone Zones from September 22 through September 27, 2015.

"If you plan to attend any of the Papal visit events, please leave your drone at home," said FAA Administrator Michael Huerta. "Anyone flying a drone within the designated restricted areas may be subject to civil and criminal charges."

The FAA has flight restrictions in place in and around [Washington, DC, New York](#) and Philadelphia

([Notice 1](#), [Notice 2](#)). Flying a drone anywhere Pope Francis will visit is against the law. Any unmanned aircraft (UAS) – including radio-controlled model aircraft/UAS – are subject to FAA requirements.



FEDERAL AVIATION
ADMINISTRATION

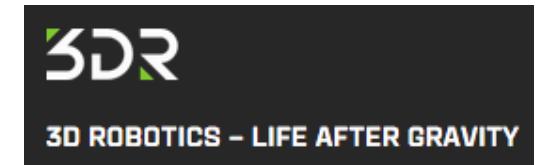
The Skies will be Abuzz with Drones!



Gartner's 2015 Hype Cycle for Emerging Technologies



acquired by



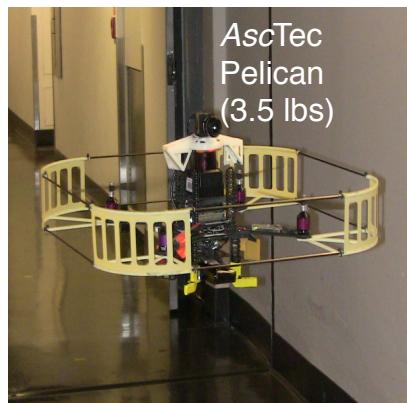
Micro Aerial Vehicles



KMeI kNanoQuad
(0.12 lb)



Hummingbird (1 lb)



AscTec
Pelican
(3.5 lbs)



Boeing
ScanEagle
(20 lbs)



Gen. Atomics
Predator (2,250
lbs)



Gen. Atomics MQ-9
Reaper (10,000 lbs)



Northrop-Grumman
Global Hawk
(32,200 lbs)

Mass

0 1 10 100 1,000 10,000 100,000

Types of Micro Air Vehicles

- Fixed wing



- Flapping wing

- Insect flight
- Avian flight



- Rotor crafts

- Helicopter
- Ducted fan
- Co-axial
- Quadrotor
- Hexrotor



Quadrotor



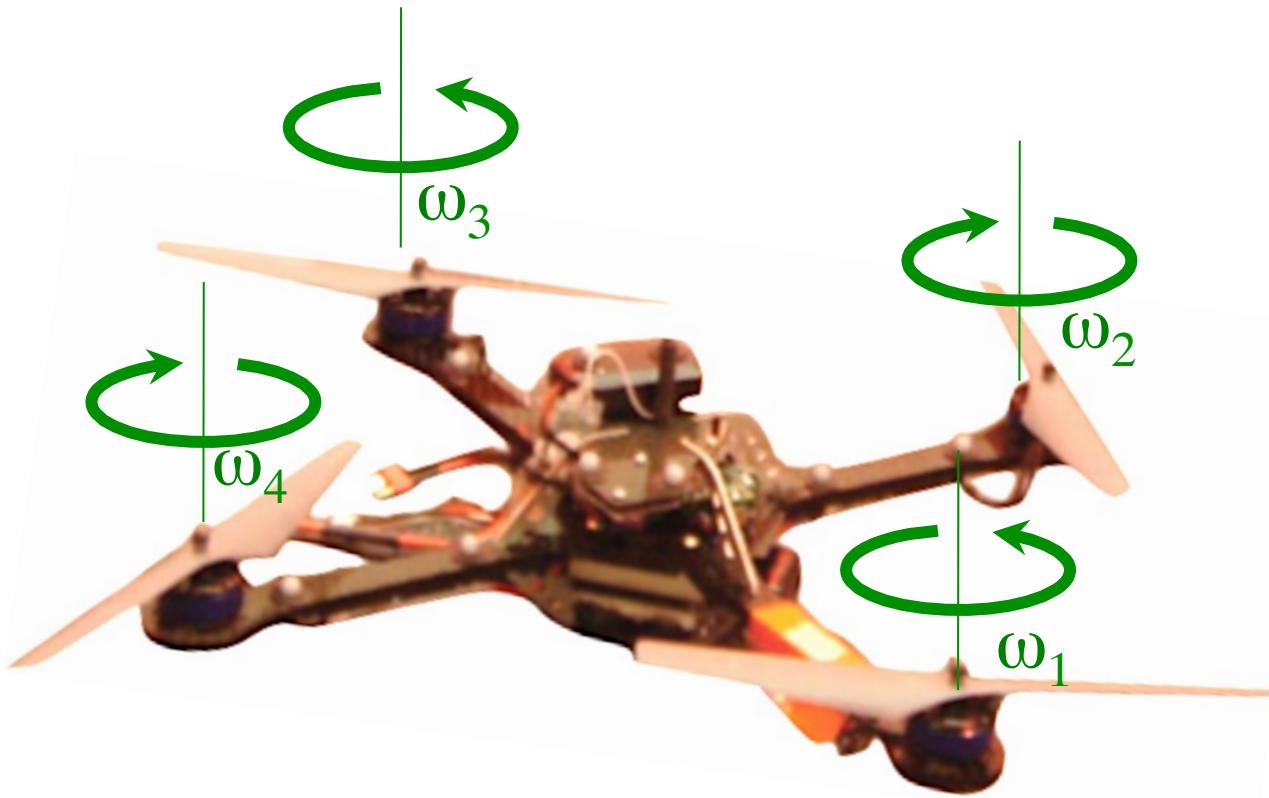
Quadrotor



Quadrotor

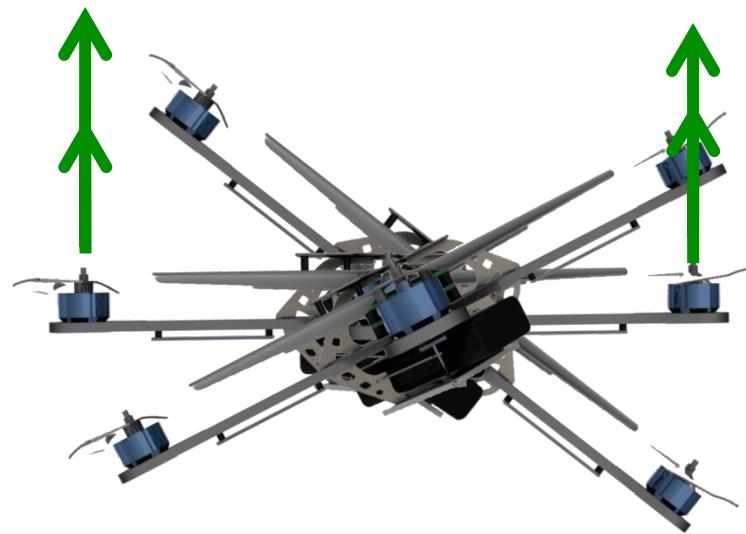


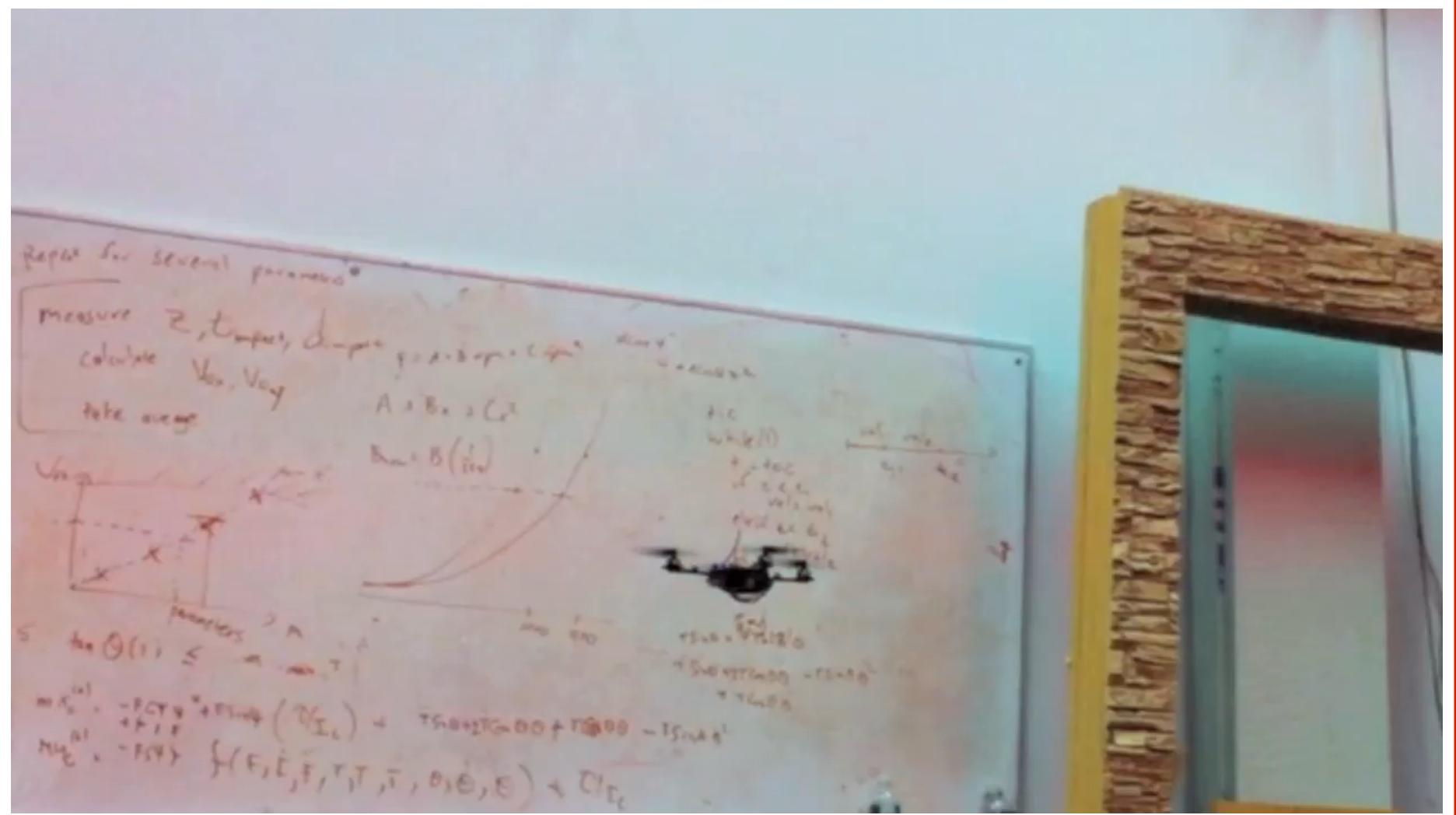
Quadrotor



Vary the speeds of the rotors to control the position and orientation of the robot

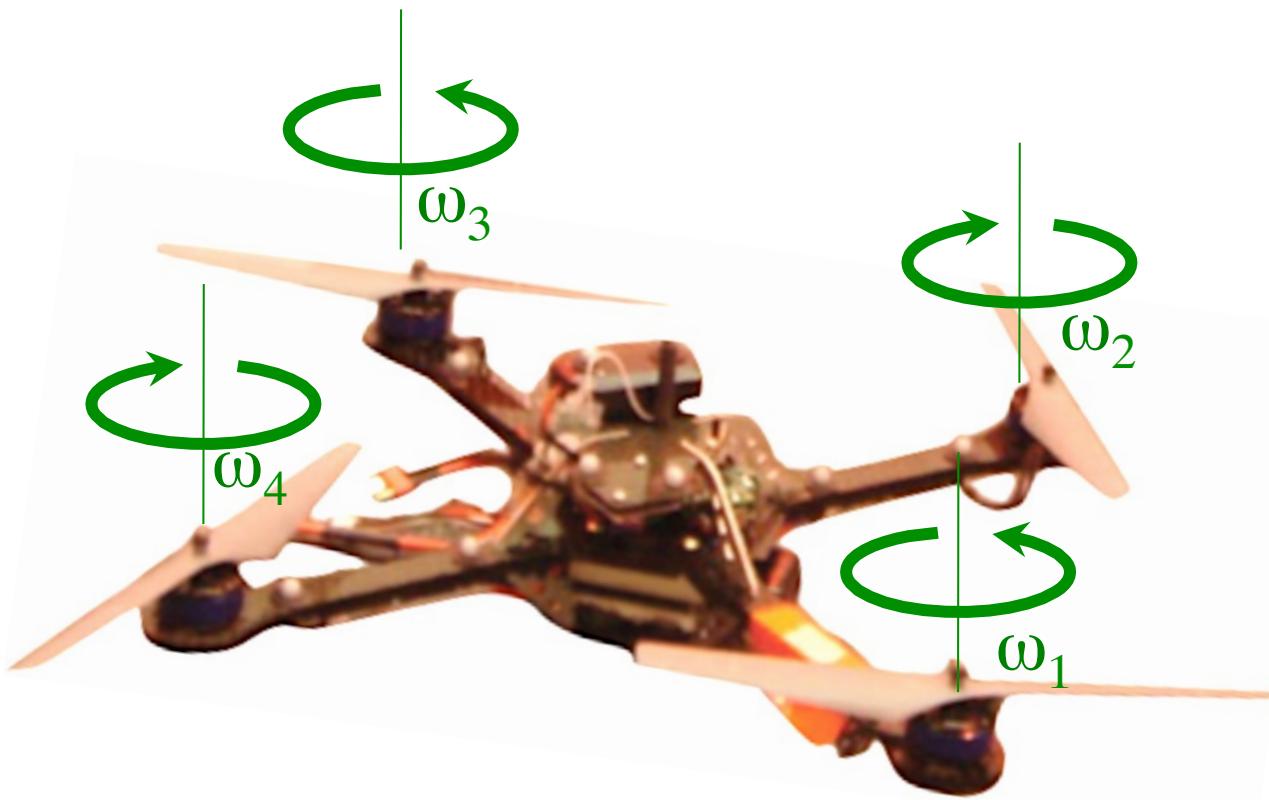
Roll and Pitch





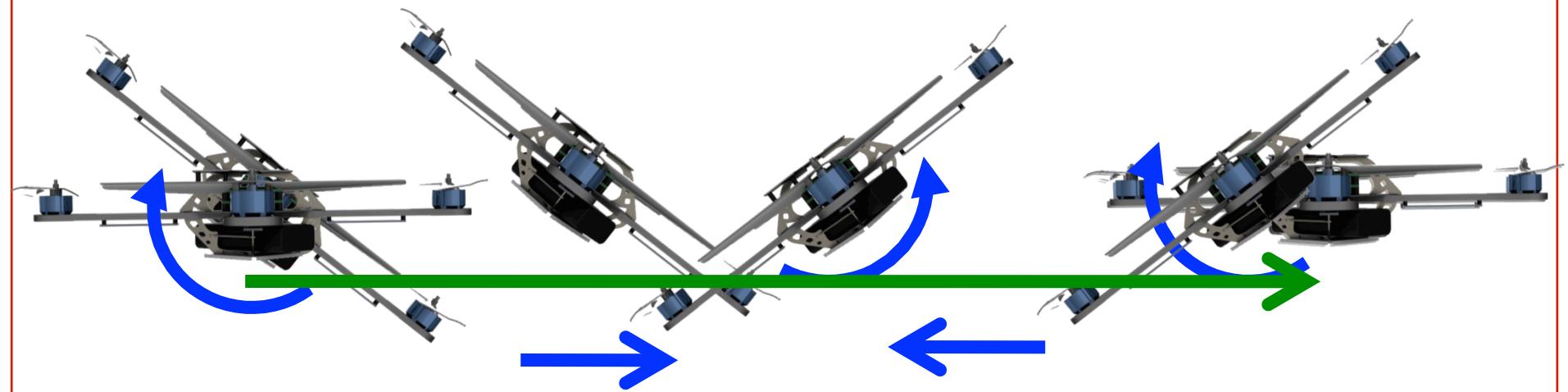
KMel Nano
Maximum pitch velocity of 2000 deg/sec

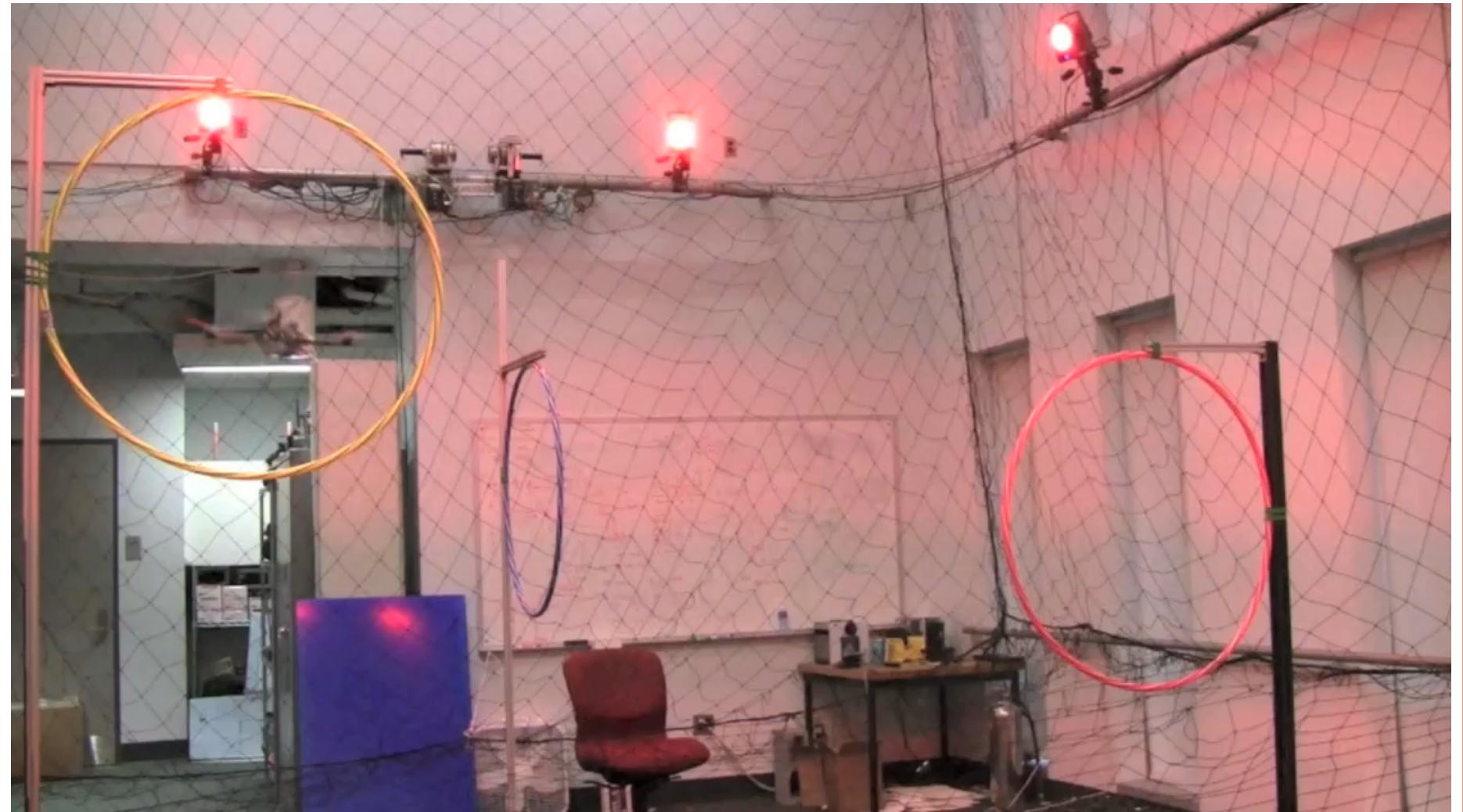
Yaw



How do you get the robot to
yaw (steer)?

Translation





The robot has six degrees of freedom!

How many different ways can you rotate
or translate the robot?

Key Components for Autonomous Flight

In any system, ask how the following components work!

- State Estimation
- Control
- Mapping
- Planning

In any system, ask how the following components work!

- State Estimation
 - Control
 - Mapping
 - Planning
- estimate the position and velocity
(including rotation and angular
velocity of the robot)*

In any system, ask how the following components work!

- State Estimation

- Control

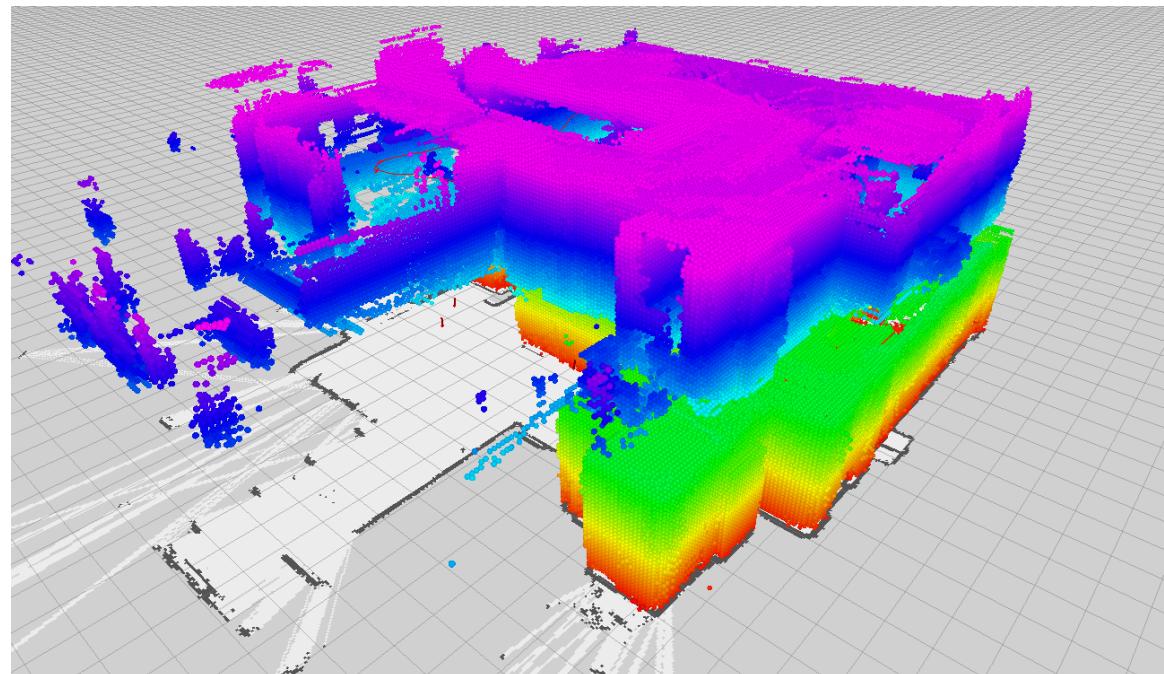
command motors and produce desired actions in order to navigate to desired state

- Mapping

- Planning

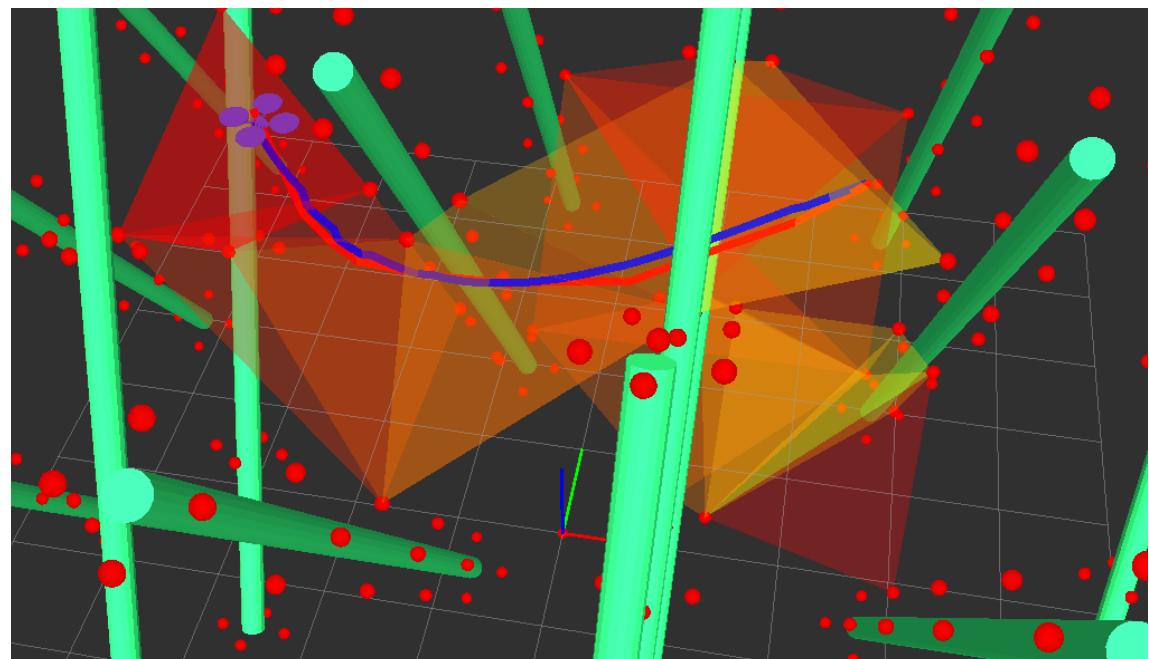
In any system, ask how the following components work!

- State Estimation
- Control
- Mapping
- Planning



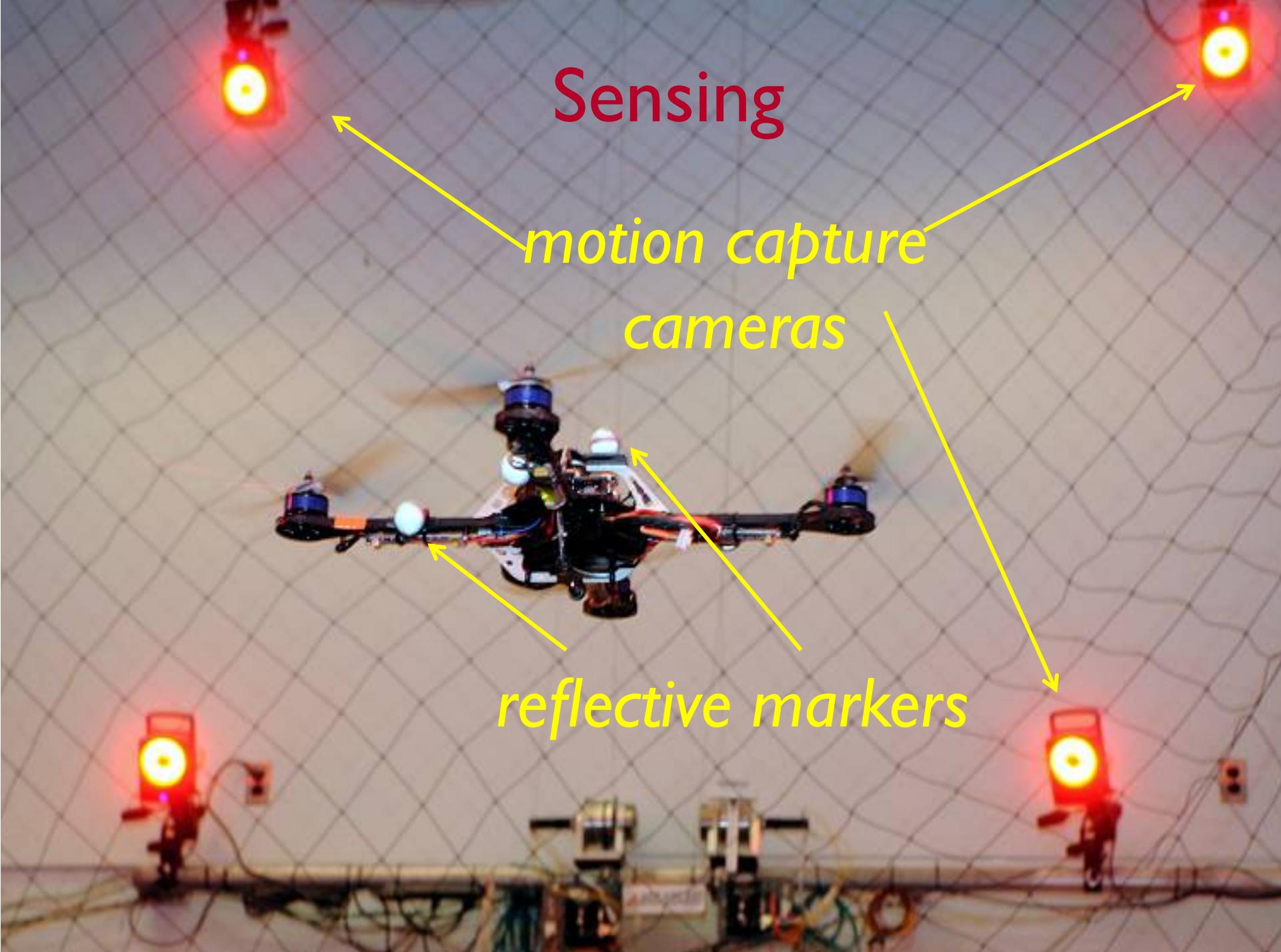
In any system, ask how the following components work!

- State Estimation
- Control
- Mapping
- Planning



State Estimation

*obtain reliable estimates of
position and velocity*



Sensing

*motion capture
cameras*

reflective markers



X-47B Wins Popular Mechanics Magazine 2013 Breakthrough Award

Photograph by Joe McNally



Unreliable GPS



Operate indoors and outdoors

No GPS

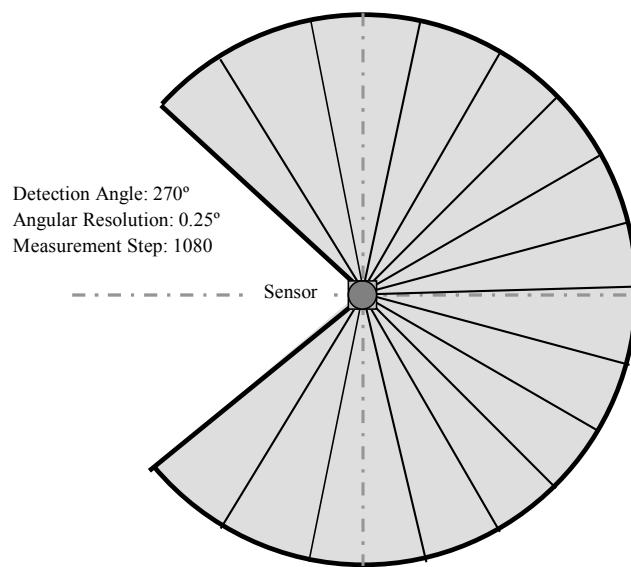
Small, maneuverable

Agile, fast

How to navigate without GPS or external motion capture cameras?



Hokuyo UTM-30LX
Scanning Laser
Rangefinder

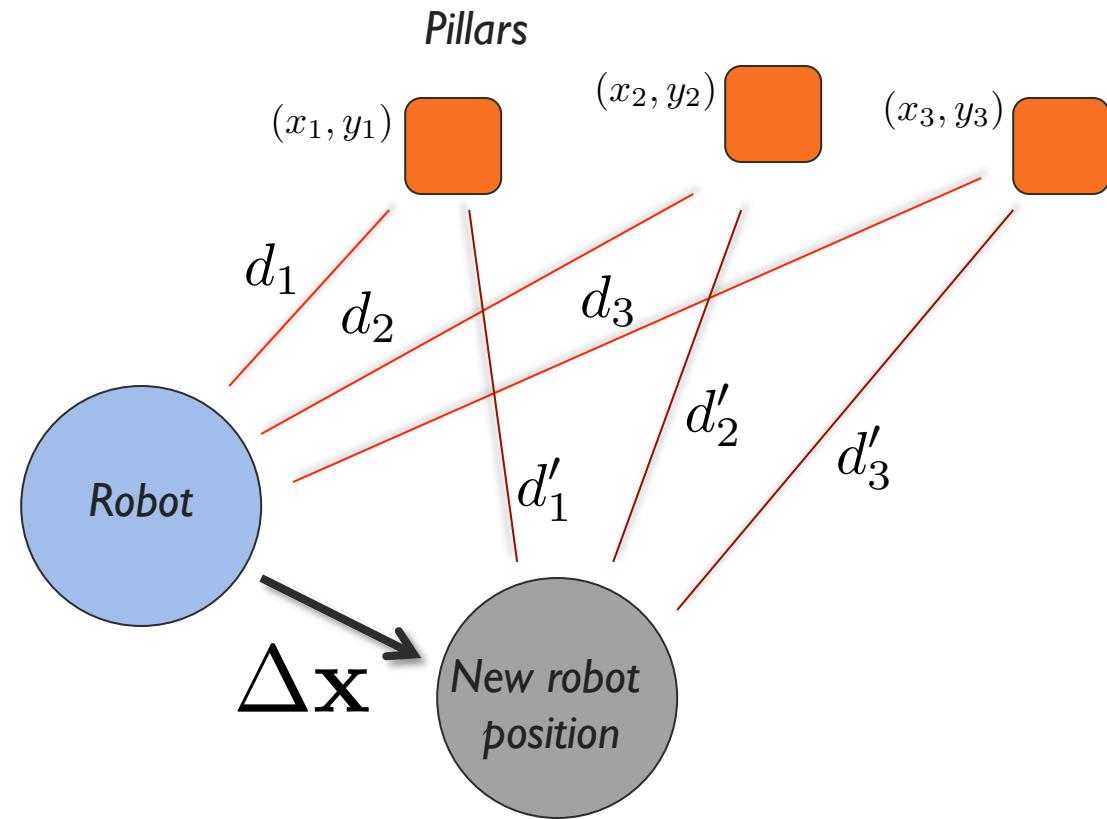


mvBlueFox USB
camera

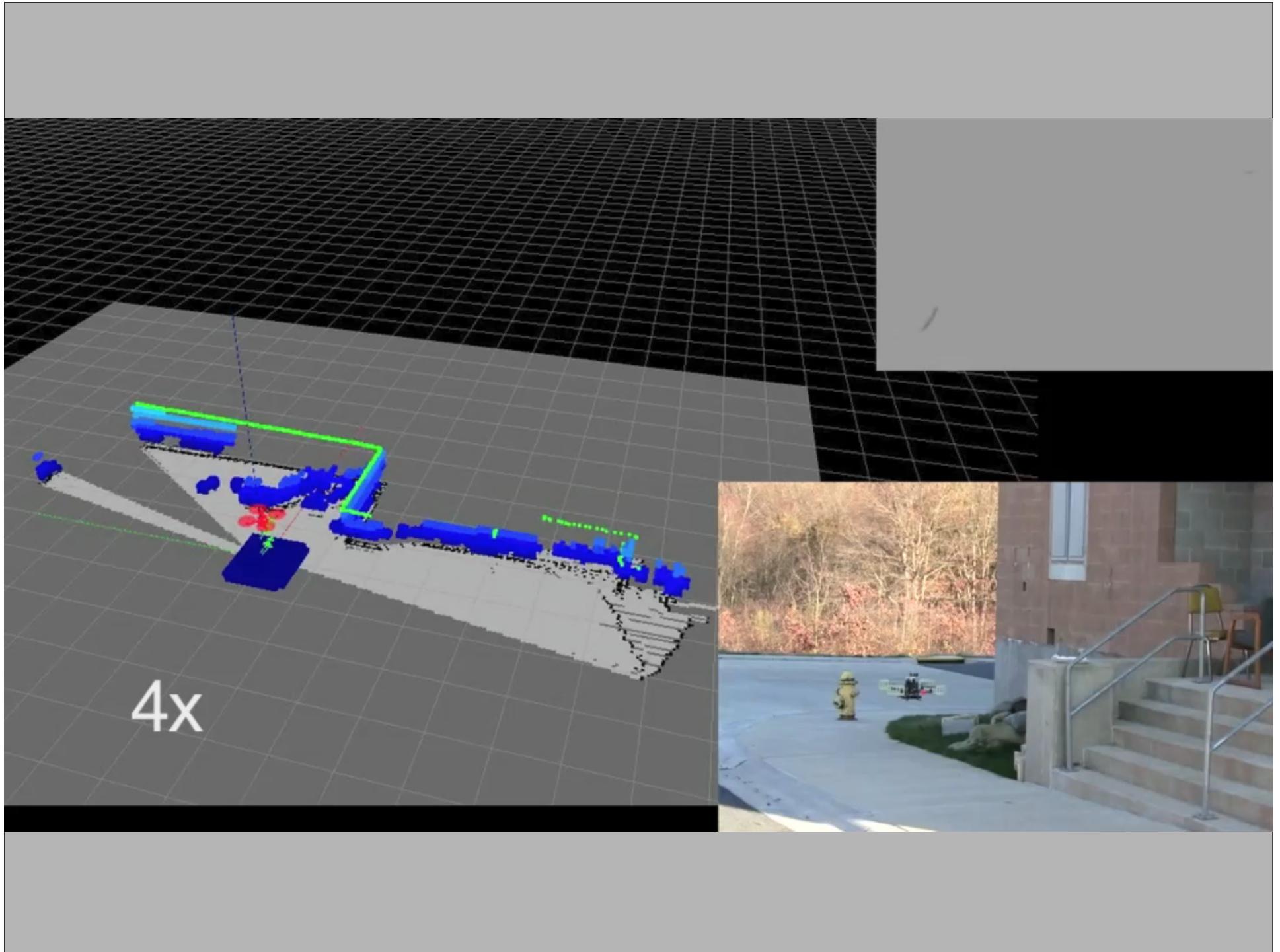


Kinect sensor for
XBOX 360

Simultaneous Localization And Mapping



Concurrently estimate
Locations of pillars (6)
Displacement of the robot (2)





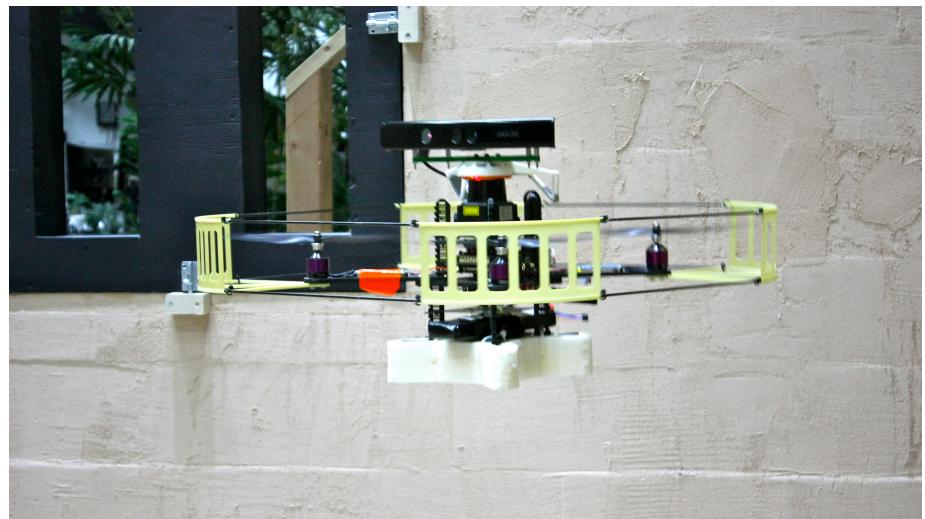
1750 g (laser, 3 cameras, GPS, IMU)



650 g (camera, IMU)

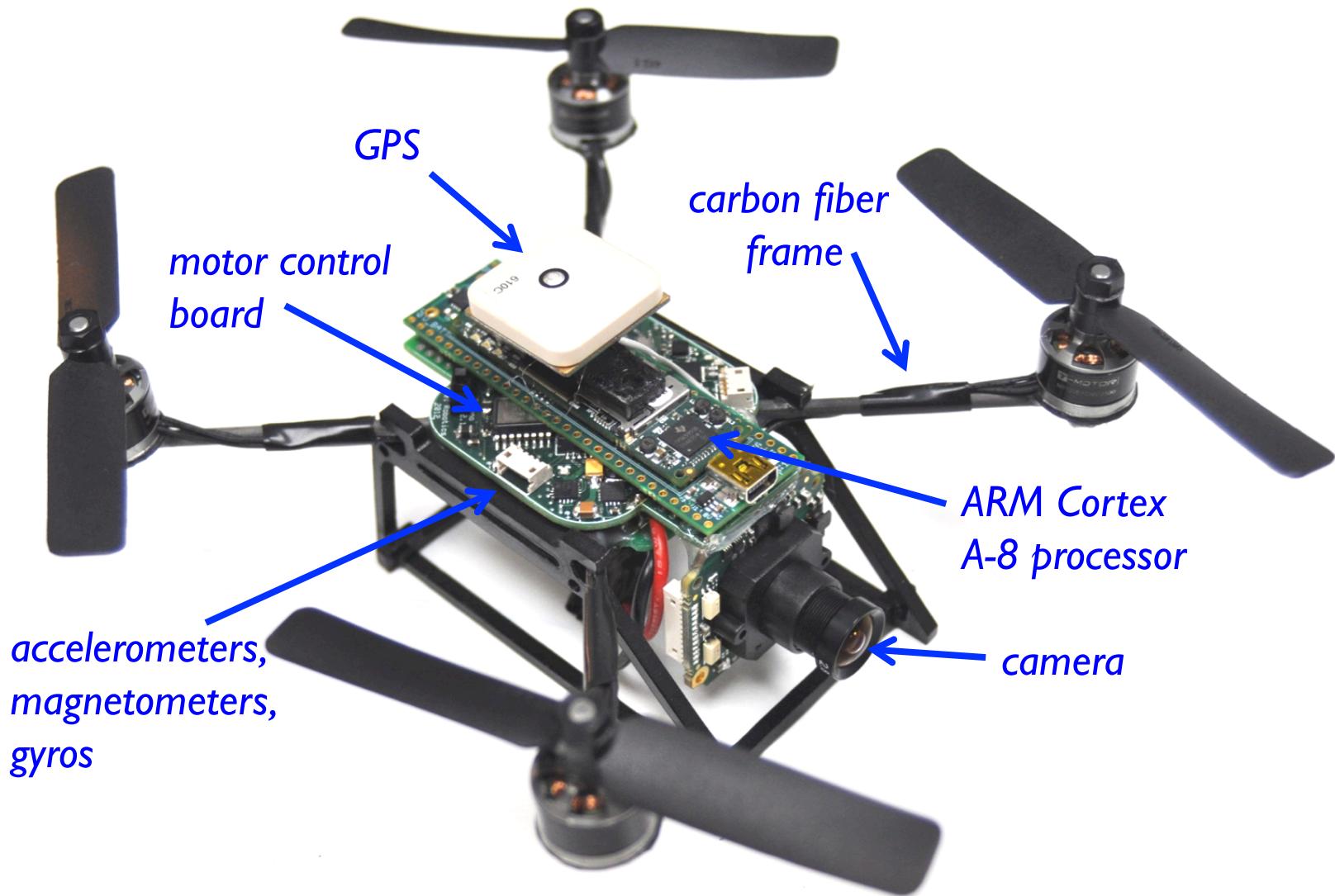


740 g (2 cameras, IMU)



1800 g (laser, Kinect, IMU)

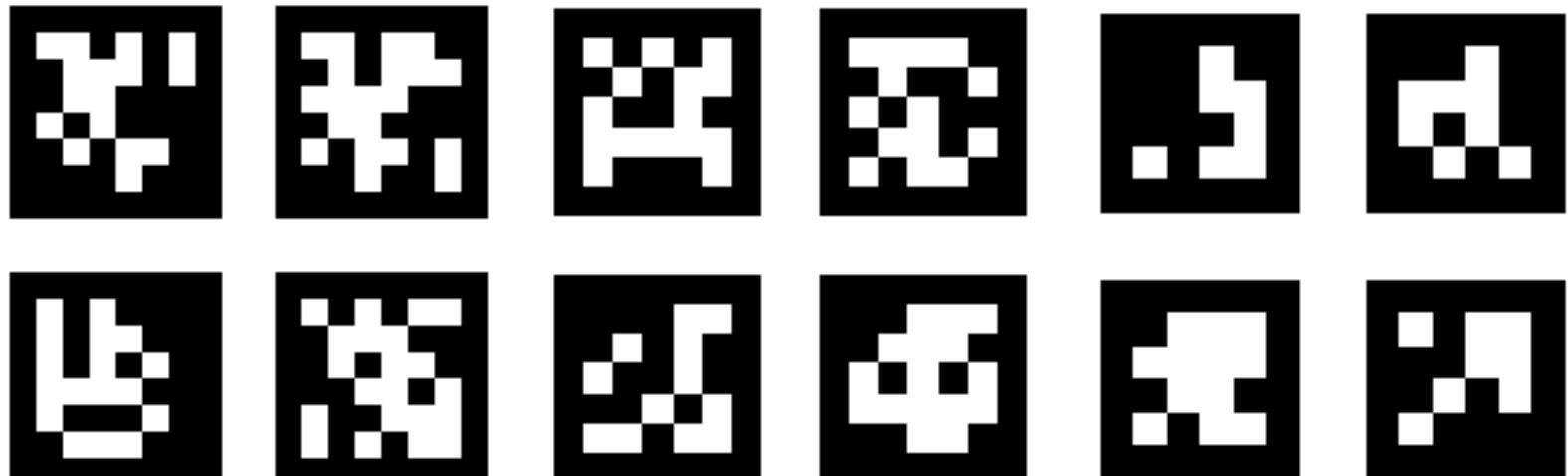
Experimental Platform



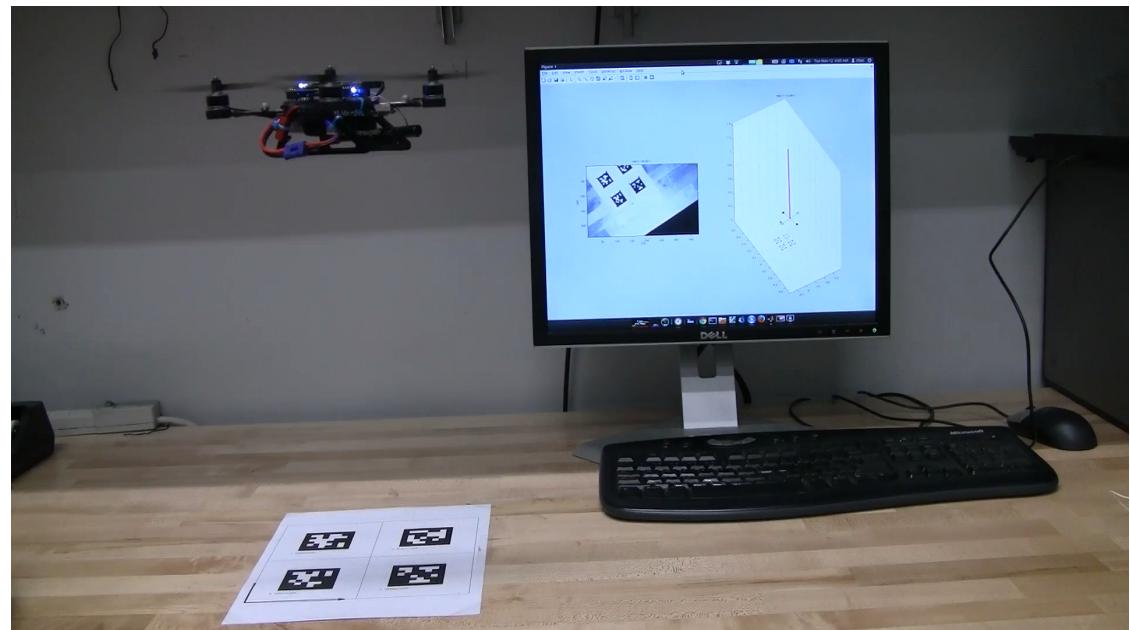
KMel Robotics

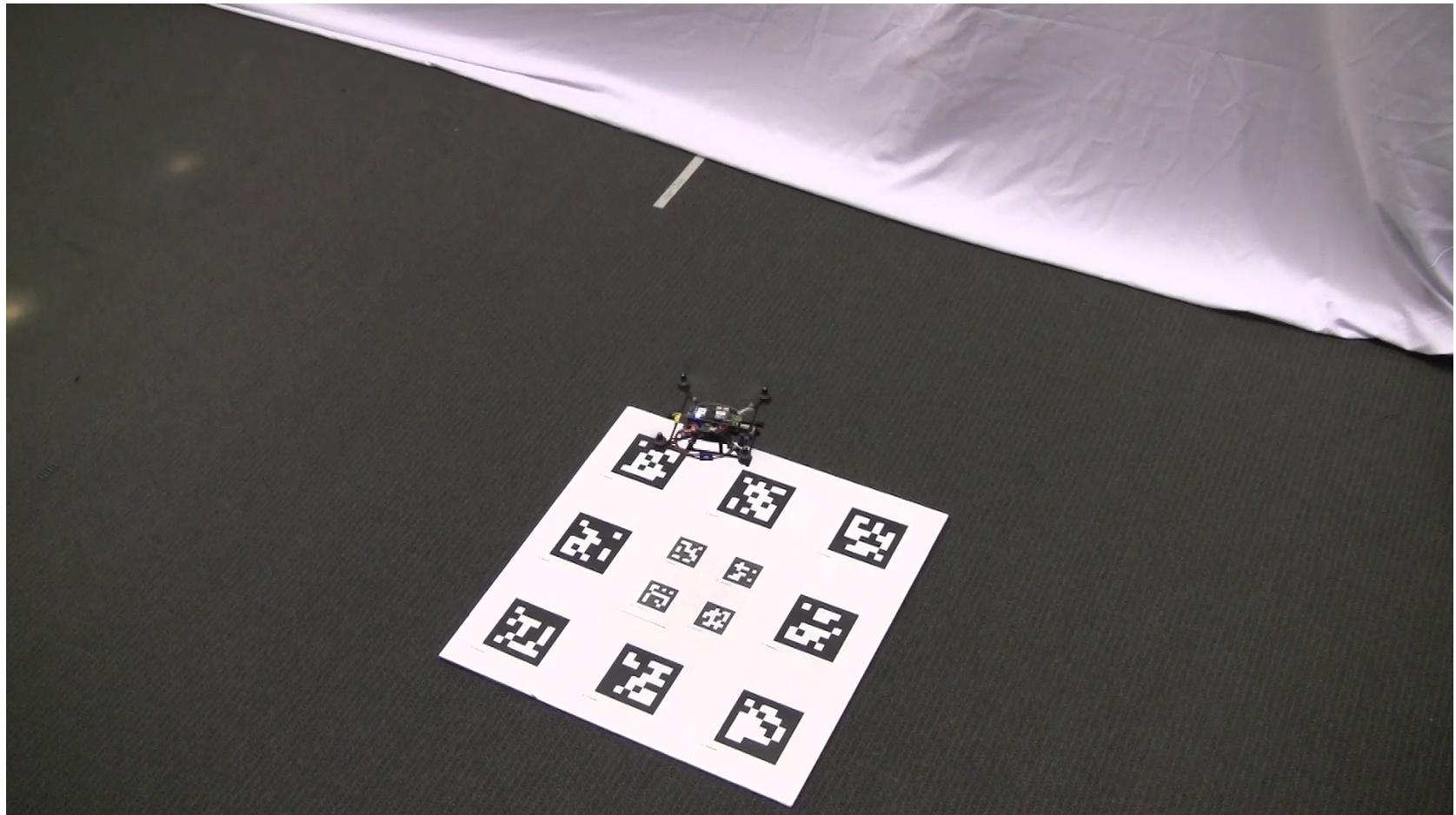
www.kmelrobotics.com

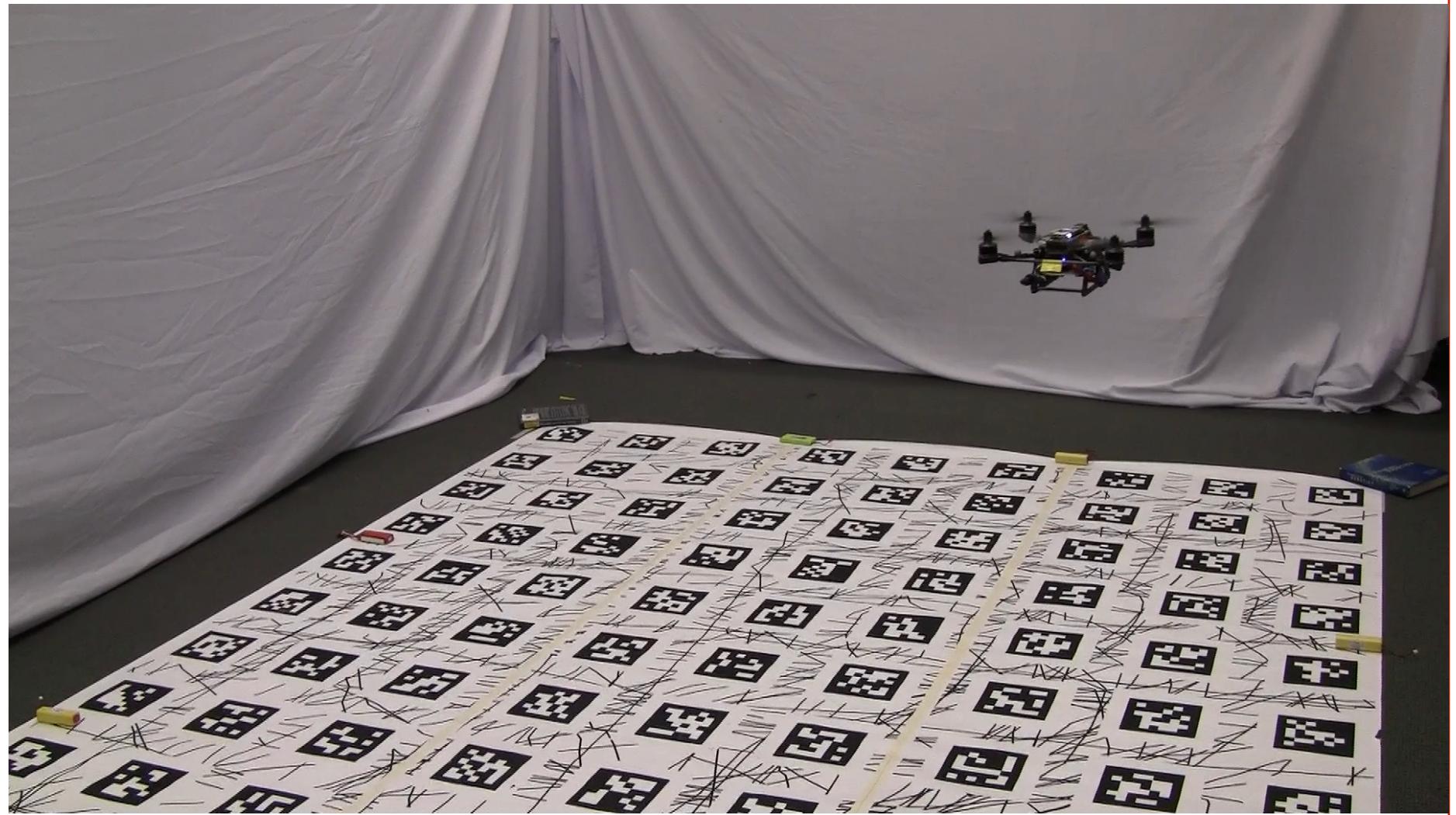
Beacons that are easy to recognize



<http://april.eecs.umich.edu/wiki/images/9/94/Tagsampler.png>

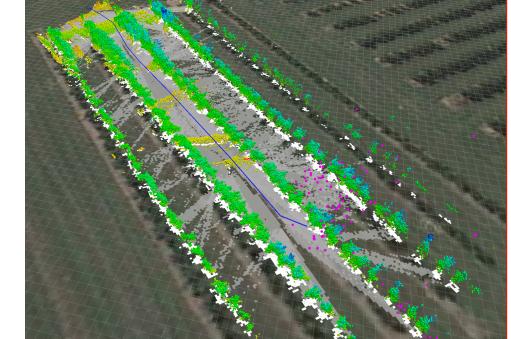
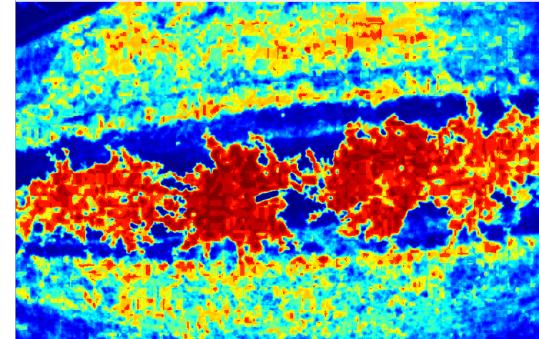
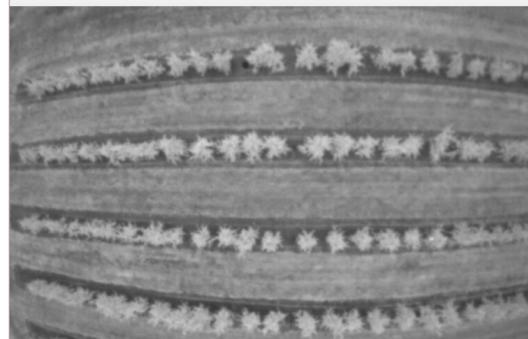








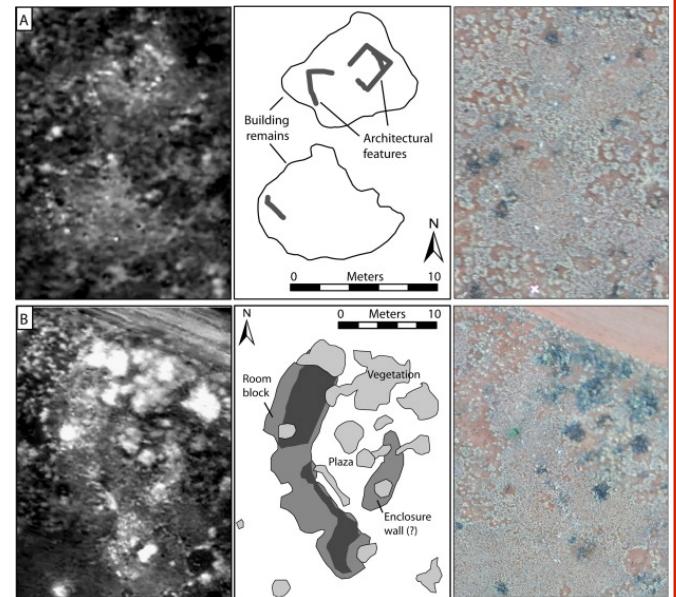
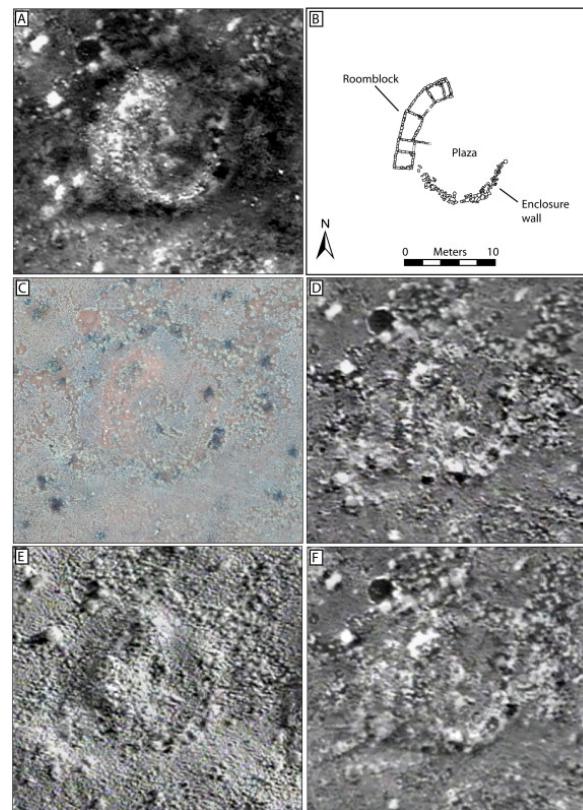
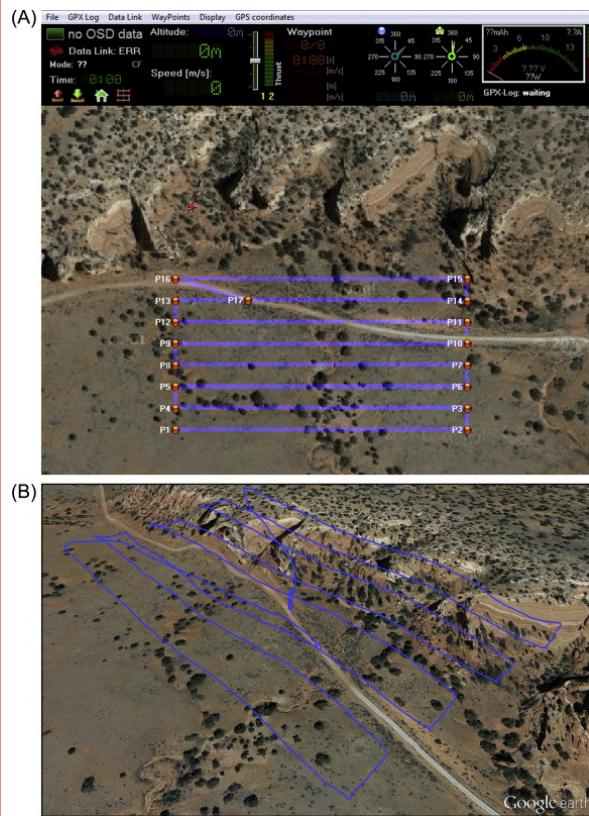
Precision Farming



Construction



Archeology



Photography



© Capungaero/Dronestagram



© Postandfly/Dronestagram



© Svetlin Marinov/Dronestagram

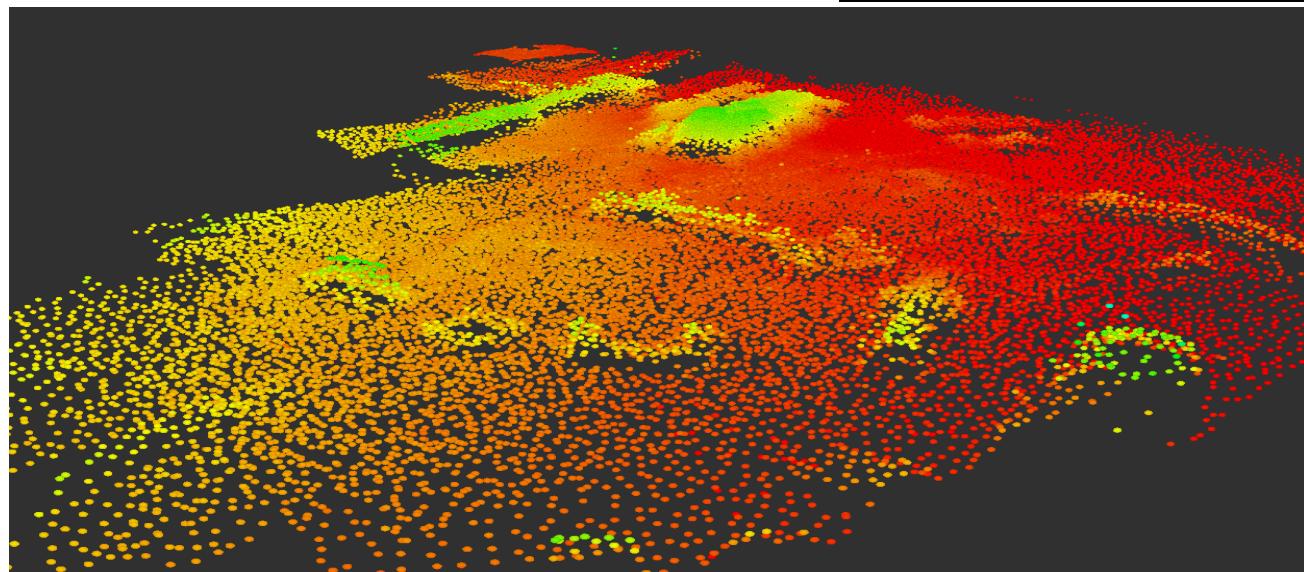
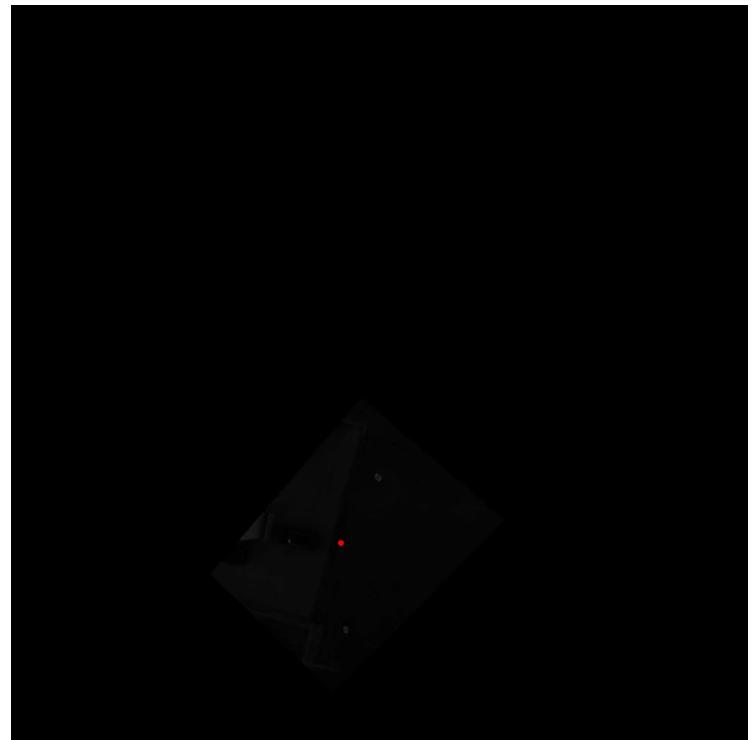
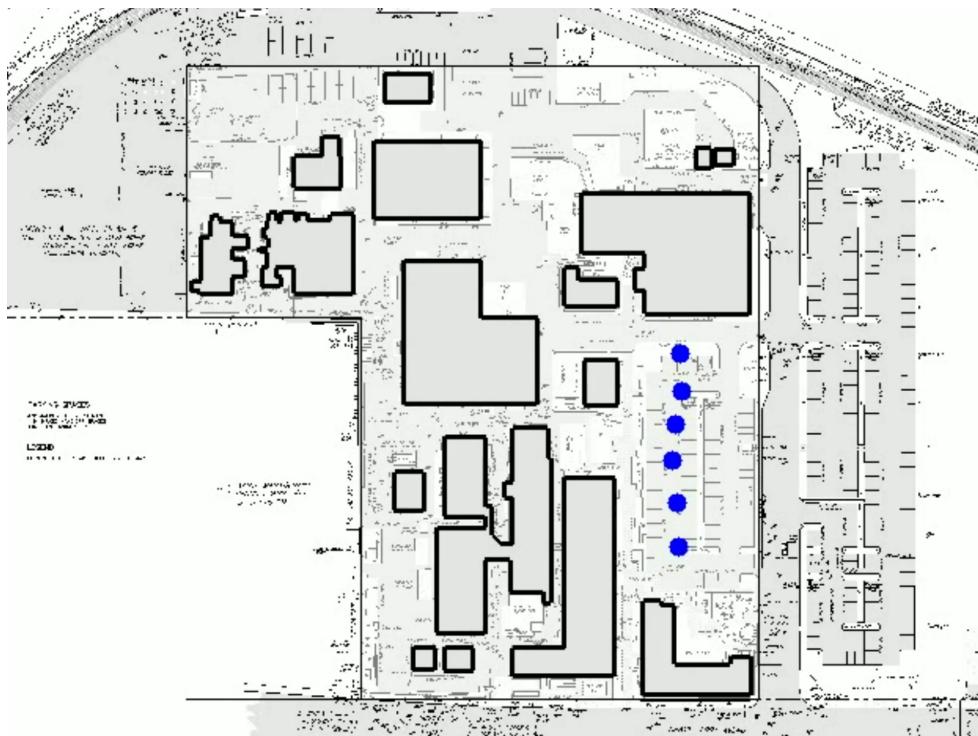


Robot First Responders



Kartik Mohta, Matthew Turpin, Alex Kushleyev, Daniel Mellinger, Nathan Michael, and Vijay Kumar,

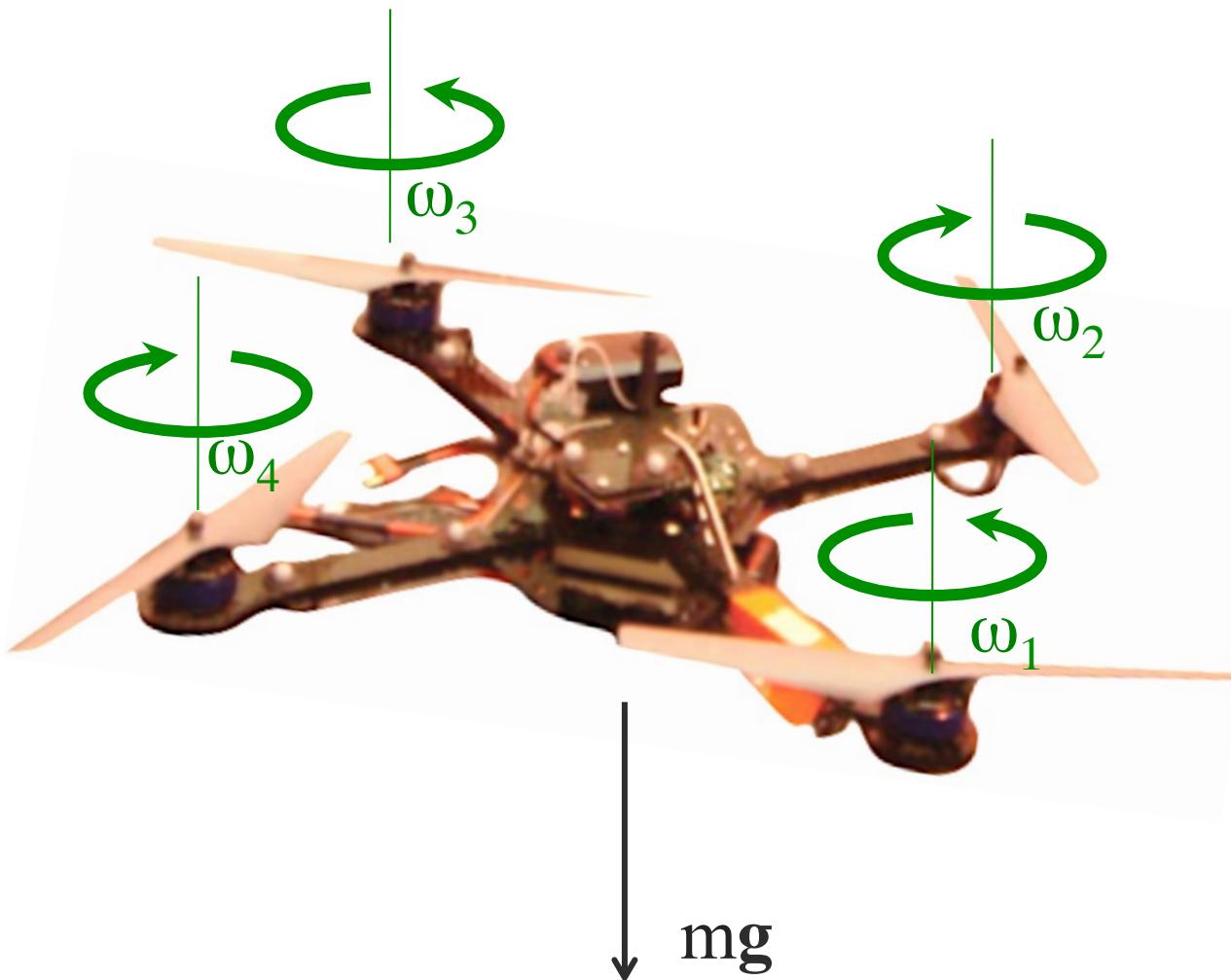
 Penn Engineering “QuadCloud: A Rapid Response Force with Quadrotor Teams,” *Int. Symp. on Experimental Robotics (ISER)*, 2014. 56



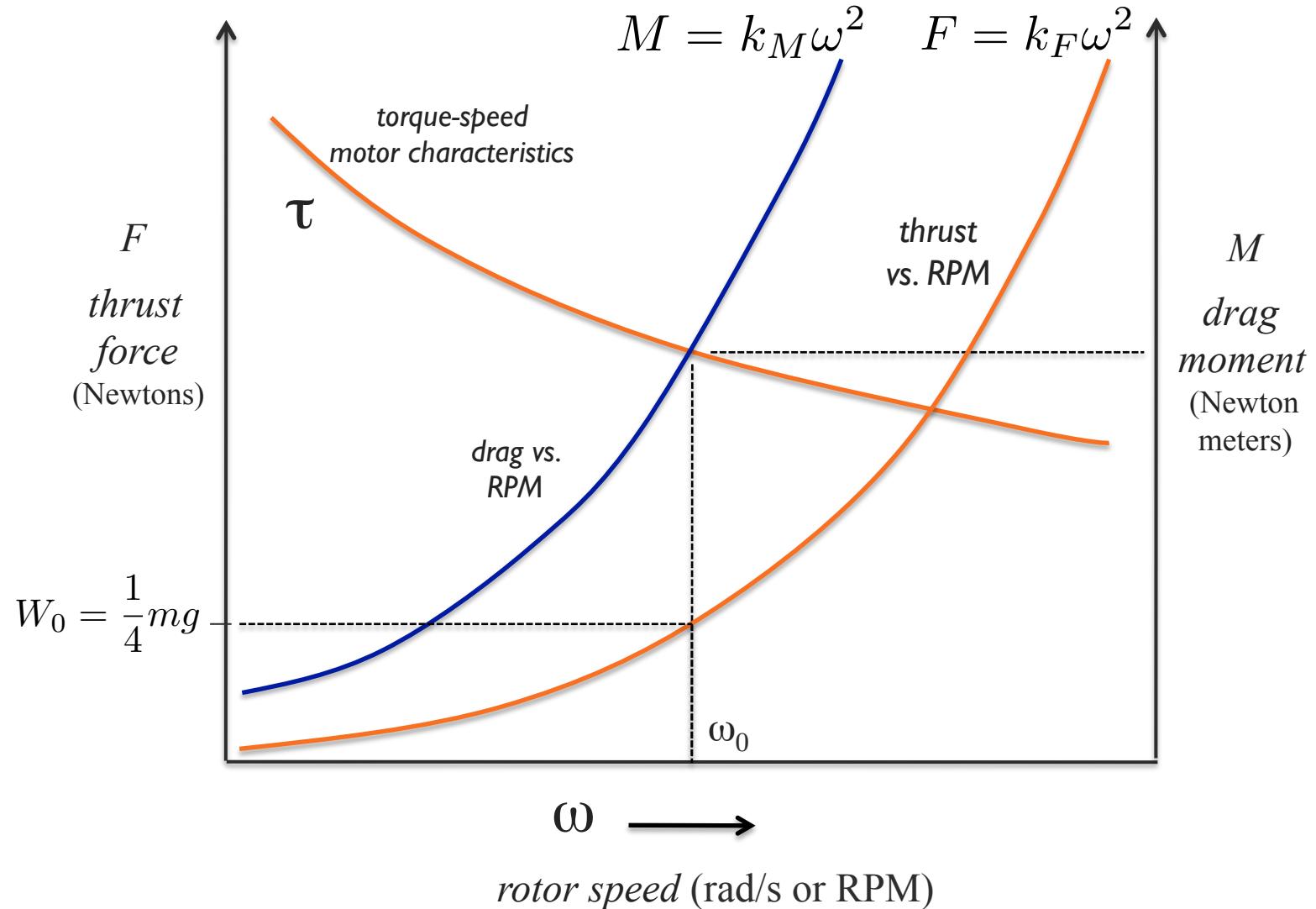
Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

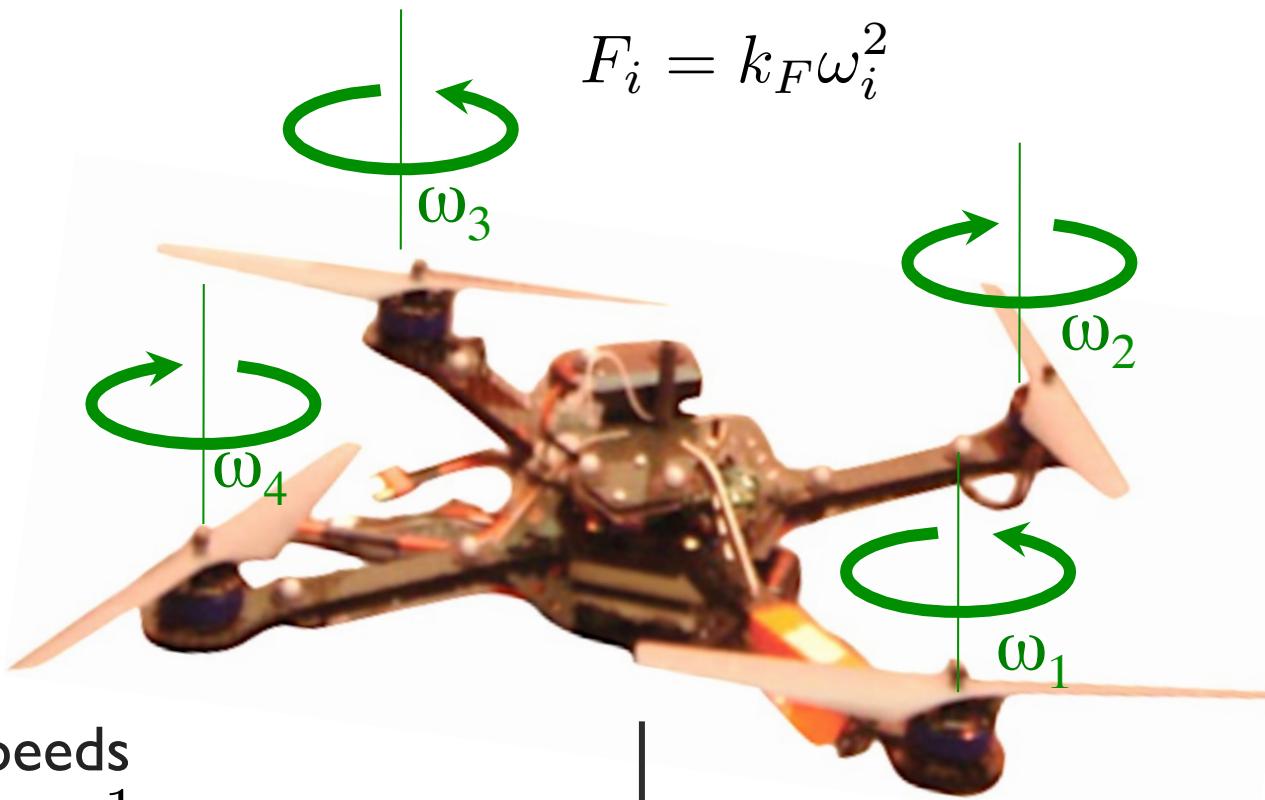
Basic Mechanics



Rotor Physics



Basic Mechanics (Hover)



Motor Speeds

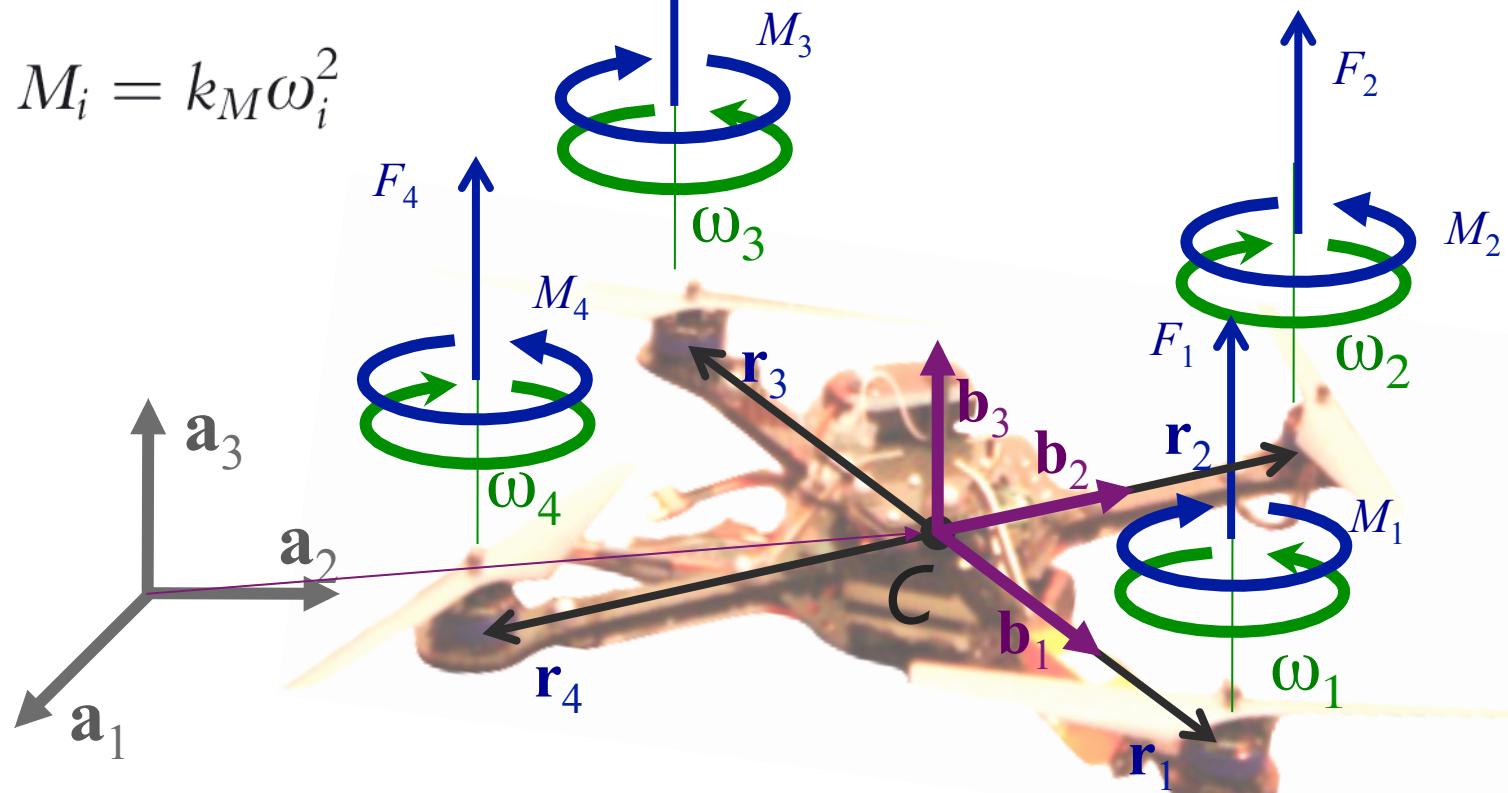
$$k_F \omega_i^2 = \frac{1}{4} mg$$

Motor Torques

$$\tau_i = k_M \omega_i^2$$

$$F_i = k_F \omega_i^2$$

$$M_i = k_M \omega_i^2$$



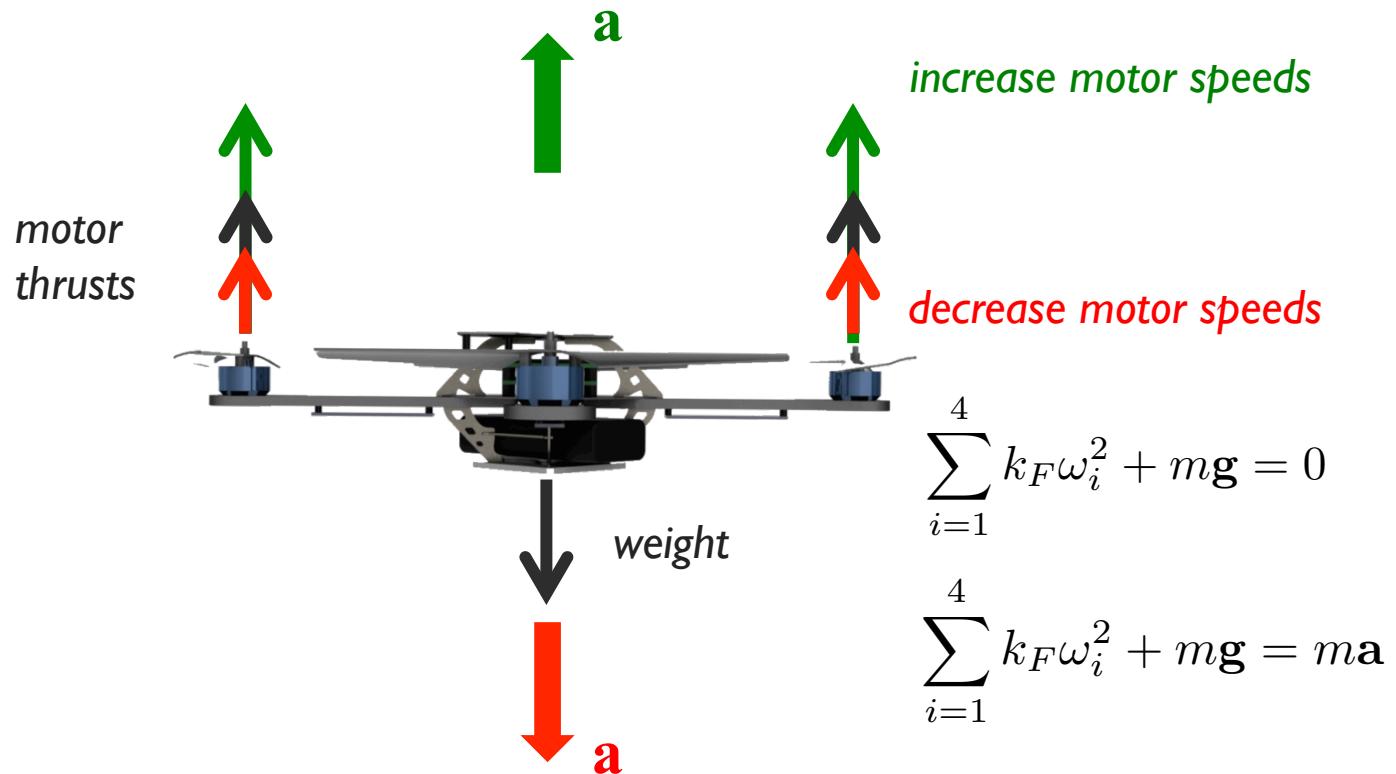
Resultant Force

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 - mg\mathbf{a}_3$$

Resultant Moment

$$\mathbf{M} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3 + \mathbf{r}_4 \times \mathbf{F}_4 + \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$

Acceleration (in the vertical direction)



Goals

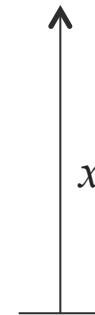
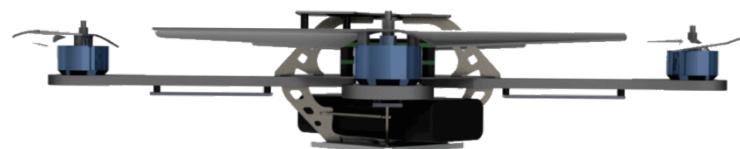
- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Control of height

$$\sum_{i=1}^4 k_F \omega_i^2 + mg = m\mathbf{a} \rightarrow a = \frac{d^2x}{dt^2} = \ddot{x}$$

Input $u = \frac{1}{m} \left[\sum_{i=1}^4 k_F \omega_i^2 + mg \right]$

Second order dynamic system $u = \ddot{x}$



Control of a linear second-order system

Problem

State, input $x, u \in \mathbb{R}$

Plant model $\ddot{x} = u$

Want x to follow the desired trajectory $x^{des}(t)$

General Approach

Define error, $e(t) = x^{des}(t) - x(t)$

Want $e(t)$ to converge exponentially to zero

Strategy

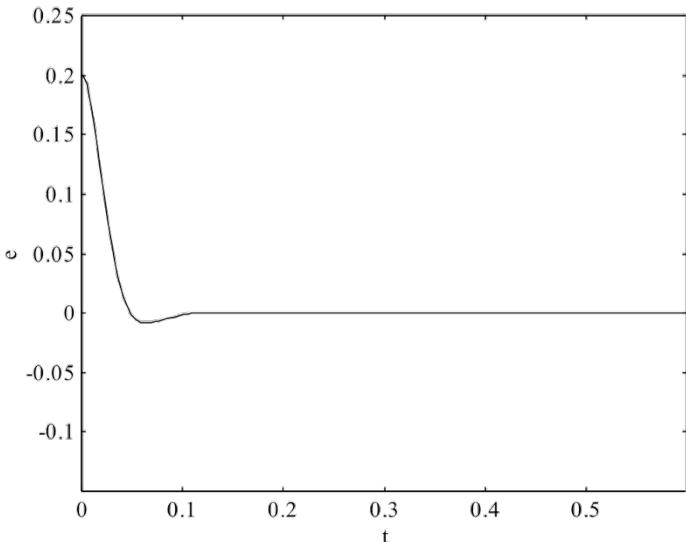
Find u such that

$$\ddot{e} + K_v \dot{e} + K_p e = 0 \quad K_p, K_v > 0$$

$$u(t) = \ddot{x}^{des}(t) + K_v \dot{e}(t) + K_p e(t)$$

Feedforward

Derivative



Proportional

Control for trajectory tracking in a simple second-order system

PD control

$$u(t) = \ddot{x}^{\text{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Proportional control acts like a spring (capacitance) response

Derivative control is a viscous dashpot (resistance) response

Large derivative gain makes the system overdamped and the system converges slowly

PID control

In the presence of disturbances (e.g., wind) or modeling errors (e.g. unknown mass), it is often advantageous to use PID control

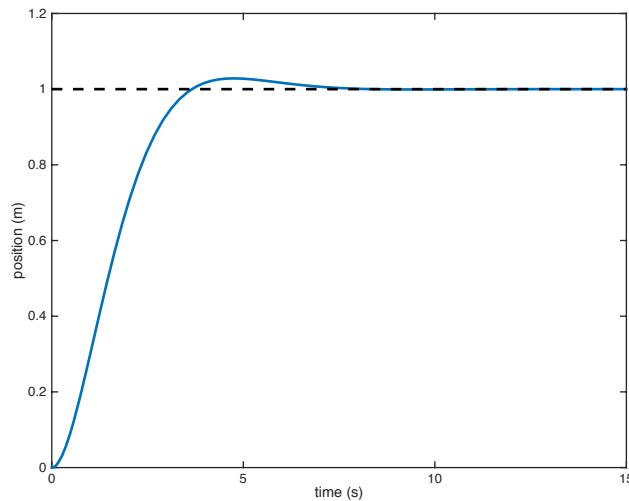
$$u(t) = \ddot{x}^{\text{des}}(t) + K_v \dot{e}(t) + K_p e(t) + K_i \int_0^t e(\tau) d\tau$$

↑
Integral

PID control generates a third-order closed-loop system

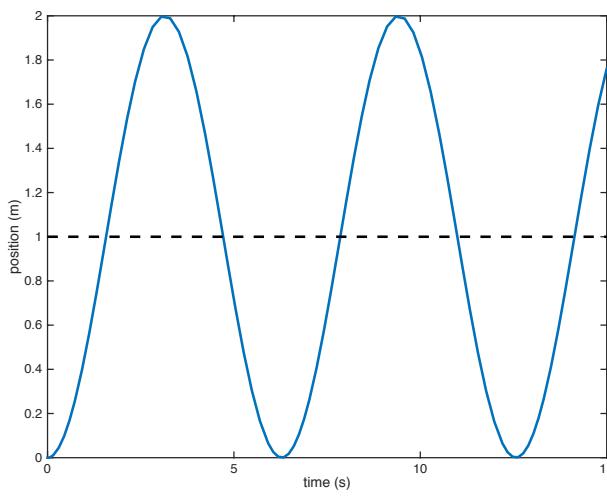
Integral control makes the steady-state error go to zero

Effects of Gains for a PD Control System



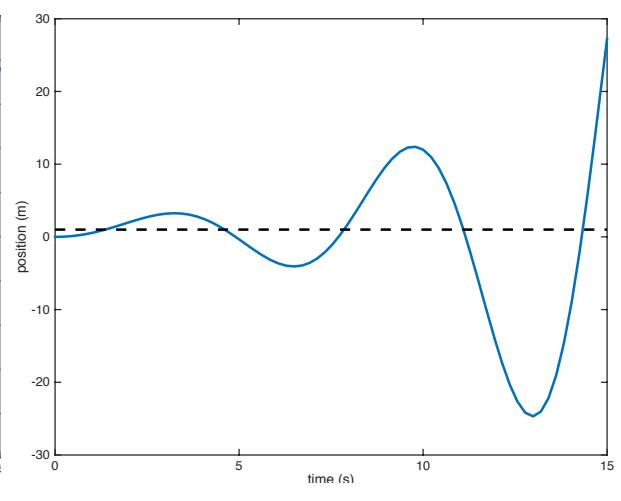
Stable

$$K_p, K_v > 0$$



Marginally Stable

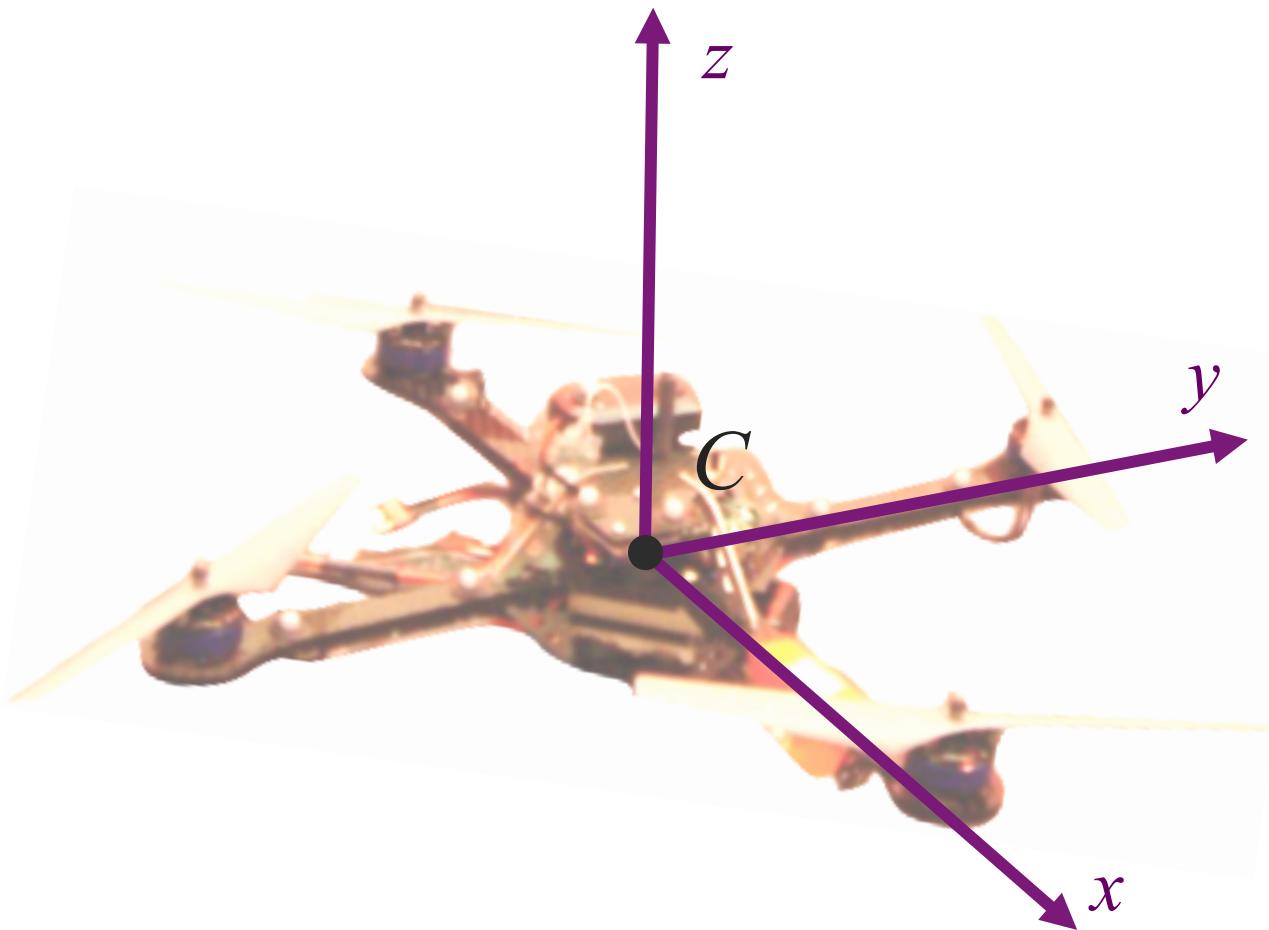
$$K_p > 0, \quad K_v = 0$$



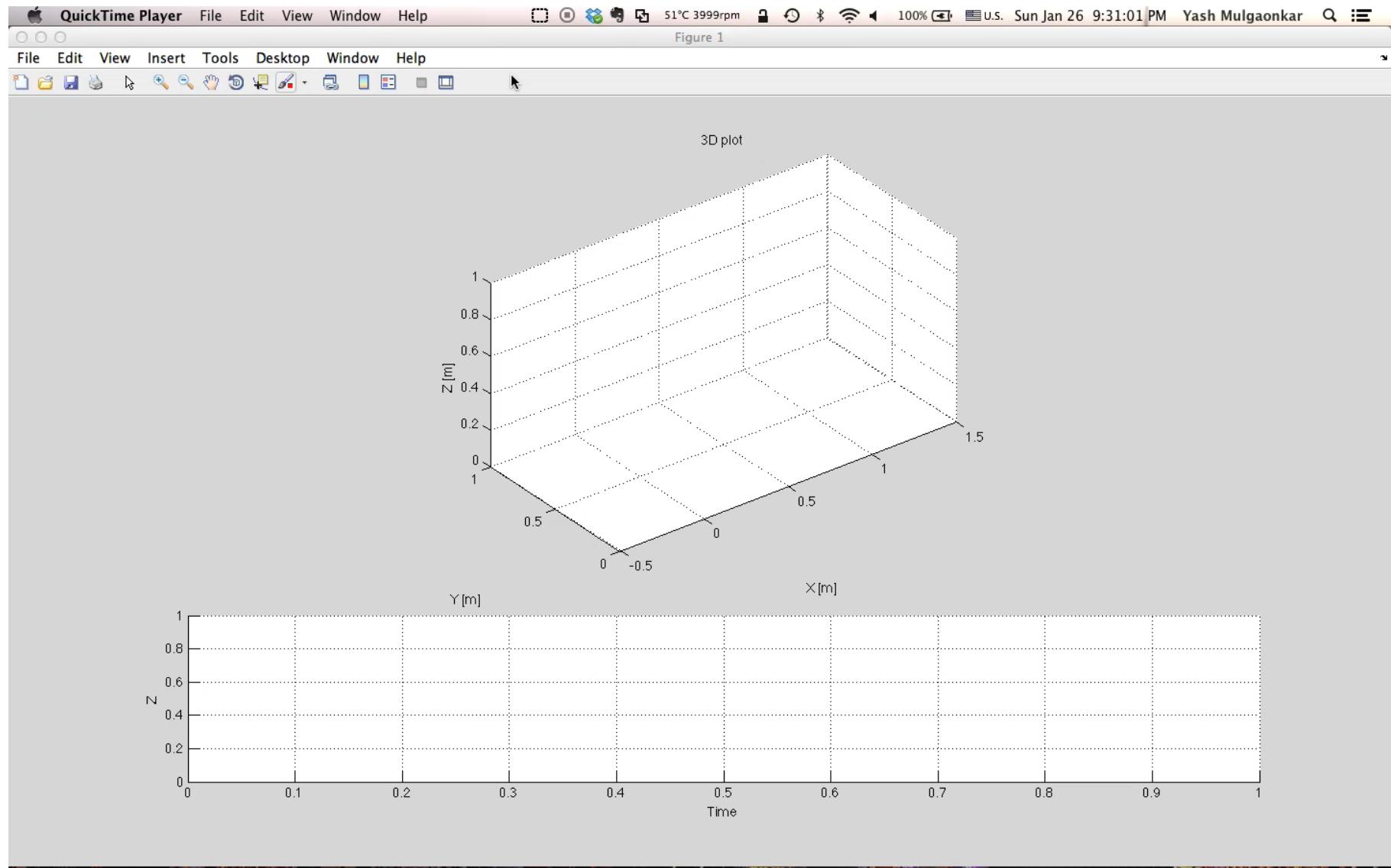
Unstable

$$K_p \text{ or } K_v < 0$$

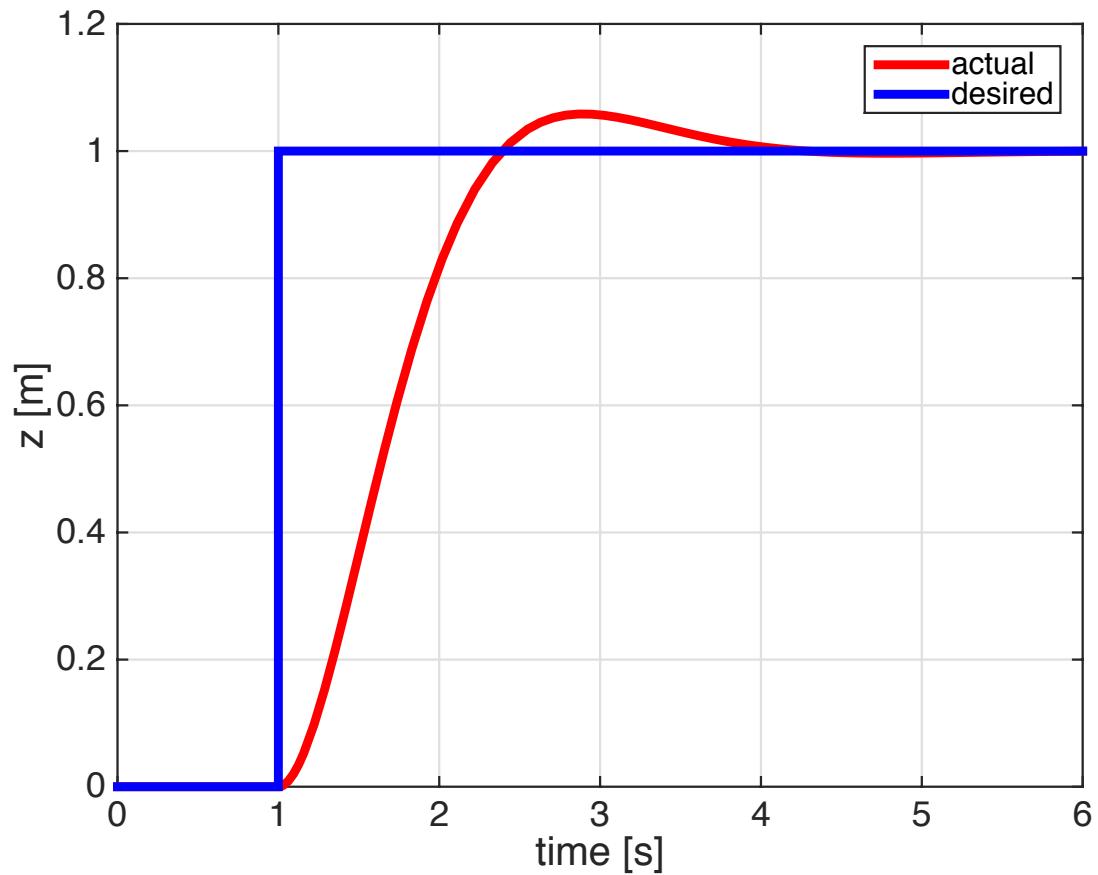
Control of quadrotor height



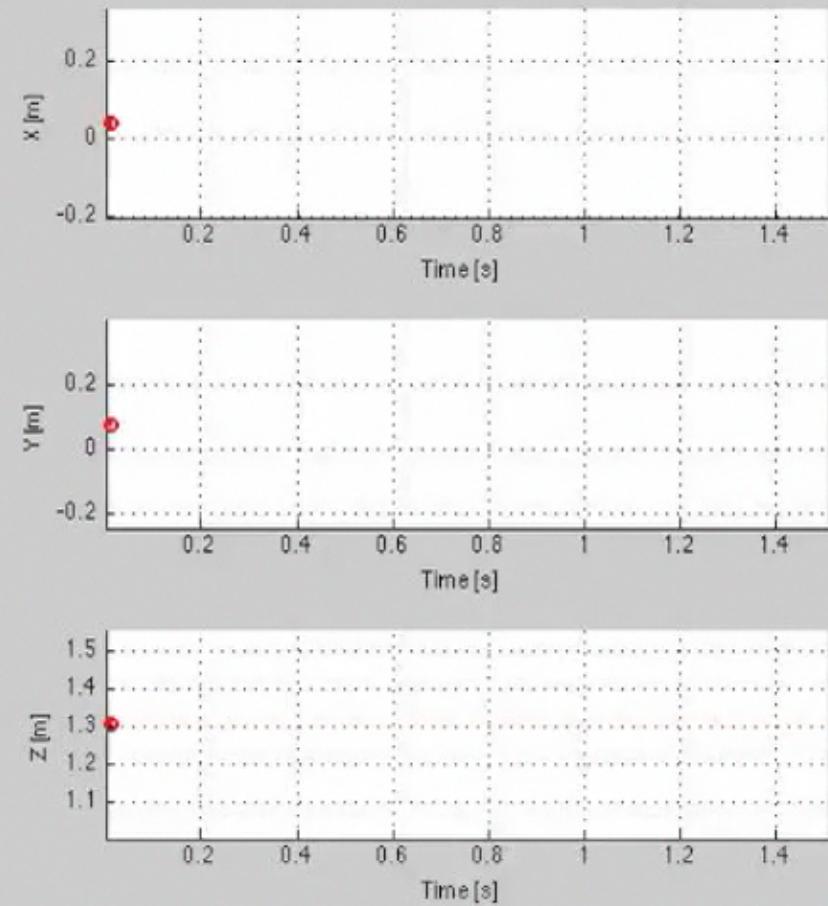
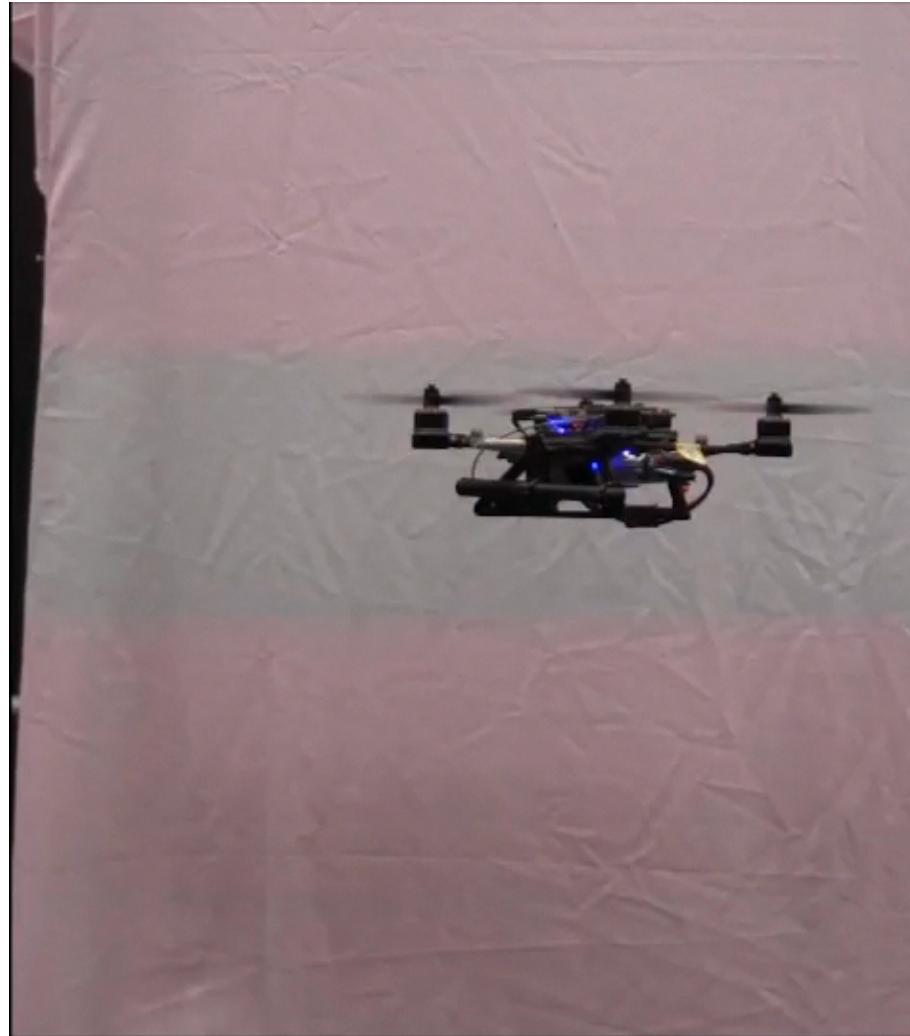
Simulation - PD Control of height



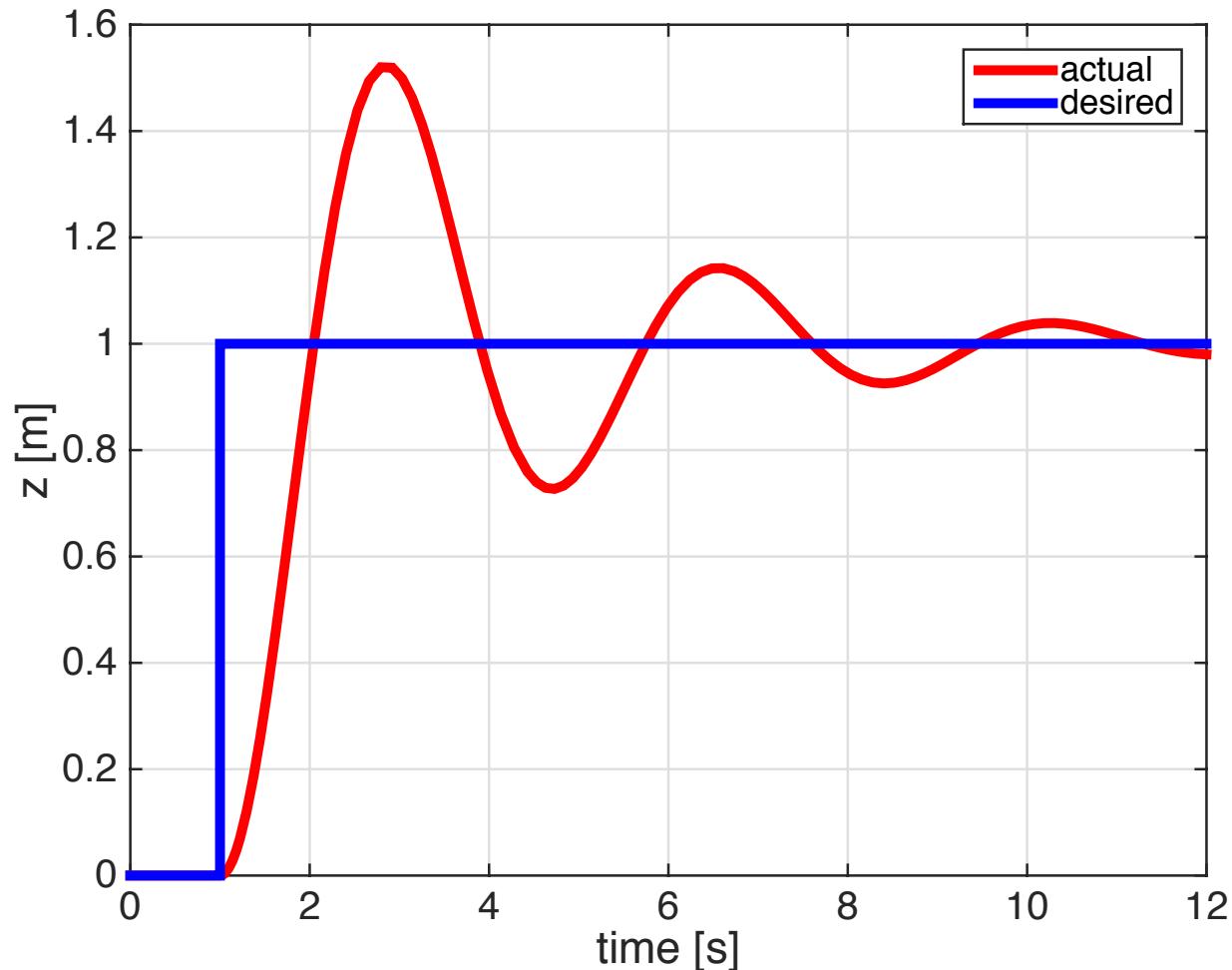
PD Controller

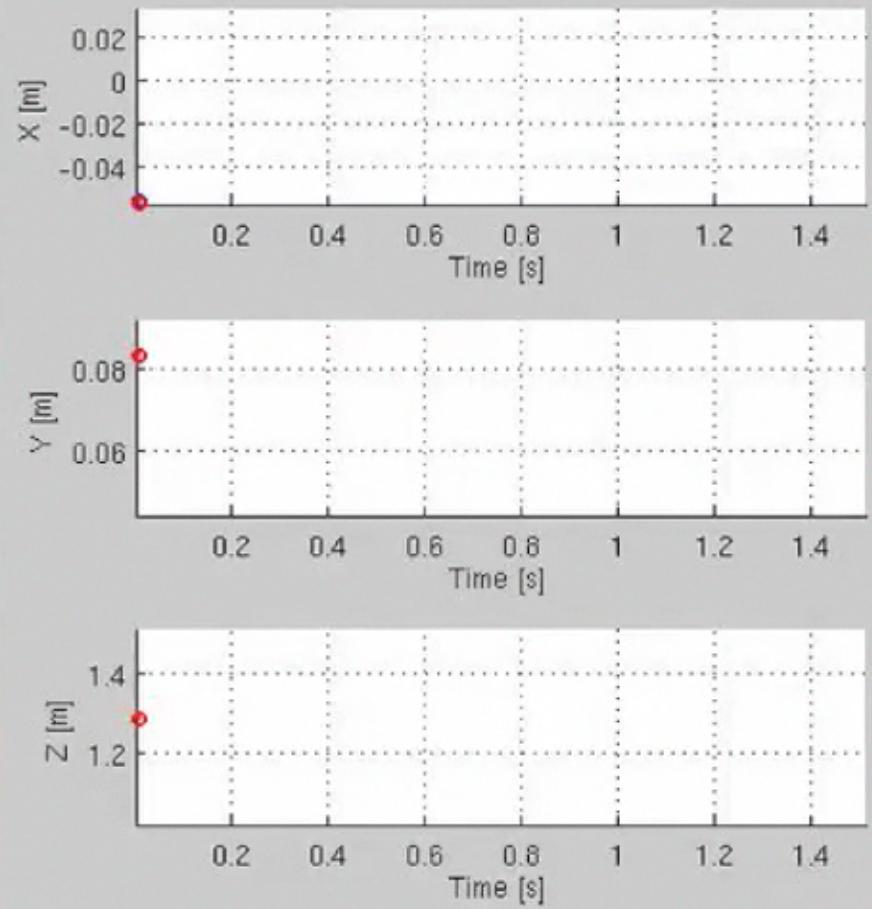
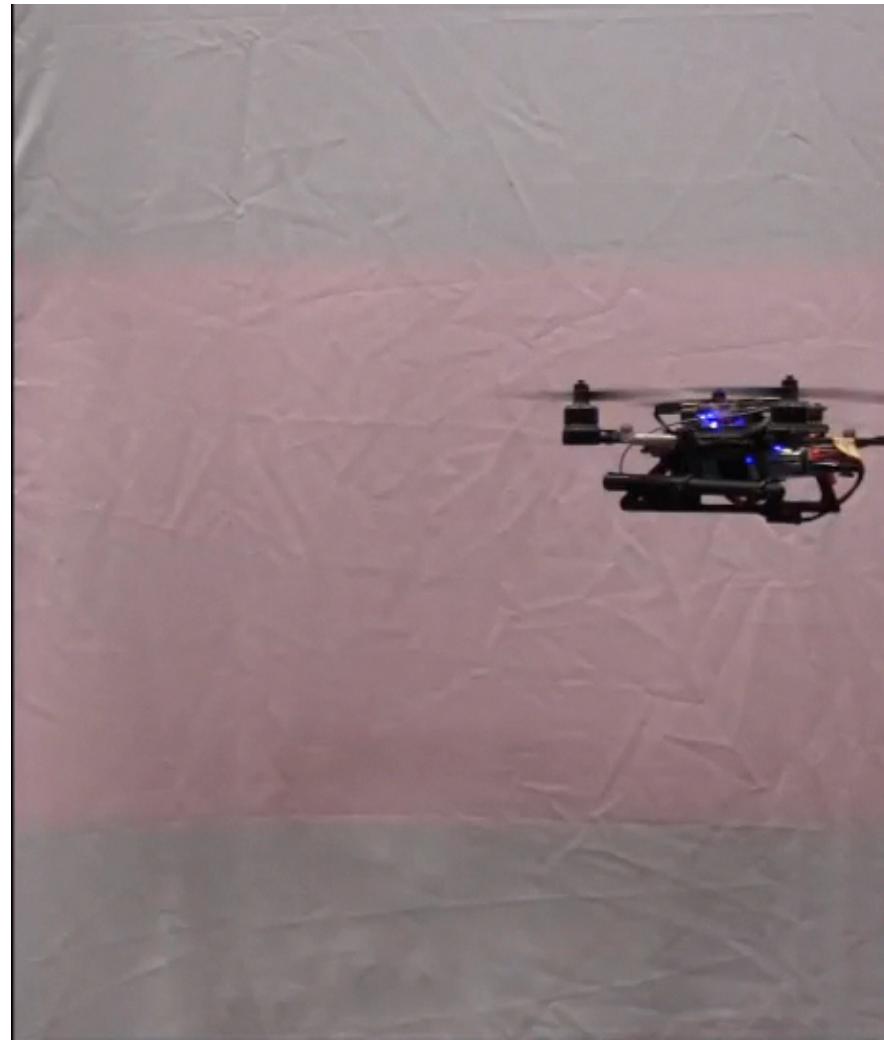


PD Controller

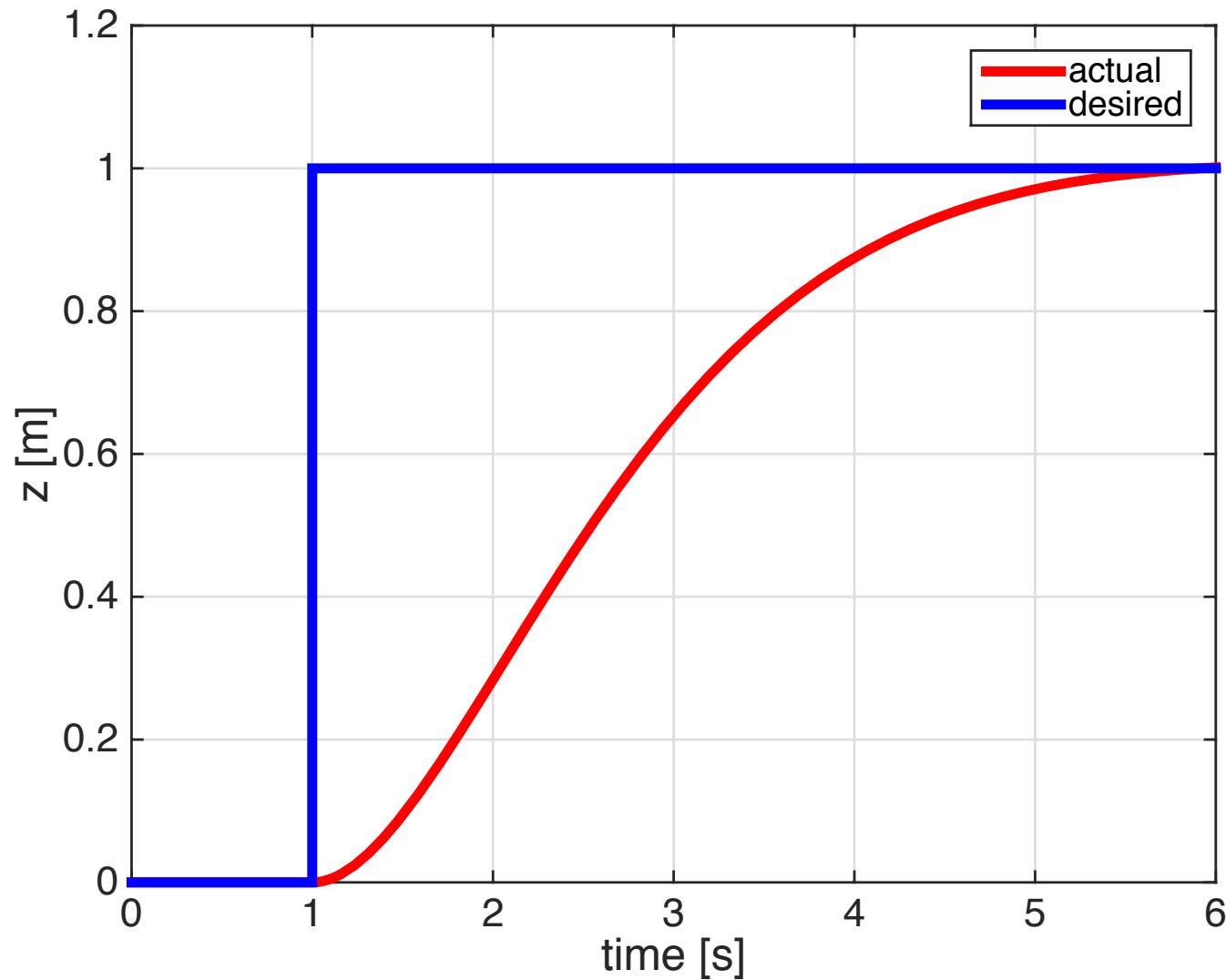


High K_p

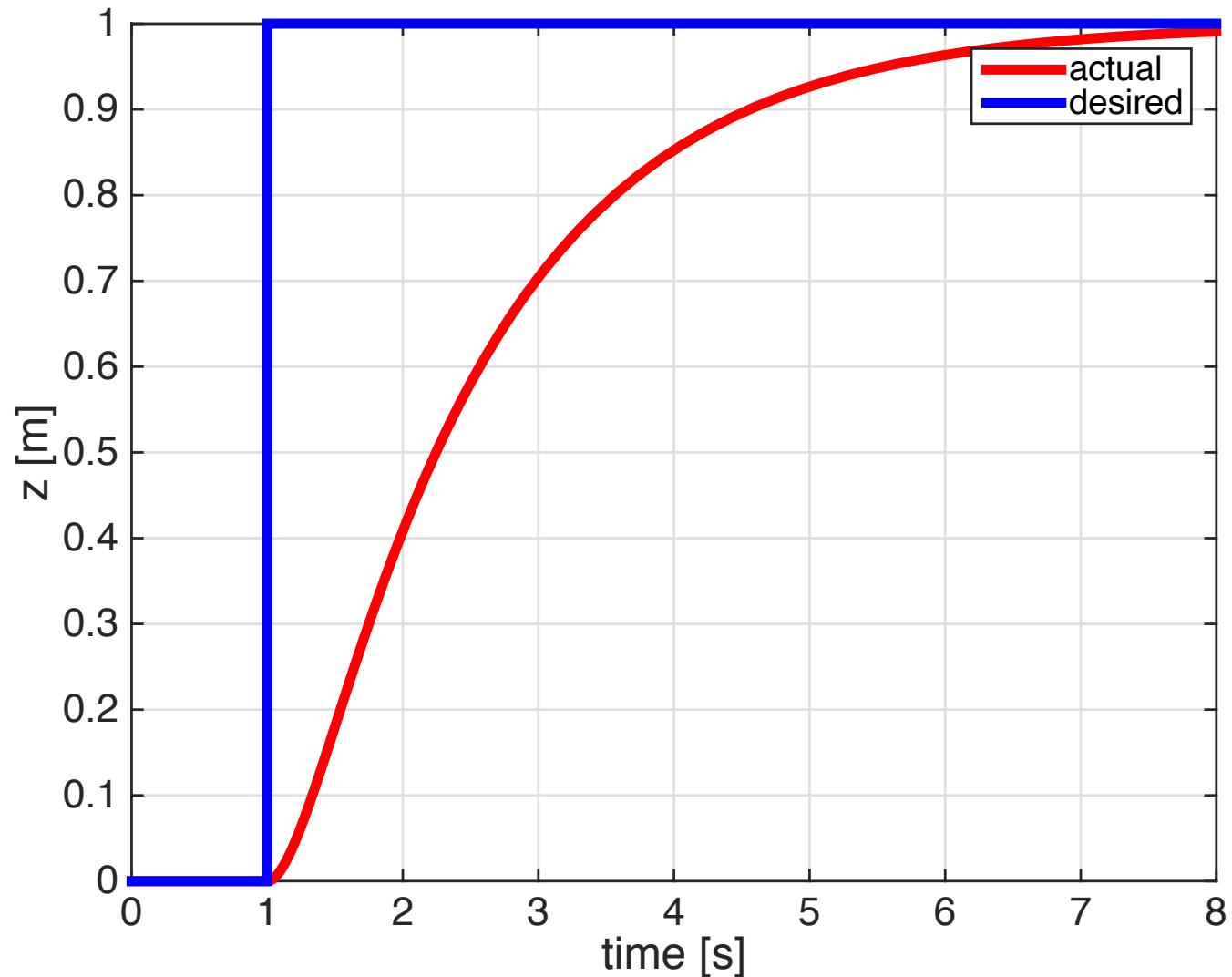


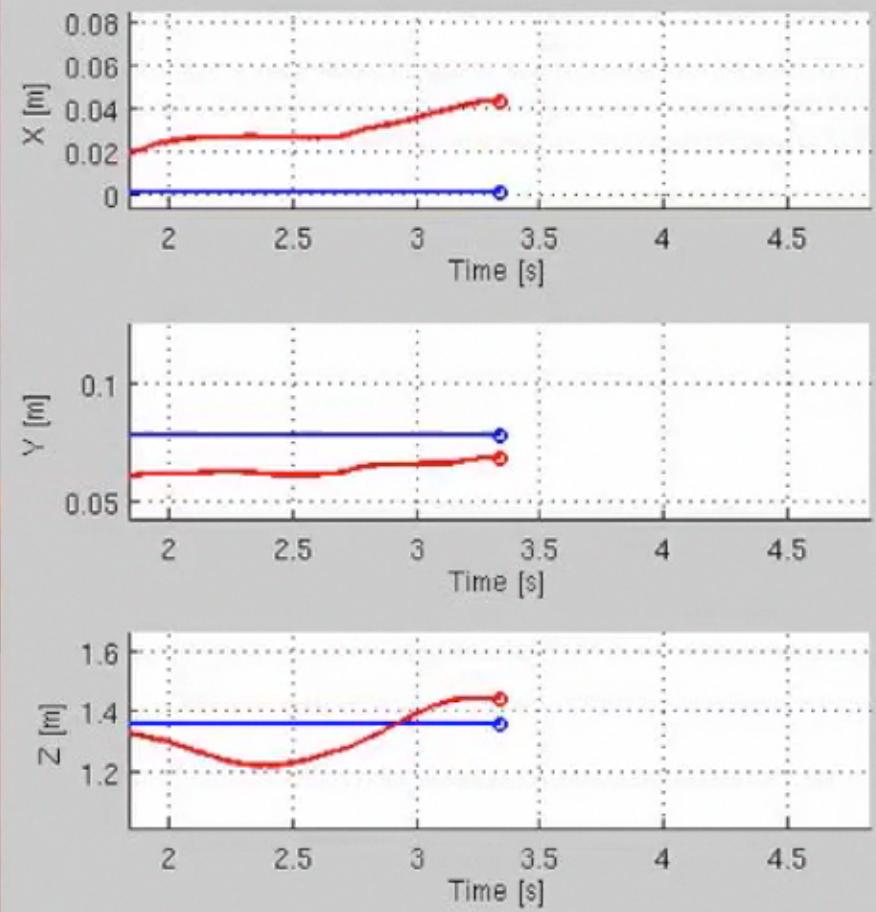
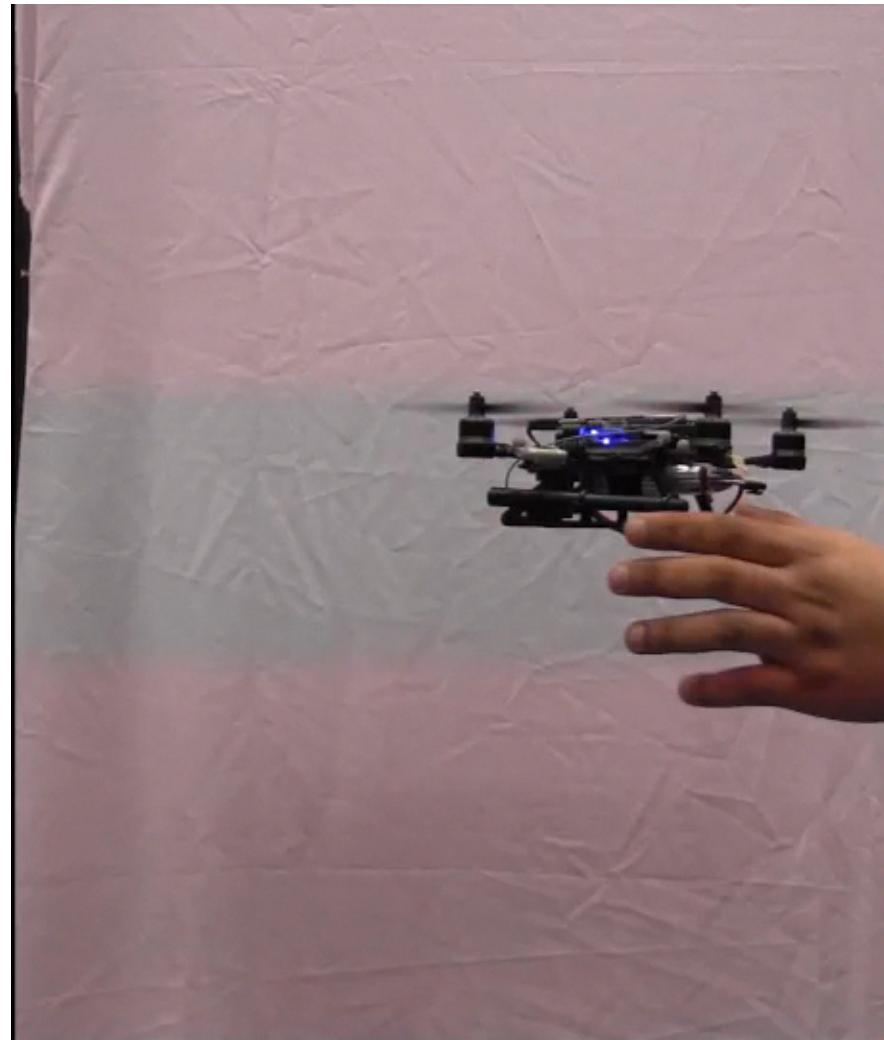


Low K_p (soft response)



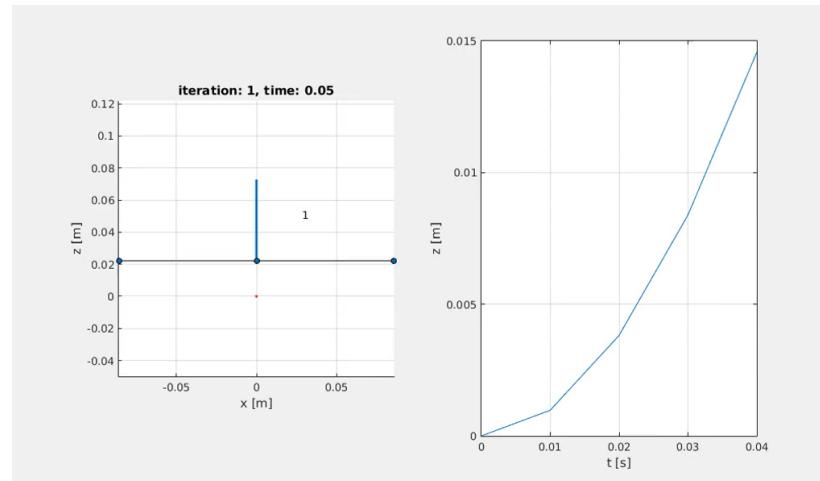
High K_v (overdamped)





Exercise

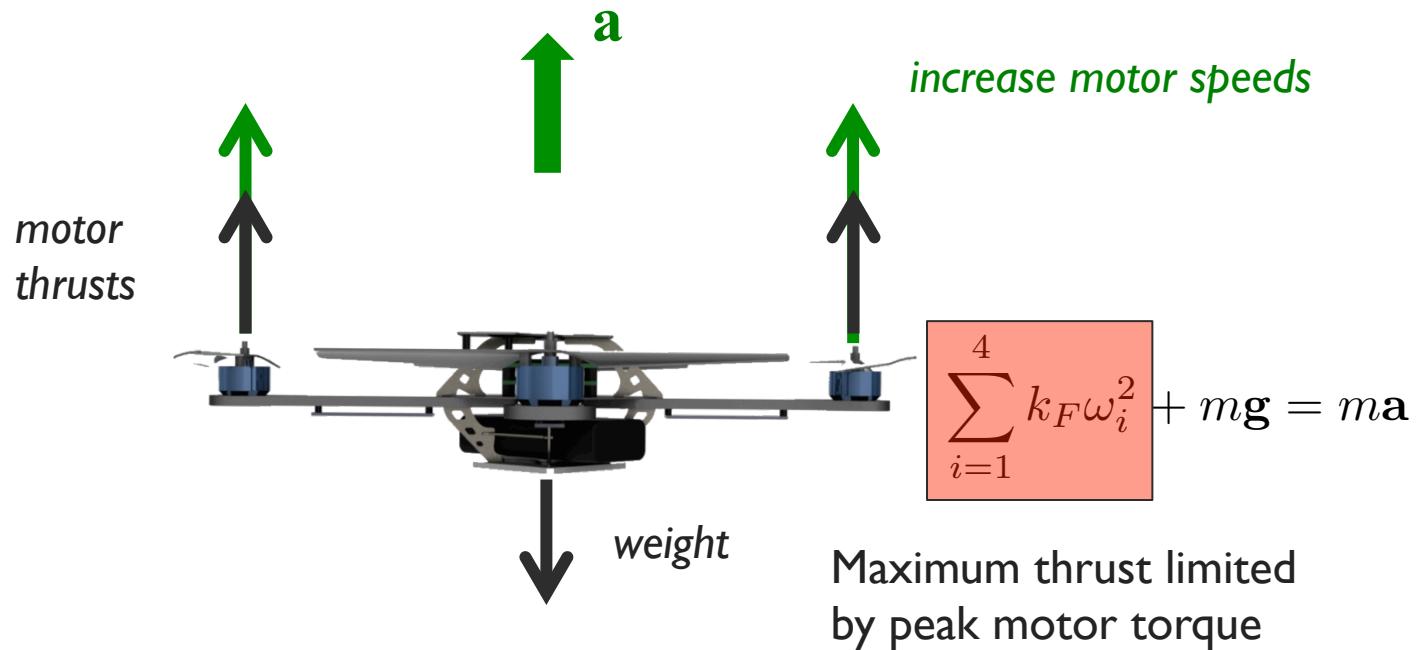
- You are given a simulator which models a PD controller for the height of a quadrotor.
- The aim of the exercise is to tune the proportional gain (K_p) of the controller in order to get a desired response from the system. The derivative gain (K_d) is kept constant.
- You should aim to get a response which has a rise time of less than 1s and a maximum overshoot of less than 5% similar to the one shown in the video below.



Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Effect of Maximum Thrust



Maximum thrust T_{max}

Maximum acceleration a_{max}

Control with Thrust Limitations

Effect of Maximum Thrust on Input

$$u = \frac{1}{m} \left[\sum_{i=1}^4 k_F \omega_i^2 + mg \right]$$

Input, defined in terms of the thrust

$$= \frac{1}{m} [T \uparrow + mg \downarrow]$$

T_{max}

Maximum thrust, as determined by peak motor torque

$$u_{max} = \frac{1}{m} [T_{max} \uparrow + mg \downarrow]$$

Maximum input, as determined by maximum thrust

PD control

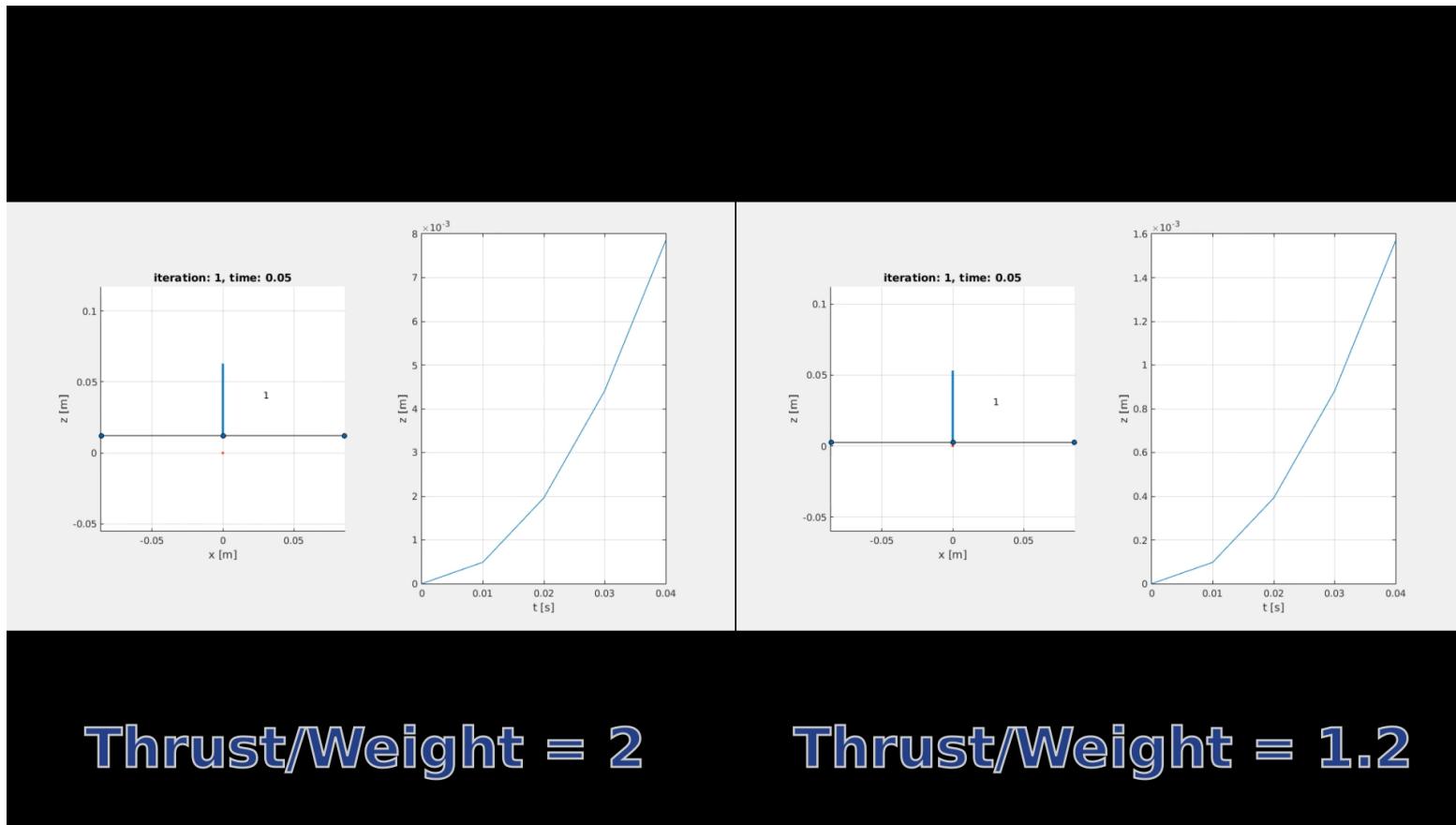
$$u(t) = \min(\ddot{x}^{des}(t) + K_V \dot{e}(t) + K_P e(t), u_{max})$$

PID control

$$u(t) = \min(\ddot{x}^{des}(t) + K_V \dot{e}(t) + K_P e(t) + K_I \int_0^t e(\tau) d\tau, u_{max})$$

Effect of the Thrust/Weight Ratio

What happens if the payload of the robot is increased (with the same motors and propellers)?



Exercise

- We'll use the same height controller from the previous exercise but now we have a limit on the max thrust for the robot.
- In this exercise, you will explore how the thrust/weight ratio affects the control response of the quadrotor. Change the mass (payload) of the robot and see how the response changes.
- We may be interested in the maximum payload that the robot can carry before the response time is degraded significantly. Using the simulation determine the maximum mass for which the rise time is less than 1s?

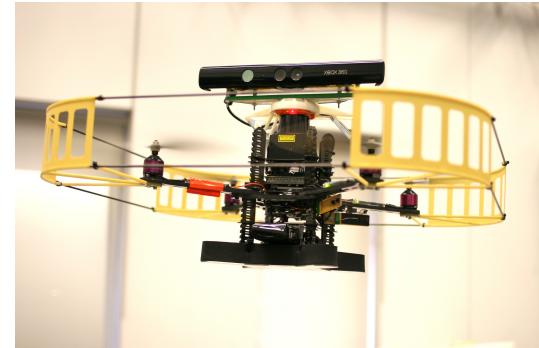
Power and Thrust



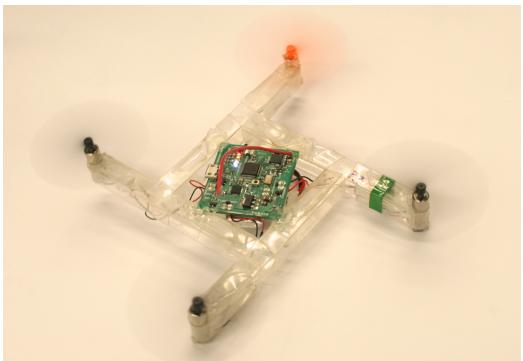
AscTec Hummingbird¹
<http://www.asctec.de/>



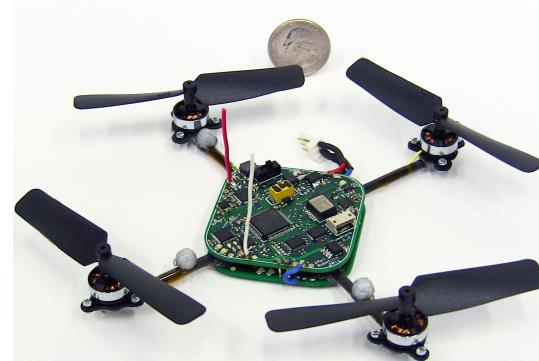
AscTec Pelican (unloaded)
<http://www.asctec.de/>



AscTec Pelican (loaded)²
<http://www.asctec.de/>



PPR Folded Quadrotor³



KMel Nano⁴



KMel kQuad 500⁵

¹Daniel Mellinger, Nathan Michael, and Vijay Kumar. Trajectory Generation and Control for Precise Aggressive Maneuvers with Quadrotors. *International Journal of Robotics Research*, Apr. 2012.

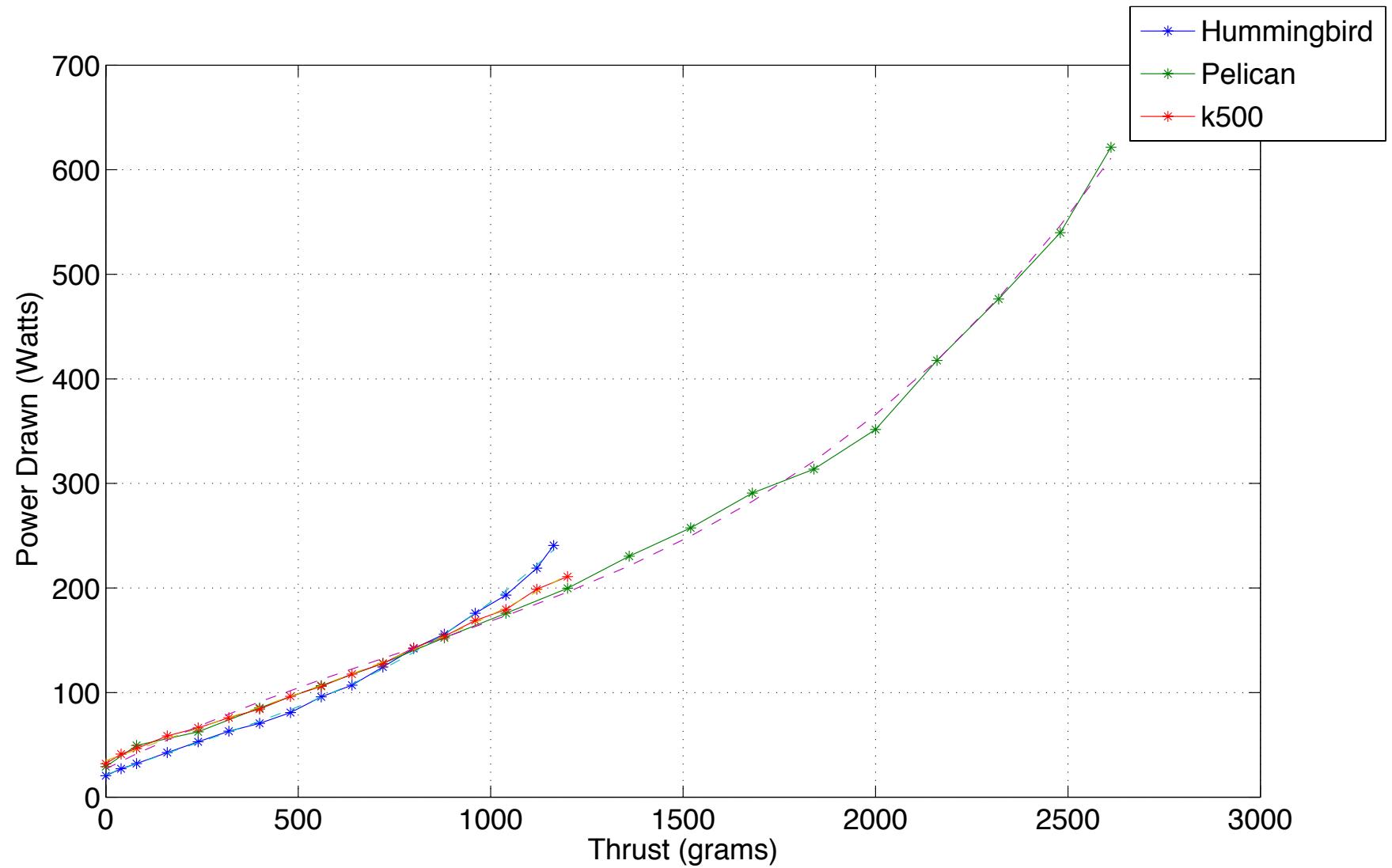
³A. Mehta, D. Rus, K. Mohta, Y. Mulgaonkar, M. Piccoli, and V. Kumar, "A Scripted Printable Quadrotor: Rapid Design and Fabrication of a Folded MAV" Proc. 16th International Symposium on Robotics Research, Singapore, Dec. 2013.

⁴Aleksandr Kushleyev, Daniel Mellinger, Caitlin Powers, and Vijay Kumar, "Towards a swarm of agile micro quadrotors," *Autonomous Robots*, Vol. 35, No. 4, Pg. 287-300, 2013.

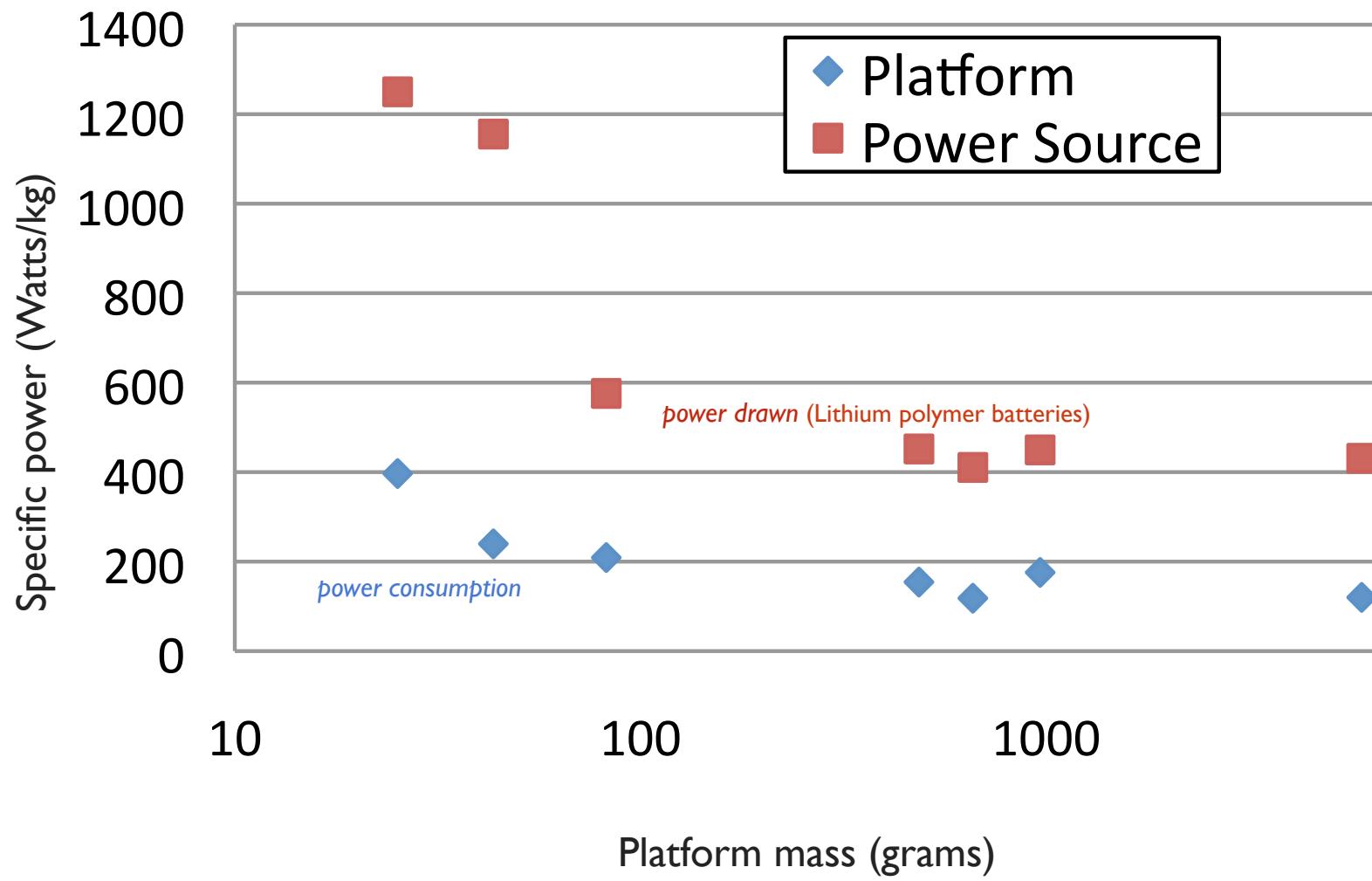
²S. Shen, N. Michael, and V. Kumar, "Stochastic differential equation-based exploration algorithm for autonomous indoor 3D exploration with a micro-air vehicle," *Intl. J. Robot. Research*, Vol. 31, No. 12, pp. 1431-1444, 2012.

⁵K. Mohta, M. Turpin, A. Kushleyev, D. Mellinger, N. Michael, and Vijay Kumar, "QuadCloud: A Rapid Response Force with Quadrotor Teams," *International Symposium on Experimental Robotics*, Morocco, 2014

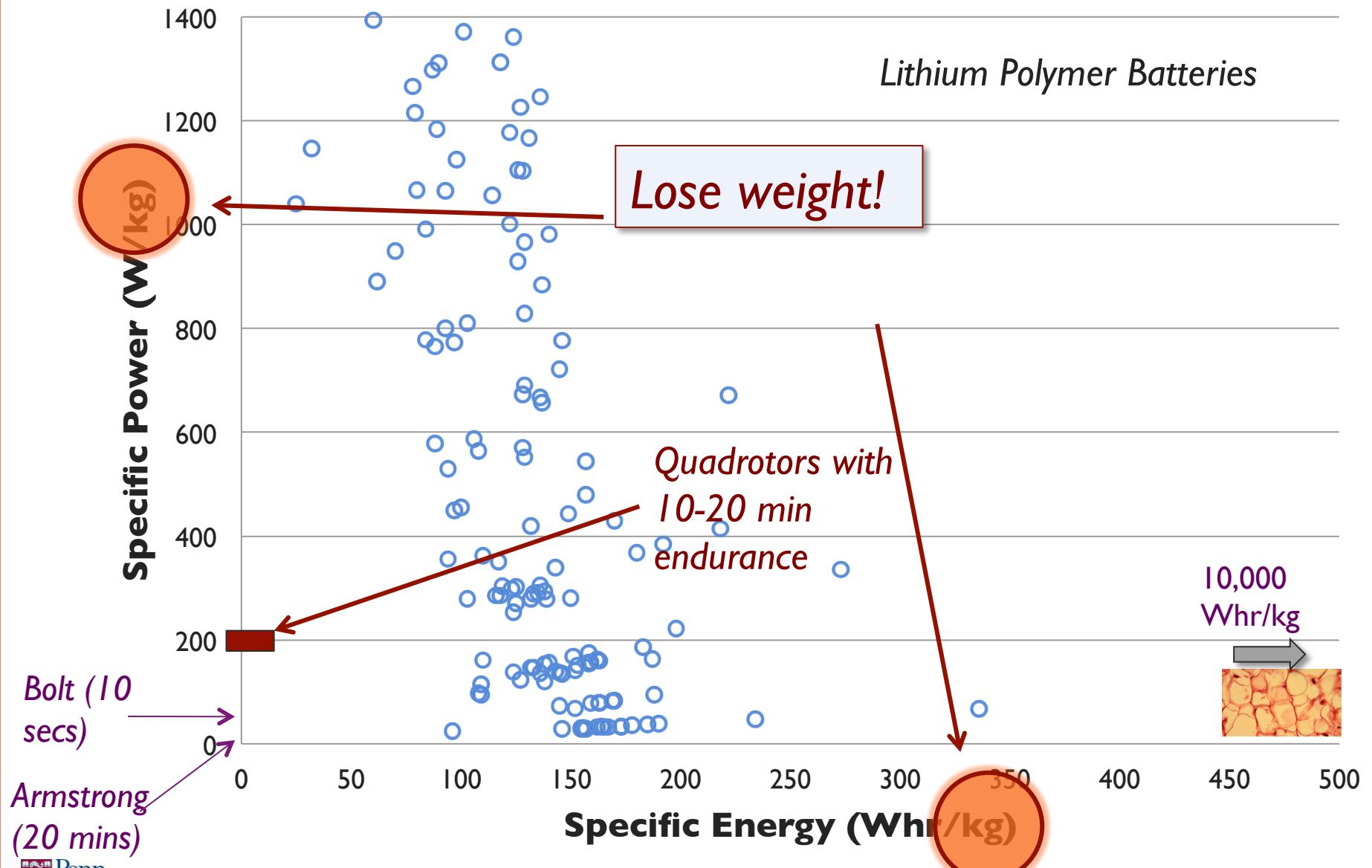
Power Consumption



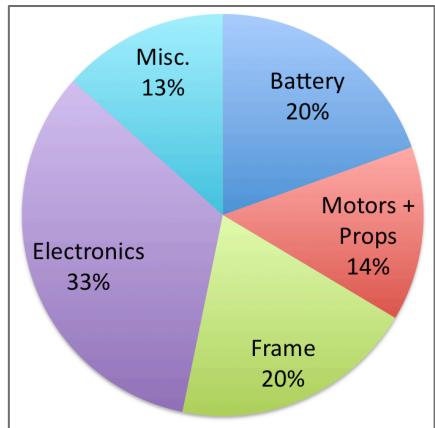
Power Consumption



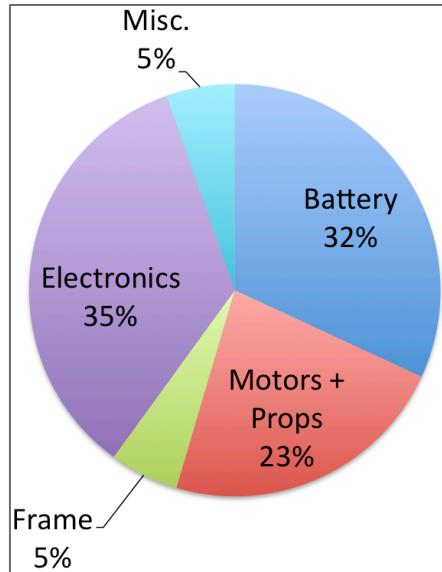
Power and Energy



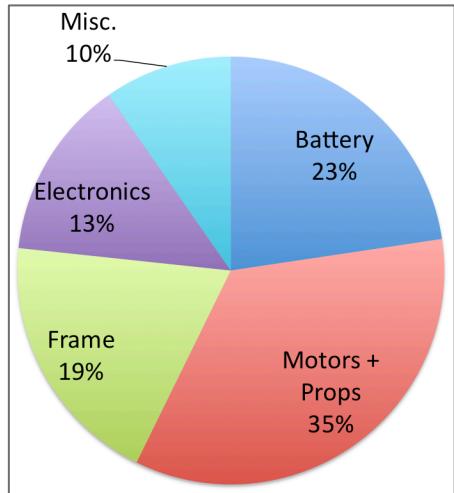
Mass Distribution



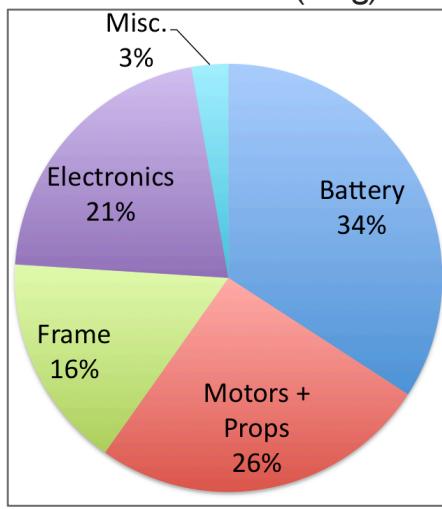
Ascending Tech.
Pelican (1937 g)



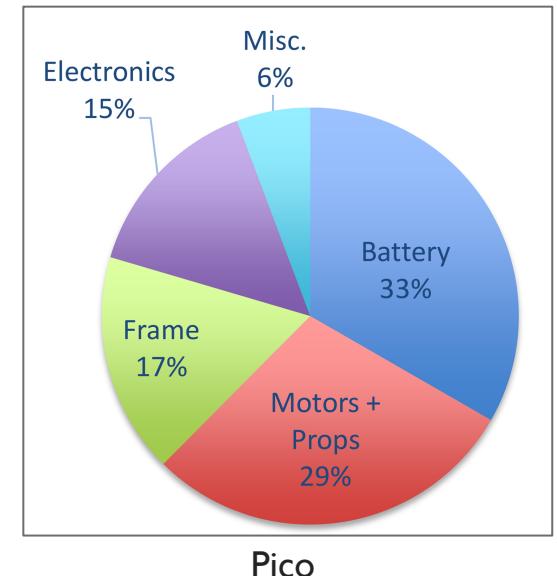
KMel Nano (82 g)



KMel kQuad (920 g)



Ascending Tech.
Hummingbird (486 g)



Pico

Batteries ~ 33% mass
Motors ~ 25% mass

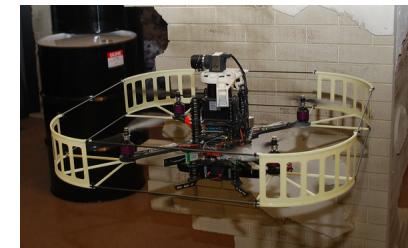
Sensors and Power

- Laser scanner

- 270 gm

- 10 W for operation plus 50-60 W for mobility

- Range 30 m



- Cameras

- 80 gm (including frame, each camera 25 g)

- 1.5 W for operation plus 15 W for mobility

- Range 10-15 m



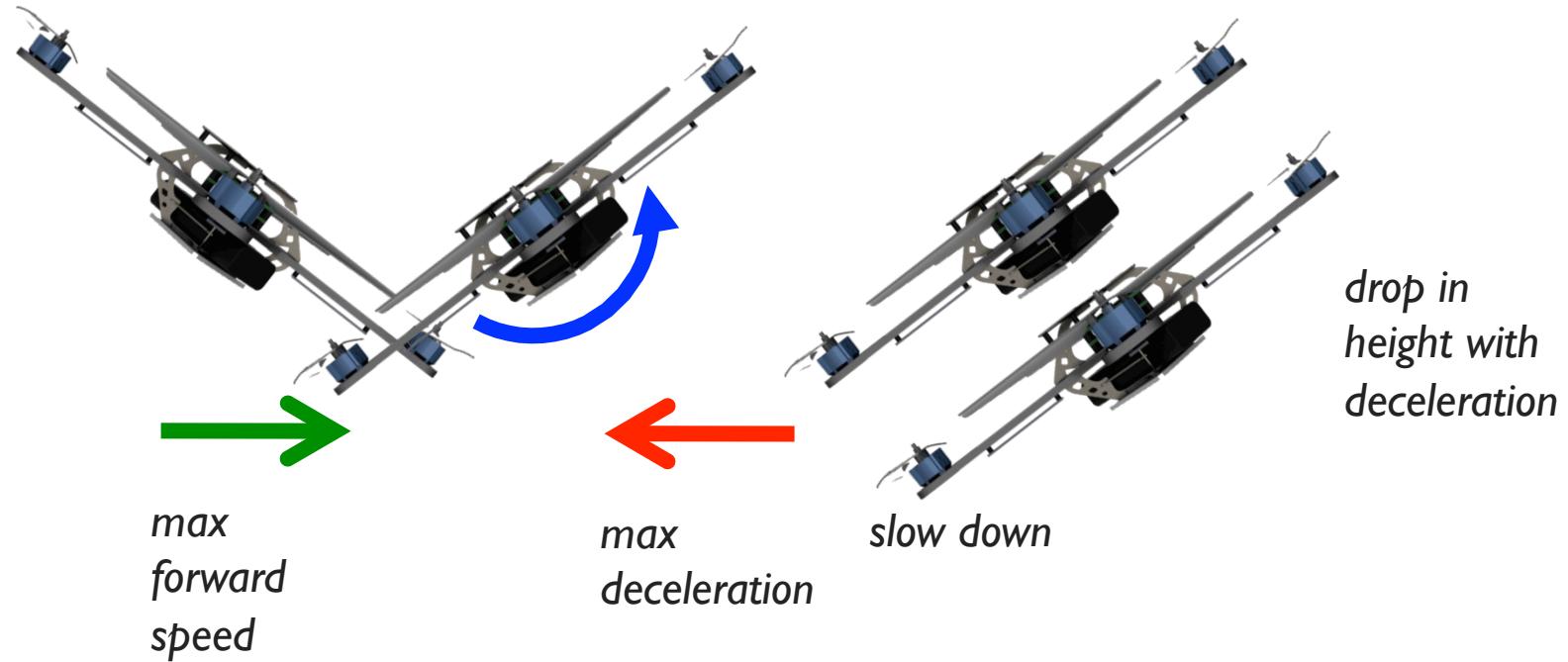
Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Agility

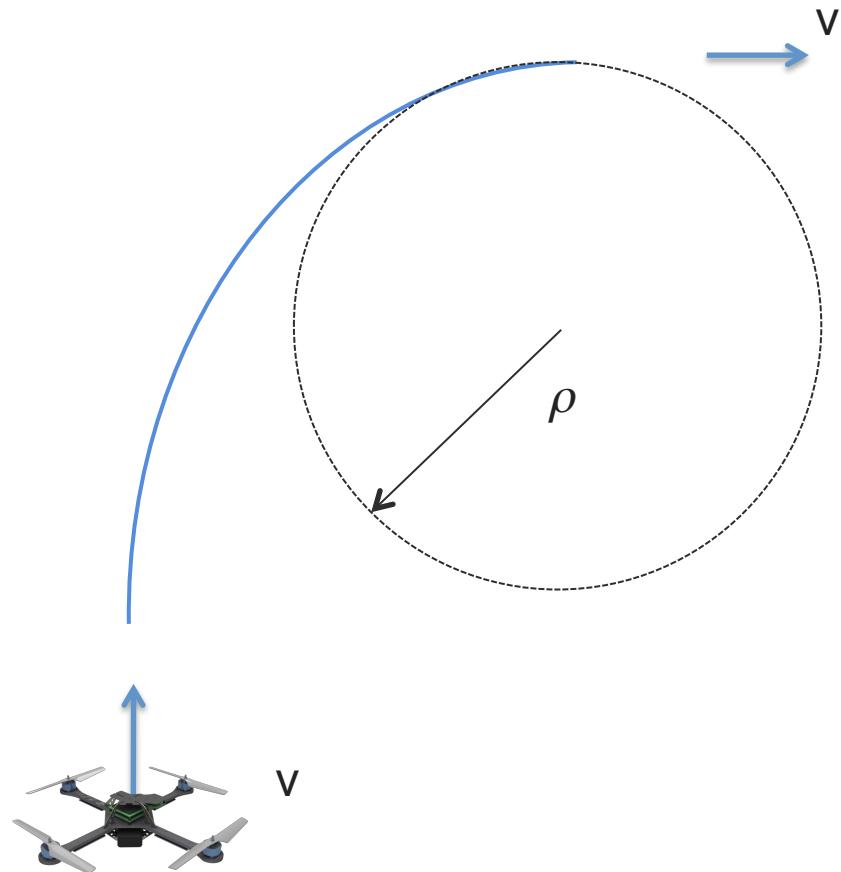


A. Maximum Velocity to Rest



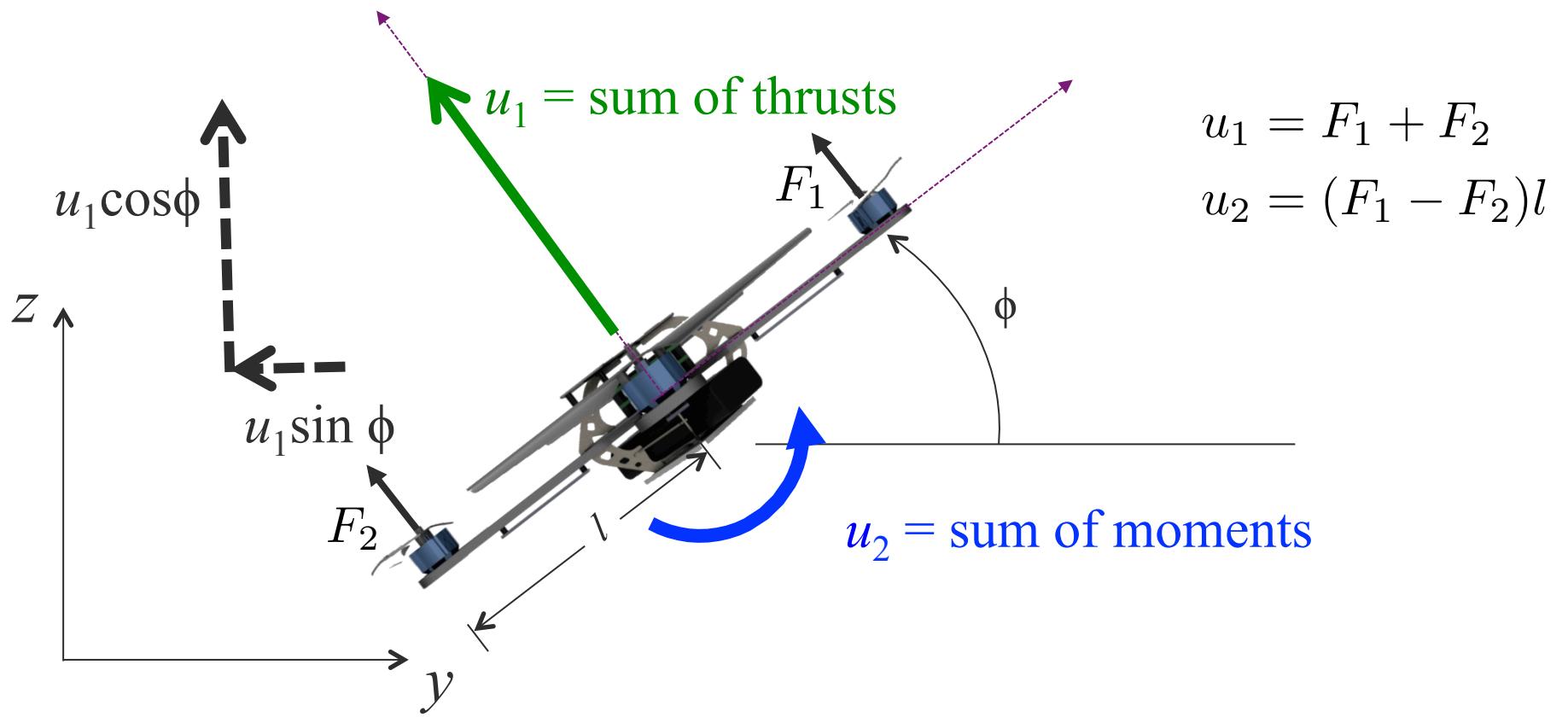
Maximize Agility: Minimize stopping distance

B. Turn Quickly without Slowing Down



Maximize Agility: Minimize minimum turning radius

Quadrotor in a Vertical Plane



$$u_1 = F_1 + F_2$$
$$u_2 = (F_1 - F_2)l$$

linear acceleration, a

angular acceleration, α

$$\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{m} \sin \phi & 0 \\ \frac{1}{m} \cos \phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Agility

Two key ideas

- Accelerate quickly

maximize a_{max}

linear acceleration

$$\text{maximize } \frac{u_{1,max}}{W}$$

- Roll/pitch quickly

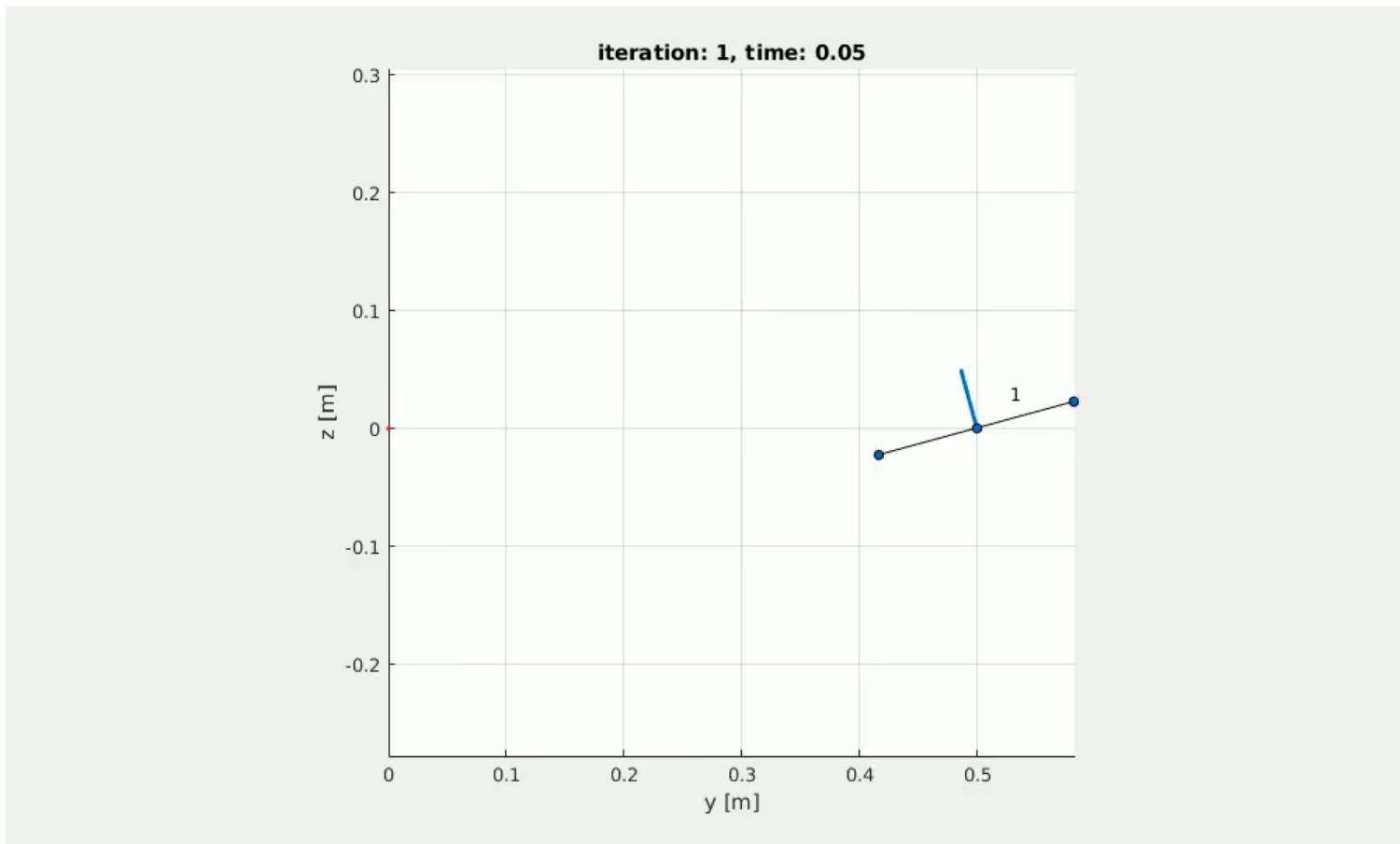
maximize α_{max}

angular acceleration

$$\text{maximize } \frac{u_{2,max}}{I_{xx}}$$

Simulation

Max forward speed to zero speed



Stopping Distance

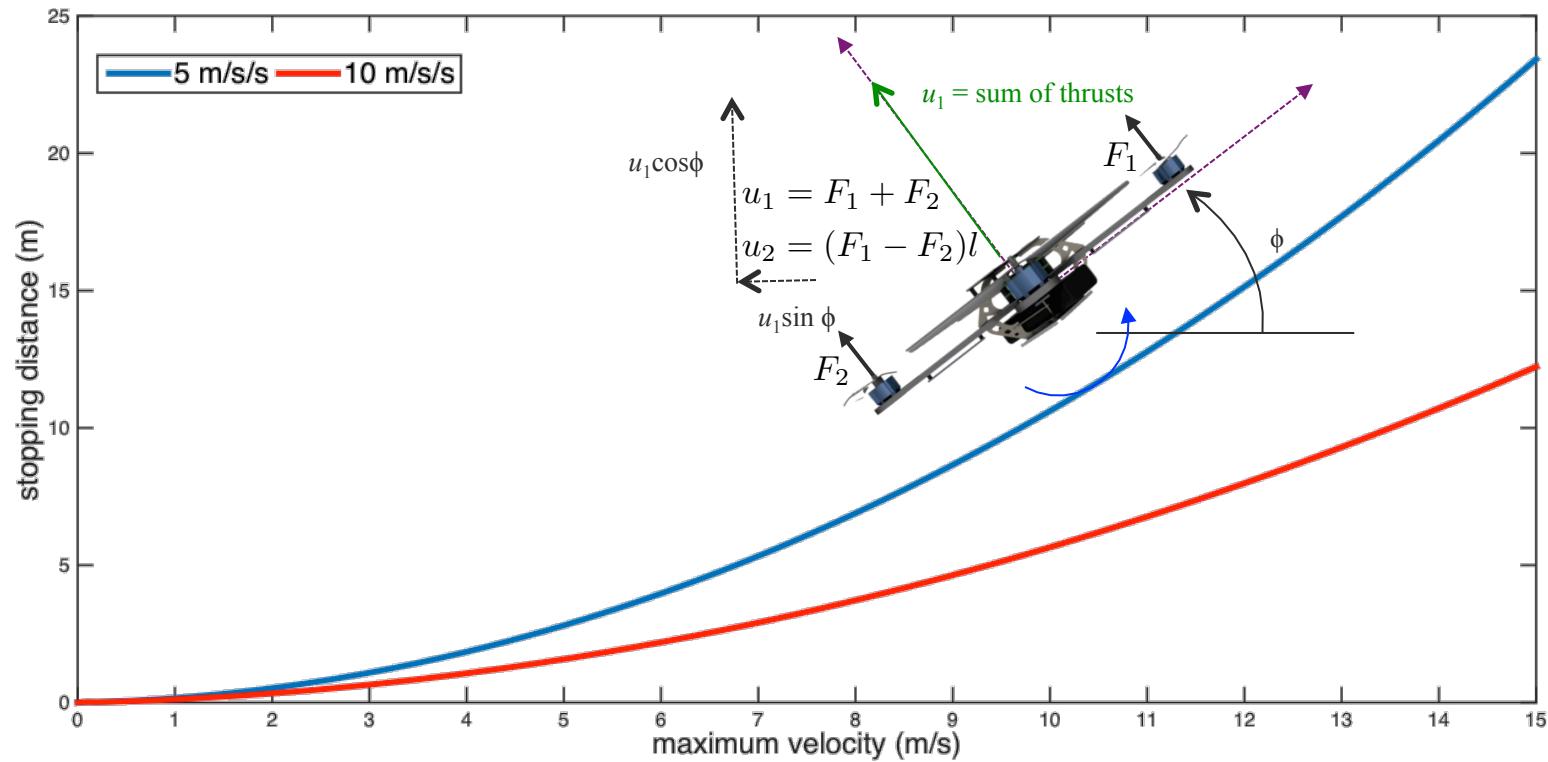
Assumptions

Thrust/weight ratio = 2

Assume robot can drop in height while turning

0 to 90 deg ~ 0.25 sec (1), 0.5 sec (2)

Conventional technology (e.g., dc motors, carbon fiber frame, li-po batteries)



Matlab Exercise

- In this exercise, we'll study how the initial velocity affects the stopping distance
- The robot is moving horizontally with the given initial velocity and it is commanded to stop
- You can change the initial velocity of the robot and run the simulation to find out the distance required* for stopping.
- What is the maximum initial velocity for which the stopping distance is less than 6m?

*Note that during this maneuver, the robot will also lose height.

Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Frame, motors and propellers

	FRAME + PIXHAWK + PROPULSION	BATTERY	PAYOUT	TOTAL WEIGHT	MAX THRUST	THRUST/ WEIGHT	Propeller
3DR X8+	1855	817	600	3272	10560	3.227	11" x 4.7"
DJI F550 + E600	1494	721	600	2815	9600	3.410	12" x 4.2"
DJI F450 + E310	826	400	600	1826	3200	1.752	9.4" x 5"
DJI F450 + E600	970	721	600	2291	6400	2.794	12" x 4.2"
DJI F550 + E310	1278	400	600	2278	4800	2.107	9.4" x 5"
DJI F550 + E310 @ 4 cell	1278	600	600	2478	5316	2.145	9.4" x 5"
DJI F550 + E305 @ 4 cell	1134	600	600	2334	5100	2.185	9.4" x 5"



<http://www.dji.com/>

Basic Hardware



Pixhawk

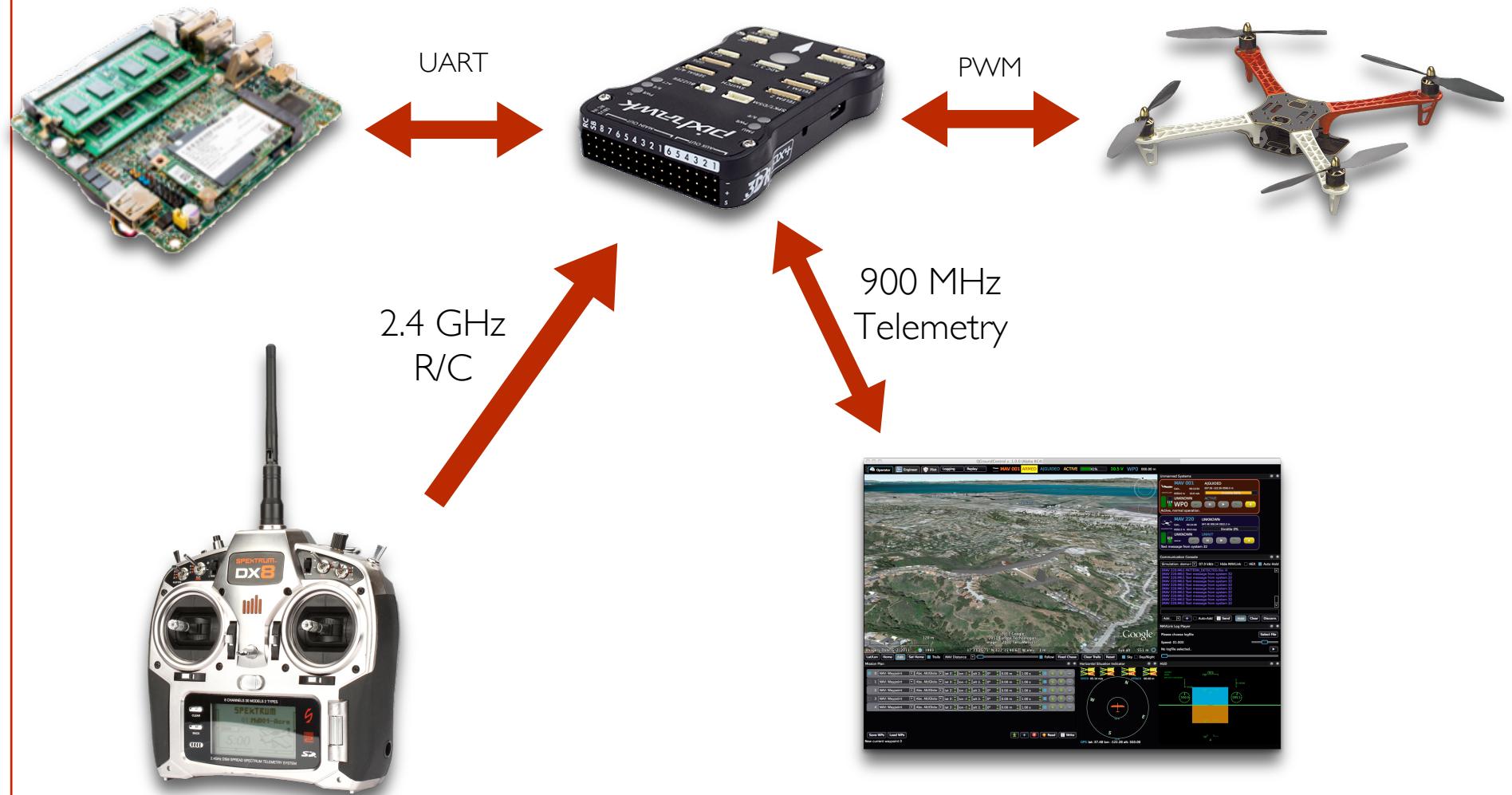
- \$200, 38g
- 168 MHz / 252 MIPS Cortex-M4F
- Sensors: 3D ACC / Gyro / MAG / Baro
- Integrated backup, override and failsafe processor with mixing
- microSD slot, 5 UARTs, CAN, I2C, SPI, ADC, etc



Intel NUC i7

- \$480, 200g
- 5th Generation Intel Core i7-5557U processor, 3.1 Ghz
- Max memory 16GB
- 4 x USB3, 2 x USB2 ports
- Internal support for M.2 SSD card & SATA3 for 2.5" HDD/SSD
- 12V DC

Processing and Communication



Outdoor Platform



Outdoor Test



DJI F450 platform + E600 motors + 600 gram payload + 721 gram battery
Thrust/Weight ratio greater than 2.7

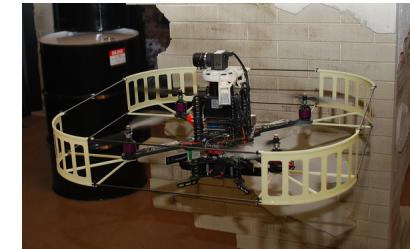
Sensors and Power

- Laser scanner

- 270 gm

- 10 W for operation plus 50-60 W for mobility

- Range 30 m



- Cameras

- 80 gm (including frame, each camera 25 g)

- 1.5 W for operation plus 15 W for mobility

- Range 10-15 m



Examples



1750 g (laser, 3 cameras, GPS, IMU)



650 g (camera, IMU)



Penn Engineering 740 g (2 cameras, IMU)



1800 g (laser, Kinect, IMU)

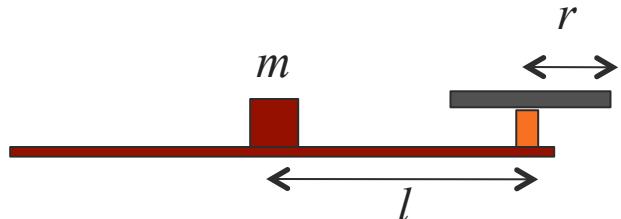
Goals

- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

Agility with Scaling

● mass, inertia

$$m \sim l^3, I \sim l^5$$



since $r \sim l$

● thrust

$$F \sim \pi r^2 \times (\omega r)^2$$

*rotor
angular
speed*

$$F \sim l^2 v^2$$

$$a \sim \frac{F}{m} \sim l^3$$

● moment

$$M \sim Fl$$

$$M \sim l^3 v^2$$

$$\alpha \sim \frac{M}{I} \sim l^5$$

maximum
accelerations

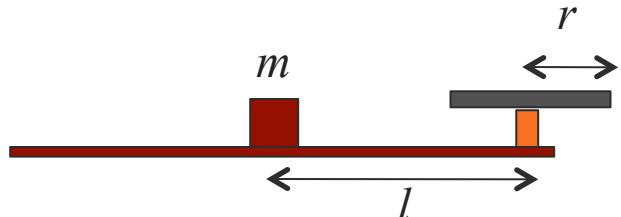
$$a \sim \frac{v^2}{l}$$

$$\alpha \sim \frac{v^2}{l^2}$$

Agility with Scaling

● mass, inertia

$$m \sim l^3, I \sim l^5$$



since $r \sim l$

● thrust

$$F \sim r^2 v^2$$

*blade tip
speed*

$$F \sim l^2 v^2$$

$$a \sim \frac{F}{m} \sim l^3$$

● moment

$$M \sim Fl$$

$$M \sim l^3 v^2$$

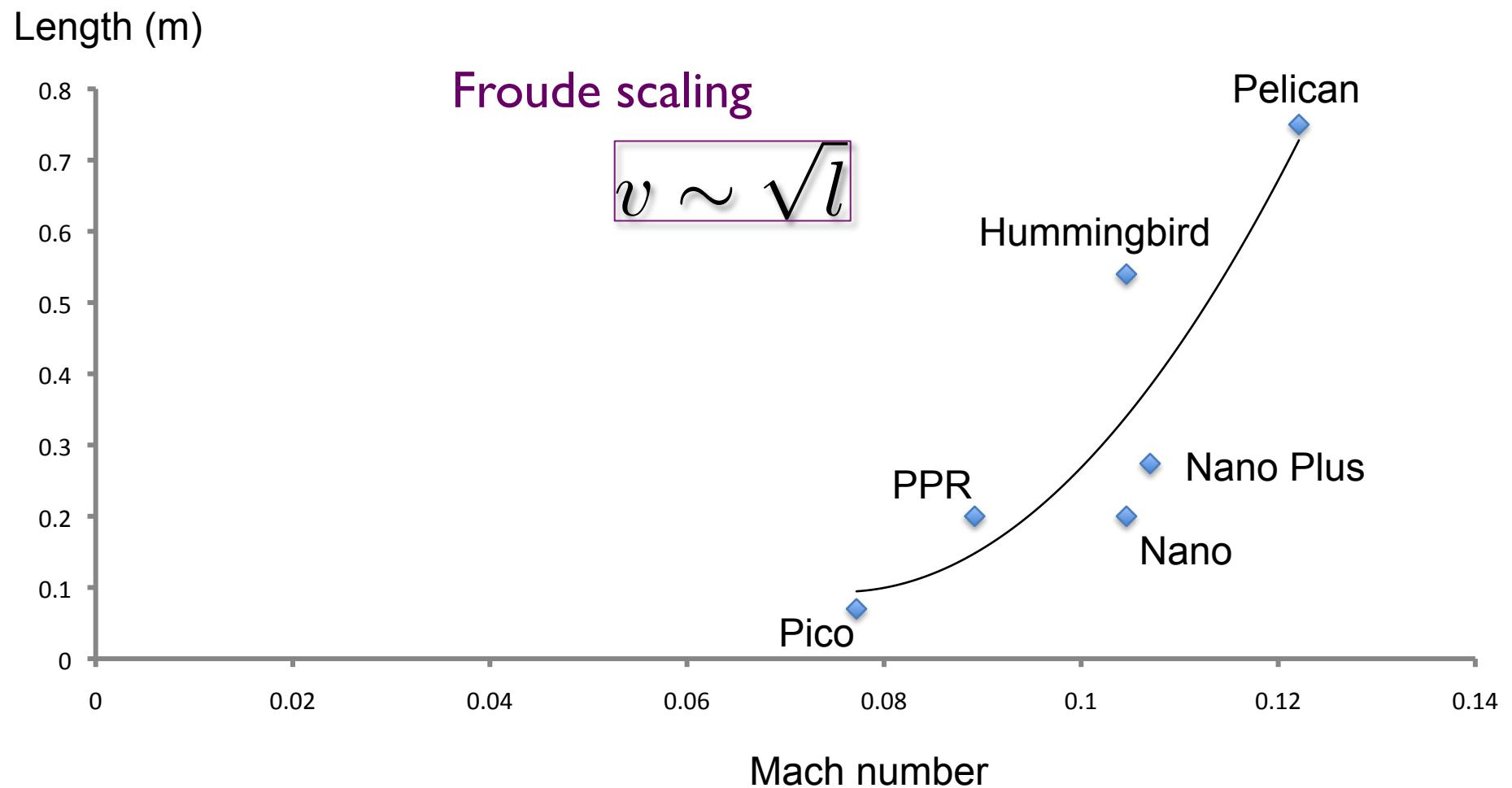
$$\alpha \sim \frac{M}{I} \sim l^5$$

maximum accelerations

$$a \sim \frac{v^2}{l}$$

$$\alpha \sim \frac{v^2}{l^2}$$

Scaling Experiments

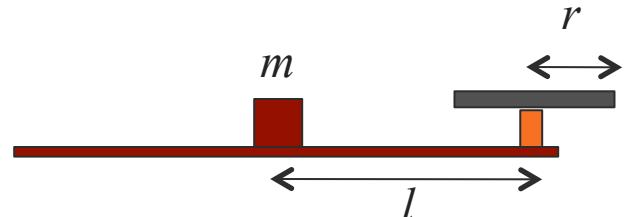


Agility with Scaling

● Froude scaling

$$v \sim \sqrt{l}$$

$$F \sim l^3$$



$$a \sim 1, \alpha \sim \frac{1}{l}$$

● Mach scaling

$$v \sim 1$$

$$F \sim l^2$$

$$a \sim \frac{1}{l}, \alpha \sim \frac{1}{l^2}$$

Dynamical Systems

Dynamical Systems

Systems where the effects of actions do not occur immediately

State: a collection of variables that completely characterizes the motion of a system

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Systems where the effects of actions do not occur immediately

State: a collection of variables that completely characterizes the motion of a system

- $x(t)$ gives the values of these states over time

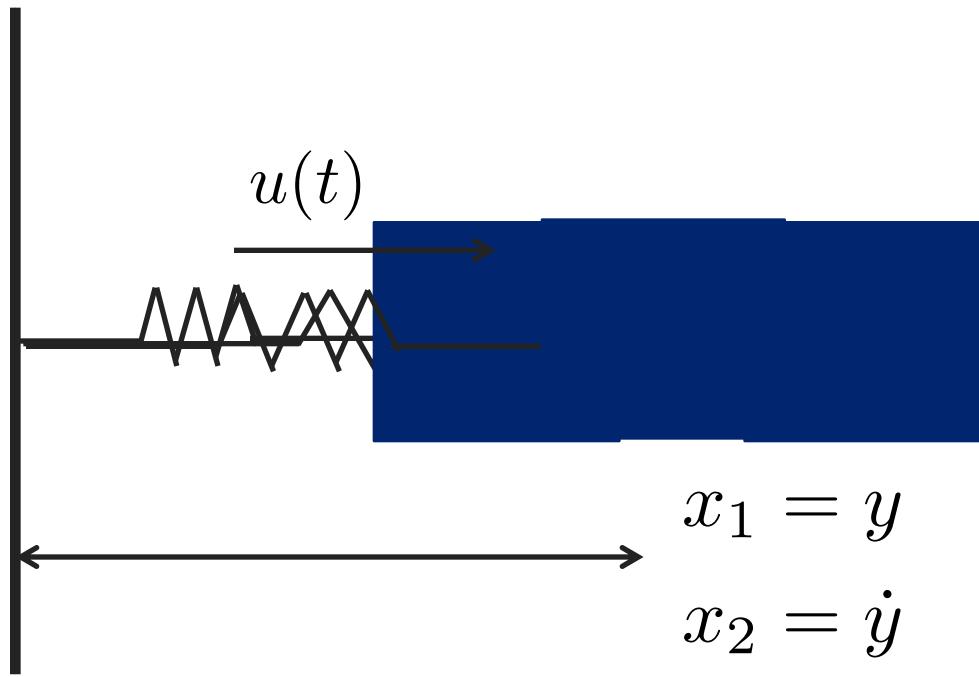
Dynamical Systems

Evolution of these states over time is often given by a set of governing ordinary differential equations

- Order: highest derivative that appears in the equations

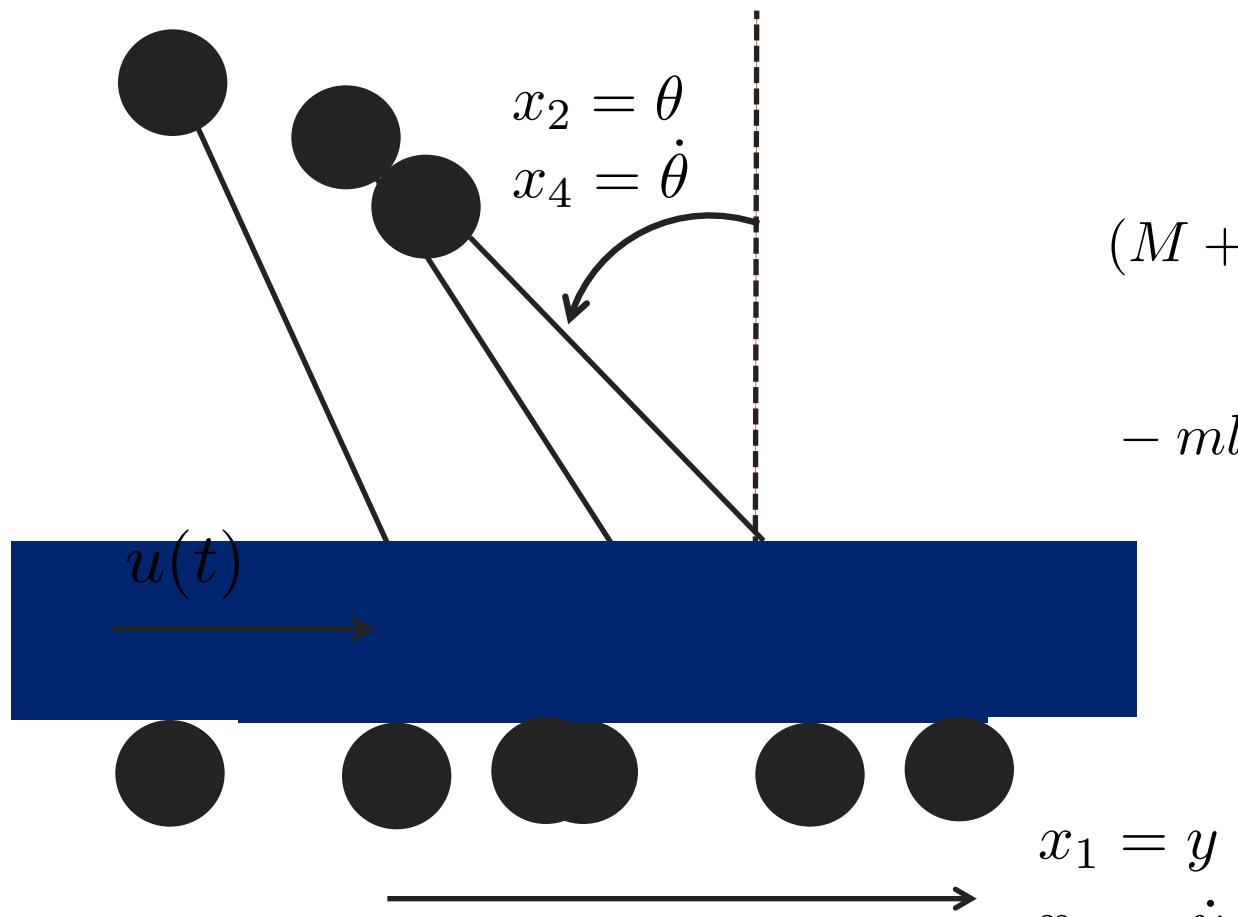
$$\boxed{\ddot{x}(t)} = u(t) \quad \text{Second-order system}$$

Example I: Mass-Spring System



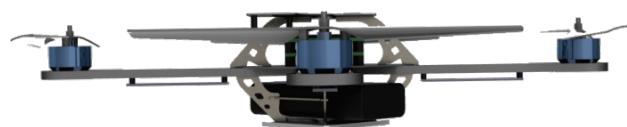
$$m\ddot{y}(t) + ky(t) = u(t)$$

Example 2: Pendulum on a Cart



$$\begin{aligned}(M + m)\ddot{y}(t) - ml \cos(\theta(t))\ddot{\theta}(t) \\ + ml \sin(\theta(t))\dot{\theta}(t)^2 = u(t) \\ - ml \cos(\theta(t))\ddot{y}(t) + (J + ml^2)\ddot{\theta}(t) \\ - mgl \sin(\theta(t)) = 0\end{aligned}$$

Example 3: Quadrotor



$$x_1 = x \quad x_7 = \dot{x}$$

$$x_2 = y \quad x_8 = \dot{y}$$

$$x_3 = z \quad x_9 = \dot{z}$$

$$x_4 = \phi \quad x_{10} = p$$

$$x_5 = \theta \quad x_{11} = q$$

$$x_6 = \psi \quad x_{12} = r$$

Rates of Convergence

Feedback Control

Recall the control problem

Determine the appropriate input that will cause the error between the desired state and the actual state of a dynamical system to eventually reach 0.

$$e(t) = x^{des}(t) - x(t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

Rates of Convergence

How fast do we want this error to go to 0?

- The error *exponentially converges* to 0 if there exists constants α and β and time t_0 , such that for all $t \geq t_0$:

$$\|e(t)\| \leq \alpha e^{-\beta t}$$

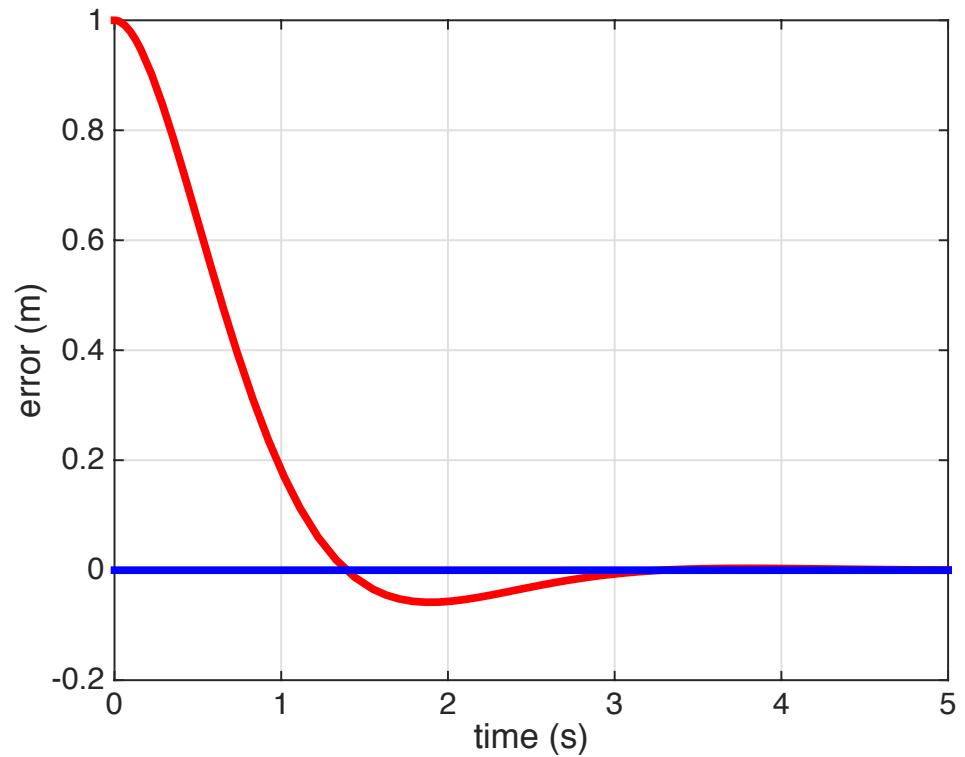
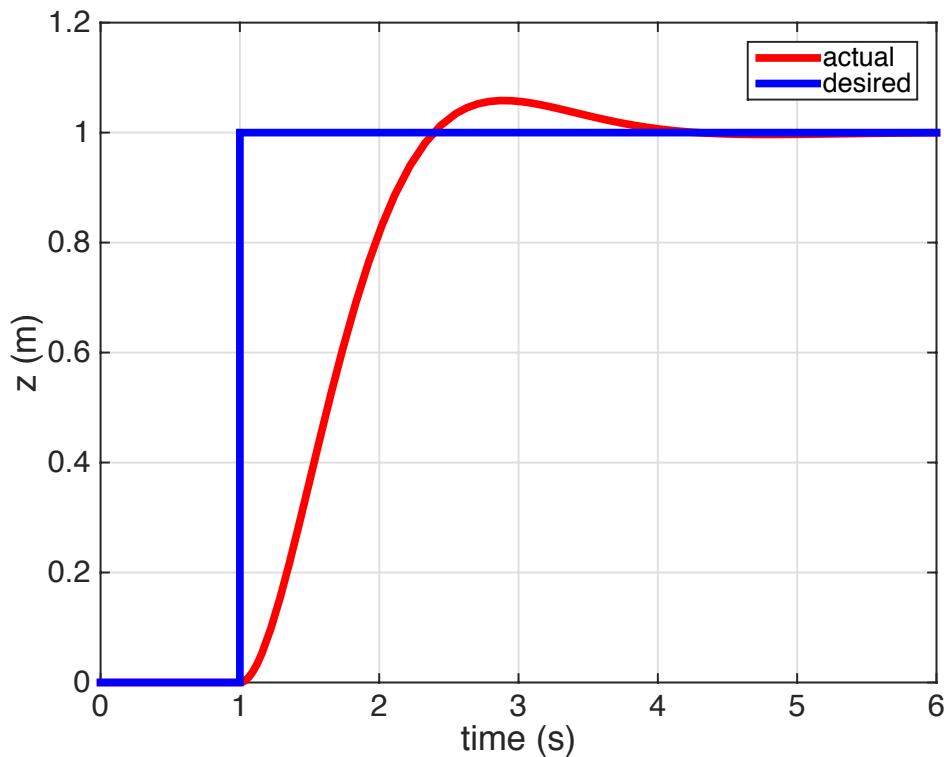
Feedback Control

Here we will accomplish this using a PD (or PID) controller.

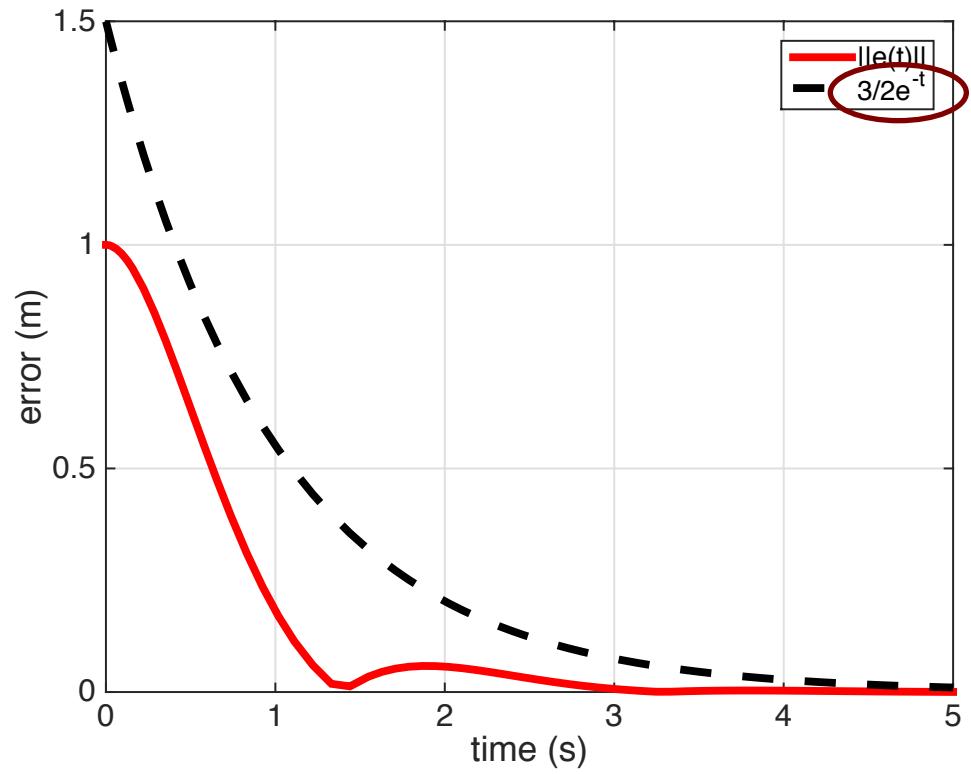
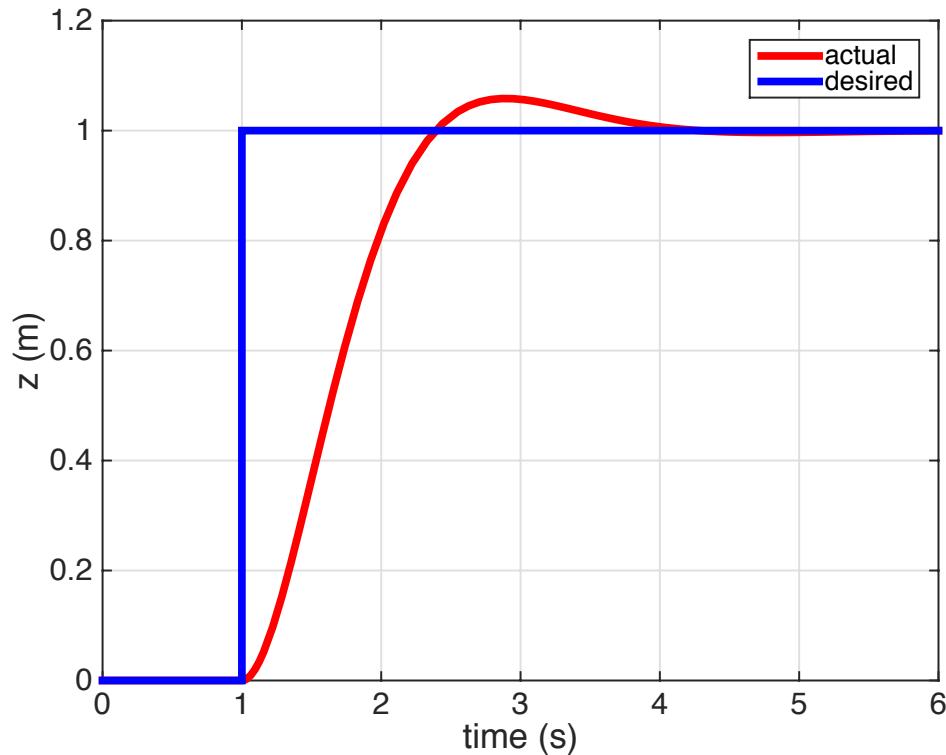
$$u(t) = \ddot{x}^{\text{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Consider the controllers we used before to control the height of a quadrotor.

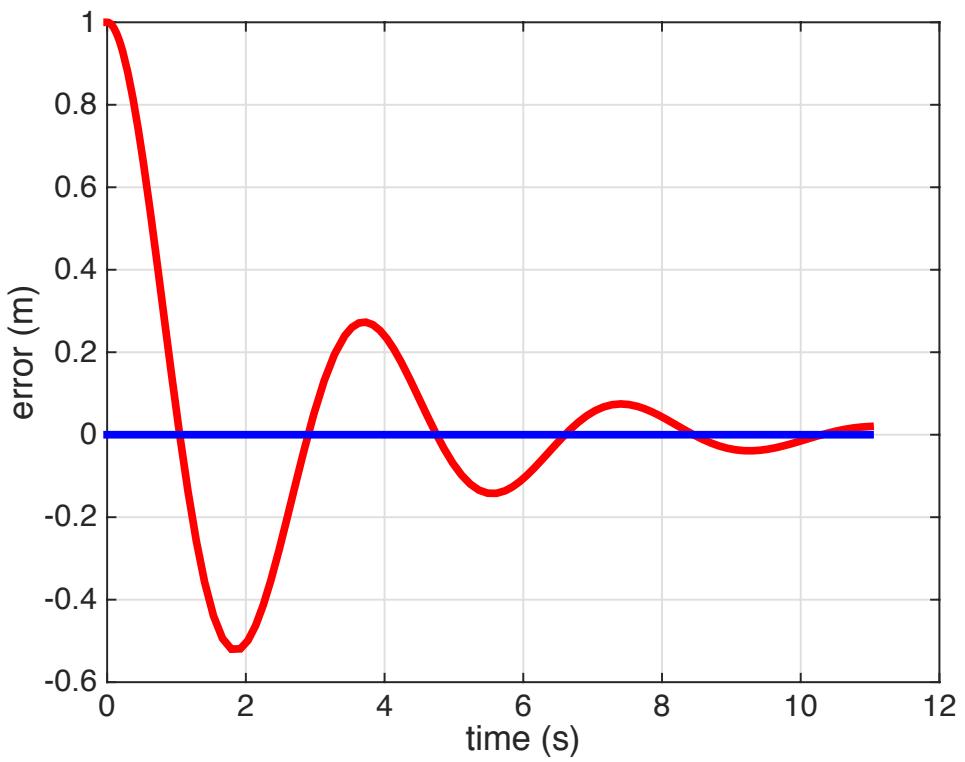
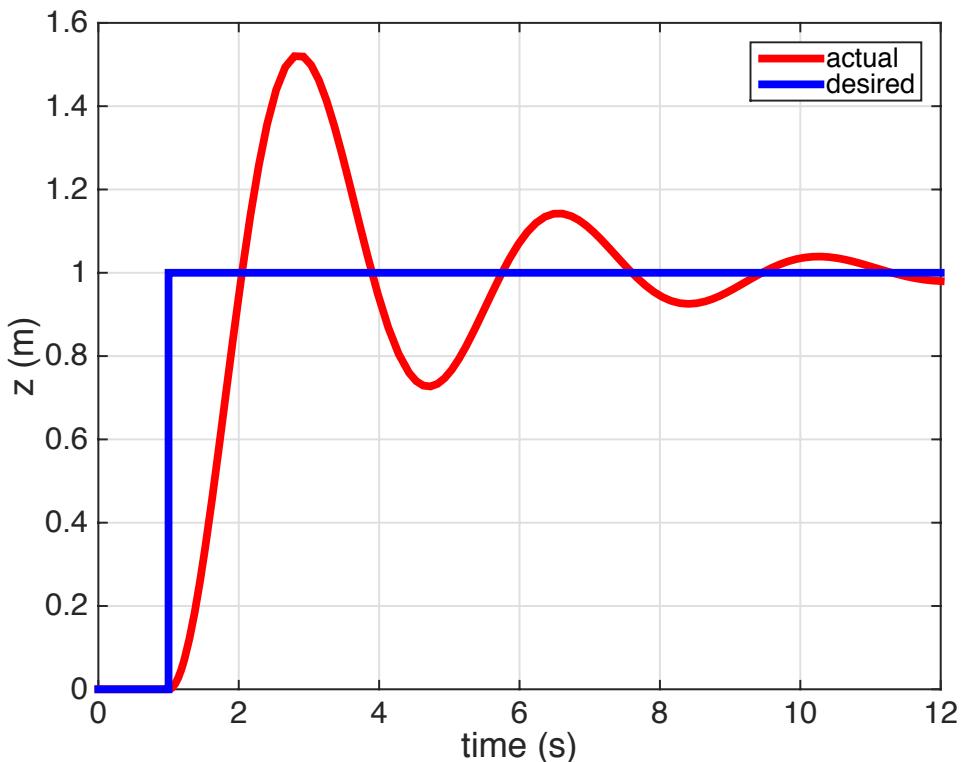
Example I: PD Controller



Example I: PD Controller



Example 2: High K_p



Example 2: High K_p

